Correlated states with broken time reversal symmetry in graphene (i) monolayer doped to saddle point (ii) bilayer at charge neutrality

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ICTP, Trieste, 06/29/11

Collaboration

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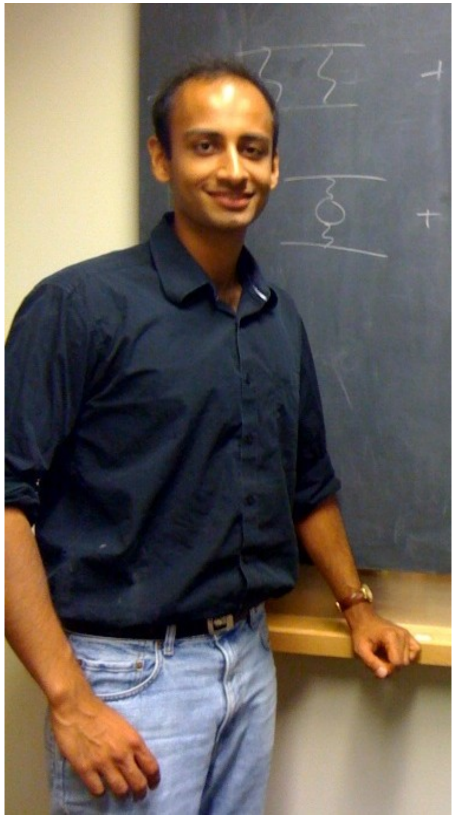
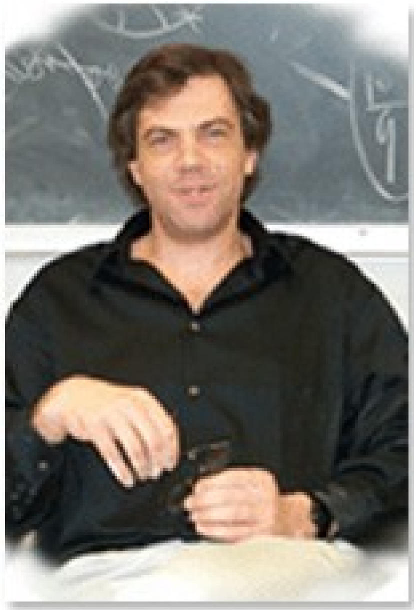
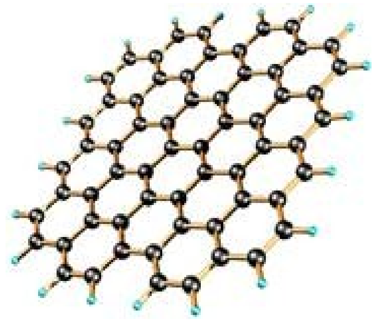
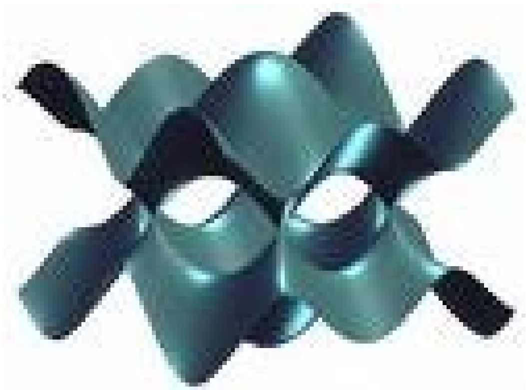




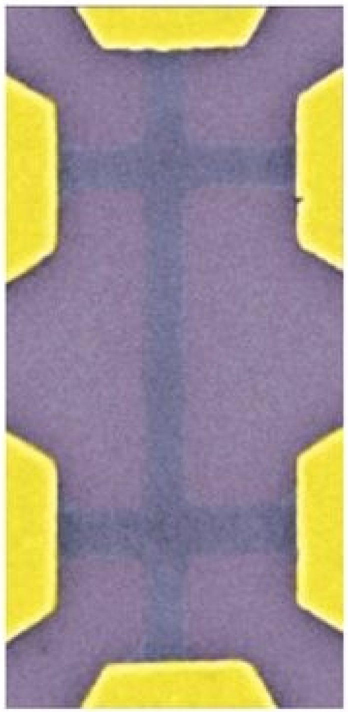
Electronic states in a single carbon layer

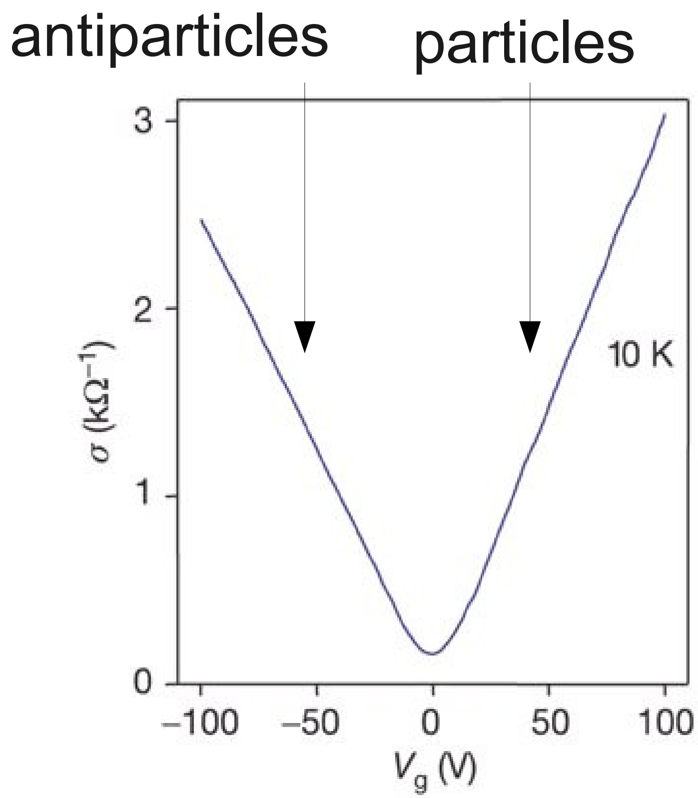
A. Field-effect enabled by gating:
tunable carrier density,
B. high mobility, no temperature dependence
C. conductivity linear in density





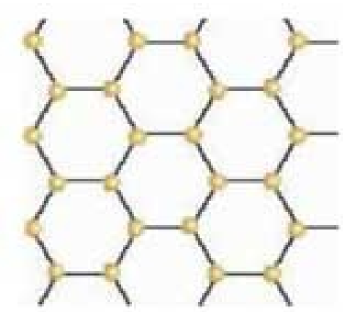
Novoselov et al, 2004,
Zhang et al, 2005



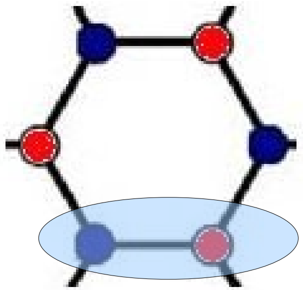


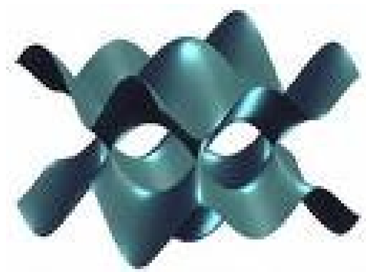
Electronic states in graphene

Two sublattices



$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$





Pseudo-spin (sublattice)

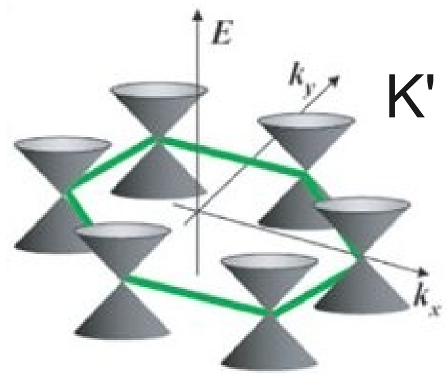
unit cell

$$\hat{H} = v_F \hat{\boldsymbol{\sigma}} \mathbf{p}$$

$$E = \sqrt{c^2 p^2 + \cancel{m^2 c^4}}$$

No gap (cond-mat) ≡ No mass (hep)

K'



K

$$v_F \approx 10^6 m/s = \frac{c}{300}$$

Slow, but ultrarelativistic Dirac fermions

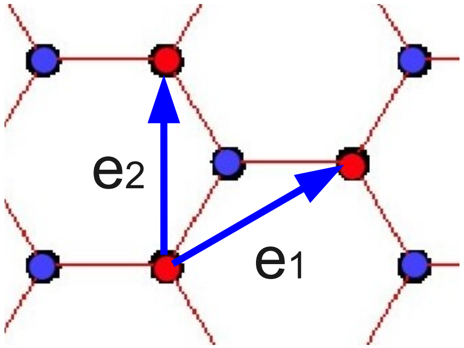
4-fold degenerac

Four flavors
(spin + KK')

The tight-binding model

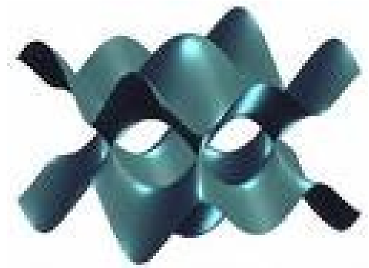
Tight-binding Hamiltonian with nearest-neighbor hopping $t_0 \approx 3.1\,eV$

In momentum representation:



$$H = \sum_{\boldsymbol{k}} t_{\boldsymbol{k}} \psi^{+}_{k,A} \psi_{k,B} + c.c.$$

$$t_{\boldsymbol{k}} = t_0 \left(1 + e^{-i\,\boldsymbol{k}\boldsymbol{e}_1} + e^{-i\,\boldsymbol{k}\boldsymbol{e}_2}\right)$$



Tight-binding Hamiltonian: $H_K = \begin{pmatrix} 0 & t_{\boldsymbol{k}} \\ t_{-\boldsymbol{k}} & 0 \end{pmatrix}$

Expand in the vicinity of points K, K'=-K:
massless Dirac Hamiltonian

$$t_k \approx v_F(\mp k_x - i\,k_y)$$

$$v_F = \frac{3}{2} t_0 a$$

Effects of interaction

- Wanted: kinetic energy << potential energy
- Large fine structure constant $\alpha = e^2/\hbar v_F \approx 2.5$
- But: low density of states at the Dirac point
- Interaction effects weak in undoped graphene,

  both a blessing and a curse
- Ways to strengthen effects of interaction:

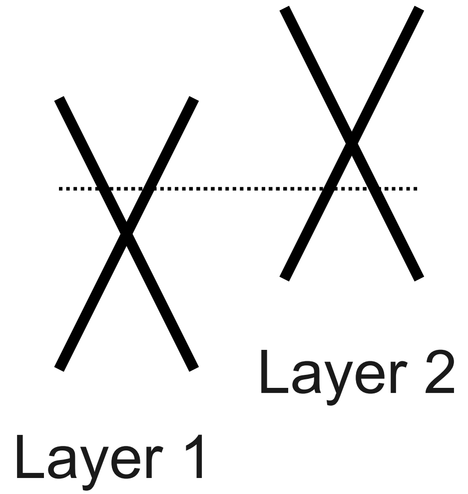
  (i) alter electronic states using external field;

  (ii) single layer doped to saddle point,

  (iii) bilayer at charge neutrality

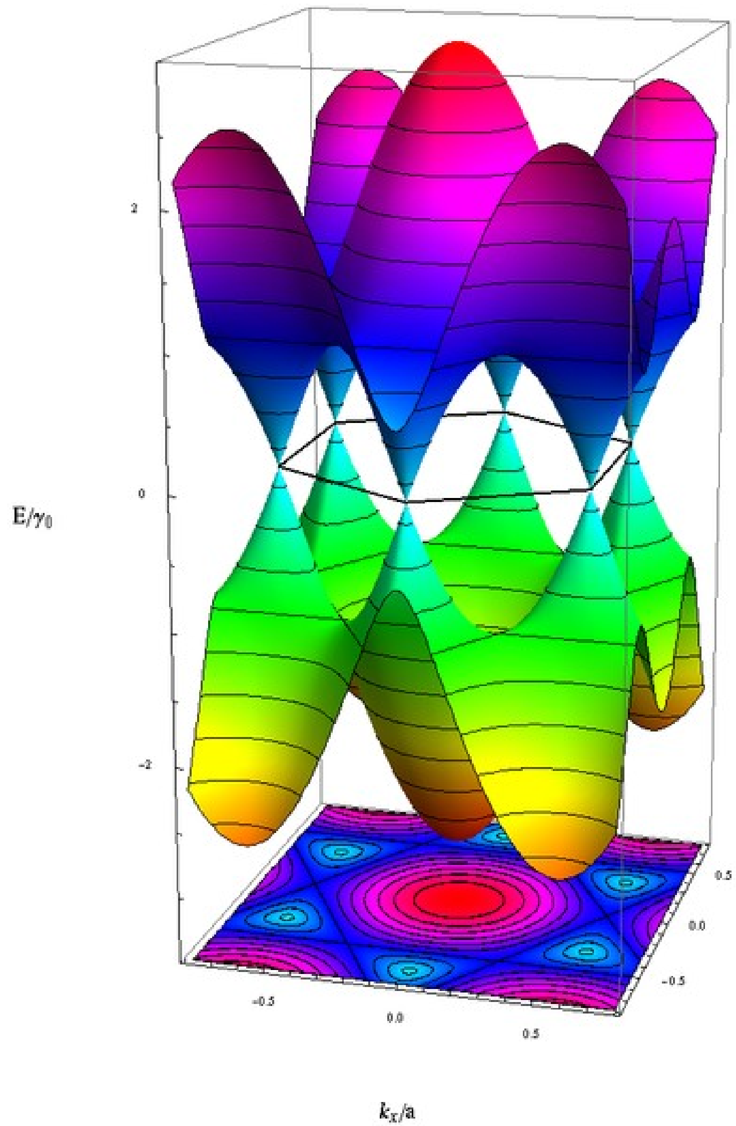
Proposals for gapped/ordered states

- In a single layer, gapped states can be induced by B field: M-catalysis (Gusynin, Miransky et al 1994), QHFM (Nomura & MacDonald 2006), has been observed (Checkelsky et al 2008)
- BCS-like excitonic instability in a pair of single layers (Min et al 2008, Kharitonov et al 2008): the energy scales Δ,Tc exponentially small b/c small DOS near the Dirac point
- BCS pairing in weakly doped monolayer (Uchoa, Castro Neto 2007, Kopnin, Sonin 2010)
- Graphene on lattice matched SiC or BN substrate
- Peierls instability and sublattice ordering of adatoms (Abanin, Shytov & LL 2010, Cheianov et al 2010)

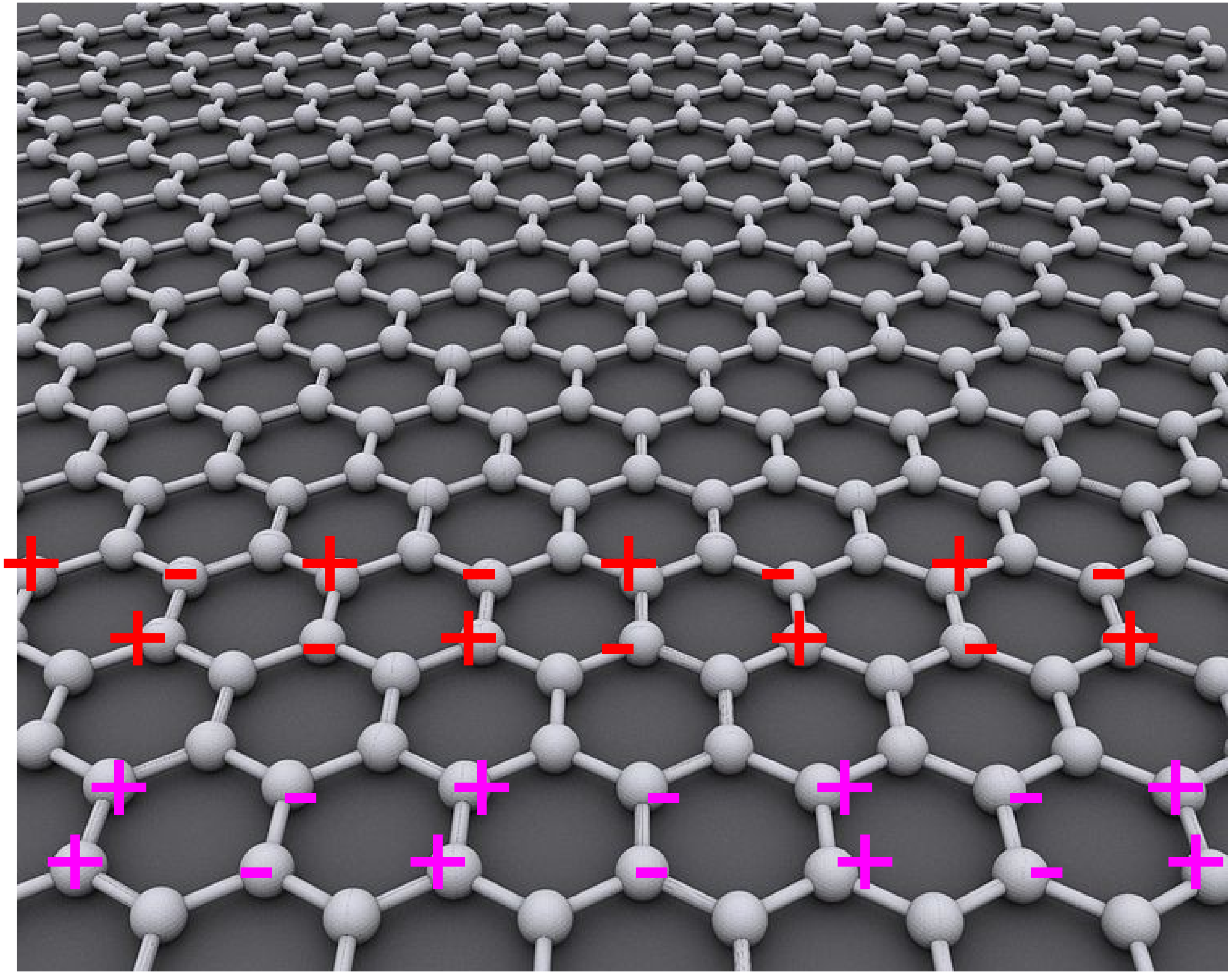


Electronic states in strongly doped graphene

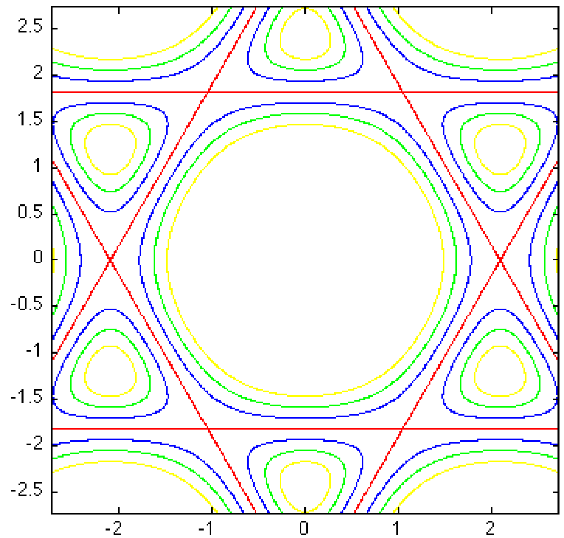
- Quadratic dispersion near saddle points at $E=+t_0,-t_0$
- Logarithmic V-H singularity
- Hexagonal FS @ n=3/8,5/8
- Nesting, enhancement of interaction effects
- *a la* square lattice @ half-filling
- Various competing orders: CDW, SDW, superconductivity, nematic order (Pomeranchuk)



Hexagonal Fermi surface: stripe states

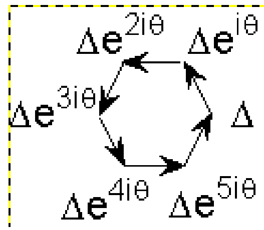


- Translational invariance, hexagonal FS
- 6 equivalent groups
- *a la* stripes @ square lattice



Chiral superconductivity from repulsive interactions

- Pairing gap winds in phase by multiples of 2π around the Fermi surface



- Induced by (weak) repulsive interactions
- d-wave pairing wins over s-wave pairing
- d+id state with *broken time reversal symmery*
- Once a candidate for high Tc, long abandoned
- Very rich phenomenology, as for p+ip states in 3He films, SrRuO, FQHE ν=5/2 (Volovik 1988, Laughlin 1998, Fu, Kane 2008, SC Zhang 2009):
  (i) charge QHE at B=0; (ii) spin and thermal QHE
  (iii) boundary charge current in B field
  (iv) Majorana states @ vortices and boundaries
  (v) Kerr effect, interesting Andreev states, etc

Previous work

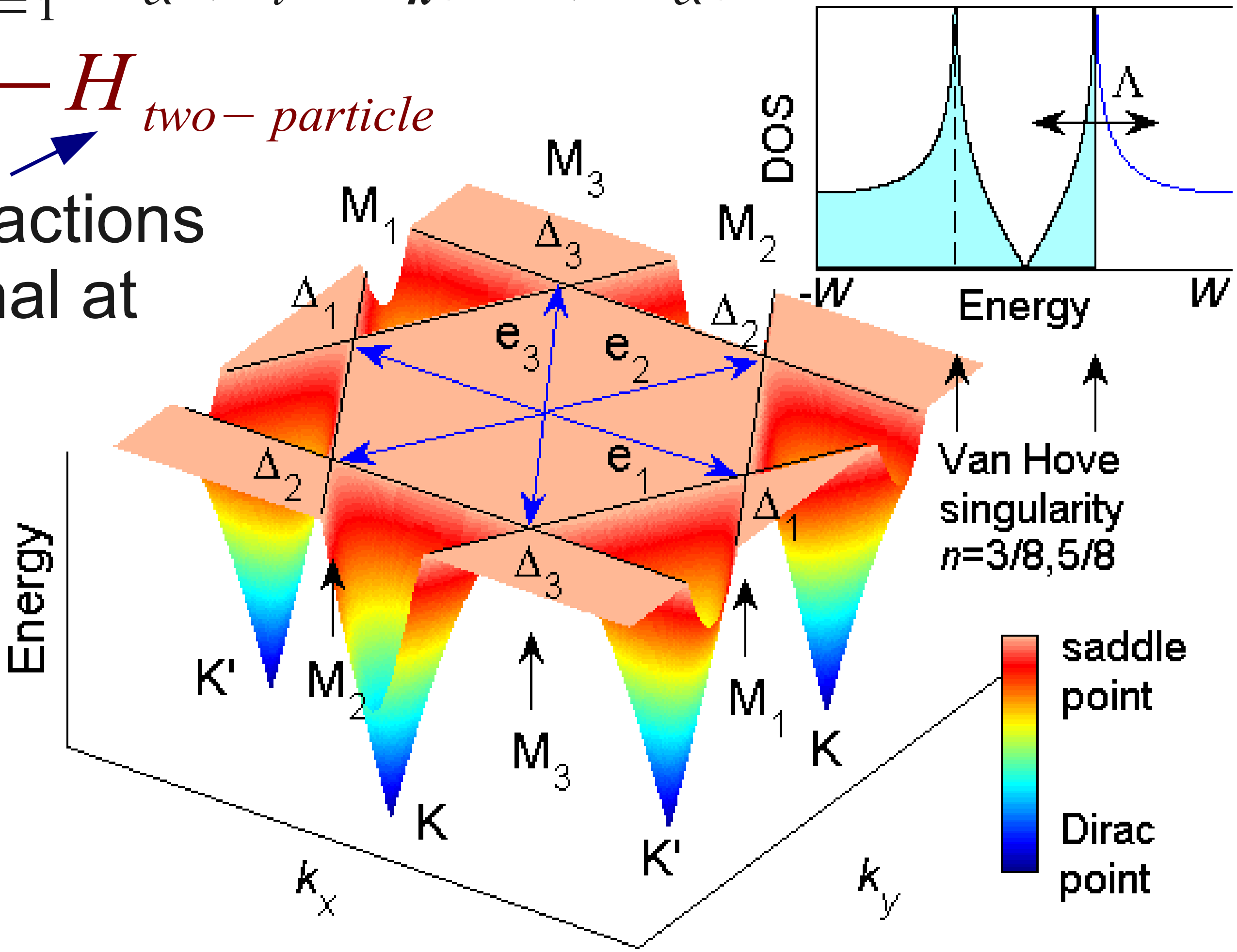
- d-wave pairing, Kohn-Luttinger framework
  (Gonzalez 2008)
- Pomeranchuk (nematic) order, mean field
  (Valenzuelo, Vozmediano 2008)
- SDW order, mean field
  (Li arxiv:1103.2420, Makogon et al arxiv:1104.5334)
- Legitimate mean-field states: superconductor, metal, insulator
- Need RG to compare these orders on equal footing

Low energy description: three inequivalent patches

$$L = \sum_{\alpha=1}^{3} \psi_\alpha^{+} \left(\partial_t - \epsilon_{\boldsymbol{k}} + \mu \right) \psi_\alpha$$

$$- H_{two-particle}$$

four interactions
(i) marginal at tree level
(ii) log's



Weak coupling theory

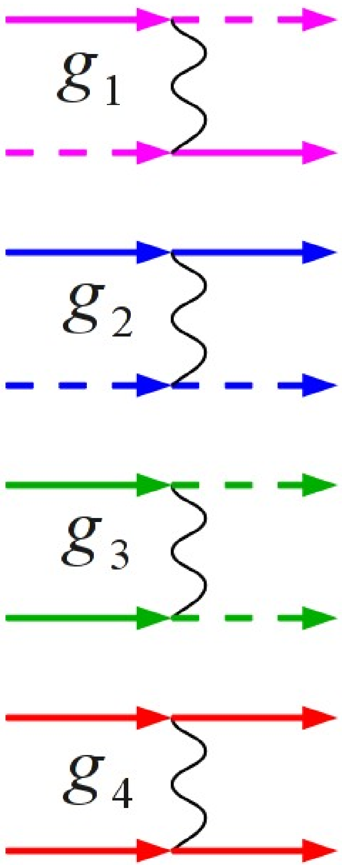
- Two sources of log divergence: DOS (Van Hove divergence), pairing susceptibility (BCS divergence)
- log^2 divergent diagrams
- One-loop RG summation of leading log-divergent contributions
- Similar to square lattice at half filling:
  Shulz 1987, Dzyaloshinskii 1987, Furukawa, Rice 1998
- Different scenario for competition of SDW and SC

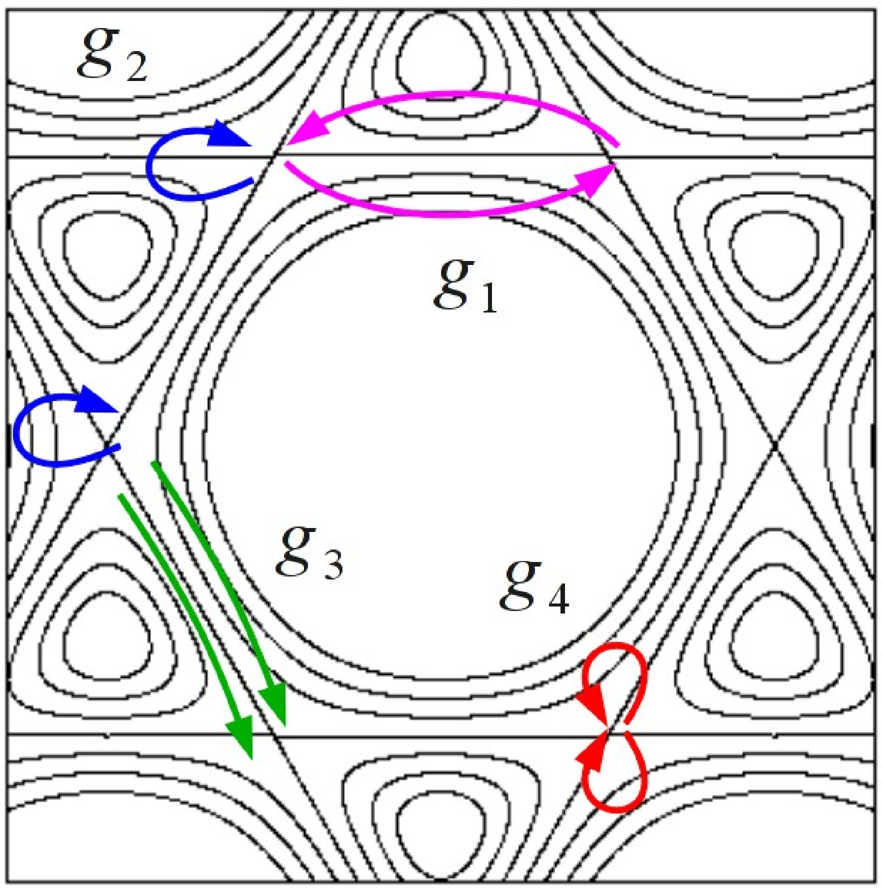
$$L=\sum_{\alpha=1}^{3}\psi_{\alpha}^{+}\left(\partial_{t}-\epsilon_{k}+\mu\right)\psi_{\alpha}-H_{two-particle}$$

Two-particle inter-patch scattering processes

$$H_{two-particle} = \sum_{\alpha,\beta=1}^{3} \frac{g_1}{2} \psi_\alpha^+ \psi_\beta^+ \psi_\alpha \psi_\beta + \frac{g_2}{2} \psi_\alpha^+ \psi_\beta^+ \psi_\beta \psi_\alpha + \frac{g_3}{2} \psi_\alpha^+ \psi_\alpha^+ \psi_\beta \psi_\beta$$

$$+ \sum_{\alpha=1}^{3} \frac{g_4}{2} \psi_\alpha^+ \psi_\alpha^+ \psi_\alpha \psi_\alpha$$





Diverging susceptibilities

SC pairing (spin-up, spin-down)

$$\Pi_{pp}(0)=\frac{\nu_0}{4}\ln\frac{\Lambda}{max(\mu,T)}\ln\frac{\Lambda}{T}$$

SDW susceptibility

$$\Pi_{ph}(Q_i)=\frac{\nu_0}{4}\ln\frac{\Lambda}{max(\mu,T)}\ln\frac{\Lambda}{max(\mu,T,t_3)}$$

Imperfect nesting

Lesser susceptibilities:

$$\Pi_{pp}(Q_i),\Pi_{ph}(0)=\frac{\nu_0}{4}\ln\frac{\Lambda}{max(\mu,T)}$$

RG flow of coulpings for n patches

$$\frac{dg_1}{dy} = 2d_1 g_1 (g_2 - g_1) \qquad \frac{dg_2}{dy} = 2d_1 (g_2^2 + g_3^2) \qquad \frac{dg_4}{dy} = -(n-1) g_3^2 - g_4^2$$

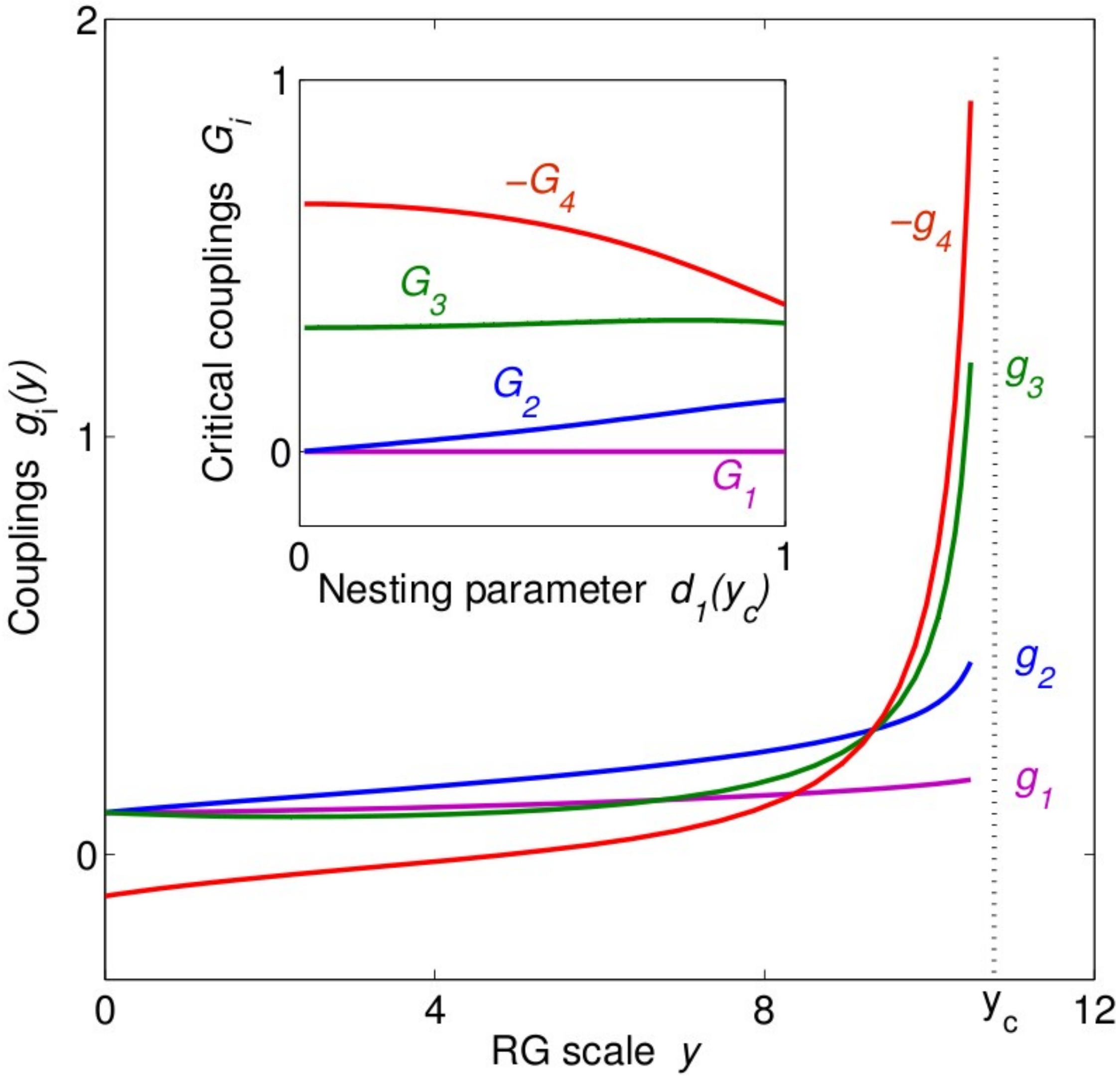
$$\frac{dg_3}{dy} = -(n-2) g_3^2 - 2g_3 g_4 + 2d_1 g_3 (2g_2 - g_1)$$

RG time $y = \Pi_{pp}(0)$

Nesting parameter $d_1 = \frac{d\,\Pi_{ph}(Q)}{d\,\Pi_{pp}(0)} < 1$

Critical couplings $g_i(y) \approx \frac{G_i}{y_c - y}$

Initial values $g_i(y=0) \approx 0.1$



RG flow features

- agrees with the square lattice (n=2)
- one stable fixed point
- g1, g3, g2 cannot change sign, stay positive
- g4 decreases & reverses sign
- g3-g4 large & positive, drives SC instability
- positive g3 penalizes s-wave,

  favors d-wave SC
- Analyze susceptibilities:

  χ_{sc} diverges faster than χ_{sdw}
- In contrast to the square lattice,

  SC a clear winner

Enhancement of Tc at weak coupling

$$T_c \approx \Lambda\, e^{-\frac{A}{\sqrt{g_0 \nu_0}}}$$

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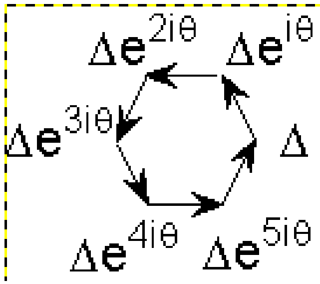
$$T_c \approx \Lambda e^{-\frac{A}{\sqrt{g_0 \nu_0}}}$$

Competition of d-wave orders below Tc

- By symmetry, two degenerate d-wave states
- Ginzburg-Landau analysis of competiton

$$\Delta = \Delta_a (x^2 - y^2) + \Delta_b 2xy$$

$$F(\Delta_a, \Delta_b) = \alpha (T - T_c)(|\Delta_a|^2 + |\Delta_b|^2) + K_1 (|\Delta_a|^2 + |\Delta_b|^2)^2 + K_2 \left|\Delta_a^2 + \Delta_b^2\right|^2$$



- Calculation of GL functional yields $K_2 > 0$
- d+id and d-id ground states $\Delta_a = \pm \Delta_b$
- Superconductivity with TRS breaking

Summary: chiral superconductivity in highly doped graphene

- Interaction driven instability in graphene doped at saddle points
- Weak repulsive interaction stabilizes chiral superconducting state d+id or d-id
- Enhanced Tc
- Topological superconductor with broken TRS
- Zoo of interesting phenomena
- Graphene is exceptionally easy to combine with other materials into hybrid structures and heterostructures: pathway to applications of chiral superconductivity

we're not done yet

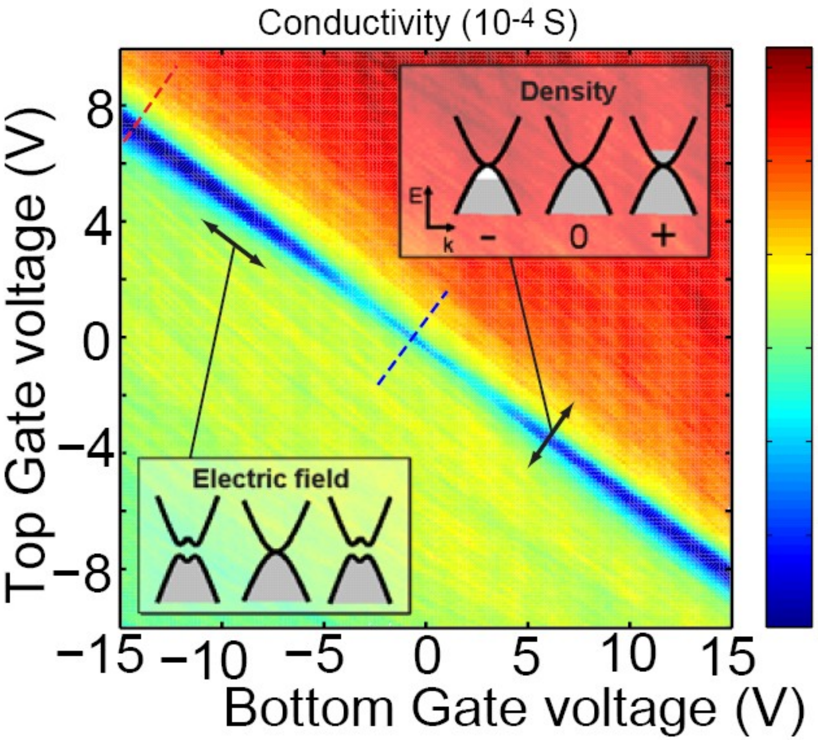
Spontaneously ordered states in bilayer graphene

- Excitonic instability in bilayer graphene
- SU(4) flavor symmetry and relation between different proposals
- Time Reversal Symmetry Breaking at E=B=0 (Anomalous Hall Insulator)
- Recent measurements (Yacoby's group)

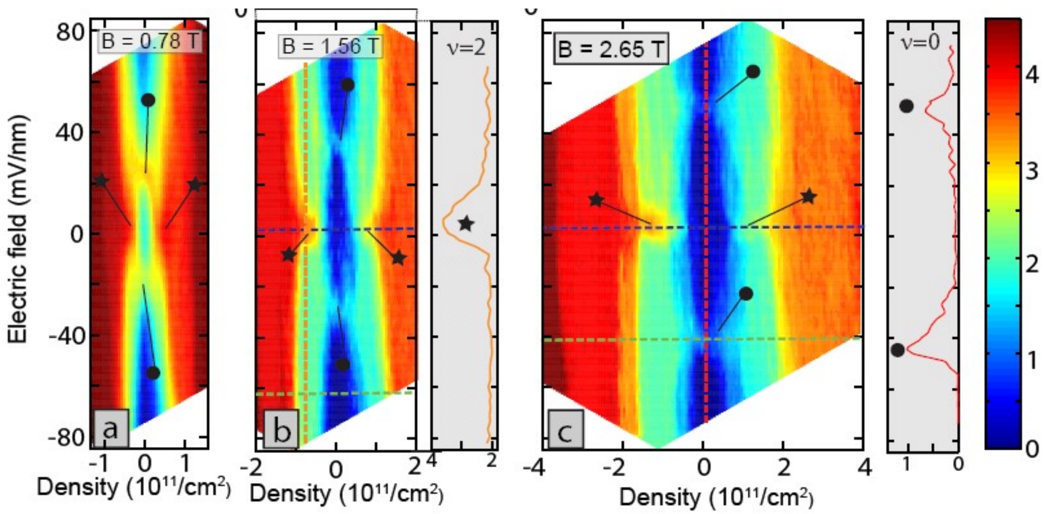
Nandkishore & LL, PRL 104, 156803 (2010), PRB 82, 115124 (2010)

Gapped states in double gated suspended bilayer

Different states realized at different E and B field values:
QHFM at high B/E, layer-polarized state at high E/B



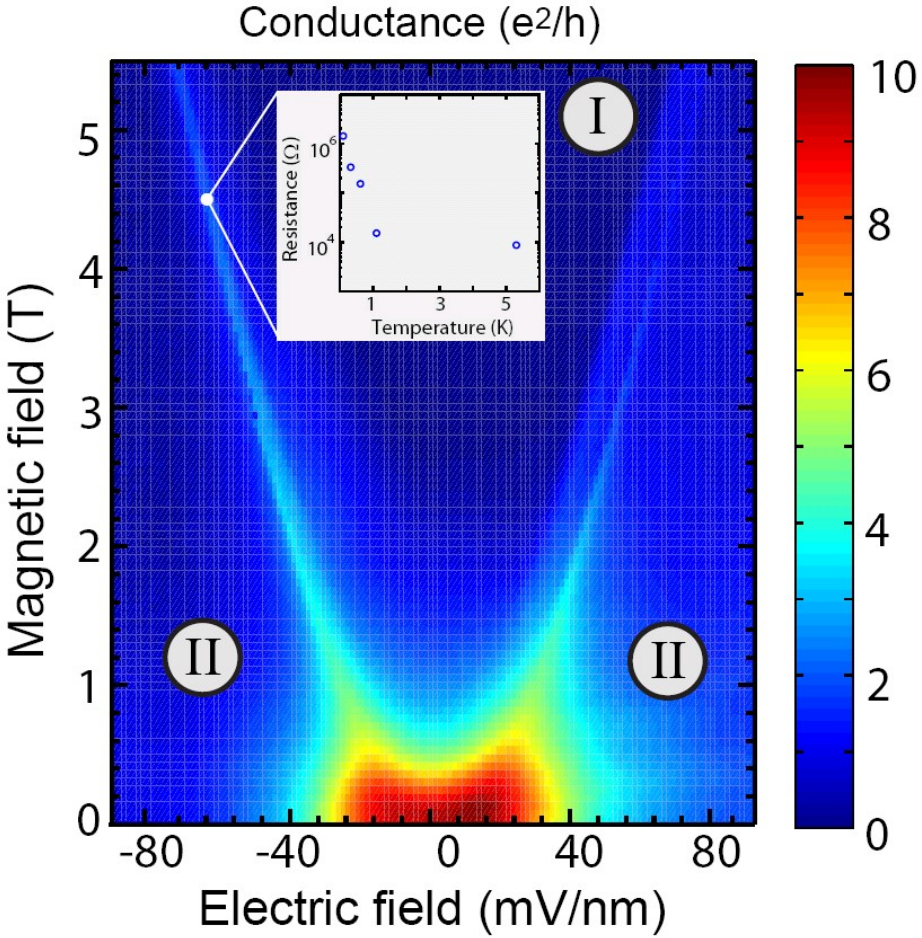
Feldman et al 2009, Allen et al 2010



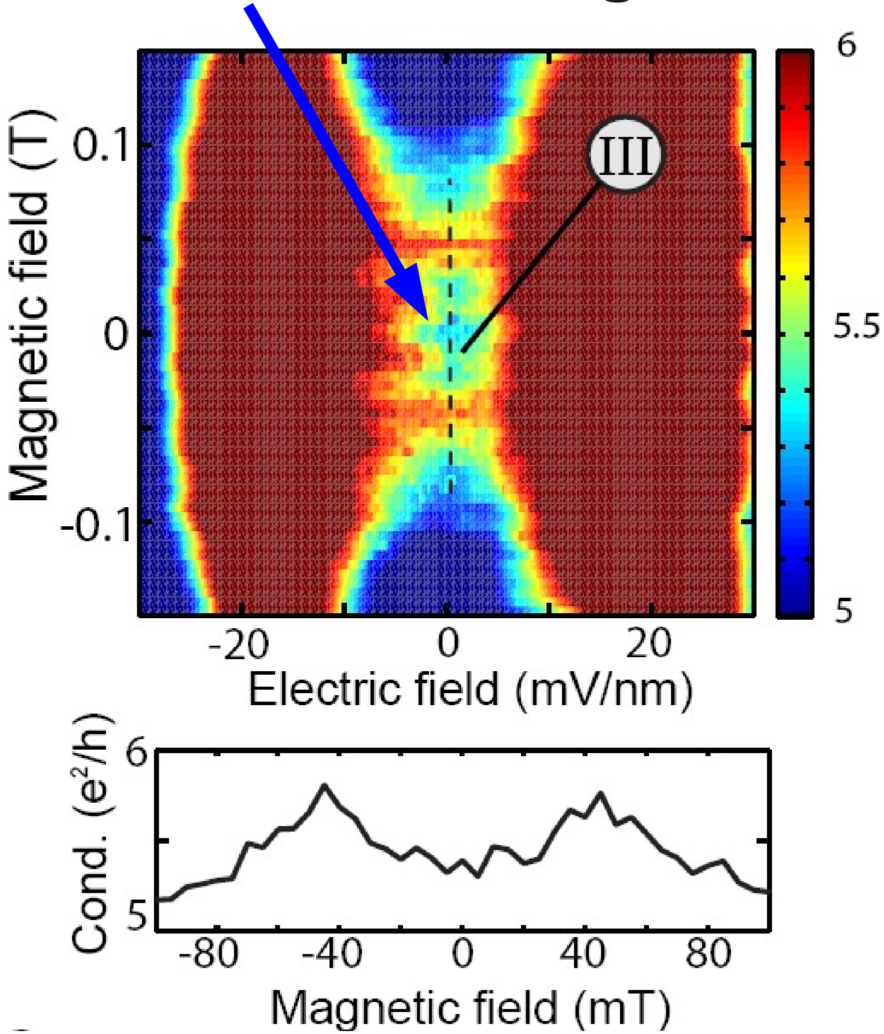
Gap at ν=0 for B>0.1 Tesla; also, suspended monolayer (X. Du et al 2010)

Measured phase diagram

- Distinct gapped states at B=0 and high B
- Phase boundary: linear E vs. B



New insulating state?



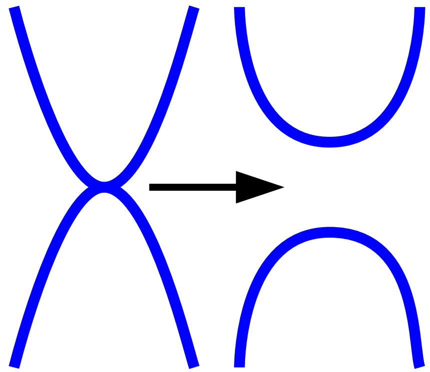
Allen et al 2010

Natural (indigenous) Excitonic Insulator at B,E=0

Min et al 2008;
Nandkishore & LL 2010
Zhang et al 2010

- Particle-hole pairing instability
- BCS-like condensate, no superfluifity, phase locking:

$$\Delta = \pm \Delta_0$$



- Gapped spectrum
- Another candidate: “nematic” order, gapless spectrum, broken rotational symmetry

Vafek, Yang 2010;
Lemonik et al 2010

$$H_{nema} = \begin{pmatrix} 0 & \frac{p_-^2}{2m} + \Delta \\ \frac{p_+^2}{2m} + \Delta & 0 \end{pmatrix} \qquad H_{gaped} = \begin{pmatrix} \Delta & \frac{p_-^2}{2m} \\ \frac{p_+^2}{2m} & -\Delta \end{pmatrix}$$

Excitonic order induced by 1/r interaction

Dynamically generated UV cutoff (characteristic "Rydberg energy" and "Bohr radius")

$$E_0 = \frac{m e^4}{\kappa^2} = \frac{1.47}{\kappa^2} eV \qquad a_0 = \frac{\kappa}{m e^2} = \kappa \times 1.1\, nm$$

- Semimetal with parabolic dispersion, finite DOS at low energies
- Weak interactions can trigger instability: gap opening, 'which-layer' symmetry breaking
- The gap Δ scales as a power law of interaction
- Δ may reach 10-20 K in a clean system

$$\Delta \approx 10^{-3} E_0$$

Analysis for 1/r interaction in:
Nandkishore & LL, PRL 104, 156803 (2010)

Large variety of possible states

$$H_K = \begin{pmatrix} \Delta_K & p_-^2/2\mathrm{m} \\ p_+^2/2\mathrm{m} & -\Delta_K \end{pmatrix} \qquad H_{K'} = \begin{pmatrix} \Delta_{K'} & p_+^2/2\mathrm{m} \\ p_-^2/2\mathrm{m} & -\Delta_{K'} \end{pmatrix}$$

$$\Delta_{K,\sigma} = \pm\Delta_{K',\sigma} = \pm\Delta_{K,-\sigma} = \pm\Delta_{K',-\sigma} \qquad p_\pm = p_1 \pm i\, p_2$$

- Four-fold spin/valley degeneracy
- Many gapped states: valley “antiferromagnet”, ferromagnetic, ferrimagnetic, ferroelectric, etc (Min et al 2008, Nandkishore & LL 2010, Zhang et al 2010)
- Degeneracy on a mean field level: instability threshold **the same for all states**: short-range interaction, screened long-range interaction models
- SU(4) symmetry?

Opposite chirality of two valleys conceals SU(4) symmetry, made manifest by performing unitary transformation

$$H_0 = \frac{p_+^2}{2m}\tilde{\tau}_- + \frac{p_-^2}{2m}\tilde{\tau}_+,$$

Approximate SU(4) symmetry (weakly broken by trigonal warping and capacitor energy)

$$H = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^{\dagger} H_0 \psi_{\mathbf{p}} + \frac{1}{2}\sum_{\mathbf{q}} V_+(q)\rho_{\mathbf{q}}\rho_{-\mathbf{q}} + V_-\lambda_{\mathbf{q}}\lambda_{-\mathbf{q}},$$

Strategy: Diagonalise SU(4) invariant Hamiltonian and incorporate anisotropies perturbatively

General mean field description of gapped states

$$H = \frac{p_+^2 \tau_- + p_-^2 \tau_+}{2m} + \Delta \tau_3 Q,$$

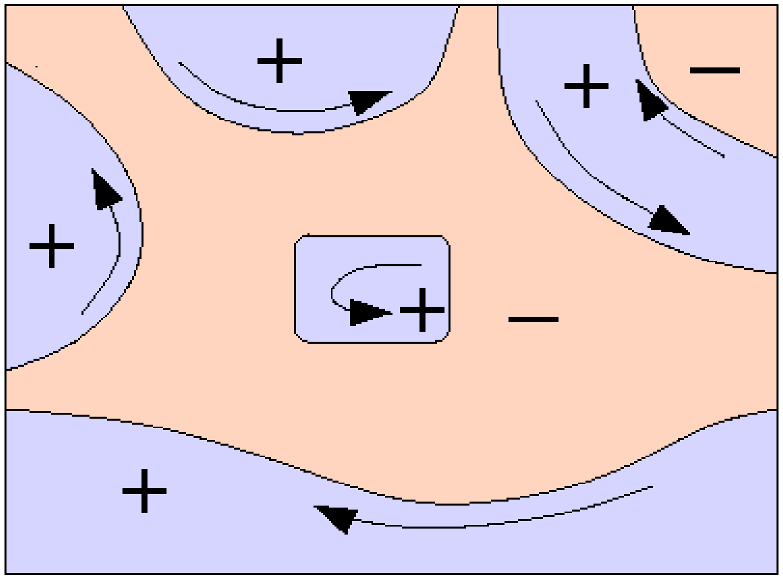
Classification into manifolds (4,0), (3,1), (2,2) and distinction between symmetry protected and accidental degeneracies

$$\sigma_{xy} = (M_> - M_<) \frac{e^2}{h},$$

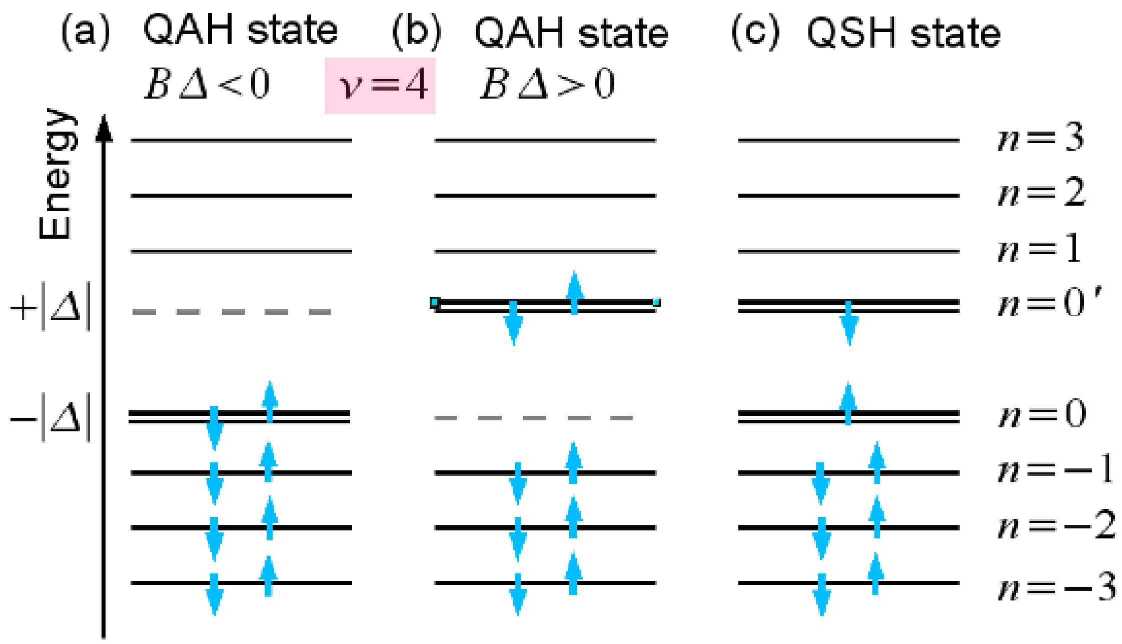
Therefore, (4,0) and (3,1) are QAH states, exhibit QHE at B=0

Edge state picture of various states

- 'Which-layer' symmetry breaking
- Domains of + and – polarization in a uniform system, domain size controled by long-range dipole interactions
- Charge, valley or spin polarized current along domain boundaries, QHE, VQHE, SQHE, etc



Inducing QAH state with external B field



Broken T symmetry at B=0, broken particle-hole symmetry at finite B. Therefore, at small non zero B and ν=4, the QAH state is preferred.

Generic mechanism/However, no spontaneous breaking of T symmetry at finite B...

- Investigate lifting of accidental degeneracies by thermal and zero point fluctuations, a `saddle point plus quadratic fluctuations' approximation.
- Thermal fluctuations favor (2,2) states (entropic reasoning)
- Zero point fluctuations favor QAH state (influence of nematic fluctuations which are unique to BLG).
- Therefore, QAH favored below critical temperature (of order Δ). *Possible realization of QAH state in BLG.*
- Caveat: Fluctuation analysis is uncontrolled.

Signatures of AHI state

Two anomalous Landau levels per flavor: total 2x4=8

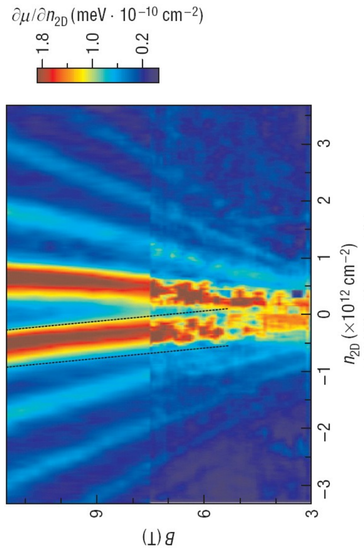
Each zero-energy state filled (unfilled) contributes +1/2(-1/2) of an electron macroscopically: (1/2)xLL density

Expect incompressible states at ν=+4 and ν=-4 even @ B=0

Local compressibility measurement

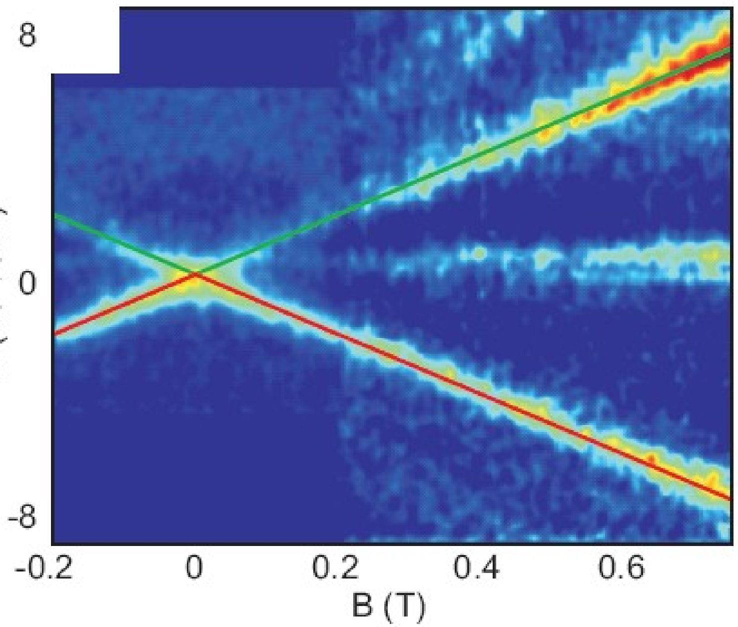
Martin et al 2010

High field



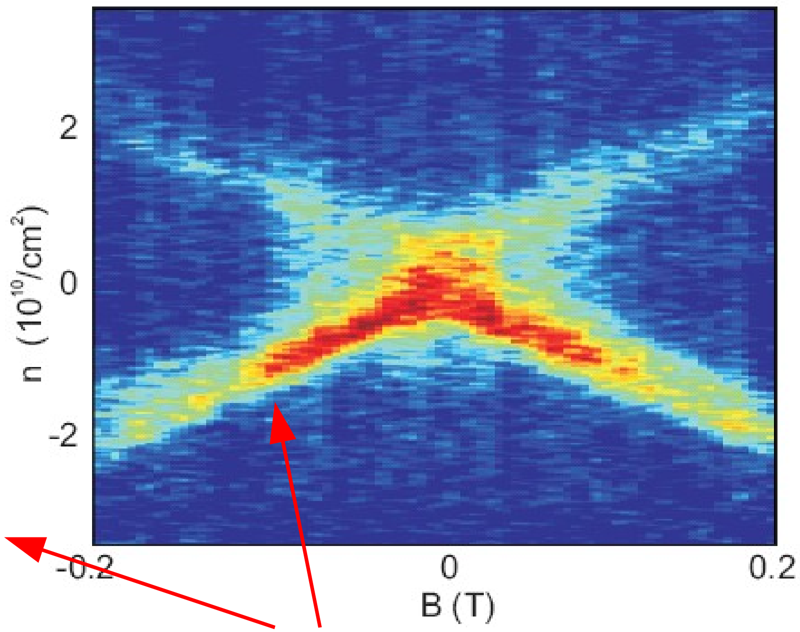
Low field

inverse compressibility dμ/dn



Zero field

inverse compressibility dμ/dn



$$n = \pm 4 \times eB/h$$

- Incompressible ν=4 states at **very low B**:
- Suggests anomalous quantized Hall effect in the B=E=0 state with

$$\sigma_{xy} = \pm 4\, e^2/h$$

Experimental signatures

- Gold standard: measurement of QHE at B=0; requires four-probe measurment on suspended BLG at low T
- Contactless, optical detection of TRS breaking (prediction of large polar Kerr effect in QAH state, Nandkishore & LL arxiv: 1105.5142)
- TRS breaking via violation of Onsager symmetry B,-B in a four-probe measurement

QAH state not yet observed

Experiments compatible with QAH state

- Incompressible (bulk gap)+finite two-probe conductivity; distinguishes QAH state from (2,2) state but not from nematic state or trigonal warping
- Phase transition at zero ν, finite B to (2,2) QHFM state (likewise)
- Incompressible regions at low B, n=4 (if field induced), n=+4 and n=-4 (if intrinsic); no such feature at higher filling factor (unlike nematic or other states)
- Phase transition at finite E to trivial insulator (Ising universality class)

Summary

- Inducing QAH state with B fieldRich pattern of phases, SU(4) classification
- Possibility of realizing QAH state at low T
- Inducing QAH state with B field
- Experimental signatures