# Stream Function and Mean Meridional circulation Variability

Somia A. Mohamed

Mohamed I. Omar

Zeinab S. Mahmoud

Tamer A. Nada

Antonios F. Mekhiel

# Objective

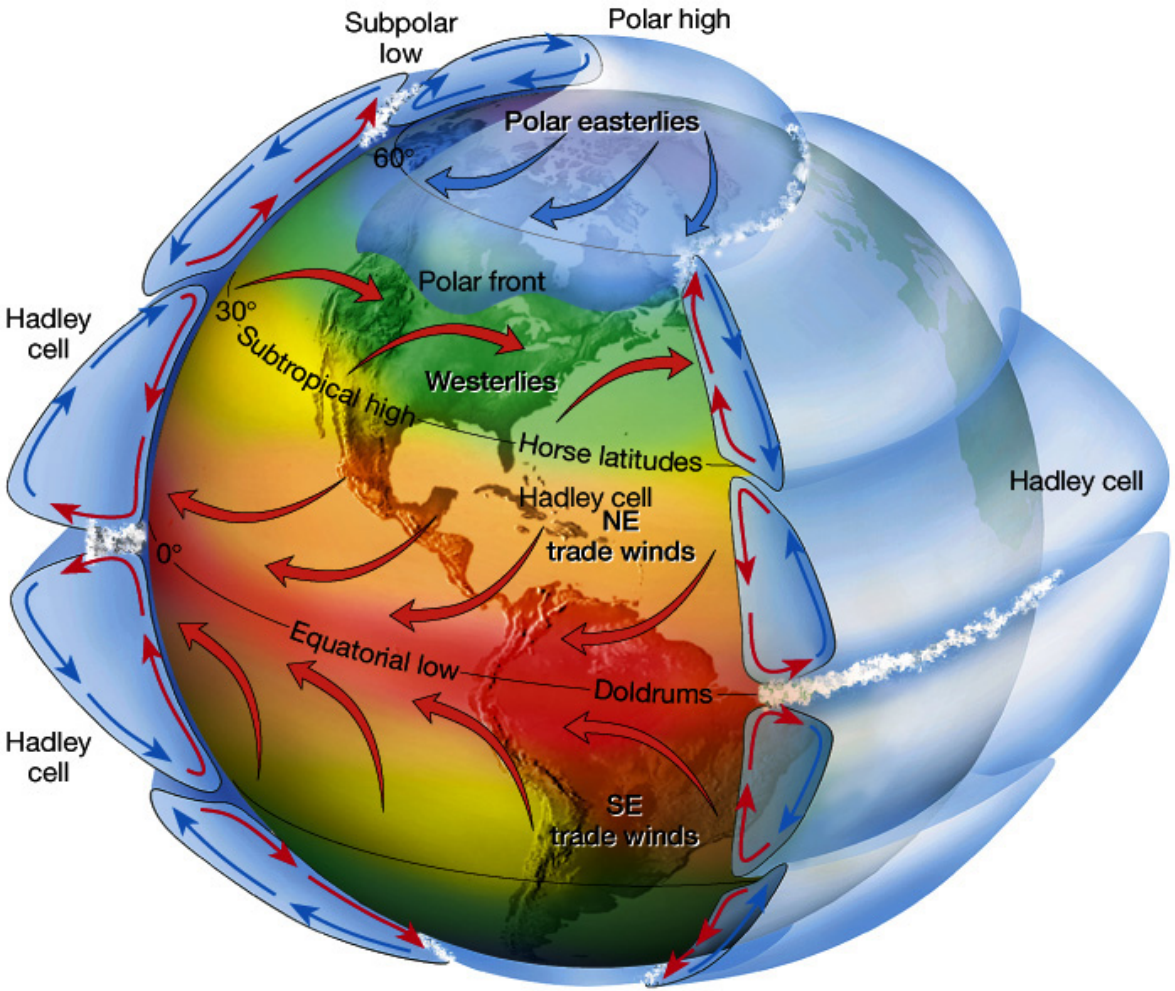
- Understanding Basic features of General Circulation

**What do we mean by circulation?**

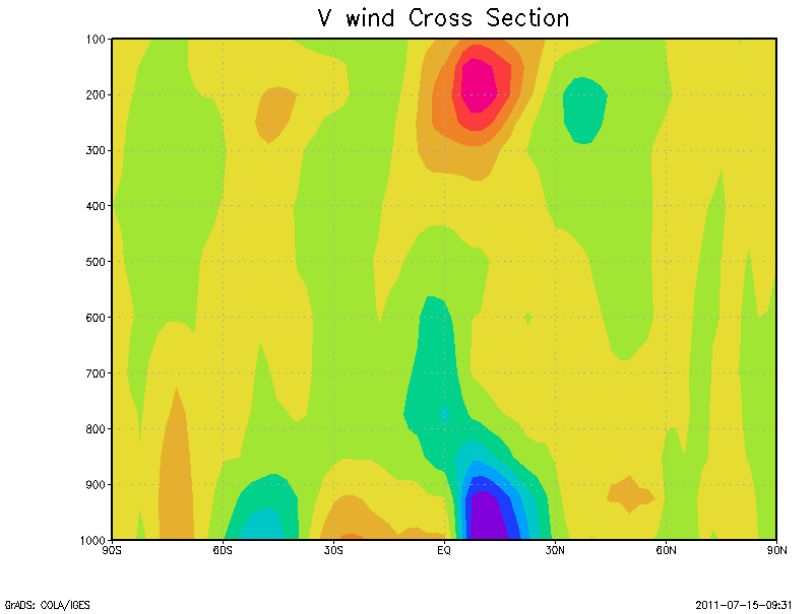
# Outlines

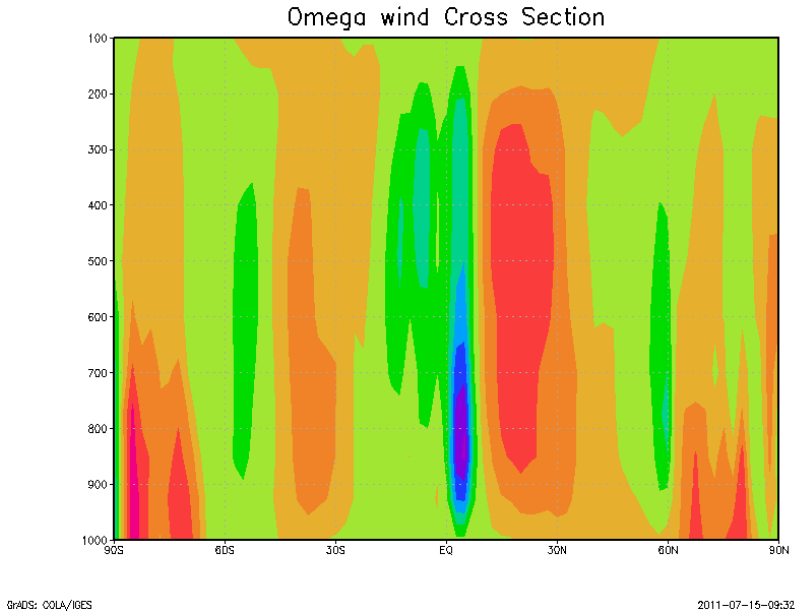
- Introduction to General Atmospheric Circulation
- Hadley Cell
- Ferrel Cell

Cartoon of proposed general circulation



# Meridional circulation (v,w)




Are not useful to see the circulation

# Stream Function (Ψ)

The mass divergence in the meridional plane is zero (conservation of mass)

$$\nabla \cdot V = 0$$

The stream funciton is a scalar field whose relationship to V is carefully selected to automatically satisfied the continuity

$$[v] = \frac{g}{2\pi \cos\varphi} \frac{\partial \psi}{\partial p}$$

$$[\omega] = -\frac{g}{2\pi \cos\varphi} \frac{\partial \psi}{\partial \varphi}$$

$$\psi_p(\phi, p) = \frac{2\pi \cos\phi}{g} \int_p^{ps} [v] dp$$

# Properties of Stream Function:

1. Stream function always satisfies the continuity equation.

2. Stream function is constant for a stream line.

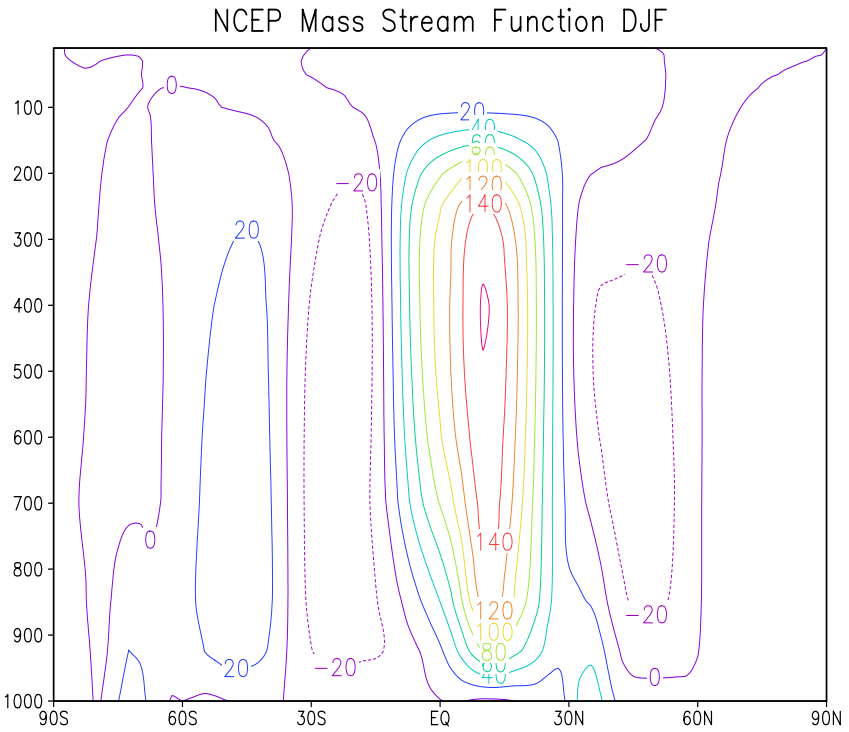
3. Stream function values vary from one stream line to another.

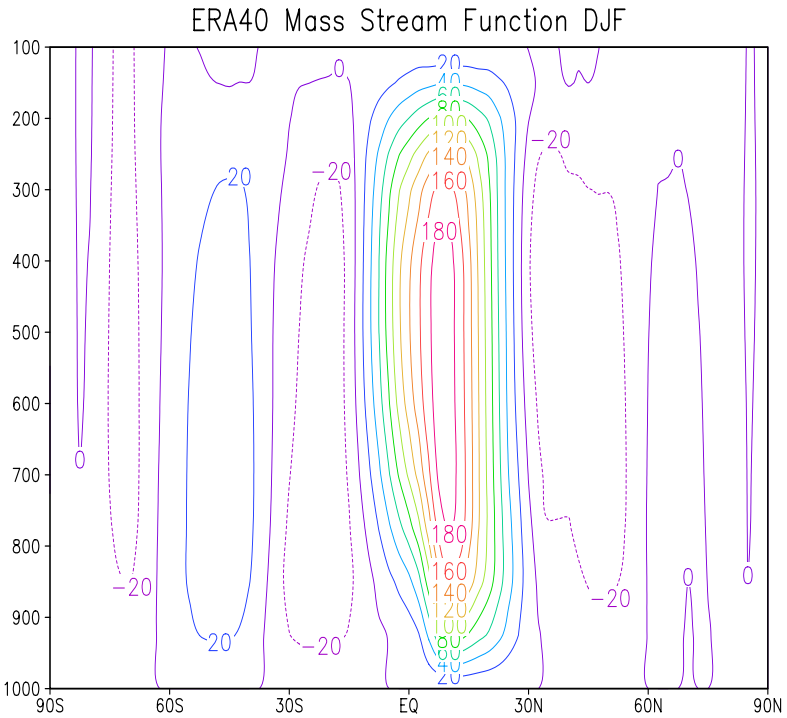
4. Stream function represents volumetric flux .i.e. volumetric flow rate per unit area. Units of stream function are kg/ms.

# Zonal Average Views

Zonal Average of x = [x]
x - [x] = x* = deviation from the zonal average
u-[u] = u*
v-[v] = v*
t-[t] = t*

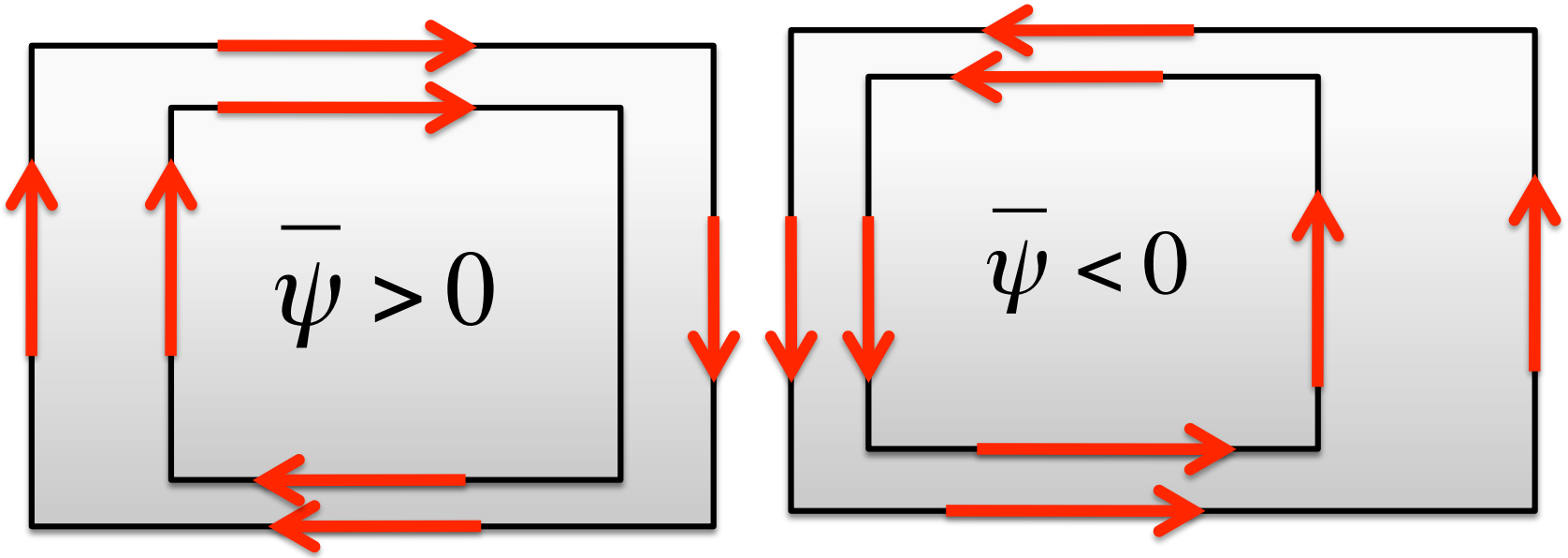
# Zonal mean Stream function



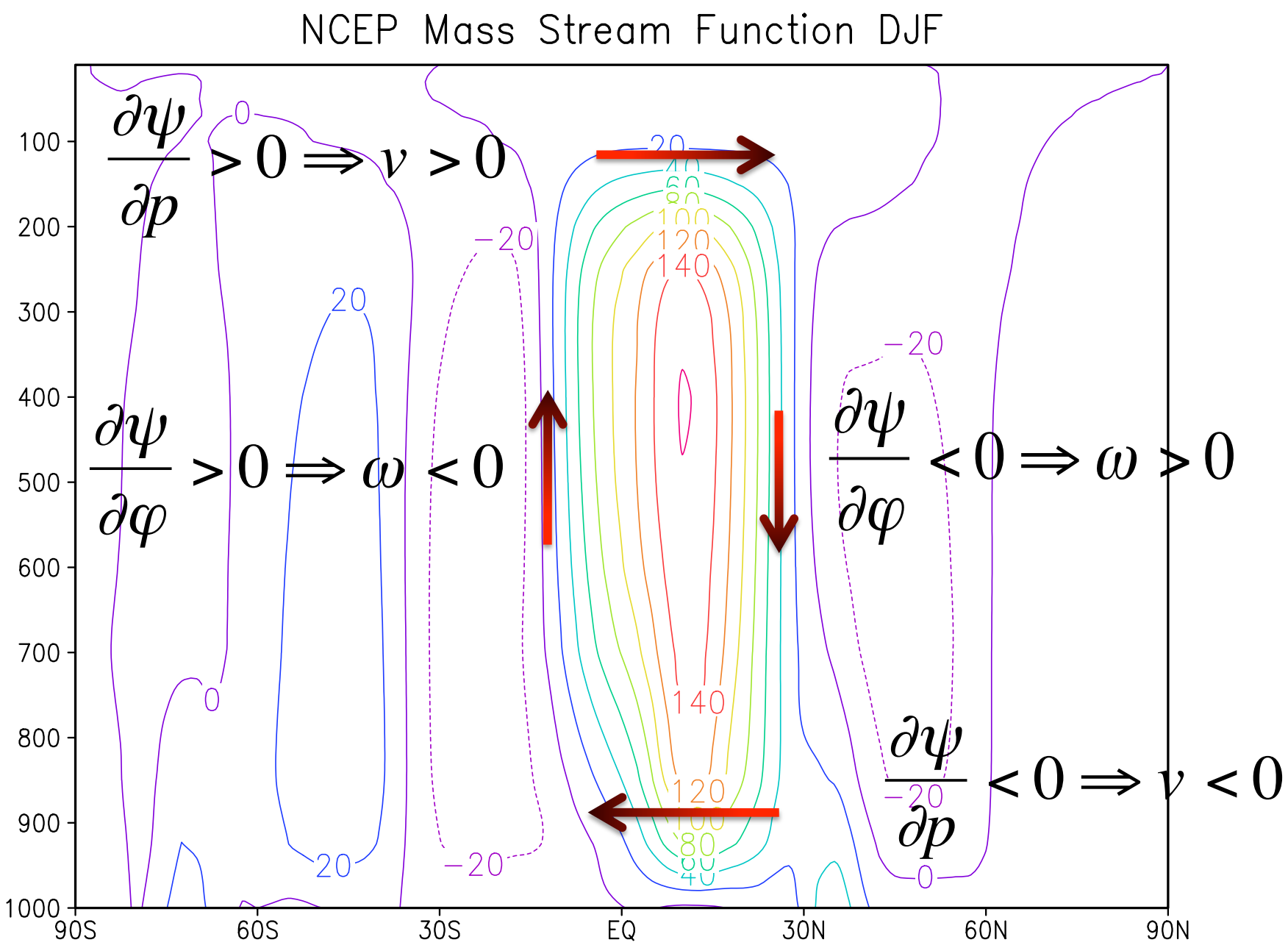

$$[\omega] = -\frac{g}{2\pi\cos\varphi}\frac{\partial\psi}{\partial\varphi}$$

$$[v] = \frac{g}{2\pi\cos\varphi}\frac{\partial\psi}{\partial p}$$

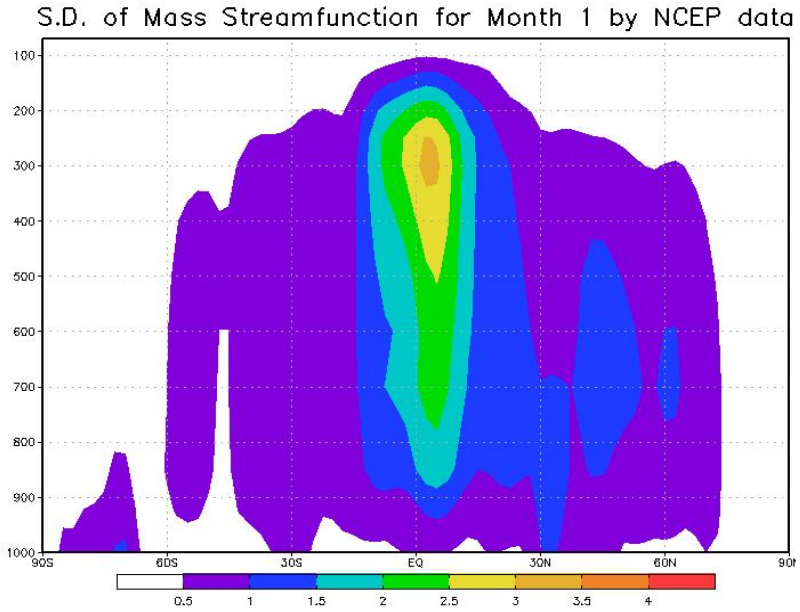


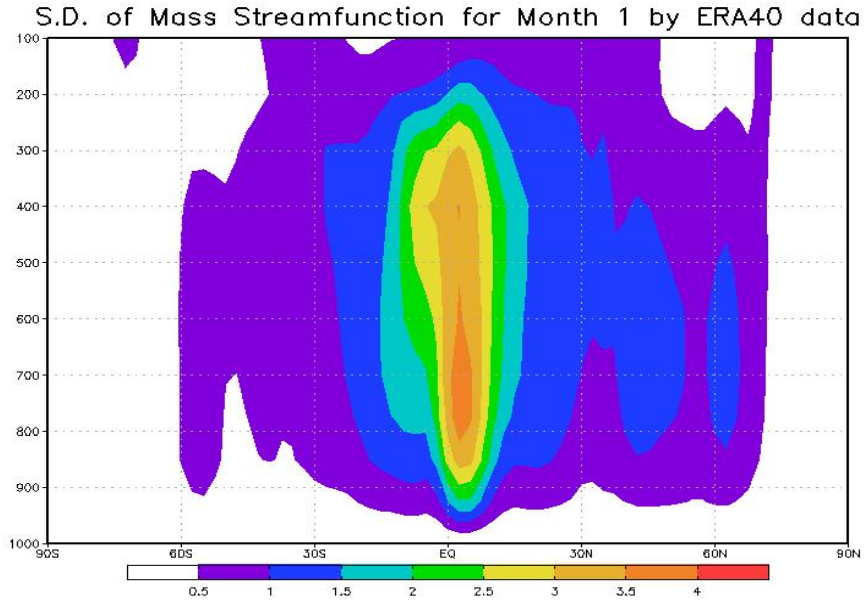

Thermally induced cell

Indirect cell


NCEP Mass Stream Function DJF
$\frac{\partial \psi}{\partial p} > 0 \Rightarrow v > 0$
$\frac{\partial \psi}{\partial \varphi} > 0 \Rightarrow \omega < 0$
$\frac{\partial \psi}{\partial \varphi} < 0 \Rightarrow \omega > 0$
$\frac{\partial \psi}{\partial p} < 0 \Rightarrow v < 0$
100
200
300
400
500
600
700
800
900
1000
90S
60S
30S
EQ
30N
60N
90N
0
20
−20
140
120
80
40

# Hadley Cell Standard deviation



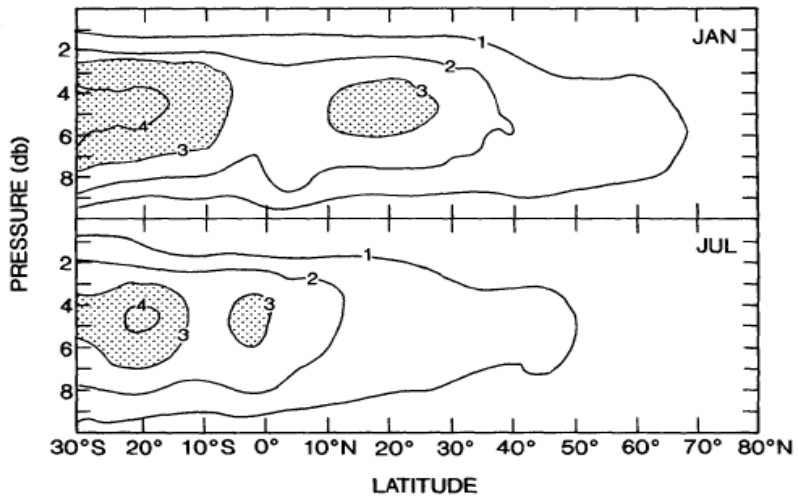




FIG. 4. Interannual standard deviation of the zonal-mean mass streamfunction $\sigma(\psi)$ for January and July in units of $10^{10}$ kg s$^{-1}$.

Oort and Yienger 1996 J. of Climate

Hadley cell Standard deviation shows maximum variability however, Ferrel cell standard deviation shows minimum variability. Since geostrophic balance and hydrostatic balance mostly hold all the time in midlatitude.

# Ferrel Cell (indirect cell)
# Eddy transport

Departure from mean zonal flow (subtropical jet stream) destroys thermal wind balance, which induces the meridioanl circulation to maintain thermal wind balance again

$$\frac{\partial^2 \overline{\chi}}{\partial y^2} + \frac{f_\circ^2}{N^2}\rho_\circ \frac{\partial}{\partial z}\left(\frac{1}{\rho_\circ}\frac{\partial \overline{\chi}}{\partial z}\right) = \frac{\rho_\circ}{N^2}\left[\frac{\partial}{\partial y}\left(\frac{\kappa \overline{J}}{H} - \frac{R}{H}\frac{\partial}{\partial y}\left(\overline{v'T'}\right)\right)\right]$$

$$-f_\circ\left(\frac{\partial^2}{\partial z \partial y}\left(\overline{u'v'}\right) - \frac{\partial \overline{X}}{\partial z}\right)$$

Diagnosis equation for stream function

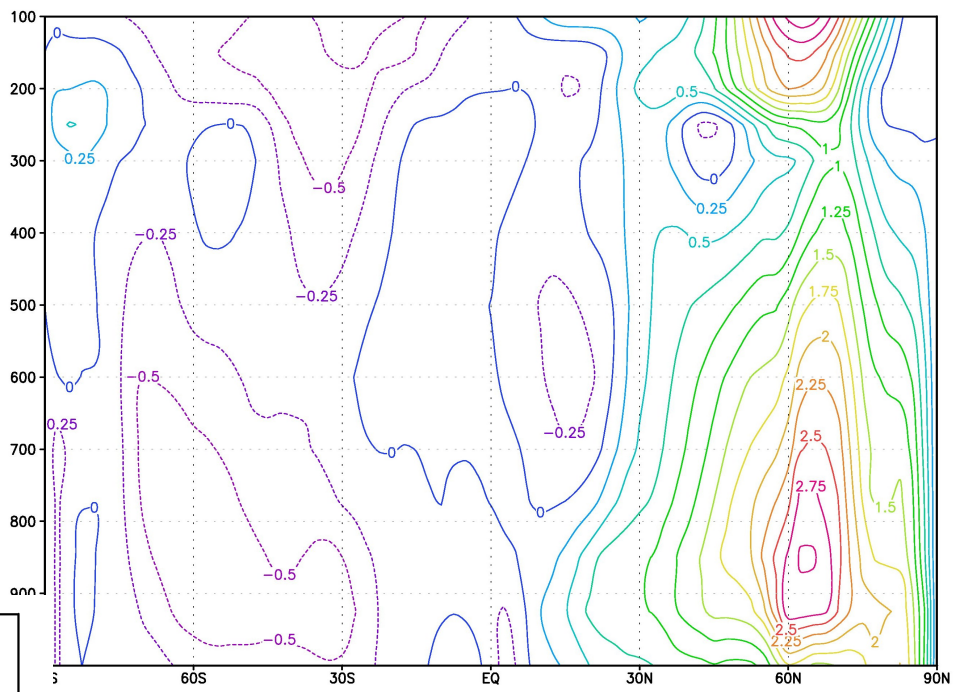
$$\bar{\chi}\ \alpha\ -\frac{\partial}{\partial y}\ (\text{diabatic heating})\ +\ \frac{\partial^2}{\partial y^2}\ (\text{large scale eddy heat flux})$$

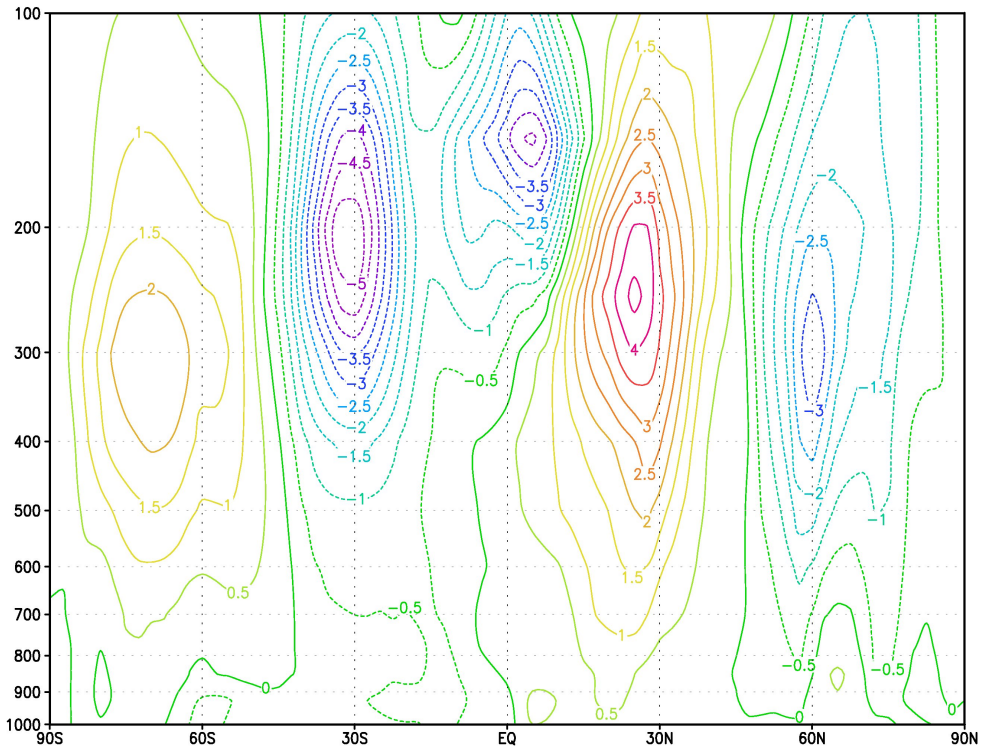
$$+\ \frac{\partial^2}{\partial y \partial z}\ (\text{large scale eddy momentum flux})\ +\ \frac{\partial}{\partial z}\ (\text{zonal drag force})$$

$$\frac{\partial^2 \bar{\chi}}{\partial y^2} + \frac{f_\circ^2}{N^2}\rho_\circ \frac{\partial}{\partial z}\left(\frac{1}{\rho_\circ}\frac{\partial \bar{\chi}}{\partial z}\right) = \frac{\rho_\circ}{N^2}\left[\frac{\partial}{\partial y}\left(\frac{\kappa \bar{J}}{H} - \frac{R}{H}\frac{\partial}{\partial y}\left(\overline{v'T'}\right)\right)\right]$$

$$-f_\circ\left(\frac{\partial^2}{\partial z \partial y}\left(\overline{u'v'}\right) - \frac{\partial \bar{X}}{\partial z}\right)$$

Eddy heat flux maximum at 50 N




Eddy momentum flux

# References

1-J. Holton, An Introduction to Dynamical Meteorogology
2-D. Strauss, presentation
3-Orrt and Yienger, *observed interanual in the hadley circulation and its connection to* ENSO, J. of Climate, 1996

Thanks
for your Attentions