



**The Abdus Salam  
International Centre for Theoretical Physics**



**2252-S-1**

**Advanced Workshop on Non-Standard Superfluids and Insulators**

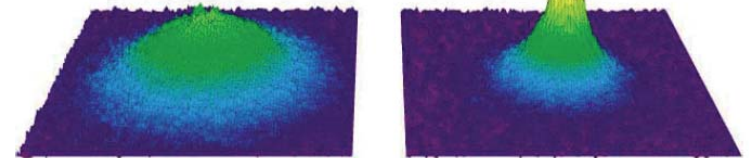
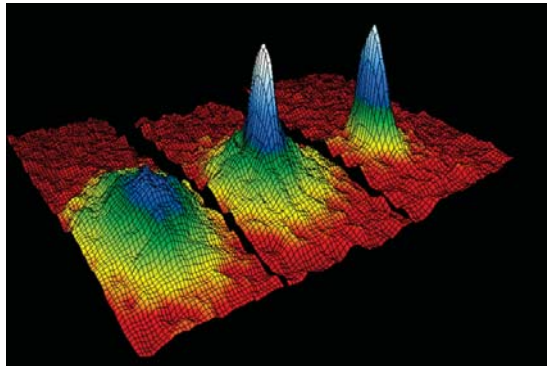
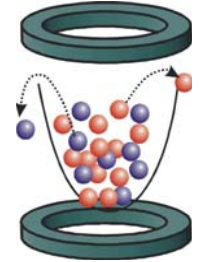
*18 - 22 July 2011*

**Finite-momentum superfluidity and phase transitions in a p-wave resonant Bose gas**

L. Radzihovsky

*University of Colorado at Boulder  
USA*

# Finite-momentum superfluidity and phase transitions in p-wave resonant Bose gas



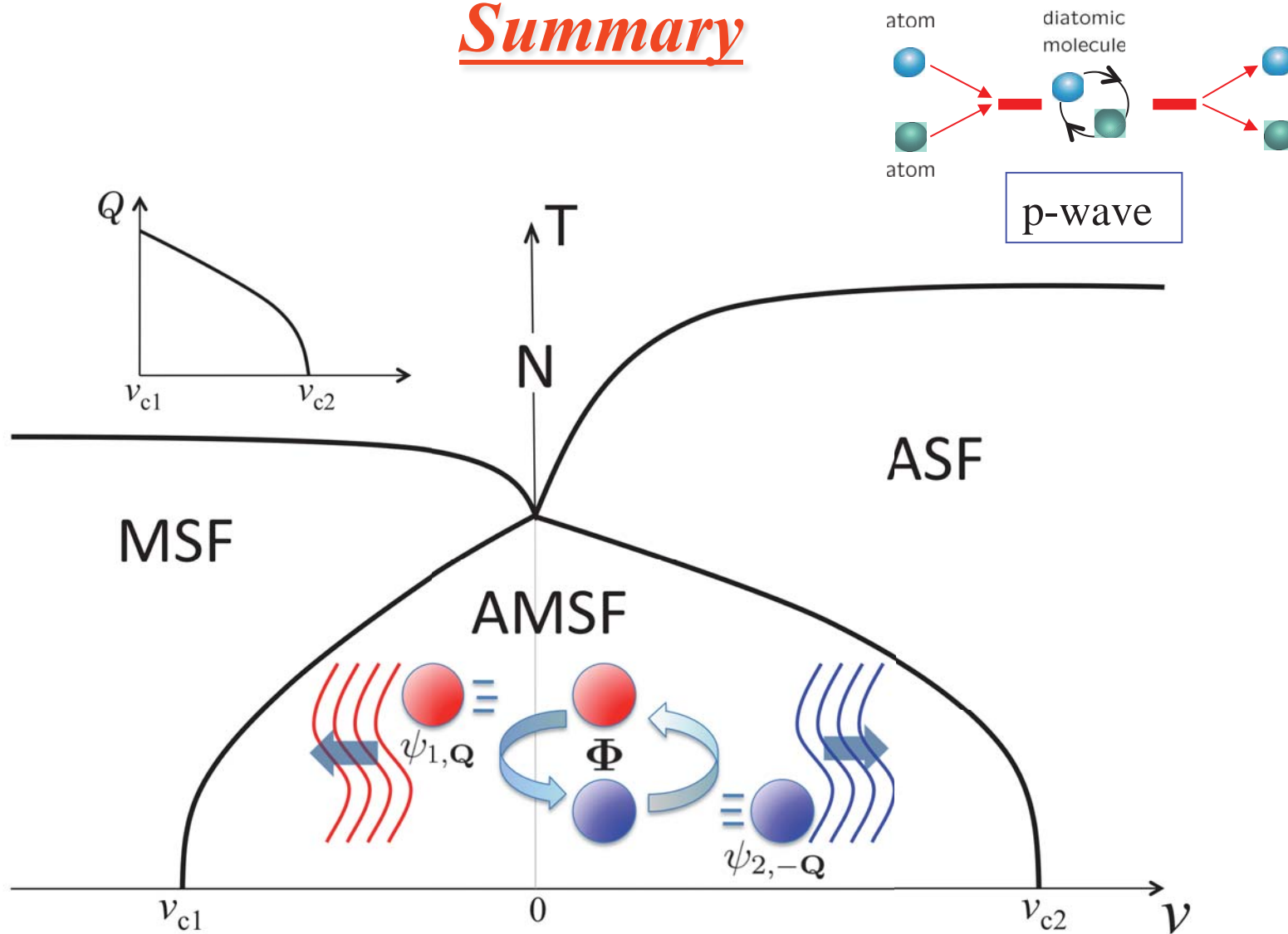
with *Sungsoo Choi*

(for details see: [arXiv:1106.5765](https://arxiv.org/abs/1106.5765) and PRL '09  
also see: Liu, Wu, and Kuklov '06)

support by: *NSF Materials Theory*

ICTP Tieste, July 2011

# Summary



- atomic (ASF) and spinor-molecular (MSF) superfluids
- atomic-molecular superfluid (AMSF) with finite momentum atomic BEC
- quantum and thermal phase transitions

# *Outline*

- Motivation
- Feshbach resonances
- Model
- Phases and transitions
- Excitations
- Conclusions

## *Motivation*

- **Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,...**  
→ **ultracold coherent bosonic atom-molecule mixtures**
- Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.
- Allow SF-SF quantum phase transitions (cf. just crossover for fermions)

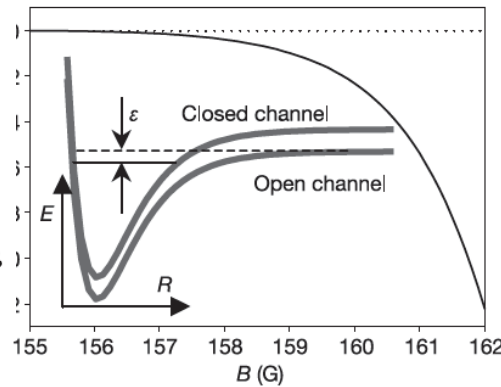
# Rb85 Feshbach resonances

## Atom-molecule coherence in a Bose-Einstein condensate

Elizabeth A. Donley, Neil R. Claussen, Sarah T. Thompson & Carl E. Wieman

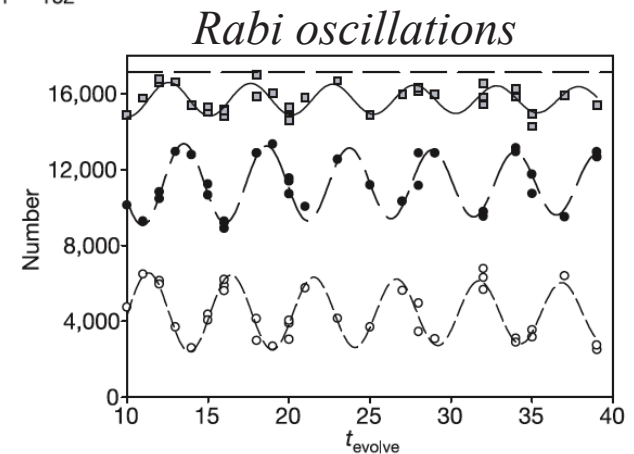
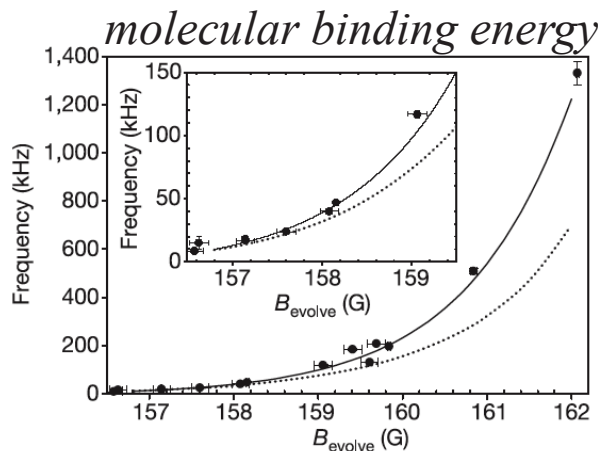
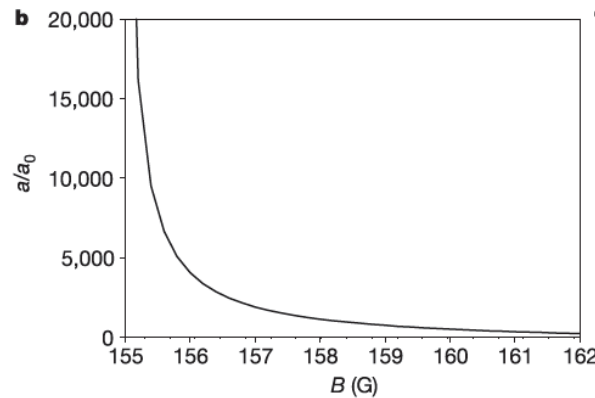
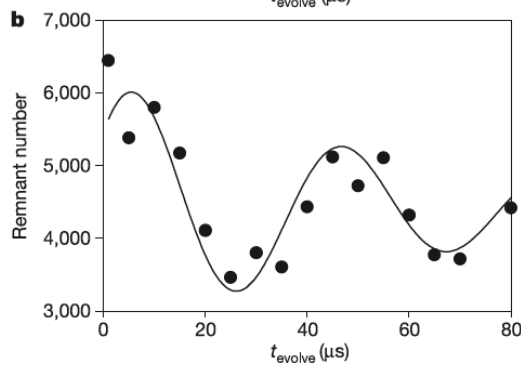
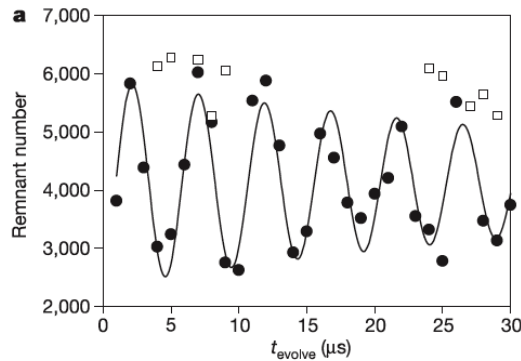
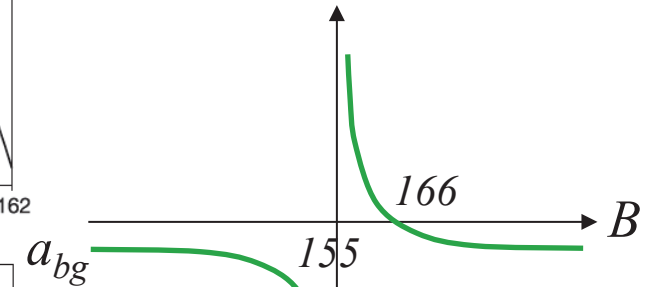
JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

NATURE | VOL 417 | 30 MAY 2002

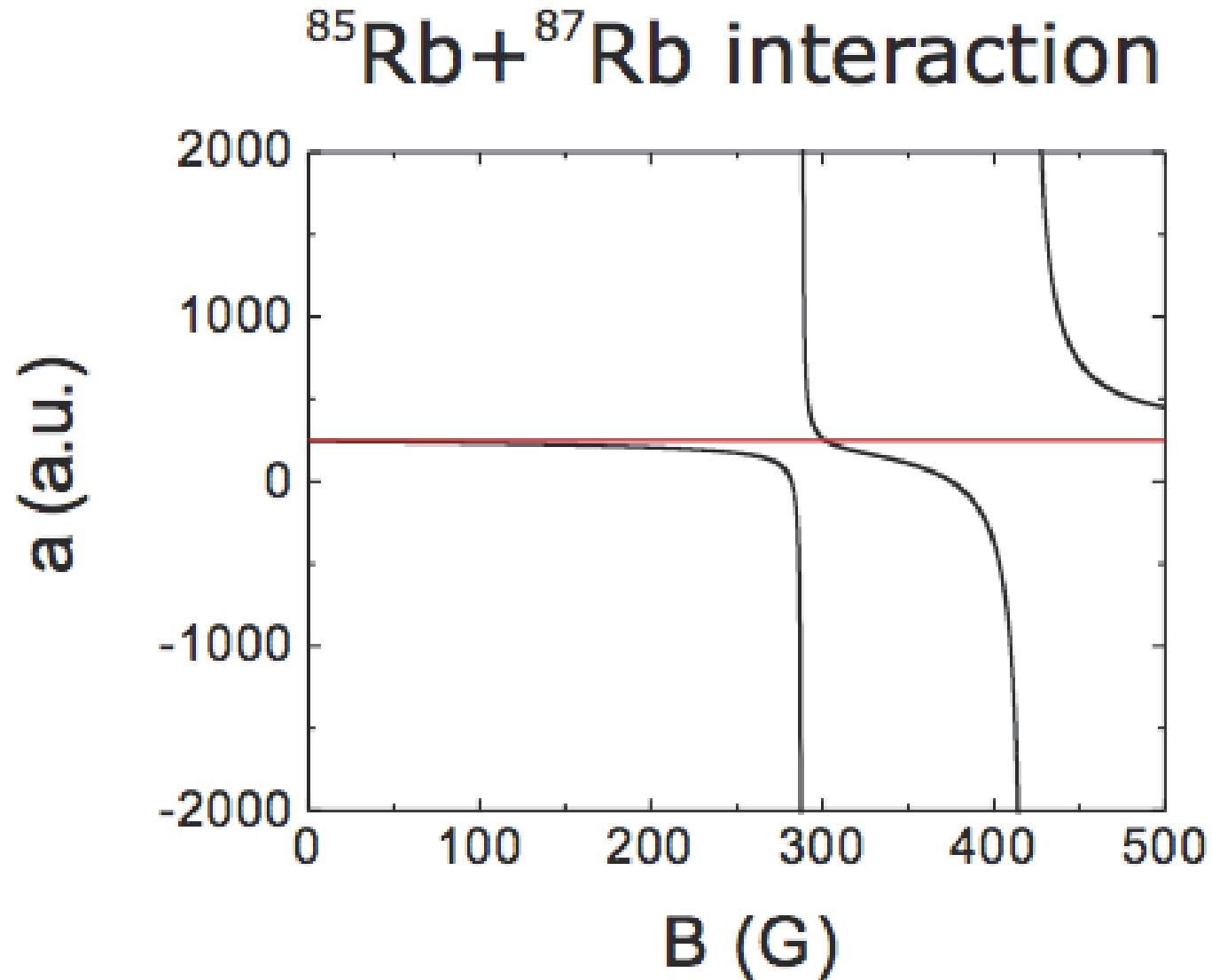


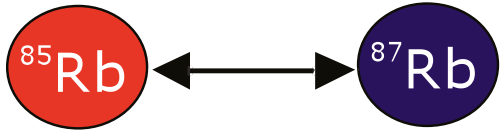
$$|F = 2, m_F = -2\rangle$$

$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right)$$



# *Rb85-Rb87 Feshbach resonances*

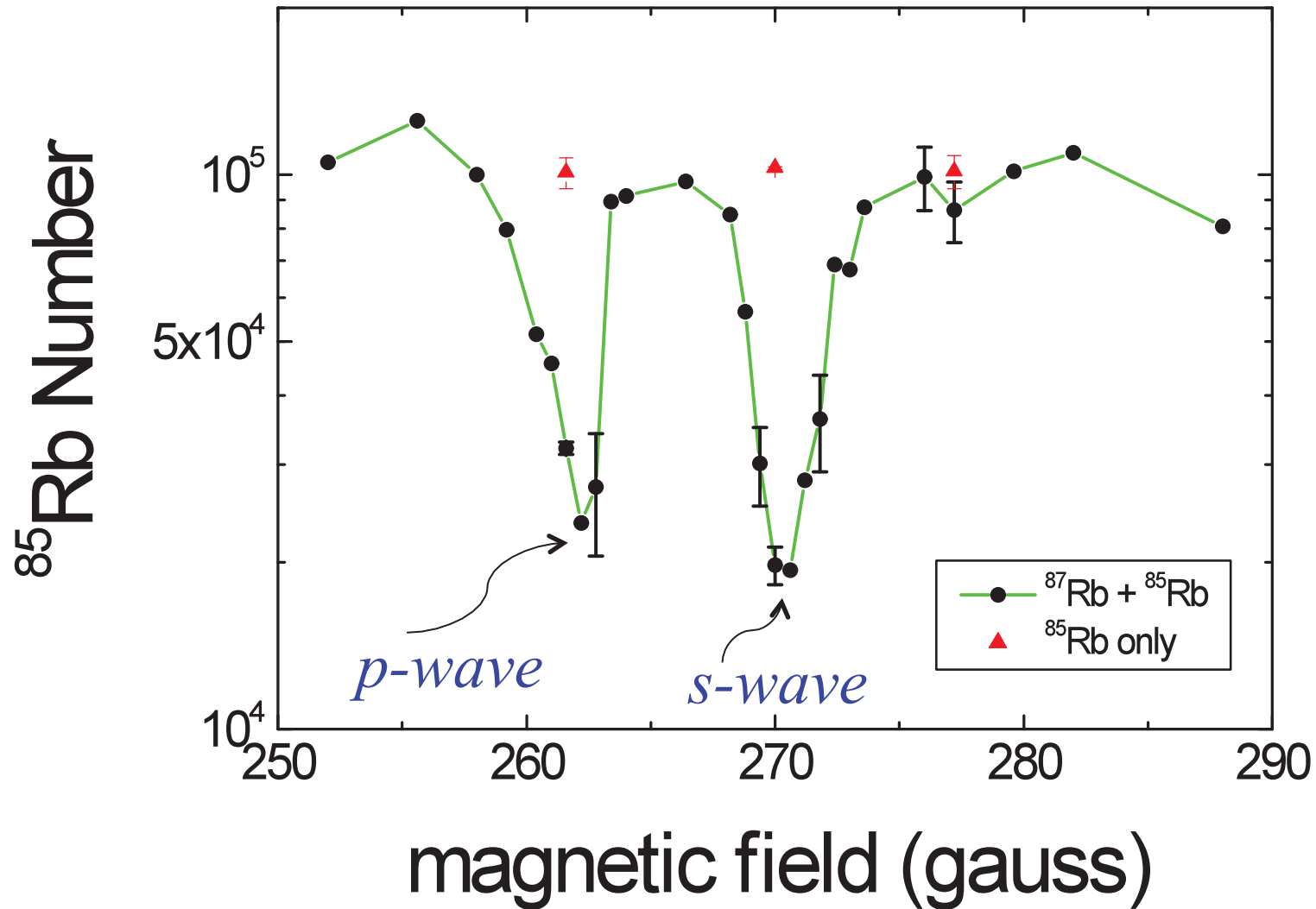




# Feshbach resonances

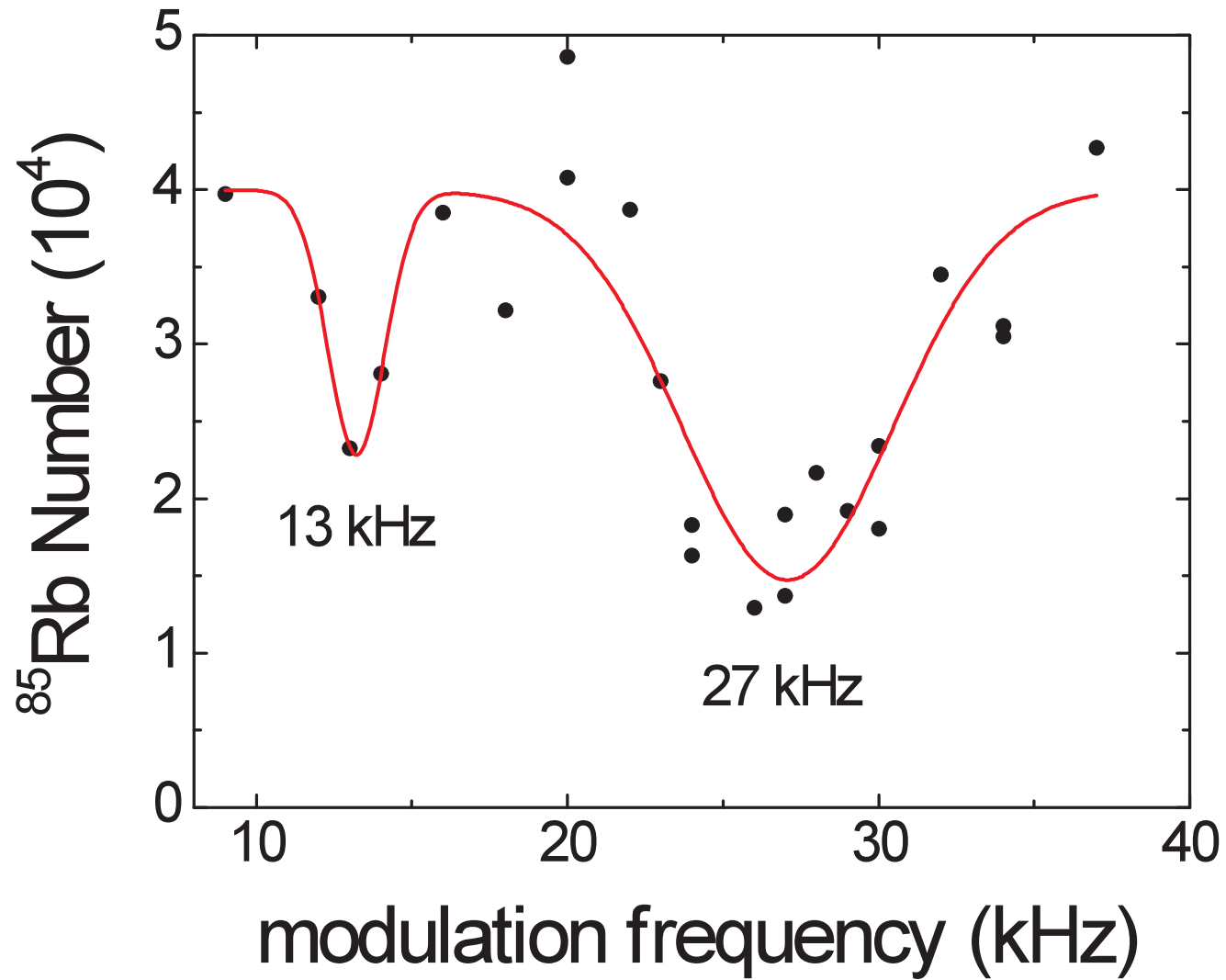
$^{85}\text{Rb}: F = 2, m_F = -2$

$^{87}\text{Rb}: F = 1, m_F = -1$

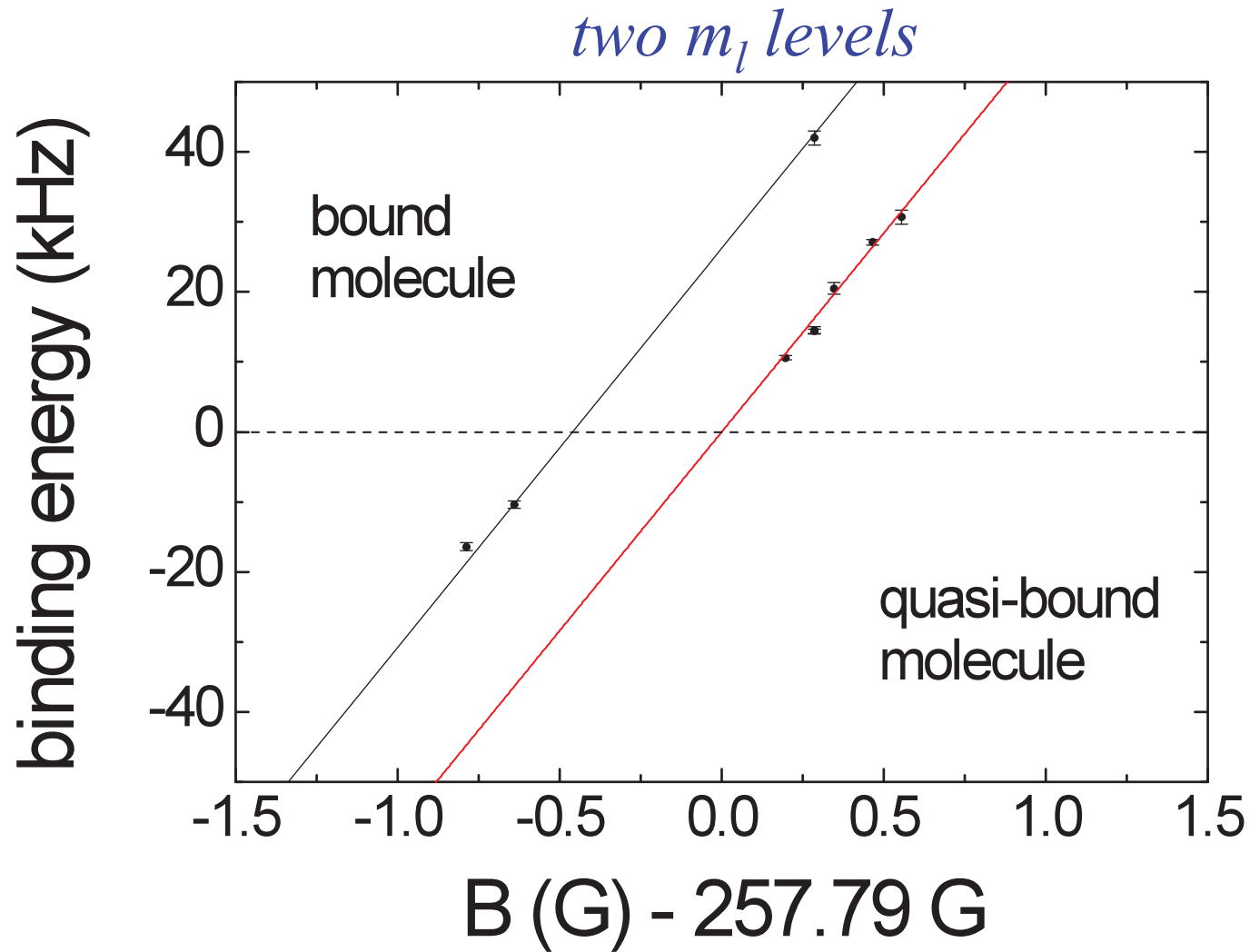




# *p-wave resonant modulation*



# *Rb85-Rb87 p-wave molecules*



## *Motivation*

- Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,...  
→ ultracold coherent bosonic atom-molecule mixtures
- **Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.**
- Allow SF-SF quantum phase transitions (cf. just crossover for fermions)

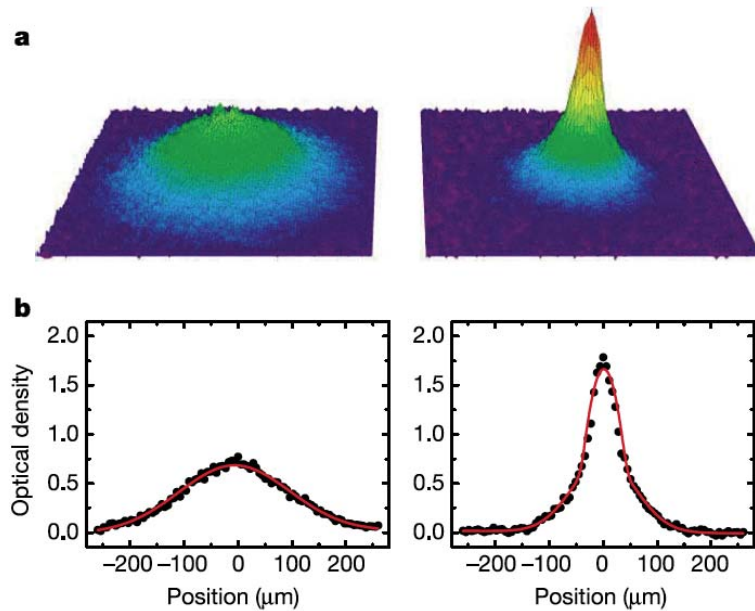
# Feshbach resonance (Fermi)

## Emergence of a molecular Bose-Einstein condensate from a Fermi gas

Markus Greiner<sup>1</sup>, Cindy A. Regal<sup>1</sup> & Deborah S. Jin<sup>2</sup>

<sup>1</sup>JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado, <sup>2</sup>Quantum Physics Division, National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

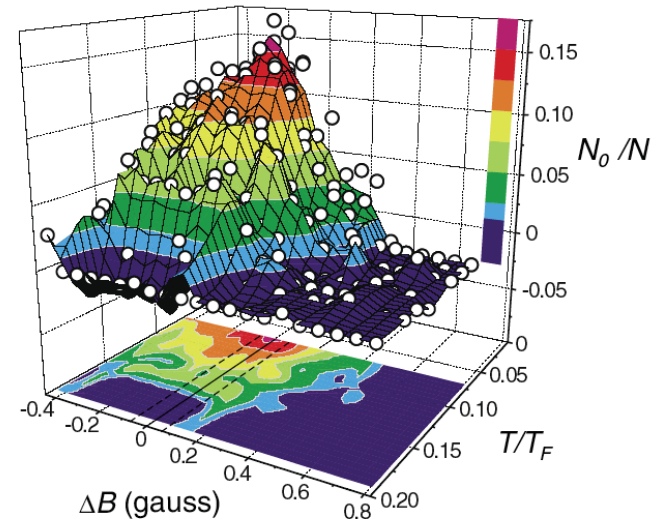
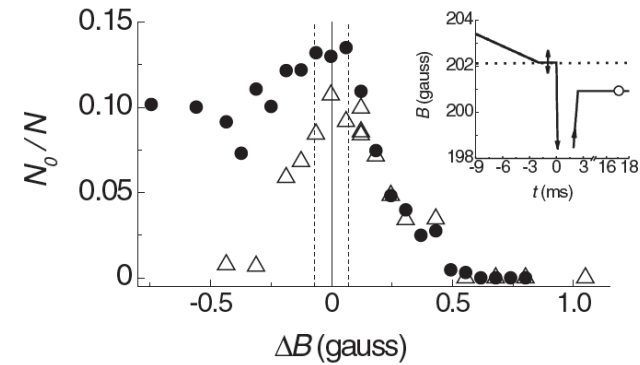
NATURE | VOL 426 | 4 DECEMBER 2003



## Observation of Resonance Condensation of Fermionic Atom Pairs

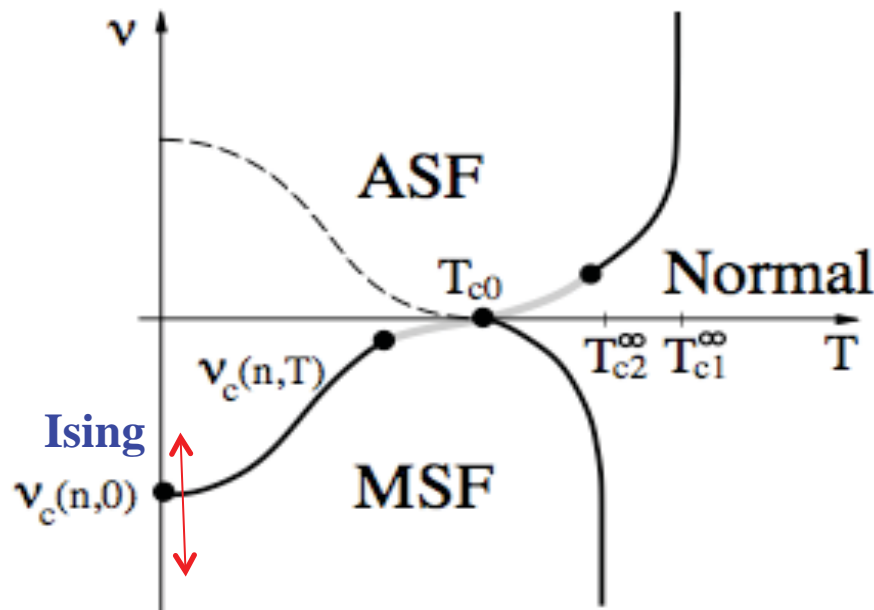
C. A. Regal, M. Greiner, and D. S. Jin\*

Physical Review Letters 92, (2004)



# Motivation

- Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,...  
→ ultracold coherent bosonic atom-molecule mixtures
- Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.
- **Allow SF-SF quantum phase transitions (cf. just crossover for fermions) even for s-wave resonance** -

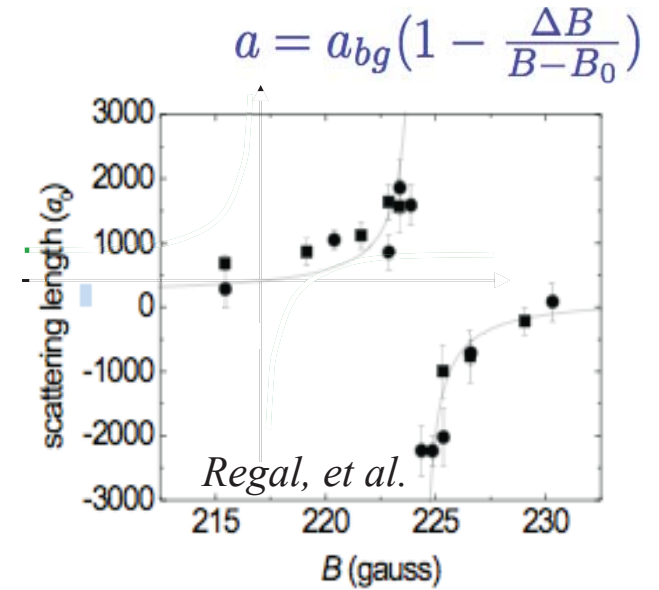
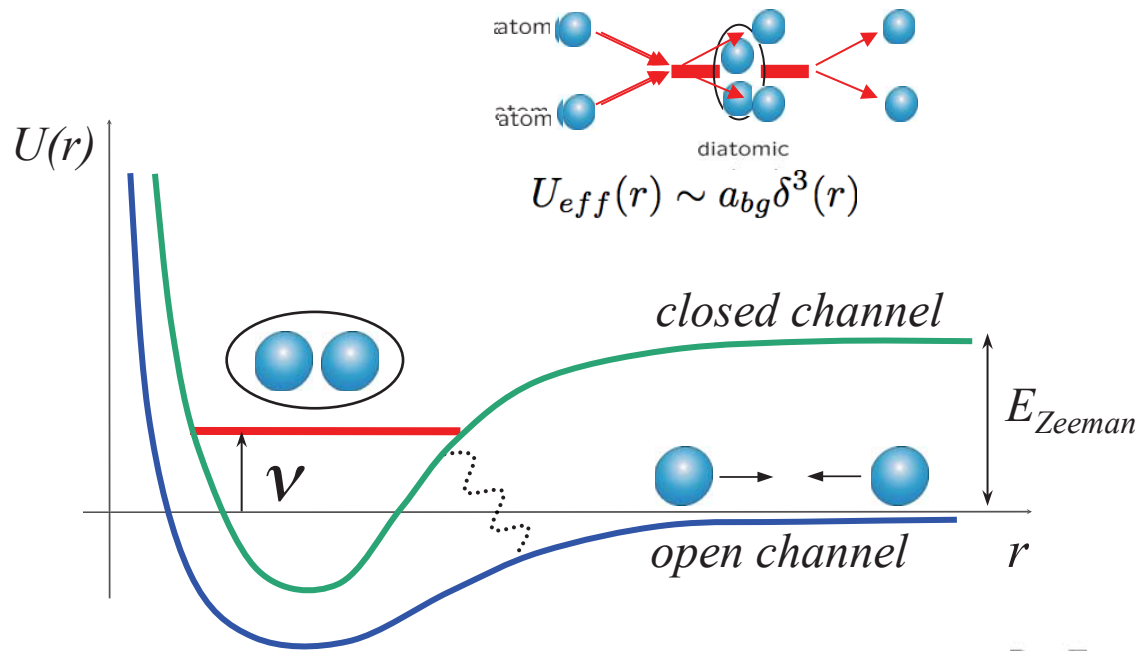


**original proposal in continuum:**  
*LR, Park, Weichman, PRL '04*  
*Romans, et al, PRL '04*

**more recently on lattice:**  
- Diehl, et al, 2010  
- Ejima, et al, 2011 (DMRG)  
- Bonnes, Wessel, 2011 (QMC)

# Feshbach resonance

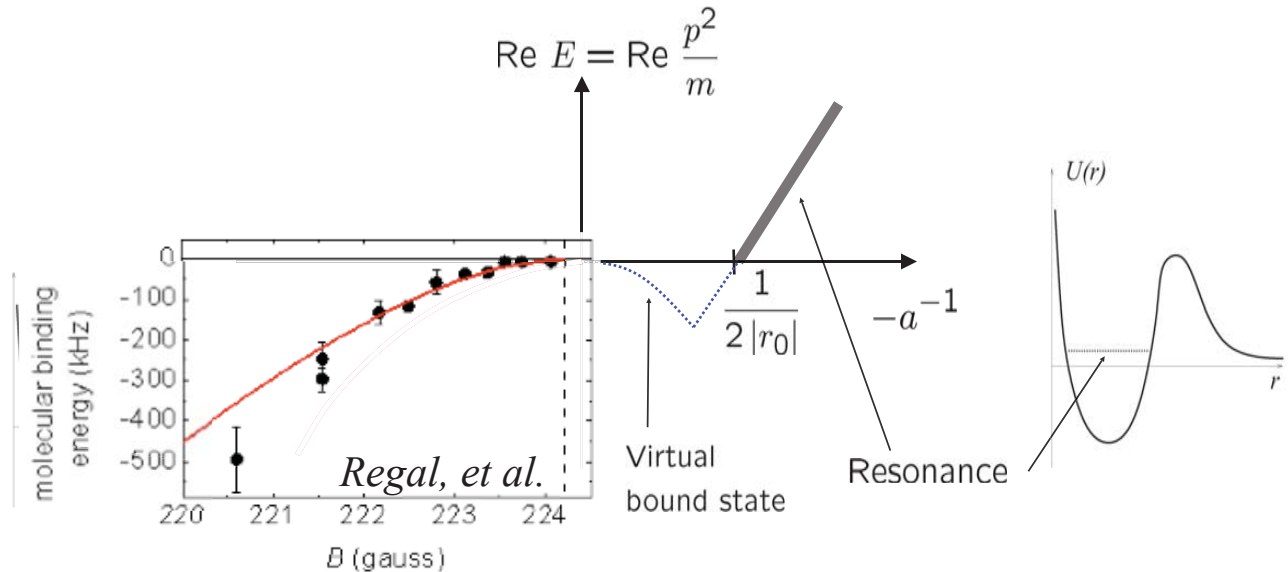
- **tunability** (strength and sign) of interactions (sudden and adiabatic) via Feshbach resonances



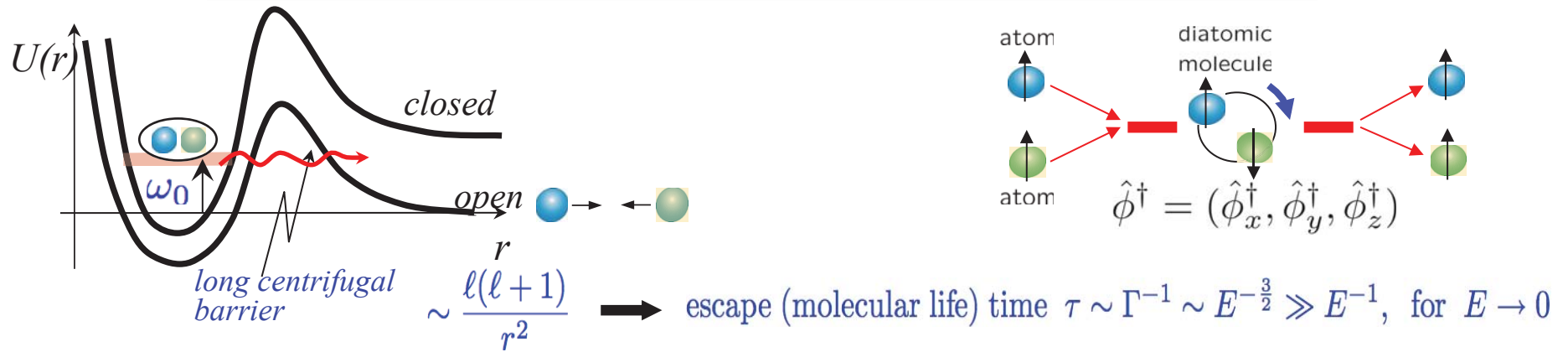
$$f_s(k) = \frac{1}{-a^{-1} + \frac{r_0}{2} k^2 - ik}$$

with  $a \sim -\frac{g^2}{\omega_0} \sim a_{bg} \frac{\Delta B}{B_0 - B}$

$r_0 \sim -\frac{1}{g^2}$



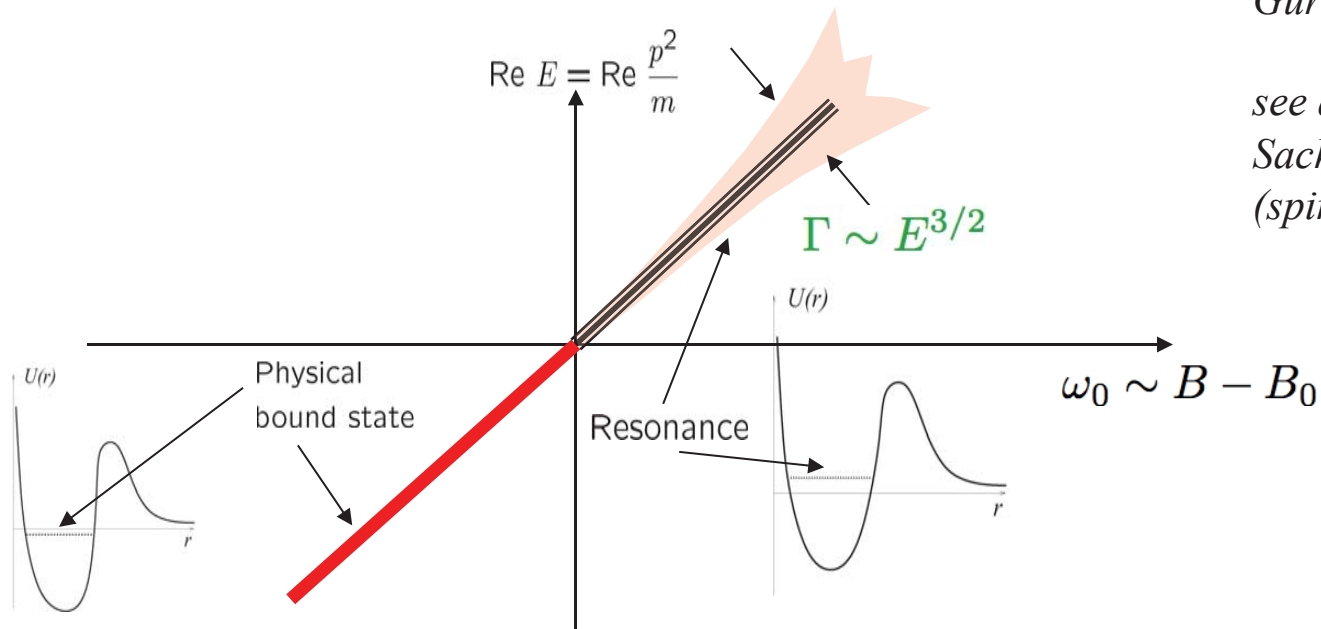
# P-wave Feshbach resonant scattering



$$H = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m} \psi_\sigma + \vec{\phi}^\dagger \cdot \left( \frac{\hat{p}^2}{4m} + \nu_0 \right) \vec{\phi} - i\alpha \vec{\phi}^\dagger \cdot \hat{\psi}_1 \vec{\nabla} \psi_2 + h.c.$$

Gurarie, L.R., AOP '09

see also:  
 Sachdev + Read '91  
 (spin liquids)



# *p-wave resonant Bose model*

- two distinguishable open-channel bosonic atoms:  $\hat{\psi}_\sigma^\dagger = (\hat{\psi}_1^\dagger, \hat{\psi}_2^\dagger)$
- p-wave closed-channel molecule:  $\hat{\phi}^\dagger = (\hat{\phi}_x^\dagger, \hat{\phi}_y^\dagger, \hat{\phi}_z^\dagger)$

- model:  $H = H_a + H_m + H_{am} + H_{FR}$

two species BEC:  $H_a = \sum_{\sigma=1,2} \left( \psi_\sigma^\dagger \left( -\frac{\nabla^2}{2m} - \mu_\sigma \right) \psi_\sigma + \frac{\lambda_\sigma}{2} \psi_\sigma^{\dagger 2} \psi_\sigma^2 \right) + \lambda_{12} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$

spinor=1 condensates:  $H_m = \vec{\phi}^\dagger \left( -\frac{\nabla^2}{4m} - \mu_m \right) \vec{\phi} + \frac{g_1}{2} |\vec{\phi}^\dagger \cdot \vec{\phi}|^2 + \frac{g_2}{2} |\vec{\phi} \cdot \vec{\phi}|^2$

$$\mu_m = \mu_1 + \mu_2 - \nu$$

$\nu$  - detuning

nonresonant interaction:  $H_{am} = g_{am} \psi_\sigma^\dagger \psi_\sigma \vec{\phi}^\dagger \cdot \vec{\phi}$

Feshbach resonant interaction:

$$H_{FR} = -i \frac{\alpha}{2} \left[ \vec{\phi}^\dagger \cdot (\psi_1 \vec{\nabla} \psi_2 - \psi_2 \vec{\nabla} \psi_1) + h.c. \right]$$



## Landau theory

$$\begin{aligned} F \approx & -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2} (\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2} |\vec{\Phi} \cdot \vec{\Phi}|^2 \\ & + \left( \frac{Q^2}{2m} - \mu_\sigma \right) |\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2} |\Psi_{Q,\sigma}|^4 + \dots \\ & + \alpha (\vec{\Phi}^* \cdot \vec{Q}) \Psi_{Q,1} \Psi_{-Q,2} + c.c. \end{aligned}$$

*conserved:*

$$\begin{array}{l} n_1 + n_m \\ n_2 + n_m \end{array} \quad \mu_m = \mu_1 + \mu_2 - \nu$$

## Landau theory

large negative detuning  $\longrightarrow \mu_\sigma < 0, \mu_m > 0$

$$F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2} (\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2} |\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+ \left( \frac{Q^2}{2m} - \mu_\sigma \right) |\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2} |\Psi_{Q,\sigma}|^4 + \dots$$

$$+ \alpha (\vec{\Phi}^* \cdot \vec{Q}) \Psi_{Q,1} \Psi_{-Q,2} + c.c.$$

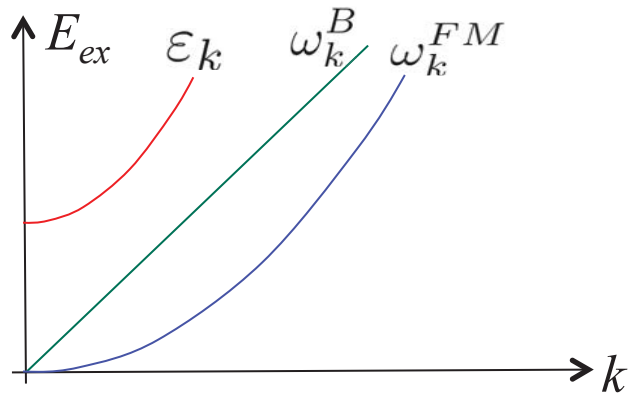
conserved:  $n_1 + n_m$   $\mu_m = \mu_1 + \mu_2 - \nu$   
 $n_2 + n_m$

# $L=1$ molecular superfluid (MSF)

$$\vec{\Phi} \neq 0, \quad \Psi_\sigma = 0$$

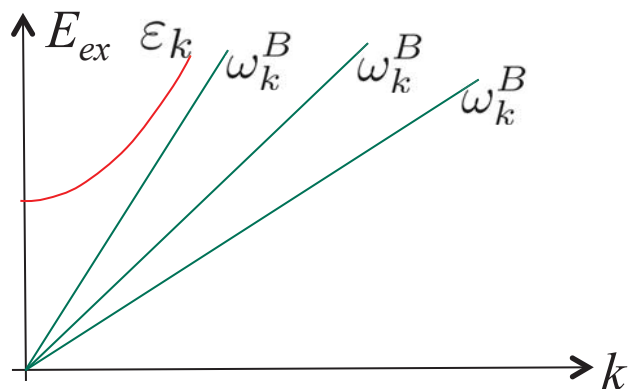
large negative detuning  $\longrightarrow \mu_\sigma < 0, \quad \mu_m > 0$

$$F_{MSF} \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2} (\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2} |\vec{\Phi} \cdot \vec{\Phi}|^2$$



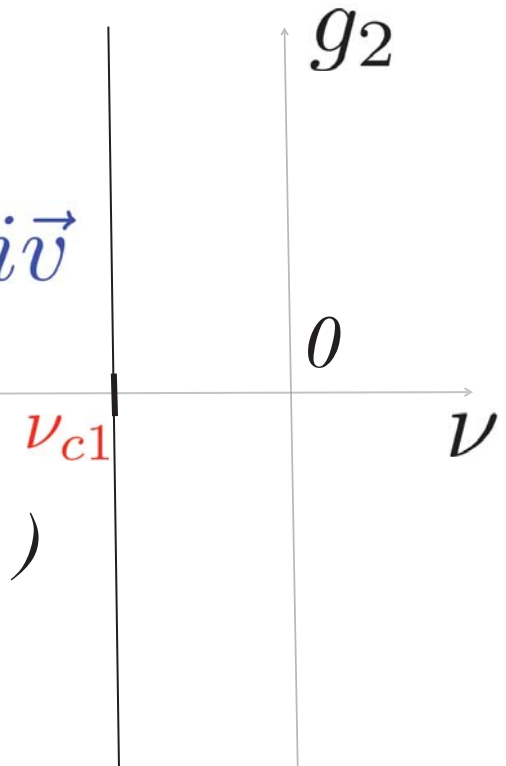
$MSF_{FM} (l_z = 1)$

$$\vec{\Phi} = \vec{u} + i\vec{v}$$



$MSF_{Polar} (l_z = 0)$

$$\vec{\Phi} = \vec{u}$$



## Landau theory

large positive detuning  $\longrightarrow \mu_\sigma > 0, \mu_m < 0$

$$F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2} (\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2} |\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+ \left( \frac{Q^2}{2m} - \mu_\sigma \right) |\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2} |\Psi_{Q,\sigma}|^4 + \dots$$

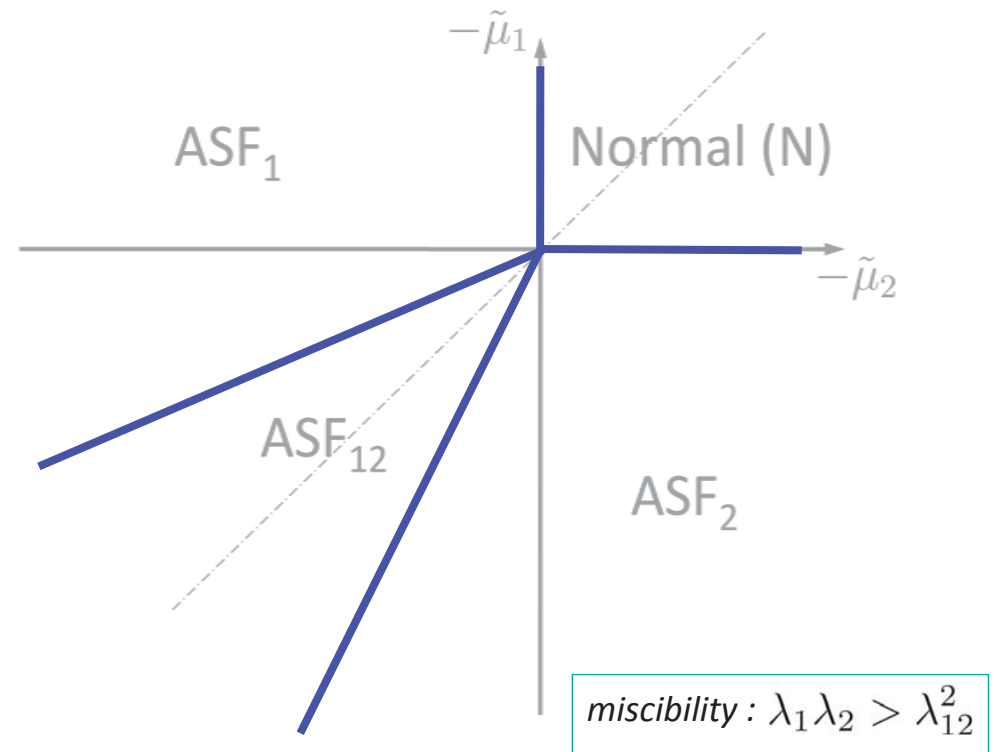
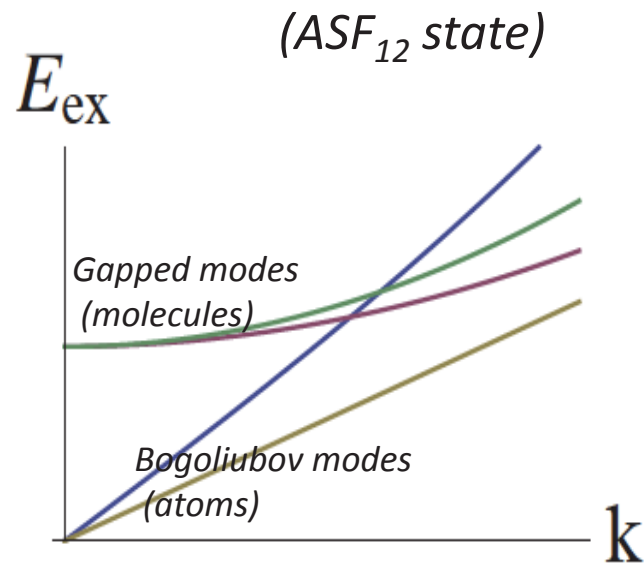
$$+ \alpha (\vec{\Phi}^* \cdot \vec{Q}) \Psi_{Q,1} \Psi_{-Q,2} + c.c.$$

# Atomic superfluid (ASF)

$$\vec{\Phi} = 0, \quad \Psi_\sigma \neq 0$$

large positive detuning  $\longrightarrow \mu_\sigma > 0, \quad \mu_m < 0$

$$F_{ASF} \approx -\mu_\sigma |\Psi_\sigma|^2 + \frac{\lambda_\sigma}{2} |\Psi_\sigma|^4 + \frac{\lambda_{12}}{2} |\Psi_1|^2 |\Psi_2|^2$$



## Landau theory

intermediate detuning  $\nu_{c1} < \nu < \nu_{c2} \longrightarrow \mu_\sigma < 0, \mu_m > 0$

$$F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2} (\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2} |\vec{\Phi} \cdot \vec{\Phi}|^2 \\ + \left( \frac{Q^2}{2m} - \mu_\sigma \right) |\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2} |\Psi_{Q,\sigma}|^4 + \dots$$

$$+ \alpha (\vec{\Phi}^* \cdot \vec{Q}) \Psi_{Q,1} \Psi_{-Q,2} + c.c.$$

# Atomic-molecular superfluid (AMSF) $\vec{\Phi} \neq 0, \Psi_{Q,\sigma} \neq 0$

intermediate detuning  $\nu_{c1} < \nu < \nu_{c2} \longrightarrow \mu_\sigma < 0, \mu_m > 0$

$$F_{AMSF} \approx \left( \frac{Q^2}{2m} - \mu_\sigma \right) |\Psi_{Q,\sigma}|^2 + \alpha (\vec{\Phi}^* \cdot \vec{Q}) \Psi_{Q,1} \Psi_{-Q,2} + \dots$$

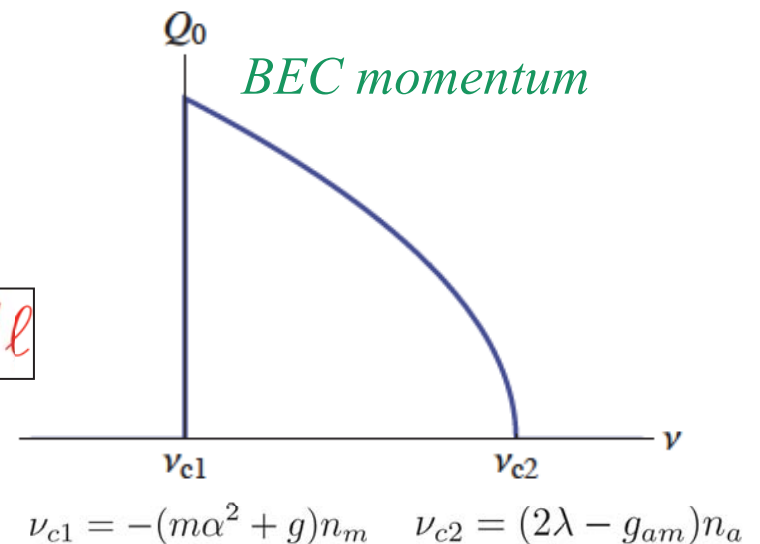
$$\approx -\frac{\mu_+}{2} |\Psi_Q^{(+)}|^2 - \frac{\mu_-}{2} |\Psi_Q^{(-)}|^2 + \dots$$

$$\mu_\pm = -\left( \frac{Q^2}{2m} - \mu \right) \pm \alpha |\vec{\Phi} \cdot \vec{Q}|$$

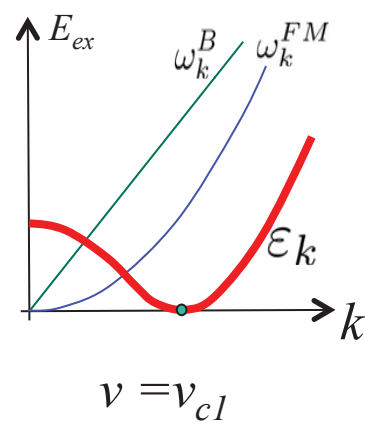
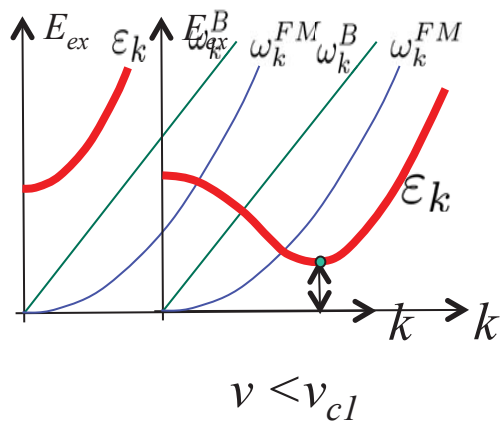
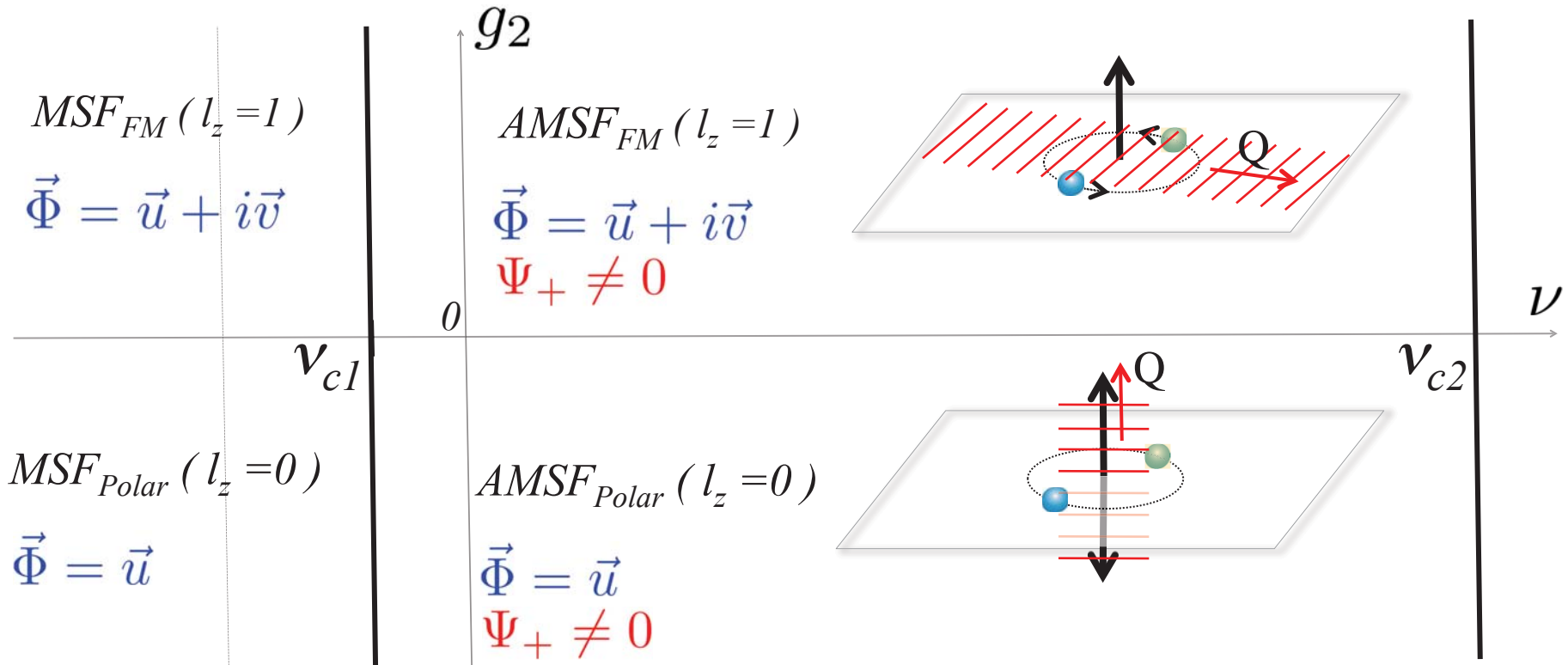
$\longrightarrow$  transition to AMSF at  $\nu_{c1}$  ( $\mu_+ = 0 > \mu_-$ )  $\Psi_\pm = \Psi_{Q,1} \pm e^{i\varphi} \Psi_{-Q,2}^*$

physics of  $Q \neq 0$ :  $\frac{Q^2}{2m} \sim \alpha \vec{Q} \cdot \vec{\Phi}$

$\longrightarrow$   $Q \approx \alpha m \sqrt{n_m} \sim \sqrt{\gamma_p \ell n_m} \lesssim \sqrt{\gamma_p / \ell}$   
(tunable with  $\nu$ )



# Near MSF-AMSF transition



$$H_a^{MSF} = \begin{pmatrix} \frac{k^2}{2m} - \mu & \alpha \vec{\Phi} \cdot \vec{k} \\ \alpha \vec{\Phi}^* \cdot \vec{k} & \frac{k^2}{2m} - \mu \end{pmatrix}$$

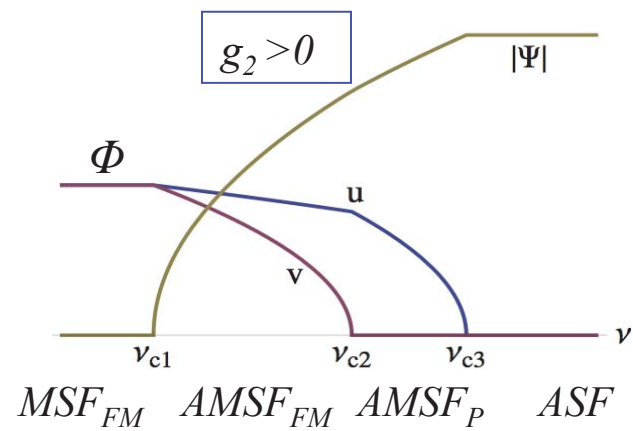
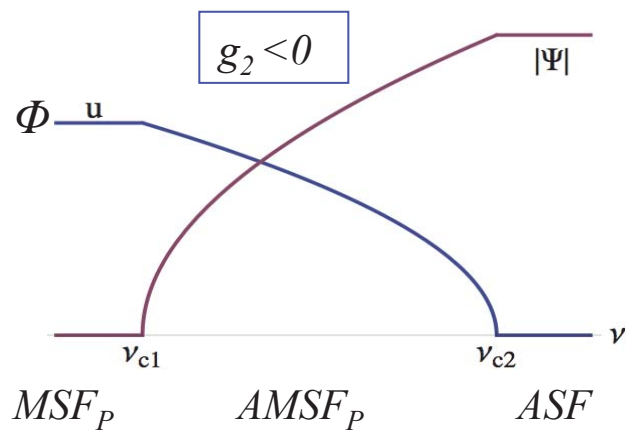
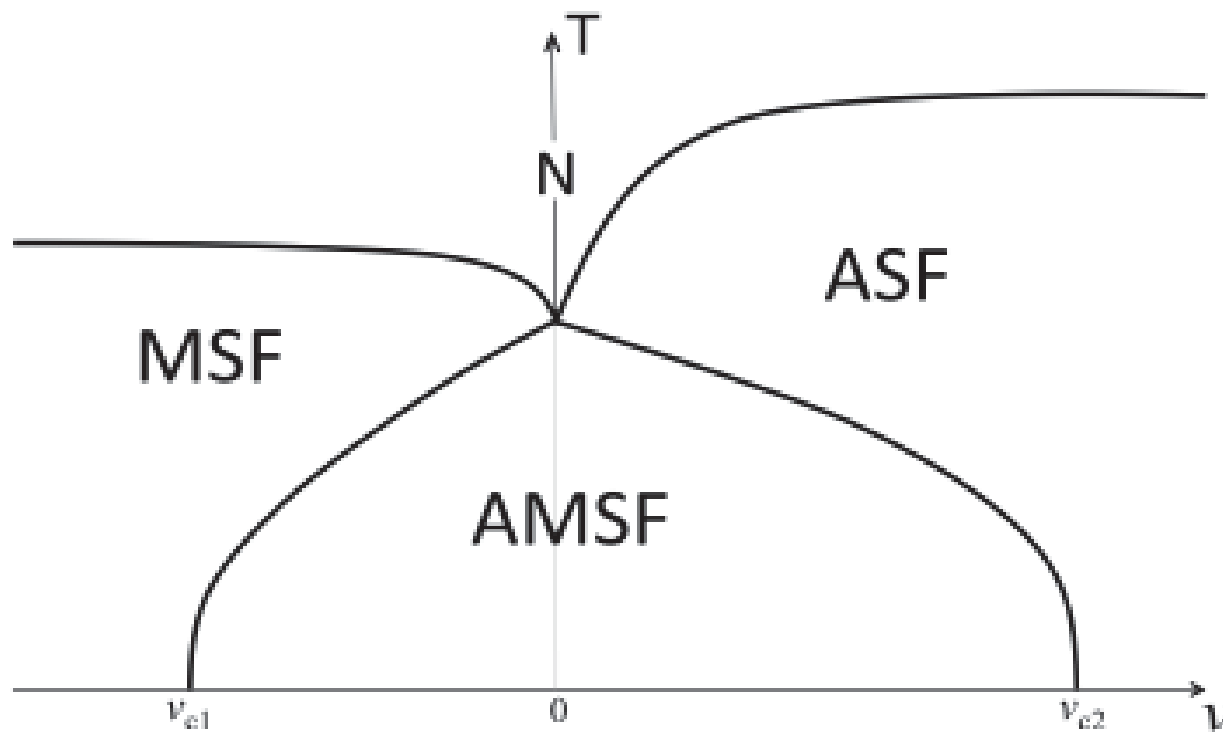
$$E_{ex} = \sqrt{\mu_+ \mu_-}$$

$$= \sqrt{\left(\frac{k^2}{2m} - \mu\right)^2 - |\alpha \vec{\Phi} \cdot \vec{k}|^2}$$

→ co-linear (FF) for  $MSF_{Polar}$



# Global phase diagram



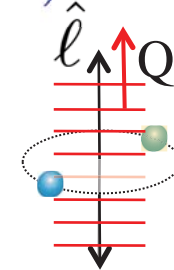
# Symmetries, order parameters, Goldstone modes

## • AMSF<sub>Polar</sub>

▪ *OP*:  $\vec{\Phi} = e^{i\phi} \hat{\ell}$ ,  $\Psi = \sum_Q \Psi_Q e^{i\theta_Q + i\vec{Q}\cdot\vec{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \approx e^{i\theta} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \cos(\vec{Q}\cdot\vec{r} + Qu)$

▪ *breaks*:  $U_N(1) \times U_{\Delta N}(1) \times SO(3) \times T_{\varepsilon\hat{\ell}}$

▪ *GM*:  $\theta_1, \theta_2, \phi, \hat{\ell} \rightarrow$  Higgs'ed:  $\theta_c, \theta_s$



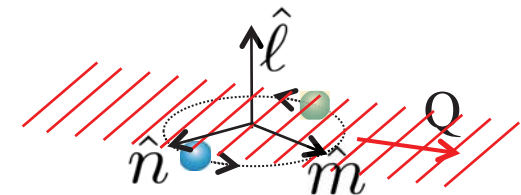
$$\mathcal{L}_p = \frac{n_c}{2} (\partial_\mu \theta_c)^2 + \frac{\chi_s}{2} (\partial_\tau \theta_s)^2 + \frac{n_s}{2} (\partial_{\parallel} \theta_s)^2 + \frac{K}{2} (\nabla_{\perp}^2 \theta_s)^2$$

## • AMSF<sub>FM</sub>

▪ *OP*:  $\vec{\Phi} = \hat{n} + i\hat{m}$ ,  $\Psi = \sum_Q \Psi_Q e^{i\theta_Q + i\vec{Q}\cdot\vec{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \approx e^{i\theta} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \cos(\vec{Q}\cdot\vec{r} + Qu)$

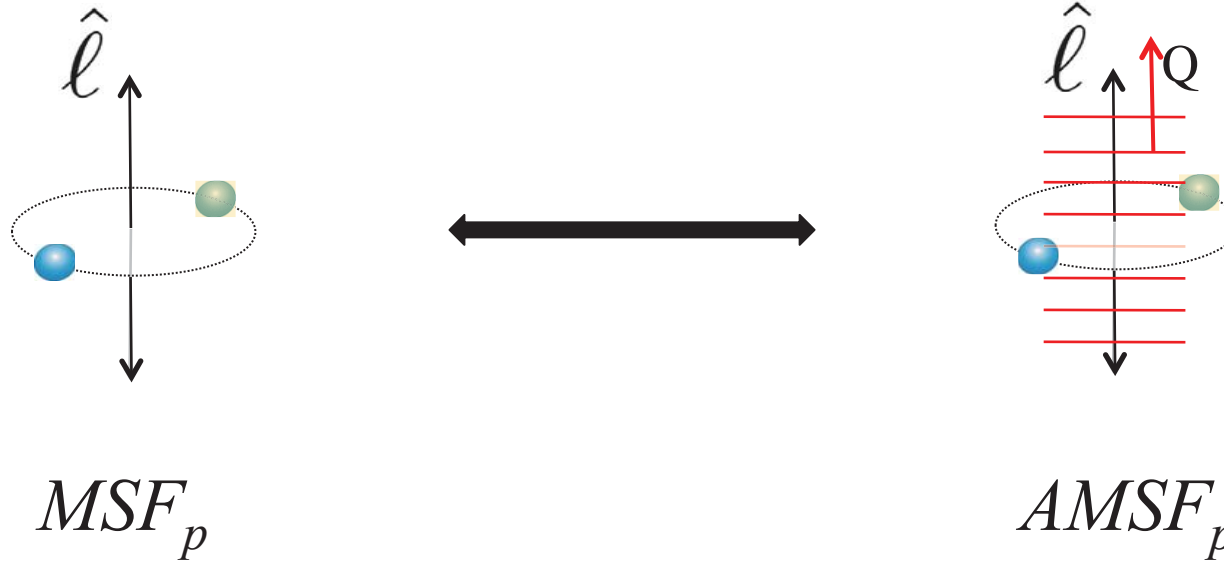
▪ *breaks*:  $U_N(1) \times U_{\Delta N}(1) \times SO(3) \times T_{\varepsilon\hat{\ell}} \times \Theta$

▪ *GM*:  $\theta_1, \theta_2, \phi, \hat{n}, \hat{m} \rightarrow$  Higgs'ed:  $\theta_c, \theta_s, \gamma$



$$\mathcal{L}_{fm} = \frac{n_c}{2} (\partial_\mu \theta_c)^2 + \frac{\chi_s}{2} (\partial_\tau \theta_s)^2 + \frac{n_s}{2} (\partial_{\parallel} \theta_s)^2 + \frac{K}{2} (\nabla_{\perp}^2 \theta_s)^2 + i\kappa \partial_y \theta_s \partial_\tau \gamma + \frac{J}{2} (\nabla \gamma)^2$$

## *MSF – AMSF transition*



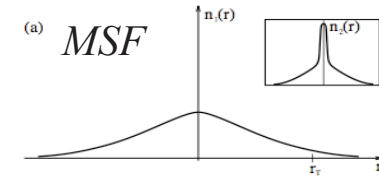
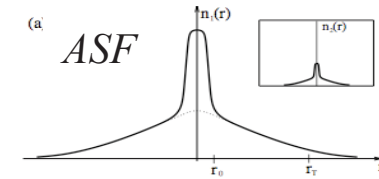
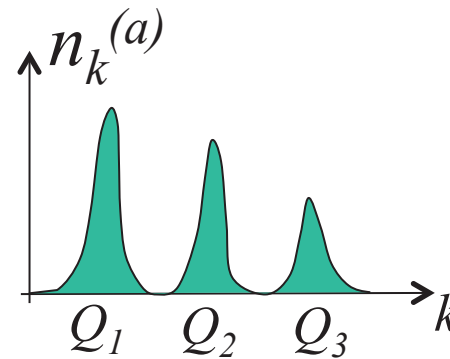
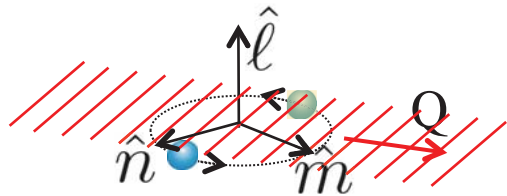
Abelian Higgs (quantum de Gennes) model:

$$\mathcal{L}_p = |\partial_\tau \psi|^2 + \frac{1}{2m} |(i\nabla - Q\delta\hat{\ell})\psi|^2 + \epsilon_+ |\psi|^2 + \frac{\lambda}{2} |\psi|^4 + \frac{1}{2g_\ell} (\partial_\mu \hat{\ell})^2 + \frac{1}{2g_\varphi} (\partial_\mu \varphi)^2$$

# Experimental signatures

- momentum distributions  $n_k^{(a)}$ ,  $n_k^{(m)}$

- Bragg peaks at  $Q_n$  in AMSF

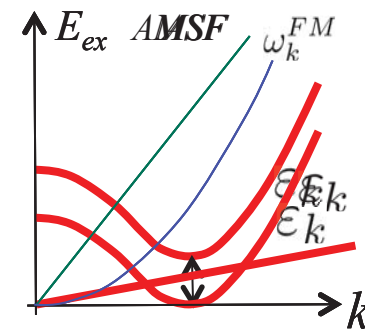


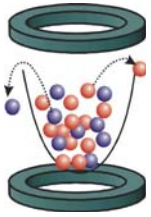
- thermodynamic singularities at transitions

- excitation spectra (phonons, Bogoluibov and spin-wave modes)

via Bragg spectroscopy

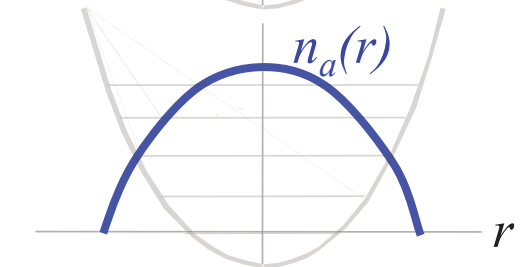
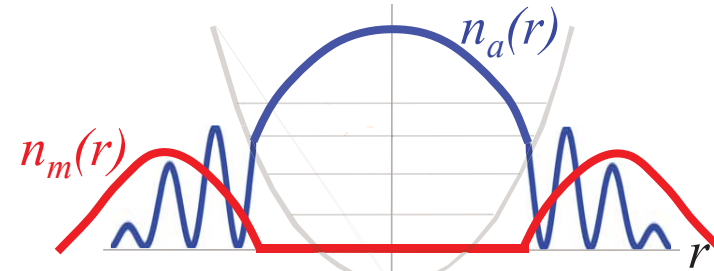
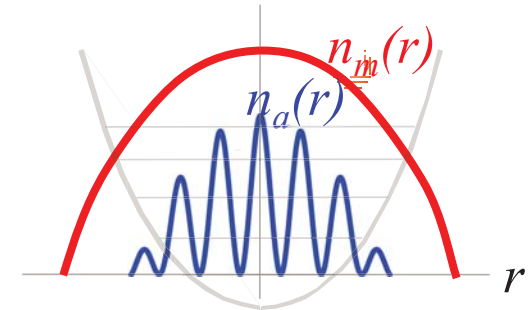
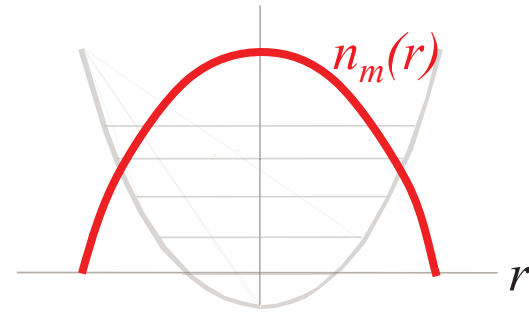
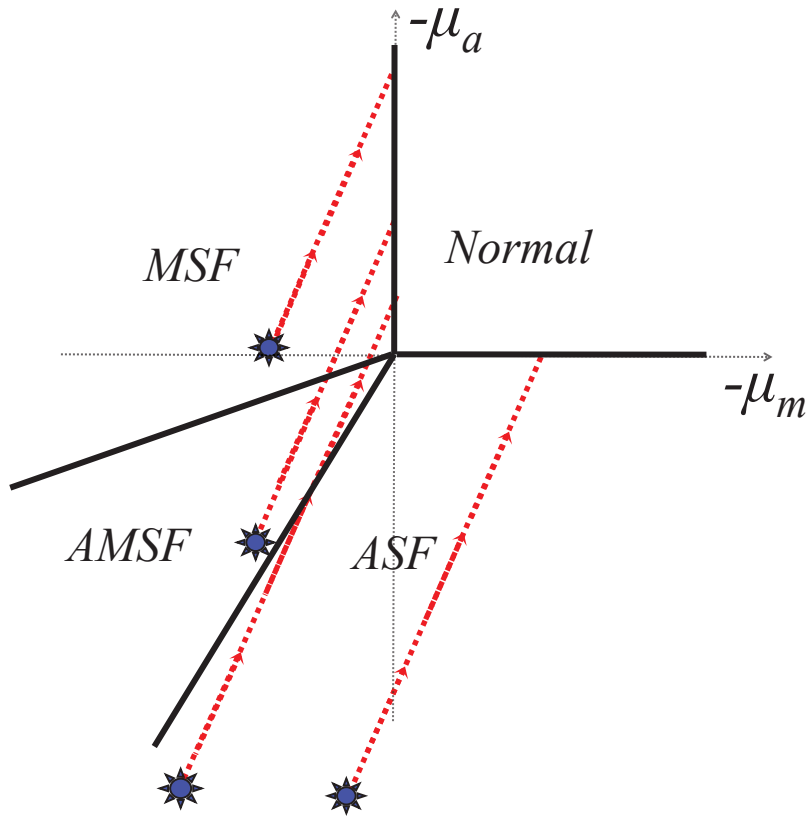
- novel vortices and dislocations





# Trapped profiles via LDA

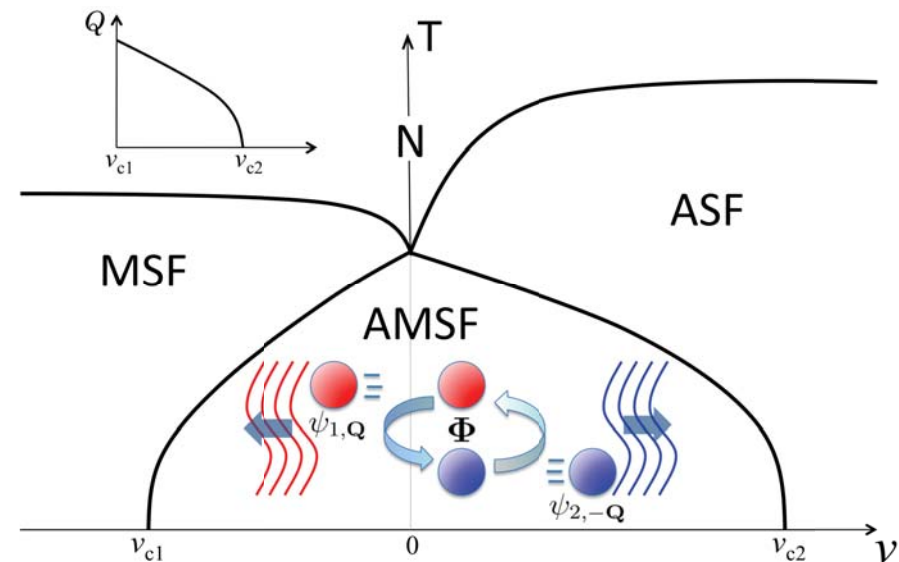
$$\mu \rightarrow \mu_{\text{eff}}(r) = \mu - \frac{1}{2}m\omega^2 r^2$$



# Summary and conclusions

- resonantly interacting Bose gas:

- atomic and molecular superfluids
- atomic supersolid, tunable  $Q(\nu)$
- quantum, thermal transitions
- topological defects...



- questions:

- nature of the AMSF solidity: vortex lattice? 3d crystal?
- stability? expect short lifetime due to 3-body instabilities
- ...

- fixes:

- optical lattice?
- avoid immediate vicinity of FBR?