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Flavor Physics - I

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FLAVOUR PHYSICS IN THE PRECISION ERA

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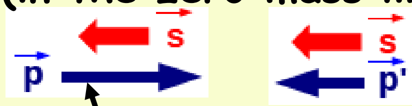
Standard Model (SM)

The SM:

- describes **strong** and **electroweak interactions** of elementary particles

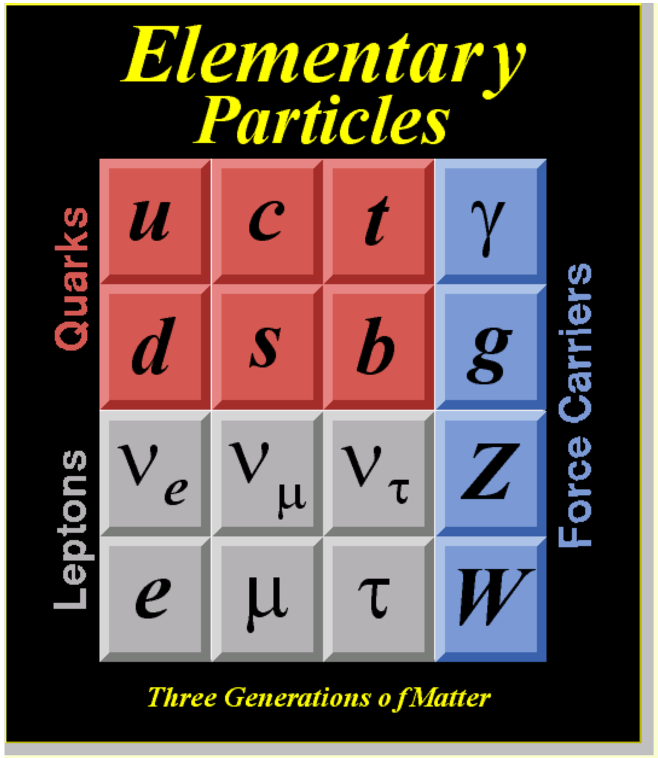
- is a **gauge theory** based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$

(in the zero mass limit)



$$\psi = \psi_L + \psi_R$$

Left-handed (L)
and Right-handed (R)
fields belong to different
group representations



→ Vectorial interaction

→ "

→ V-A & V+A

→ V-A only

+ Higgs (?)
soon an answer from the LHC

Matter fields (either quarks or leptons)
exist in **three families**:
3 flavours of up-type and
3 of down-type

Flavour Physics studies transitions between different flavours

Weak Interactions

Let's focus on the quark sector

Within the SM, quark masses are generated by Spontaneous Symmetry Breaking (Higgs mechanism)

e.g., the mass term for down-type quarks is: $\overline{D_L'}\, M_d\, d_R'$
(M_d 3x3 matrix in flavour space)

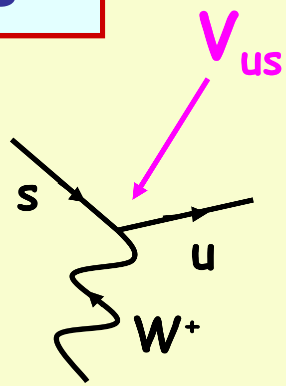
The mass matrix can be diagonalized by rotating (indipendently) Left (L) and Right (R) fields:

$$D_L' = V_d\, D_L \qquad \Longrightarrow \qquad \overline{D_L}\, \underbrace{V_d^{\dagger}\, M_d\, U_d}_{\text{diagonal}}\, d_R$$

$$d_R' = U_d\, d_R$$

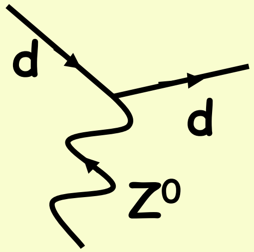
Mass eigenstates

Weak Charged and Neutral Currents in the basis of mass eigenstates



Charged currents are NOT diagonal
$\leftrightarrow$change flavour $\quad \overline{U}_L \; \gamma^\mu \; \underbrace{V_u^\dagger V_d}_{V_{CKM}} \; D_L \; W_\mu^+$

Cabibbo-Kobayashi-Maskawa
(mixing matrix in flavour space)



Neutral currents are diagonal
$\leftrightarrow$Flavour Changing Netural Currents (FCNC)
are **absent** at tree-level in the **SM**,
due to the **GIM mechanism** (Glashow-Iliopoulos-Maiani)

$$\overline{D}_L \; \gamma^\mu \; \underbrace{V_d^\dagger V_d}_{\mathbb{1}} \; D_L \; Z_\mu^0 + \ldots$$

FCNC can appear only through loops within the SM and are thus suppressed and a privileged place to look for New Physics

Similarly in the Lepton Sector

- Neutrino Oscillations have been measured showing that neutrinos are massive and not degenerate in mass
- Lepton charged currents are NOT diagonal in the mass eigenstates as well as for quarks
- The mixing matrix here is V_{PMNS}
  (Pontecorvo-Maki-Nakagawa-Sakata)

Neutrino Physics is a very interesting and active field, whose discussion would require separate lectures!

Weak eigenstates

Mass eigenstates

The CKM Matrix

$$\bar{U}_L \, \gamma^\mu \, \underbrace{V_u^\dagger V_d}_{V_{CKM}} \, D_L \, W_\mu^+$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- 3x3 unitary matrix
- 4 parameters: 3 angles and 1 phase
- The phase is responsible of CP-violation

(With exact CPT, CP is equivalent to T, T is a antiunitary operator ⇨ T $V_{CKM} \rightarrow V^*_{CKM}$ which differs from V_{CKM} due to the phase)

First important aim of Flavour Physics:
Accurate determination of the CKM parameters

At present an accuracy of few % has been achieved!

Nowadays, an even more important aim:
searching for New Physics effects

The SM turns out to be very successful
in describing essentially all processes
But
It is expecetd to be an effective theory valid up to a cutoff scale
as it has some important limits

- The SM is a quantum theory for strong and electroweak interactions
 but NOT for gravitation (quantum effecs in gravitation are expected
 to become important at very high energies (M_{Pl}~10^{23} GeV))

- There is cosmological evidence of Dark Matter
 (not made up of SM particles) in the Universe

- ...

•…

•In order to explain the dominance of matter over anti-matter in the Universe it is crucial to have CP-violation: the phase in the CKM matrix is not enough to explain the required amount for baryogenesis

•In the SM the Higgs mass receives large radiative corrections, quadratic in the cutoff $\Lambda \sim M_{Pl} \sim 10^{23}$ GeV (energy scale where the SM fails).
In order to have a Higgs mass of O(100 GeV) as known indirectly from electroweak precision tests, an innatural fine-tuning is required (hierarchy problem)

•In the SM neutrinos (ν) are assumed massless while the evidence of oscillations shows that they are massive. The most credited mechanism to generate ν masses is the so called SEE-SAW, where ν masses are small because inversely proportional to a large scale.
It is reasonable to identify this scale with a NP scale where strong and electroweak interactions unify (GRAND-UNIFICATION)

•…

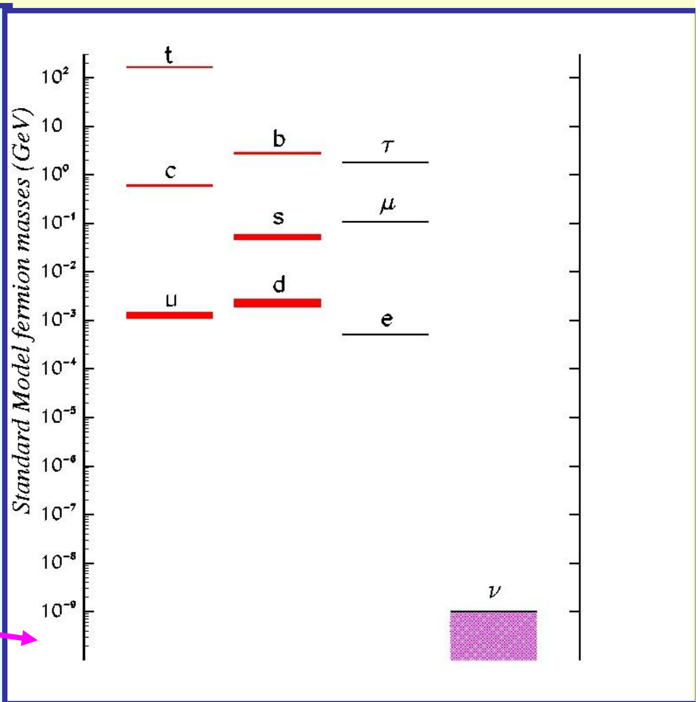
And, in particular, our understanding of flavour is UNSATISFACTORY

- The SM has too many unpredicted parameters to be fundamental
  (10 only in the quark sector)

- The CKM matrix turns out to be very hierarchical
  (while this is not the case for the PMNS matrix in the lepton sector)

- The quark masses have a very peculiar pattern
  (over 5 orders of magnitude)

- The number of families (3) remains unexplained



It is reasonable and desirable to think that an explanation is provided by New Physics (NP) beyond the SM!

Moreover, the solution doesn't seem to be trivial:
the FLAVOUR PROBLEM

"NP is expected at the TeV scale
(in order to solve the hierarchy problem)
but in flavour processes NP effects are not observed
(hinting for NP at higher scales)"

$\Downarrow$

The flavour structure of the NP model
cannot be generic

We will discuss this in the 4° lecture


Tools
for theoretical predictions
in Flavour Physics

Process calculations are based on an effective theory approach

Basic idea:

- Processes involving energies of O(1 GeV) can be more simply described integrating out the heavy degrees of freedom
  (M_W, M_Z, m_t, m_{NP} >> 1 GeV)

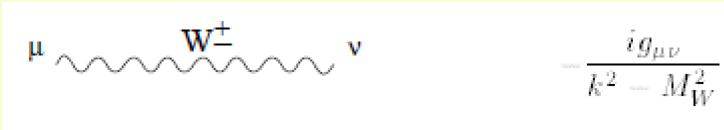
- Heavy propagators reduce to local interactions, e.g.:

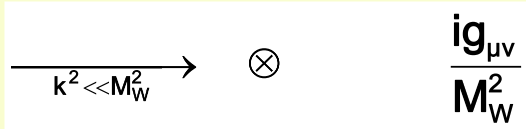
$$-\frac{ig_{\mu\nu}}{k^2 - M_W^2} \xrightarrow[k^2 \ll M_W^2]{} \otimes \qquad \frac{ig_{\mu\nu}}{M_W^2}$$

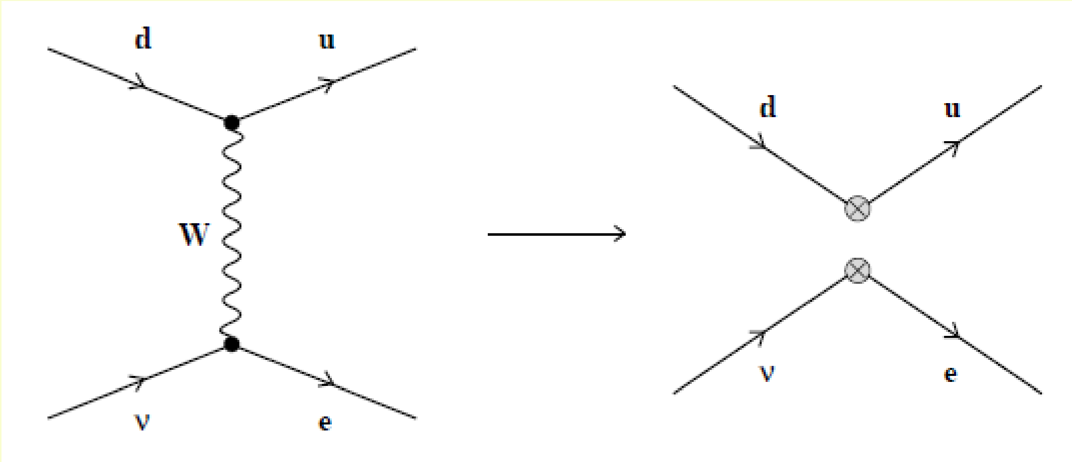
- H_{eff} is a series of effective vertices (Q_i) multiplied by effective coupling constants (C_i)
  [the so-called Operator Product Expansion (OPE), where typically the first term in the series is the only relevant one]

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) Q_i$$

A simple and famous example:
Fermi Theory for β decay







$$H_{eff}^{(\beta)} = \frac{G_F}{\sqrt{2}} V_{ud}\, \bar{u}\gamma^{\mu}(1-\gamma_5)\, d \otimes \bar{e}\gamma_{\mu}(1-\gamma_5)\, \nu_e$$

The effective theory has important advantages w.r.t. the original theory

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) Q_i$$

The separation of energy scales:

•$C_i(\mu)$ contains contributions from scales higher than μ and thus, due QCD asymptotic freedom, can be computed in perturbation theory.
(Nowadays at NLO and often at NNLO in QCD)

•In particular $C_i(\mu)$ summarize the effect of the heavy particles integrated away. Also NP effects can be included in this (simple) way

•Q_i are local operators (like $\bar{u}\gamma^\mu(1-\gamma_5)d \otimes \bar{e}\gamma_\mu(1-\gamma_5)\nu_e$)
Their matrix elements contain contributions from scales lower than μ and thus require non perturbative calculations. The main approach comes from Lattice QCD
(true QCD in a finite and discrete volume (see lecture 3)).
Other methods (introducing some model dependence) exist:
QCD sum rules, quark models, large-Nc,…

An important help for the computation of matrix elements (e.g. in fitting the mass dependence of lattice data) comes again from specific effective theories:

- Chiral Perturbation Theory (ChPT)
  for systems with light quarks (u,d,s)

- Heavy Quark Effective Theory (HQET)
  for systems with a heavy quark (b)

- Non-relativistic QCD (NRQCD)
  for heavy quarkonium systems

- Soft Collinear Effetive Theory (SCET)
  for systems involving widely different energies

…

- The Wilson coefficients $C_i(\mu)$ depend on the renormalization scale μ

- μ is typically chosen of the order of the mass of the decaying hadron ($O(m_b)$, $O(m_c)$, $O(1-2\ \mathrm{GeV})$) for B-, D- and K-mesons)

- The μ-dependence of C_i is cancelled in physical observables by the μ-dependence of the matrix elements $\langle Q_i(\mu)\rangle$

- The cancellation is not exact due to the truncation of the perturbative series and the residual μ-dependence is an estimate of higher order effects

- C_i contain $\log(\mu^2/M_W^2)$ (and similar), which are large logarithms

- Large logarithms can be resummed at all orders in α_s thanks to the Renormalization Group (as we will see later)


Let's discuss in some detail
the procedure to determine
the Wilson coefficients

Step n.1

Determination of the Wilson coefficients C_i at the high scale $\mu=m_W$ (where $\log(\mu^2/m_W^2)$ vanish) from a matching condition

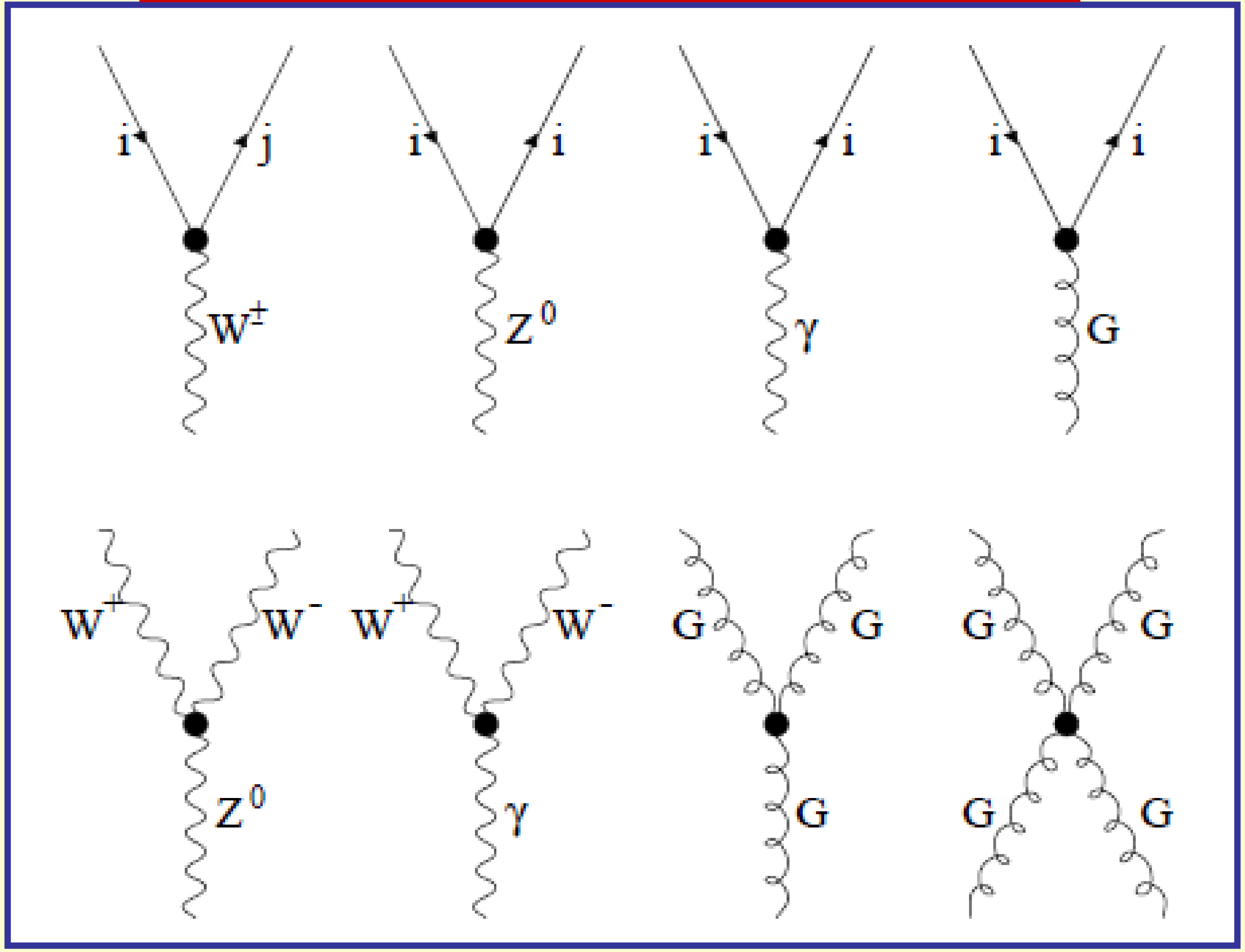
$$A_{full}=A_{eff}$$

- choose the same external (elementary particle) states, with the same configuration of momenta for A_{full} and A_{eff}
- calculate the amplitude in the original full theory (i.e. with the usual Feynman rules)

  [OBS. with external quark and lepton states it can be computed, the difficutly which is overcome thanks to the effective theory approach is the computation of non local interactions with hadronic states]
- calculate the amplitude within the effective theory (i.e. with vertices given by the Q_i)
- read the C_i from the matching equation $A_{full} = A_{eff}$

Within the full theory

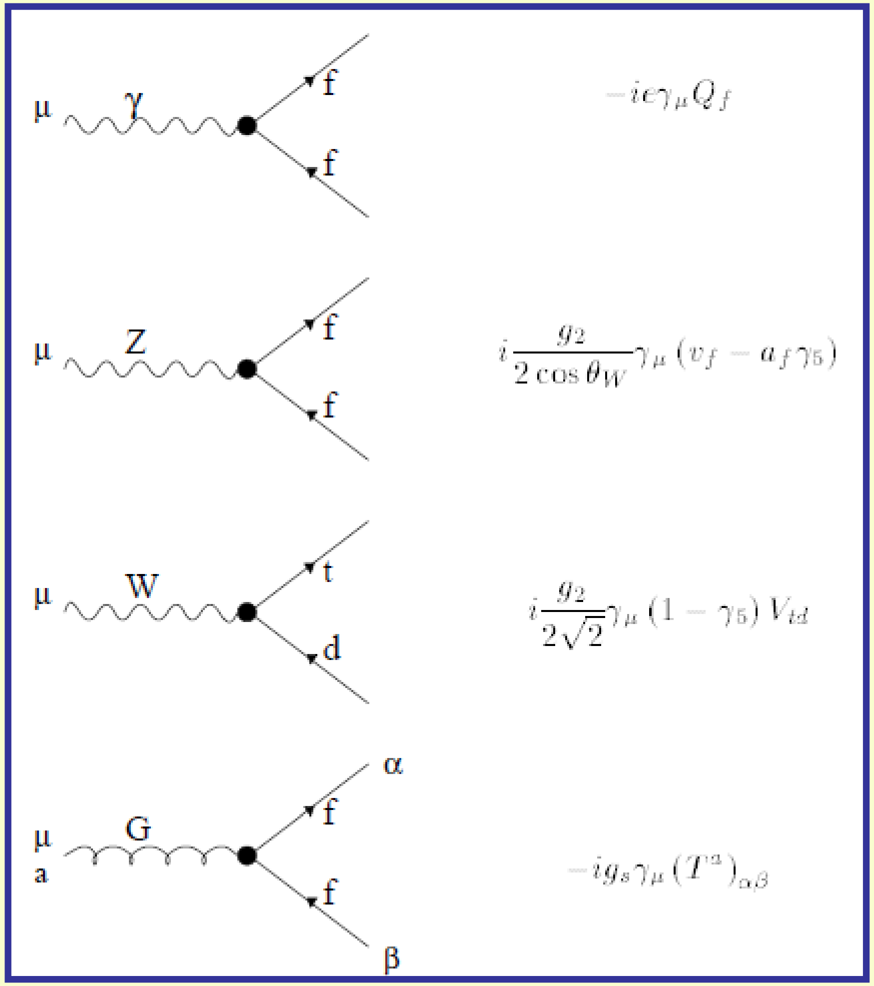
The (SM) Elementary vertices



The well-known (SM) Feynman rules

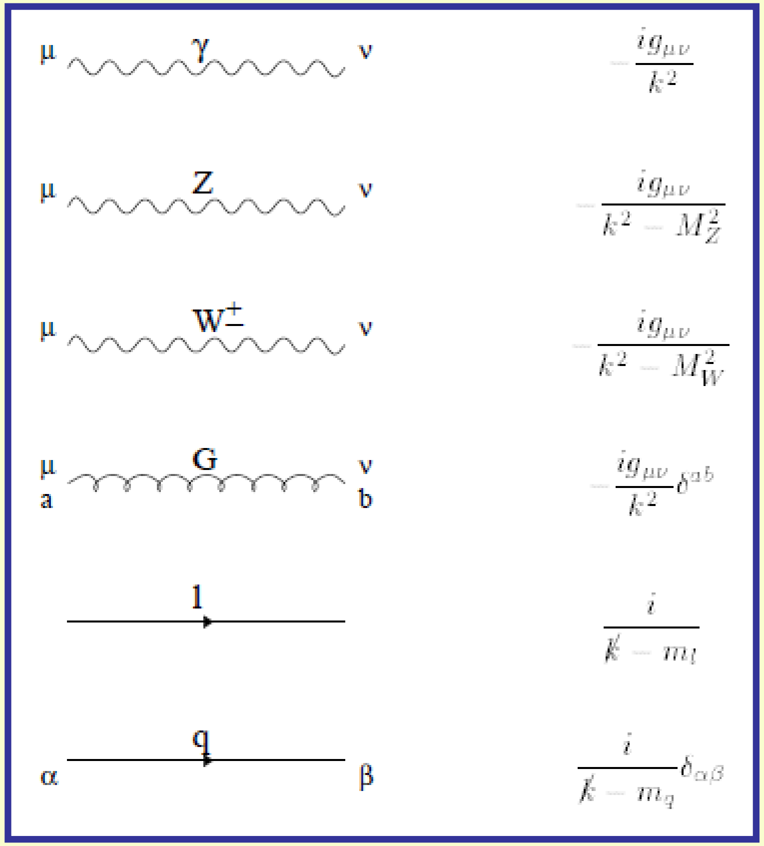
Vertices

(3 gauge bosons and Higgs vertices are not shown as do not enter the FCNCs)



Propagators

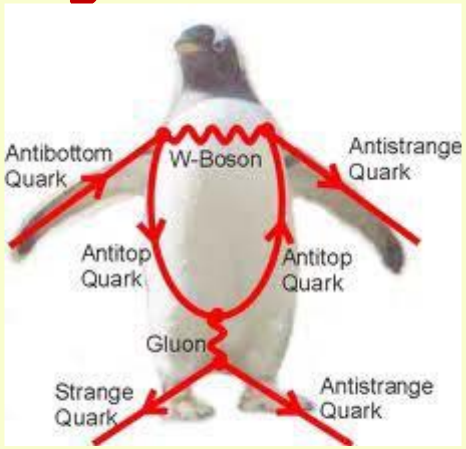
(Higgs propagator is not shown as does not enter the FCNCs)



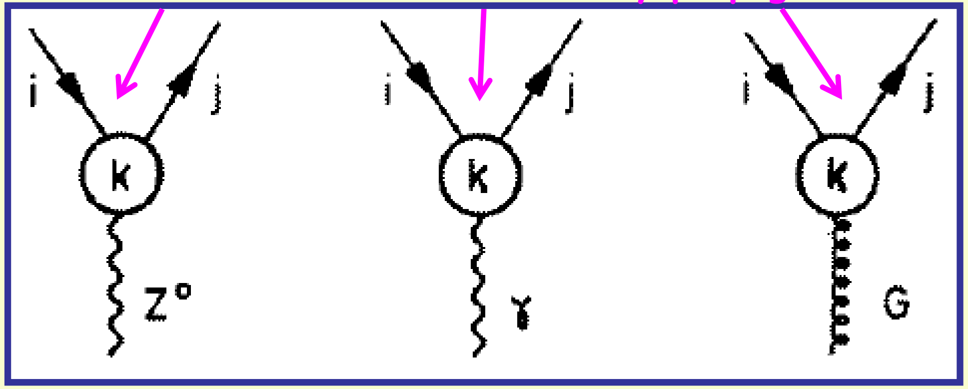
Within the effective theory

The effective vertices

The so-called Penguin Vertices

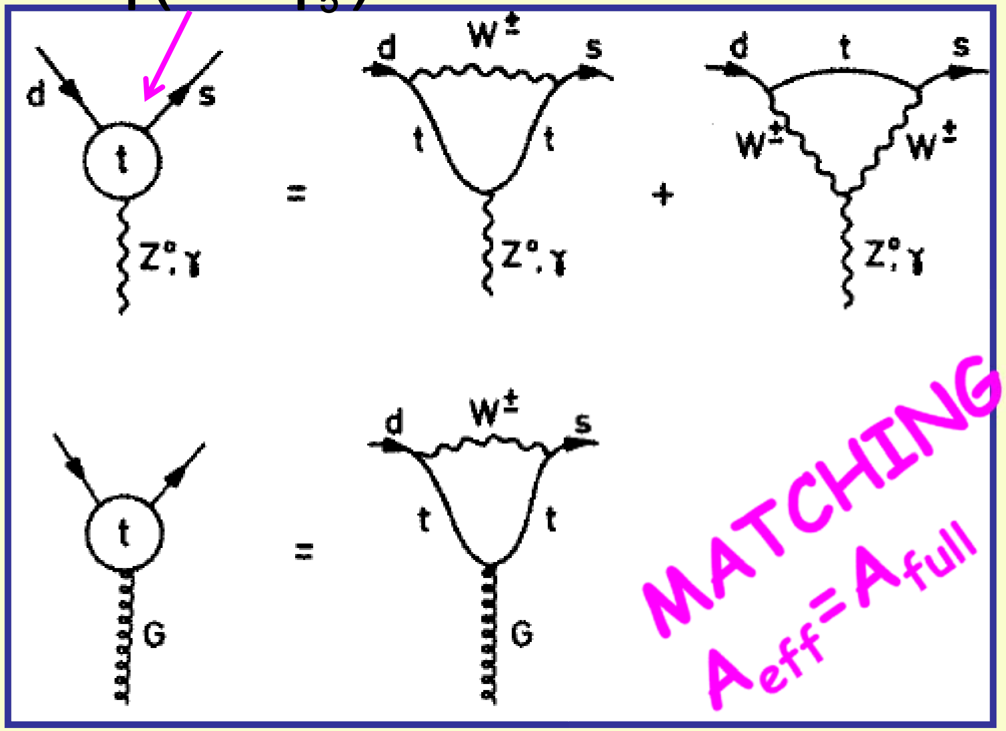


blobs stand for contracted heavy propagators

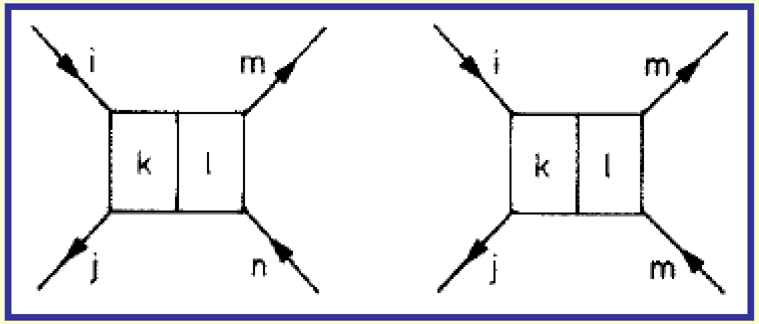


e.g.

$$A^{\Delta s=1}_{eff} = C \cdot \bar{s}\gamma^{\mu}(1-\gamma_5)\, d$$

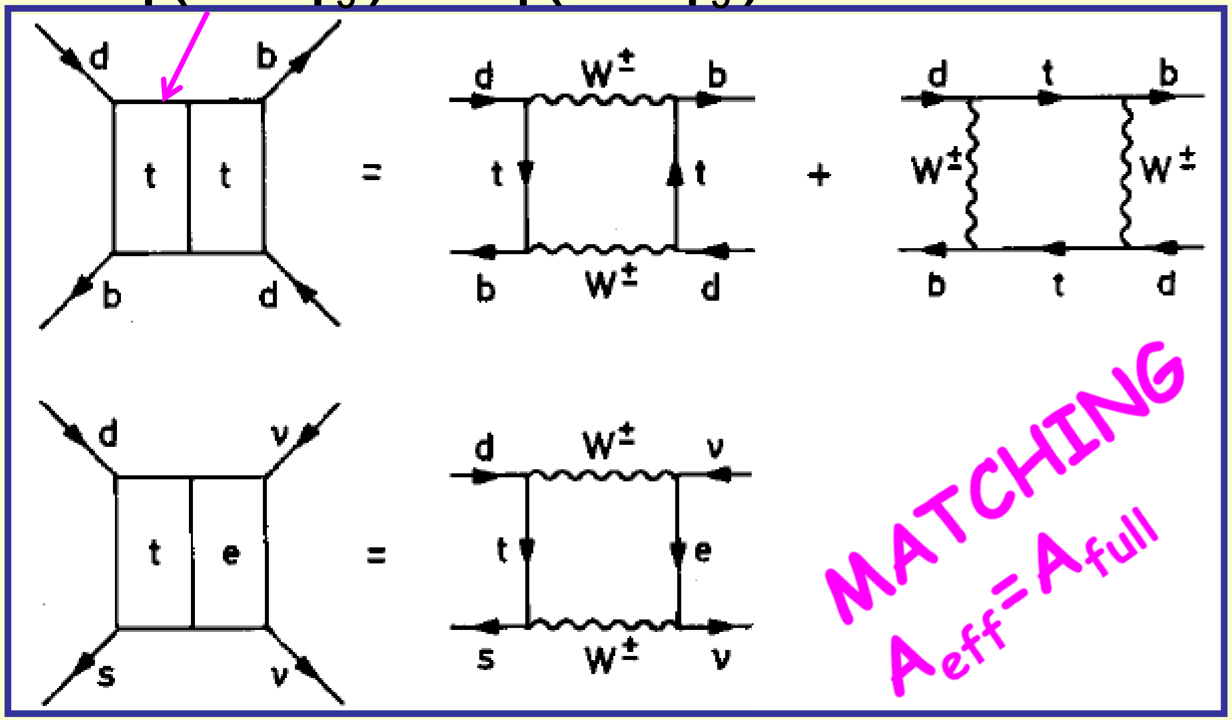


The so-called
Box Vertices



e.g.

$$A_{eff}^{\Delta B=2} = C \cdot \bar{b}\gamma^{\mu}(1-\gamma_5)\, d\; \bar{b}\gamma_{\mu}(1-\gamma_5)\, d$$



The effective Feynman rules

$\lambda_i = V_{is}^* V_{id} \qquad \bar{\lambda}_i = V_{is}^* V_{ib}$

$$\mathrm{Box}(\Delta S = 2) \;=\; \lambda_i^2 \frac{G_F^2}{16\pi^2} M_W^2 S_0(x_i)(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$$

For $A_{eff}^{\Delta S=1}$

OBS. From A to H a factor "i" appears

(S=exp[-i H], A↔ S⇨H ↔i A]

$$\mathrm{Box}(T_3 = 1/2) \;=\; \lambda_i \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\Theta_W}[-4B_0(x_i)](\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

$$\mathrm{Box}(T_3 = -1/2) \;=\; \lambda_i \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\Theta_W} B_0(x_i)(\bar{s}d)_{V-A}(\bar{\mu}\mu)_{V-A}$$

$$\bar{s}Zd \;=\; i\lambda_i \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} M_Z^2 \frac{\cos\Theta_W}{\sin\Theta_W} C_0(x_i)\bar{s}\gamma_\mu(1-\gamma_5)d$$

$$\bar{s}\gamma d \;=\; -i\lambda_i \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} D_0(x_i)\bar{s}(q^2\gamma_\mu - q_\mu \not{q})(1-\gamma_5)d$$

$$\bar{s}G^a d \;=\; -i\lambda_i \frac{G_F}{\sqrt{2}} \frac{g_s}{8\pi^2} E_0(x_i)\bar{s}_\alpha(q^2\gamma_\mu - q_\mu \not{q})(1-\gamma_5)T^a_{\alpha\beta} d_\beta$$

For $A_{eff}^{\Delta B=1}$

OBS. Prime stands for on-shell photon and gluon (e.g. important in b→ s γ)

$$\bar{s}\gamma' b \;=\; i\bar{\lambda}_i \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} D_0'(x_i)\bar{s}[i\sigma_{\mu\lambda}q^\lambda[m_b(1+\gamma_5)]]b$$

$$\bar{s}G'^a b \;=\; i\bar{\lambda}_i \frac{G_F}{\sqrt{2}} \frac{g_s}{8\pi^2} E_0'(x_i)\bar{s}_\alpha[i\sigma_{\mu\lambda}q^\lambda[m_b(1+\gamma_5)]]T^a_{\alpha\beta} b_\beta\,,$$

Properties of the effective vertices

- They are **higher order** in the gauge couplings and thus **suppressed** w.r.t elementary transitions

- Beacuse of the internal W^{+-} exchanges they are all (V-A) as the W^{+-} interactions

- They **depend on** the CKM elements, due to W vertices

- They **depend on the masses of internal quarks** (or leptons) and are thus calculable functions of $x_i = m_i^2/M_W^2$

- The **dependence on** CKM factors and x_i governs the **strength** of the vertices

- A new feature of these effective vertices is that they **depend on the gauge** used for the W propagator
(but gauge-independent combinations enter physical observables)

Basic short-distance functions (Wilson coefficients) entering the effective vertices

Defined omitting mass-independent terms, which cancel in FCNC processes due to GIM (as we will see in a blackboard calculation)

$$B_0(x_t) = \frac{1}{4}\left[\frac{x_t}{1-x_t} + \frac{x_t \ln x_t}{(x_t-1)^2}\right]$$

$$C_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-6}{x_t-1} + \frac{3x_t+2}{(x_t-1)^2}\ \ln x_t\right]$$

$$D_0(x_t) = -\frac{4}{9}\ln x_t + \frac{-19x_t^3+25x_t^2}{36(x_t-1)^3} + \frac{x_t^2(5x_t^2-2x_t-6)}{18(x_t-1)^4}\ln x_t$$

$$E_0(x_t) = -\frac{2}{3}\ln x_t + \frac{x_t^2(15-16x_t+4x_t^2)}{6(1-x_t)^4}\ln x_t + \frac{x_t(18-11x_t-x_t^2)}{12(1-x_t)^3}$$

$$D_0'(x_t) = -\frac{(8x_t^3+5x_t^2-7x_t)}{12(1-x_t)^3} + \frac{x_t^2(2-3x_t)}{2(1-x_t)^4}\ln x_t$$

$$E_0'(x_t) = -\frac{x_t(x_t^2-5x_t-2)}{4(1-x_t)^3} + \frac{3}{2}\frac{x_t^2}{(1-x_t)^4}\ln x_t$$

$$S_0(x_t) = \frac{4x_t-11x_t^2+x_t^3}{4(1-x_t)^2} - \frac{3x_t^3\ln x_t}{2(1-x_t)^3}$$

$$\left.\begin{aligned} &S_0(x_c) = x_c \\ &S_0(x_c,x_t) = x_c\left[\ln\frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2\ln x_t}{4(1-x_t)^2}\right]\end{aligned}\right\}$$

Neglecting higher orders in $x_c \ll 1$

Subscript "0" stands for LO in QCD (without including yet gluon corrections)

We will calculate $S_0(x_t)$, which is the function entering B-$\bar{B}$ mixing in lecture n.2

Gauge-independent combinations of the short-distance functions

$$C_0(x_t,\xi) - 4B_0(x_t,\xi,1/2) = C_0(x_t) - 4B_0(x_t) = X_0(x_t)$$

$$C_0(x_t,\xi) - B_0(x_t,\xi,-1/2) = C_0(x_t) - B_0(x_t) = Y_0(x_t)$$

$$C_0(x_t,\xi) + \frac{1}{4}D_0(x_t,\xi) = C_0(x_t) + \frac{1}{4}D_0(x_t) = Z_0(x_t).$$

Z-penguin and box

Z- and γ-penguin

$$X_0(x_t) = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\,\ln x_t\right]$$

$$Y_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-4}{x_t-1} + \frac{3x_t}{(x_t-1)^2}\ln x_t\right]$$

$$\begin{aligned} Z_0(x_t) \quad &= \quad -\frac{1}{9}\ln x_t + \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t-1)^3} + \\ &\quad + \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t-1)^4}\ln x_t. \end{aligned}$$

The set of gauge-independent short-distance functions, which govern FCNC process, is:

$$S_0(x_t), \quad X_0(x_t), \quad Y_0(x_t), \quad Z_0(x_t), \quad E_0(x_t), \quad D_0'(x_t), \quad E_0'(x_t)$$

Including gluon corrections, i.e. at NLO in QCD, some complications arise

UV divergences appear
(Wilson coefficients, being short-distance, cannot be IR divergent.
Possible IR divergences of the full theory appear in the
operator matrix elements within the effective theory)
(Quark field renormalizations cancel in matching.
Only the renormalization constant of the effective vertex is relevant)

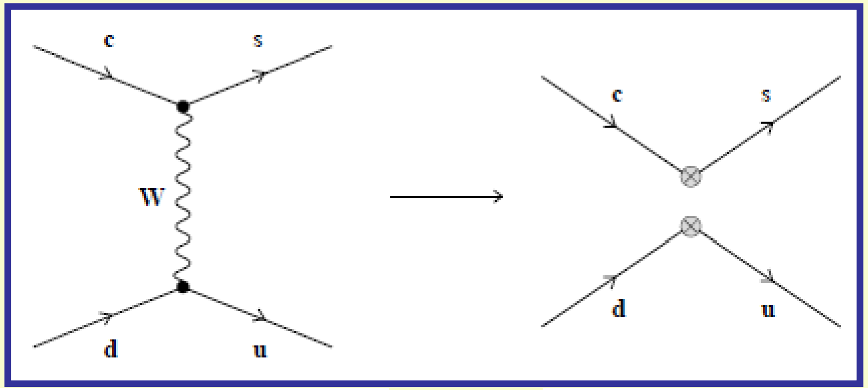
- **Regularization** (typically Dimensional Regularization, with some care in presence of γ_5: naïve, DRED, HV)

- **Renormalization** (typically $\overline{MS}$ scheme)

- **Resummation of large logs** through **Renormalization Group**

A simple example: $c \to s\, u\, \bar{d}$

At LO in QCD



$$A = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{M_W^2}{k^2 - M_W^2} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} \Rightarrow \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \mathcal{O}(\frac{k^2}{M_W^2})$$

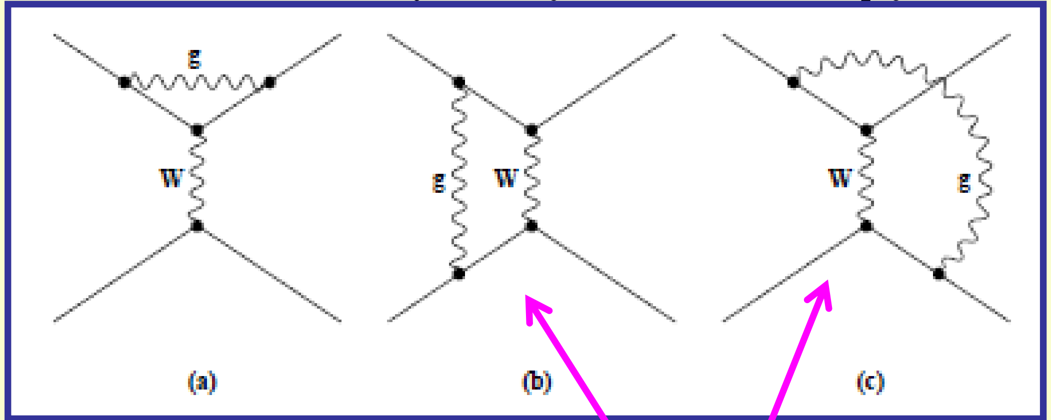
Already including the "i" to get H_{eff}

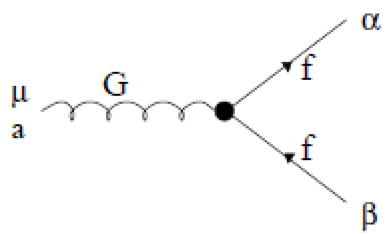
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \text{High D Operators}$$

Only one operator at LO

$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

At NLO in QCD (full theory)





$$T^a_{\alpha\beta}T^a_{\gamma\rho} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$$

$$-ig_s\gamma_\mu(T^a)_{\alpha\beta}$$

A second operator, Q_1, appears

$$Q_1 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}$$

$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}$$

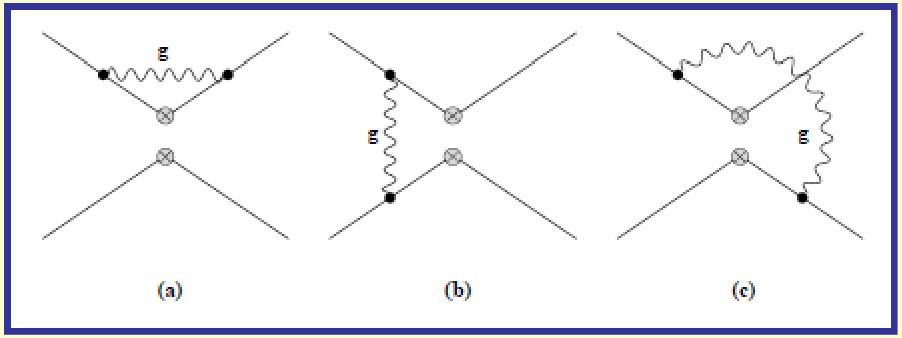
The calculation of A_{full} provides

$$A_{full} = \frac{G_F}{\sqrt{2}}V^*_{cs}V_{ud}\Big[\Big(1 + 2C_F\frac{\alpha_s}{4\pi}(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2})\Big)S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2}S_2 - 3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2}S_1\Big]$$

$$S_1 \equiv \langle Q_1\rangle_{tree} = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}$$

$$S_2 \equiv \langle Q_2\rangle_{tree} = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}$$

Within the effective theory (at NLO in QCD) with external states chosen as in the full theory (for simplicity: all p equal, null external quark masses)



The calculation of A_{eff} provides

$$
\begin{aligned}
\langle Q_1\rangle^{(0)} &= \left(1+2C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)\right)S_1+\frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_1 \\
&\quad -3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_2 \\
\langle Q_2\rangle^{(0)} &= \left(1+2C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)\right)S_2+\frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_2 \\
&\quad -3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon}+\ln\frac{\mu^2}{-p^2}\right)S_1
\end{aligned}
$$

Operator mixing at NLO

A closer look at divergences: RENORMALIZATION

Full theory

$$A_{full} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}\Big[\Big(1 + 2C_F\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2} S_2 - 3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2} S_1\Big]$$

Effective theory

$$\langle Q_1\rangle^{(0)} = \Big(1 + 2C_F\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big) S_1 - 3\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big) S_2$$

$$\langle Q_2\rangle^{(0)} = \Big(1 + 2C_F\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big) S_2 - 3\frac{\alpha_s}{4\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\Big) S_1$$

Cancelled by Z_q

$$q^0 = Z_q^{1/2} q$$

$$Z_q = 1 - \frac{\alpha_s}{4\pi} C_F \frac{1}{\varepsilon} + O(\alpha_s^2)$$

Require **renormalization of the effective vertices** (i.e. of Q_1 and Q_2)

$$Q_i^{(0)} = Z_{ij} Q_j, \qquad i,j = 1,2$$

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}$$

2x2 matrix due to mixing

after renormalization

$$\langle Q_1\rangle = \Big(1 + 2C_F\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2}\Big) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2} S_1 - 3\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2} S_2$$

$$\langle Q_2\rangle = \Big(1 + 2C_F\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2}\Big) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2} S_2 - 3\frac{\alpha_s}{4\pi}\ln\frac{\mu^2}{-p^2} S_1$$

From the matching condition

$$A_{full} = C_1(\mu)\langle Q_1(\mu)\rangle + C_2(\mu)\langle Q_2(\mu)\rangle$$

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}, \qquad C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}$$

Evidence for scale separation

$$\begin{aligned} A_{full} &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Big[\Big(1 + 2C_F \frac{\alpha_s}{4\pi} \cancel{\big(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2}\big)} \Big) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} S_2 \\ &\quad -3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} S_1 \Big] \end{aligned}$$

$$C_1(\mu) = -3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2} \,, \qquad C_2(\mu) = 1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2}$$

Schematic dependence on the scales

$$(1 + \alpha_s G \ln \frac{M_W^2}{-p^2}) \doteq (1 + \alpha_s G \ln \frac{M_W^2}{\mu^2}) \cdot (1 + \alpha_s G \ln \frac{\mu^2}{-p^2})$$

$$A_{full} \quad = \quad C(\mu) \quad \times \quad \langle Q(\mu) \rangle$$

RENORMALIZATION GROUP EQUATION

$$\vec{C}^T = (C_1, C_2)\ , \qquad \vec{Q}^T = (Q_1, Q_2)$$

$$\vec{C}^{(0)} = \hat{Z}_c \vec{C} \qquad \vec{Q}^{(0)} = \hat{Z}\vec{Q} \xrightarrow{\vec{C}^{(0)T}\vec{Q}^{(0)} \equiv \vec{C}^T\vec{Q}} \hat{Z}_c^T = \hat{Z}^{-1}$$

RG equation:

$\vec{C}^{(0)}$ does not depend on μ

$$\frac{d\vec{C}^{(0)}}{d\ln\mu} = 0$$

$$\frac{d\vec{C}(\mu)}{d\ln\mu} = \hat{\gamma}^T(\alpha_s)\vec{C}(\mu)$$

$$\hat{\gamma} = \hat{Z}^{-1}\frac{d\hat{Z}}{d\ln\mu}$$

$$\vec{C}(\mu) = \hat{U}(\mu, M_W)\vec{C}(M_W)$$

$$\hat{U}(\mu, M_W) = \exp\left[\int_{g(M_W)}^{g(\mu)} dg' \frac{\hat{\gamma}^T(g')}{\beta(g')}\right]$$

$\beta(g) = dg/d\ln\mu$

α_s adim $\mu^{2\varepsilon}$

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}$$

$$\hat{\gamma}(\alpha_s) = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} = \frac{\alpha_s}{4\pi}\begin{pmatrix} -6/N & 6 \\ 6 & -6/N \end{pmatrix}$$

Evolution matrix at LO

$$\hat{U}(\mu, M_W) = \exp\left[\int_{g(M_W)}^{g(\mu)} dg' \frac{\hat{\gamma}^T(g')}{\beta(g')}\right]$$

$$\beta(g) = -\beta_0 g^3/16\pi^2$$

$$\hat{\gamma}(\alpha_s) = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} = \frac{\alpha_s}{4\pi}\begin{pmatrix} -6/N & 6 \\ 6 & -6/N \end{pmatrix}$$

$$\alpha_s(\mu) = \frac{\alpha_s}{1+(\beta_0\alpha_s/4\pi)\ln(\mu^2/M_{ren}^2)}$$

$$\hat{U}^{(0)}(\mu, M_W) = \hat{V}\left(\boxed{\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]}^{\frac{\vec{\gamma}^{(0)}}{2\beta_0}}\right)_D \hat{V}^{-1}$$

resumming large logs

$$\hat{V} = \hat{V}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Rotation matrix which diagonalizes $\vec{\gamma}^{(0)}$ (i.e. removing mixing)

$$\vec{C}(\mu_c) = \hat{U}^{(f=4)}(\mu_c, \mu_b)\hat{U}^{(f=5)}(\mu_b, \mu_W)\vec{C}(\mu_W)$$

$$\frac{\alpha_s(\mu)_f}{4\pi} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2_{f\overline{MS}})}$$

Evolution crosses quark mass thresholds

$\Lambda_{QCD} \rightarrow \alpha_s \rightarrow U$ depend on the active flavours f

GOING TO HIGHER ORDERS

- It is important to reduce the μ-dependence

- It follows the same procedure with some complications (more difficult diagrams, $1/\varepsilon^2$ divergences appear requiring additional *evanescent* operators ($\rightarrow 0$ for $\varepsilon \rightarrow 0$)

- Nowadays most of the Wilson coefficients are known at NNLO

Wilson coefficients: Theoretical Status (within the SM)

M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 [arXiv:hep-ph/9808385].

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In the '90s basically the NLO QCD corrections to all relevant decays and transitions have been calculated (for $\Delta\Gamma_{d,s}$ in 2003)

In the last decade, NNLO QCD corrections to several flavour processes have been calculated

A. J. Buras, M. Gorbahn, U. Haisch and U. Nierste, "Charm quark contribution to $K^+ \to \pi^+ \nu\bar{\nu}$ at next-to-next-to-leading order," JHEP **0611** (2006) 002 [arXiv:hep-ph/0603079].

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C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, arXiv:hep-ph/0312090.

A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, Nucl. Phys. B **648** (2003) 254 [arXiv:hep-ph/0208088].

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M. Beneke, T. Feldmann and D. Seidel, NLO," Eur. Phys. J. C **41** (2005) 173 [arXiv:hep-ph/0412400].