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Workshop on Cosmic Rays and Cosmic Neutrinos: Looking at the Neutrino Sky

20 - 24 June 2011

Quantifying uncertainties in the high energy neutrino cross-section

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The high energy neutrino cross-section in the Standard Model and its uncertainty

Philipp Mertsch

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NUSKY 2011, ICTP, Trieste
24 June 2011

Science & Technology Facilities Council

UNIVERSITY OF OXFORD

Why do we want predictions for the ν cross-section?

ν astronomy

- want to measure flux J_ν
- event rate $R \propto J_\nu \, \sigma_\nu$
- even considering the attenuation: $R/J_\nu \propto \sigma_\nu^{0.45}$

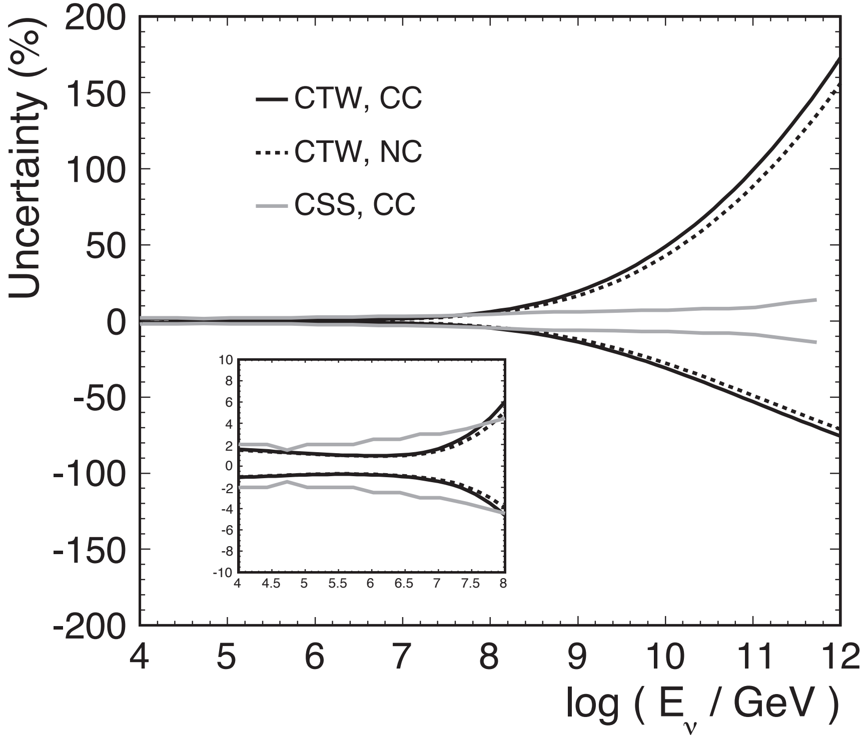
particle physics

- want to measure cross-section σ_ν
- test standard model at c.m. energies up to $\sim 10^3$ TeV: gluon saturation, colour glass condensate, black holes?

- there are a few observables which are independent of σ
  $\rightarrow$ measure flux **and** cross-section

Kusenko and Weiler, PRL **88**, 161101 (2002); Anchordoqui *et al.*, PRD **74** (2006) 043008

How accurately can we predict the ν cross-section?

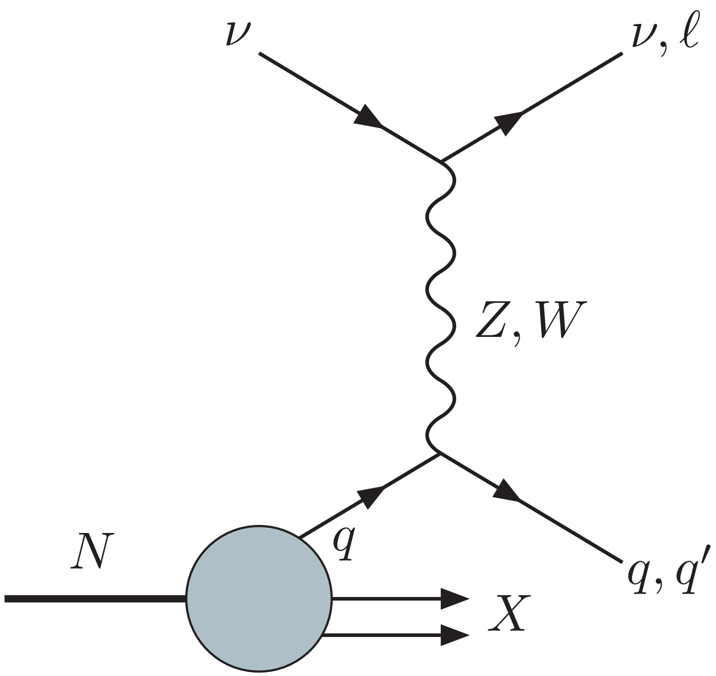


CSS: Cooper-Sarkar and Sarkar, JHEP **01** (2008) 075

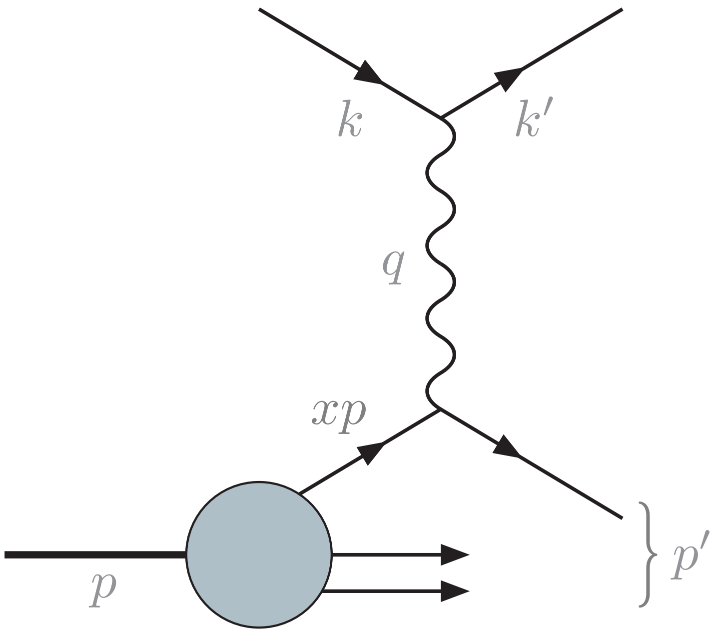
CTW: Connolly *et al.*, arXiv:1102.0691

Does the uncertainty **really** blow up to $\mathcal{O}(1)$?

DIS



DIS



four Lorentz invariants:

- centre of mass energy $\sqrt{s}$
  $$s = (p + k)^2$$
- momentum transfer
  $$Q^2 = -q^2 = -(k - k')^2$$
- Bjorken scaling variable
  $$x = Q^2/(2p \cdot q)$$
- inelasticity
  $$y = p \cdot q/(p \cdot k)$$

ν cross-section

Double differential cross-section

$$\frac{\mathrm{d}^2\sigma(\nu(\bar{\nu})N)}{\mathrm{d}x\ \mathrm{d}Q^2} = \frac{G_\mathrm{F}^2 M_W^4}{4\pi(Q^2 + M_W^2)^2 x}\sigma_\mathrm{r}(\nu(\bar{\nu})N)$$

with reduced cross-section

$$\sigma_\mathrm{r}(\nu(\bar{\nu})N) = \left[Y_+ F_2^\nu(x, Q^2) - y^2 F_\mathrm{L}^\nu(x, Q^2) \pm Y_- x F_3^\nu(x, Q^2)\right]$$

where $Y_\pm = 1 \pm (1-y)^2$.

Total cross-section

$$\sigma = \int \mathrm{d}x \int \mathrm{d}Q^2 \, \frac{\mathrm{d}^2\sigma(\nu(\bar{\nu})N)}{\mathrm{d}x\ \mathrm{d}Q^2}$$

LO (quark parton model)

$q(x)$: probability density for quark q with momentum fraction x

structure functions

combination of quark PDFs,

$$
\begin{aligned}
F_2^\nu &= x(u + d + 2s + 2b + \bar{u} + \bar{d} + 2\bar{c})\,, \\
F_L^\nu &= 0\,, \\
xF_3^\nu &= x(u + d + 2s + 2b - \bar{u} - \bar{d} - 2\bar{c})\,,
\end{aligned}
$$

and similar for $\bar{\nu}$.

NLO

$q(x, Q^2)$ now scale-dependent, **no** probability density

structure functions

$$F_2 = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \left[\sum_i e_i^2 \, x q_i(\xi, Q^2) C_q \left(\frac{x}{\xi}, \alpha_s \right) + \left(\sum_i e_i^2 \right) x g(\xi, Q^2) C_g \left(\frac{x}{\xi}, \alpha_s \right) \right]$$

$$F_L = \ldots$$

$$x F_3 = \ldots$$

C_q, C_g: coefficient functions

PDF fitting: idea

problem

- ideally, would like to calculate PDFs from first principles
- however, interactions of partons are soft ($Q^2 \lesssim \Lambda^2_{\mathrm{QCD}}$)

$\rightarrow$ non-perturbative regime

- lattice?

DGLAP evolution

- however, can calculate the evolution of PDFs in the perturbative regime ($Q^2 \gg \Lambda^2_{\mathrm{QCD}}$)
- assume parametric form at input scale and evolve to other scale

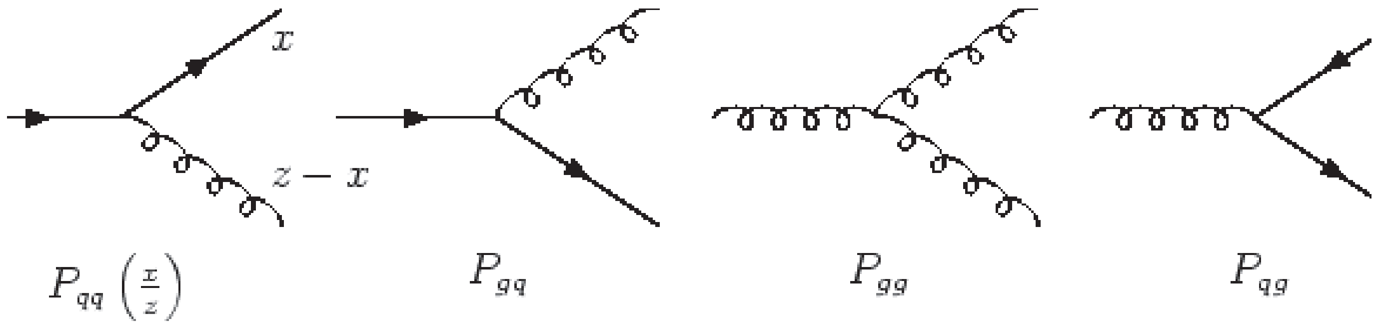
DGLAP evolution

$$\frac{\partial q^{\mathsf{NS}}(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \left(q^{\mathsf{NS}} \otimes P_{qq} \right)$$

$$\frac{\partial \Sigma(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \left(\Sigma \otimes P_{qq} + g \otimes 2 n_f P_{qg} \right)$$

$$\frac{\partial \Sigma(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \left(\Sigma \otimes P_{gq} + g \otimes 2 n_f P_{gg} \right)$$

Σ and q^{NS} are convenient linear combinations of quark PDFs.



PDF fitting: procedure

- chose parametrisation at input scale Q_0^2, e.g.

$$xg = x^{\lambda_g}(1-x)^{\eta_g} P_g(x)$$
$$xS = x^{\lambda_S}(1-x)^{\eta_S} P_S(x)$$
$$\ldots$$

- evolve to scale of measurement: $Q_0^2 \to Q^2$
- calculate F_2, F_L and xF_3 functions and (differential) cross sections
- determine parameters λ_i, η_i, $P_i(x)$ by fitting to data

PDF errors

experimental uncertainties

- many experimental errors correlated
- correlation matrix diagonalised

$\rightarrow$ linearly independent eigenvectors = variations of best-fit PDF

- can add errors from eigenvectors in quadrature

model/parameter uncertainities

- some parameters/model assumptions get fixed before fit
- vary these parameters within c.l. interval

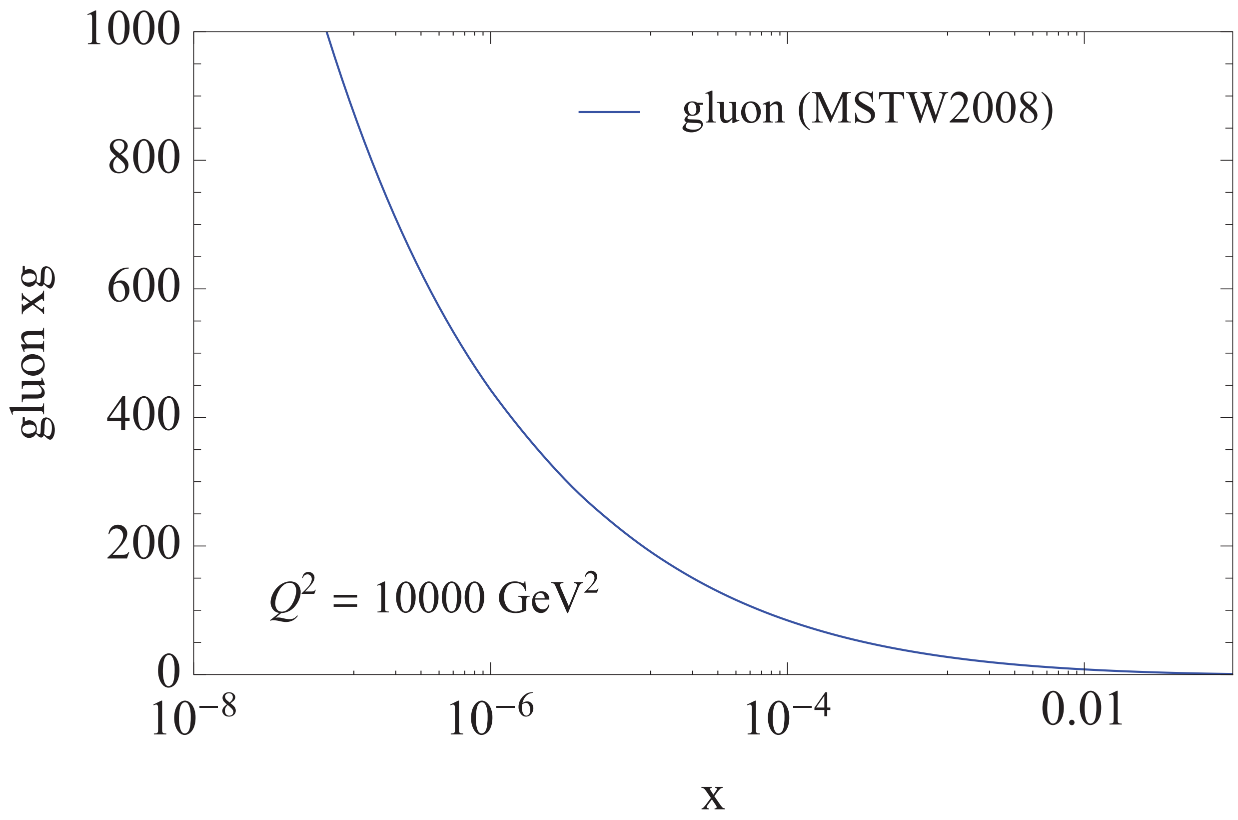
$\rightarrow$ variations of best-fit PDF

α_s uncertainties

- α_s determines how quickly PDFs rise at low x

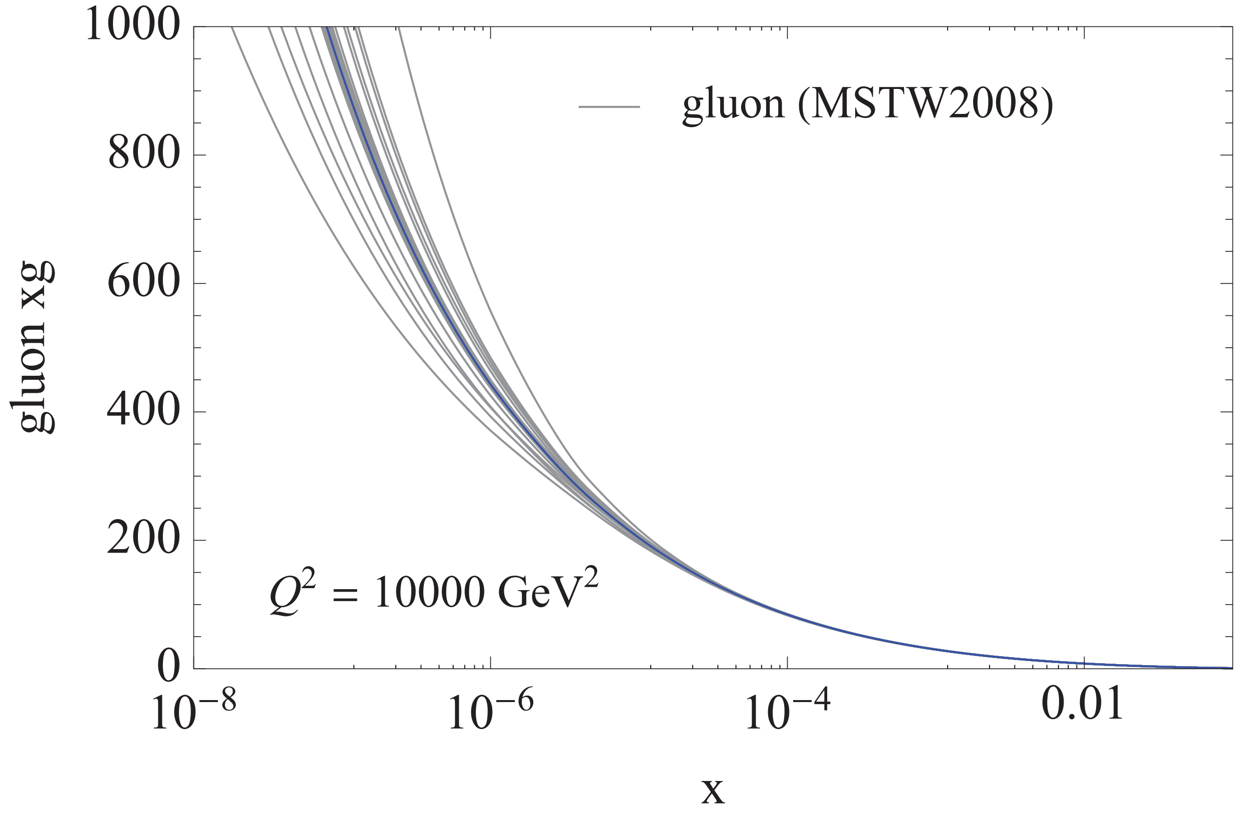
$\rightarrow$ possibly large effect

PDF errors



1000
800
600
400
200
0
gluon xg
gluon (MSTW2008)
Q^2 = 10000 GeV^2
10^{-8}
10^{-6}
10^{-4}
0.01
x

PDF errors



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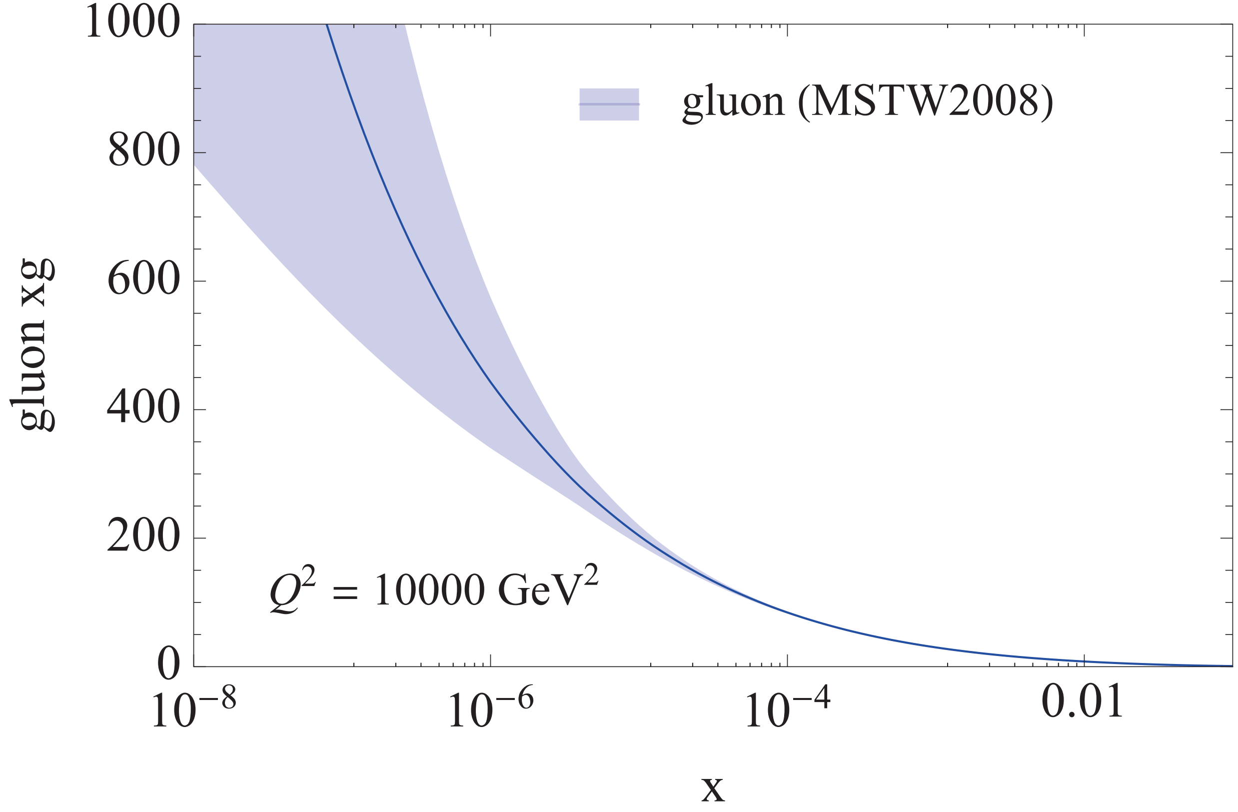

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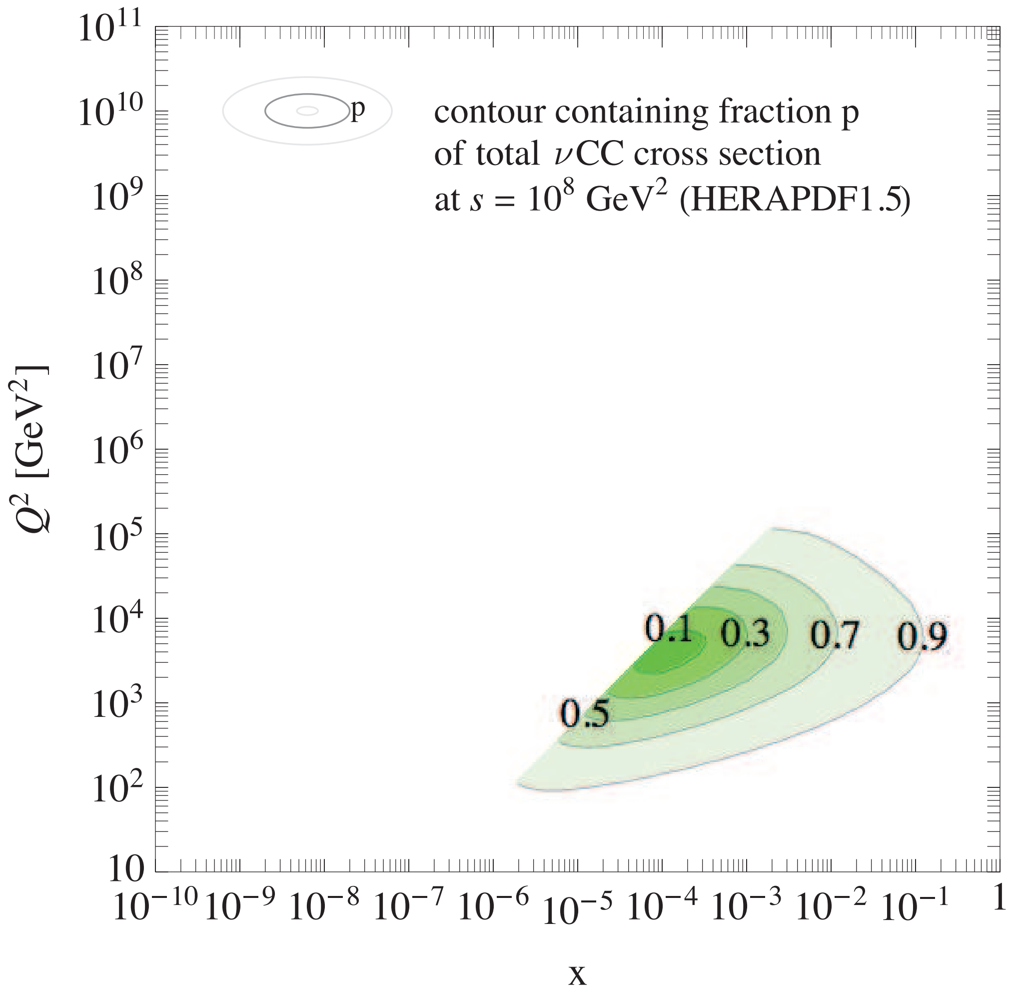
A detailed comparison

- use only up-to-date PDFs:
  - HERAPDF1.5 ✓
  - CT10 ✓
  - MSTW2008 ✗ (does not include combined HERA data)
- work **consistently** at NLO
- use only publicly available tools (e.g. LHAPDF)
- highlight different contributions to uncertainty **within DGLAP**:
  - experimental
  - parameters
  - model

The kinematic range



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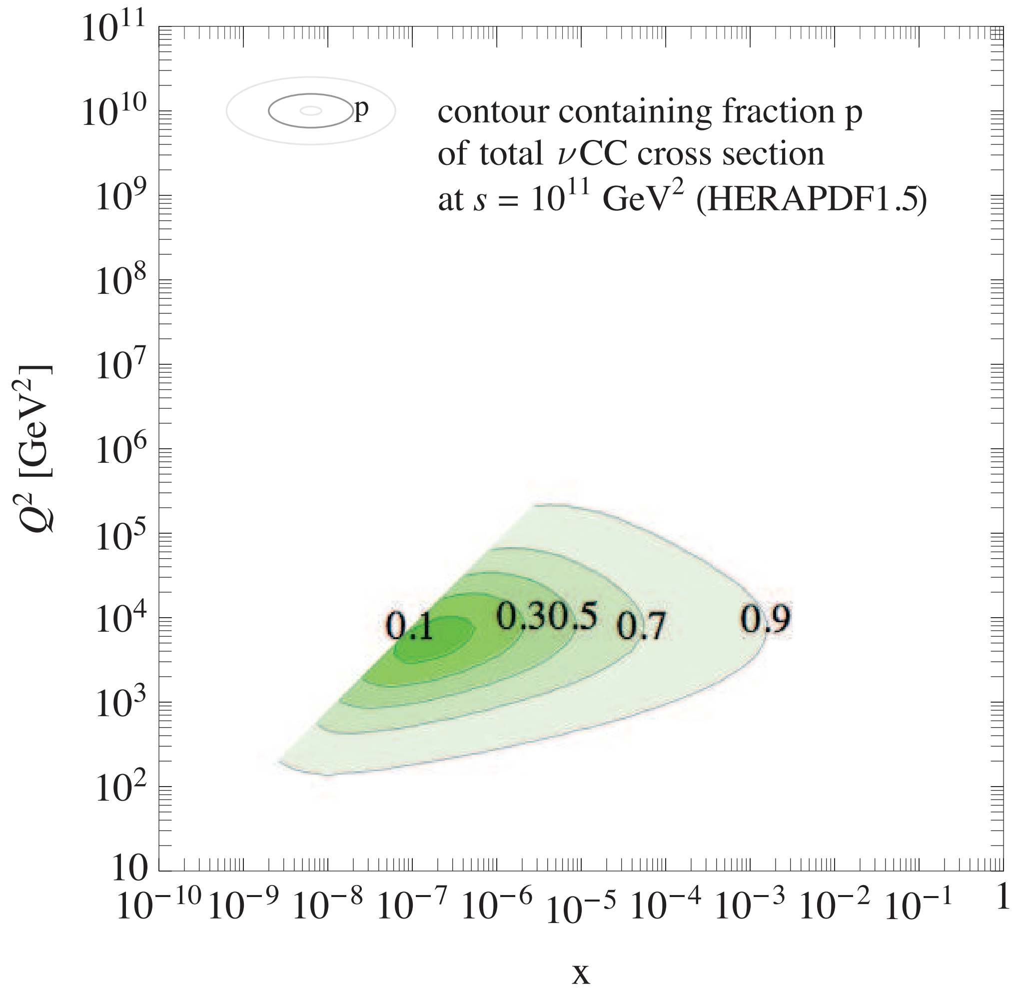


Pitfalls

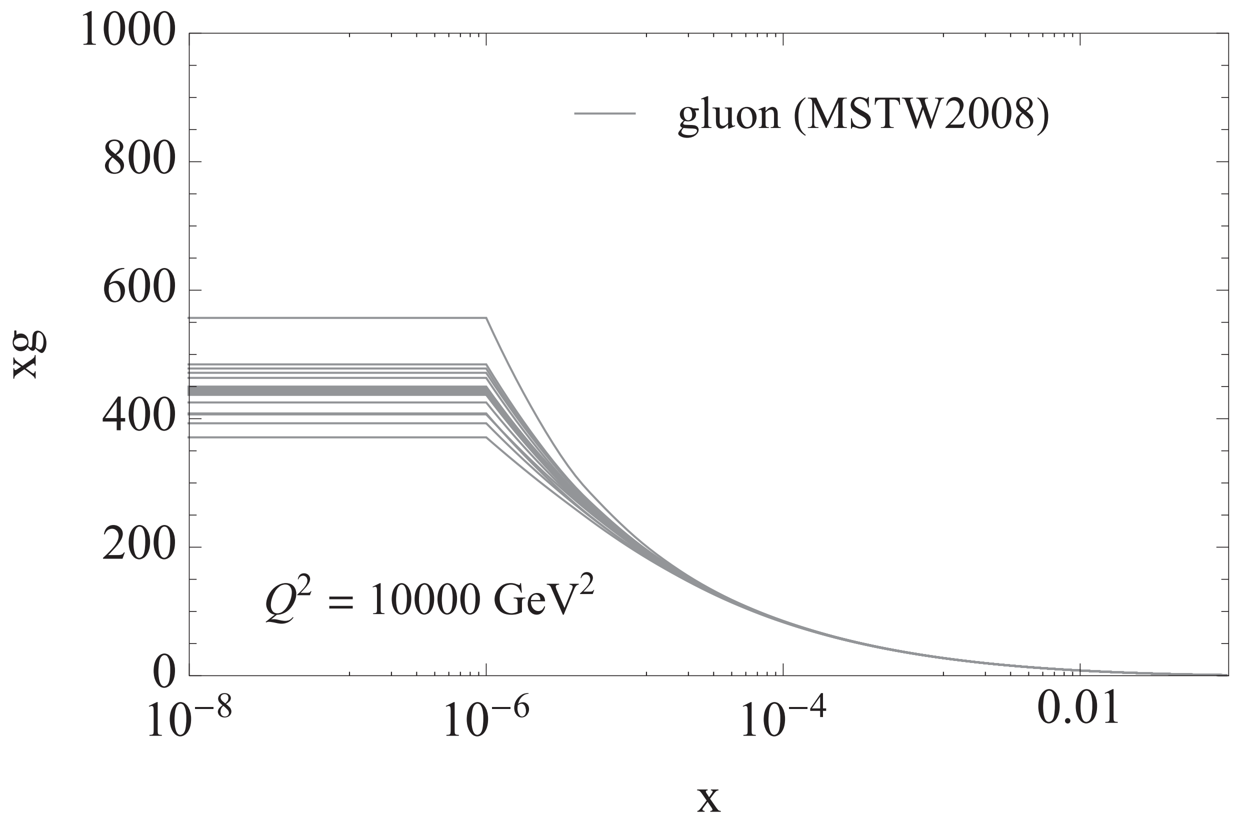
event generators, e.g. PYTHIA

- are for the most part LO
- using NLO PDFs: inconsistent ✗

LHAPDF

- PDFs provided on a limited grid of points (x, Q^2)
- going beyond this grid: PDFs "freeze" ✗

Pitfalls



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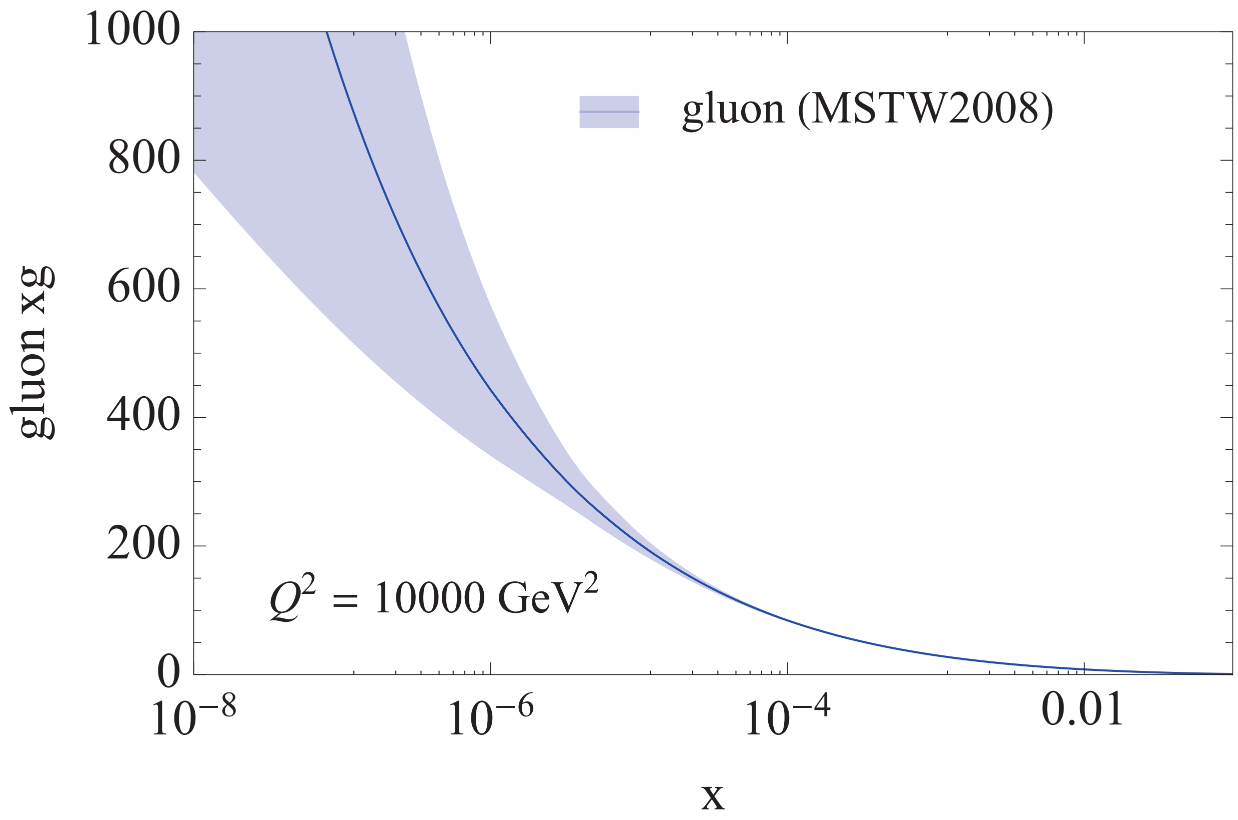
gluon parametrisation

- some groups choose a general parametrisation
- gluon PDF can go negative: meaning?

Example: MSTW2008 gluon momentum distribution



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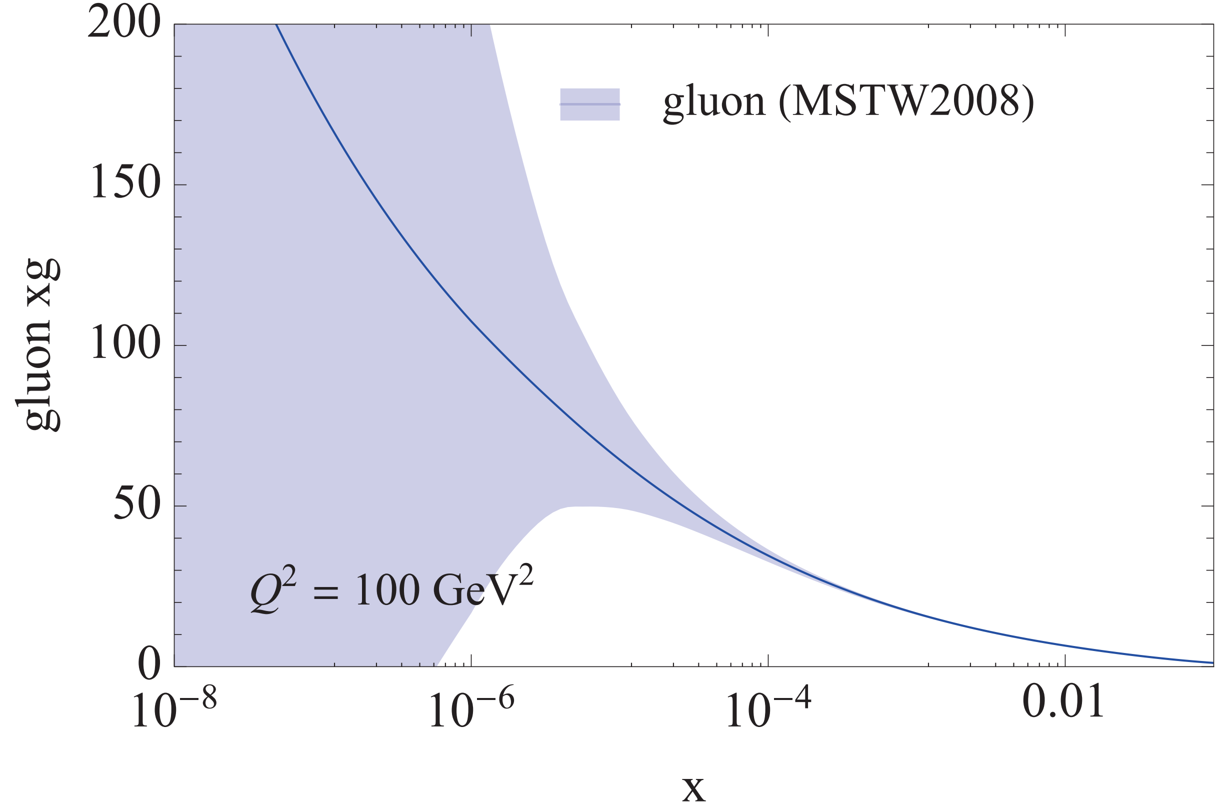
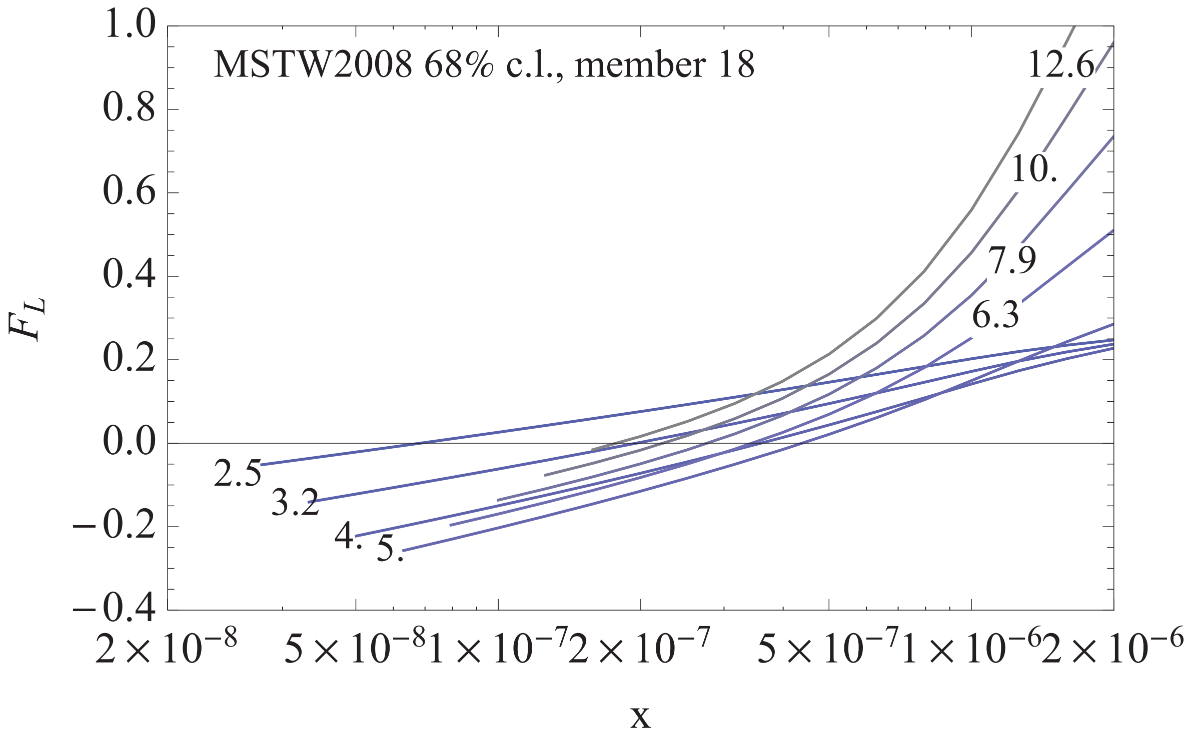


Could the gluon become negative?

at NLO, the gluon **could** become negative

however longitudinal structure function F_L **must** stay positive



With MSTW2008, F_L **does** go negative!

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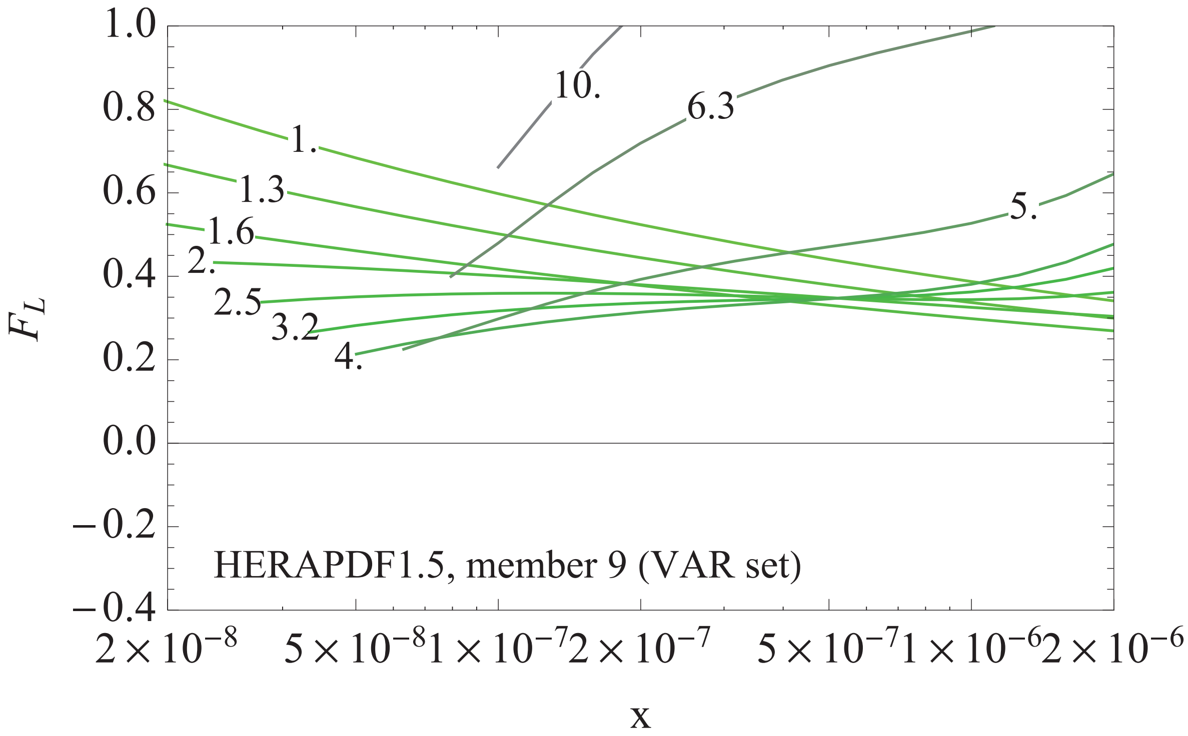
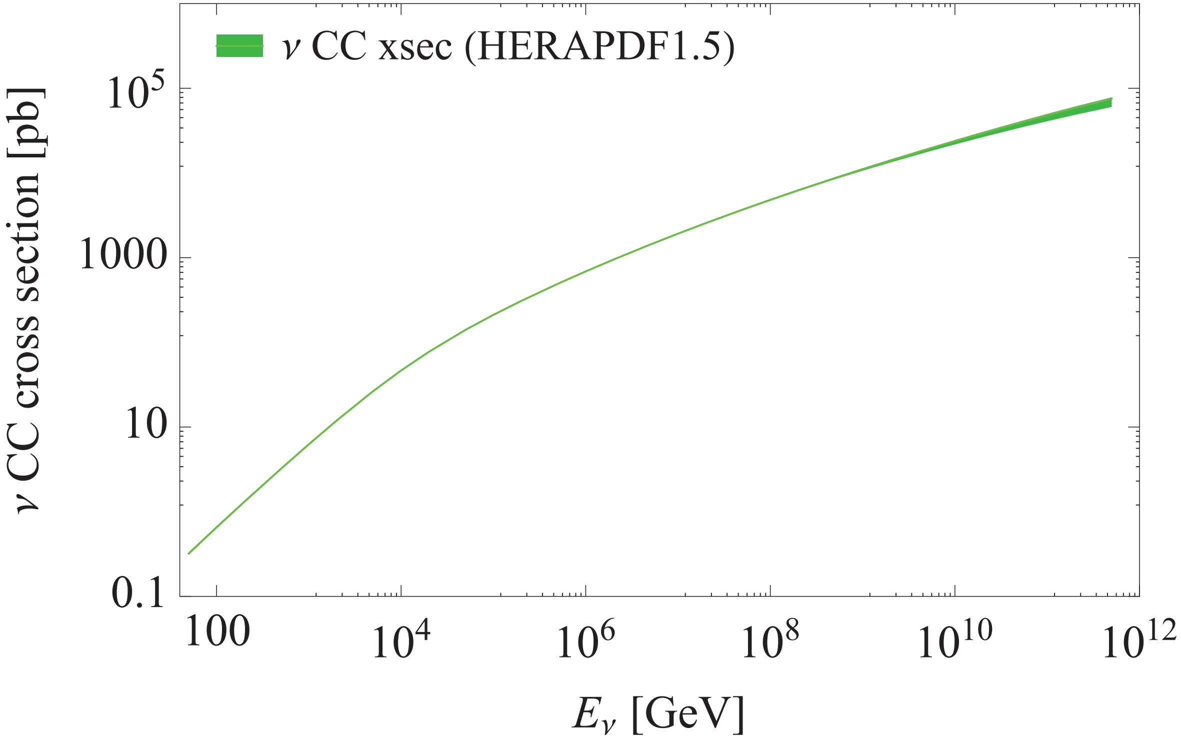
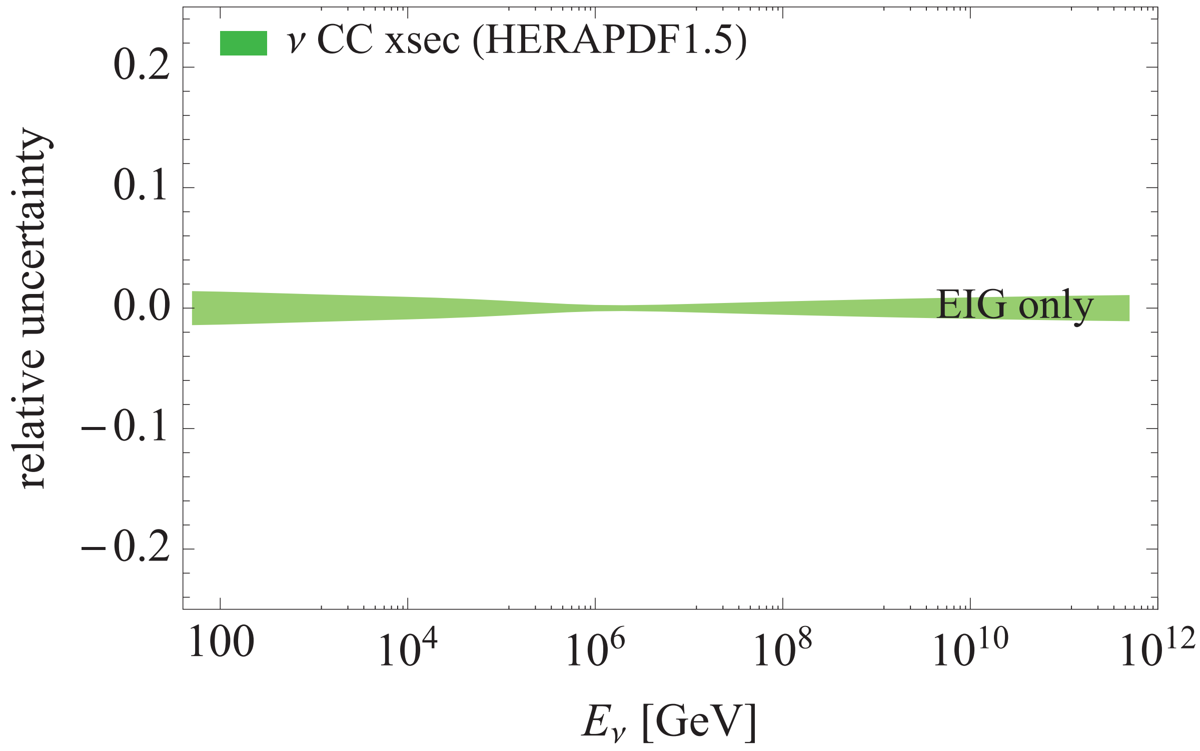


With HERAPDF1.5, F_L **does** stay positive!

Total ν CC cross-section (HERAPDF1.5)



ν CC cross-section uncertainty (HERAPDF1.5)



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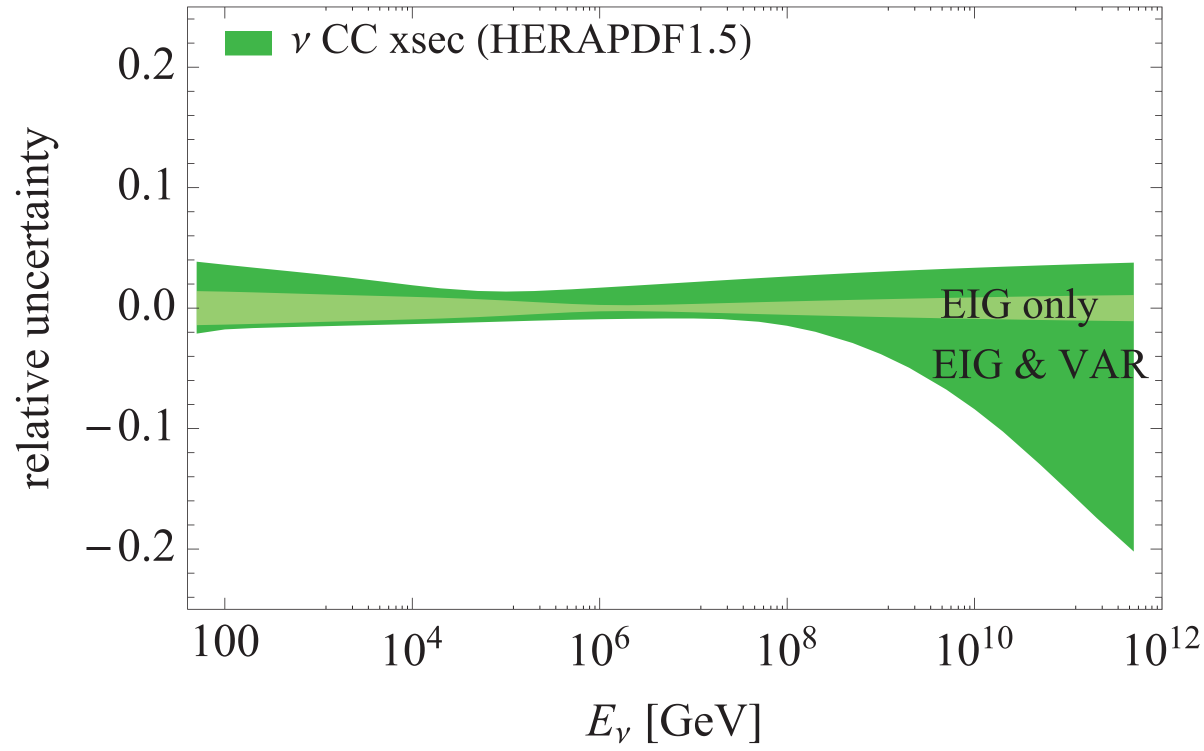

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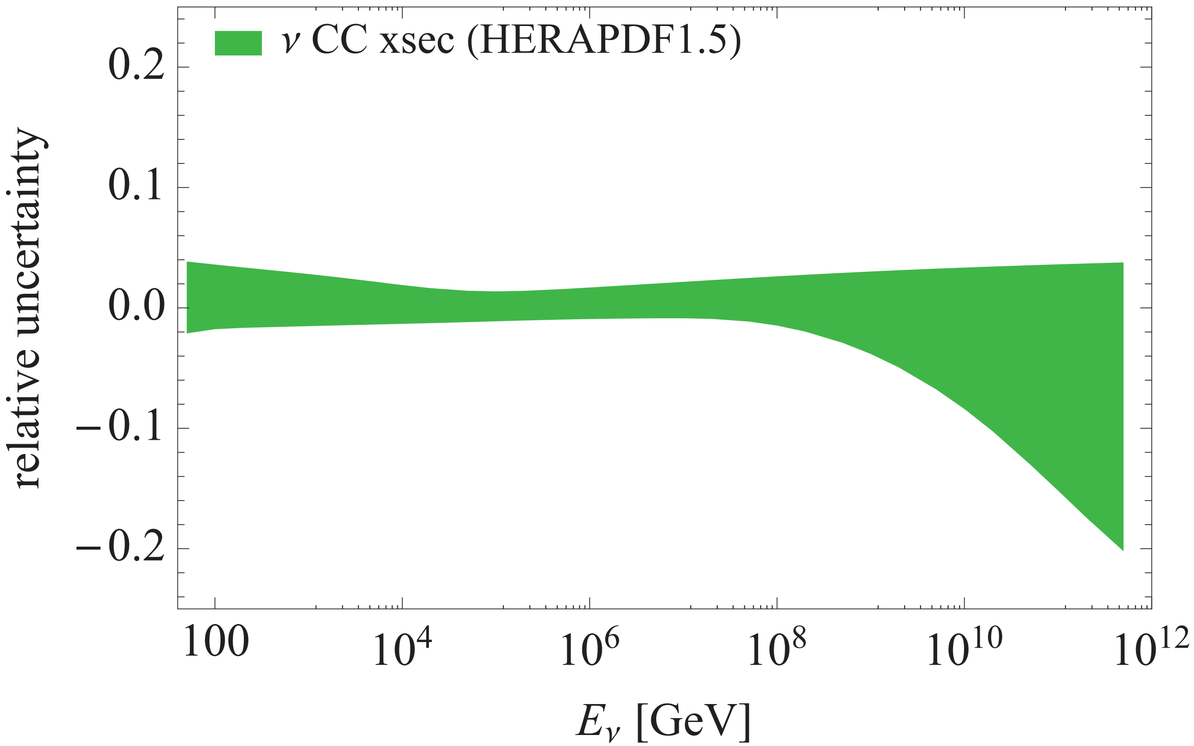

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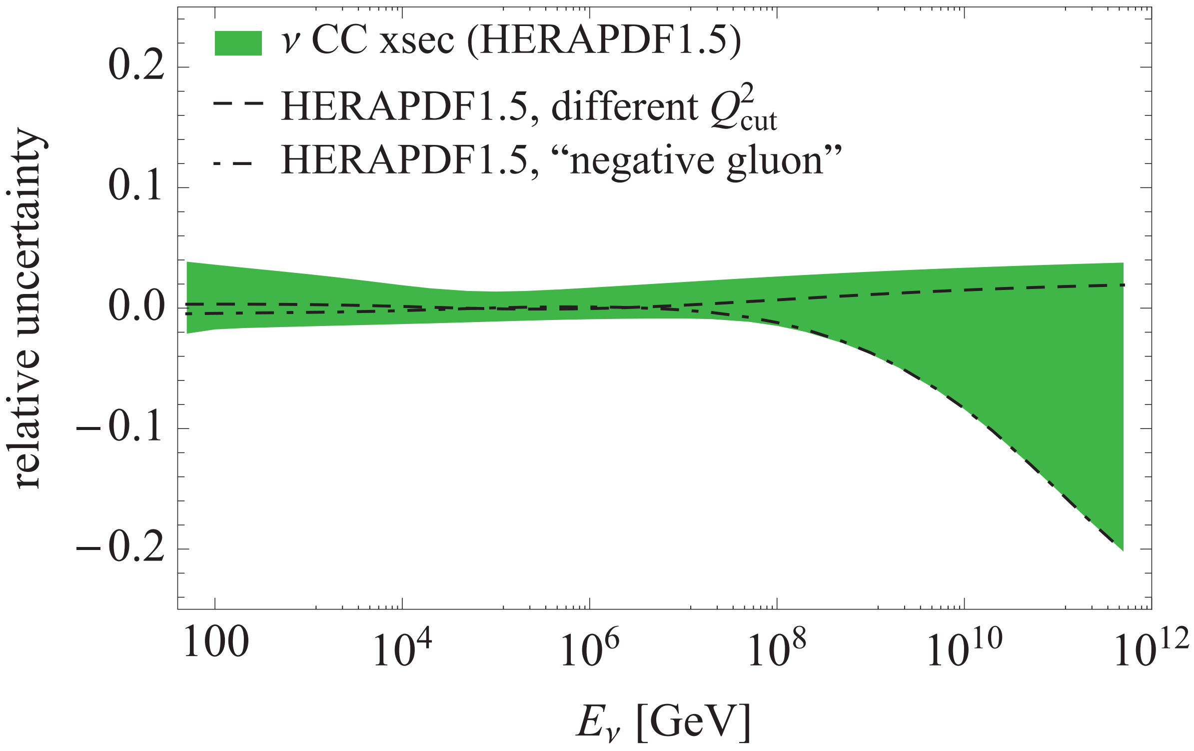

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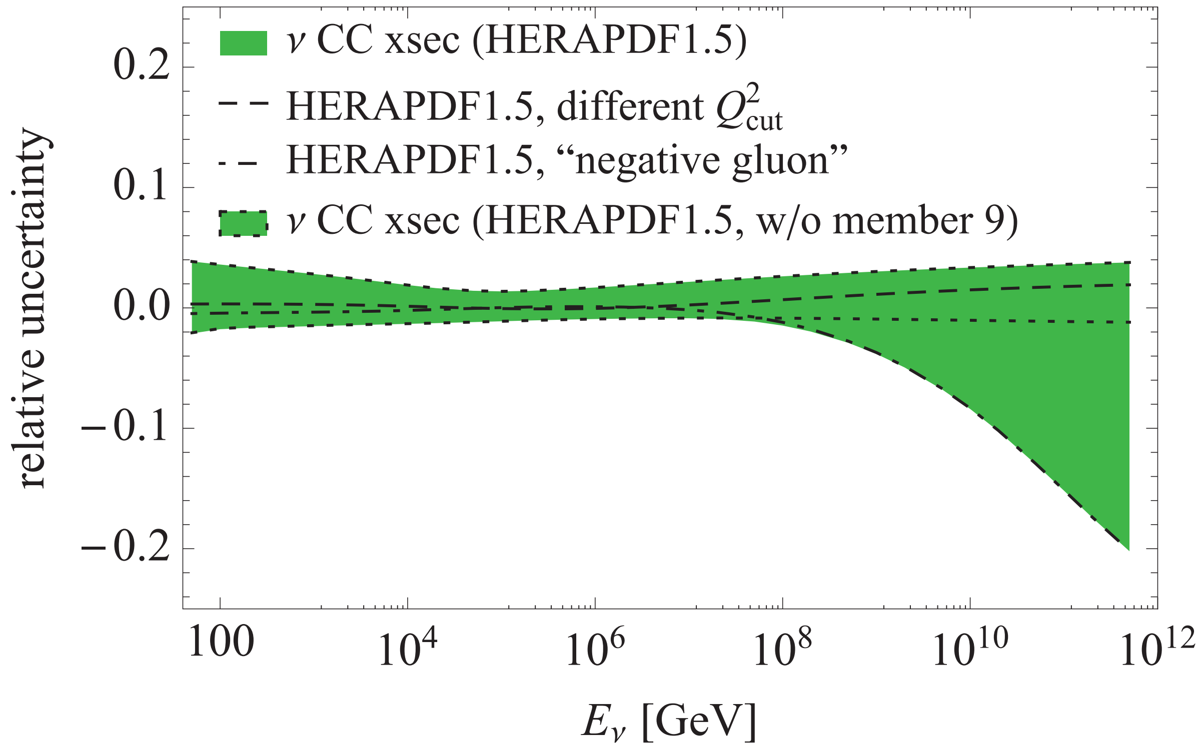
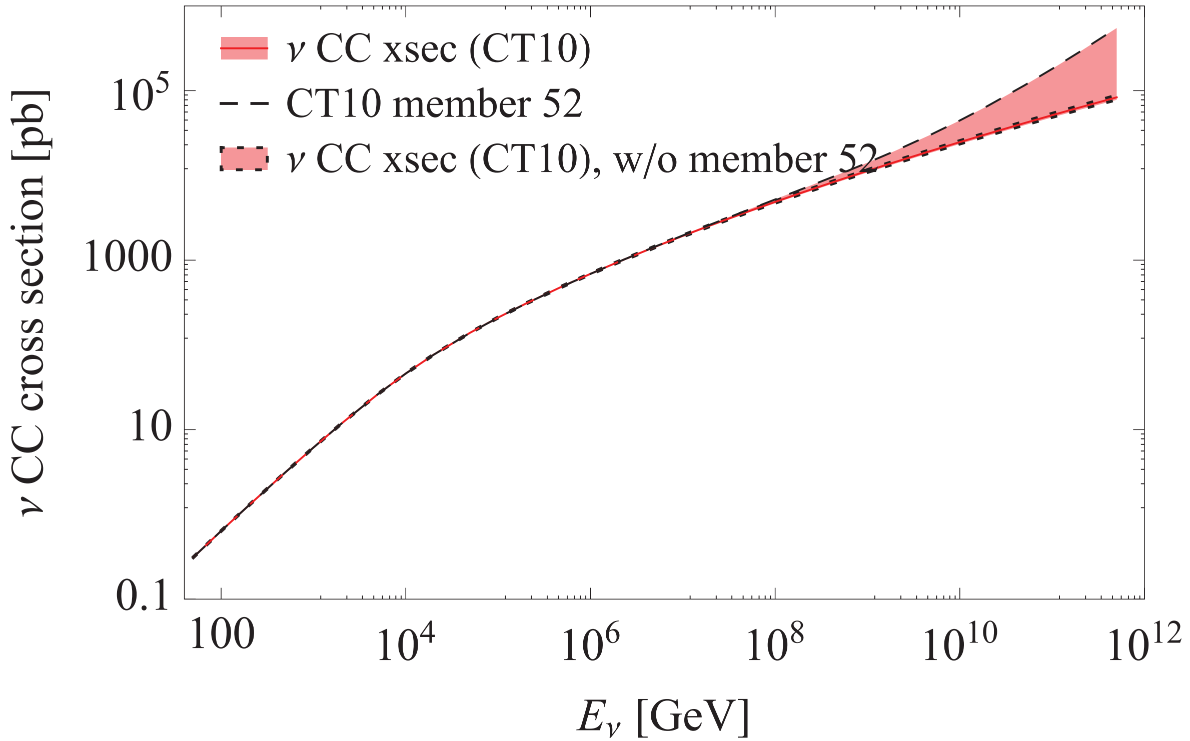
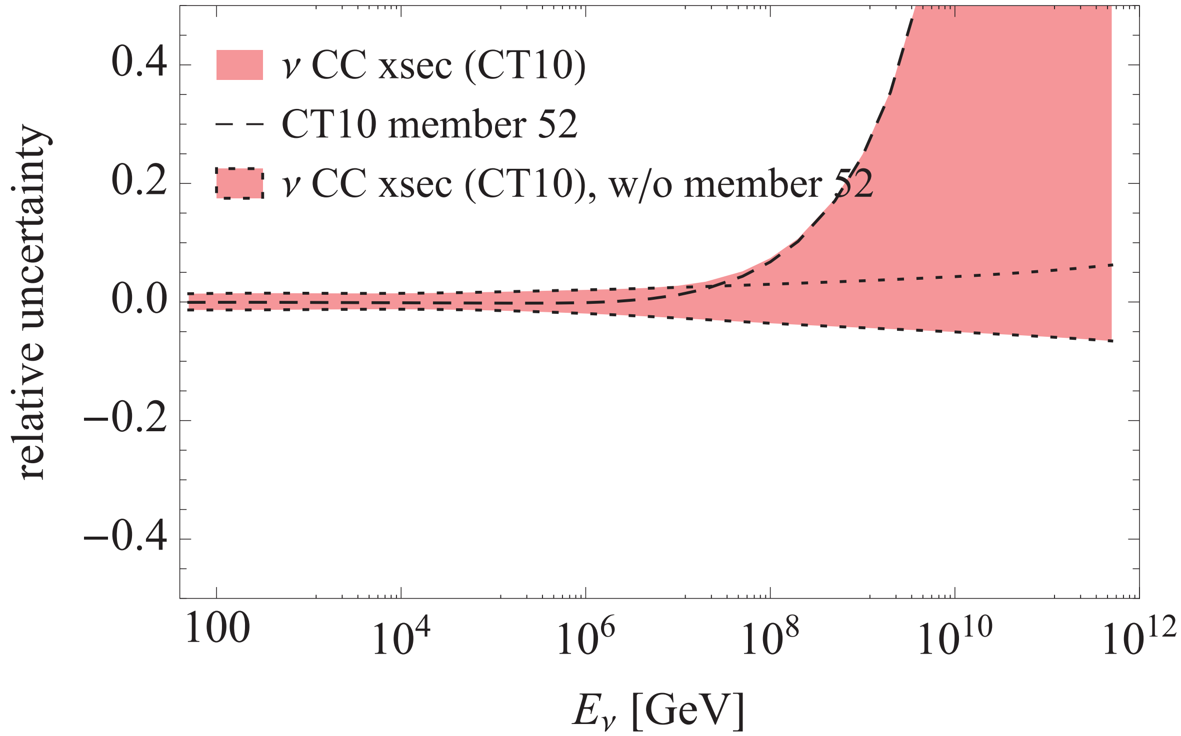


Total ν CC cross-section (CT10)

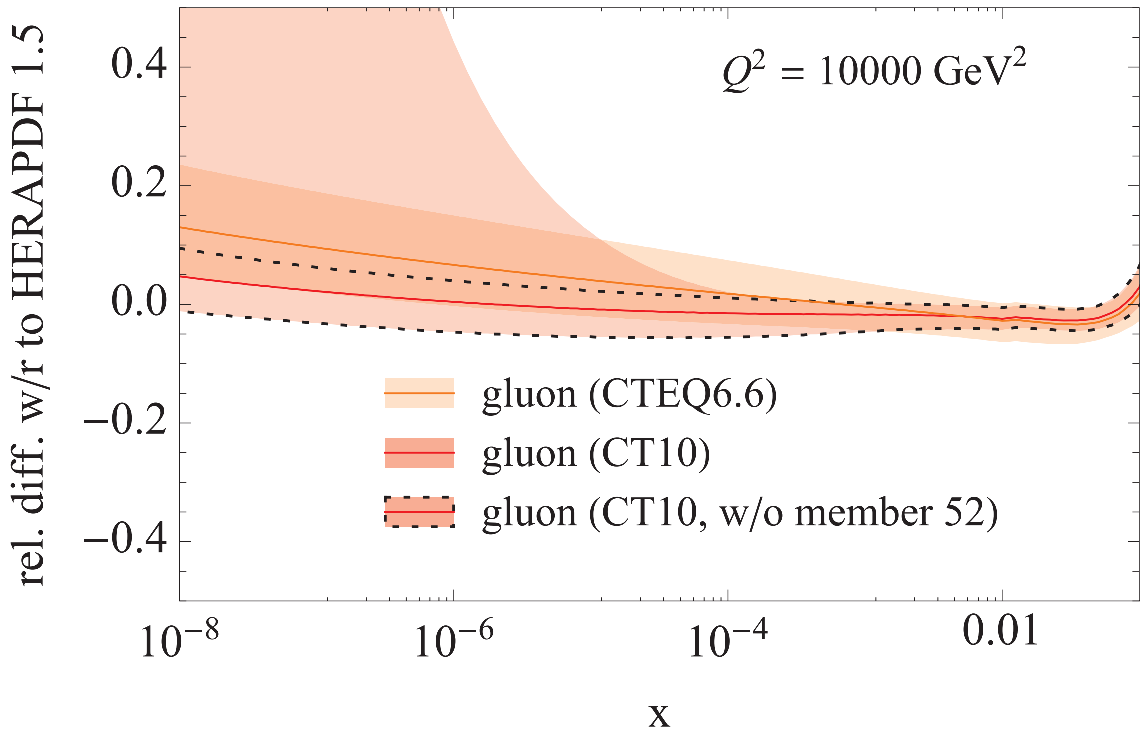


cross-section for member 52 rises $\propto E_\nu^{0.7}$; central member $\propto E_\nu^{0.3}$

ν CC cross-section uncertainty (CT10)

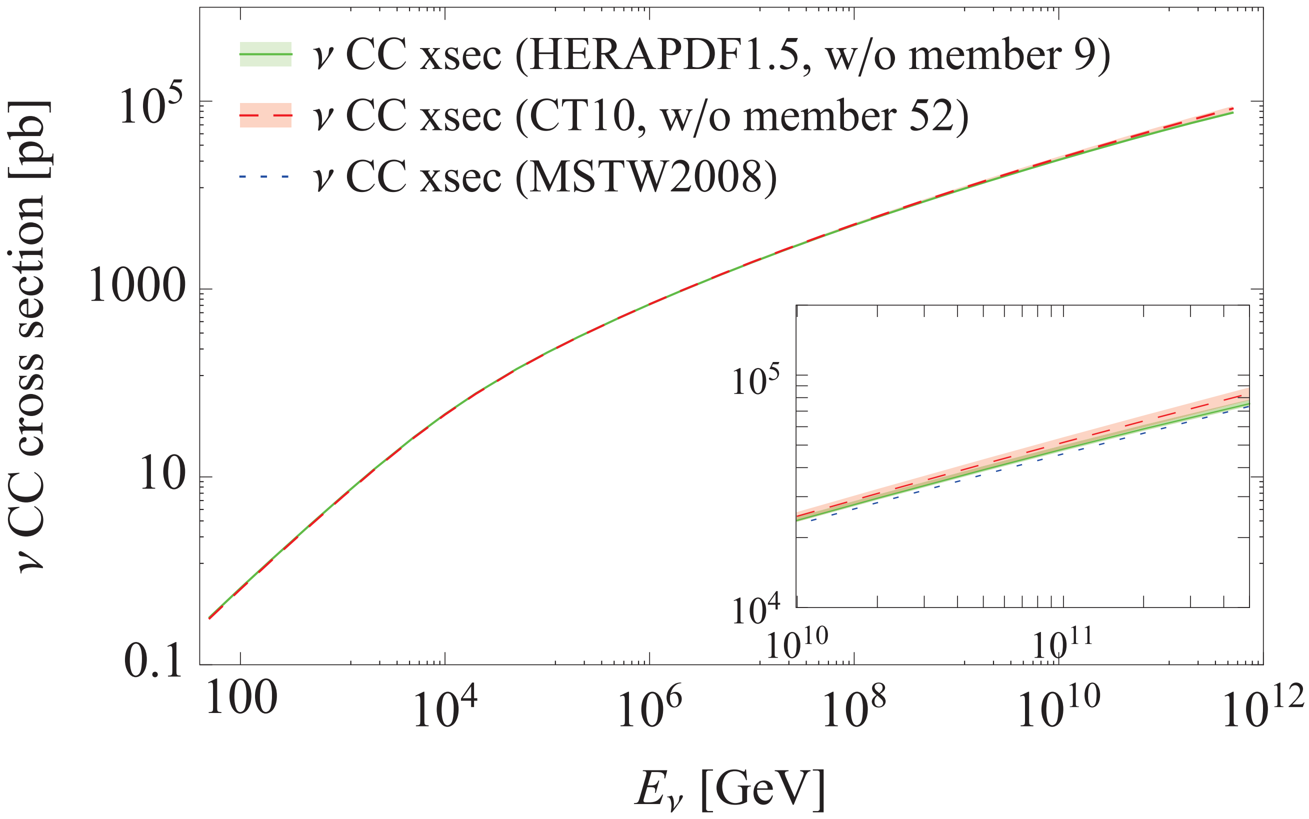


member 52 of CT10

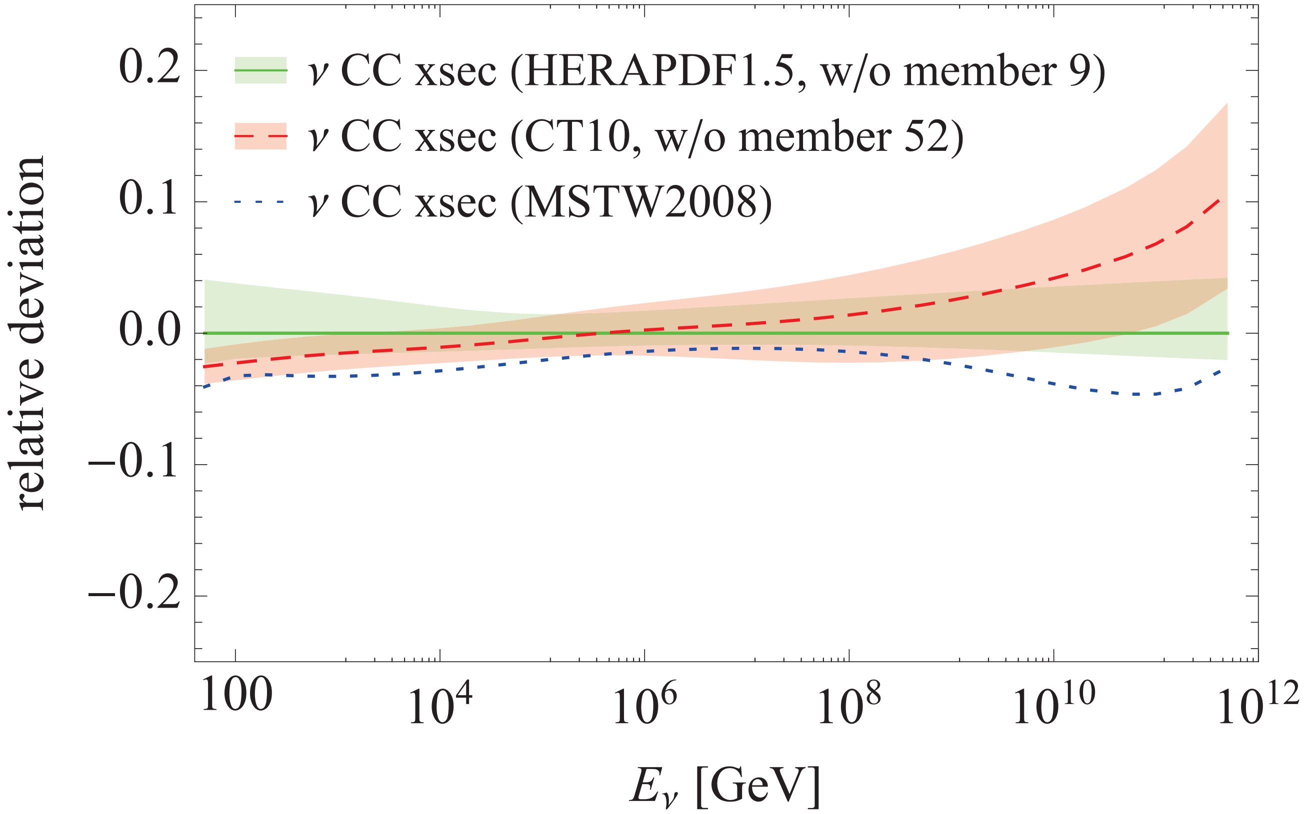


member 52 put in by hand

ν CC cross-section (excluding rogue members)



ν CC cross-section uncertainty (excluding rogue members)



Using these results . . .

more details in our paper

Cooper-Sarkar, Mertsch and Sarkar, arXiv:1106.3723

- more results: ν and $\bar{\nu}$, CC and NC
- comparison with event generators and other calculations
- tabulated total cross sections and uncertainties

future plans

updating the widely used event generator `ANIS` with the new cross-sections

A. Gazizov, M. P. Kowalski, Comput. Phys. Commun. **172** (2005) 203.

gluon at low x

$\ln 1/x$ resummation

- DGLAP contains terms $\sim (\alpha_s \ln x_0/x)^n$
- at low x this becomes larger than 1

$\rightarrow$ need to resumm $\ln 1/x$ terms

Froissart bound

- DGLAP predicts $xg \propto x^{-\delta}$ at low x

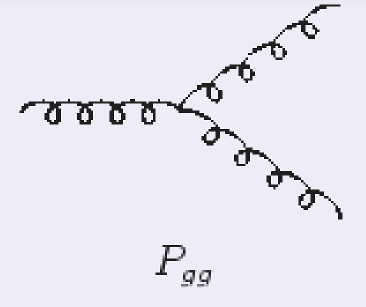
$\rightarrow$ $\sigma \propto s^{\delta}$ at large s

- however, unitarity demands s, $\sigma \propto (\ln s/s_0)^2$ at most

non-linear effects

- DGLAP eqns. are linear
- however, in DGLAP gluon and sea quark density large at small x

$\rightarrow$ gluon saturation? gluon recombination?



$\rightarrow$ would tame the rise of σ

example

colour glass condensate

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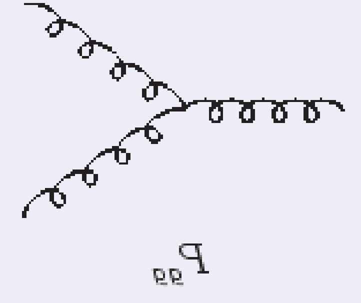

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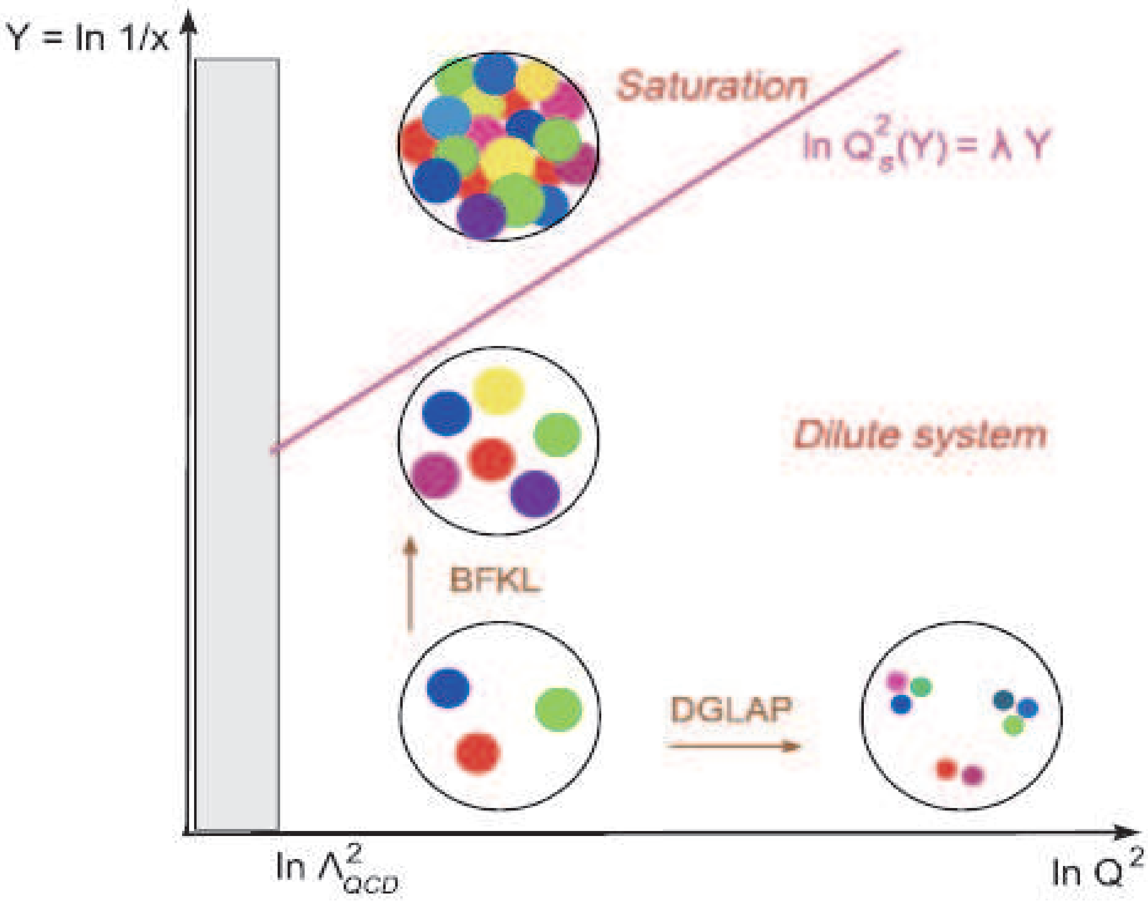


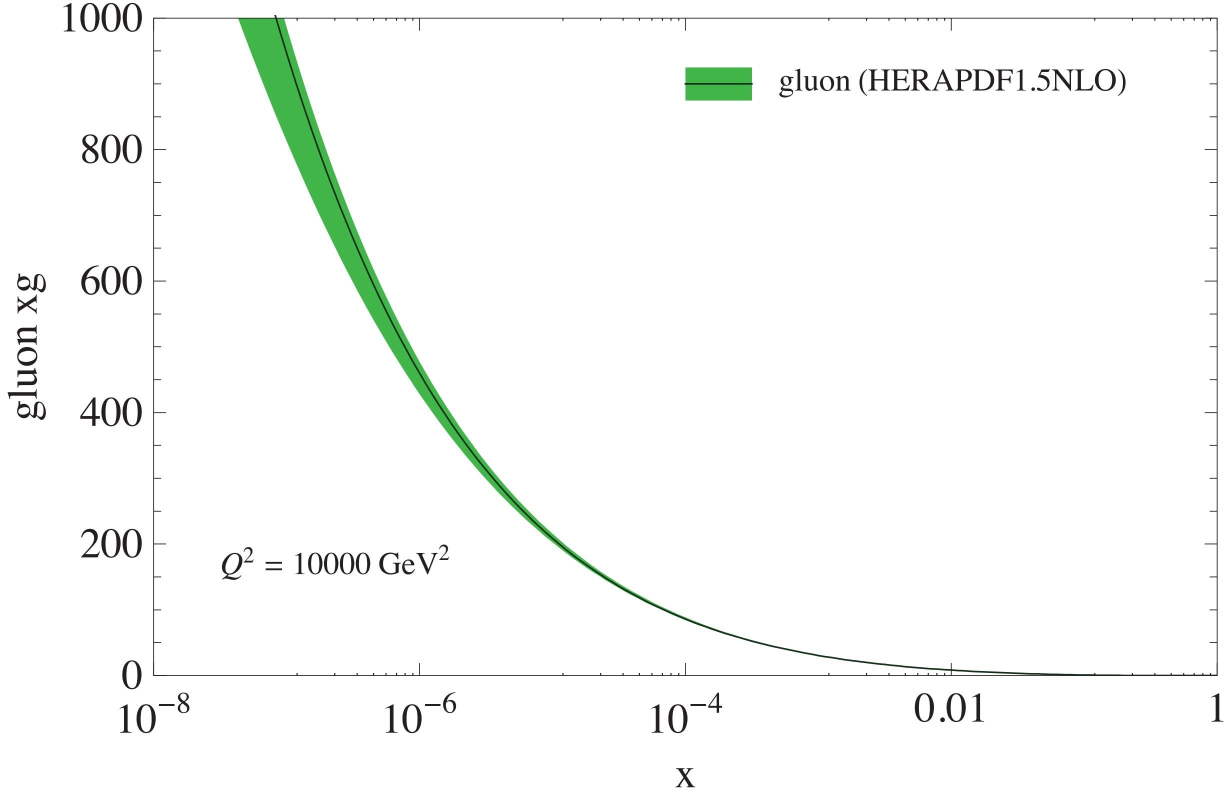
Conclusion

- cross-sections central values for
  - HERAPDF1.5
  - CT10
  - MSTW2008

  agree very well
- for HERAPDF1.5 and CT10 (under moderate assumptions) uncertainty is $\lesssim 10\,\%$, even at $E_\nu \sim 10^{20}$ GeV
- many pitfalls...e.g tabulated PDFs in LHAPDF "freeze" below some x value etc. $\rightarrow$ Don't try to do this at home!
- Any measured deviation from these cross-sections would signal the need for new physics!

Backup slides

Example: gluon momentum distribution



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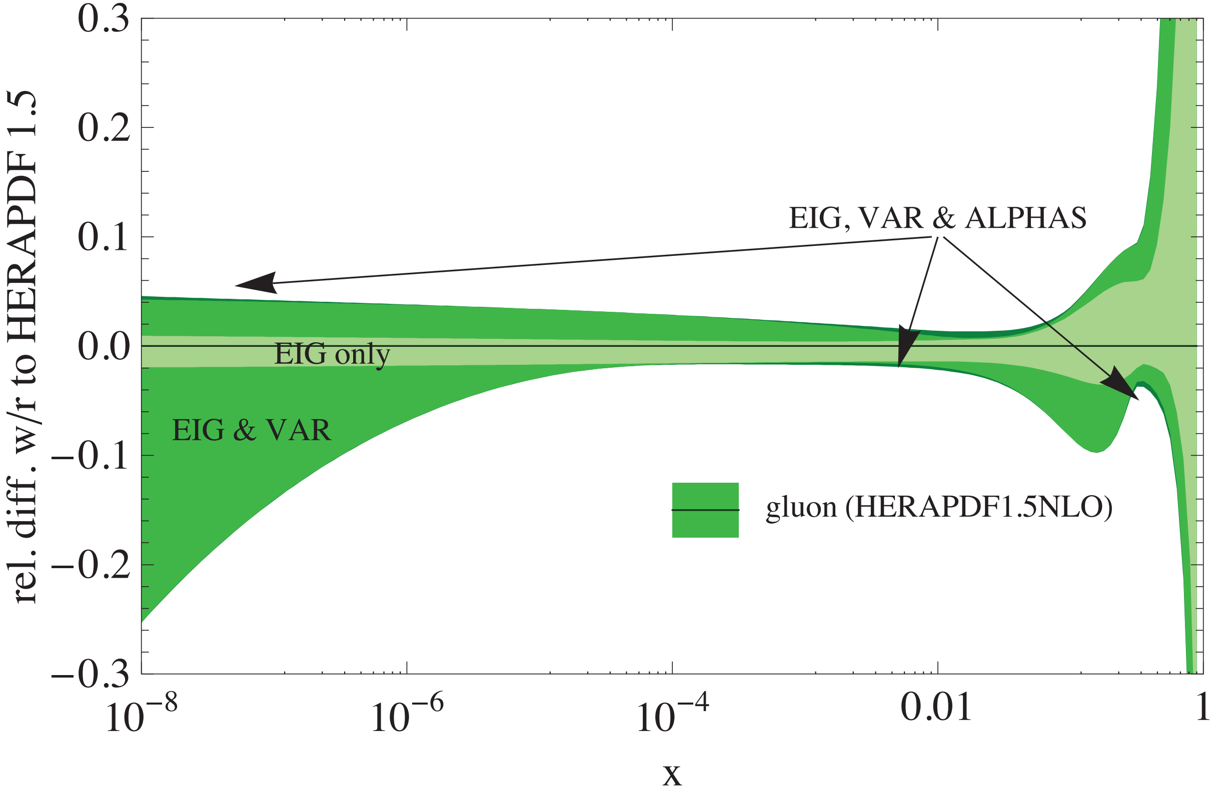
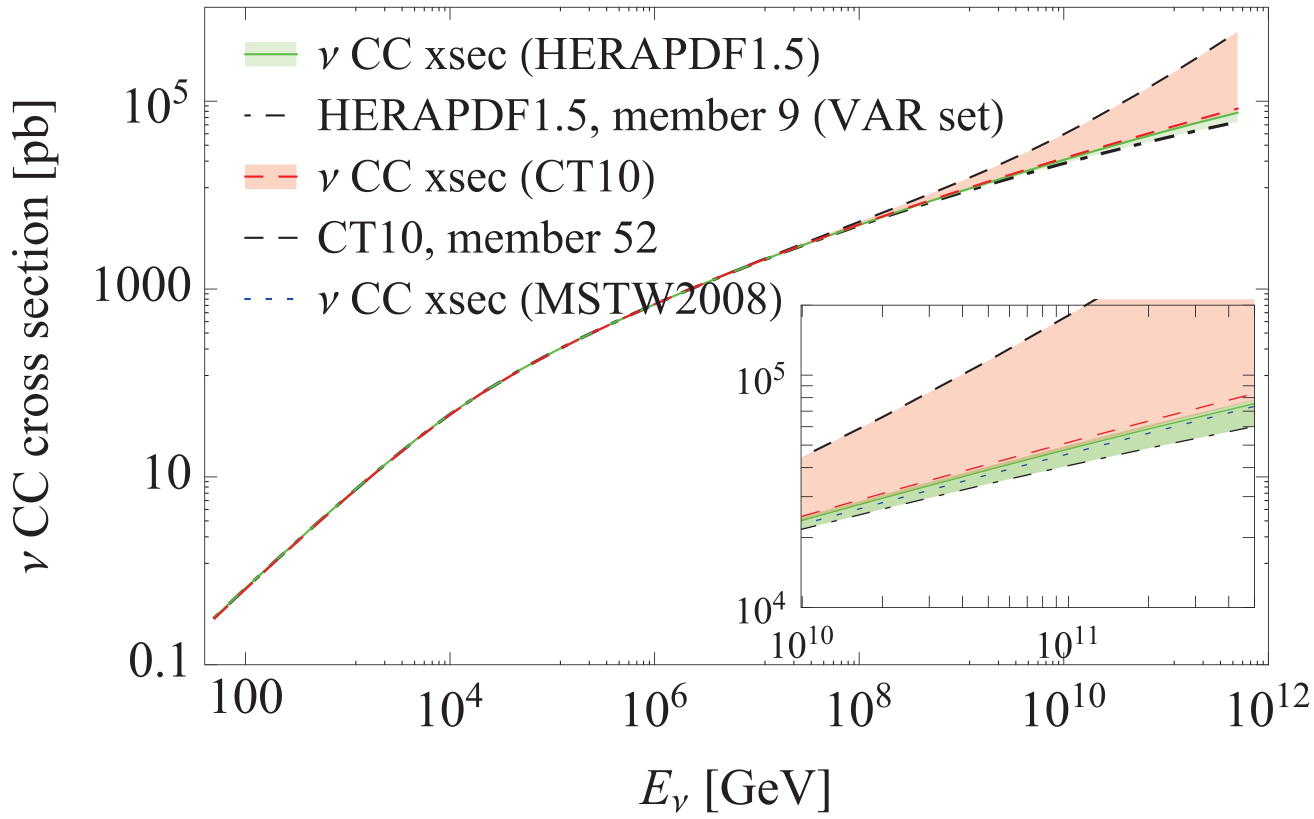
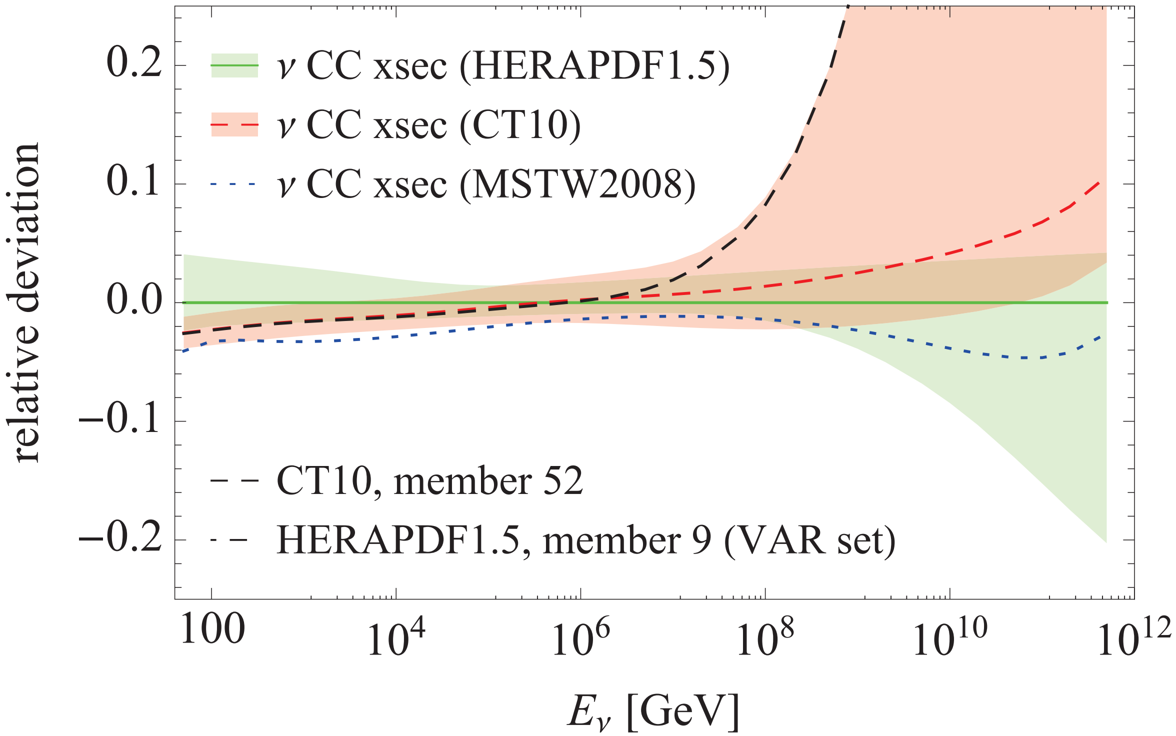


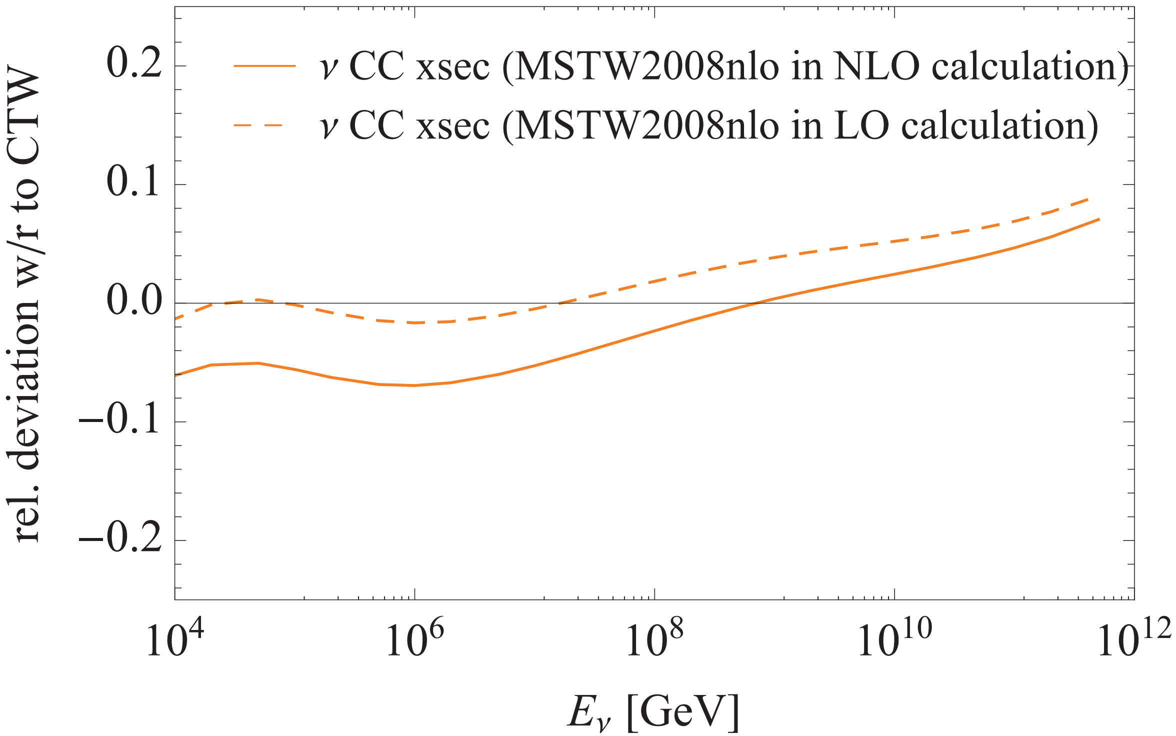
ν CC cross-section



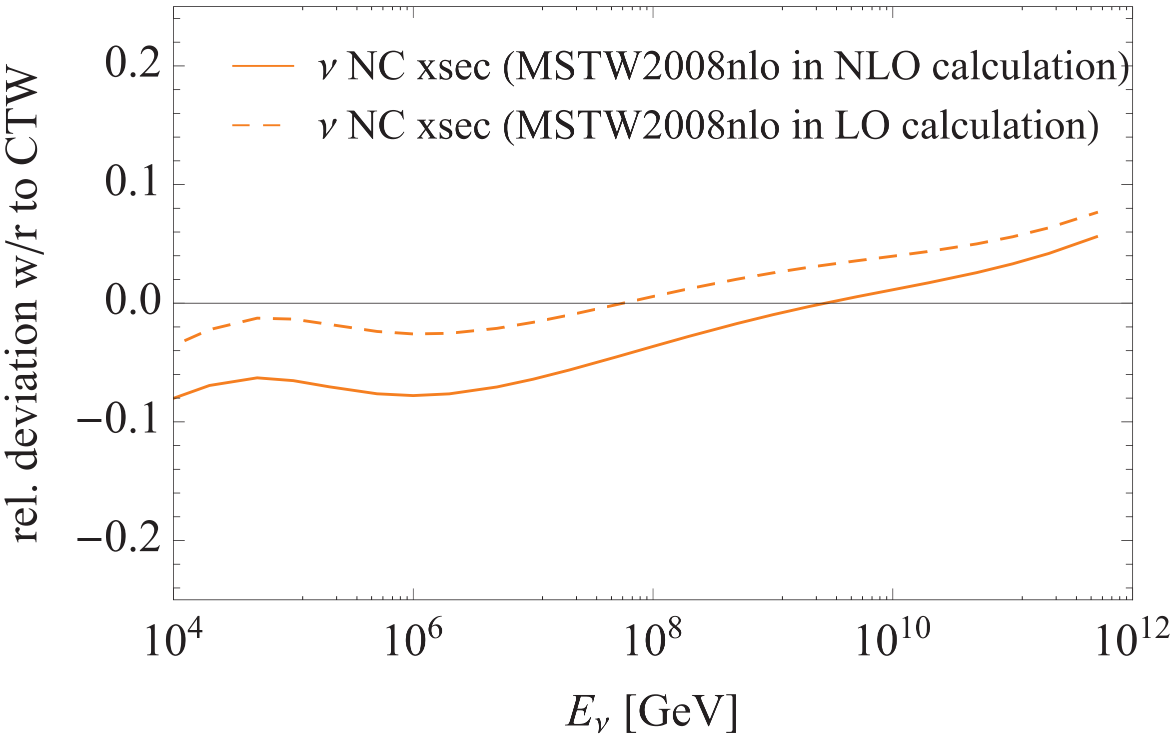
ν CC cross-section uncertainty



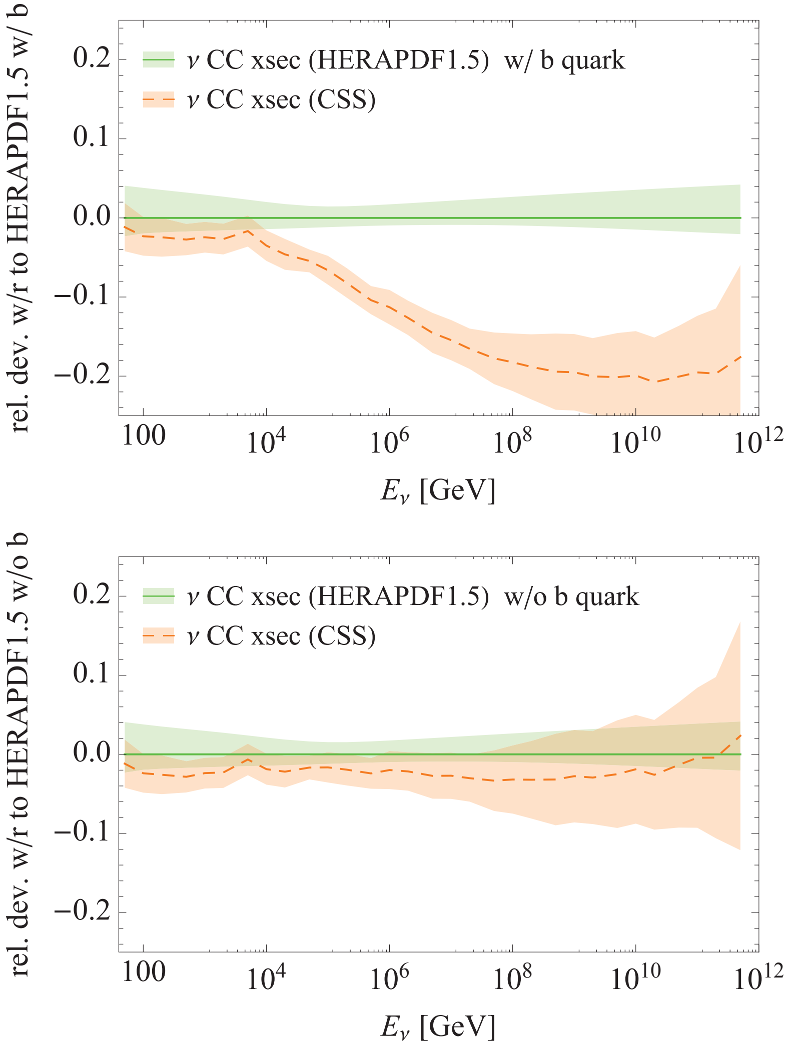
Comparison with CTW



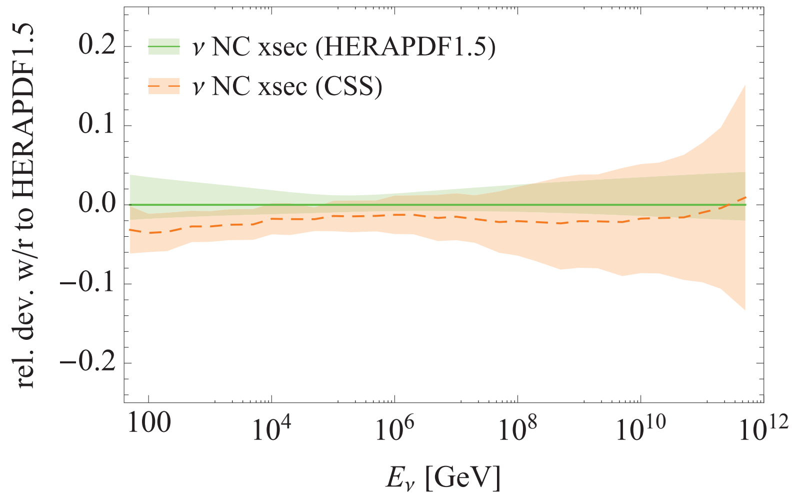
Comparison with CTW



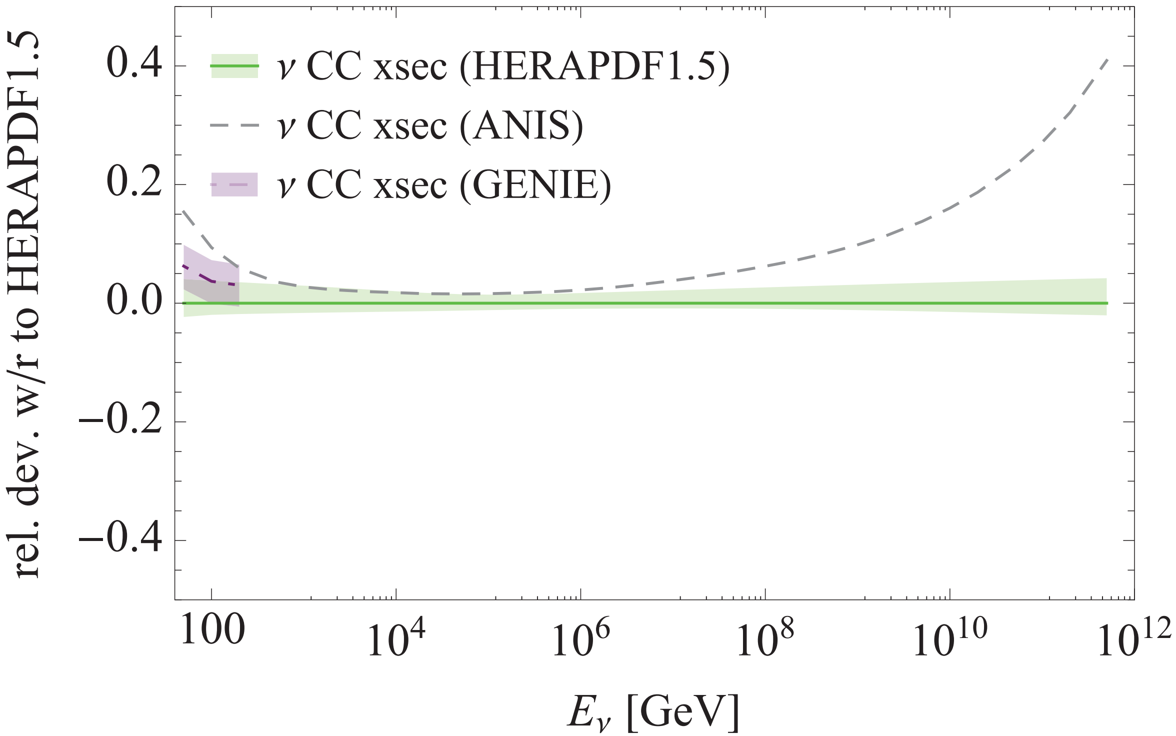
Comparison with CSS



Comparison with CSS



Comparison with ANIS



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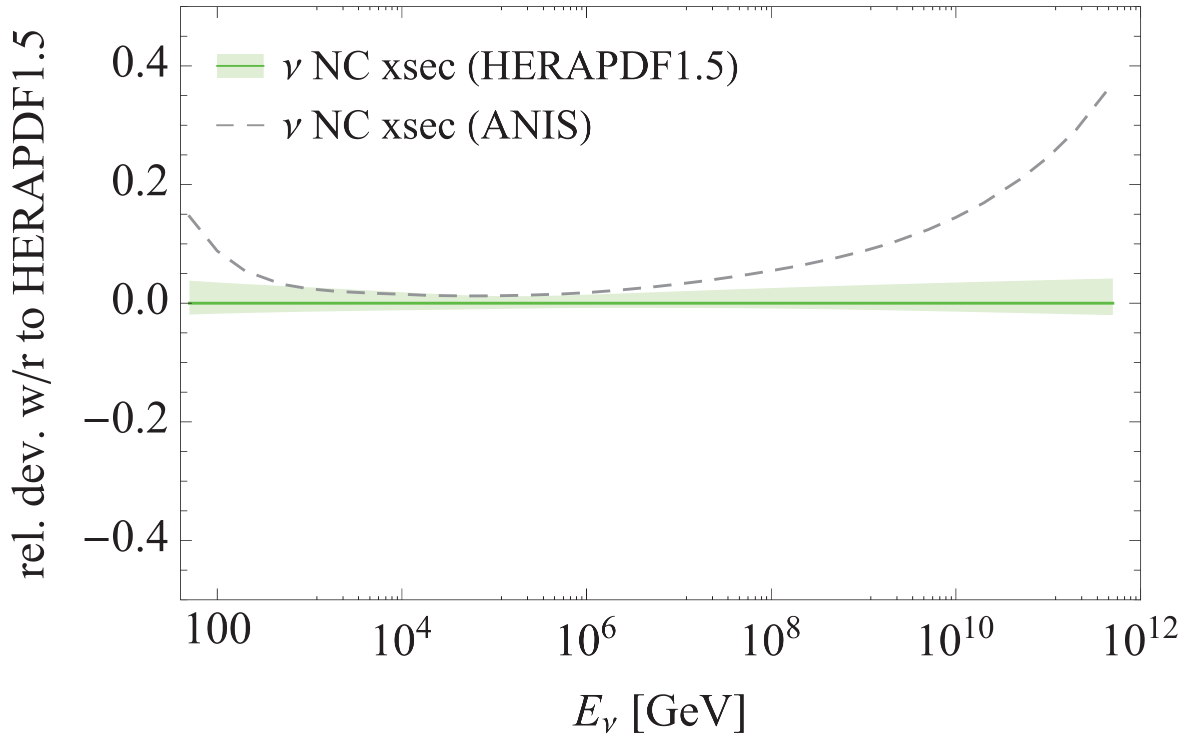

The kinematic range

TBD: new figure w/o THERA!