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Advanced Workshop on Non-Standard Superfluids and Insulators

18 - 22 July 2011

p-band superfluid and insulator phases in optical lattices

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USA

Advanced Workshop on Non-standard Superfluids and Insulators, ICTP Trieste, 18-22 July 2011

p-orbital band superfluid and insulator phases in optical lattices

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[View of Pittsburgh---source: PittsburghSkyline.com]

Acknowledgement

Our Work:

- PRA 2006
- PRL 2006
- PRL 2008a
- PRL 2008b
- PRL 2010
- PRA 2010
- PRA 2011
- Nature Phys (news & view, 2011)
- arXiv:1011.4301
- arXiv:1103.5964

group members on p-band:

Students: Chiu-Man Ho *(→UC Berkeley→Vanderbilt)*
H. H. Hung *(→UCSD→UIUC)*
Xiaopeng Li *(current)*
Vladimir Stojanovic *(Carnegie-Mellon→ U Basel)*
Zixu Zhang *(current)*

Postdoc: Chungwei Lin *(moving to UT Austin/2nd postdoc),*
Erhai Zhao *(now assistant professor at George Mason Univ, Fairfax, VA)*

External collaborators

Theory: Sankar Das Sarma, Ivan Deutsch, Krittika Goyal, Philipp Hauke, Maciej Lewenstein, Joel Moore, Kai Sun, Congjun Wu
Exp: Andreas Hemmerich

Thanks for Support by
U.S. ARO (individual) and
DARPA-OLE/ARO (Hulet/Rice team)

Outline

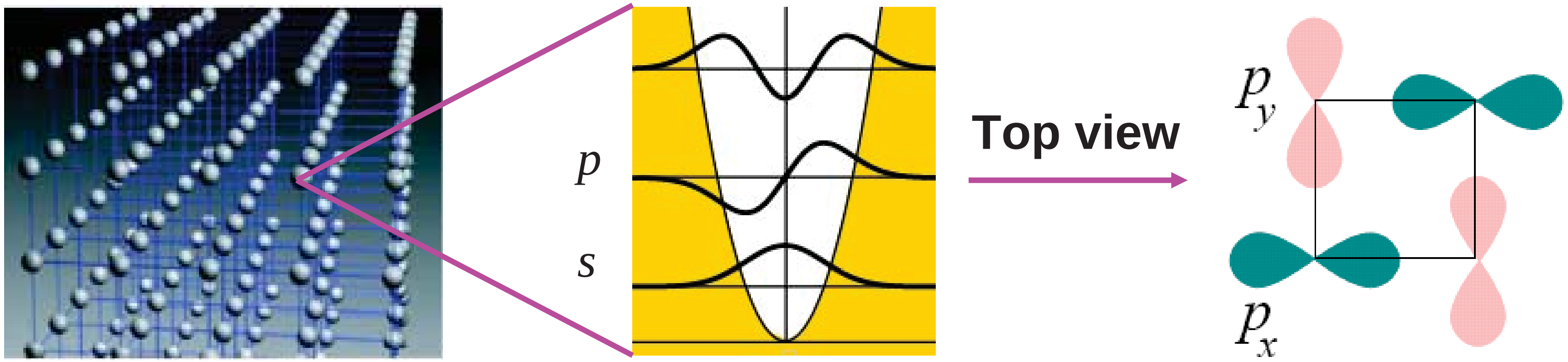
1. Introduction
   - Beyond s-orbital, the basics of p-orbital? Why p-orbital?
   - Recent experimental progress

2. p-band bosons (including examples of no prior analogue in solids).
   - Finite momentum BEC with px+ipy orbital order
   - Mott insulator phases and orbital order
   - Elongated Orbitals and mapping to quantum dimer model

3. p-band fermions
   - Single species. Mott states. Orbital-only model. Realization of Kitaev-like model?
   - Topological states (quantum Hall like). Topological semimetal (non-interacting), and topological insulator (turn on interaction). Time reversal breaking driven by interaction, no magnetic field!

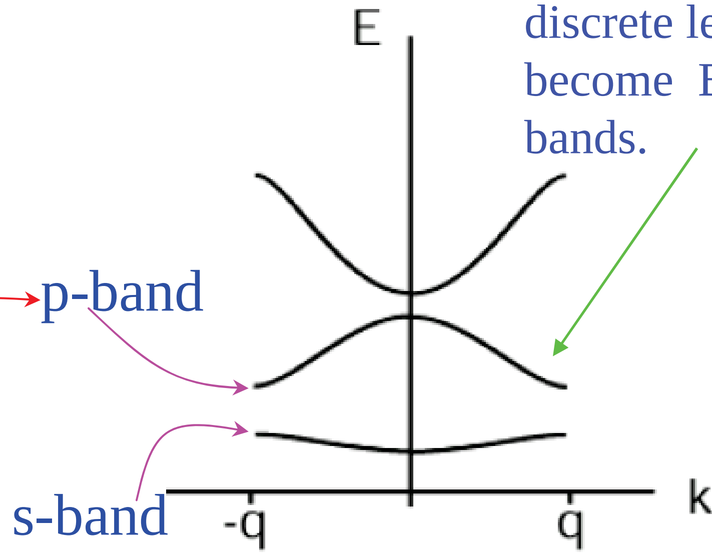
Part 1. Introduction:
What is the p-band, Why?

p-band of an optical lattice (1D illustration)



[lattice picture from web]

With tunneling, discrete levels become Bloch bands.



- **Bose-Hubbard model**

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{1}{2}U\sum_i b_i^\dagger b_i^\dagger b_i b_i$$

theory: Jaksch et al, PRL 1998

exp.: Bloch et al, Nature 2002

- **This talk**

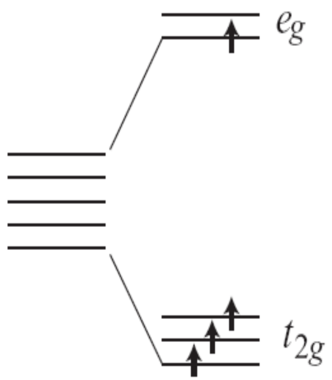
Orbital ordering of d-electrons

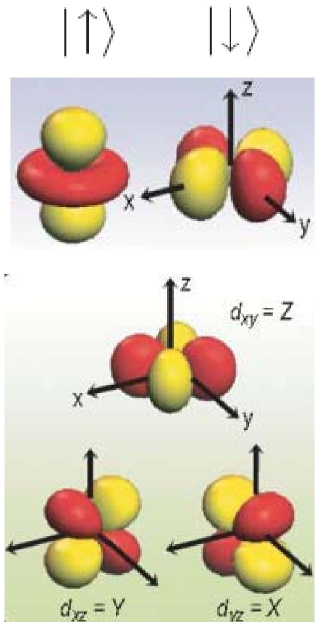
[see, for example, review by Tokura and Nagaosa, science 288, 462, (2000).]

orbital: shape of the electron cloud

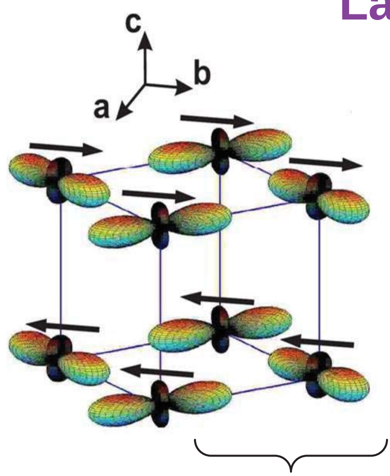
Orbital degeneracy in transition metal oxides:

pseudospin 1/2:





$LaMnO_3$



spin

superposition of e_g orbitals

Kugel-Khomskii superexchange

$$H = \sum_{\langle i,j \rangle} \hat{J}_{ij}^{(\gamma)} (\vec{S}_i \vec{S}_j + \frac{1}{4}),$$

$$\hat{J}_{ij}^{(\gamma)} = J(T_i^{(\gamma)} T_j^{(\gamma)} - \frac{1}{2} T_i^{(\gamma)} - \frac{1}{2} T_j^{(\gamma)} + \frac{1}{4})$$

$$T_i^{(a/b)} = \frac{1}{4}(-\sigma_i^z \pm \sqrt{3}\sigma_i^x), \qquad T_i^{(c)} = \frac{1}{2}\sigma_i^z$$

- Charge, spin, orbital, and lattice degree of freedom entangled together
- Complex phase diagrams

A new direction: p-orbital physics in optical lattices

- **Orbital degeneracy** (*px, py, pz orbitals*) is considerably less understood in comparison. Implies emergent symmetry.
- **Similar to spin physics** but is different fundamentally.
- **Strong anisotropy**: Anisotropy is an interesting new feature, not a problem!
- ***p*-orbitals are different than *d*-orbital in solids:** Parity ODD. New possibility---*p*-orbital bosons as opposed to *d*-electrons (fermions) in solids
- unique to cold atom systems, "non-standard" condensed matter systems. For instance, Orbital physics of bosons has no prior analogue in CM physics?!

→ **new quantum phases (a main motivation of our study).**

Theoretical studies on the excited bands An incomplete list!! *[red=our work]*

On multi-orbital

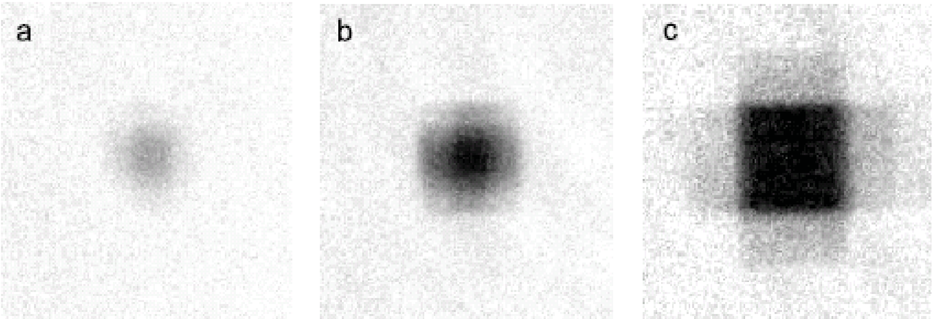
- V. Scarola, S. Das Sarma, *Phys. Rev. Lett.* (2005)
- O. E. Alon, A. I. Streltsov, L. S. Cederbaum, *Phys Rev Lett* (2005)
- …

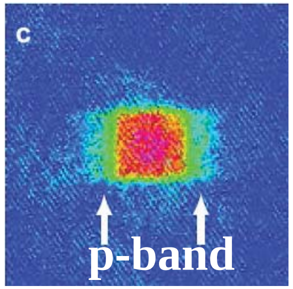
On p-orbital

- A. Isacsson and S. Girvin, *Phys. Rev. A* 72, 053604 (2005).
- A. B. Kuklov, *Phys. Rev. Lett.* (2006)
- WVL and C. Wu, *Phys. Rev. A* (2006)
- C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. Rev. Lett.* (2006).
- A. F. Ho, arXiv:cond-mat/0603299
- C. Xu and M. P. A. Fisher, *Phys. Rev. B* 75, 104428 (2007)
- C. Wu, D. Bergman, L. Balents, and S. Das Sarma, *Phys. Rev. Lett.* (2007)
- A. Kantian, A. J. Daley, P. Törmä and P. Zoller, *New J. Phys.* (2007)
- L. Guo, Y. Zhang, and S. Chen, *Phys. Rev. A (2007)*
- E. Zhao and WVL, *Phys. Rev. Lett.* (2008);
- R. O. Umucallar and M. Ö. Oktel, *Phys. Rev. A* (2008)
- K. Wu and H. Zhai, *Phys. Rev. B* (2008)
- L. Wang, X. Dai, S. Chen, X. C. Xie, *arXiv:0805.2719* (2008)
- R. M. Lutchyn, S. Tewari, S. Das Sarma, *arXiv:0812.0815 (2008)*
- V. Stojanovic, C. Wu, WVL and S. Das Sarma, *Phys. Rev. Lett.* (2008)
- …
- K. Sun. E. Zhao, WVL, *Phys. Rev. Lett.* (2010)
- Z. Zhang, H. H. Hung, C.M. Ho, E. Zhao, WVL, *Phys. Rev. A (2010)*
- A. Collin, J. Larson, and J.-P. Martikainen, *Phys. Rev. A* (2010)
- ...

Orbital physics of interacting cold atoms

- **Fermions in p-band** M. Köhl et al, PRL **94**, 080403 (2005)

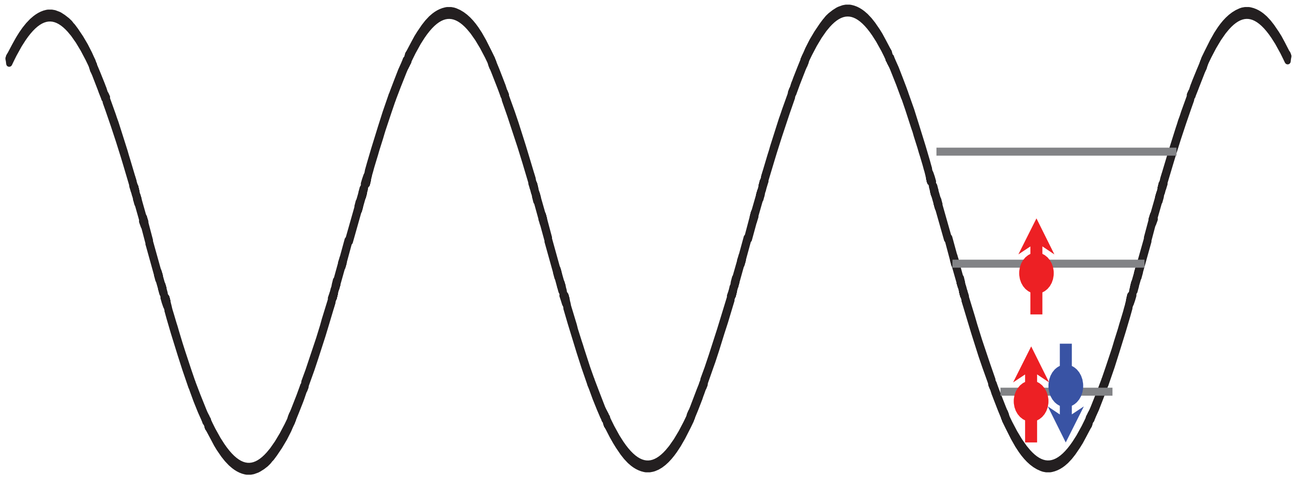




Fermi Surface vs band filling

Fermions are transferred into the p-band using a sweep across the Feshbach resonance, i.e., by strong interaction.

Preparation of p-band fermions

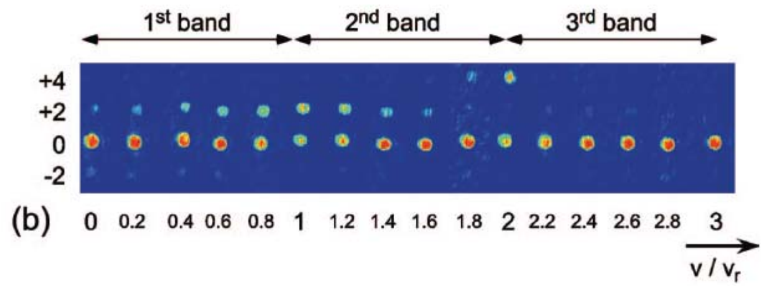


- ***Even simpler:*** Fill the lowest s-band with the specific species (say spin up). Just by having more than 2 particles/site for two-components
- p-band fermions should have no problem in lifetime (Pauli exclusion principle)

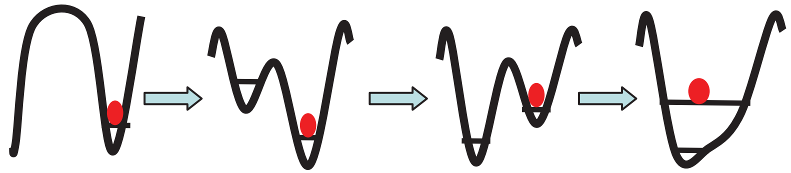
Bosons in the optical lattice p-band

Experiments

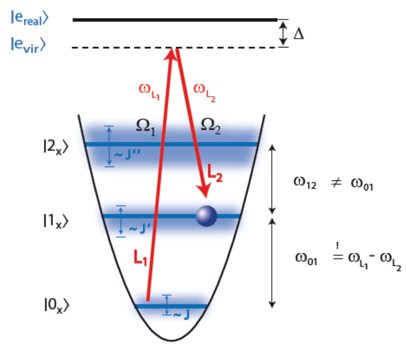
- By moving lattices [A. Browaeys, W. D. Phillips, et al, PRA **72**, 053605 (2005)]



- Dynamically deforming the double-well lattice [NIST Porto/Phillips groups: PRA (2006); J. Phys. B (2006); PRL 2007; Nature 2007; …]



- Pumping bosons by Raman transition [T. Mueller, I. Bloch et al., PRL 2007]



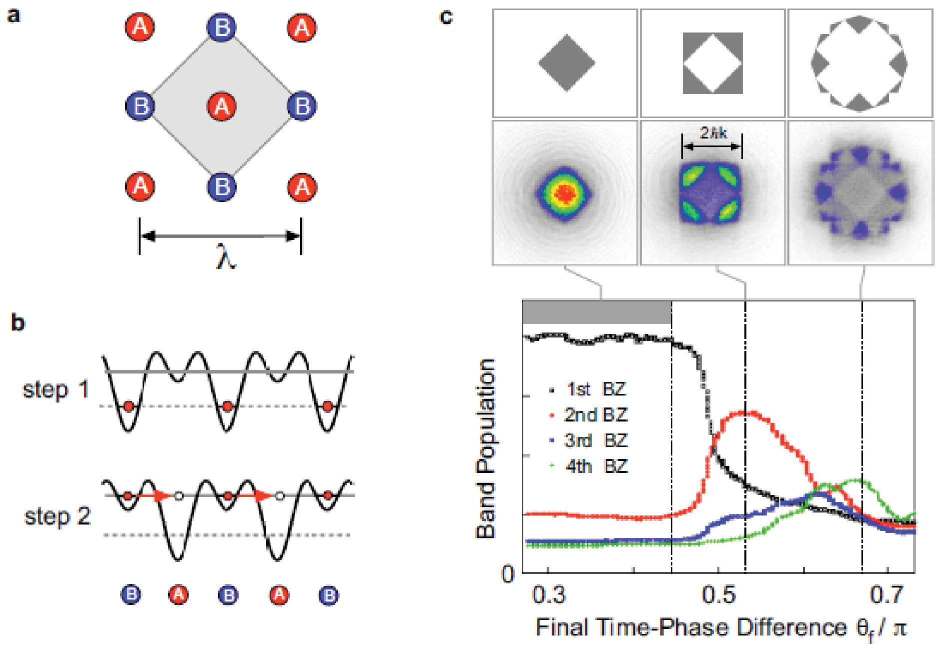
Theories:

- Isacsson & Girvin, PRA 05
- Kuklov, PRL 06
- WVL & Wu, PRA 06

Lifetime issue: to be discussed.

New p- and f-band experiments – double well lattices

Hamburg/Hemmerich group



- "P-band superfluidity+orbital order in chequerboard (double well) lattice", long life time [G. Wirth. M. Olschlager, A. Hemmerich, *Nature Physics* 2011]
- "F-band" [M. Olschlager, G. Wirth, A. Hemmerich, PRL 2011]

Another recent exp: hexagonal lattice

- Reported "Unconventional (complex) multi-orbital superfluidity" [P. Soltan-Panahi, D. Luhmann, J. Struck, P. Windpassinger, and K. Sengstock, arxiv:1104.3456]

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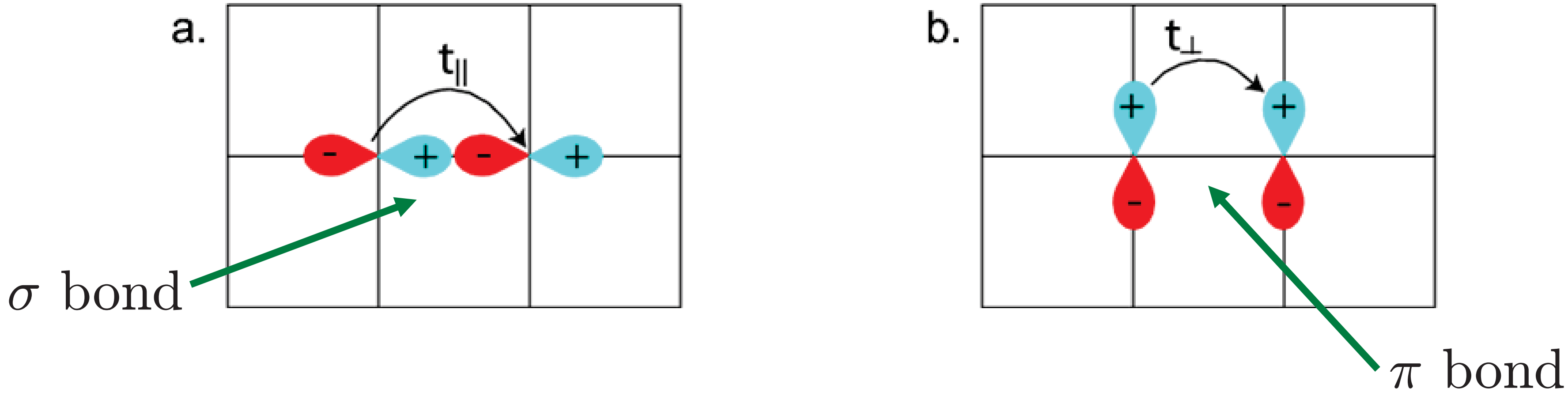
p-orbital Bose-Hubbard model: 3D cubic lattice
[derived in WVL and C. Wu, PRA (2006)]

$$H = \sum_{\mathbf{r}\mu} [t_{\|}\delta_{\mu\nu} - t_{\perp}(1-\delta_{\mu\nu})] \left(b^{\dagger}_{\mu,\mathbf{r}+a\mathbf{e}_{\nu}} b_{\mu\mathbf{r}} + h.c. \right) + \frac{1}{2} U \sum_{\mathbf{r}} \left[n_{\mathbf{r}}^2 - \frac{1}{3}\mathbf{L}_{\mathbf{r}}^2 \right]$$



$\mu, \nu = x, y, z$ or p_x, p_y, p_z

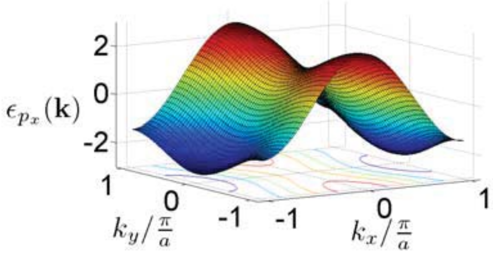
Density field operator $n_{\mathbf{r}} = \sum_{\mu} b^{\dagger}_{\mu\mathbf{r}} b_{\mu\mathbf{r}}$

Angular momentum operator: $L_{\mu\mathbf{r}} = -i \sum_{\nu\lambda} \epsilon_{\mu\nu\lambda} b^{\dagger}_{\nu\mathbf{r}} b_{\lambda\mathbf{r}}$

Weak coupling theory

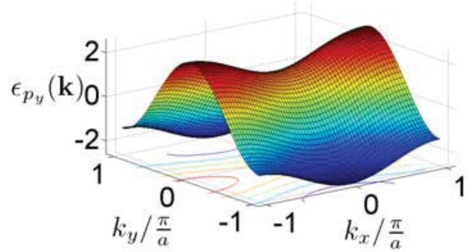
-p_x orbital band

$$\varepsilon_{px} = 2t_{\parallel}\cos(k_x) - 2t_{\perp}\cos(k_y)$$



-p_y orbital band

$$\varepsilon_{py} = -2t_{\perp}\cos(k_x) + 2t_{\parallel}\cos(k_y)$$



$\mathbf{Q}_x = (-\frac{\pi}{a}, 0)$ ⟵ **band minima** ⟶ $\mathbf{Q}_y = (0, -\frac{\pi}{a})$

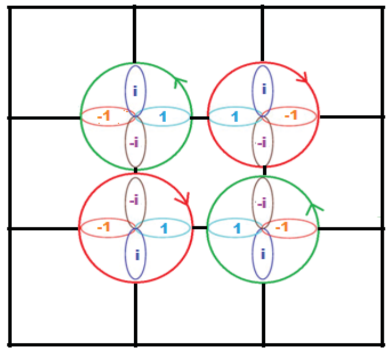
$$\langle p_x(\mathbf{r})\rangle = \varphi_x(\mathbf{r}) \sim \sqrt{n_x}e^{i\theta}e^{i\Delta/2}\mathbf{e}^{i\mathbf{Q}_x\cdot\mathbf{r}} \qquad \langle p_y(\mathbf{r})\rangle = \varphi_y(\mathbf{r}) \sim \sqrt{n_y}e^{i\theta}e^{-i\Delta/2}\mathbf{e}^{i\mathbf{Q}_y\cdot\mathbf{r}}$$

-inter-band phase locking

$$E(\Delta) \sim 2n_x n_y U \cos(2\Delta)$$

Gives a Bose-Einstein condensate of: $p_x \pm ip_y$

-pattern of TSOC superfluidity in real space



M. A. Lewenstein and WVL, Nat. Phys 7, 101(2011)

The *p*-orbital BEC (*p*-OBEC)

Parameterization of Order parameter:

$$\begin{pmatrix} \langle b_{x\mathbf{k}=\mathbf{Q}_x} \rangle \\ \langle b_{y\mathbf{k}=\mathbf{Q}_y} \rangle \\ \langle b_{z\mathbf{k}=\mathbf{Q}_z} \rangle \end{pmatrix} = \rho e^{i\varphi - i\vec{T}\cdot\vec{\theta}} \begin{pmatrix} \cos\chi \\ i\sin\chi \\ 0 \end{pmatrix}$$

U(1) phase — SO(3) orbital — T-reversal

[$\vec{T}$'s are three 3x3 matrices]

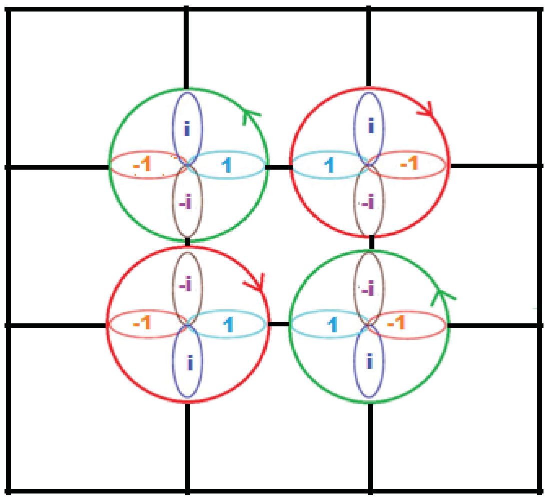
For a dilute lattice gas of U>0, the condensate is found to be a **non-zero-momentum** p_x+ip_y **BEC.**

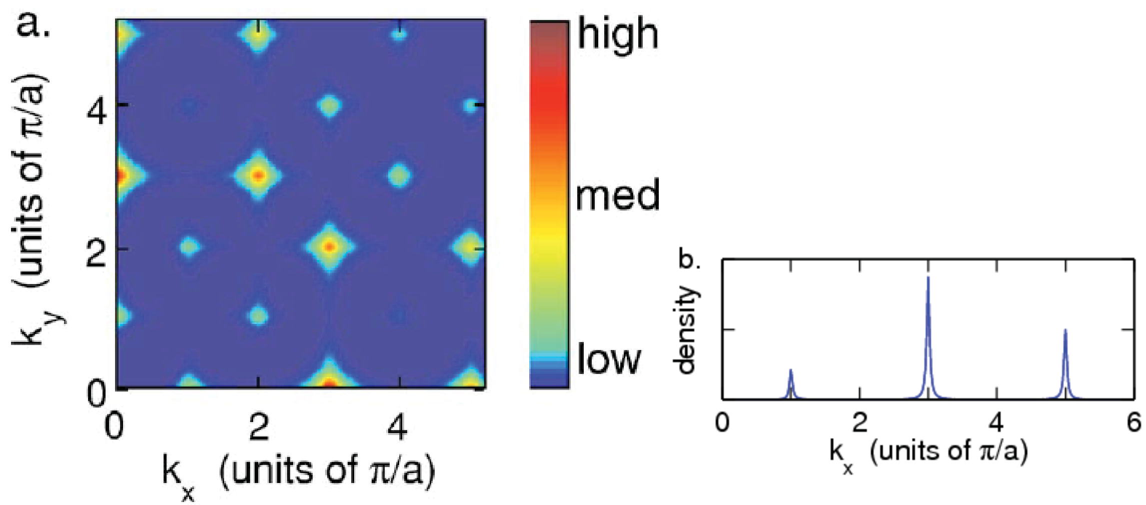
$$\begin{pmatrix} \langle b_{x\mathbf{k}=\mathbf{Q}_x} \rangle \\ \langle b_{y\mathbf{k}=\mathbf{Q}_y} \rangle \\ \langle b_{z\mathbf{k}=\mathbf{Q}_z} \rangle \end{pmatrix} = \sqrt{\frac{\text{Vol.} \times n_0^b}{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

- A new concept that defies the paradigm of BEC!
- A metastable BEC wavefunction with nodes---not contradict Feynman's argument.

Features of p-orbital BEC phase

- **Spontaneous Time reversal symmetry breaking**
- **Transversely staggered orbital current in real space (TSOC)**
- **Coherent peaks at finite momenta (finite momentum BEC)**
- **Violation of Feynman “no-node theorem” but meta-stable**

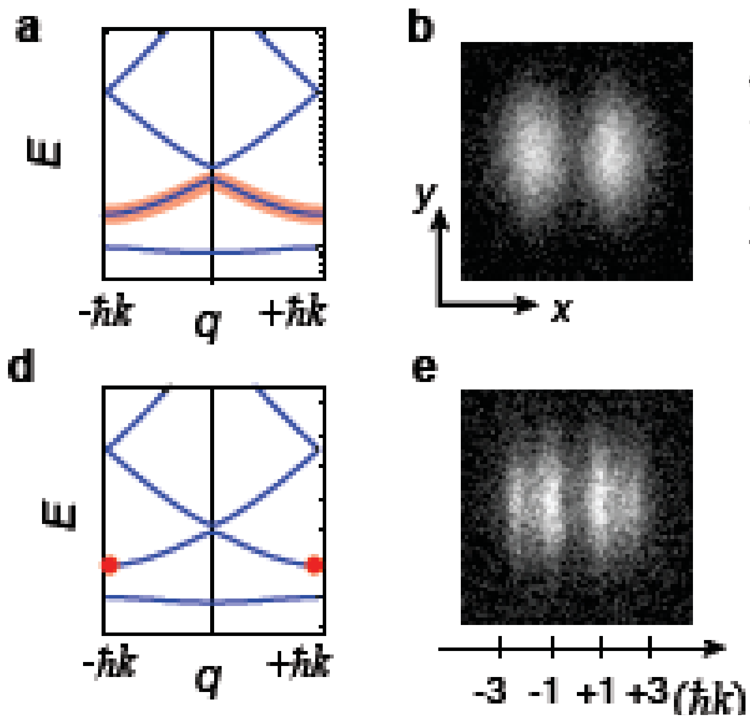




WVL and C. Wu Phys. Rev. A 74, 013607 (2006)

[Related results independently by: A. Isacsson, S. Girvin, PRA (2005); A. B. Kuklov, PRL 97, 110405 (2006)]

Experimental observation I: finite momentum BEC
by the Mainz/Bloch group [Mueller, Bloch, et al, PRL, 2007]



Initial time (incoherent) $20\mu s$

Later time (coherent) 1ms

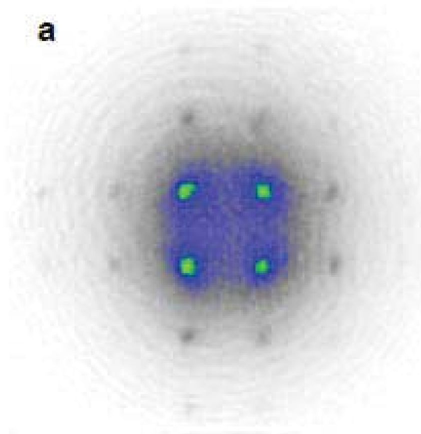
Mueller, Bloch et al [PRL 2007] and theoretical prediction [Isacsson-Girvin, 2005; Kuklov, 2006; WVL, Wu 2006] **agrees**!

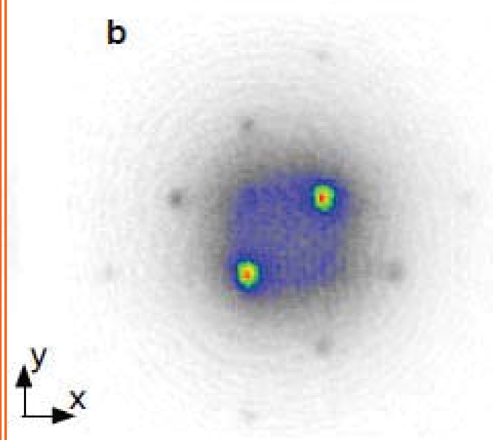
Experimental observation II: complex p-orbital superfluids

G. Wirth, M. Olschlager, A. Hemmerich, Nat. Phys. 2011

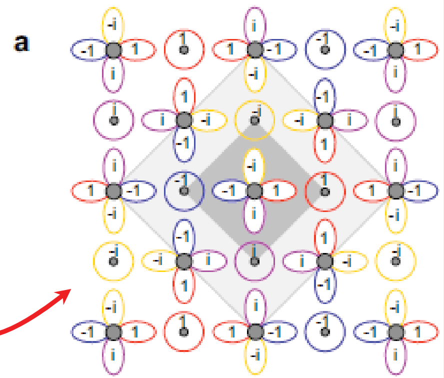
Isotropic | With small anisotropy

Observed momentum distribution

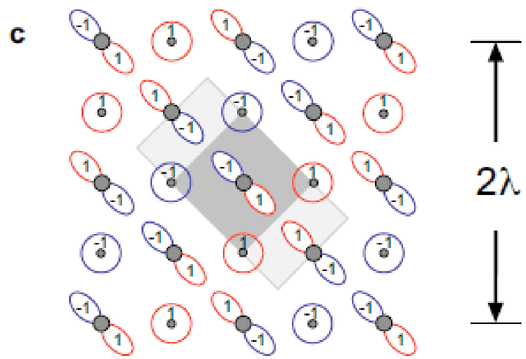




Interpretation:
Nature of orbital order



complex $p_x \pm i p_y$



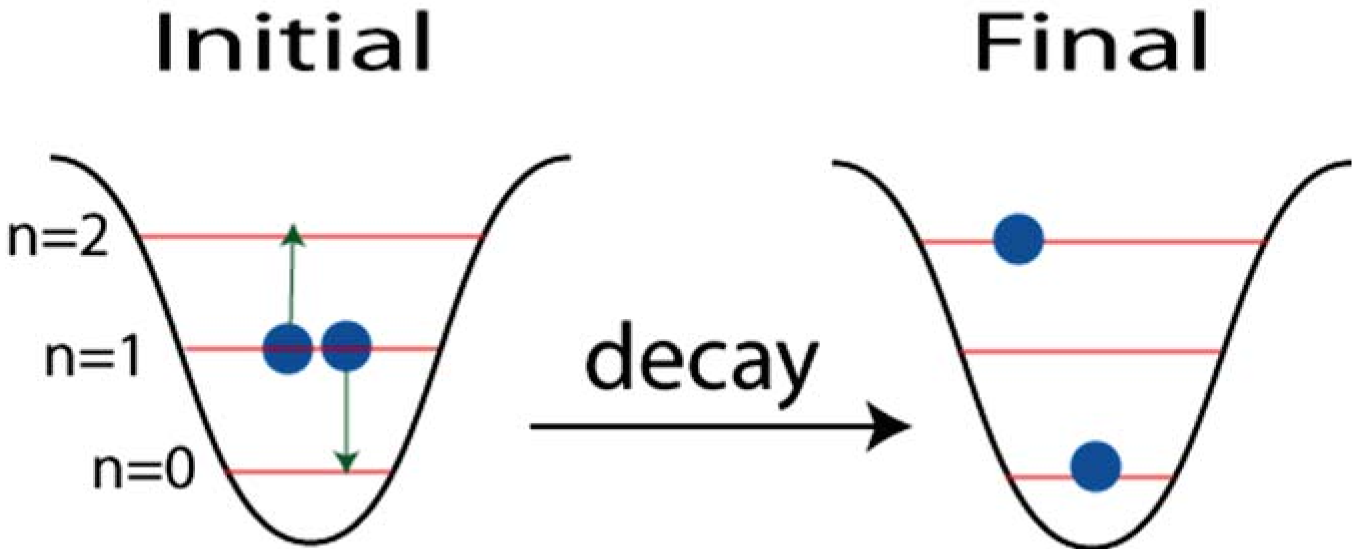
p_x rotated by 45°

Experimental finding consistent with prediction by [WVL, C. Wu, PRA 2006]

The decay problem of p-orbital bosons

The decay process where two p-bosons collide, promoting one to the 2nd excited band and bringing one down to the s-band.

My crude cartoon picture:



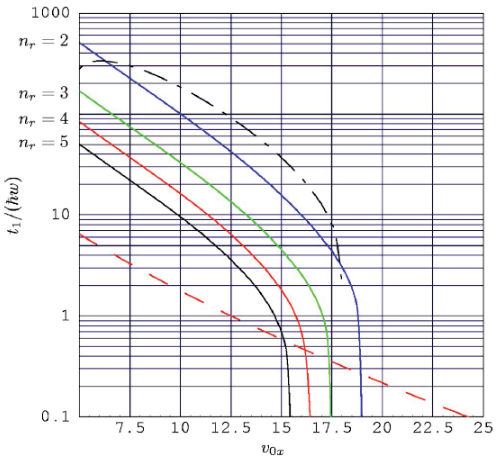


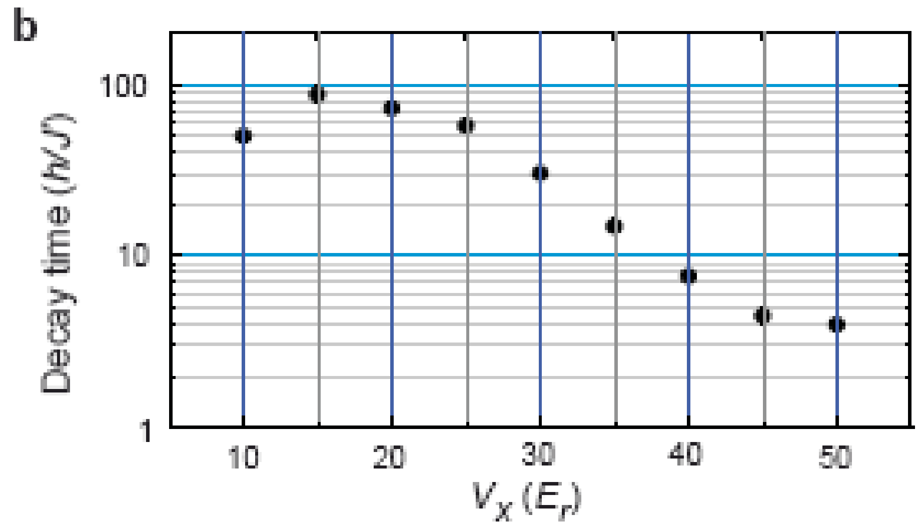
FIG. 15. (Color online) First-order lifetime w^{-1} for a 1D system with filling factor $\nu_1 = 1$ and $(a_s/a) = 1/100$ in the narrowband limit

First Studied by [Isacsson and Girvin (2005).]

p-band decay time measured by the Mainz group

[T. Mueller, I. Bloch, et al, PRL 2007]

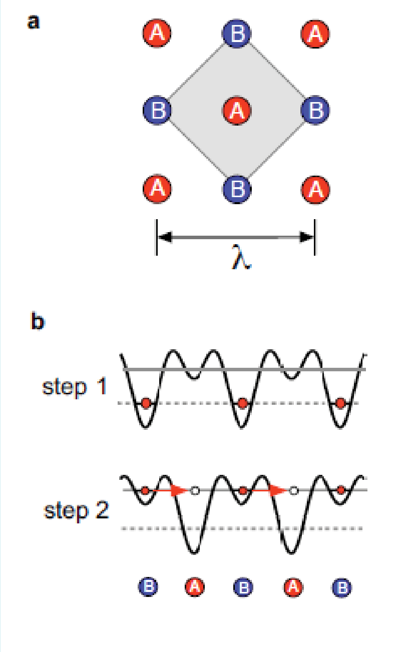
Decay time in units of
tunneling time scale
vs
lattice depth

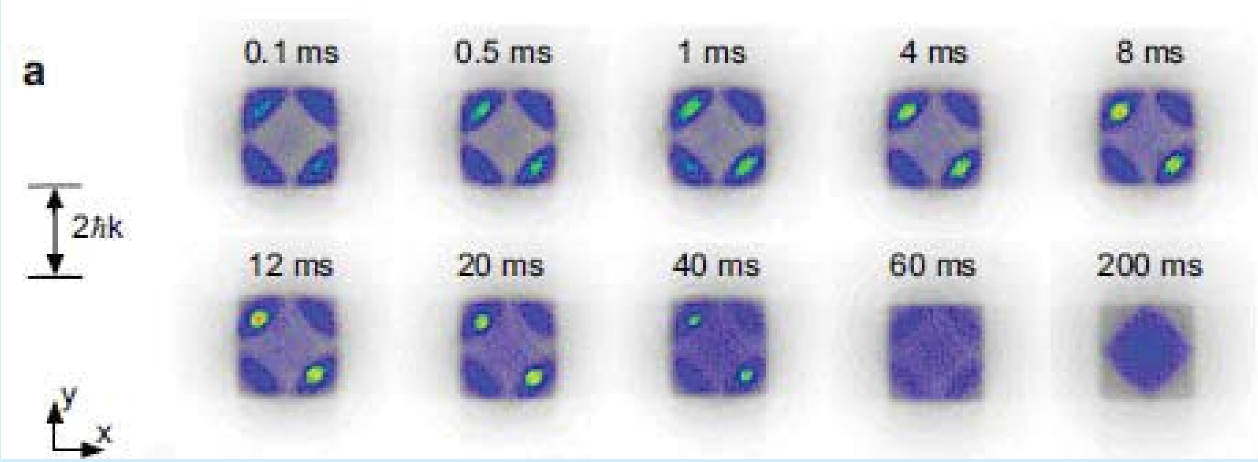


An idea to slow decay by Bloch et al: anharmonicity

p-band long life time observed in the double-well lattice

G. Wirth, M. Olschlager, A. Hemmerich, Nat. Phys. 2011





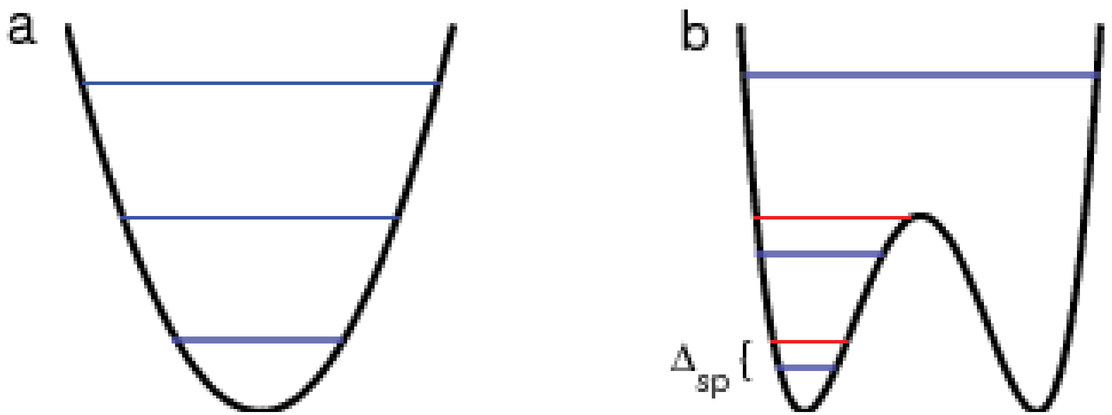
Note:
This **double-well lattice** exhibits a life-time of tens of milliseconds (and up to half second from private communication with A Hemmerich) with coherence.

Why? See possible explanation next slide

"Energy-blocking" mechanism to suppress the decay

[WVL and C. Wu, PRA (2006)]

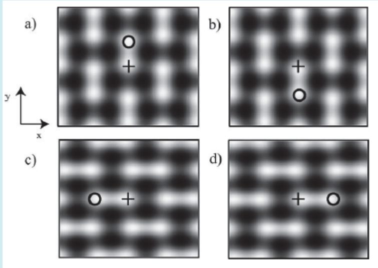
single well lattice ***double well lattice***

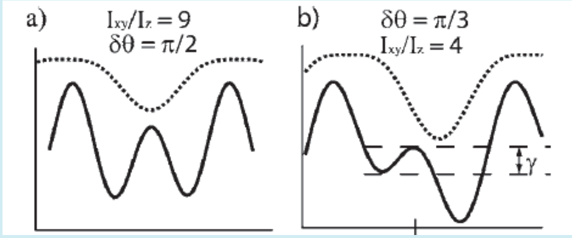


- Low energy motion of bosons is an effective two-band model;
- *p*-orbital bosons cannot decay to the "s" by energy conservation.

Early and recent experiments of double-well superlattices

NIST group (Porto/Phillips)





[J. Sebby-Strabley, et al, PRA (2006); M. Anderlini, et al, J. Phys. B (2006); Nature (2007); …]

Does not preserved original geometric symmetry

Bloch group

[S. Trotzky et al, Science (2008); P. Cheinet et al, Phys. Rev. Lett. 101, 090404 (2008) ; Y. Chen et al, arXiv: 1003.4956 (2010); …]

Time-Resolved Observation and Control of Superexchange Interactions with Ultracold Atoms in Optical Lattices
S. Trotzky, *et al.*
Science **319**, 295 (2008);

Stamper-Kurn group

Cf. his talk at this workshop

Superlattice (red and blue lattices overlapped)
→ Kagome

p-band Mott insulator and superfluid phases and finite temperature

- A. Isacsson and S. M. Girvin, PRA 72, 053604 (2005)
- A. Collin, J. Larson, and J.-P. Martikainen, Phys. Rev. A 81, 023605 (2010)
- X. Li, E. Zhao, WVL, Phys. Rev. A 83, 063626 (June 2011)

[different results on p-band Mott states]

Strong coupling theory
--physical picture

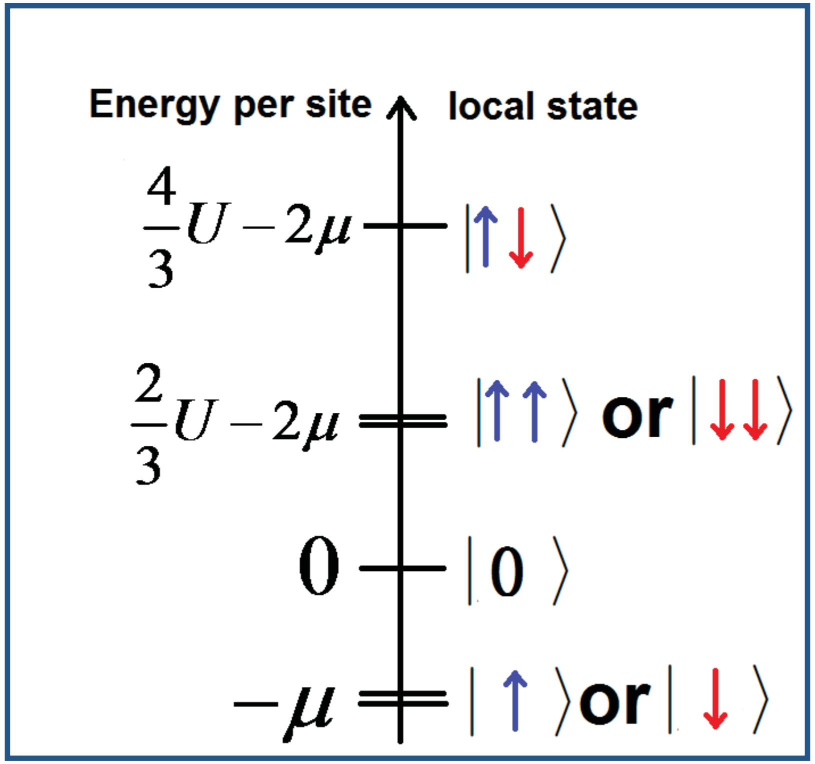
-Strong coupling limit

$$H_{\rm int} = \frac{1}{2}U\sum_{\mathbf{r}}\{n(\mathbf{r})[n(\mathbf{r})-\frac{2}{3}]-\frac{1}{3}L_z^{\,2}(\mathbf{r})\}$$

$$\varepsilon(n,m)=\frac{U}{2}\{(n+m)[(n+m)-\frac{2}{3}]-\frac{1}{3}(n-m)^2\}-\mu(n+m)$$

- $\uparrow = p_x + ip_y$, $\downarrow = p_x - ip_y$
- n– the occupation number of $|\uparrow\rangle$
- m– the occupation number of $|\downarrow\rangle$

-Energy per site in the strong coupling limit



Single particle/hole gap ~ U

The particle/hole excitations are greatly suppressed by the gap in the strongly coupling regime.

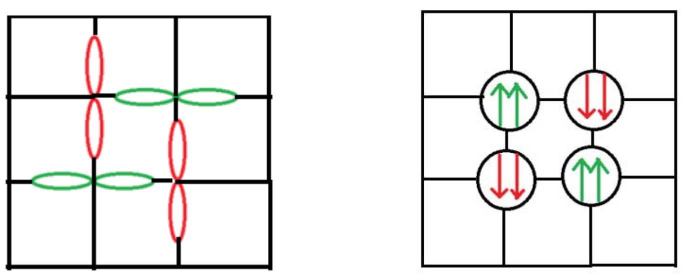
Phase diagram

[X. Li, E. Zhao and WVL, PRA 83, 063626 (2011)]

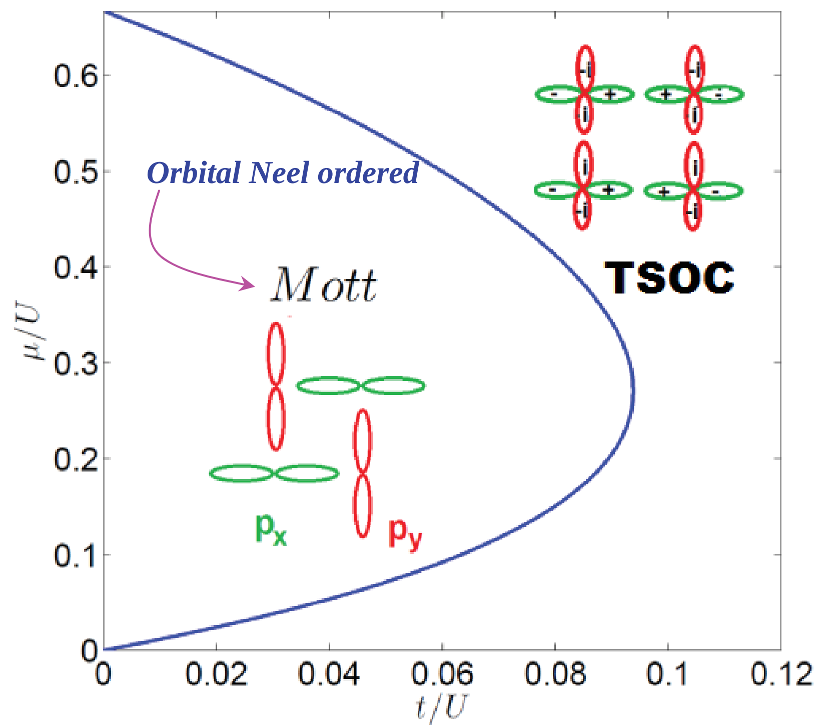
[See also early Mott phase results by A. Isacsson and S. M. Girvin, PRA 72, 053604 (2005); and A. Collin, J. Larson, and J.-P. Martikainen, Phys. Rev. A 81, 023605 (2010)]

-Mott phases at commensurate filling

1. filling =1
2. filling >1



-phase boundary of Mott with one particle per site



Compare Mott results

- Our theory agrees with Isacsson-Girvin MFT for filling=1
- Differs Isacsson-Girvin at higher integer fillings [who predicted p_x-p_y alternating]
- Collin-Larson-Martikainen disagrees with both Isacsson-Girvin and us

Compare: the usual s-band Mott phase is featureless.

Finite temperature phase transitions of the TSOC (p-orbital BEC) phase --- XY model plus Ising

$$\begin{bmatrix} \varphi_x(\mathbf{r}) \\ \varphi_y(\mathbf{r}) \end{bmatrix} = \sqrt{n_s/2} \begin{bmatrix} e^{i\theta(\mathbf{r})}(-)^{s_x(\mathbf{r})} \\ e^{i\theta(\mathbf{r})+\frac{\pi}{2}}(-)^{s_y(\mathbf{r})} \end{bmatrix}$$

$$s_{x,y} = \text{even, odd}$$

$$E = -J_\theta \sum_{\mathbf{r}} [\cos(\theta(\mathbf{r}) - \theta(\mathbf{r}+x)) + \cos(\theta(\mathbf{r}) - \theta(\mathbf{r}+y))]$$

$$+\sum_{\mathbf{r}} [-J_1 e^{i\pi s_y(\mathbf{r})} e^{i\pi s_y(\mathbf{r}+y)} - J_2 e^{i\pi s_y(\mathbf{r})} e^{i\pi s_y(\mathbf{r}+x)}]$$

$$+\sum_{\mathbf{r}} [-J_1 e^{i\pi s_x(\mathbf{r})} e^{i\pi s_x(\mathbf{r}+y)} - J_2 e^{i\pi s_x(\mathbf{r})} e^{i\pi s_x(\mathbf{r}+x)}]$$

Isotropic XY

$$J_\theta = (|t_\parallel| + |t_\perp|) n_s / 2$$

Anisotropic Ising

$$J_1 = \frac{1}{2} |t_\parallel| n_s$$

$$J_2 = \frac{1}{2} |t_\perp| n_s$$

Ising transition [L. Onsager, Phys. Rev. (1944)]

Kosterlitz-Thouless transition [J. Phys. C (1973)]

X. Li, E. Zhao and WVL, Phys. Rev. A 83, 063626 (2011)

Topological phase of dipolar bosons in elongated Wannier orbitals

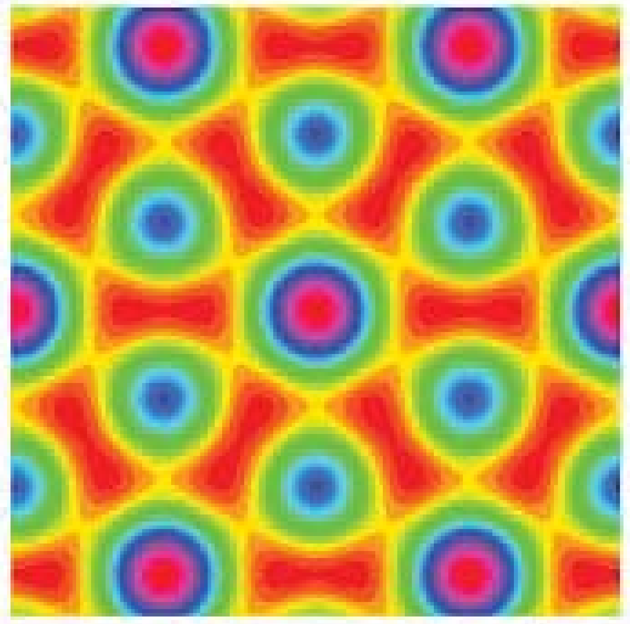
Orbital + Dipolar interaction to realize a quantum dimer model

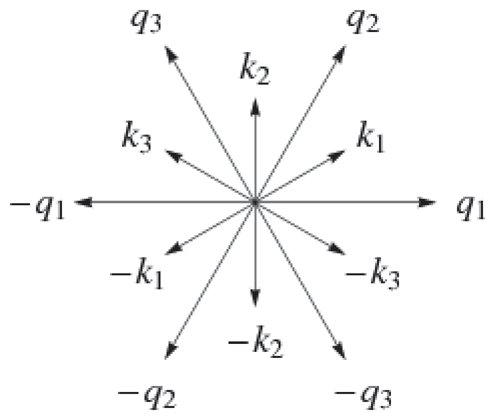
[Kai Sun, Erhai Zhao, WVL, *Phys. Rev. Lett.* 104, 165303 (2010)]

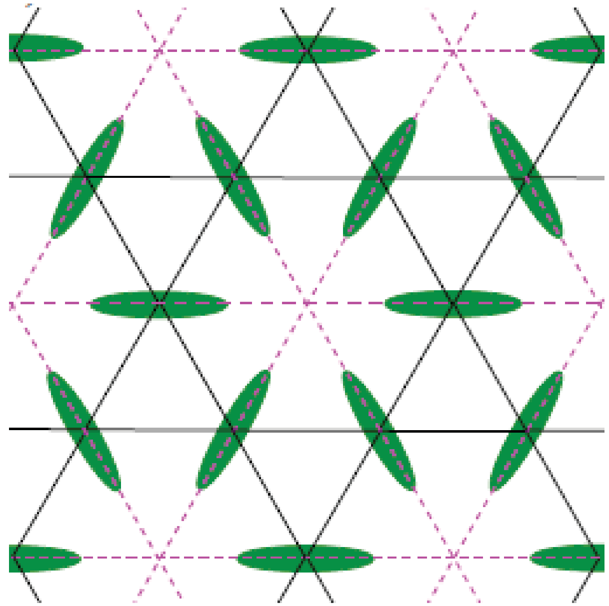
2D Optical lattice with elongated orbital

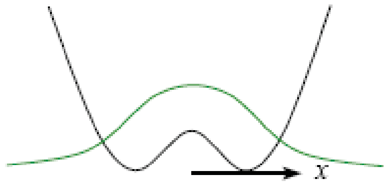
$$I = I_1 \sum_{i=1,2,3} \cos^2(\mathbf{k}_i \cdot \mathbf{r}) + I_2 \sum_{j=1,2,3} \cos^2(\mathbf{q}_j \cdot \mathbf{r})$$

$$|\mathbf{q}_j| = \sqrt{3}|\mathbf{k}_i|,\ \theta_{\mathbf{q}_j} = 2j\pi/3,\ \theta_{\mathbf{k}_i} = (2i-1)\pi/6$$







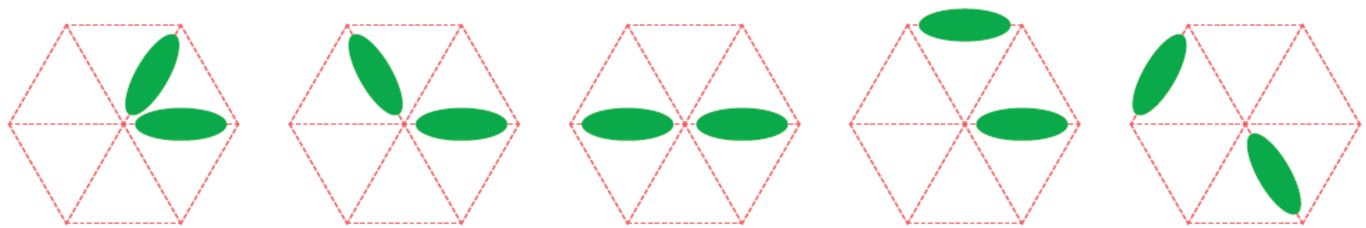


- Elongated Wannier orbital wave-function is a "dimer"
  - use charge degree of freedom not spin
- Site of the Kagome lattice ⟺ bond of the triangular lattices
- Hard-core constraint? Polar molecules with dipolar interaction (next slide)

[Kai Sun, Erhai Zhao, WVL, Phys. Rev. Lett. 2010]

Polar Molecules

Dipoles aligned in z direction (2D $1/r^3$ repulsive interaction)

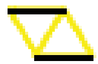
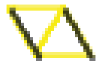


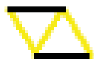
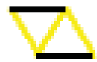
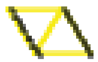
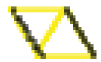
V_1 V_2 V_3 >> $V_{//}$ V_S

1/6 filling mapped to Orbital Quantum Dimer Model

- Energy scale: 1nk (by our estimate)
- Charge (no spin)
- Either Fermion or Boson? (Having fermionic dimers is new and unexplored.)

The dual effective Hamiltonian.
Model of quantum dimers

$H = -\,t_{\text{eff}} \sum \Big(|$  $\rangle\langle$  $| + h.c. \Big)$

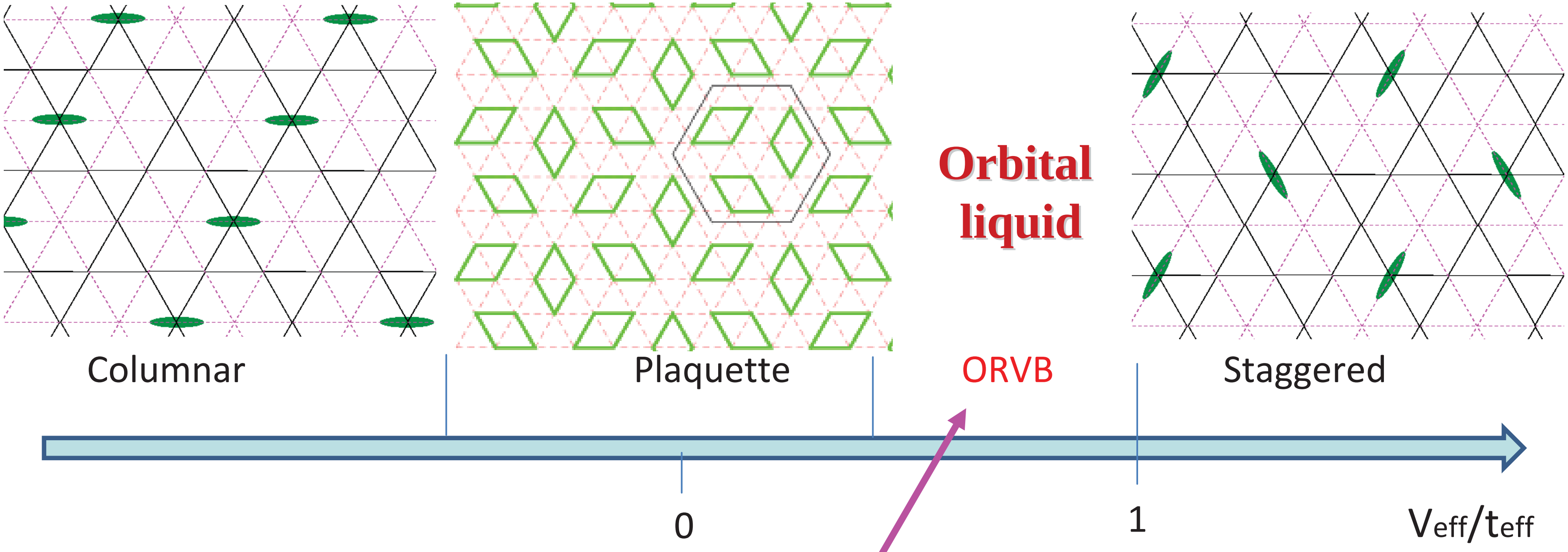
$+\,V_{\text{eff}} \sum \Big(|$  $\rangle\langle$  $| + |$  $\rangle\langle$  $| \Big)$

$+\ \ldots$ (perturbative terms)

$t_{\text{eff}} \sim \dfrac{t^2}{V_i}$ with t = molecule hopping amplitude

$$V_{\text{eff}} \sim V_{\|} - V_S + O(t^2/V_i)$$

Phase Diagram for Bosons



- Breaks no symmetry (no local order parameter), outside the paradigm of Landau-Ginsburg-Wilson theory
  - No symmetry-related signature, c.f. other three phases
- Fractional excitation: holons
  - carries ½ quantum number of the particle
  - can be fermions or bosons (microscopic details)
    - Fermionic quasiparticles in a bosonic system!
- Topological degeneracy

Unexplored Questions

- Phase transition to superfluid upon doping
- Fermionic dimers. use fermionic molecules.
- Dimers with polarity. Fill fermions onto the p-orbital?
- Challenge: Experimental detection of holons (for cold gases)

Outline

1. Introduction
   - Beyond s-orbital, the basics of p-orbital? Why p-orbital?
   - Recent experimental progress

2. p-band bosons (including examples of no prior analogue in solids).
   - Finite momentum BEC with px+ipy orbital order
   - Mott insulator phases and orbital order
   - Elongated Orbitals and mapping to quantum dimer model

3. p-band fermions
   - Single species. Mott states. Orbital-only model. Realization of Kitaev-like model?
   - Topological states (quantum Hall like). Topological semimetal (non-interacting), and topological insulator (turn on interaction). Time reversal breaking driven by interaction, no magnetic field!

Single species fermions in p-bands

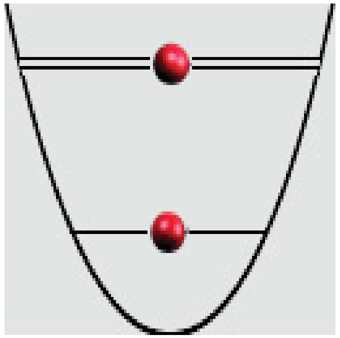
[E. Zhao and WVL, PRL (2008); C. Wu, PRL 2008.]

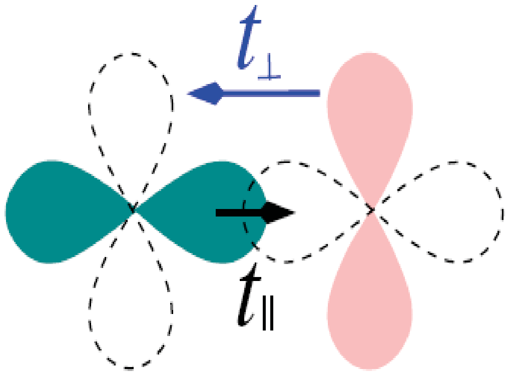
Motivation: *Study the simplest possible case, without spin or lattice or charge effect*

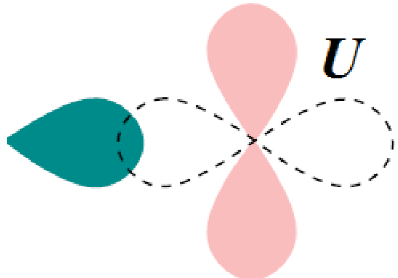
"plain-vanilla" orbital ordering & orbital only models

Single species (spinless) fermions interact in p-wave channel

Mott state: 2 fermion per site, one occupies the s-orbital, the other one occupies p-orbital (p_x or p_y, two fold degenerate for 2D lattice)







Strong coupling regime: *U/t* can be large by tuning close to a p-wave Feshbach resonance [e.g., talks of Leo Radzihovsky, G. Shlyapnikov]

Multiband (s and p) Hubbard → p-band Hubbard model with anisotropic hopping

Mott insulator phase - Effective orbital exchange

***p*-band Hubbard:**

$$H_p = \sum_{i;\mu,\nu=x,y} t_{\mu\nu}[c^\dagger_{i,\mu} c_{i+\nu,\mu} + h.c.] + U \sum_i n_{ix} n_{iy}$$

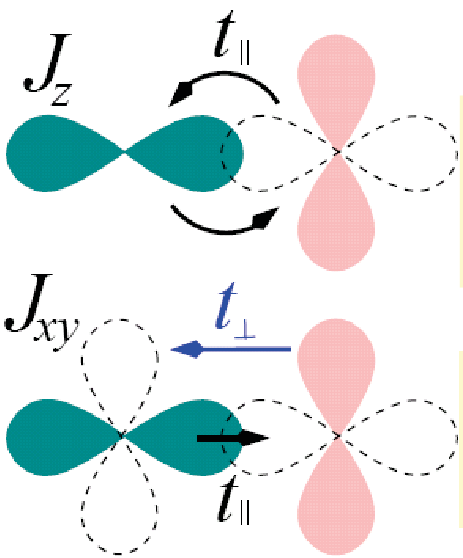
Hopping anisotropy: $t_{\mu\nu} = t_\parallel \delta_{\mu\nu} - t_\perp (1 - \delta_{\mu\nu}$

$U \gg t_\parallel$

Virtual hopping → ***orbital exchange:***

$$H_{\rm orb} = \sum_{<i,j>} [J_{xy}(T_i^+ T_j^- + h.c.) + J_z T_i^z T_j^z]$$

$J_{xy} \ll J_z$



$J_z = 2(t_\perp^2 + t_\parallel^2)/U$

Antiferro-orbital Ising

$J_{xy} = -2t_\perp t_\parallel / U$

Ferro-orbital XY

$$H_{\rm orb} \simeq J_z \sum_{<i,j>} T_i^z T_j^z$$

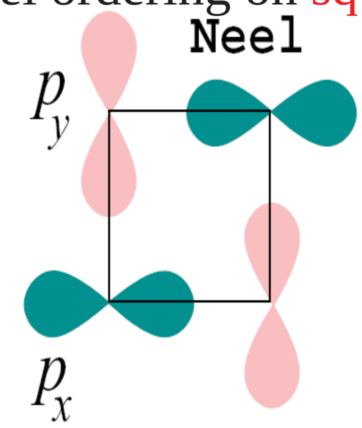
$$T^z = (c_x^\dagger c_x - c_y^\dagger c_y)/2$$

$$T^+ = c_x^\dagger c_y, \quad T^- = (T^+)^\dagger$$

Neel orbital order and Frustration

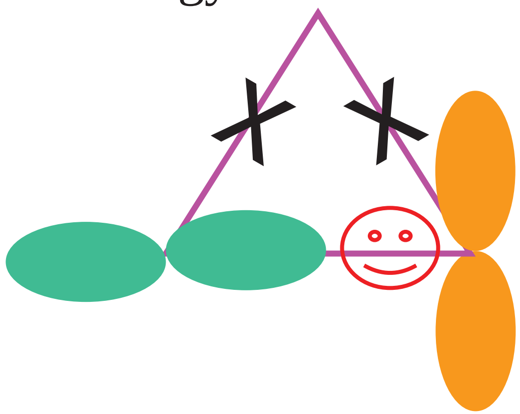
Antiferro-orbital exchange favors perpendicular alignment of nn orbitals along bonds

Neel ordering on square lattice



Triangle, honeycomb, Kagome …?

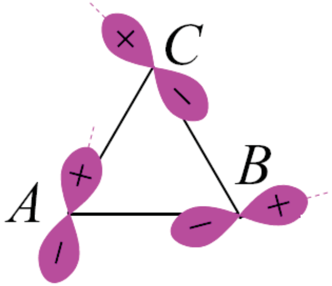
Non-orthogonal bonds can not achieve the lowest energy simultaneously

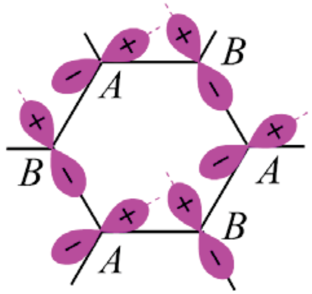


Orbital quantum 120º model

[E. Zhao and WVL, PRL (2008); See also independent work by C. Wu, PRL 2008.]

Pseudo-spin operators on frustrated lattices (triangular, honeycomb, Kagome, …)



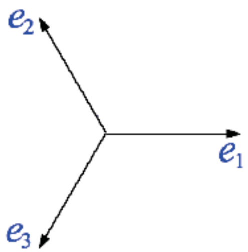


$|p_x\rangle$ $|p_y\rangle$

$$T_\mu = \frac{1}{2}\left(c_x^\dagger, c_y^\dagger\right)\sigma_\mu \begin{pmatrix} c_x \\ c_y \end{pmatrix}$$

$$\mu, \nu = x, y, z$$

The orbital exchange Hamiltonian is:



$$H_{120} = J_z \sum_{\mathbf{R},j} T_j(\mathbf{R}) T_j(\mathbf{R} + \hat{e}_j)$$

lattice sites

j =1,2,3

where

$$T_1 = T_z,\ T_2 = -\tfrac{1}{2}T_z - \tfrac{\sqrt{3}}{2}T_x$$

$$T_3 = -\tfrac{1}{2}T_z + \tfrac{\sqrt{3}}{2}T_x$$

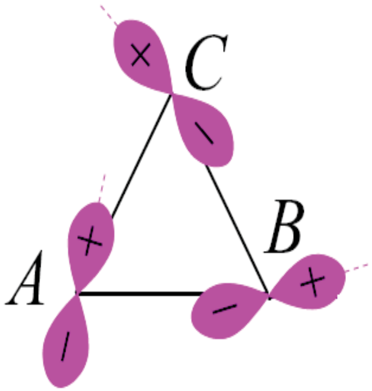
This quantum 120º model is closely related to the compass model and Kitaev model.

Quantum 120º model of electron e_g orbitals: van den Brink, New J. Phys. 6, 201 (2004).

(classical) Orbital phase on Kagome lattice

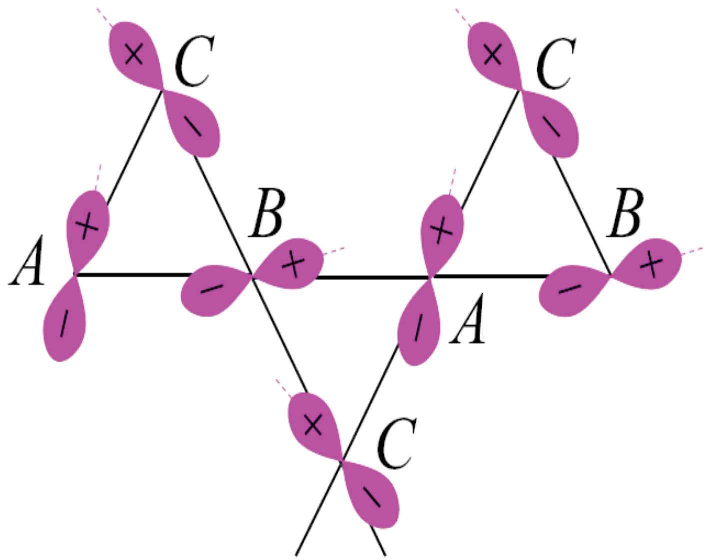
Strategy: large pseudospin T expansion, first solve the classical problem (mean field), then consider $1/T$ quantum fluctuation corrections.

Single triangle: minimize the classical energy



- Orbital at 15^{o} with the bond
- Equivalent configuration: all orbitals rotated by 90^{o}

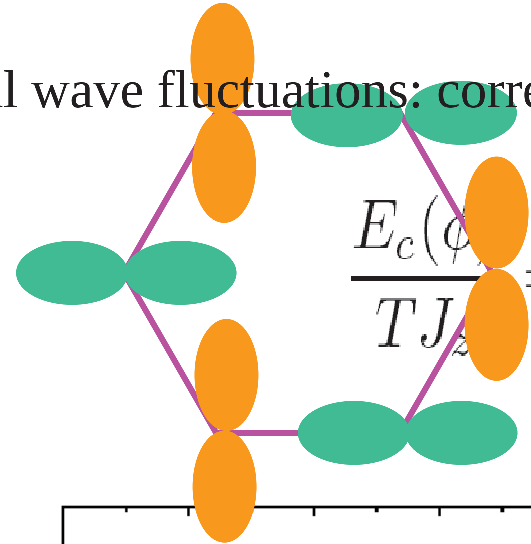
Kagome lattice: corner shared triangles



Order by disorder: honeycomb lattice

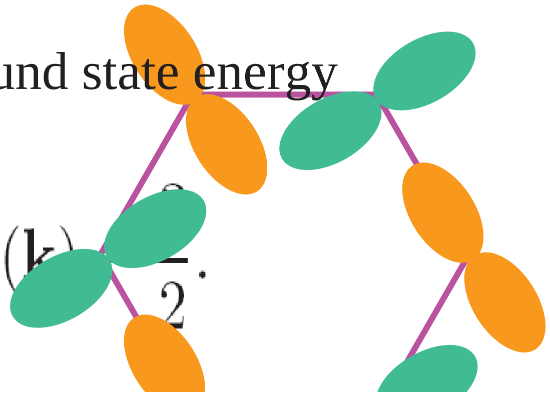
Classical energy is minimized for any Neel configuration, continuous degeneracy

Orbital wave fluctuations: corresction to ground state energy

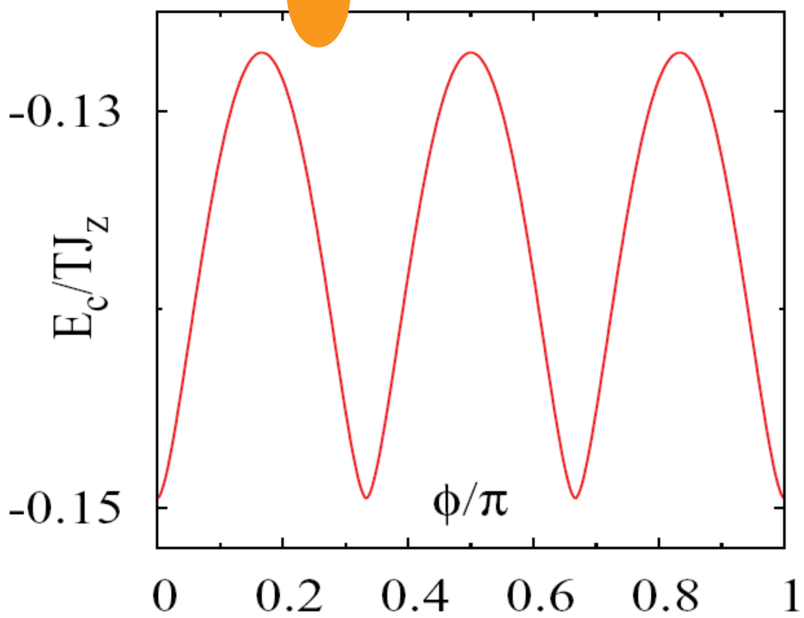


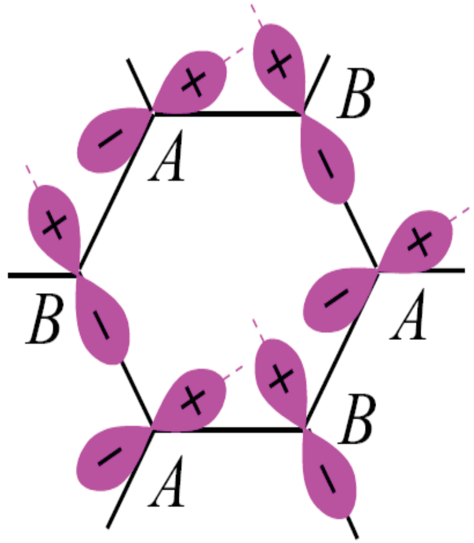
$$\frac{E_c(\phi)}{TJ_z} = \frac{1}{N}\sum_{\mathbf{k},\lambda} \frac{\omega_\lambda(\mathbf{k})}{2}.$$

or



or …





Orbital at sublattice A at angle n*30°

Mott phase of 3D (*orbital only!*) lattice model

Interaction different for three p-wave channels

- 3D lattice
- 1/3 filling, three p-states per site

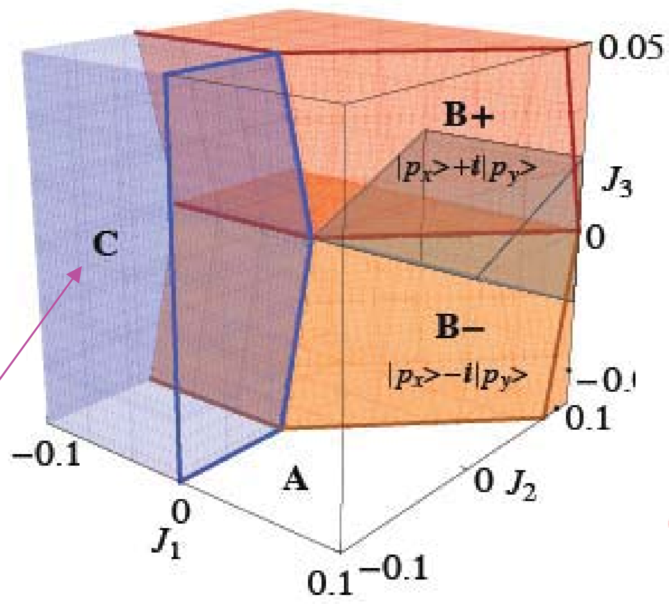
$$H_{\text{eff}} = -\sum_{i}\Big[\sum_{\mu=x,y,z}\sum_{\delta=\pm e_\mu} J_\mu n_{\mu,i}\left(1-n_{\mu,i+\delta}\right) + \sum_{\mu=x,y}\sum_{\delta=\pm e_\mu}\left(J_2-J_1\right) n_{\mu,i} n_{z,i+\delta} + \sum_{\delta=\pm e_z} J_3(i c^\dagger_{y,i} c_{x,i} n_{z,i+\delta} + h.c.)\Big]$$

$$J_x = J_y \equiv J_1, \quad J_z \equiv J_2$$

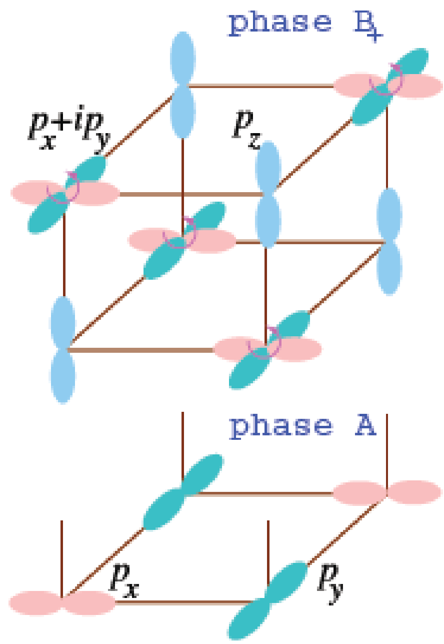
- Effective model for the system of Goyal-Reichenbach-Deutsch [PRA 2010] with optical p-wave Feshbach resonance (e.g., ^{171}Yb)
- m= -1,0,+1 channels separate
- Explicit Time-Reversal breaking term

[with P. Hauke, E. Zhao, K. Goyal. I. Deutsch, M. Lewenstein, arXiv:1103.5964]

p-band fermions at 1/3 filling in 3D – *Phase diagram*



Homogeneously filled phase



***T*-breaking**

***T* not breaking**

[with P. Hauke, E. Zhao, K. Goyal. I. Deutsch, M. Lewenstein, arXiv:1103.5964]

Topological Semimetal: a probable new state of cold gases in optical lattices

protected by D4 symmetry

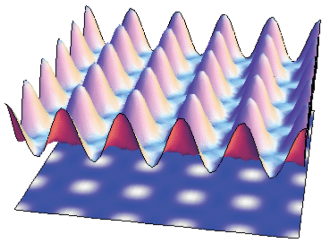
Work done (in collaboration) with:

- Kai Sun (Maryland)
- Sankar Das Sarma (Maryland)
- Andreas Hemmerich (Hamburg)

[arXiv:1011.4301]

Motivations and Summery of Results

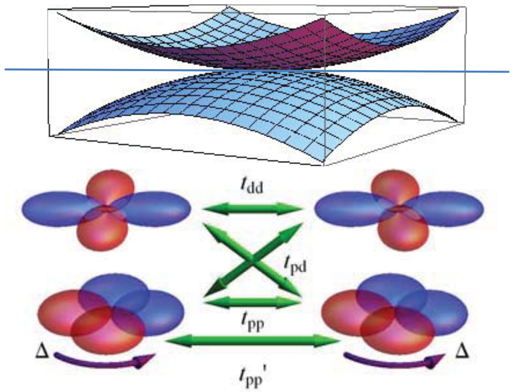
A. Fermions on a square lattice of double-wells



$$V(x,y) = -V_1[\cos(kx)+\cos(ky)] + V_2[\cos(kx+ky)+\cos(kx-ky)]$$

Will show how to experimentally realize [with Hemmerich]

B. Mixing of p and d orbitals: Semi-metal with topological winding number 2



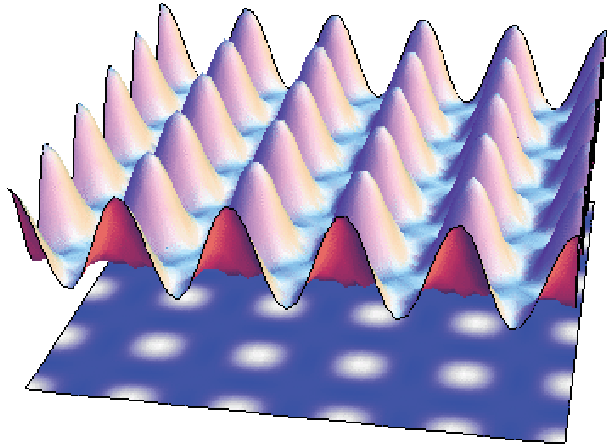
- Beyond Fermi liquid: $k_F=0$
- Graphene: (Nobel Prize 2010)
- Here: a different example of semi-metal

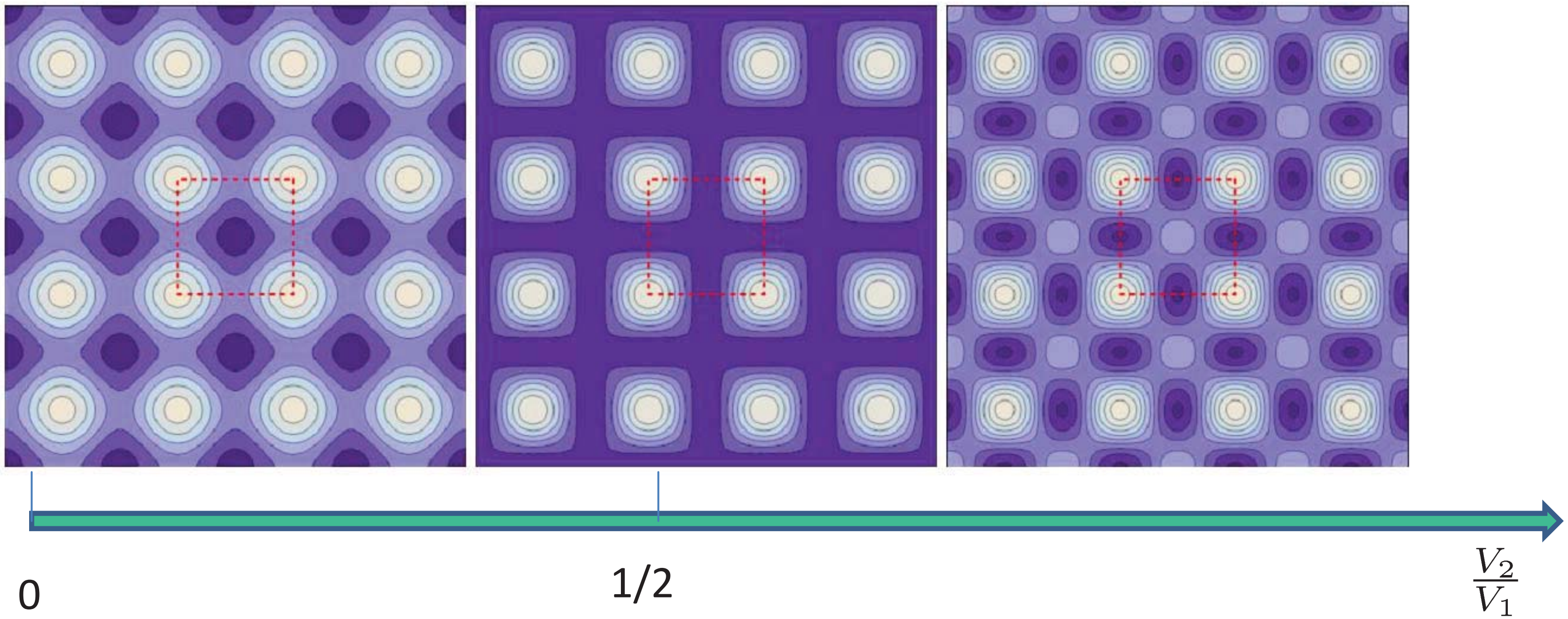
C. Interaction turns this topological semimetal into topological insulator. Mechanism differs from:

- Rotating fermions
- Synthetic gauge fields: I. B. Spielman (2009)

Optical lattice and Band degeneracy

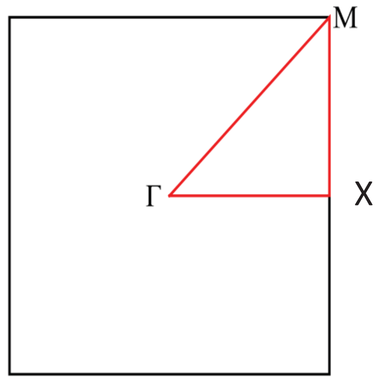
$$V(x,y) = -V_1[\cos(kx) + \cos(ky)] + V_2[\cos(kx+ky) + \cos(kx-ky)]$$





Band structure (from Numerical)

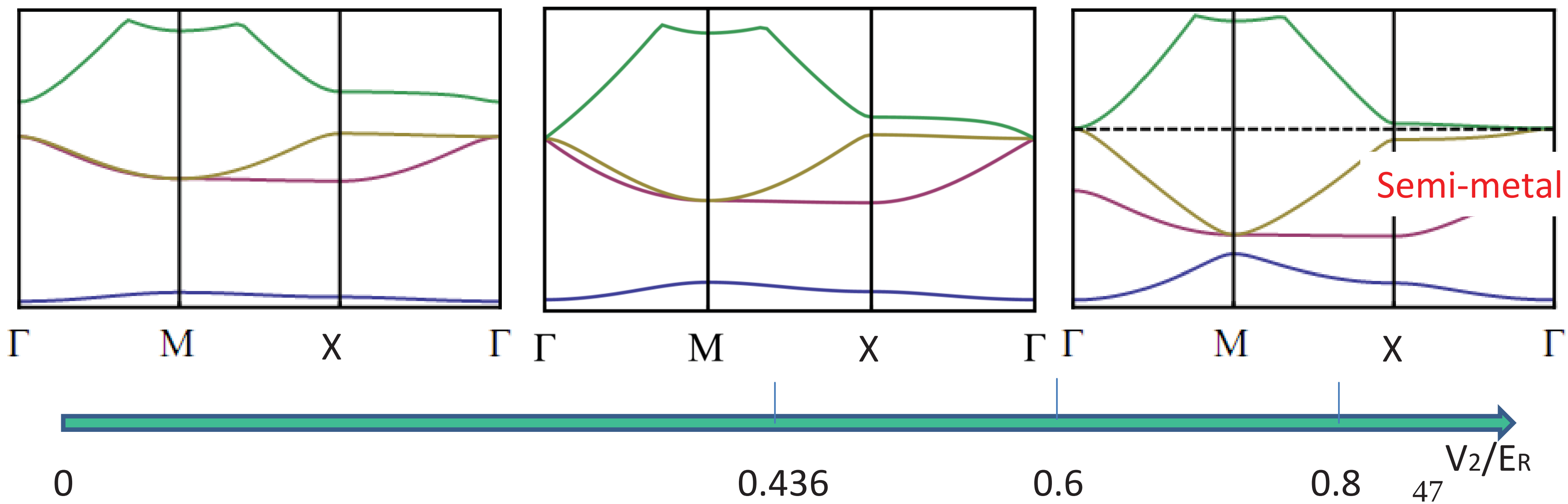
$$V(x,y) = -V_1[\cos(kx)+\cos(ky)] + V_2[\cos(kx+ky)+\cos(kx-ky)]$$



$V_1 = 1.2\ E_R$

$V_2 < 0.8\ E_R$

Band Degeneracy: Very stable (symmetry/ topologically protected)



Topology of the band degenerate point Γ

Expand momentum around point Γ:

$$\mathcal{H} = \frac{t_1 + t_2}{2}(k_x^2 + k_y^2)I + 2t_3 k_x k_y \sigma_x + \frac{t_1 - t_2}{2}(k_x^2 - k_y^2)\sigma_z$$

$$t_1 = t_{pp} + \frac{4t_{pd}^2}{2t_{pp} - 2t'_{pp} + 4t_{dd} - \delta}, \quad t_2 = -t'_{pp}, \quad t_3 = \frac{2t_{pd}^2}{2t_{pp} - 2t'_{pp} + 4t_{dd} - \delta}$$

Map Hamiltonian to a planar vector

$$\vec{h} = \left(2t_3 k_x k_y, \frac{t_1 - t_2}{2}(k_x^2 - k_y^2)\right)$$

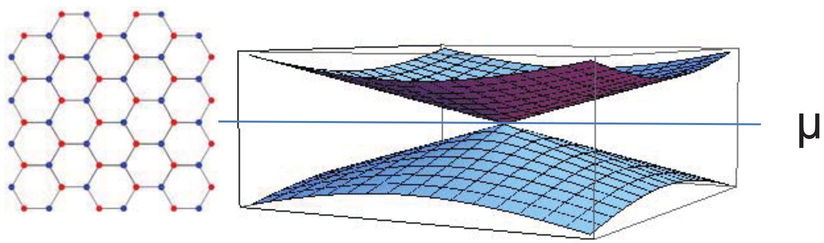
Winding number

$$W = \frac{1}{2\pi}\oint_{\mathcal{C}} d\mathbf{k} \cdot \left[\epsilon_{ij}\frac{h_i}{|\vec{h}|}\nabla\left(\frac{h_j}{|\vec{h}|}\right)\right]$$

Found W=2 for Γ point

Interaction

Dirac Point (graphene)



• Linear dispersion relation

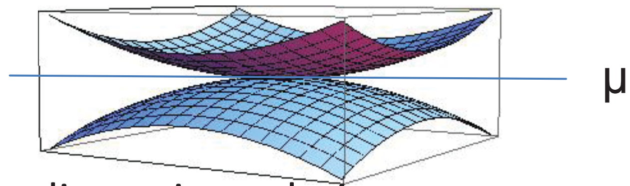
• Appear in pairs at k and –k (same energy)

• Observed in real systems

At low T

• Stable against short-range interactions

Semi-metal found here



• Quadratic dispersion relation

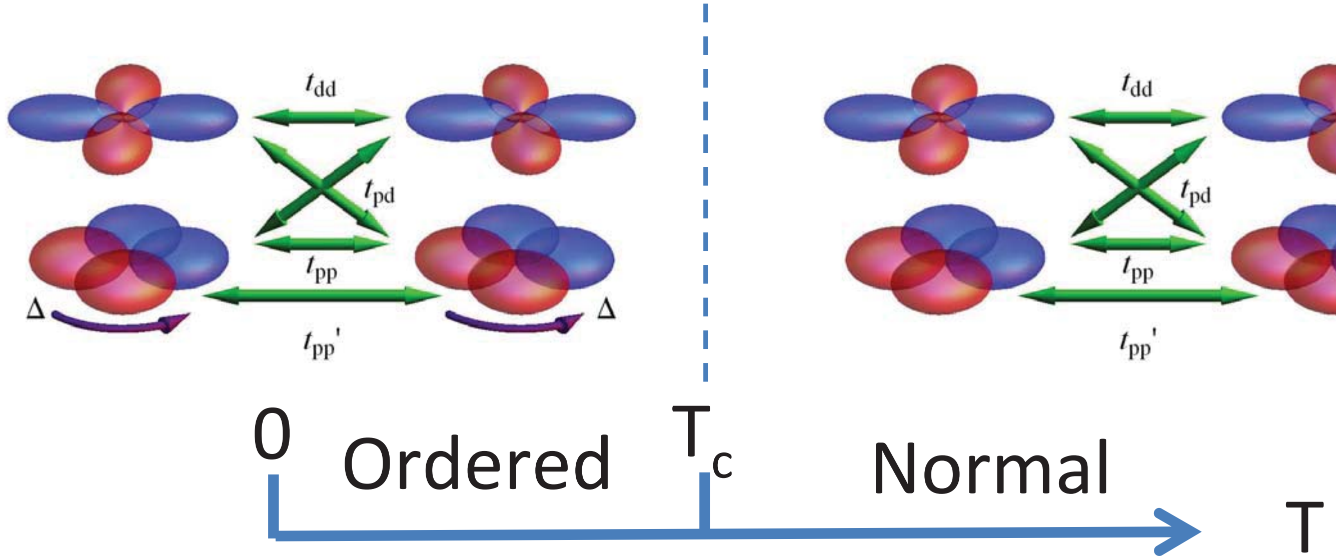
• no pairs at –k (can split into two Dirac point)

• Not observed in real systems

At low T

• Unstable against short-range repulsions (by usual Renormalization group analysis, applied to quadratic dispersion, time scaling dim=2 opposed to 1 in usual Fermi liquid)

Phase diagram



$$H_{\text{int}} = V \sum_{\vec{r}} p^{\dagger}_{x,\vec{r}} p_{x,\vec{r}} p^{\dagger}_{y,\vec{r}} p_{y,\vec{r}} = -\frac{V}{2} \sum_{\vec{r}} (L^z_{\vec{r}})^2$$

$$L^z_{\vec{r}} = -i(p^{\dagger}_{x,\vec{r}} p_{y,\vec{r}} - p^{\dagger}_{y,\vec{r}} p_{x,\vec{r}})$$

$$L^z_{\vec{r}} |p_x + ip_y\rangle = |p_x + ip_y\rangle$$

$$L^z_{\vec{r}} |p_x - ip_y\rangle = - |p_x - ip_y\rangle$$

Low T ordered phase:

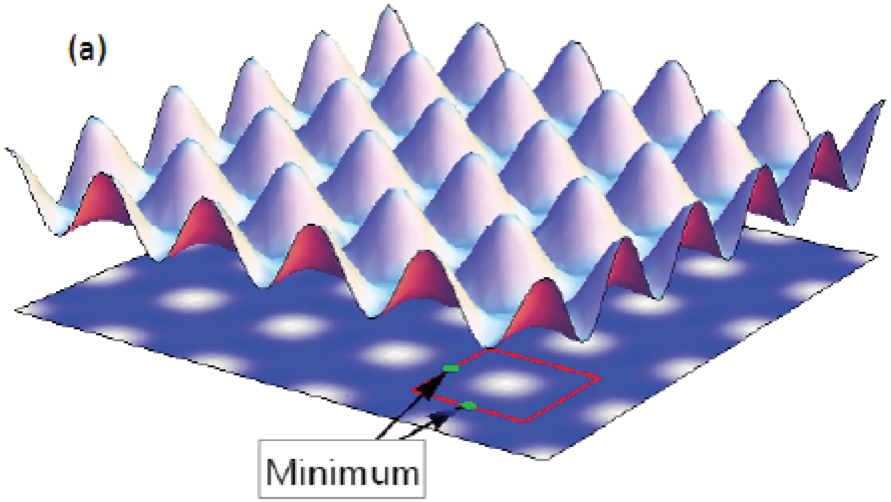
- Breaks Time-reversal symmetry
- Topological state of matter
- Quantum Hall induced by interaction
  - Anomalous Quantum Hall [1-2]
  - Chern number=1 [2]

1. Haldane PRL 1988; S. Raghu, et.al. PRL 2008.
2. K. Sun, et al. PRL 2009, PRB 2008.

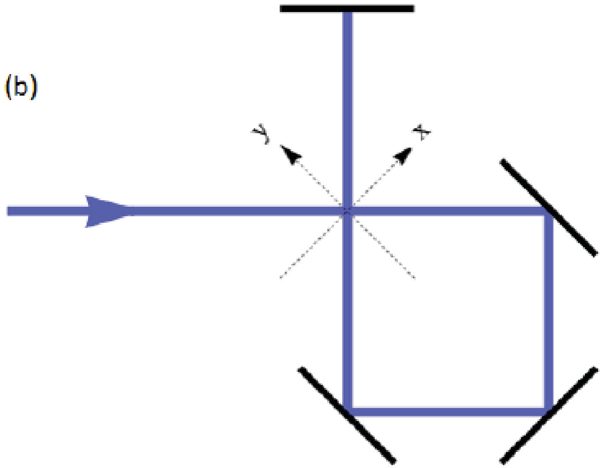
How to realize the proposed lattice?

[due to A. Hemmerich]

(a)



(b)



$$E = \epsilon \begin{pmatrix} 0 \\ \sin\alpha \\ \cos\alpha \end{pmatrix} \cos\left[k(x+y)/2\right] - \epsilon \begin{pmatrix} \sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix} \cos\left[k(x-y)/2\right]$$

by $U(x,y) = -\chi |E(x,y)|^2$

$$\begin{aligned} U(x,y) = & - V_1[\cos(kx) + \cos(ky)] \\ & + V_2[\cos(kx+ky) + \cos(kx-ky)] + \chi\epsilon^2 \end{aligned}$$

$$\begin{aligned} V_1 = & - \chi\epsilon^2 \cos^2\alpha, \\ V_2 = & - \chi\epsilon^2/2. \end{aligned}$$

Conclusion

- Degenerate p-orbitals. Emergent symmetry. Orbital physics.

- Novel superfluid and Mott states – no prior analogue in condensed matter physics.
  - *Examples: T-breaking px+ipy finite-momentum BEC. Real or complex orbital Neel order in Mott states. Topological semimetal. Interaction-driven quantum Hall-like topological insulator, etc.*

- Can also emulate condensed matter orbital models *(e.g., quantum 120^o model)*

Interested? Perspectives in:

OPTICAL LATTICES

Orbital dance

Emulating condensed-matter physics with ground-state atoms trapped in optical lattices has come a long way. But excite the atoms into higher orbital states, and a whole new world of exotic states appears.

Maciej Lewenstein and W. Vincent Liu [Nature Phys. 7, 101 (Feb 2011) (news and views)]