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IAEA
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Advanced Workshop on Non-Standard Superfluids and Insulators

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Disordered Commensurate Bosons

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Disordered Commensurate Bosons

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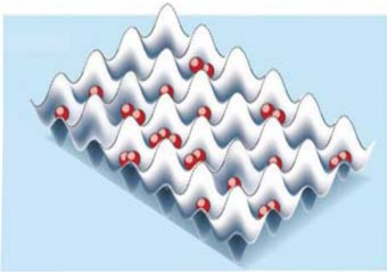
ICTP, Trieste, 19 July 2011

Bose Hubbard model with bounded disorder at a commensurate filling

$$H = -t\sum_{\langle i,j\rangle} a_i^+ a_j + \frac{U}{2}\sum_i n_i(n_i-1) + \sum_i \varepsilon_i n_i$$



$\varepsilon_i \in [-\Delta, \Delta]$ random on-site potential

$\nu \equiv \overline{n}_i = 1$ (or other integer)

Superfluid (SF)

Mott insulator (MI) gapped insulator

Bose glass (BG) compressible insulator

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M.P.A. Fisher, P.B. Weichman, G. Grinstein, and D.S. Fisher, Phys. Rev. B **40**, 546 (1989).

Q1: *Does disorder change the phase diagram at* $\Delta \ll U, t$ *?*

Q2: *Is disorder a relevant perturbation for SF-insulator transition?*

Experiment on disordered bosons in optical lattices

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J. Billy, V. Josse, Z. Zuo, A. Bernard, B.Hambrecht, P. Lugan, D. Clement1, L. Sanchez-Palencia, P. Bouyer, and A. Aspect, Nature 453, 891 (2008).

G. Roati, C. D'Errico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno, and M. Inguscio, Nature 453, 895 (2008).

Y. P. Chen, J. Hitchcock, D. Dries, M. Junker, C. Welford, and R. G. Hulet, Phys. Rev. A 77, 033632 (2008).

M. White, M. Pasienski, D. McKay, S.Q. Zhou, D. Ceperley, and B. DeMarco, Phys. Rev. Lett. 102, 055301 (2009).

M. Pasienski, D. McKay, M. White, and B. DeMarco, Nature Physics 6, 677 (2010).

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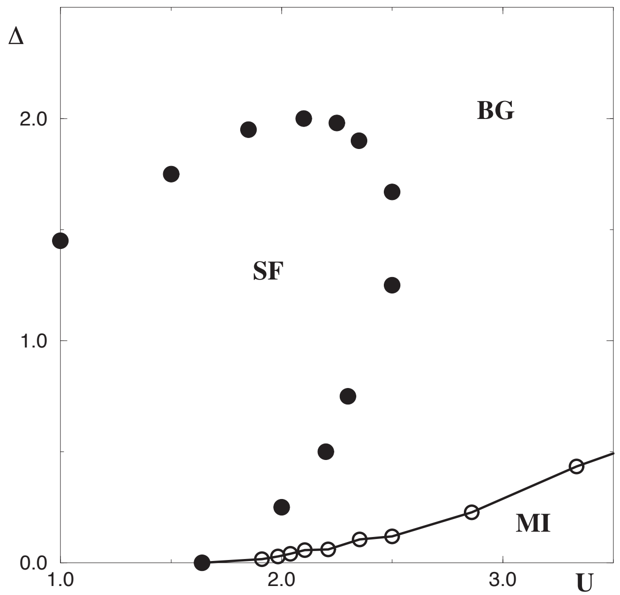
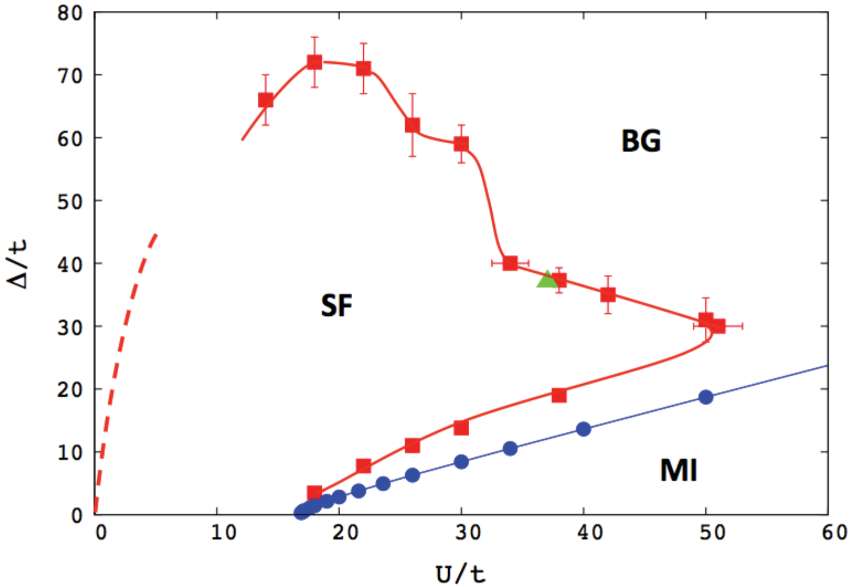
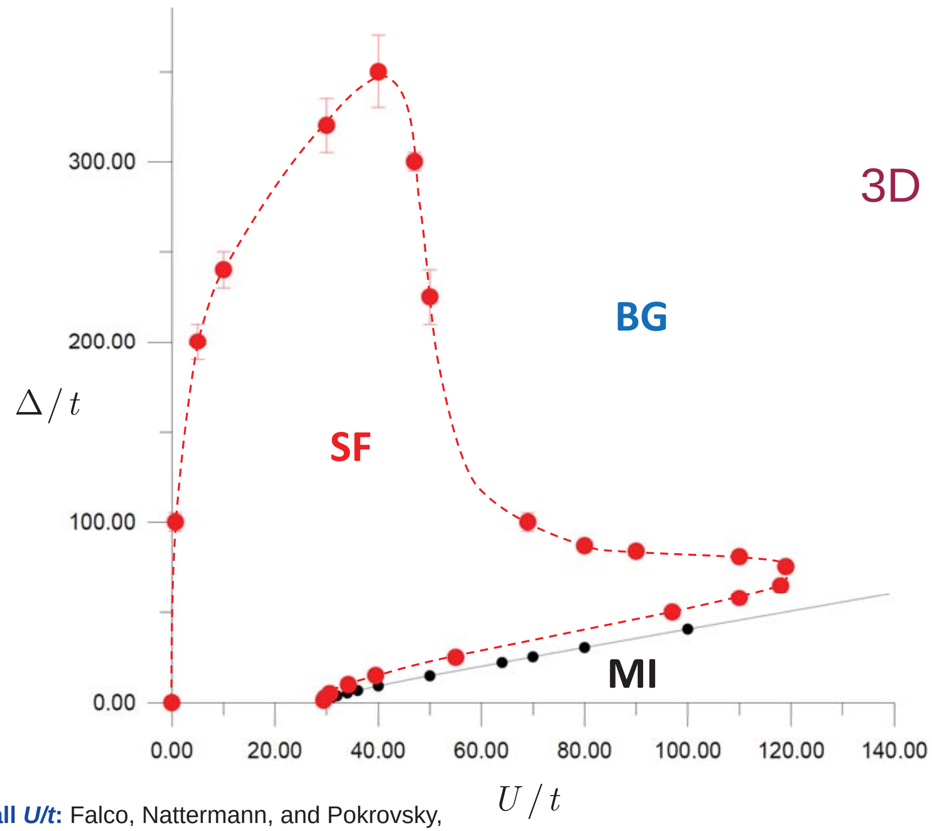
1D

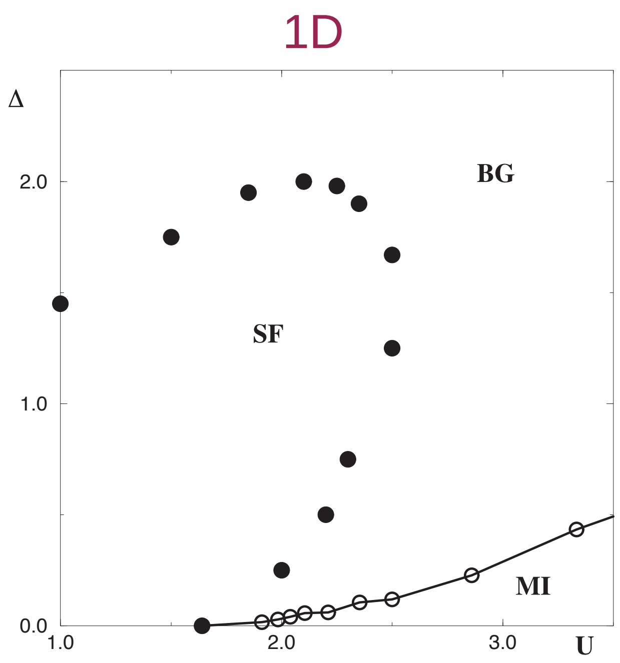


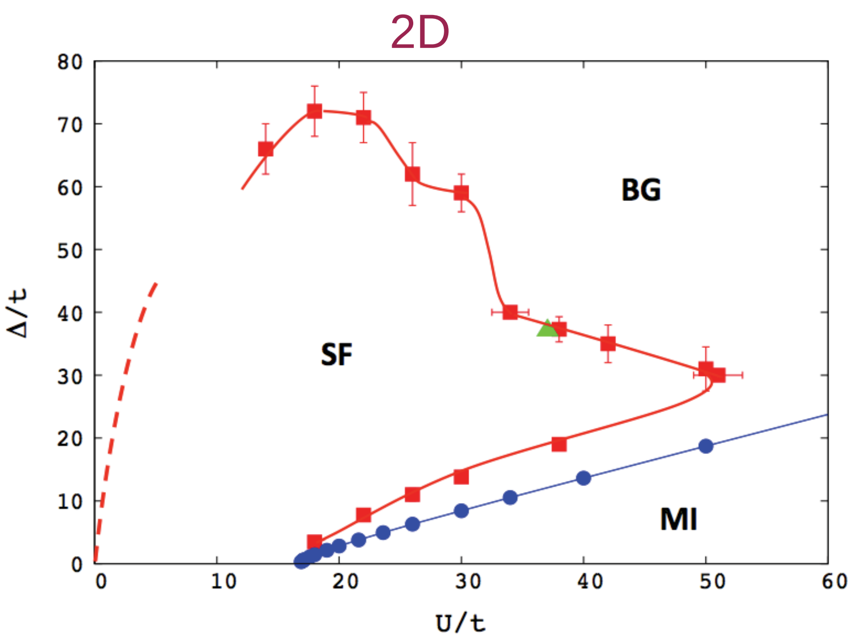
2D

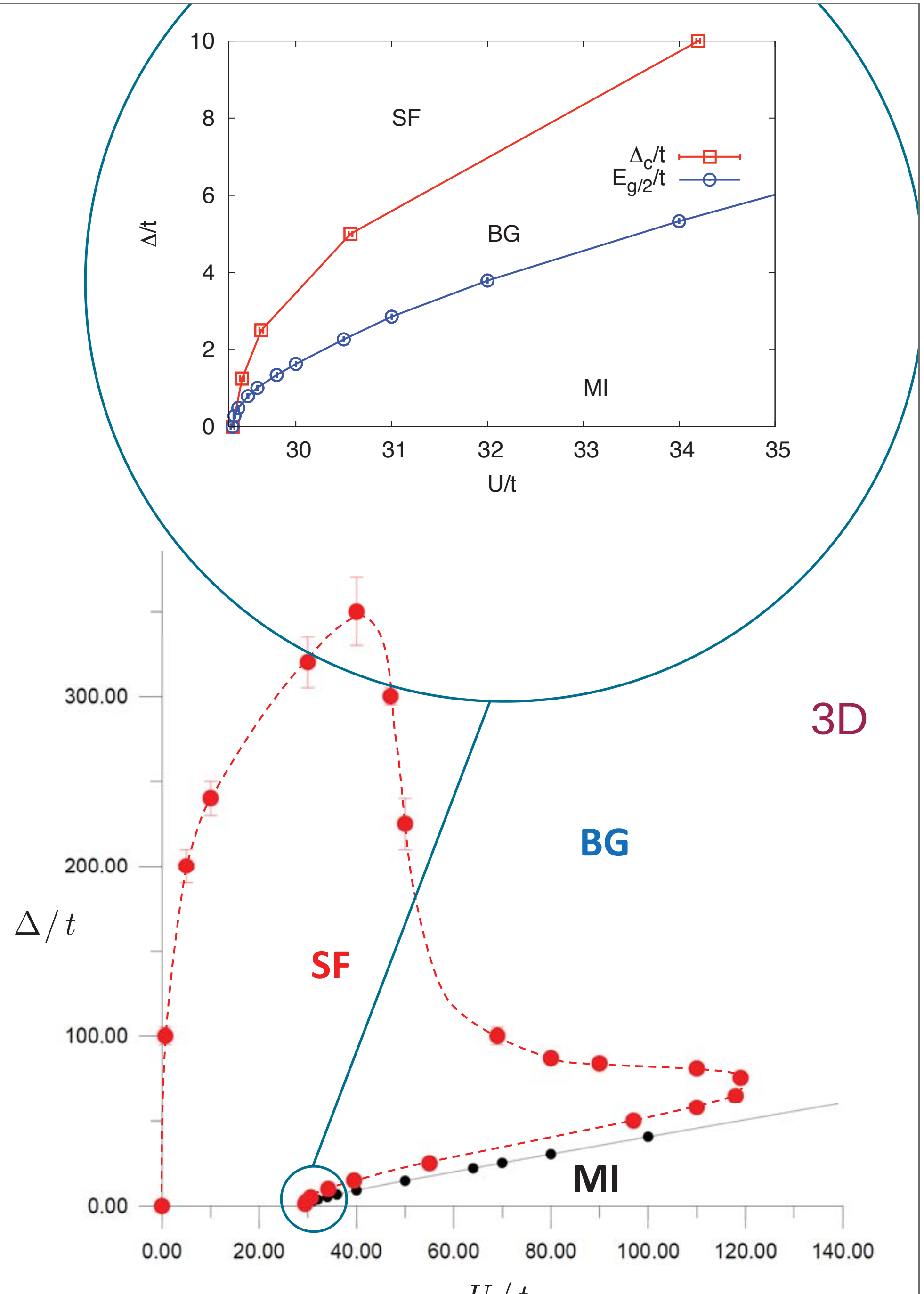




Small *U/t*: Falco, Nattermann, and Pokrovsky, Phys. Rev. B **80**, 104515 (2009).

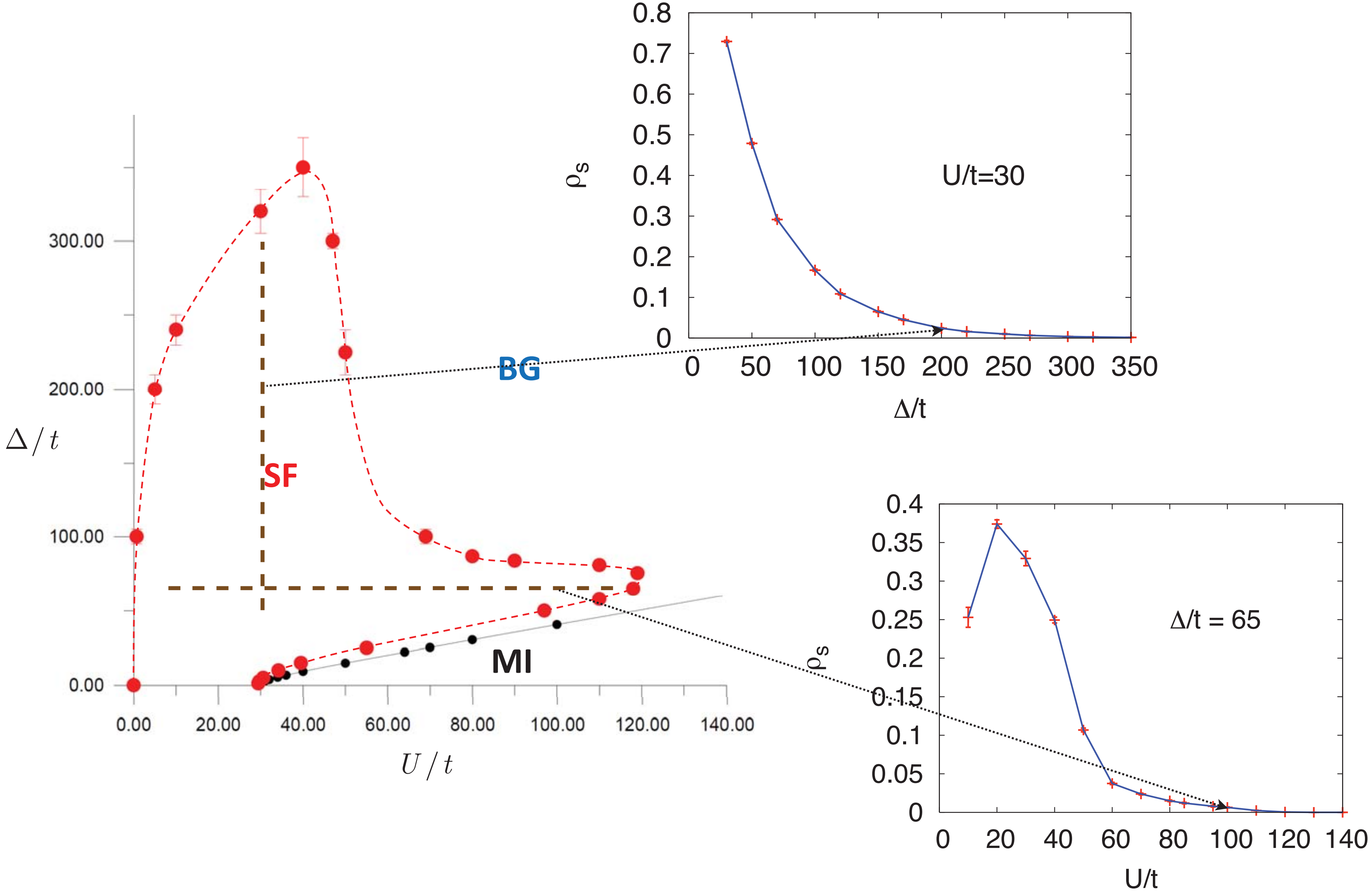




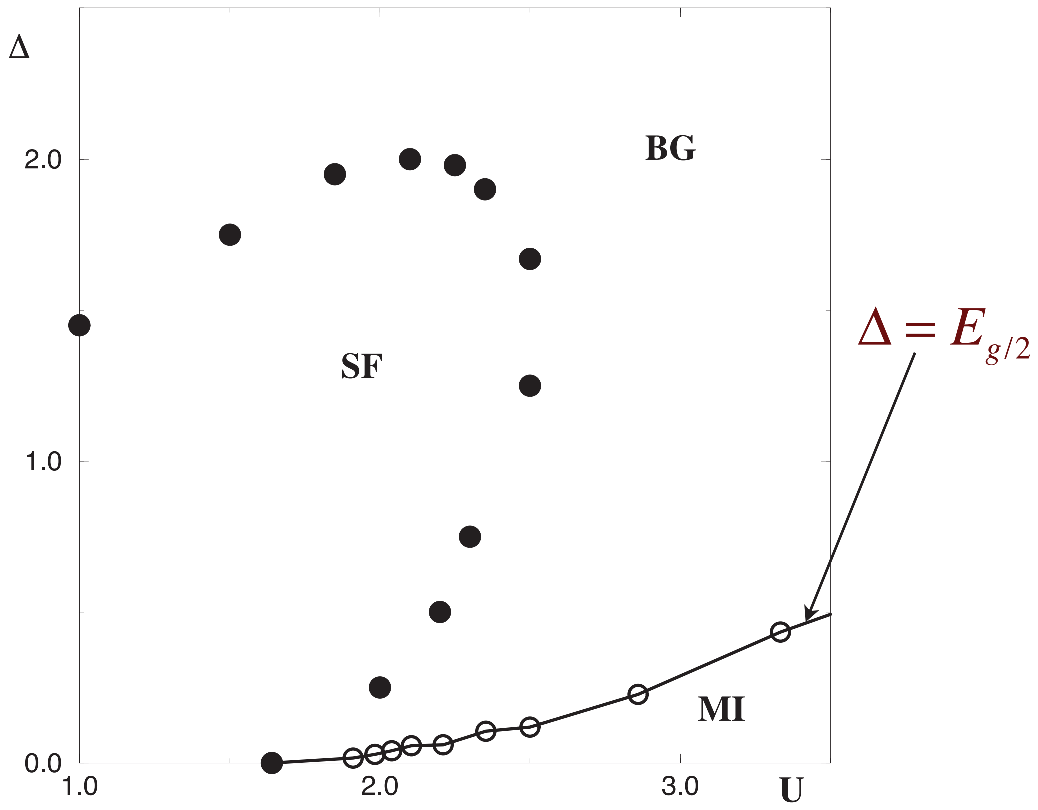


Small *U/t*: Falco, Nattermann, and Pokrovsky, Phys. Rev. B **80**, 104515 (2009).

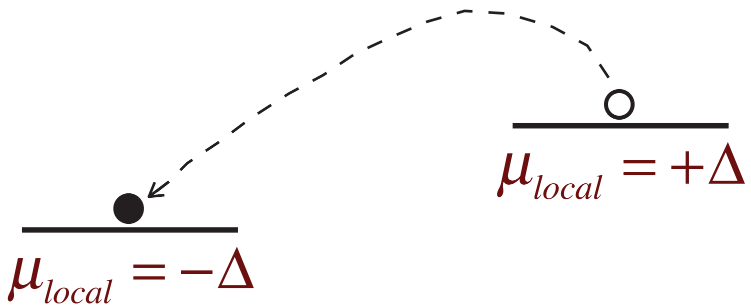
Large numbers = fragile superfluidity !



The line $\Delta = E_{g/2}$



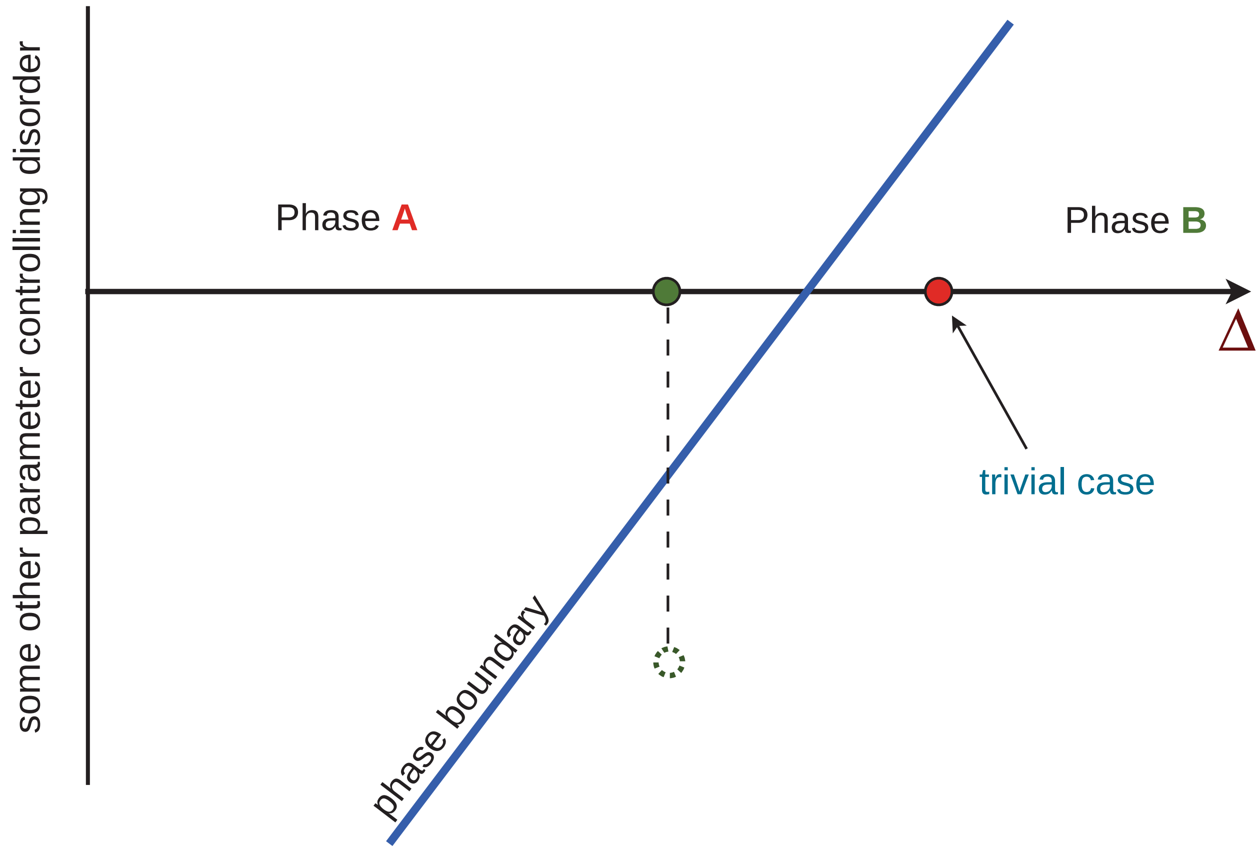
$E_{g/2}$ is the half of the gap in the ***pure*** system.



Theorem of Inclusions

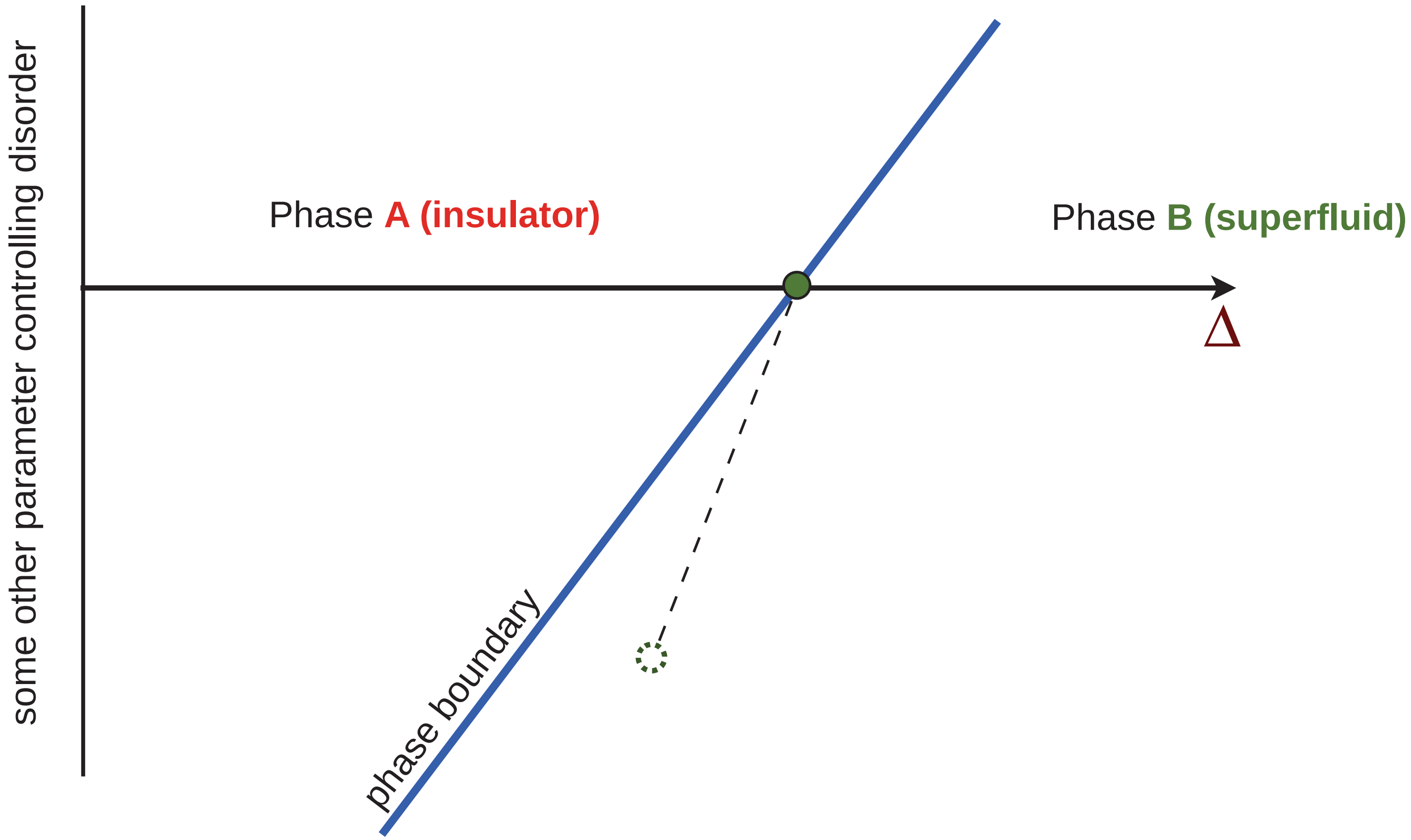
Def. ***Generic disorder:*** Probability density for any (local) realization is finite.

Theorem of Inclusions: Absence of SF-to-gapped-insulator transition



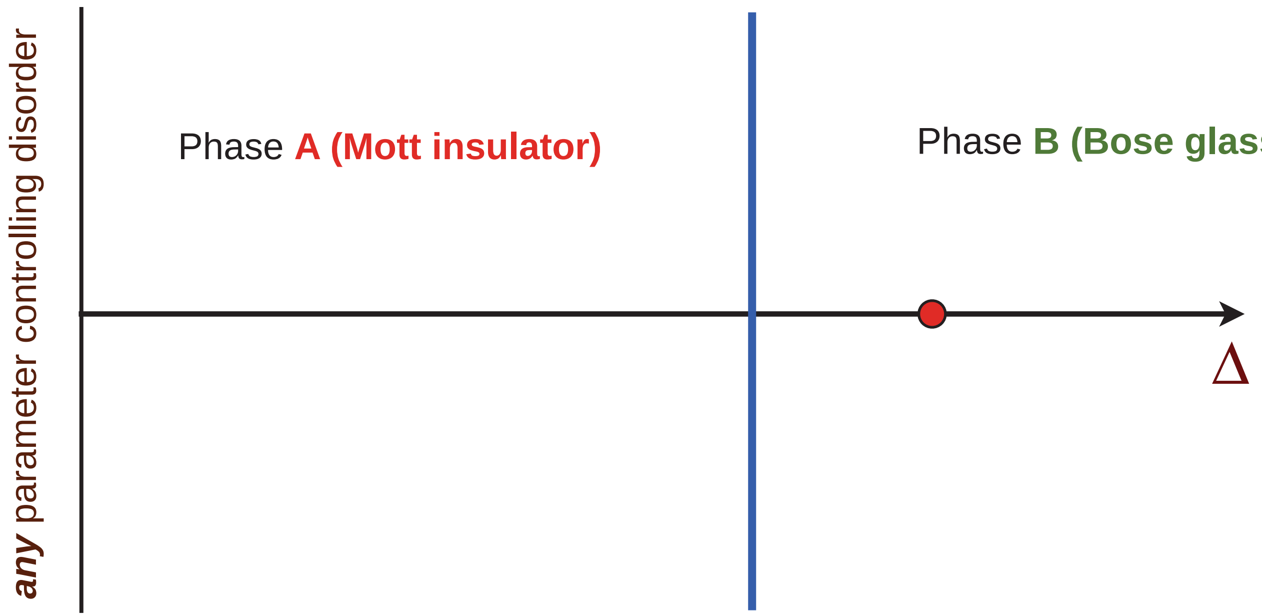
In the presence of generic bounded disorder there exist rare, but arbitrarily large, regions of the competing phase across the transition line.

Enhanced version: Finite compressibility at the critical point



There exist arbitrarily large regions with actual bound smaller than the global one by a finite value.

An exception implied by the rule: MI to BG transition is of the Griffiths type



The phase boundary line is generically vertical if, and only if, the transition is of the Griffiths type, i.e. driven by rare regions in which disorder emulates some regular external perturbation with the amplitude Δ .

Some important details on how disorder gets relevant in the vicinity of the SF-MI point

1D case: vortex instantons in (1+1) dimensions

$$S[\Phi]=\int dx\,d\tau\left\{i\bar{n}(x)\,\partial_\tau\Phi+\frac{1}{2}\Lambda_s\left(\partial_x\Phi\right)^2+\frac{1}{2}\kappa\left(\partial_\tau\Phi\right)^2\right\}$$

Λ_s: superfluid stiffness

$\kappa = dn/d\mu$ compressibility

$$S[\Phi]=\int dx\,dy\left\{i\bar{n}(x)\,\partial_y\Phi+\frac{1}{2\pi K}\left(\nabla\Phi\right)^2\right\}$$

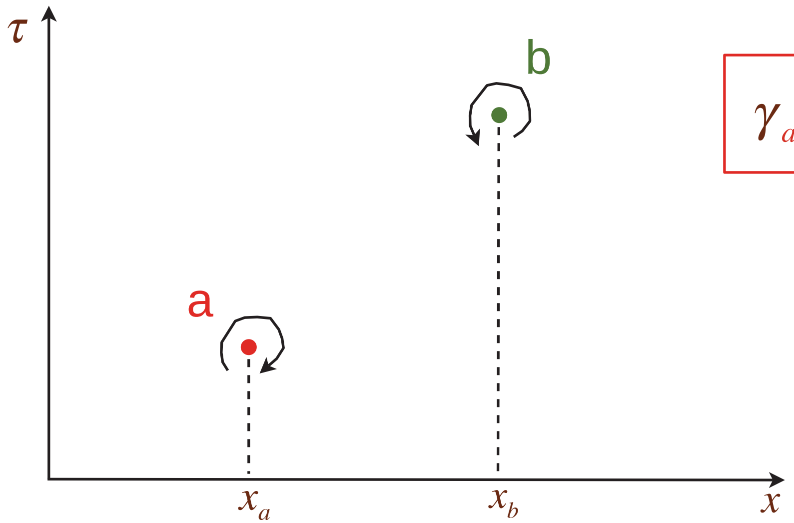
K: Luttinger-liquid parameter

$y=c\tau,\quad c=\sqrt{\Lambda_s/\kappa},\quad K^{-1}=\pi\sqrt{\Lambda_s\kappa}$

$$S[\Phi]=\frac{1}{2\pi K}\int\left(\nabla\Phi\right)^2dx\,dy-i\sum_j p_j\gamma(x_j)$$

$p_j=\pm1,\pm2,\pm3,\ldots$ instanton charge

$\gamma(x)=2\pi\int^x\bar{n}(x')dx'$ instanton phase



$$\gamma_{ab}=2\pi\int_{x_a}^{x_b}\bar{n}(x)dx$$

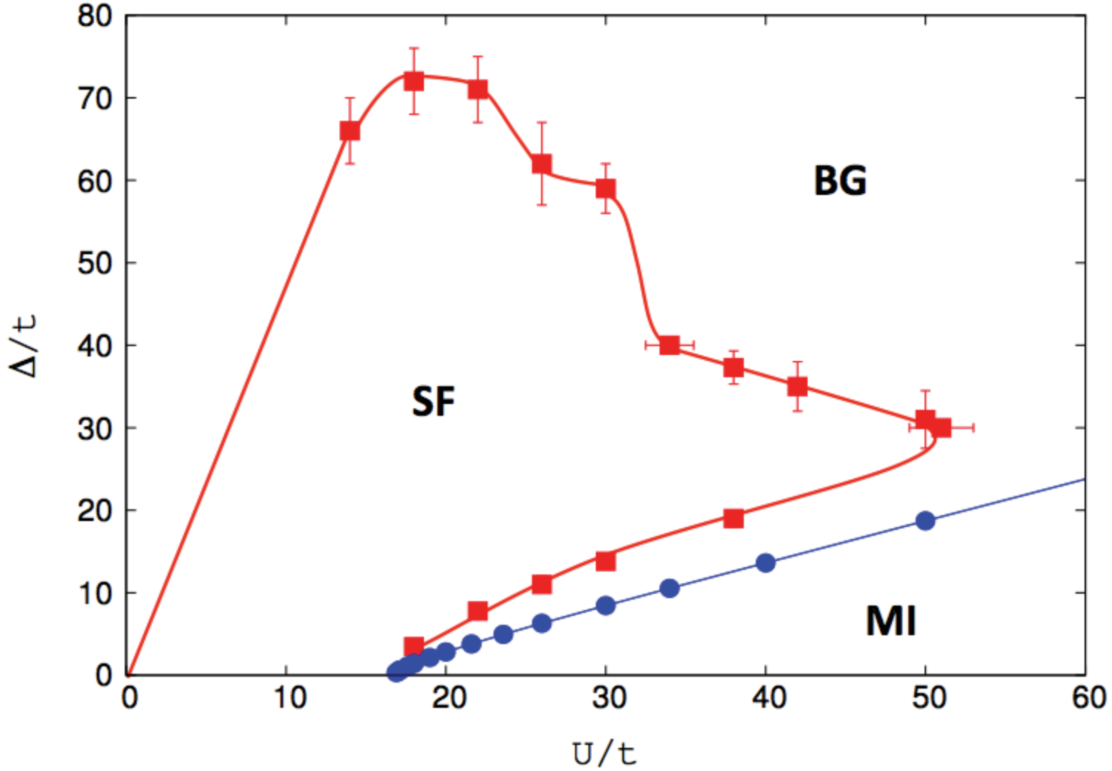
phase of the instanton pair

$K_c=1/2$ SF-MI

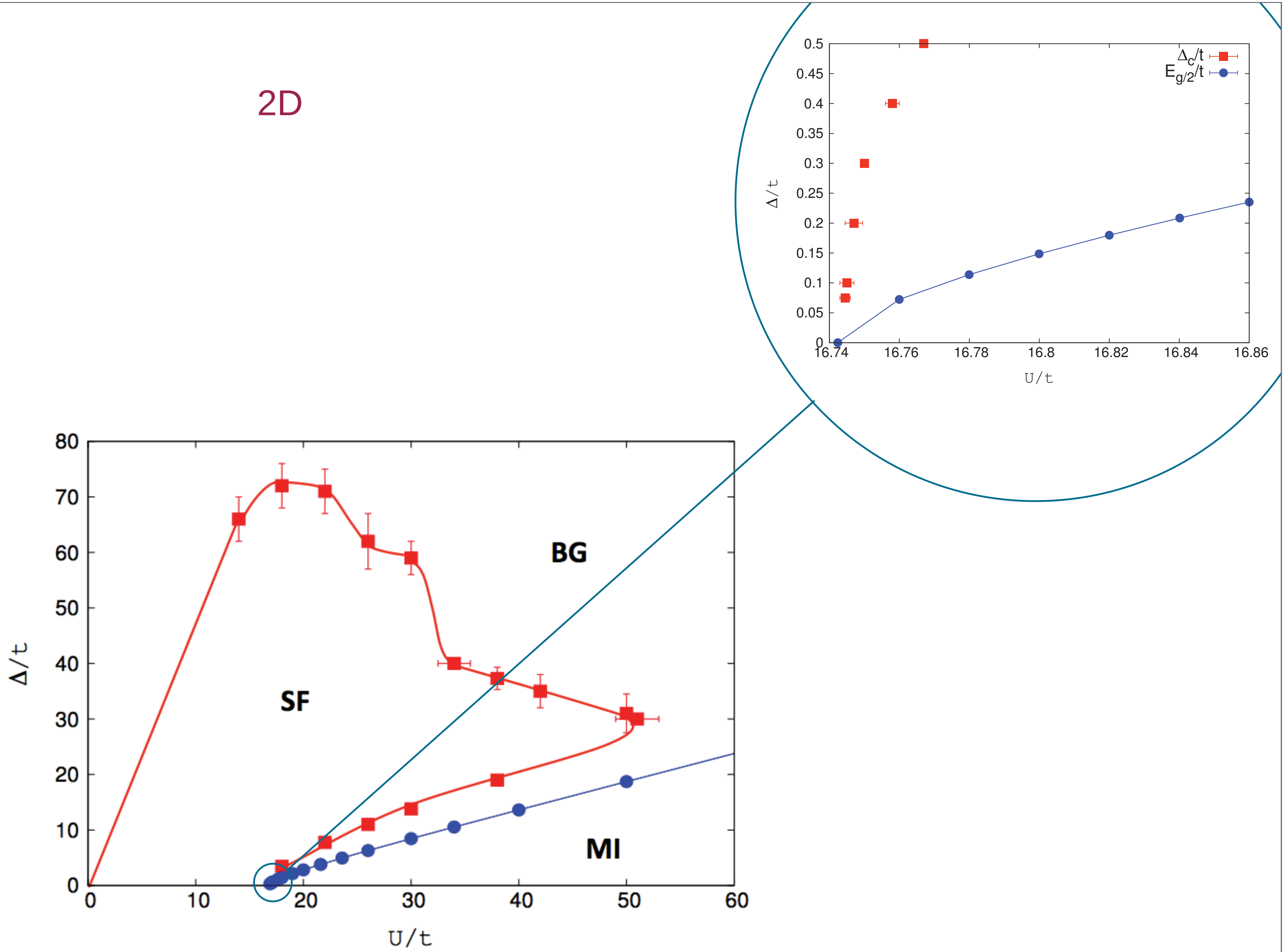
$K_c=2/3$ SF-BG

Shape of the SF-BG in the vicinity of the SF-MI point

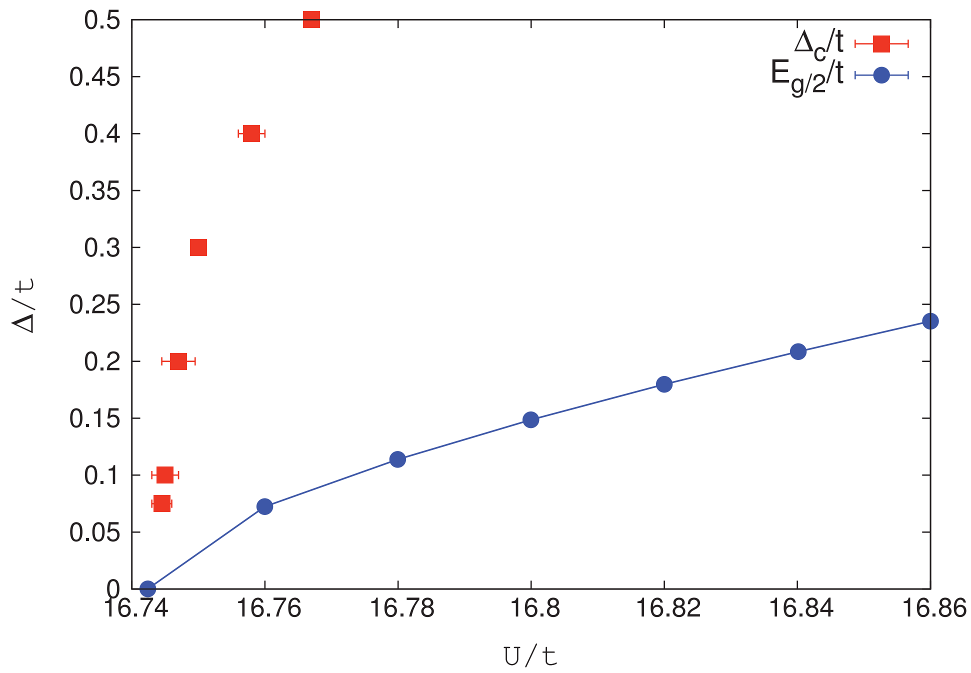
2D



2D



2D



Δ_c/t
$E_{g/2}/t$
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
16.74
16.76
16.78
16.8
16.82
16.84
16.86
Δ/t
U/t

Main points

Exact numerics for Bose Hubbard model at unity filling in d=1,2, and 3.

Theorem of Inclusions: Absence of SF-to-gapped-insulator transitions

Enhanced version: Finite compressibility at the critical point

Extended version: MI to BG transition is of the Griffiths type

Open question: *Shape of the SF-BG curve in the vicinity of the SF-MI point in d=2,3.*