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Higgs and Electroweak Symmetry Breaking - I

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Sally Dawson, BNL
Introduction to Electroweak Symmetry Breaking
Trieste, 2011

- Introduction to Electroweak Symmetry Breaking
  - Review of the SU(2) x U(1) Electroweak theory
  - Constraints from Precision Measurements
  - Experimental Searches for the Higgs
- Theoretical problems with the Standard Model
- Beyond the SM
  - Why are we sure there is physics BSM?
  - What do the LHC and Tevatron tell us?

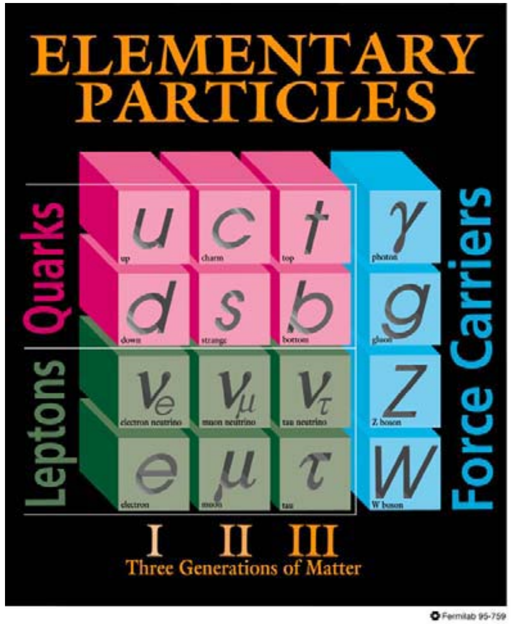
Large Hadron Collider (LHC)

- proton-proton collider at CERN running now!
- 7 TeV total energy
- Typical energy of quarks and gluons 1-2 TeV

If there is a Higgs boson, we expect it soon!

What we know

- The photon and gluon appear to be massless
- The W and Z gauge bosons are heavy
  - M_W=80.399 ± 0.023 GeV
  - M_Z =91.1875 ± 0.0021 GeV
- There are 6 quarks
  - M_t=173.3 ±1.1 GeV
  - M_t >> all the other quark masses
- There appear to be 3 distinct neutrinos with small but non-zero masses
- The pattern of fermions appears to replicate itself 3 times
  - Why not more?



Abelian Higgs Model

- Why are the W and Z boson masses non-zero?
- U(1) gauge theory with single spin-1 gauge field, A_μ

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

- U(1) local gauge invariance:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x)$$

- Mass term for A would look like:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

- Mass term violates local gauge invariance
- We understand why $M_A = 0$

Gauge invariance is guiding principle

Abelian Higgs Model, 2

- Add complex scalar field, φ, with charge –e:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| D_\mu \phi \right|^2 - V(\phi)$$

- Where $D_\mu = \partial_\mu - ieA_\mu$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

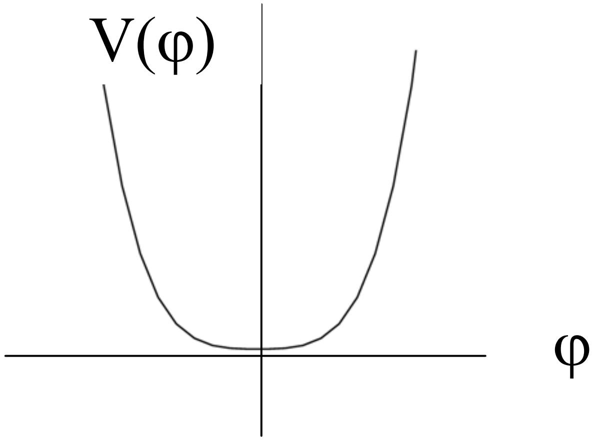
$$V(\phi) = \mu^2 |\phi|^2 + \lambda \left(|\phi|^2 \right)^2$$

- L is invariant under local U(1) transformations:

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$$
$$\phi(x) \to e^{-ie\eta(x)} \phi(x)$$

Abelian Higgs Model, 3

- Case 1: $\mu^2 > 0$
  - QED with $M_A=0$ and $m_\varphi=\mu$
  - Unique minimum at $\varphi=0$



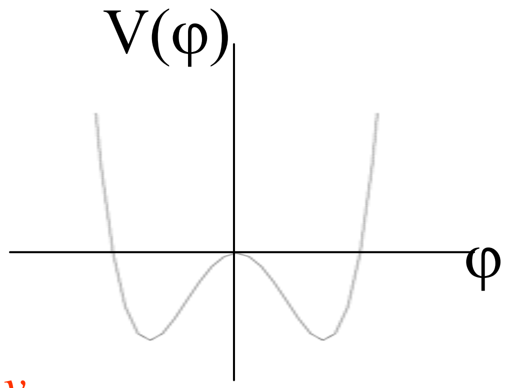
$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|D_\mu \phi\right|^2 - V(\phi)$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$V(\phi) = \mu^2\left|\phi\right|^2 + \lambda\left(\left|\phi\right|^2\right)^2$$

By convention, $\lambda > 0$

Abelian Higgs Model, 4



- Case 2: $\mu^2 < 0$

$$V(\phi) = -|\mu^2||\phi|^2 + \lambda\left(|\phi|^2\right)^2$$

- Minimum energy state at $<\phi> = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$

- Physical particle has minimum energy state at 0:

$$\varphi' = \varphi - <\varphi>$$

Vacuum breaks U(1) symmetry

Aside: What fixes sign (μ^2)?

v is termed vacuum expectation value (VEV)

Abelian Higgs Model, 5

- Rewrite $\phi \equiv \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}} (v+h)$

χ and h are the 2 degrees of freedom of the complex Higgs field

- h has minimum at 0
- L becomes:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - evA_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A^\mu A_\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h + 2\mu^2 h^2\right)$$
$$+ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \text{ interactions})$$

- Theory now has:
  - Photon of mass M_A=ev
  - Scalar field h with mass-squared $-2\mu^2 > 0$
  - Massless scalar field χ *(Goldstone Boson)*

Abelian Higgs Model, 6

- What about mixed χ-A propagator?
  - Remove by gauge transformation $A'_\mu \equiv A_\mu - \frac{1}{ev}\partial_\mu \chi$
  - χ field disappears
  - We say that it has been *eaten* to give the photon mass
  - χ field called Goldstone boson
  - *This is Abelian Higgs Mechanism*
  - This gauge (unitary) contains only physical particles

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2v^2}{2}A'^{\mu}A'_{\mu} + \frac{1}{2}\left(\partial_\mu h \partial^\mu h\right) - V(h)$$

Higgs Mechanism summarized

Spontaneous breaking of a gauge theory by a non-zero VEV of a scalar field results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson

Gauges

Choice above called unitarity gauge

- No χ field
- Bad high energy behavior of A propagator

$$\Delta_{\mu\nu}(k) = -\frac{i}{k^2 - M_A^{\ 2}}\left(g_{\mu\nu} - \frac{k^\mu k^\nu}{M_A^{\ 2}}\right)$$

- R_ξ gauges more convenient:
- $L_{GF} = (1/2\xi)(\partial_\mu A^\mu + \xi e v \chi)^2$

$$L_2 = -\frac{1}{2}A_\mu\left(-g^{\mu\nu}\partial^2 + \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu - e^2v^2\right)A_\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h + 2\mu^2 h^2\right)$$
$$+\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{\xi}{2}e^2v^2\chi^2$$

More on R_ξ gauges

Mass of Goldstone boson χ depends on ξ

- $\xi=1$: Feynman gauge with massive χ
- $\xi=0$: Landau gauge
- $\xi\to\infty$: Unitary gauge

Gauge Boson, A

$$\frac{-i}{k^2-M_A^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2-\xi M_A^2}(1-\xi)\right)$$

Higgs, h

$$\frac{i}{k^2-M_h^2}$$

Goldstone Boson, χ

$$\frac{i}{k^2-\xi M_A^2}$$

Unitary Gauge (no Goldstone bosons)

- We started with:
  - Massless photon (2 transverse polarizations)
  - Complex scalar (2 degrees of freedom)
- After redefining scalar so it has minimum energy state at 0, we have:
  - Massive gauge boson (2 transverse, 1 longitudinal polarization)
  - Physical scalar h
- Degrees of freedom preserved
- Must find h to confirm

Non-Abelian Higgs Mechanism

- Vector fields $A^a_\mu(x)$ and scalar fields $\varphi_i(x)$ of SU(N) group

$$\Phi = \begin{pmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_N \end{pmatrix} \qquad L_\Phi = (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi), \qquad V(\Phi) = \mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$$

i=1…N

- L is invariant under the non-Abelian symmetry:

$$\phi_i \to (1 - i\eta^a \tau^a)_{ij} \phi_j$$

$$D_\mu \phi = \left(\partial_\mu - ig\tau^a A^a_\mu\right)\phi$$

- τ_a are group generators, a=1…N^2-1 for SU(N)

For SU(2): $\tau^a = \sigma^a/2$ $\qquad$ σ are Pauli matrices

Non-Abelian Higgs Mechanism, 2

- In exact analogy to the Abelian case $D_\mu \phi = \left(\partial_\mu - i g \tau^a A_\mu^a\right)\phi$

$$(D_\mu \Phi)^+ (D^\mu \Phi) \quad \rightarrow \quad \ldots + g^2 (\tau^a \phi^+)_i (\tau^b \phi)_i A_\mu^a A^{b\mu} + \ldots$$

$$\xrightarrow{\phi \rightarrow \phi_0} \ldots + g^2 (\tau^a \phi_0{}^+)_i (\tau^b \phi_0)_i A_\mu^a A^{b\mu} + \ldots$$

- $\tau^a \varphi_0 \neq 0$

  $\Rightarrow$ Massive vector boson + Goldstone boson
- $\tau^a \varphi_0 = 0$

  $\Rightarrow$ Massless vector boson + massive scalar field

Non-Abelian Higgs Mechanism, 3

- Consider SU(2) example
  - Gauge field A^a in triplet SU(2) representation, a=1…3
  - Scalar field in doublet representation

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad D_\mu \phi = \left(\partial_\mu - ig \frac{\sigma^a}{2} A_\mu^a \right) \phi$$

← Matrix equation

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Non-Abelian Higgs Mechanism, 4

- SU(2) example cont. $$D_\mu \phi = \left(\partial_\mu - ig \frac{\sigma^a}{2} A_\mu^a \right) \phi$$
- Suppose φ gets a VEV: $$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$
- Gauge boson mass term $$\left| D_\mu \phi \right|^2 = \frac{1}{2} g^2 (0, v) \tau^a \tau^b \begin{pmatrix} 0 \\ v \end{pmatrix} A_\mu^a A^{b\mu}$$
- Using the property of group generators, $\{\tau^a, \tau^b\} = \delta^{ab}/2$
- Mass term for gauge bosons:

$$\boxed{L_{mass} = \frac{g^2 v^2}{8} A_\mu^a A^{a\mu}}$$

Standard Model Synopsis

- Group: SU(3) x SU(2) x U(1)

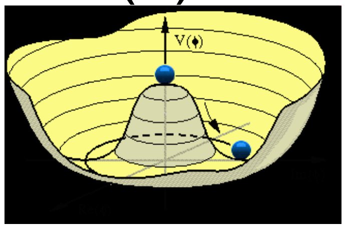
  QCD (SU(3)) — Electroweak (SU(2) x U(1))

- Gauge bosons:
  - SU(3): G_μ^i, i=1…8
  - SU(2): W_μ^i, i=1,2,3
  - U(1): B_μ
- Gauge couplings: g_s, g, g'
- Complex SU(2) Higgs doublet: Φ

Ignore SU(3) in these lectures

SM Higgs Mechanism

- Standard Model includes complex Higgs SU(2) doublet



$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$$

- With SU(2) x U(1) invariant scalar potential

$$V = \mu^2\Phi^+\Phi + \lambda(\Phi^+\Phi)^2$$

Invariant under $\Phi \rightarrow -\Phi$

- If $\mu^2 < 0$, then spontaneous symmetry breaking
- Minimum of potential at:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$$

$$\Phi \rightarrow e^{i\varpi^a \cdot \sigma^a / v}\begin{pmatrix}0 \\ \frac{h+v}{\sqrt{2}}\end{pmatrix}$$

  – Choice of minimum breaks gauge symmetry

More on SM Higgs Mechanism

- Couple Φ to SU(2) x U(1) gauge bosons (W_i^μ, i=1,2,3; B^μ)

$$L_S = (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi)$$

$$D_\mu = \partial_\mu - i\frac{g}{2}\sigma^i W^i{}_\mu - i\frac{g^{'}}{2} B_\mu$$

- Gauge boson mass terms from:

$$(D_\mu \Phi)^+ D^\mu \Phi \rightarrow \ldots + \frac{1}{8}(0, v)(g W_\mu^a \sigma^a + g' B_\mu)(g W^{b\mu}\sigma^b + g' B^\mu)\begin{pmatrix} 0 \\ v \end{pmatrix} + \ldots$$

$$\rightarrow \ldots + \frac{v^2}{8}\left(g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-g W_\mu^3 + g' B_\mu)^2\right) + \ldots$$

More on SM Higgs Mechanism

- With massive gauge bosons:

$$W_\mu^\pm = \left(\frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}} \right)$$

$$Z_\mu^0 = \left(\frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \right)$$

$$M_W = \frac{gv}{2}$$

$$M_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2}$$

- Orthogonal combination to Z is massless photon

$$A_\mu^0 = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$$

More on SM Higgs Mechanism

- Weak mixing angle defined :

$Z = -\sin\theta_W B + \cos\theta_W W^3$

$A = \cos\theta_W B + \sin\theta_W W^3$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

➡ Natural Relationship: $M_W = M_Z \cos\theta_W$

$$\rho = \frac{M_W}{M_Z \cos\theta_W} = 1$$

Recap of SM Higgs Mechanism

- Generate mass for W,Z using Higgs mechanism
  - Higgs VEV breaks SU(2) x U(1)
  - Single Higgs doublet is minimal case
- Before spontaneous symmetry breaking:
  - Massless W_i, B, Complex Φ
- After spontaneous symmetry breaking:
  - Massive $W^{\pm}$, Z; massless γ; physical Higgs boson h

W, Z, Higgs Couplings

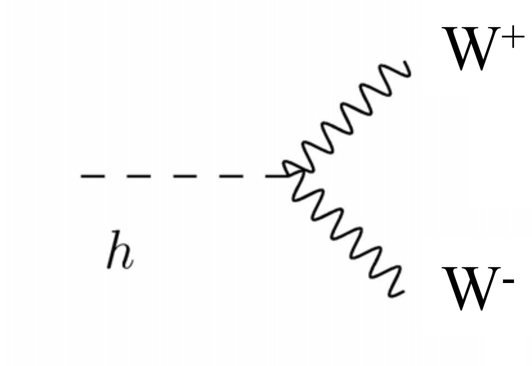
- Lagrangian in terms of massive gauge bosons and Higgs boson:

$$L = gM_W W^{+\mu} W_\mu^- h + \frac{gM_Z}{\cos\theta_W} Z^\mu Z_\mu h$$

- Higgs couples to gauge boson mass
- Spontaneous symmetry breaking gives W/Z mass $\Rightarrow$ longitudinal polarization

Example: $h \to W^+W^-$

- Rest frame of h:
  - $p_h=(M_h,0,0,0)$
  - $p_{W+}=M_h/2(1,0,0,\beta)$
  - $p_{W-}=M_h/2(1,0,0,-\beta)$
  - $\varepsilon_\pm(W^+)=(0,1,\pm i,0)/\sqrt{2}$
  - $\varepsilon_\pm(W^-)=(0,1,\mp i,0)/\sqrt{2}$
  - $\varepsilon_L(W^+)=(M_h/2M_W)(\beta,0,0,1)$
  - $\varepsilon_L(W^-)=(M_h/2M_W)(\beta,0,0,-1)$



$$A(h \to W^+W^-) = -gM_W\,\varepsilon(W^+)\cdot\varepsilon(W^-)$$

$$A(h \to W^+W^-)_{longitudinal} \approx g\frac{M_h^2}{4M_W}$$

$$A(h \to W^+W^-)_{transverse} \approx gM_W$$

The action is in the longitudinal sector!

$\beta^2=1-4M_W^2/M_h^2$