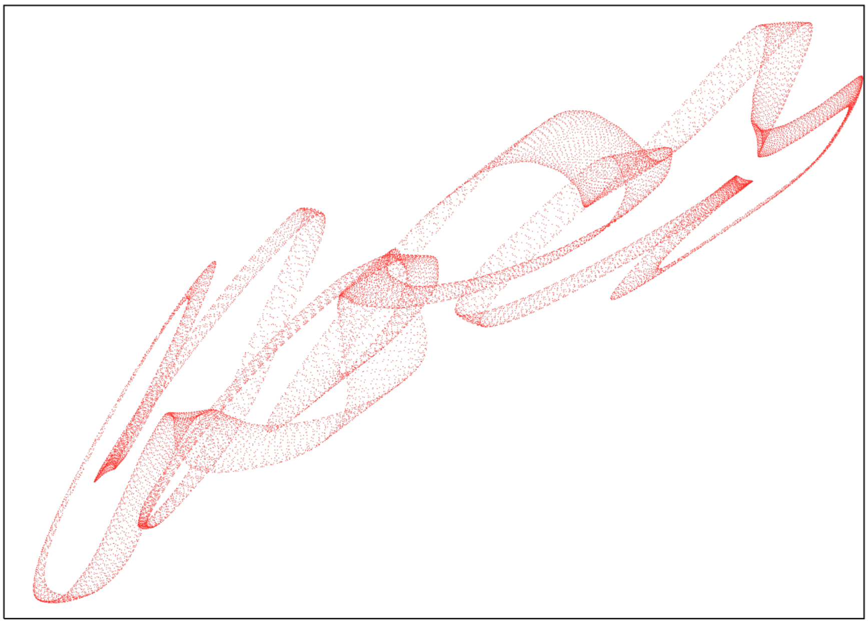
The Abdus Salam
International Centre for Theoretical Physics

School and Conference on Computational Methods in Dynamics

20 June - 8 July 2011

ICTP, Trieste, Italy

List of Abstracts



Strada Costiera 11, 34151 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Monodromy computation and its application to bifurcation analysis

Zin Arai (arai@cris.hokudai.ac.jp)
Japan Science and Technology Agency, Hokkaido University
Creative Research Initiative Sousei, Sapporo 001-0021, Japan

Abstract:
In this talk, we discuss a rigorous computational method for computing the monodromy action of dynamical systems associated to loops in the hyperbolic locus of the complexified paramter space of the system. In this case, the monodromy takes values in the automorphism group of symbolic dynamics. It will be shown that monodromy along a loop carries the information of bifurcations taking place inside the loop. Therefore, we can apply our computational method for the study of bifurcations.

Parameter-sweeping techniques for studying complex systems: numerical and rigorous results

Roberto Barrio Gil (rbarrio@unizar.es)
Universidad de Zaragoza, Depto. de Matemática Aplicada,
E-50009 Zaragoza, Spain

Abstract:
In the last few years a large plethora of numerical methods have been designed for the study of dynamical systems and for the complement of theoretical studies. In this talk we present some brute-force methods, based on parameter-sweeping techniques using the state-of-the-art numerical ODE solver TIDES. We show examples in dissipative systems like Rossler and Lorenz, and the combination with more specific methods (spiking-counting techniques and duty-cycle analysis) in the study of neuronal models. Finally, we show how in some cases we may give computer assisted proofs of some of the numerical studies, like in the case of rigorous skeletons of periodic orbits.

Verified integration of flows for large ranges of initial conditions and parameters and applications to computer assisted proofs in the Lorenz system

M. Berz and K. Makino
Michigan State University, East Lansing MI, United States of America

Abstract:
We review Taylor-model based methods and the rigorous verified integrator COSY-VI for the verified integration of ODEs and flows. Particular emphasis is given to the suppression of the wrapping effect to very high accuracy while representing large ranges of initial conditions and parameters by automatic domain decomposition methods.

We discuss various applications for computer assisted proofs of the study of the Lorenz system. We determine rigorous representations for the flow for a very large box of initial conditions containing all of the commonly studied dynamics. With rather moderate computational effort, the entire flow can be represented uniformly to an accuracy of less than 1E-6. The resulting flows can be used for the study of dynamics using discretizations and graph-based methods.

Furthermore, we present methods to rigorously enclose invariant manifolds of flows of dynamical systems. First, local approximate representations obtained from normal forms are rigorously verified using self-inclusion techniques on Taylor models. The resulting small piece of initial manifold is

integrated forward in an automatic fashion using domain decomposition, leading to tight rigorous enclosures of invariant manifolds of the Lorenzmsystem for very extended ranges.

Furthermore, we study heuristically determined parameter dependent topological rectangles on a fixed surface that appear to lead to horseshoe crossings in the second return to the surface. Using the verified integrator, the parameter dependent boundaries of the rectangles are transported, projected to the Poincare surfaces, and used for the verification of the Markov crossing property. This is achieved with a uniform accuracy below 1E-10. As a result, chaoticity is proved for the entire interval of rho values for a range of rho from about 25 to 95.

**Part of this work is jointly with Alexander Wittig, Sheldon Newhouse, and Yuting Zou.*

A numerically accessible criterion for the breakdown of quasi-periodic solutions

Renato Calleja (calleja@math.mcgill.ca)
McGill University, Department of Mathematics and Statistics
805 Sherbrooke Street West, Montreal H3A 2K6 Quebec, Canada

Abstract:
We formulate and justify rigorously a numerically efficient criterion for the computation of the analyticity breakdown of quasi-periodic solutions in Symplectic maps and 1-D Statistical Mechanics models. Depending on the physical interpretation of the model, the analyticity breakdown may correspond to the onset of mobility of dislocations, or of spin waves (in the 1-D models) and to the onset of global transport in symplectic twist maps. The criterion we propose here is based on the blow-up of Sobolev norms of the hull functions. The justification of the criterion suggests fast numerical algorithms that we have implemented in several examples.

Computer Assisted Proof for Invariant Manifolds in the Restricted Three Body Problem

Maciej Capinski (mcapinsk@agh.edu.pl)
Department of Applied Mathematics, AGH University of Science and Technology
Al. Mickiewicza 30, 30-059 Krakow, Poland

Abstract:
We first present a computer assisted proof for detection of a family of periodic orbits around an equilibrium point in the Planar Restricted Circular Three Body Problem. The method provides very tight estimates and can be applied over a broad range of energies. We then present a method for detection of fibers of stable/unstable manifolds associated with the orbits. The method is based on a combination of a parameterization method together with topological arguments. We also discuss propagation of the fibers along the flow to prove transversal intersections of the manifolds.

Attraction domain of a nonlinear system using interval analysis and Lyapunov Theory

Nicolas Delanoue (nicolas.delanoue@univ-angers.fr)
Université d'Angers, avenue Notre Dame du Lac, 49000 Angers, France

Abstract:
Consider a given dynamical system, described by \dot{x} = f (x) (where f is a nonlinear function) and [x0] a subset of R^n. We present an algorithm, based on interval analysis, able to show that there exists a unique equilibrium state x^\infty in [x0] which is asymptotically stable. The effective method also provides a set [x] (subset of [x0]) which is included in the attraction domain of x^\infty.
In a second time, the flow of the ordinary differential equation x = f(x) is discretized, inclusion methods are combined with graph theory to compute a set which is included in the attraction domain of x^\infty.

Period doubling for area preserving maps II: Rigidity

Denis Gaidashev (cccgaidash@kth.se)
University of Bergen, Department of Mathematics
Johs. Bruns Gt. 12, N-5008 Bergen, Norway

Abstract:
Period-doubling renormalization for two-dimensional highly dissipative Henon-like maps has been extensively studied by M. Lyubich, M. Martens et al. In particular, for "infinitely renormalizable" maps they have shown existence of invariant hyperbolic Cantor sets on which dynamics is homeomorphic to an "adding machine", but is not rigid: dynamics of two maps is conjugate by a transformation which is not generally smooth.

We demonstrate that rigidity is recovered for infinitely renormalizable area-preserving maps: conjugacy of dynamics on the "stable" Cantor sets is C^{1+alpha} were alpha>0.
A crucial ingredient of the proof are rigorous computer bounds on the renormalization spectrum.
This is a joint work with Tomas Johnson and Marco Martens.

High-precision computations of asymptotic series and homoclinic phenomena

Vassili Gelfreich (V.Gelfreich@warwick.ac.uk)
Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK

Abstract:
In this talk we will described how high-precision arithmetics can be used to study transversality of invariant manifolds in the region of parameters were the separatrix splitting is exponentially small compared to a perturbation parameter. The approach will be illustrated by results for model families of symplectic diffeomorphisms in dimensions 2 and 4.
The talk is based on a joint work with C.Simo and A.Vieiro.

Computation of normal form coefficients of codimension 2 bifurcations of limit cycles

Willy Govaerts (Willy.Govaerts@UGent.be)
(Ghent University, Belgium)

This is joint work with Fabio Della Rossa (Politecnico di Milano, I), Virginie De Witte (Ghent University, B) and Yuri A. Kuznetsov (Utrecht University, NL).

Abstract:
There are 11 generic codimension 2 bifurcations of limit cycles where the limit cycle at the bifurcation point still exists; this excludes other cases like homoclinic connections. The critical multipliers are the eigenvalues of the monodromy matrix with modulus 1. The number nc of critical multipliers is also the dimension of the critical center manifold in which the bifurcation takes place.

It is natural to order the codimension 2 bifurcations by critical dimensionality. Two bifurcations have nc = 2, namely CPC (cusp of cycles) and GPD (generalized period doubling). Six have nc = 3, namely CH (Chenciner), LPPD (limit point+period doubling), and the 4 resonance cases R1, R2, R3, R4. Two have nc = 4, namely LPNS and PDNS and one only has nc = 5, namely NSNS.

In principle, the codimension 2 bifurcations can be understood with the help of their Poincar´e maps which in turn can be studied by their reduction to normal form. This requires partial derivatives of the Poincar´e map up to order 5. In practice, in the case of limit cycles these derivatives cannot be computed to sufficient accuracy.

We discuss an alternative method which has been developed and implemented in the case of codimension 1 bifurcations by Yu. A. Kuznetsov, W. Govaerts, E. J. Doedel and A. Dhooge, Numerical periodic normalization for codim 1 bifurcations of limit cycles, SIAM J. Numerical Analysis, 43(4):1407−1435, 2005. In this approach, the Poincar´e map or its derivatives are not computed at all. Instead, the computation of the normal form coefficients is reduced to solving certain linear boundary value problems where only (derivatives of) the right-hand side function f(u) of the dynamical system $\dot{u} = f(u)$ is used.

Our work is based on the theoretical results obtained by G. Iooss, Global char- acterization of the normal form for a vector field near a closed orbit, J. Differential Equations, 76(1):47−76, 1988. However, the application of these ideas to obtain ex-plict formulas for the normal form coefficients is a tedious work that has to be done in each bifurcation case separately.

We have implemented these algorithms in the cases where nc = 2 and nc = 3
in the software package MatCont. In this implementation the boundary value problems are discretized by orthogonal collocation with piecewise polynomial functions.

Although the computation of the normal form coefficients is a big task in itself, and still incomplete in the sense that three codimension 2 cases have not been dealt with, it is only the first step in a comprehensive study of codimension 2 bifurcations. The natural next step would be the initialization in a two-parameter setting of curves of codimension 1 bifurcations that are rooted in codimension 2 bifurcation points.

Nevertheless, the preceding implementation already allows to perform sophisti- cated numerical bifurcation studies of dynamic models, in particular to distinguish between different bifurcation scenarios. For example, we studied cascades of R2 bi- furcation points and NS curves in a periodically forced predator-prey model. We also showed how a higher order degeneracy (a swallowtail bifurcation) in a Lorenz84 system can be captured.

Reliable computation of invariant tori (and beyond)

Alex Haro (alex@maia.ub.es)
Dpt. Matematica Aplicada i Analisi
Universitat de Barcelona, Gran Via de les Corts Catalanes 585
08007 Barcelona, Spain

(with Jordi-Lluis Figueras)

Abstract:
We present a methodology to perform rigorous (computer assisted) proofs of existence and local uniqueness of normally hyperbolic invariant tori in quasiperiodically forced systems. We apply these techniques in several scenarios where the invariant tori are about to break.
We also explore numerically (non-rigorously) those scenarios, in order to give some insight into the mechanisms of destruction of invariant tori.

Is the Outer Solar System Chaotic?

Wayne Hayes.
Visiting Fellow, Oxford Centre for Collaborative Applied Mathematics, and Associate Professor of Computer Science, University of California, Irvine.

Abstract:
The stability of our Solar System has been debated since Newton devised the laws of gravitation to explain planetary motion. Newton himself doubted the long-term stability of the Solar System, and the question has remained unanswered despite centuries of intense study by generations of illustrious names such as Laplace, Langrange, Gauss, and Poincare. Finally, in the 1990s, with the advent of computers fast enough to accurately integrate the equations of motion of the planets for billions of years, the question has finally been settled: for the next 5 billion years, and barring interlopers, the shapes of the planetary orbits will remain roughly as they are now. This is called "practical stability": none of the known planets will collide with each other, fall into the Sun, or be ejected from the Solar System, for the next 5 billion years. Although the Solar System is now known to be practically stable, it may still be "chaotic". This means that we might---or might not---be able to precisely predict positions of the planets within their orbits, for the next 5 billion years. The precise positions of the planets can effect the tilt of each planet's axis, and so can have a measurable effect on the climate. For the past 15 years, there has been some debate about whether the Solar System exhibits chaos or not: when performing accurate integrations of the planetary motions, some astronomers observe chaos, and some do not. This is particularly disturbing because it is known that inaccurate integration can inject chaos into a numerical solution that would otherwise be stable. I will also show lots of pretty (but as yet unpublished) pictures at the end, demonstrating the fractal nature of the boundary between chaos and regularity in the outer Solar System, and the chaotic interplay between the inner and outer planets in our Solar System.

Period doubling in area-preserving maps I: Lack of elliptic islands

Tomas Johnson
Dept. of Mathematics, Cornell University, Ithaca, NY 14853, USA

Abstract:
In 1984 Eckmann, Koch, and Wittwer gave a computer-assisted proof of the existence of a universal area-preserving map - a map with orbits of all binary periods. This universal map is a fixed point of a locally hyperbolic renormalization operator. In this talk we will give a description of the dynamics of the universal map. In particular, we will describe an algorithm to compute the periodic orbits of an area-preserving twist map using its generating function. The algorithm is used to study the (lack of) ellipticity of the universal area-preserving map associated with period doubling.
The talk is based on joint work with Denis Gaidashev and Marco Martens.

Estimating long term behavior of flows without trajectory integration: the infinitesimal generator approach

Oliver Junge
Technische Universitaet Muenchen, Muenchen, Germany

Abstract:
The long-term distributions of trajectories of a flow are described by invariant densities, i.e. fixed points of an associated transfer operator. In addition, global slowly mixing structures, such as almost-invariant sets, which partition phase space into regions that are almost dynamically disconnected, can also be identified by certain eigenfunctions of this operator. Indeed, these structures are often hard to obtain by brute-force trajectory-based analyses. In a wide variety of applications, transfer operators have proven to be very efficient tools for an analysis of the global behavior of a dynamical system. The computationally most expensive step in the construction of an approximate transfer operator is the numerical integration of many short term trajectories. In this paper, we propose to directly work with the infinitesimal generator instead of the operator, completely avoiding trajectory integration. We propose two different discretization schemes; a cell based discretization and a spectral collocation approach. Convergence can be shown in certain circumstances. We demonstrate numerically that our approach is much more efficient than the operator approach, sometimes by several orders of magnitude. This is joint work with Gary Froyland (UNSW) and Péter Koltai (TUM).

Computer assisted proof of the KAM stability of the figure eight orbit in the N-body problem

Tomasz Kapela (kapela@ii.uj.edu.pl) and Carles Simó (carles@maia.ub.es)

Abstract:
We present a methodology for computer assisted proofs of the existence and the KAM stability of an arbitrary periodic orbit for Hamiltonian systems. In short, using interval arithmetics and rigorous ODE solvers we are able to get verified bounds for coefficients of the Birkhoff normal form of a suitable Poincaré map. These estimates allow to check the KAM conditions.

We applied this algorithm to the the 3-body problem and we proved the KAM stability of the well-known figure eight orbit and two selected orbits of the so called family of rotating Eights. They are examples of the so called choreographic solutions in which all bodies travel along the same curve with constant phase shift (in case of rotating Eights in suitable rotating frame.

Computer-assisted methods in dynamical systems and PDEs
(Joint work with Gianni Arioli)

Hans Koch (koch@math.utexas.edu)
University of Texas at Austin, Department of Mathematics, 1 University Station C1200
Austin TX 78712-0257, USA

Abstract:
We consider some functional equations that arise in dynamical systems and PDEs. They are solved by reducing them to fixed point problems that are appropriate for a computer-assisted analysis. Specific examples that will be discussed include a renormalization analysis related to the breakup of smooth invariant tori, the construction of a hyperbolic orbit for the unidimensional Kuramoto-Sivashinski equation (using a small-time "integrator" and shadowing techniques), and the construction of a non-symmetric low-index solution for a symmetric boundary value problem.

Graph based topological computation for global analysis of dynamical systems

Hiroshi Kokubu (kokubu@math.kyoto-u.ac.jp)
Kyoto University, Graduate School of Science, Department of Mathematics
Kitasirakawa Oiwake-cho, Sakyo Kyoto 606-8502, Japan

Abstract:
A computational method for constructing a database of global dynamics of a multiparameter dynamical system is introduced. An outer approximation of the dynamics for each subset of the parameter range is computed using rigorous numerical methods and is represented by means of a directed graph. The dynamics is then decomposed into the recurrent and gradient-like parts by fast combinatorial algorithms and is classified via the so-called Morse decompositions. These Morse decompositions are compared at adjacent parameter sets via continuation to detect possible changes in the dynamics. The Conley index is used to study the dynamics of isolated invariant sets associated with the computed Morse decompositions. The power of the developed method is illustrated with an application to several concrete examples such as the two-dimensional, density-dependent, Leslie population model. In this talk, I will discuss some recent results and attempts for further extensions of this method.
(A part of the talk is a joint work with Zin Arai, Bill Kalies, Konstantin Mischaikow, Pawel Pilarczyk, Hiroe Oka)

The role of global manifolds in the transition to chaos in the Lorenz system

Bernd Krauskopf (b.krauskopf@bristol.ac.uk)
(joint work with Hinke Osinga and Eusebius Doedel)

Abstract:
The Lorenz system still fascinates many people because of the simplicity of the equations that generate such complicated dynamics on the famous butterfly attractor. This talk addresses the role of the stable and unstable manifolds in organising the dynamics more globally. A main object of interest is the stable manifold of the origin of the Lorenz system, also known as the Lorenz manifold. This two-dimensional manifold and associated manifolds of saddle periodic orbits can be computed accurately with numerical methods based on the continuation of orbit segments, defined as solutions of suitable boundary value problems. This allows us to give a precise characterization of global manifolds during the transition from simple dyanamics, via preturbulence to chaotic dynamics, as the Rayleigh parameter of the Lorenz system is increased.

Travelling Waves and Slow Manifolds of Saddle-Type

Christian Kuehn (ck274@cornell.edu)
Cornell University, Center for Applied Mathematics
657 Rhodes Hall, Ithaca NY 14853-3801, USA

Abstract:
We start with an introduction to fast-slow systems. The geometric viewpoint of the theory will be emphasized. Then we discuss the three-dimensional FitzHugh-Nagumo (FHN) equation and its bifurcations. The singular limit bifurcation diagram of the FHN equation will be derived. The computation and interaction of different types of invariant manifolds will be emphasized to explain the dynamics of travelling wave solutions. In particular, we compute slow manifolds of saddle-type and also discuss the conditioning of the associated algorithm.

Long time computations of the Solar System dynamics

Jacques Laskar
Observatoire de Paris, Paris, France

Kolmogorov's normal form for equations of motion with some peculiar dissipative effects

Ugo Locatelli (ugo@arcetri.astro.it)
Università degli Studi di Roma "Tor Vergata", Roma, Italy

Abstract:
We focus on the equation of motion related to the "dissipative spin-orbit model", that is commonly studied in Celestial Mechanics. Actually, we study it in the slightly more general context of a 2n-dimensional action-angle phase space. Since we assume that the friction terms are linear and isotropic with respect to the actions, we succeed in proving a KAM-like theorem, by constructing the equivalent of the Kolmogorov's normal form in a very similar way to the Hamiltonian case.

We will show that also the frequency map analysis (as designed by J. Laskar for conservative systems) can nicely adapt to these dissipative systems. Therefore, we can obtain some suitable enough values of a pair of parameters, that allow us to start the algebraic manipulations explicitly constructing the Kolmogorov's normal form. This approach provides some interesting numerical results, which will be discussed.
The talk is based on a joint work with L. Stefanelli.

A boundary value approach to computing slow manifolds and canard orbits

Hinke Osinga (H.M.Osinga@bristol.ac.uk)
University of Bristol
Bristol BS8 1TR, UK
(joint work with Mathieu Desroches and Bernd Krauskopf)

Abstract:
We present a general technique for the computation of two-dimensional slow manifolds in systems with one fast and two slow variables. It is based on the continuation of a one-parameter family of orbit segments, given as solutions of a suitably-defined boundary value problem. In this way, we are able to deal effectively with the numerical challenges of strong attraction to and strong repulsion from the slow manifolds. Visualization of the computed surfaces gives unprecedented insight into the geometry of the system. In particular, our technique allows us to find and then continue canard solutions as the intersection curves of attracting and repelling slow manifolds. The method is illustrated with the self-coupled FitzHugh-Nagumo system, and a reduced Hodgkin-Huxley model. We concentrate on mixed-mode periodic orbits that are organized by canard orbits, which arise geometrically as intersection curves of two-dimensional attracting and repelling slow manifolds near a folded-node singularity. Using the continuation of orbit segments, we compute the slow manifolds and associated canard orbits. We show how the subsequent continuation of canard orbits allows us to find and investigate new types of dynamics, such as the interaction between canard orbits and a saddle periodic orbit that is generated in a singular Hopf bifurcation.

Geometric Evolutionary Dynamics of Protein Interaction Networks

Natasa Przulj
Imperial College London Department of Computing, London, UK

Abstract:
Understanding the evolution and structure of protein-protein interaction (P PI) networks is a central problem of systems biology. Since most processes in the cell are carried out by groups of proteins acting together, a theoretical model of how PPI networks develop based on duplications and mutations is an essential ingredient for understanding the complex wiring of the cell. Many different network models have been proposed, from those that follow powerlaw degree distributions and those that model complementarity of protein binding domains, to those that have geometric properties. Here, we introduce a new model for PPI network (and thus gene) evolution that produces well-fitting network models for currently available PPI networks. The model integrates geometric network properties with evolutionary dynamics of PPI network evolution.

Existence of instabilities in the elliptic RTBP -- Numerical investigations

Pablo Roldán (pablo.roldan@upc.edu)
Universitat Politecnica de Catalunya, Departament de Matematica Aplicada 1
Diagonal 647, 08028 Barcelona, Spain
Co-authors: Jacques Féjoz, Marcel Guardia, and Vadim Kaloshin.

Abstract:
We do a numerical investigation of a Normally Hyperbolic Invariant Manifold associated to a certain resonant family of periodic orbits in the RTBP. In particular, we compute the NHIM, its hyperbolic invariant manifolds, their intersection and their splitting, and the associated inner/outer dynamics. We check the numerical results by performing different tests whenever possible. The numerical study is applied to prove existence of instabilities in the elliptic RTBP.

Geometric and probabilistic descriptions of chaotic phase space transport

Shane Ross (sdross@exchange.vt.edu)
Virginia Tech Engineering Sciences Research Coordinator,
MultiSTEPS 224 Norris Hall (MC 0219), USA
http://www.shaneross.com

Abstract:
Several geometric and probabilistic methods for studying chaotic phase space transport have been developed and fruitfully applied to diverse areas from orbital mechanics to biomechanics to fluid mechanics and beyond. Increasingly, systems of interest are determined not by analytically defined model systems, but by data from experiments or large-scale simulations. This emphasis on real-world systems sharpens our focus on those features of phase space transport in finite-time systems which seem to be robust, leading to the consideration of not only invariant manifolds and invariant manifold-like objects, but also their connection with concepts such as symbolic dynamics, braids, and almost-invariant sets. This two-part talk will address both (1) systems known analytically from which phase space structures (separatrices) controlling transport and stability can be computed, and (2) approaches for identifying separatrices and quantifying transport in mechanical systems not known analytically. Applications to areas such as stability in musculoskeletal biomechanics, ship capsize prediction, and atmospheric transport will be discussed.

Azimuthal waves, their stability and connecting orbits
in thermal convection in rotating spherical shells

Juan Sánchez Umbría, Fernando Garcia González and Marta Net Marcè
Univ. Catalunya, Spain

Abstract:
The thermal convection of a pure fluid contained in a spherical shell rotating about a fixed axis is studied. The bifurcations from the conduction state give rise to azimuthal waves which have been computed by continuation methods, as steady solutions of a system for the waves, in the frame of reference of the spheres. Their stability has also been studied. We have found the coexistence of stable waves of different azimuthal wave number for the same value of the parameters of the problem. The eigenfunctions at the secondary bifurcation points reveal the nature of the modulation of the waves

when they loose stability. When the waves are unstable, connecting orbits are found by time integration, starting from the unstables states. Details of the numerical techniques employed will be given.

Numerical shadowing trajectories in nonlinear dynamics

Tim Sauer (tsauer@gmu.edu)
George Mason University, Dept. of Mathematical Sciences
Fairfax VA 22030, United States of America

Abstract:
Shadowing trajectories are exact orbits of a dynamical system that lie near a particular computer-generated orbit. We survey the development of strategies for shadowing computed trajectories of nonhyperbolic systems. The strategies prove the existence of exact solutions of maps and flows even in the presence of sensitive dependence on initial conditions, and in some cases provide numerical approximations.
Additionally, we report on the related problem of the effect of finite precision on the calculation of natural measures from long trajectories, and discuss typical obstructions to shadowing.

Guiding Simulations and Experiments using Continuation

Frank Schilder (f.schilder@mat.dtu.dk)
Technical University of Denmark Department of Mathematics Building 303
DK-2800 Lyngby, Denmark

Abstract:
When applying continuation of periodic solutions to high-dimensional finite element models one might face a dilemma. The mesh resolution and thus the dimension N of the model are typically chosen such that a given computer system can store the information necessary to perform one integration step for dimension N, but not for larger dimensions. In other words, a model is usually implemented as a carefully derived implicit integration scheme tailored for numerically stable simulations with the highest spacial resolution admitted by the computational power available. On the other hand, stable numerical methods for periodic solutions, for example, multiple shooting or collocation, typically require the simultaneous storage and manipulation of information for K>1 states, which would imply that periodic solutions cannot be computed without a significant reduction of the model's resolution. The recently developed method of control based continuation allows the continuation of periodic solutions without a reduction of the model resolution, and even directly in physical experiments. Moreover, both a simulation as well as an experiment can run asynchronously from the actual continuation method, which communicates with the simulation or experiment by setting a control target and by taking measurements. The key ideas of this approach are (1) to introduce a control scheme that locally stabilizes periodic solutions without perturbing them, and (2) to use continuation to guide the simulation or experiment around folds and through bifurcation points. In this talk we will present a Matlab toolbox for control based continuation and illustrate its application with a lab experiment of an impact oscillator that exhibits a large hysteresis loop. We will indicate current challenges with this method and how we intend to tackle them.

The dynamics of a low-order model for the Atlantic Multidecadal Oscillation

Alef Sterk (A.E.Sterk@exeter.ac.uk)∗
College of Engineering, Mathematics and Physical Sciences
University of Exeter Harrison Building North Park Road Exeter EX4 4QF, UK

Abstract:
Observations and model studies provide ample evidence for the presence of multidecadal variability in the North Atlantic sea-surface temperature known as the Atlantic Multidecadal Oscillation (AMO). This variability is characterised by a multidecadal time scale, the westward propagation of temperature anomalies, and a phase difference between the anomalous meridional and zonal overturning circulations.
We derive a low-order model which captures the characteristics of the AMO. The starting point is a minimal model consisting of a thermal wind balance and an equation for the advection of temperature in a 3-dimensional box-shaped ocean basin. The low-order model is obtained by an orthogo- nal projection onto a finite-dimensional function space. Flows are forced by restoring the sea surface temperature to an idealised atmospheric tem- perature profile with an equator-to-pole gradient Δ as a control parameter. A second control parameter, γ, interpolates between restoring ($\gamma = 0$) and prescribed heat flux ($\gamma = 1$) conditions.
For the standard values $\Delta = 20^\circ$C and $\gamma = 0$ the low-order model has a stable equilibrium which corresponds to a steady ocean flow. By increas- ing the parameter γ from 0 to 1 this equilibrium becomes unstable through a supercritical Hopf bifurcation and we find a periodic attractor with the physical signature of the AMO. In turn, this attractor can bifurcate through (cascades of) period doublings when Δ is increased. Next, we impose a time-periodic forcing, modelling annual variations in the ocean-atmosphere heat flux. In this setting the AMO appears through a Hopf-Neĭmark-Sacker bifurcation as an invariant 2-torus attractor. Hence, we have to study bifur- cations of invariant tori. For $\Delta \geq 22^\circ$C the 2-torus associated with the AMO bifurcates through a sequence of quasi-periodic period doublings, which can give birth to strange attractors of quasi-periodic Hénon-like type.
∗ *This is joint work with Henk Broer, Henk Dijkstra, Carles Simó, and Renato Vitolo.*

Computation of bifurcations of periodic points related to homoclinic bifurcations

Joan Carles Tatjer
Universitat de Barcelona, Barcelona, Spain

Abstract:
Let (f_a) be an m-parameter family of maps from R^n to itself. Suppose that for $a=a_0$ there exists a hyperbolic fixed point having a non-degenerate homoclinic tangency. Under suitable conditions it is possible to prove the existence of bifurcations of n-periodic points, for values of the parameter a_n that goes to a_0 when n tends to infinity. Moreover, for values of the parameter close to these bifurcations, attracting n-periodic orbits appear. In the one-dimensional non-invertible case, we will obtain some global `non-rigourous' results about the abundance of attracting periodic orbits in the parameter line. For other dimensions it is possible to obtain computable asymptotic expressions of the values of the bifurcation parameters, but the results are necessarily partial, due to the appearance of other phenomena, as the coexistence of attractors.

Holder shadowing on finite intervals

Sergey Tikhomirov
Free University of Berlin, Berlin, Germany

Abstract:
Hammel-Yorke-Grebogi conjectured that for a wide class of diffeomorphisms, including in particular Henon maps, any d-pseudotrajectory can be d^alpha shadowed by an exact trajectory on intervals of length 1/d^alpha} for \alpha = 1/2. We prove that Hammel-Yorke-Grebogi conjecture for alpha 1/2 holds only for structurally stable diffeomorphisms. The main technique is the reduction to inhomogeneous linear equation v_{k+1} = A_k v_k + b_{k+1} where A_k are differential of the diffeomorphism along an exact trajectory and b_k is an arbitrarily bounded sequence.

Abundance of stable periodic orbits inside homoclinic lobes

C. Simó (carles@maia.ub.es) and A.Vieiro (vieiro@maia.ub.es)
Universitat de Barcelona, Gran Via de les Corts Catalanes 585, 08007 Barcelona, Spain

Abstract:
Let Fϙ be a one-parameter family of area-preserving maps (APMs) having an elliptic fixed point E0. Typically, as ϙ evolves, different chains of resonant islands of stability bifurcate from E0. Generically, these stability islands have a pendulum-like phase space structure (that is, in a topological sense, the separatrices of the related hyperbolic Birkhoff periodic points form a Poincaré figure-eight). Note that, generically, these separatrices split, bounding an infinity number of homoclinic lobes.
In [1] we study the abundance of elliptic periodic orbits visiting the homoclinic lobes (EPL), a domain typically dominated by a chaotic behaviour. To this end, given a family of APMs Fϙ, we use the separatrix map as a model of the return to a fundamental domain containing lobes. Within this framework, it was proved in [2] that a generic family of APMs (having a figure-eight structure with split separatrices bounding homoclinic lobes) has EPL for a positive (bounded from below) set of ϙ-parameters. However, the results in [2] deal with the so-called a priori unstable case, hence they do not apply to the lobes related to resonant islands considered above. On the other hand, these results do not provide quantitative information concerning the measure of the set of ϙ-parameters for which the map has EPL.
In this talk, we will present quantitative results analogous to those in [2], but for the a priori stable situation. Several considerations concerning the differences between the a priori sta- ble/unstable cases will be given. The analytical results will be complemented with quantitative numerical studies. We will consider for illustrations the separatrix map, the standard map and the H´enon map as concrete families of APMs.
References
[1] C. Simó and A. Vieiro. Some remarks on the abundance of stable periodic orbits inside homoclinic lobes. Submitted to Physica D, 2010.
[2] C. Simó and D.V. Treschev. Stability islands in the vicinity of separatrices of near- integrable maps. Discrete Contin. Dyn. Syst. Ser. B, 10(2-3):681–698, 2008.

Numerical continuation of quasi-periodic bifurcations of invariant circles in low-dimensional systems

Renato Vitolo (Univ. Exeter) Carles Simó (Univ. Barcelona) Henk Broer (Univ. Groningen)

Abstract:
We present algorithms for the numerical computation and continuation of quasi-periodic invariant circles and of their bifurcations. Examples are given for quasi-periodic bifurcations of Hopf, saddle-node and period-doubling type in a three-dimensional map, obtained as a model for the Hopf-saddle-node bifurcation of fixed points. The essential role played by reducibility is emphasised by examining some of the phenomenona occurring near bifurcations of non-reducible invariant circles in the quasi-periodically driven Henon map.

Earliest Computations in Dynamics --- Kepler, Newton, Euler

Gerhard Wanner (Gerhard.Wanner@unige.ch)
Section Mathématiques Université de Genève
2-4 rue du Lievre, CH-1211 Genève 4, Switzerland

Computability of Thurston equivalence of branched coverings

Michael Yampolsky (yampol@math.toronto.edu)
University of Toronto, Department of Mathematics, 40 St. George Street,
Ontario Toronto M5S 2E4, Canada

Dynamics of the area preserving Henon map at 3:1 resonance

Piotr Zgliczynski (umzglicz@cyf-kr.edu.pl)
Institute of Computer Science, Jagiellonian University ul. Lojasiewicza 6,
30--348 Krakow, Poland
joint work with C. Simo and T. Kapela

Abstract:
We investigate the following area preserving Henon map
$h(x, y) = R\alpha(x, y - x2)$, $\alpha = 2\pi/3$
where $R\alpha$ is a rotation by α. We focus on $\alpha = 2\pi/3$ case, the 3:1 resonance.

I will discuss proofs of:

1. the existence of periodic islands
2. the existence of the symbolic dynamics and the existence of the 'hyper- bolic' set with some interesting properties: some points have zero Lapunov exponent, some nonzero and for others points the limit defining Lapunov exponent oscilates between zero and some nonzero value.
3. Gevrey character of the branches of unstable set for the resonant fixed point.