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Summer School on Particle Physics

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Higgs and Electroweak Symmetry Breaking - III

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Why the Standard Model isn’t Perfect

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Trieste 2011, Lecture 3

If we find a “Higgs-like” object, what then?

- We need to:
  - Measure Higgs couplings to fermions & gauge bosons
  - Measure Higgs spin/parity
  - Reconstruct Higgs potential
  - Is it the SM Higgs?
- Reminder: Many models have other signatures:
  - New gauge bosons (little Higgs)
  - Other new resonances (Extra D)
  - Scalar triplets (little Higgs, NMSSM)
  - Colored scalars (MSSM)

Is it a Higgs?

- How do we know what we've found?
- Measure couplings to fermions & gauge bosons

$$\boxed{\frac{\Gamma(h\to b\bar{b})}{\Gamma(h\to\tau^+\tau^-)}\approx 3\frac{m_b{}^2}{m_\tau{}^2}}$$

- Measure spin/parity

$$\boxed{J^{PC}=0^{++}}$$

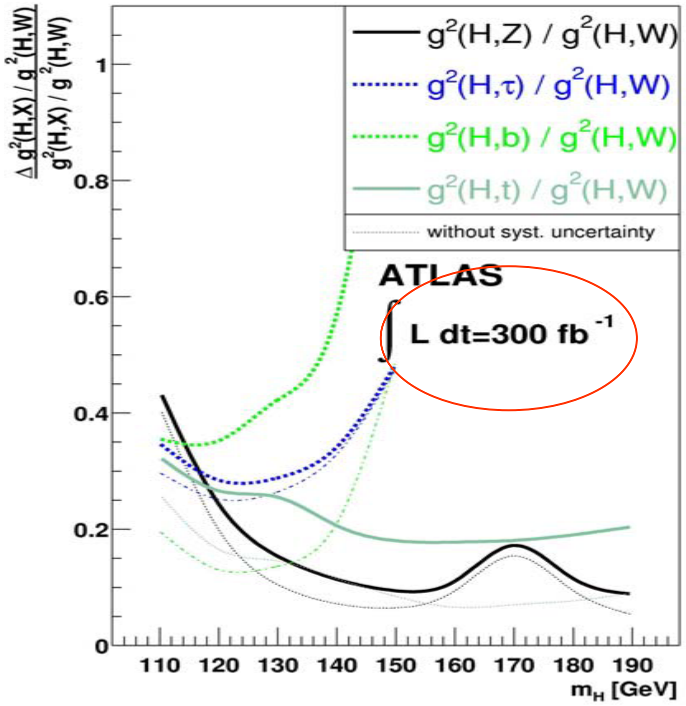
- Measure self interactions

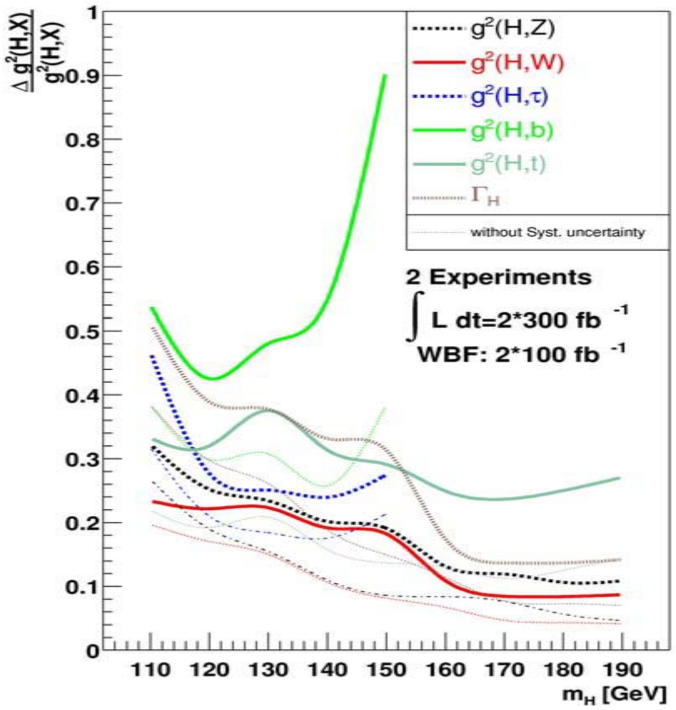
$$\boxed{V=\frac{M_h{}^2}{2}h^2+\frac{M_h{}^2}{2v}h^3+\frac{M_h{}^2}{8v^2}h^4}$$

Absolute Measurements of Higgs Couplings

- Ratios of couplings more precisely measured than absolute couplings

- 10-40% measurements of most couplings

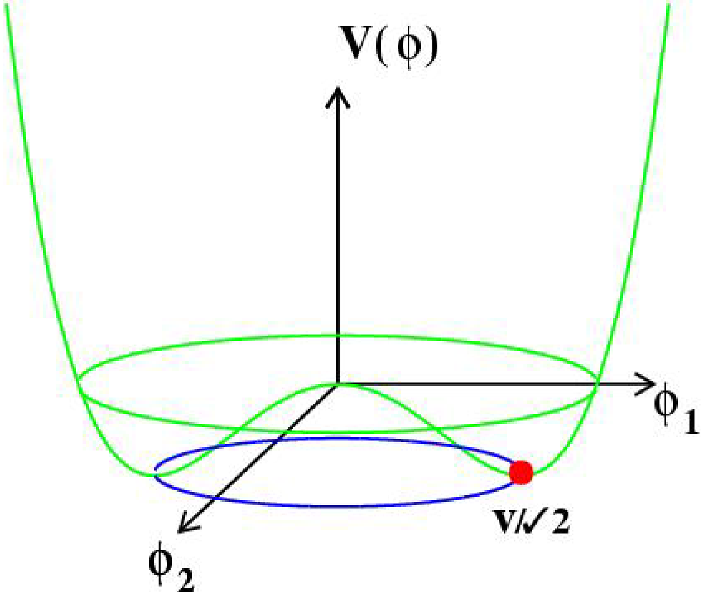




Can we reconstruct the Higgs potential?

$$V = \frac{M_h{}^2}{2} h^2 + \lambda_3 v h^3 + \frac{\lambda_4}{4} h^4$$

$$\boxed{SM:\ \lambda_3 = \lambda_4 = \frac{M_h{}^2}{2v^2}}$$



- Fundamental test of model!

- We have no idea how to measure λ_4

The Standard Model Works

- Any discussion of the Standard Model has to start with its success
- This is unlikely to be an accident!
- But it’s not perfect
  - Unitarity constraints
  - Chimney plot: Consistency of the Standard Model
  - Higgs mass renormalization

Unitarity

- Consider 2 → 2 particle elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

- a_l are the spin *l* partial waves

Unitarity

- $P_l(\cos\theta)$ are Legendre polynomials:

$$\int_{-1}^{1} dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$$

$$\sigma = \frac{8\pi}{s} \sum_{l=0}^{\infty} (2l+1) \sum_{l'=0}^{\infty} (2l'+1) a_l a_{l'}^{*} \int_{-1}^{1} d\cos\theta P_l(\cos\theta) P_{l'}(\cos\theta)$$

$$= \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) \left| a_l \right|^2$$

Sum of positive definite terms

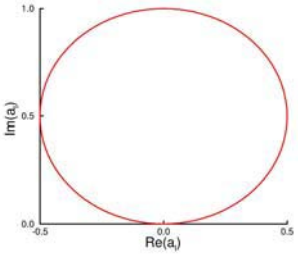
More on Unitarity

- Optical theorem $\sigma = \frac{1}{s}\mathrm{Im}\left[A(\theta=0)\right] = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)\left|a_l\right|^2$

$$\boxed{\mathrm{Im}(a_l) = \left|a_l\right|^2}$$

Optical theorem derived assuming
only conservation of probability

- Unitarity requirement:



$$\left|\mathrm{Re}(a_l)\right| \leq \frac{1}{2}$$

True for any 2→2
scattering amplitude

More on Unitarity

- Idea: Use unitarity to limit parameters of theory

Cross sections which grow with energy always violate unitarity at some energy scale

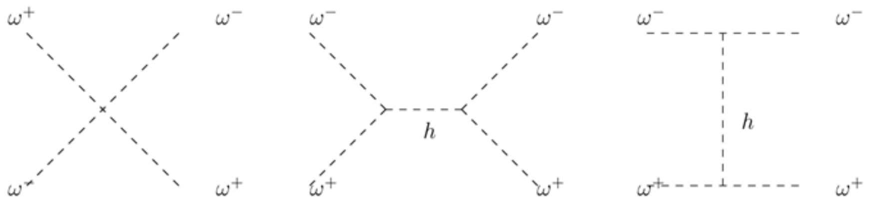
Example: $W^+W^-\to W^+W^-$

- Recall scalar potential (Include Goldstone Bosons in Feynman gauge)

$$V=\frac{M_h^2}{2}h^2+\frac{M_h^2}{2v}h\left(h^2+z^2+2\omega^+\omega^-\right)+\frac{M_h^2}{8v^2}\left(h^2+z^2+2\omega^+\omega^-\right)^2$$

- Consider Goldstone boson scattering: $\omega^+\omega^-\to\omega^+\omega$

$$iA(\omega^+\omega\to\omega^+\omega^-)=-2i\frac{M_h^2}{v^2}+\left(-i\frac{M_h^2}{v}\right)^2\frac{i}{t-M_h^2}+\left(-i\frac{M_h^2}{v}\right)^2\frac{i}{s-M_h^2}$$



$\omega^+\omega^- \rightarrow \omega^+\omega^-$

- Two interesting limits:
  - s, t >> M_h^2

$$A(\omega^+\omega^- \rightarrow \omega^+\omega^-) \rightarrow -2\frac{M_h^2}{v^2}$$

$$\boxed{a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}}$$

  - s, t << M_h^2

$$A(\omega^+\omega^- \rightarrow \omega^+\omega^-) \rightarrow -\frac{u}{v^2}$$

$$\boxed{a_0^0 \rightarrow -\frac{s}{32\pi v^2}}$$

Use Unitarity to Bound Higgs

$$\left|\mathrm{Re}(a_l)\right| \leq \frac{1}{2}$$

- High energy limit:

$$\boxed{a_0^0 \to -\frac{M_h^2}{8\pi v^2}} \Rightarrow \boxed{M_h < 800 \text{ GeV}}$$

- Heavy Higgs limit:

$$\boxed{a_0^0 \to -\frac{s}{32\pi v^2}} \Rightarrow$$

$E_c \sim 1.7$ TeV

→ New physics at the TeV scale

But don’t precision measurements require a light Higgs?

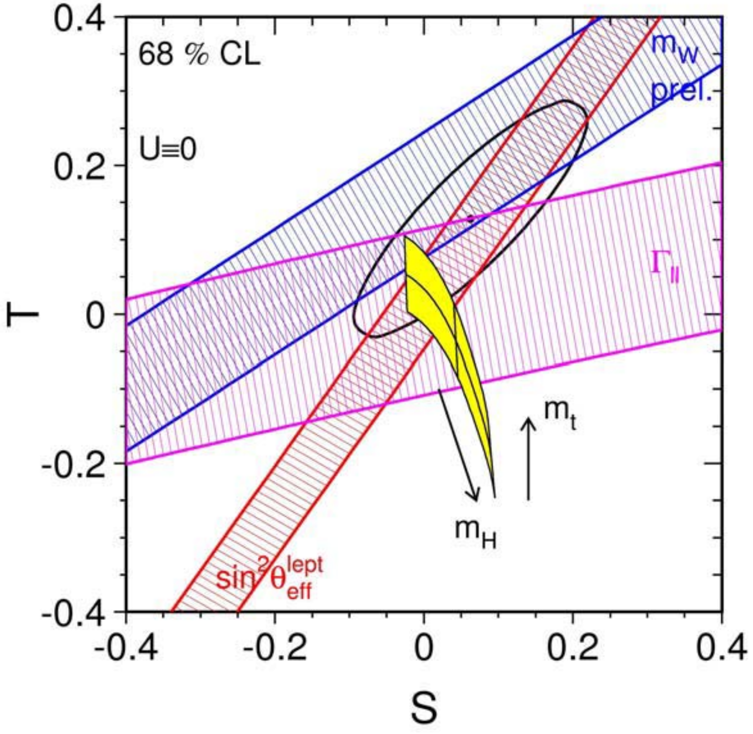
- Higgs mass limits from precision measurements assume Standard Model
  - The Standard Model is an effective low energy theory valid to scale Λ

$$\boxed{L = L_{SM} + \sum \frac{c_i}{\Lambda^2} O_i^6 + \ldots..}$$

We don’t know the high scale physics!

SM isn't only theory that can fit precision measurements

- Heavy scalar allowed with large ΔT ($\alpha\Delta T=\Delta\rho$)
- Straightforward to construct such models



Example: 4th generation of fermions with large mass splitting between charge 2/3 and charge -1/3 quarks

Is the Standard Model Self-Consistent?

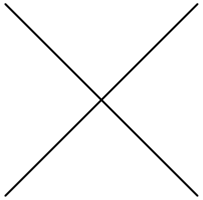
- M_h is a free parameter in the Standard Model
- Can we derive limits on the basis of consistency?
- Consider a scalar potential:

$$V = \frac{M_h^2}{2} h^2 + \frac{\lambda}{4} h^4$$

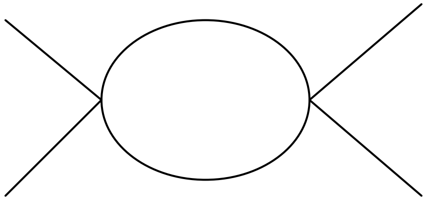
- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way

Consider hh→hh

- Tree level: A_0=-6λ_0



- One loop:



$$iA_s = (-6i\lambda_0^2)^2 \frac{1}{2}\int \frac{d^n k}{(2\pi)^2} \frac{i}{k^2 - M_h^2} \frac{i}{(k+p+q)^2 - M_h^2}$$

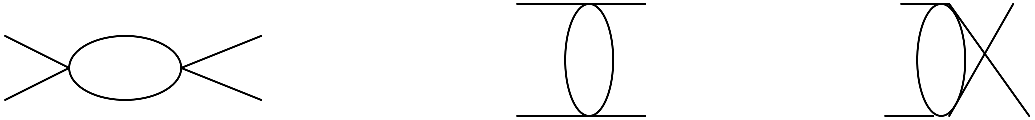
$$= \frac{9\lambda_0^2}{8\pi^2}(4\pi\mu^2)^\varepsilon \Gamma(\varepsilon)\left(M_h^2 - s\ x(1-x)\right)^{-\varepsilon}$$

$$\boxed{\Rightarrow \frac{9\lambda_0^2}{8\pi^2}\frac{1}{\varepsilon}\Gamma(1+\varepsilon) - \frac{9\lambda_0^2}{8\pi^2}\log\left(\frac{s}{M_h^2}\right) + ...}$$

$$\begin{aligned} \Gamma(\varepsilon) &= \frac{1}{\varepsilon}\Gamma(1+\varepsilon) \\ a^\varepsilon &= 1 + \varepsilon \log(a) \end{aligned}$$

hh→hh (2)

- Add 1-loop diagrams (and take s=t=u=Q^2)



- Absorb 1/ε poles

$$\boxed{\lambda_{eff} \Rightarrow \lambda - \frac{9\lambda^2}{8\pi^2}\frac{1}{\varepsilon}\Gamma(1+\varepsilon)}$$

$$\boxed{A \Rightarrow -6\lambda_{eff}\left(1 - \frac{9\lambda^2{}_{eff}}{16\pi^2}\log\left(\frac{Q^2}{M_h^2}\right) + \ldots.\right)}$$

hh→hh (3)

- Sum the geometric series to define a running coupling

$$A = -6\lambda\left(1 + \frac{9\lambda}{16\pi^2}\log\frac{Q^2}{M_h^2}\right) + \ldots$$

$$A = \frac{6\lambda}{1 - \frac{9\lambda}{8\pi^2}\log\left(\frac{Q}{M_h}\right)} \equiv 6\lambda(Q)$$

- $\lambda(Q)$ blows up as Q→∞ (called Landau pole)

hh→hh (4)

- This is independent of starting point
- BUT…. Without $\lambda\phi^4$ interactions, theory is non-interacting
- Require quartic coupling be finite

$$\frac{1}{\lambda(Q)} > 0$$

$$\frac{\lambda}{1 - \frac{9\lambda}{8\pi^2}\log\left(\frac{Q}{M_h}\right)} = \lambda(Q)$$

hh→hh (5)

- Use $\lambda = M_h^2/(2v^2)$ and approximate $\log(Q/M_h) \rightarrow \log(Q/v)$
- Requirement for $1/\lambda(Q) > 0$ gives upper limit on M_h

$$\boxed{M_h^2 < \frac{32\pi^2 v^2}{9\log\left(\frac{Q^2}{v^2}\right)}}$$

- Assume theory is valid to 10^{16} GeV
  - Gives upper limit on $M_h < 180$ GeV

High Energy Behavior of λ

- Renormalization group scaling $\frac{1}{\lambda(Q)} = \frac{1}{\lambda(\mu)} + (\ldots)\log\left(\frac{Q}{\mu}\right)$

$$\boxed{16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12 g_t^4 + (gauge)}$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right) \qquad g_t = \frac{M_t}{v}$$

- *Large λ (Heavy Higgs):* self coupling causes λ to grow with scale
- *Small λ (Light Higgs):* coupling to top quark causes λ to become negative

Does Spontaneous Symmetry Breaking Happen?

- SM requires spontaneous symmetry breaking
- This requires

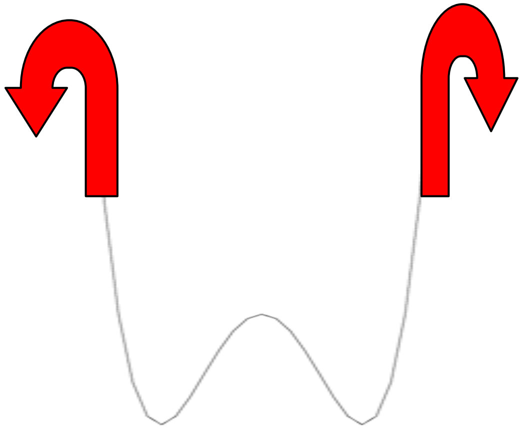
$$\boxed{V(v) < V(0)}$$

- For small λ

$$\boxed{16\pi^2 \frac{d\lambda}{dt} \approx -16 g_t^4}$$

- Solve

$$\boxed{\lambda(\Lambda) \approx \lambda(v) - \frac{3 g_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)}$$



Does Spontaneous Symmetry Breaking Happen? (2)

- $\lambda(\Lambda) > 0$ gives lower bound on M_h

$$\boxed{M_h^2 > \frac{3v^2}{2\pi^2}\log\left(\frac{\Lambda^2}{v^2}\right)}$$

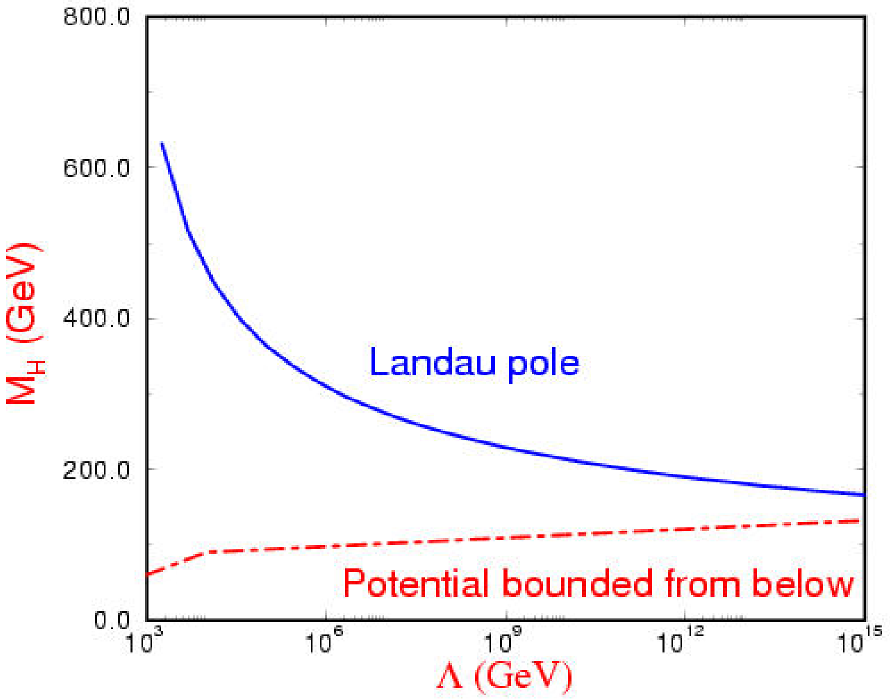
- If Standard Model valid to 10^{16} GeV

$$\boxed{M_h > 130\ GeV}$$

- For any given scale, Λ, there is a theoretically consistent range for M_h

Bounds on SM Higgs Boson

- If SM valid up to Planck scale, only a small range of allowed Higgs Masses



Problems with the Higgs Mechanism

- We often say that the SM cannot be the entire story because of the quadratic divergences of the Higgs Boson mass
- But don’t we just renormalize the mass????

Masses at one-loop

- Consider a fermion coupled to a massive complex Higgs scalar

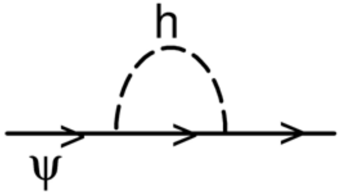
$$L = \overline{\Psi}(i\partial)\Psi + \left|\partial_\mu \phi\right|^2 - m_s\left|\phi\right|^2 - \left(\lambda_F \overline{\Psi}_L \Psi_R \phi + h.c.\right)$$

- Assume symmetry breaking as in SM:

$$\phi = \frac{(h+v)}{\sqrt{2}} \qquad\qquad m_F = \frac{\lambda_F v}{\sqrt{2}}$$

Masses at one-loop (2)

- Calculate mass renormalization for Ψ



$$-i\Sigma_F(p) = \left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k+m_F}{[k^2-m_F^2][(k-p)^2-m_s^2]}$$

Renormalized fermion mass

$$\delta m_F = \Sigma_F(p)\Big|_{p=m_F}$$

$$= i\frac{\lambda_F^2}{32\pi^4}\int_0^1 dx \int d^4k' \frac{m_F(1+x)}{[k'^2 - m_F^2x^2 - m_s^2(1-x)]^2}$$

- Do integral in Euclidean space

$$k_0 \to ik_4$$

$$d^4k' \to id^4k_E$$

$$k'^2 = k_0^2 - \left|\vec{k}\right|^2 \to k_4^2 - \left|\vec{k}\right|^2 = -k_E^2$$

$$\int d^4k_E f(k_E^2) = \pi^2\int_0^{\Lambda^2} y\,dy\,f(y)$$

Renormalized fermion mass (2)

- Renormalization of fermion mass:

$$\delta m_F = -\frac{\lambda_F^2 m_F}{32\pi^2}\int_0^1 dx\ (1+x)\int_0^{\Lambda^2}\frac{y\,dy}{[y+m_F^2 x^2+m_s^2(1-x)]^2}$$

$$= -\frac{3\lambda_F^2 m_F}{32\pi^2}\log\left(\frac{\Lambda^2}{m_F^2}\right)+\ldots..$$

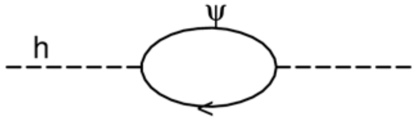
Fermion mass renormalization is logarithmic

Symmetry and the fermion mass

- $\delta m_F \approx m_F$
  - – $m_F=0$ quantum corrections vanish
  - – $m_F=0$ Lagrangian is invariant under
    - $\Psi_L \rightarrow e^{i\theta_L}\Psi_L$
    - $\Psi_R \rightarrow e^{i\theta_R}\Psi_R$
  - – $m_F \rightarrow 0$ increases the symmetry of the theory
  - – Yukawa coupling (proportional to mass) breaks symmetry and so corrections $\approx m_F$

$$\boxed{\begin{aligned} L &= \overline{\Psi}(i\partial)\Psi - \left(\frac{m_F}{v}\overline{\Psi}_L\Psi_R\phi + h.c.\right) + \ldots \\ &= \overline{\Psi}_L(i\partial)\Psi_L + \overline{\Psi}_R(i\partial)\Psi_R - \left(\frac{m_F}{v}\overline{\Psi}_L\Psi_R\phi + h.c.\right) + \ldots \end{aligned}}$$

Scalars are very different



$$-i\Sigma_s(p^2) = -\left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4}\,\frac{Tr\left[(k+m_F)((k-p)+m_F)\right]}{(k^2-m_F^2)[(k-p)^2-m_F^2]}$$

$$\boxed{\delta M_h^2 = \Sigma_S(m_s^2) = -\frac{\lambda_F^2\Lambda^2}{8\pi^2} + \left(m_s^2 - m_F^2\right)\log\left(\frac{\Lambda}{m_F}\right) + \ldots.}$$

- M_h diverges quadratically!
- This implies quadratic sensitivity to high mass scales

Scalars Masses (2)

- Higgs mass has large sensitivity to high scale physics (hierarchy problem)
- Higgs does not obey decoupling theorem
  - Which says that effects of heavy particles decouple as $M\rightarrow\infty$
  - This is because Higgs couplings are proportional to mass
- $M_h \rightarrow 0$ doesn't increase symmetry of theory
  - Nothing protects Higgs mass from large corrections

What's the problem?

- Compute M_h in dimensional regularization and absorb infinities into definition of M_h

$$M_h^2 = M_{h0}^2 + \frac{1}{\varepsilon}(\ldots)$$

- Perfectly valid approach
- Except we know there is a high scale

What physics can be lurking?

- We are about to probe a new energy scale at the LHC
- There may very well be new physics in the electroweak sector
- Let’s review what we really know
- The Standard Model provides an excellent description of physics at theTevatron energy
- BUT…..we haven’t seen the Higgs

Consider the Standard Model as an effective low energy theory

Effective Field Theory

- Effective Lagrangian

$$\boxed{L = L_{SM} + \sum \frac{c_i}{\Lambda^2} O_i^6 + \ldots..}$$

- O_i contain light fields (including Higgs for now)
- All information about heavy degrees of freedom in c_i
- Categorize operators by their dimensions, d_i
- Operators with $d_i > 4$ suppressed by powers of E/Λ

Operating assumption is that $\Lambda \gg M_Z$

Trouble…..

- Much possible new physics is excluded at the TeV scale
  - Look at possible dimension 6 operators
    - Many more operators than shown here
  - Limits depend on what symmetry is violated

New operators	Experimental limits
$\frac{(\bar{d}s)(\bar{d}s)}{\Lambda^2}$	$\Lambda > 1000\ TeV$
$\frac{m_b(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b)}{\Lambda^2}$	$\Lambda > 50\ TeV$
$\frac{(h^+ D_\mu h)^2}{\Lambda^2}$	$\Lambda > 5\ TeV$
$\frac{(D^2 h^+ D^2 h}{\Lambda^2}$	$\Lambda > 5\ TeV$

New Physics typically must be at scale Λ > 5 TeV

No Higgs?

- Remember, Higgs is used to unitarize the SM
- Unitarity violated at 1.7 TeV without a Higgs
- This sets the scale for something new
- Construct the Standard Model without a Higgs

Higgs is only piece we haven’t seen experimentally

Standard Model Revisited

- Scalar sector described by SU(2) doublet

$$\Phi = \frac{(v+h)}{\sqrt{2}} e^{i w^a \sigma^a / v} \qquad\qquad V = \frac{\lambda}{4}\left[Tr\left(\Phi^+ \Phi\right) - \frac{v^2}{2} \right]^2$$

- Scalar potential invariant under global $SU(2)_L$ x $SU(2)_R$ symmetry:

$$\Phi \rightarrow L \Phi R^+$$

- Global symmetry broken by $<\Phi>= \frac{v}{\sqrt{2}}$

Higgsless Standard Model

- Construct effective Lagrangian with 2 derivatives describing Goldstone bosons with global $SU(2)_L$ x $SU(2)_R$ symmetry
- This construction is expansion in powers of E^2/Λ^2

$$L = \frac{v^2}{4} Tr\left[\partial_\mu \Sigma \partial^\mu \Sigma^+\right]$$

$$\Sigma = e^{i w^a \sigma^a / v}$$

➡

Looks a lot like the Higgs field Φ with only the Goldstone bosons

Higgsless Standard Model (2)

Gauge theory: $L = \frac{v^2}{4} Tr\left[D_\mu \Sigma D^\mu \Sigma^+\right] + (kinetic)$

$$D_\mu \Sigma = \partial_\mu \Sigma - igW_\mu^a \frac{\sigma^a}{2}\Sigma + ig' B_\mu \Sigma \frac{\sigma_3}{2}$$

This is SM with massive gauge bosons

- At $O(E^2/\Lambda^2)$ gauge couplings identical to SM
- Since no Higgs, unitarity violated TeV scale

Higgsless Standard Model (3)

- Add $O(E^4/\Lambda^4)$ operators
  - Contributions from $O(E^2/\Lambda^2)$ operators at one loop generate infinities (SM is not renormalizable without Higgs)
  - These infinities absorbed into definitions of $O(E^4/\Lambda^4)$ operators
  - Can do this at every order in the energy expansion
- Coefficients are unknown but limited by precision measurements
- The $O(E^4/\Lambda^4)$ terms will change 3 and 4 gauge boson interactions

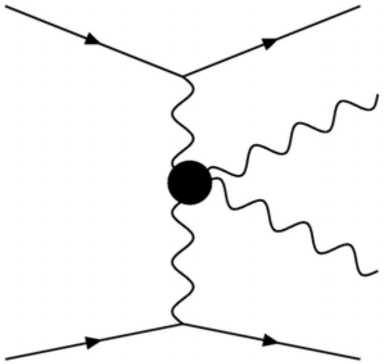
Gauge Boson Pair Production

- W^+W^-, $W^\pm\gamma$, etc, production sensitive to new physics
- Expect effects which grow with energy
  - $A_t \sim (\ldots)(s/v^2)+O(1)$
  - $A_s \sim -(\ldots)(s/v^2)+O(1)$
  - $\sigma_{TOT} \sim O(1)$

Very general result

WW scattering at LHC

- Four gauge boson interactions are sensitive to unitarity violating physics (Vector boson fusion)
- Look for W^+W^-, ZZ, $Z\gamma$, $W\gamma$ pair production in vector boson fusion



No resonance ⇨
Counting experiment ⇨
Hard

Construct Models without Higgs

- Problems with unitarity
- Something must come in to conserve unitarity if theory is to remain perturbative
- Extra dimension “Higgsless” models have tower of Kaluza Klein particles
  - These look like heavy copies of the W, Z, and photon
- Tower of KK vector bosons
  - Can be produced at LHC, e^+e^-
- Tension between:
  - Unitarity wants light KK
  - Precision EW wants heavy KK

Experimental Signatures of Higgsless Models

- Look for massive W, Z, γ like particles in vector boson fusion
  - Need small couplings to fermions to avoid precision EW constraints
  - Narrow resonances in WZ channel

LHC