The Abdus Salam International Centre for Theoretical Physics

2252-S-8

Advanced Workshop on Non-Standard Superfluids and Insulators

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Hydrodynamic and magnetic properties of dipolar Chromium Bose-Einstein condensates

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Dipolar chromium BECs

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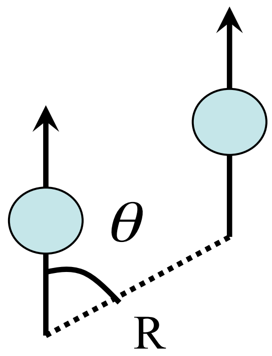
Chromium (S=3): Van-der-Waals plus dipole-dipole interactions

Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$

Long range

Anisotropic



Non local anisotropic mean-field

Hydrodynamics

Magnetization changing collisions

Spinor properties

Hydrodynamics of a dipolar BEC

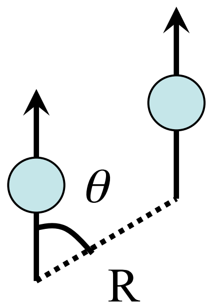
Relative strength of dipole-dipole and Van-der-Waals interactions

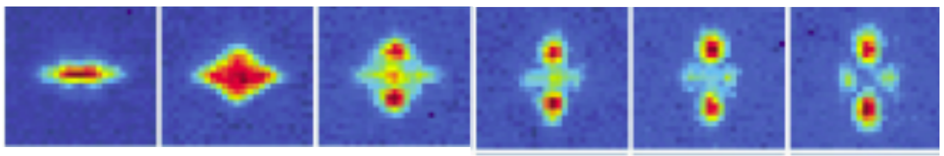
$$\varepsilon_{dd} = \frac{\mu_0 \mu_m^2 m}{12\pi\hbar^2 a} \propto \frac{V_{dd}}{V_{VdW}}$$

Cr: $\varepsilon_{dd} = 0.16$

$\varepsilon_{dd} > 1$ BEC collapses

Stuttgart: Tune contact interactions using Feshbach resonances (Nature. 448, 672 (2007))





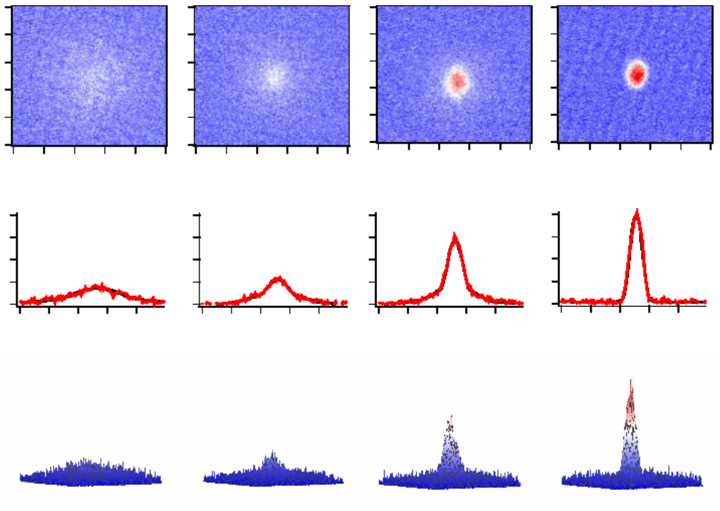
Anisotropic explosion pattern reveals dipolar coupling.

Stuttgart: d-wave collapse, PRL **101**, 080401 (2008)
Stabilization by confinement arXiv:1105.5015 (2011)

$\varepsilon_{dd} < 1$ BEC stable despite attractive part of dipole-dipole interactions

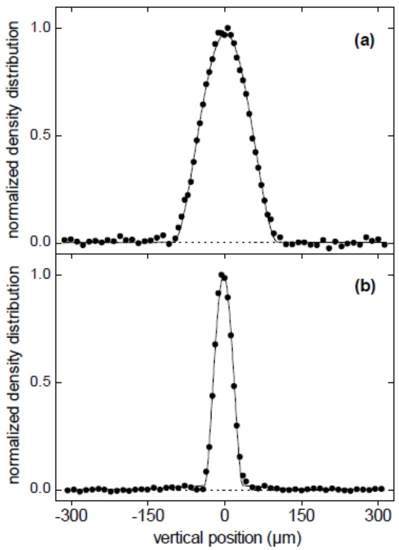
Parabolic ansatz still good. Striction of BEC. (Eberlein, PRL **92**, 250401 (2004))

Interaction-driven expansion of a BEC



A lie:

Imaging BEC after time-of-fligth is a measure of in-situ momentum distribution



Cs BEC with tunable interactions (from Innsbruck))

Self-similar, Castin-Dum expansion

Phys. Rev. Lett. **77**, 5315 (1996)

$$R_j(t) = \lambda_j(t) R_j(0)$$

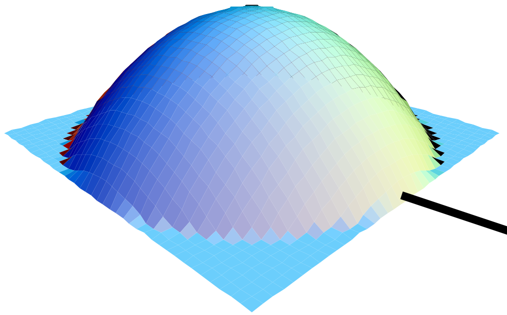
$$\ddot{\lambda}_j = \frac{\omega_j^2(0)}{\lambda_j \lambda_1 \lambda_2 \lambda_3} - \omega_j^2(t)\lambda_j$$

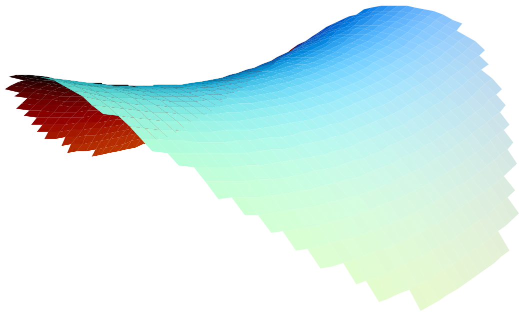
TF radii after expansion related to interactions

Modification of BEC expansion due to dipole-dipole interactions

TF profile

$$\Phi_{dd}(\vec{r}) = \int V_{dd}(\vec{r} - \vec{r}\,')n(\vec{r}\,')d^3\vec{r}\,'$$

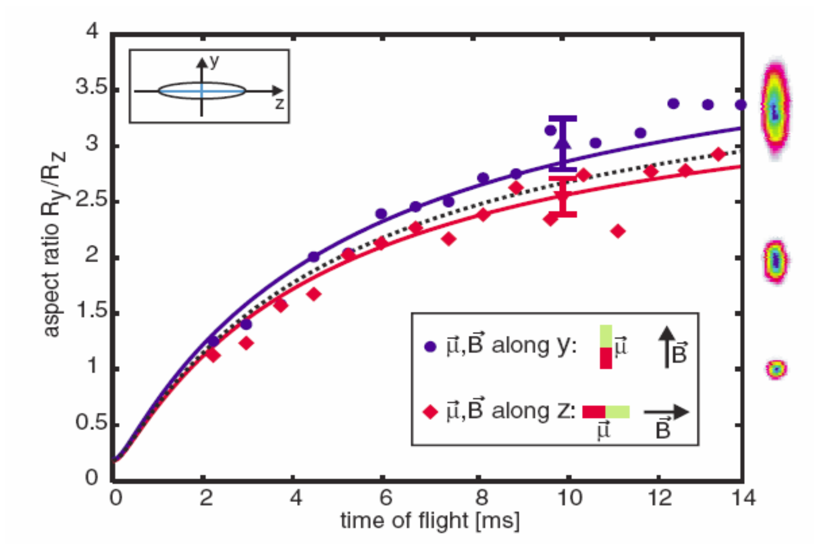




Striction of BEC

(non local effect)

Eberlein, PRL **92**, 250401 (2004)



Pfau,PRL **95**, 150406 (2005)

(similar results in our group)

Frequency of collective excitations

(Castin-Dum)

$$\ddot{\lambda}_j = \frac{\omega_j^2(0)}{\lambda_j \lambda_1 \lambda_2 \lambda_3} - \omega_j^2(t)\lambda_j$$

Consider small oscillations, then

$$\frac{d^2 \vec{\lambda}}{dt^2} = H.\vec{\lambda}$$

with

$$H = \begin{pmatrix} -3\omega_1^2 & -\omega_1^2 & -\omega_1^2 \\ -\omega_2^2 & -3\omega_2^2 & -\omega_2^2 \\ -\omega_3^2 & -\omega_3^2 & -3\omega_3^2 \end{pmatrix}$$

Interpretation

Sound velocity

$$mv_S^2 = gn$$

$$E \propto \frac{v_S}{R} \propto \omega$$

TF radius

$$\frac{1}{2} m\omega^2 R^2 = gn$$

In the Thomas-Fermi regime, collective excitations frequency independent of number of atoms and interaction strength:

Pure geometrical factor

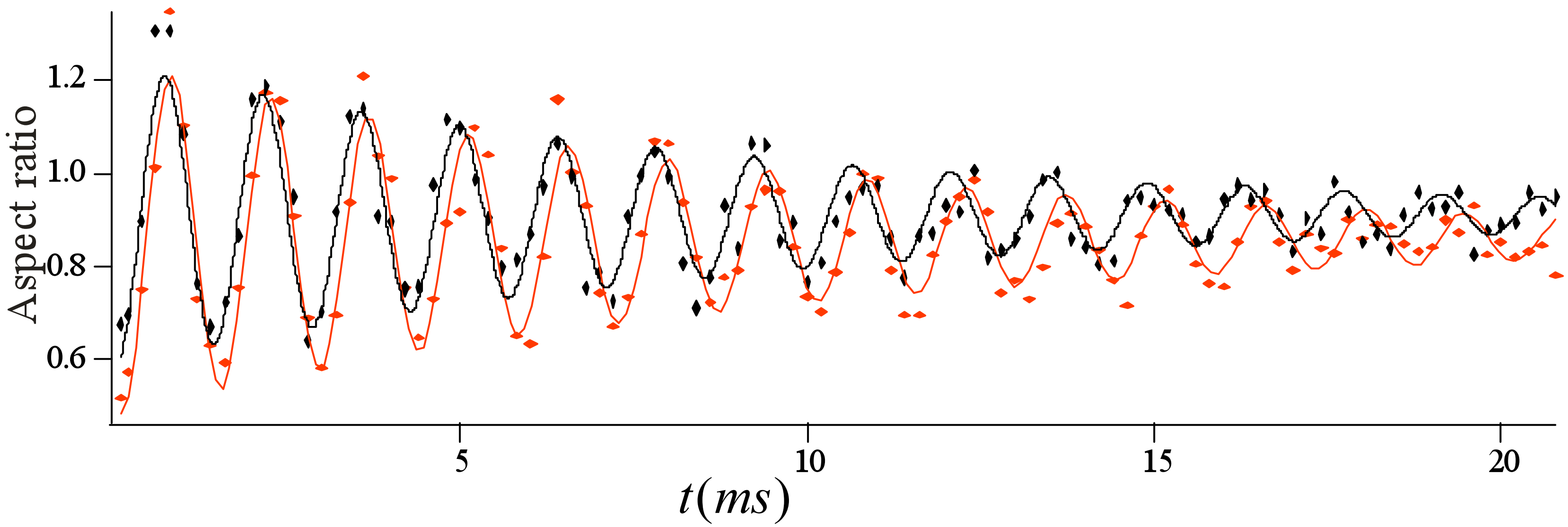
(solely depends on trapping frequencies)

Collective excitations of a dipolar BEC

Due to the anisotropy of dipole-dipole interactions, the dipolar mean-field depends on the relative orientation of the magnetic field and the axis of the trap

Parametric excitations

Repeat the experiment for two directions of the magnetic field



A small, but qualitative, difference (geometry is not all)

$$\frac{\Delta\nu}{\nu} \propto \varepsilon_{dd}$$

Phys. Rev. Lett. **105**, 040404 (2010)

Bragg spectroscopy

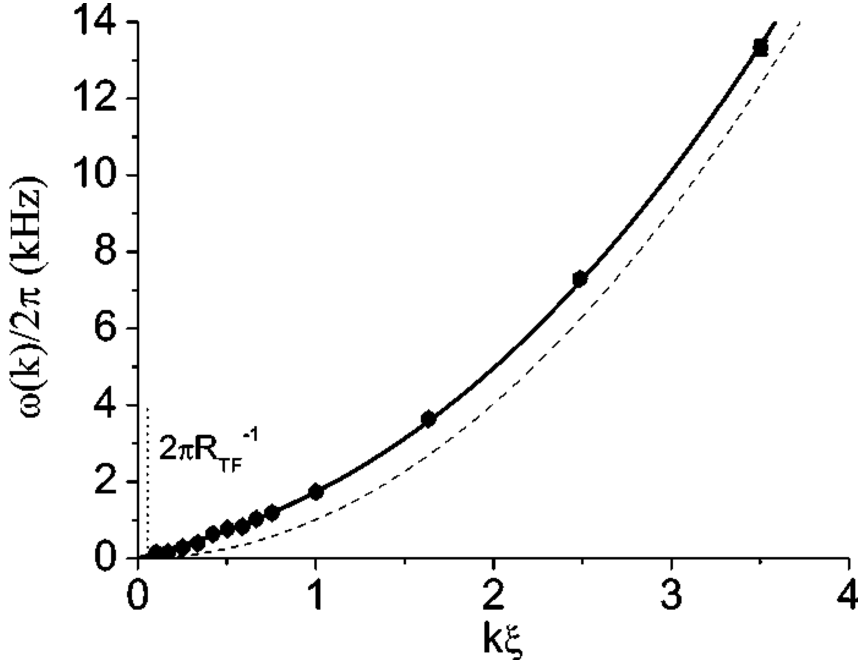
Probe dispersion law $E(k) = ck$

Quasi-particles, phonons $k\xi << 1$

c is sound velocity

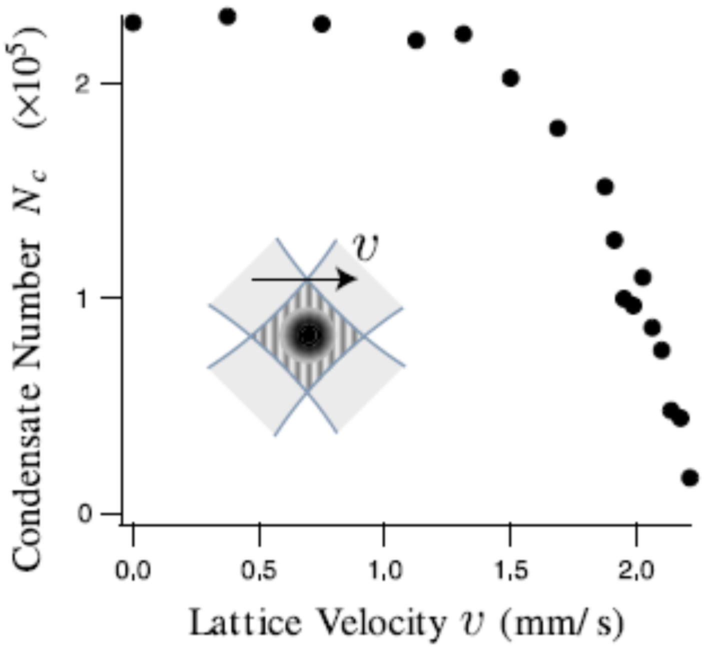
c is also critical velocity

Landau criterium for superfluidity



ξ healing length

Rev. Mod. Phys. **77**, 187 (2005)

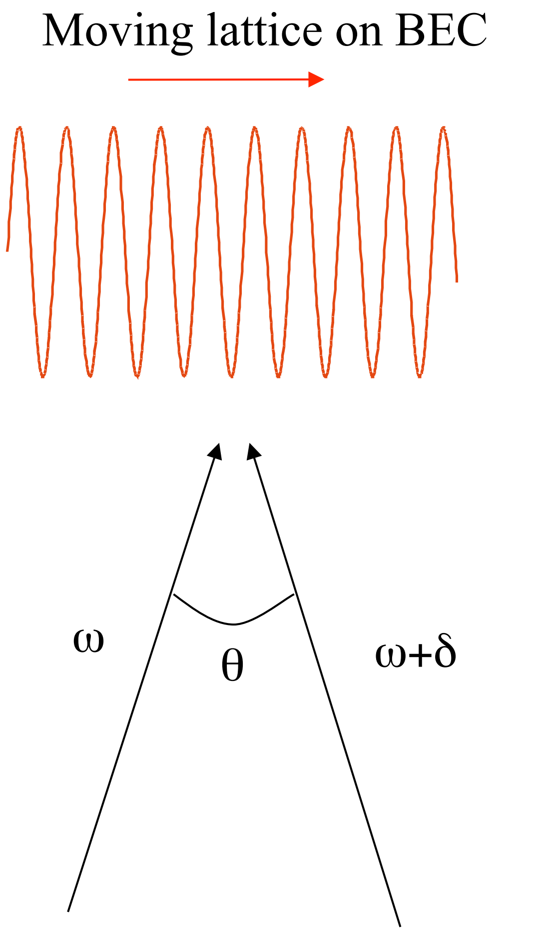


Phys. Rev. Lett. **99**, 070402 (2007)

Bogoliubov spectrum

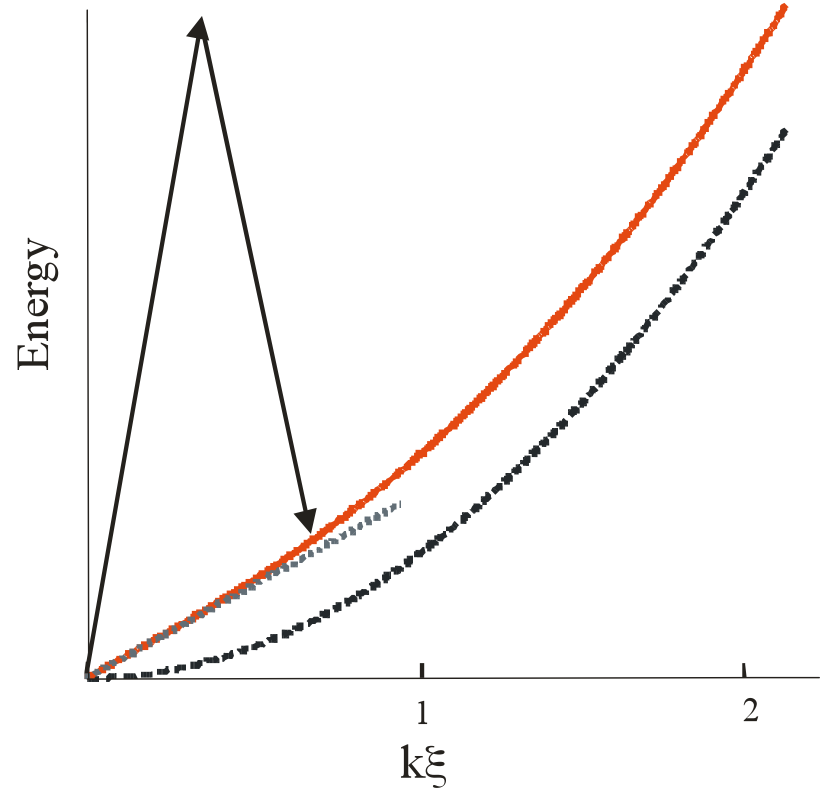
$$\varepsilon_k = \sqrt{E_k(E_k + 2n_0 g_c)}$$

Bragg spectroscopy of an anisotropic superfluid



Lattice beams with an angle.
Momentum exchange

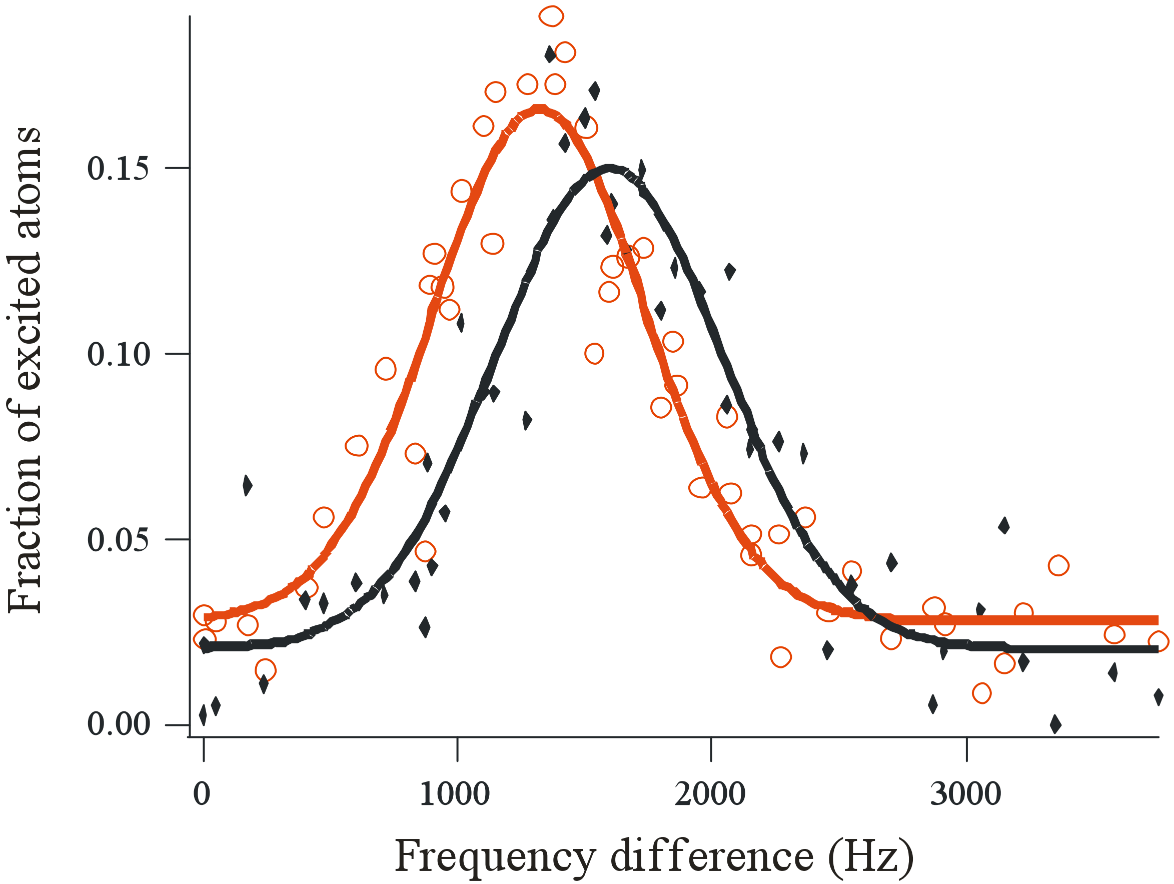
$$\hbar k = 2\hbar k_L \sin(\theta / 2)$$

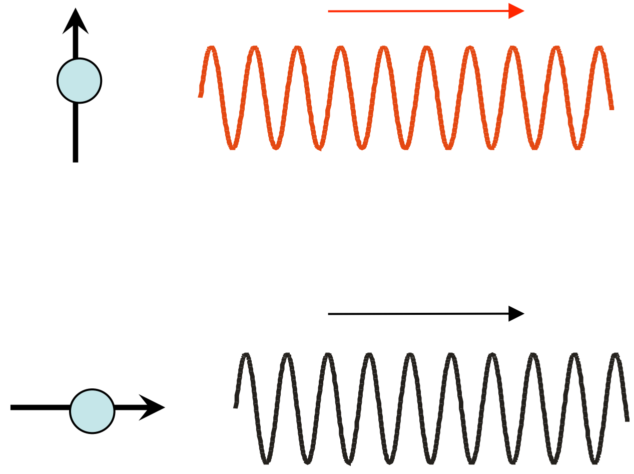


$$k\xi = 0.8$$

Resonance frequency gives speed of sound

Anisotropic speed of sound



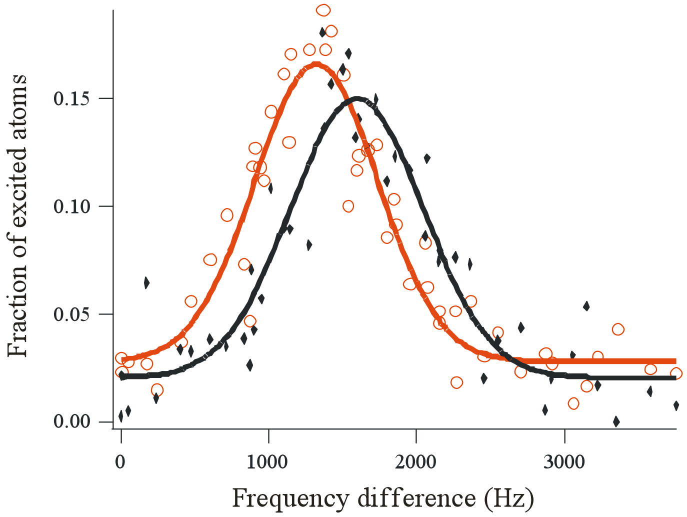


Width of resonance curve: finite size effects (inhomogeneous broadening)

Speed of sound depends on the relative angle between spins and excitation

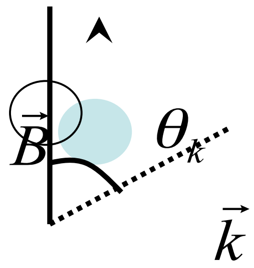
Anisotropic speed of sound

A 20% effect, much larger than the (~2%) modification of the mean-field due to DDI



An effect of the momentum-sensitivity of DDI

$$\tilde{V}(k) = \frac{4\pi d^2}{3}(3\cos^2\theta_k - 1)$$



$$\varepsilon_k = \sqrt{E_k(E_k + 2n_0(g_c + g_d(3\cos^2\theta_k - 1))}$$

Good agreement between theory and experiment:

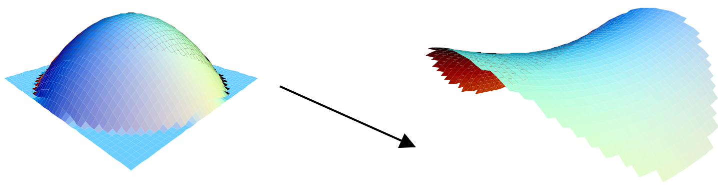
	Theo	Exp
Parallel	3.6 mm/s	3.4 mm/s
Perpendicular	3 mm/s	2.8 mm/s

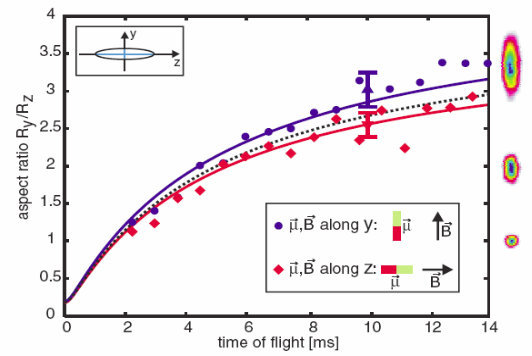
(See also prediction of anisotropic superfluidity of 2D dipolar gases : Phys. Rev. Lett. **106**, 065301 (2011))

$\varepsilon_{dd} = 0.16$

A « non-standard superfluid »…

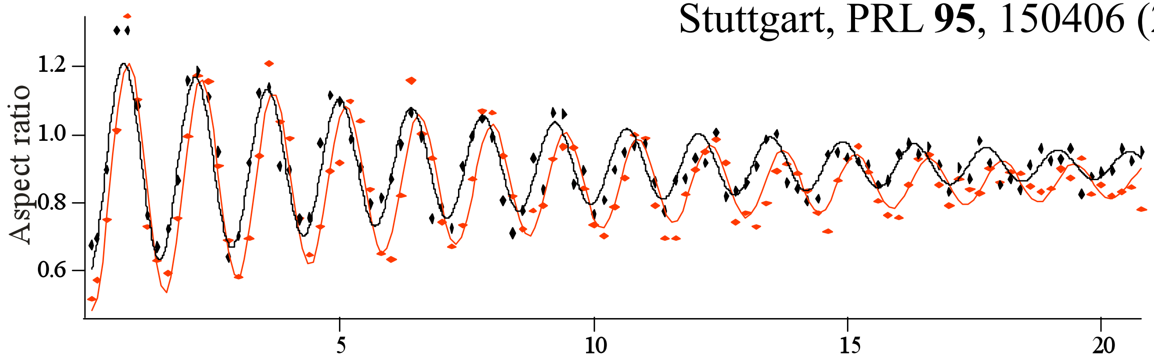
Striction





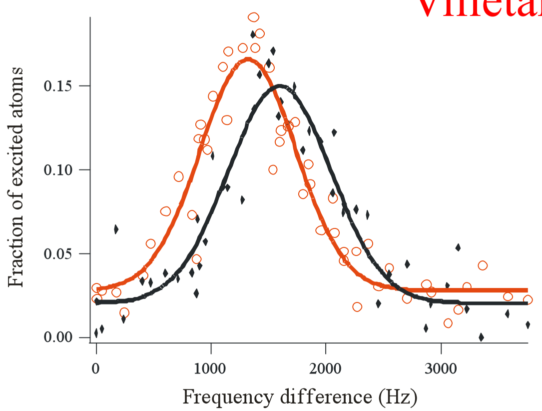
Stuttgart, PRL **95**, 150406 (2005)

Collective excitations



Villetaneuse, PRL **105**, 040404 (2010)

Anisotropic speed of sound



Villetaneuse

Non local anisotropic meanfield

-Static and dynamic properties of BECs

Small effects in Cr…

Need Feshbach resonances or larger dipoles.

With… ? Cr ? Er ? Dy ? Dipolar molecules ?

Then…, Tc, solitons, vortices, Mott physics,
new phases (checkboard, supersolid),
1D or 2D physics (rotons),
breakdown of integrability in 1D…

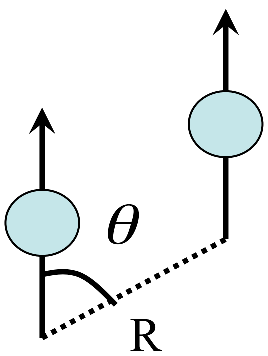
Spinor properties

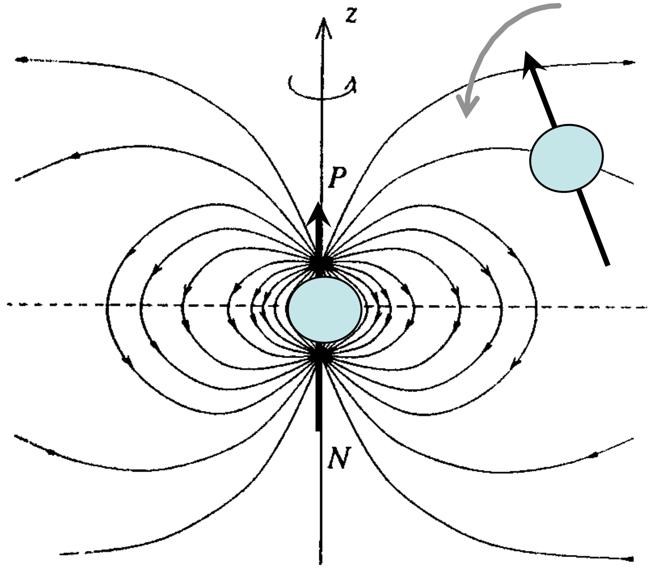
Spin degree of freedom coupled to orbital degree of freedom

- Spinor physics and magnetization dynamics

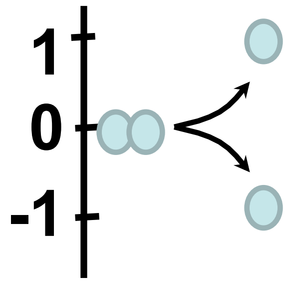
Dipole-dipole interactions

$$V_{dd} = \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 \left(1 - 3\cos^2(\theta)\right) \frac{1}{R^3}$$

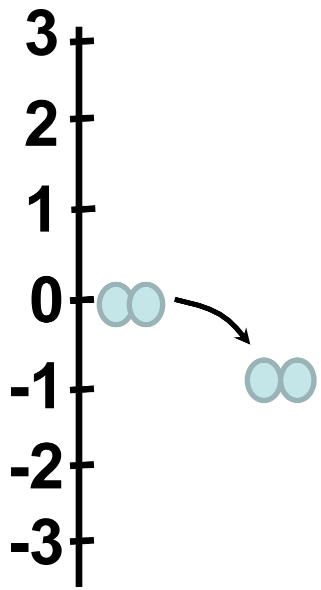




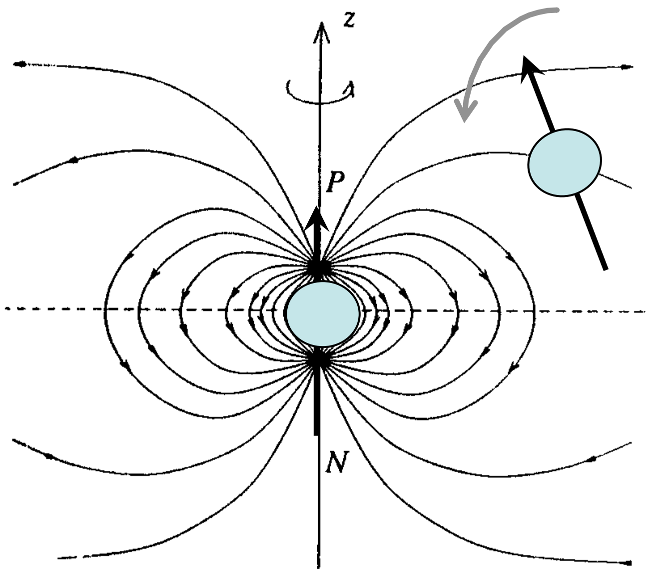
without V_{dd}

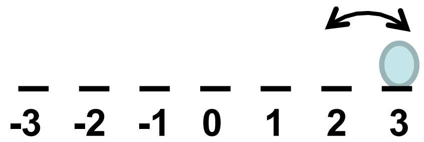


with V_{dd}



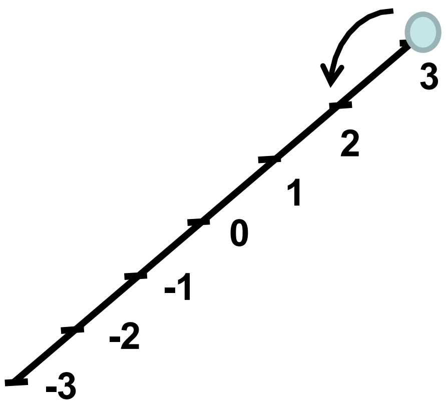
B=0: Rabi





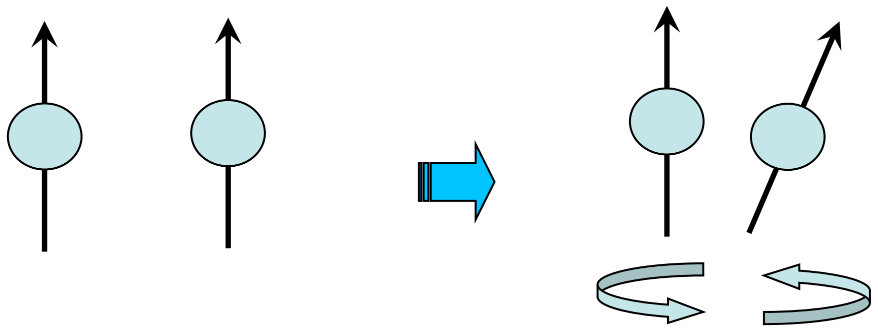
$$\hbar\Gamma \approx V_{dd}$$

In a finite magnetic field: Fermi golden rule



$$\hbar\Gamma \approx \left|V_{dd}\right|^2 \rho\left(\varepsilon_f = g\mu_B B\right)$$

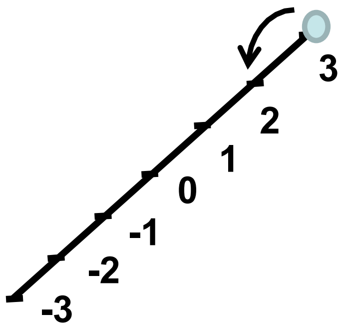
Dipolar relaxation, rotation, and magnetic field



Angular momentum conservation

$$\Delta m_S + \Delta m_l = 0$$

$$|3,3\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|3,2\rangle + |2,3\rangle\right)$$



$$\Delta \ell = 2$$

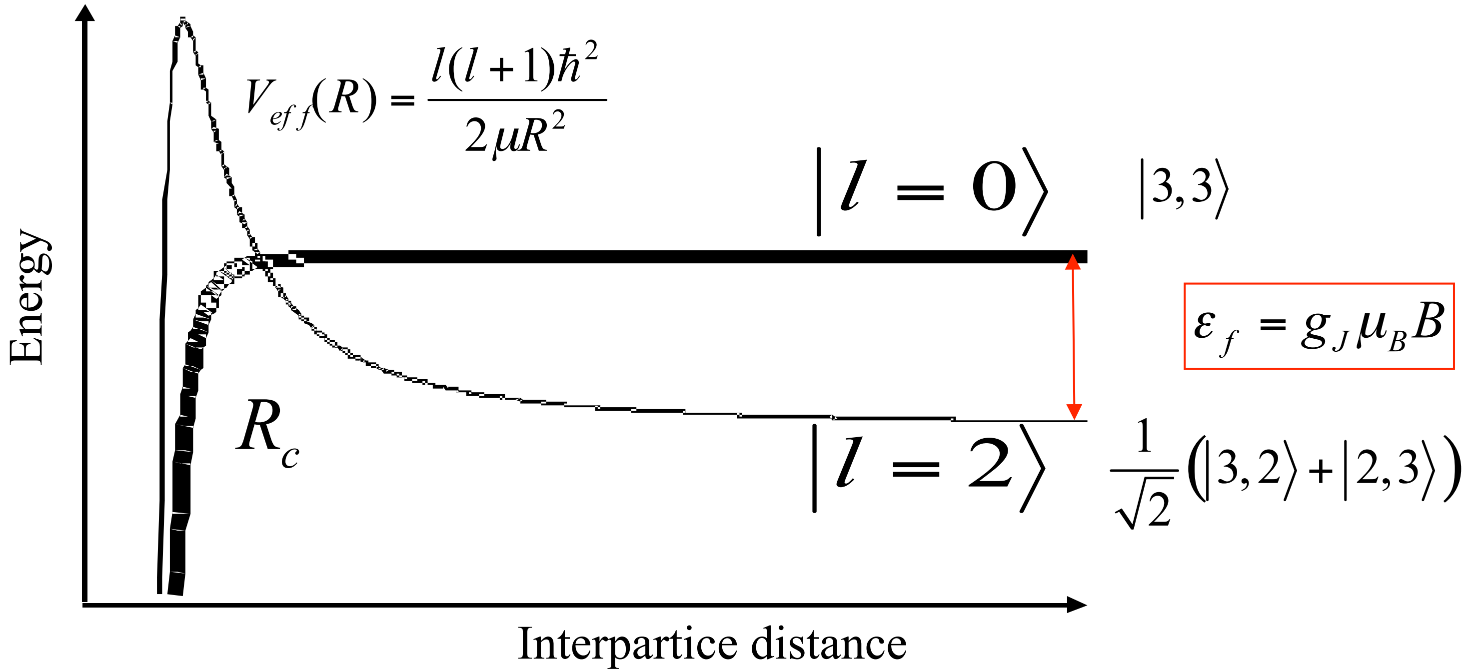
$$\Delta E = \Delta m_S g \mu_B B$$

Rotate the BEC ?
Spontaneous creation of vortices ?
Einstein-de-Haas effect

Important to control magnetic field

Ueda, PRL **96**, 080405 (2006)
Santos PRL **96**, 190404 (2006)
Gajda, PRL **99**, 130401 (2007)
B. Sun and L. You, PRL **99**, 150402 (2007)

From the molecular physics point of view



R_c = Condon radius

$$R_C \approx \sqrt{\frac{l(l+1)\hbar^2}{m g_S \mu_B B}}$$

$$\Gamma \propto |\Psi_{in}(R_c)|^2$$

PRA **81**, 042716 (2010)

Energy scale, length scale, and magnetic field

Molecular binding enery : 10 MHz

Magnetic field = 3 G

$$R_c = a_S$$

(a_S is the scattering length)

Inhibition of dipolar relaxation due to inter-atomic (VdW) repulsion

Band excitation in lattice : 100 kHz

30 mG

$$R_c = a_\perp$$

($a_\perp$ is the harmonic oscillator size)

Suppression of dipolar relaxation in optical lattices

Chemical potential : 1 kHz

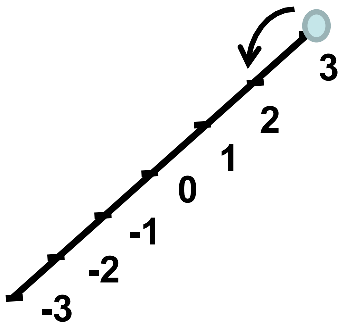
.3 mG

$$R_c = n^{-1/3}$$

Inelastic dipolar mean-field

3 Gauss

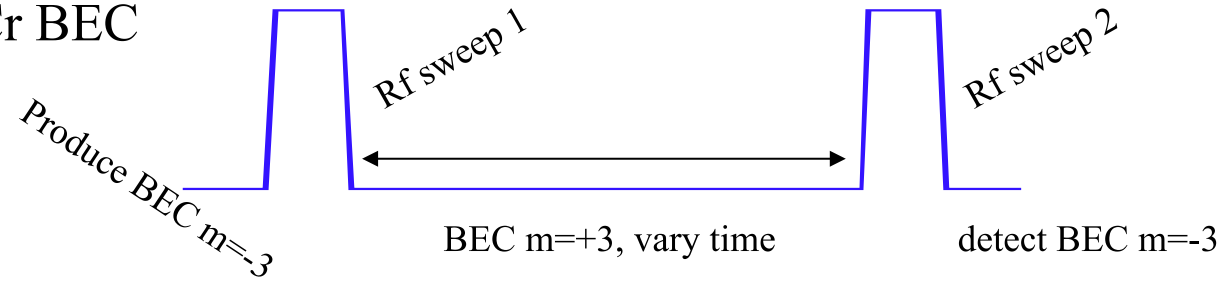
Suppression of dipolar relaxation due to inter-atomic repulsion

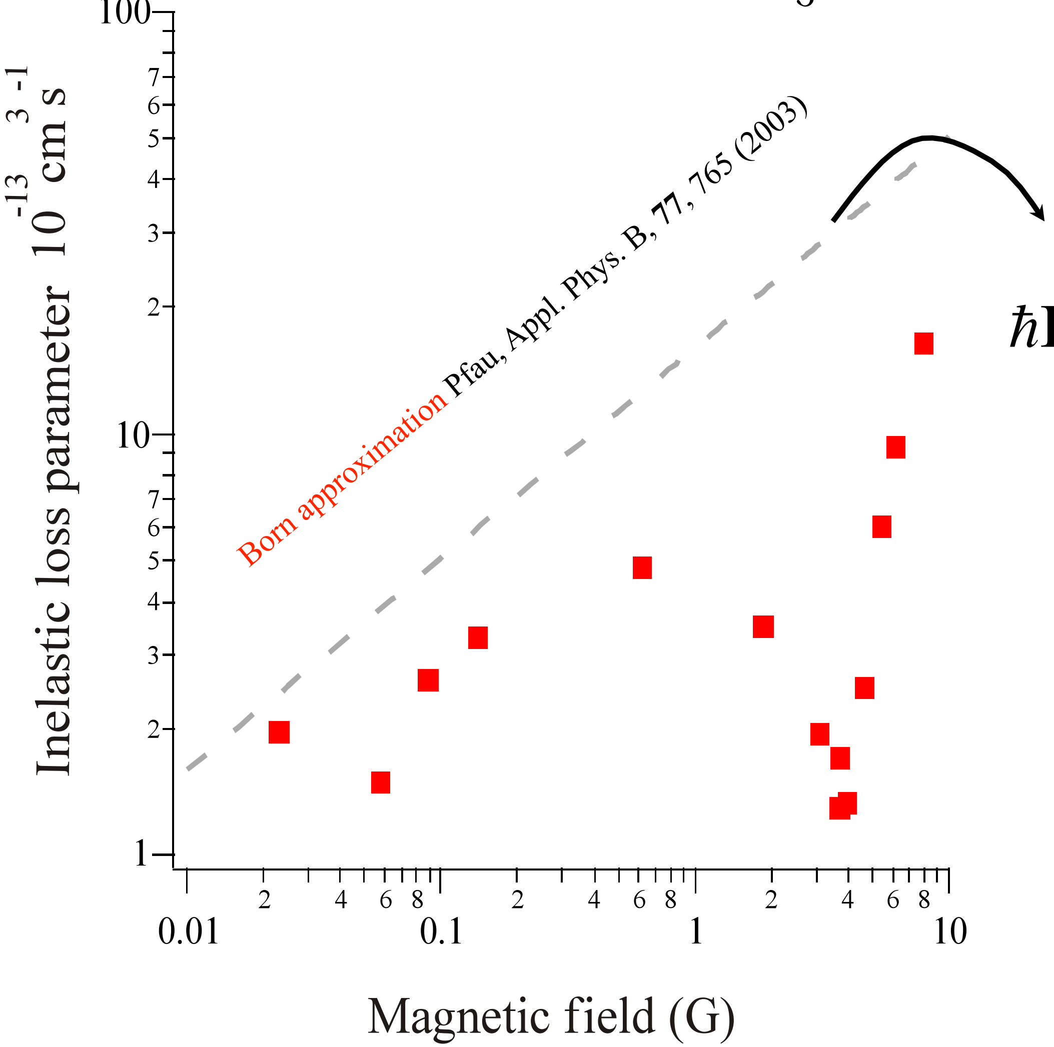


$$R_c = a_S$$

…spin-flipped atoms gain so much energy they leave the trap

Dipolar relaxation in a Cr BEC





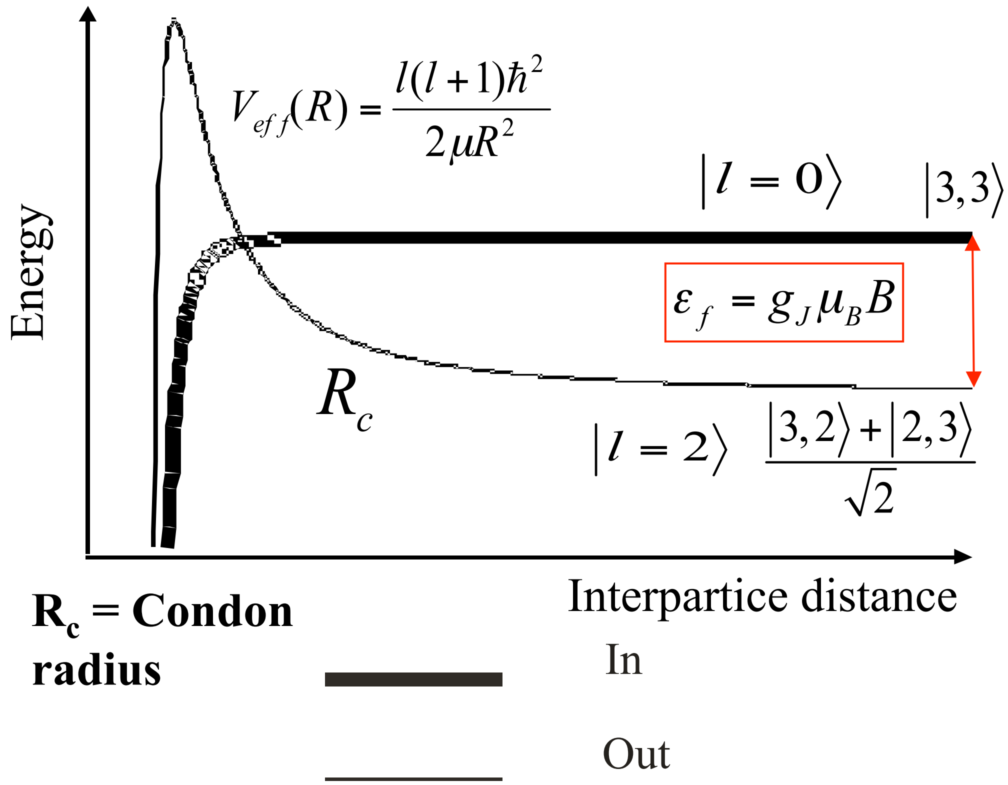
Fermi golden rule

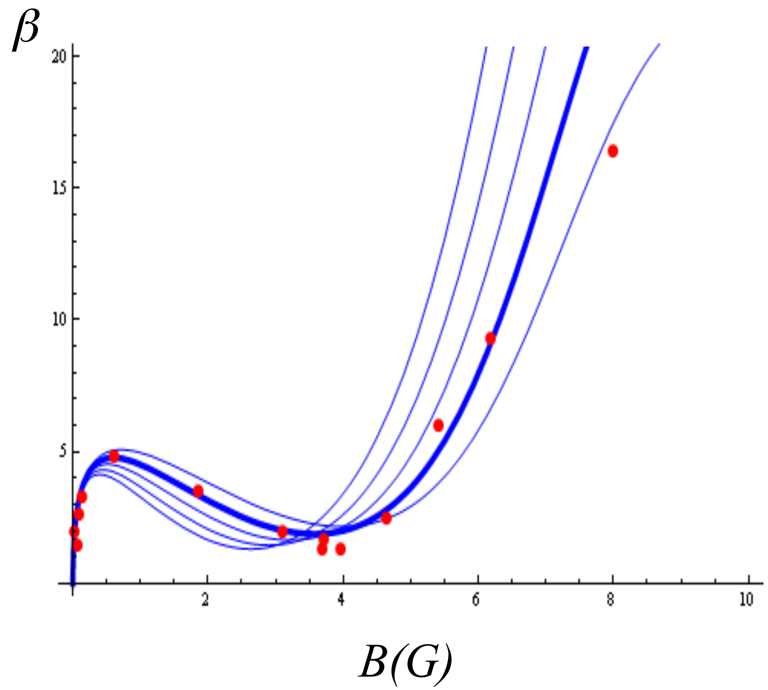
$$\hbar\Gamma \approx \left|V_{dd}\right|^2 \rho\left(\varepsilon_f\right) \qquad \varepsilon_f = g_J \mu_B B$$

$$\Gamma \propto \left|\Psi_{in}(R_c)\right|^2$$

See also Shlyapnikov PRL **73**, 3247 (1994)

Determination of Cr scattering lengths





PRA **81**, 042716 (2010)

$a_6 = 103 \pm 4\ a_0.$

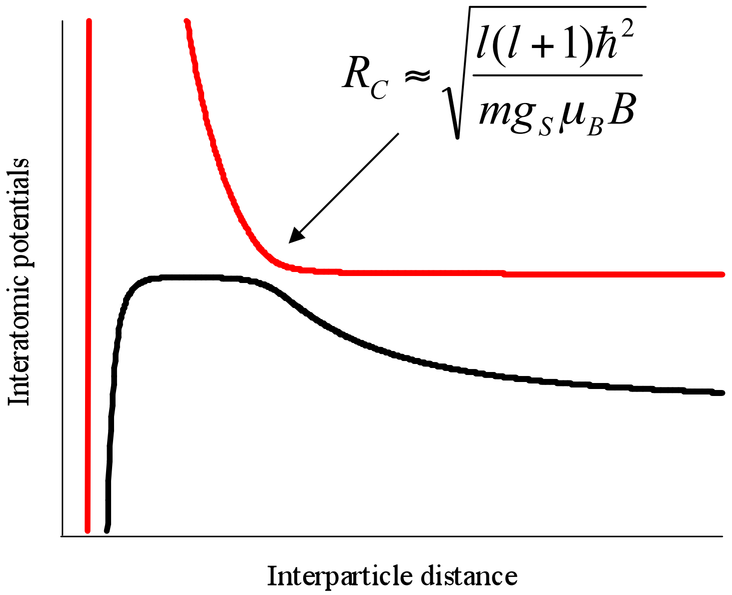
$$\Gamma \propto |\Psi_{in}(R_c)|^2$$

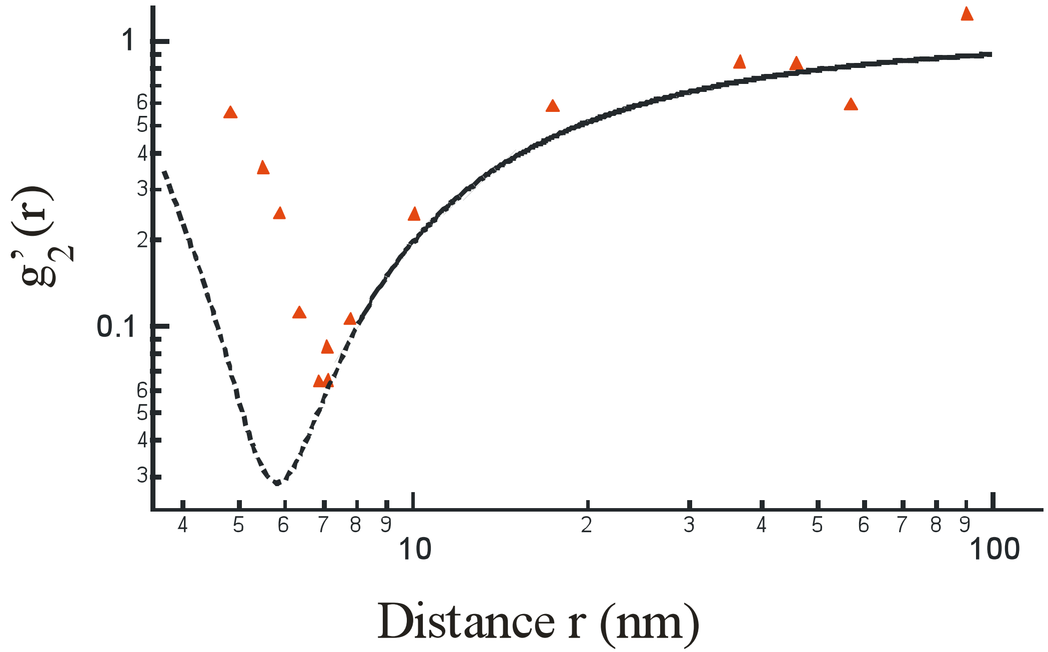
$R_c = a_S$ ⟶ **Zero coupling**

Determination of scattering lengths S=6 and S=4 (coll. Anne Crubellier)

$a_6 = 102.5 \pm 0.4\ a_0$ Feshbach resonance in d-wave PRA **79**, 032706 (2009)

Dipolar relaxation: measuring non-local correlations





$$g_2(r) \simeq (1 - a/r)^2 \qquad r >> R_{VdW}$$

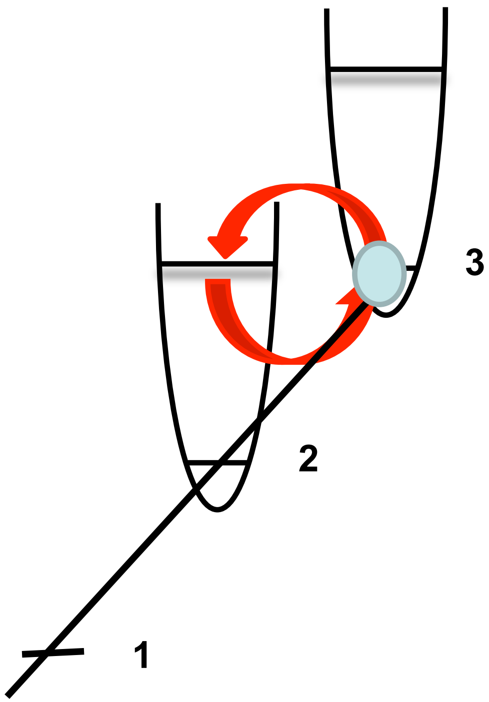
L. H. Y. Phys. Rev. **106**, 1135 (1957)

A probe of long-range correlations :

here, a mere two-body effect, yet unacounted for in a mean-field « product-ansatz » BEC model

30 mGauss

Spin relaxation and band excitation in optical lattices

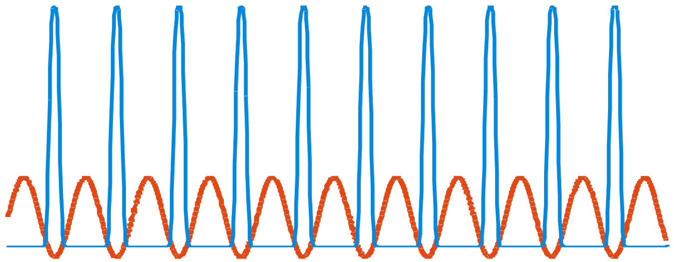


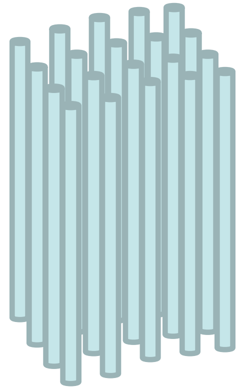
$$R_c = a_\perp$$

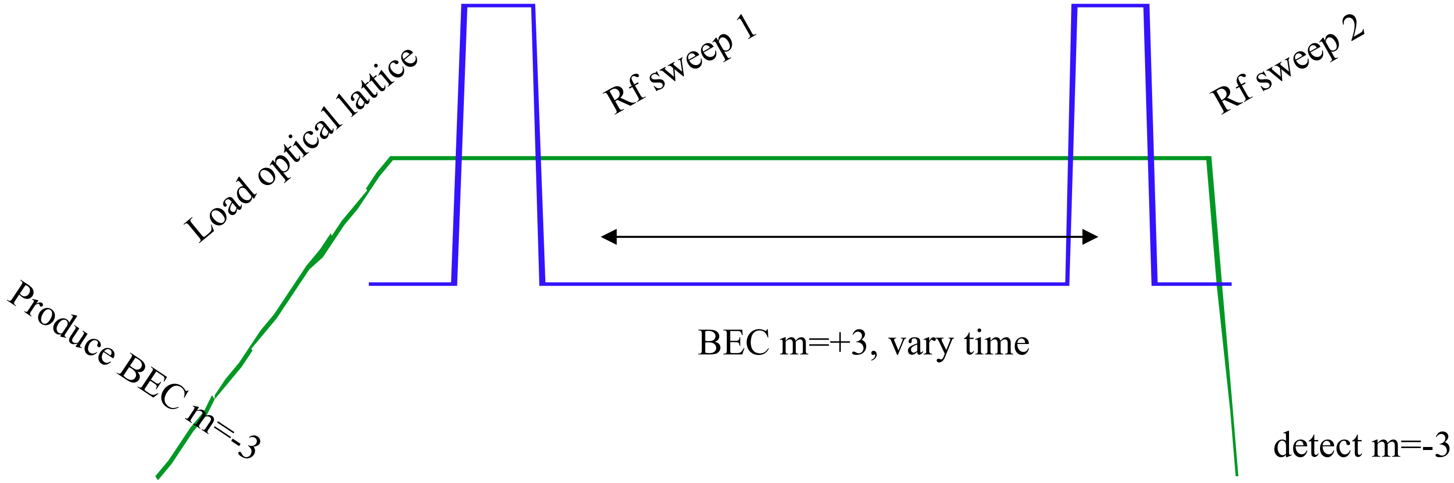
…spin-flipped atoms go from one band to another

Reduction of dipolar relaxation in optical lattices

Load the BEC in a 1D or 2D Lattice

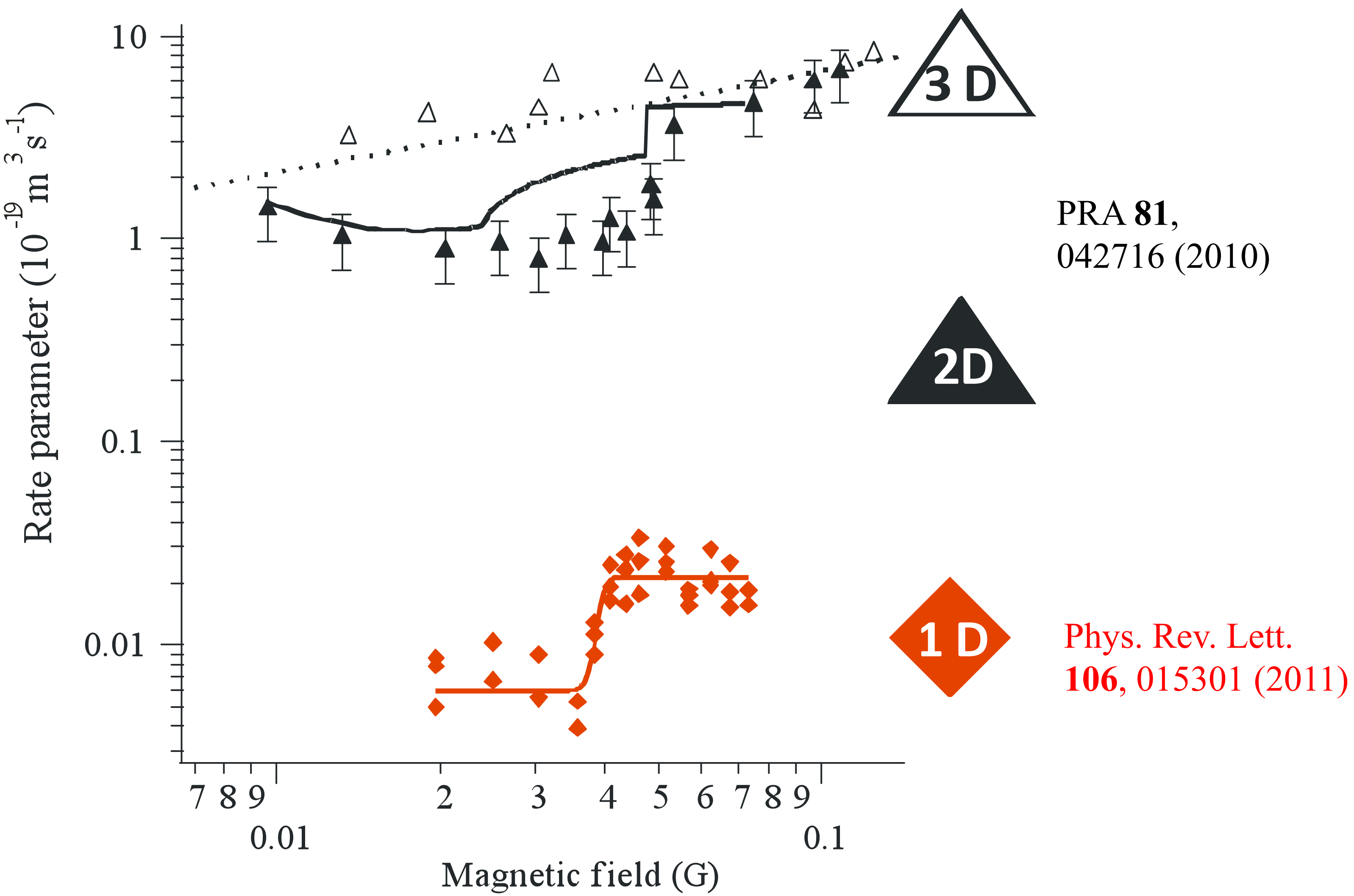






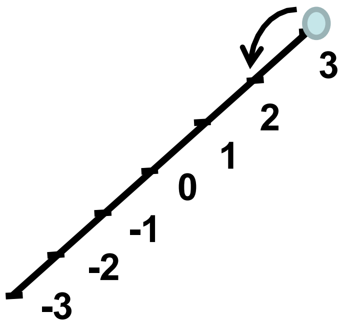
$$\hbar\Gamma \approx \left|V_{dd}\right|^2 \rho\left(\varepsilon_f\right)$$

One expects a reduction of dipolar relaxation, as a result of the reduction of the density of states in the lattice



3 D
PRA 81, 042716 (2010)
2D
1 D
Phys. Rev. Lett. 106, 015301 (2011)
Rate parameter (10^{-19} m^3 s^{-1})
10
1
0.1
0.01
7 8 9
2
3
4
5
6
7 8 9
0.01
0.1
Magnetic field (G)

What we measure in 1D:

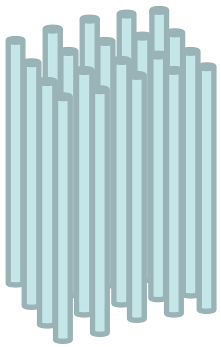


Energy released

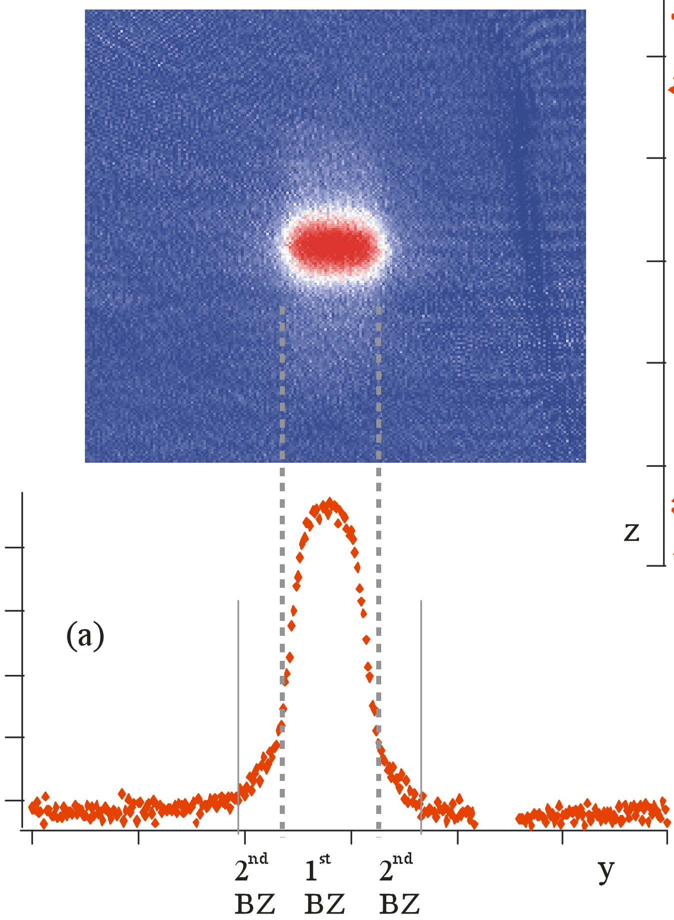
$$\varepsilon_f = g_J \mu_B B$$

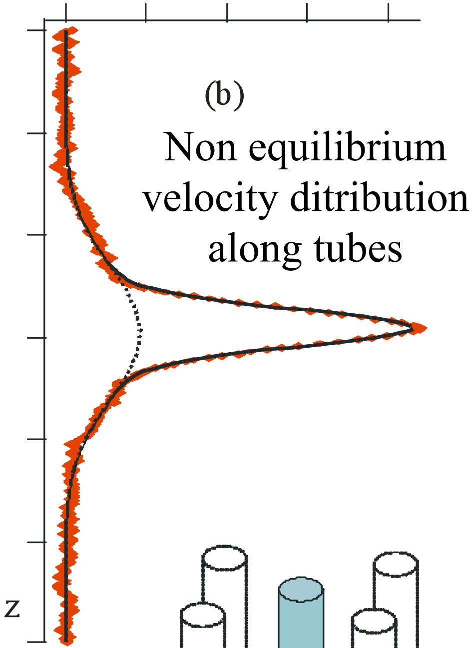
Band excitation

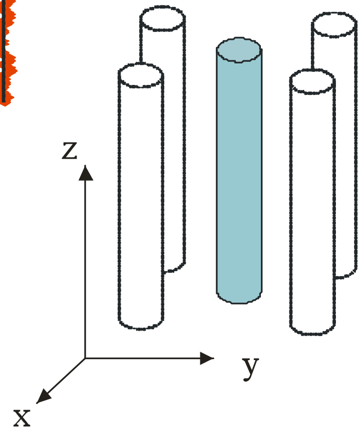
Kinetic energy along tubes



« Band mapping »



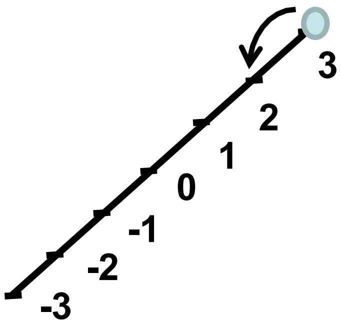




Population in different bands due to dipolar relaxation

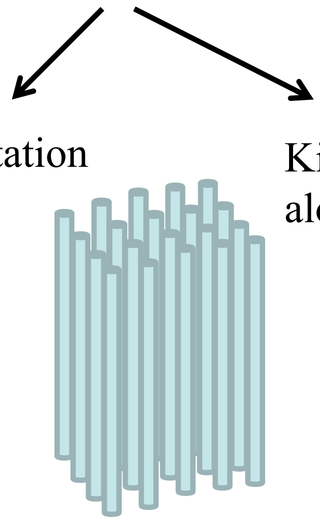
Phys. Rev. Lett. **106**, 015301 (2011)

(almost) complete suppression of dipolar relaxation in 1D at low field



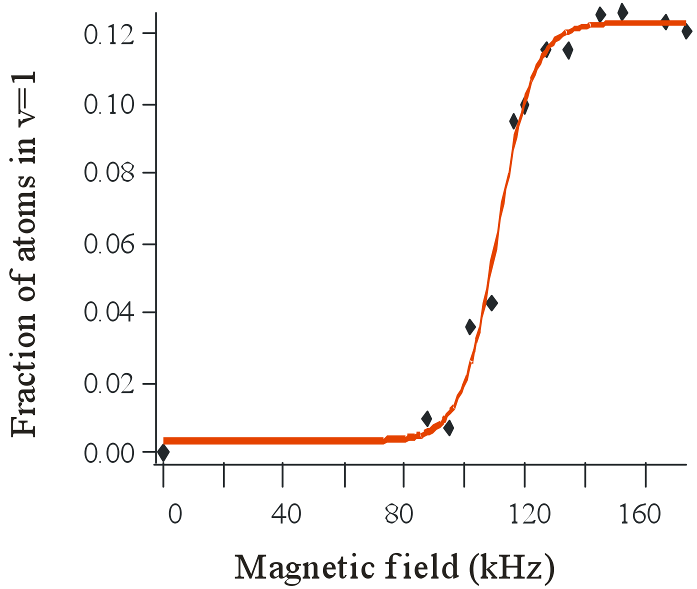
Energy released

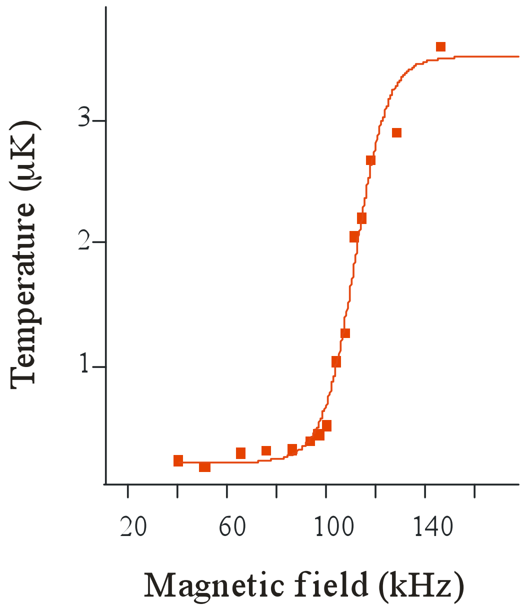
$$\varepsilon_f = g_J \mu_B B$$



Band excitation

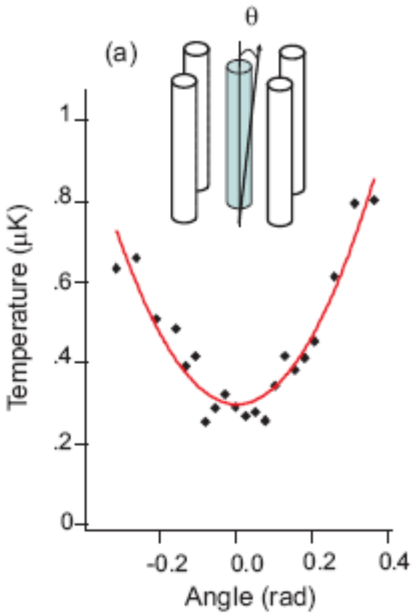
Kinetic energy along tubes





B. Pasquiou et al., Phys. Rev. Lett. **106**, 015301 (2011) (almost) complete suppression of dipolar relaxation in 1D at low field in 2D lattices:

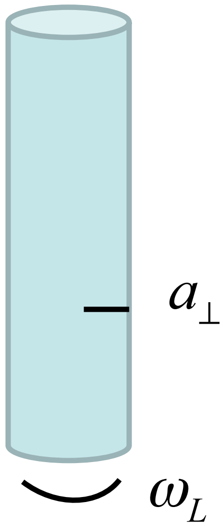
onsequence of angular momentum conservation



$$\Delta m_S + \Delta m_l = 0$$

$$\Delta m_S = -1$$

$$g\mu_B B = \Delta E > E(l=2) = \frac{\hbar^2}{m a_L^2} = \hbar\omega_L$$



Below threshold:

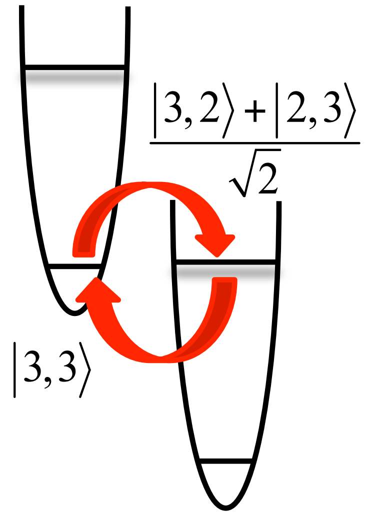
a (spin-excited) metastable 1D quantum gas ;

Interest for spinor physics, spin excitations in 1D…

Above threshold :

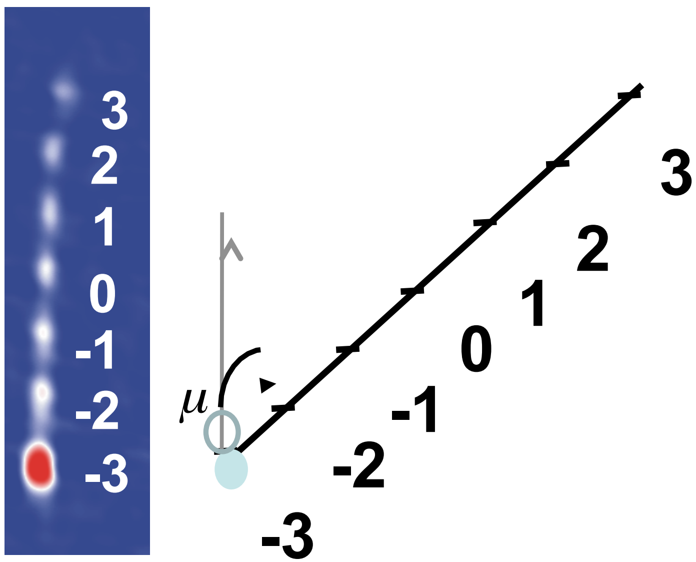
should produce vortices in each lattice site (EdH effect) (problem of tunneling)

Towards coherent excitation of pairs into higher lattice orbitals ? (Rabi oscillations)



.3 mGauss

Magnetization dynamics of spinor condensates



$$R_c = n^{-1/3}$$

…spin-flipped atoms *loses* energy

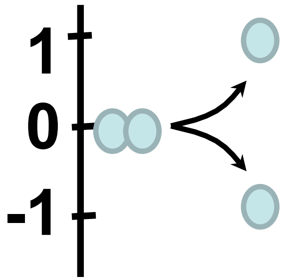
Similar to M. Fattori et al., Nature Phys. 2, 765 (2006) at large fields and in the thermal regime

S=3 Spinor physics with free magnetization

- Up to now, spinor physics with S=1 and S=2 only

- Up to now, all spinor physics at constant magnetization (exchange interactions, no dipole-dipole interactions)

- They investigate the ground state for a given magnetization
  -> Linear Zeeman effect irrelavant

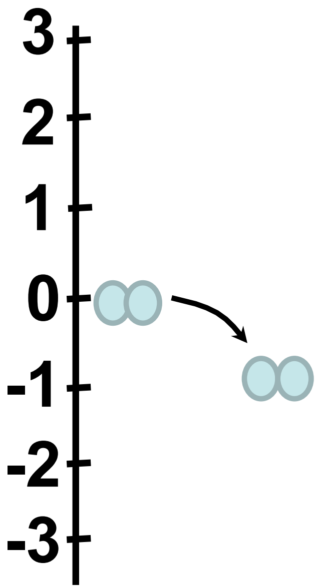


New features with Cr

- First S=3 spinor (7 Zeeman states, four scattering lengths, a_6, a_4, a_2, a_0)

- Dipole-dipole interactions free total magnetization

- Can investigate the true ground state of the system (need very small magnetic fields)

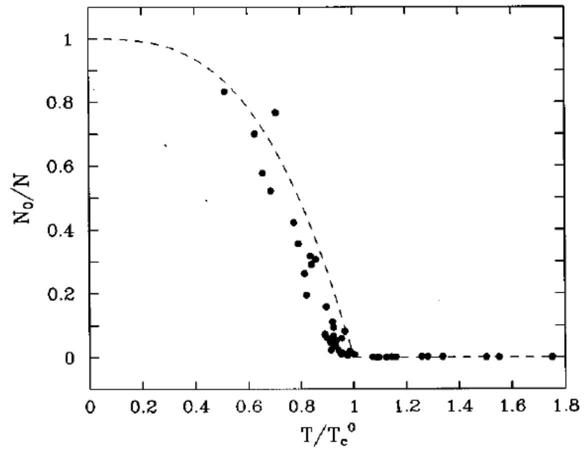


Single vs multi- component non-interacting Bose thermodynamics

Single component:

$$N_{th} = \sum_{n_x, n_y, n_z \neq 0} \frac{1}{\exp\left[\beta\left(\hbar\omega_x n_x + \hbar\omega_y n_y + \hbar\omega_z n_z - \mu\right)\right] - 1}$$

$$\beta = 1/k_B T$$

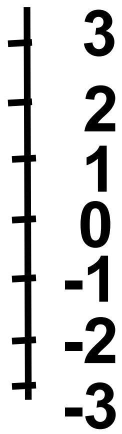


PRL **77**, 4984 (1996)

Multi-component, true (i.e. magnetization is free) thermodynamic equilibrium:

$$N_{th}^{m_S} = \sum_{n_x, n_y, n_z \neq 0} \frac{1}{\exp\left[\beta\left(\hbar\omega_x n_x + \hbar\omega_y n_y + \hbar\omega_z n_z + \underbrace{m_S g \mu_B B - \mu}_{\mu_{m_S}}\right)\right] - 1}$$

(only linear Zeeman effect included here)



Non-interacting BEC is always ferromagnetic

PRA, **59**, 1528 (1999)

Magnetization-fixed Bose thermodynamics

When magnetization is fixed (negligible dipole-dipole interactions), the system does not go to its true thermodynamic equilibium, but to an equilibrium for a fixed magnetization

$$N_{th}^{m_S} = \sum_{n_x, n_y, n_z \neq 0} \frac{1}{\exp\left[\beta\left(\varepsilon(n_x, n_y, n_z) + m_S g \mu_B B_{eff} - \mu\right)\right] - 1}$$

Magnetic-chemical potential (fixes magnetization) → $m_S g \mu_B B_{eff}$

Chemical potential → μ

J. Phys. Soc. Jpn, **69,** 12, 3864 (2000)

Magnetization-free vs magnetization-fixed Bose thermodynamics

Reduction of Tc due to more degrees of freedom

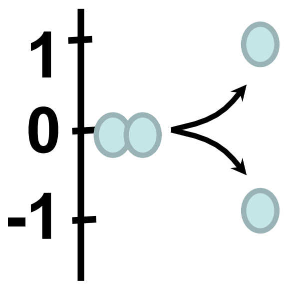
Three possible phases:

A: Thermal gas

B: a BEC only in the majority component

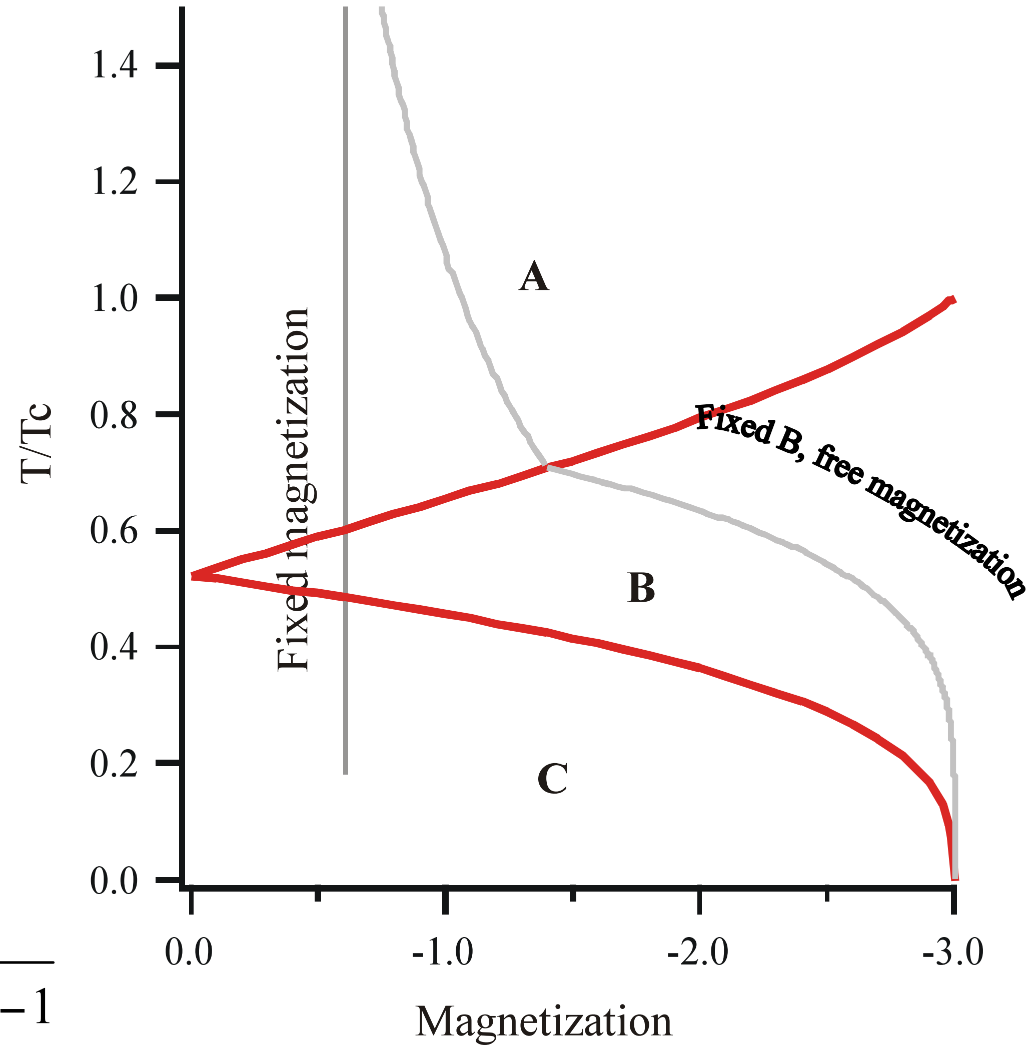
C: a multi-component BEC (a BEC component in all Zeeman states)

$$\mu_{i+1} + \mu_{i-1} = 2\mu_i$$



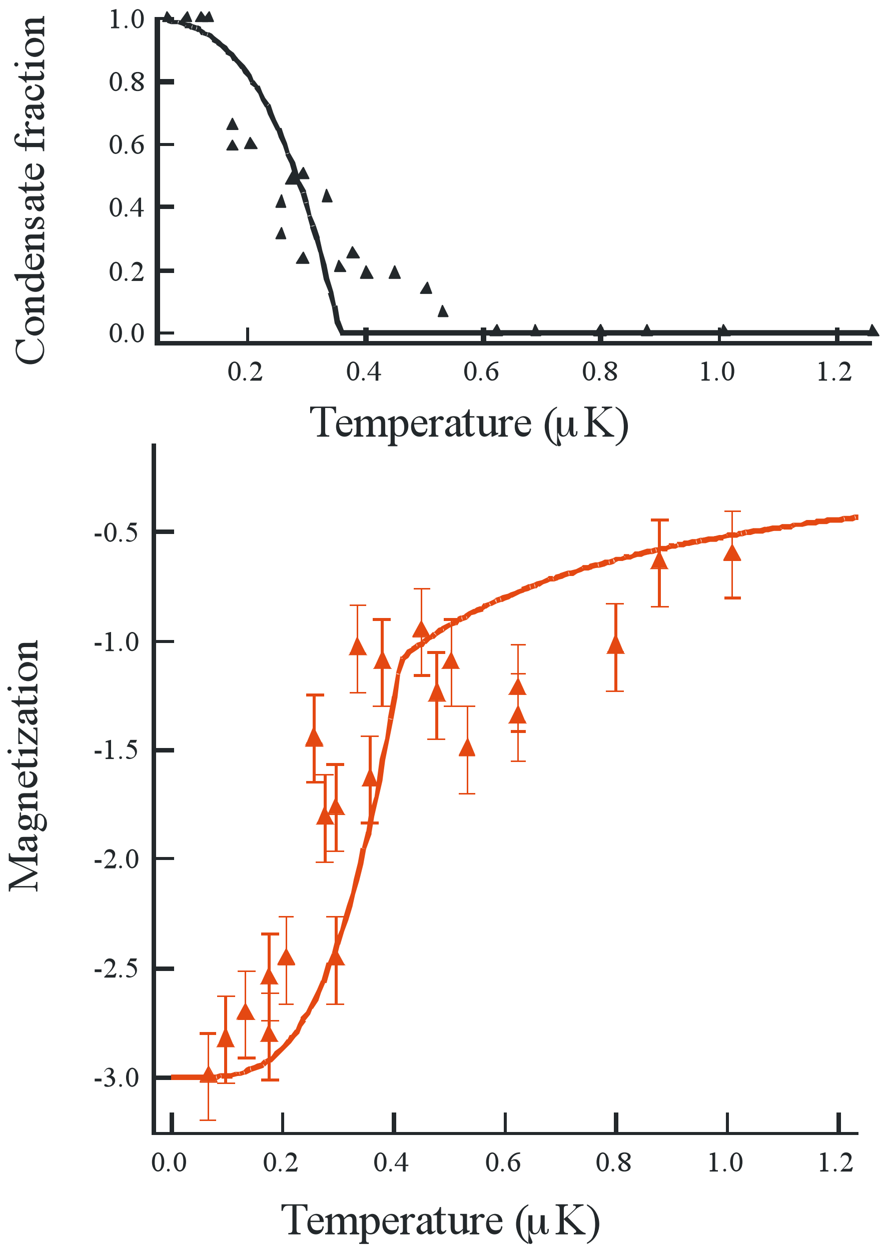
$$N_{th}^{i} = \sum_{n_x, n_y, n_z \neq 0} \frac{1}{\exp\left[\beta\left(\varepsilon(n_x, n_y, n_z) + \mu_i\right)\right] - 1}$$

(only linear Zeeman effect included here)



J. Phys. Soc. Jpn, **69,** 12, 3864 (2000)

Thermodynamics: a BEC is ferromagnetic



$B = 900\mu G$

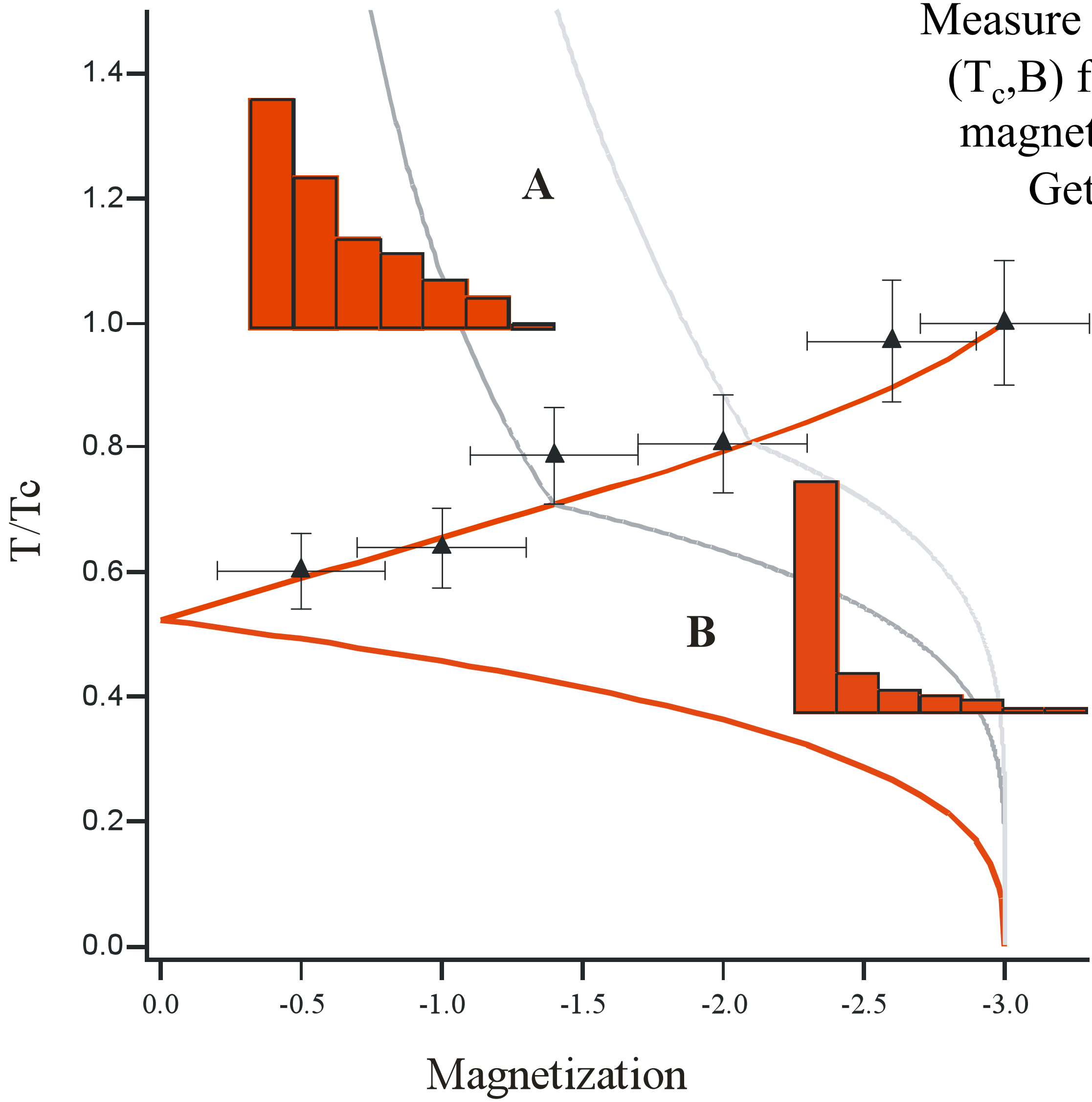
BEC only in m_S=-3

↓

Cloud spontaneously polarizes !

Thermodynamics: phase diagram of a ferromagnetic BEC

Quasi-Boltzmann distribution



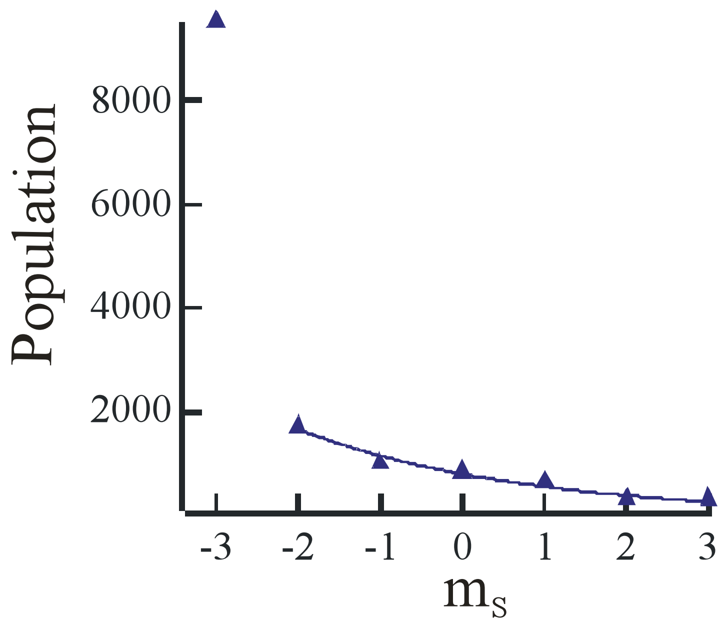
Measure $T_c(B)$ and M (T_c,B) for different magnetic fields B Get $T_c(M)$

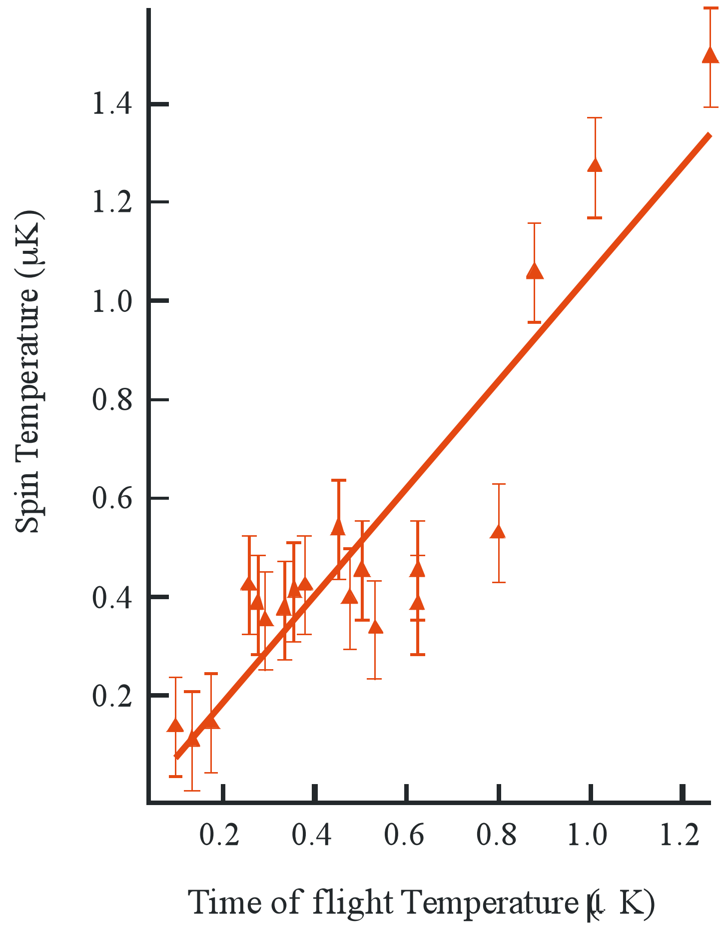
Bi-modal spin distribution

The C (spinor-)phase is always avoided !

Spin thermometry, and new cooling method ?

a bi-modal spin distribution



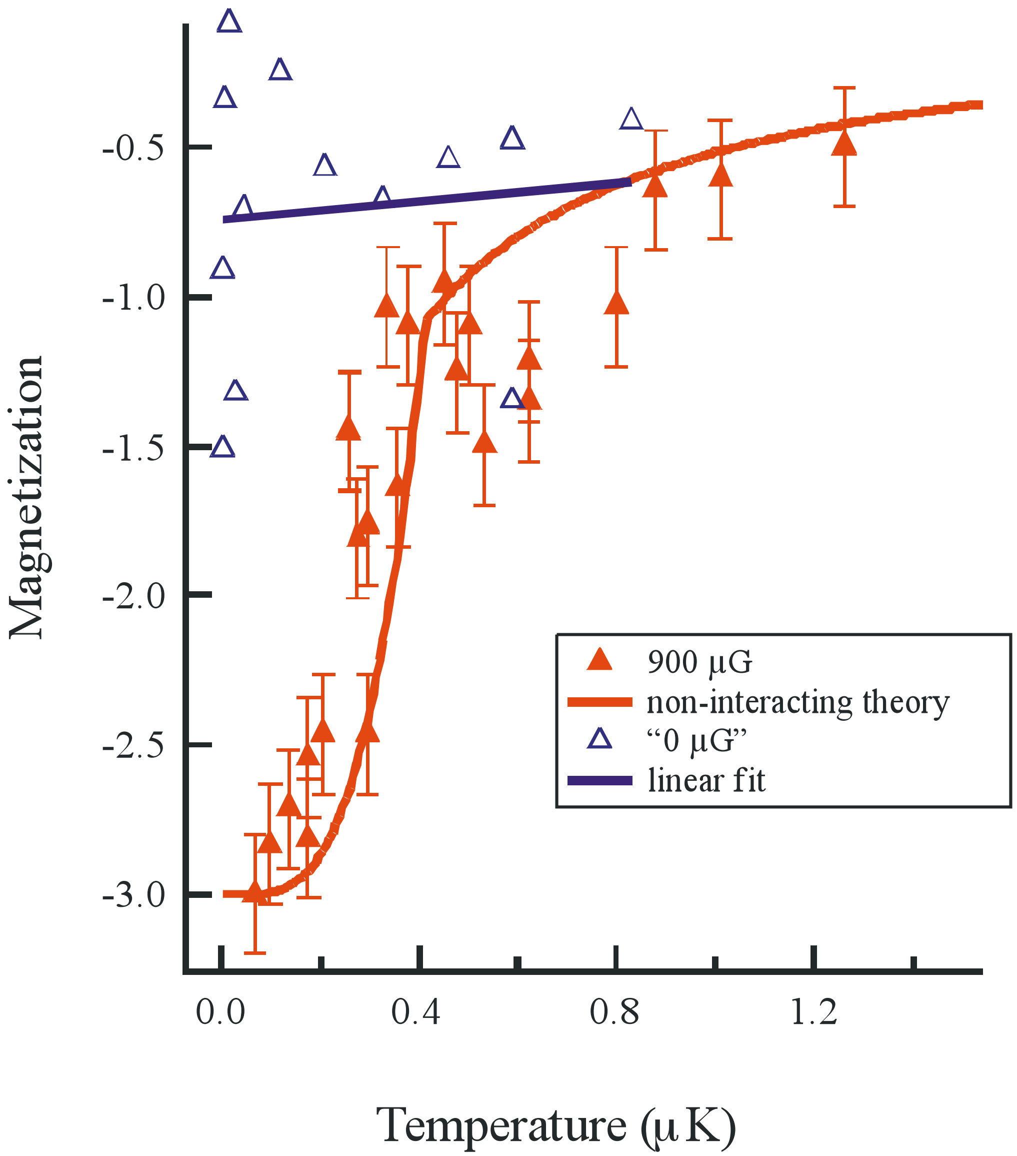


Only thermal gas depolarizes…

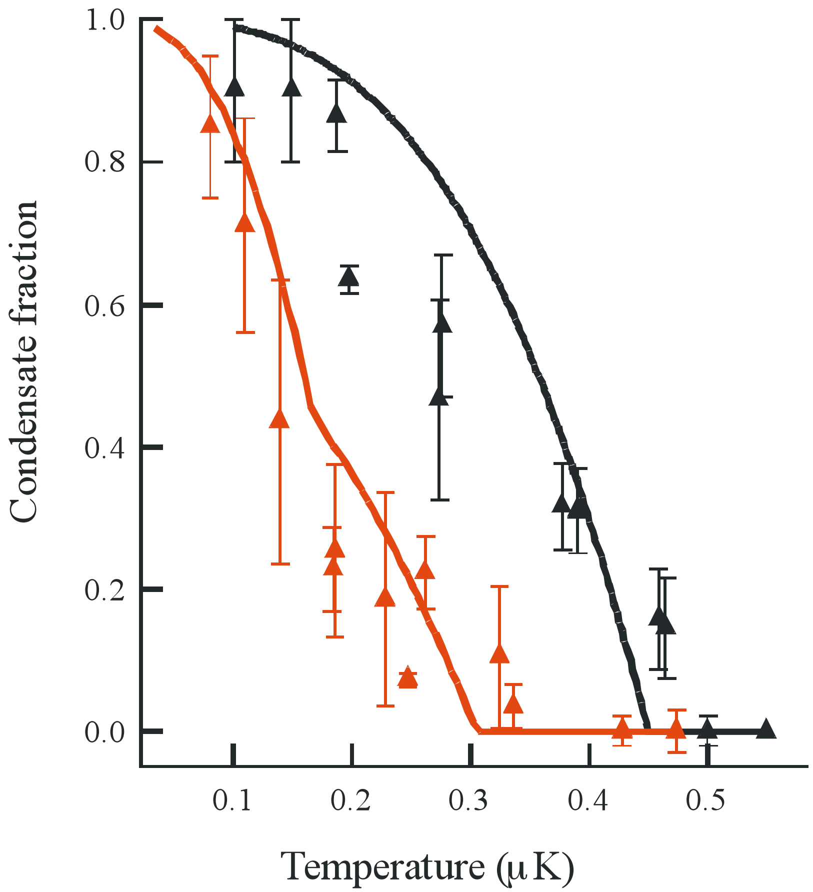
get rid of it ?
(field gradients)

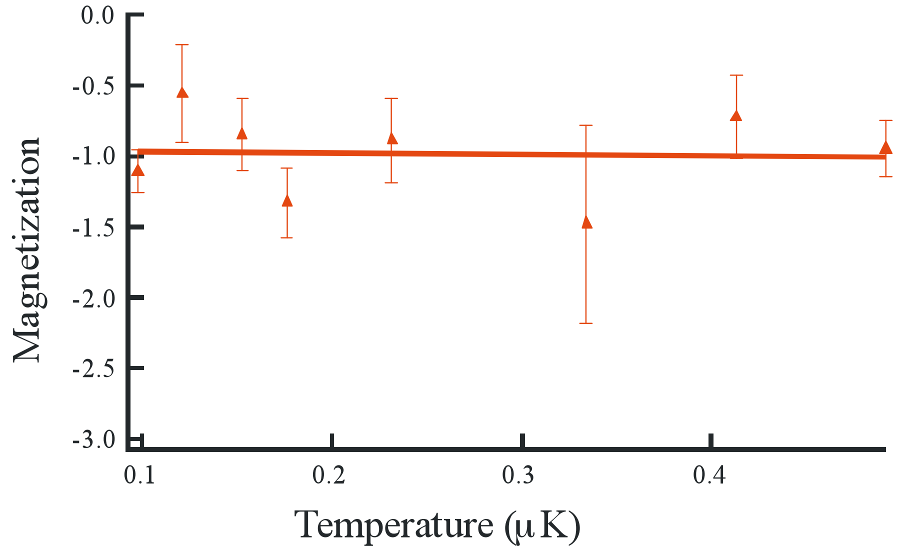
We measure spin-temperature by fitting (in practice) a Boltzmann distribution to the m_S population

Below a critical magnetic field: the BEC ceases to be ferromagnetic !



Thermodynamics near B=0





B=0

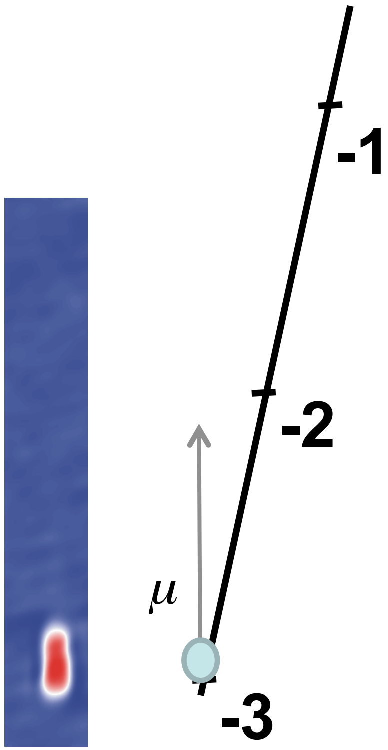
B=20 mG

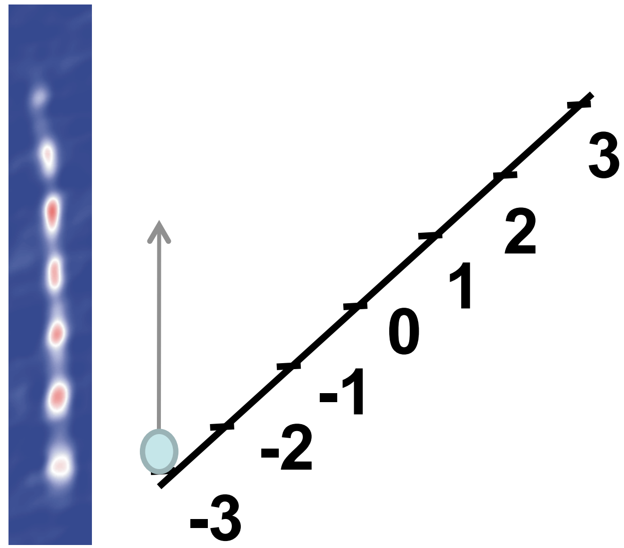
-Magnetization remains small even when the condensate fraction approaches 1

!! Obervation of a depolarized condensate !!

Necessarily an interaction effect

Cr spinor properties at low field

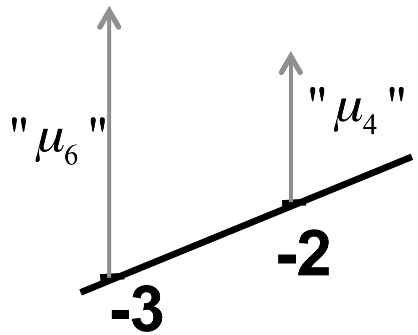




Large magnetic field : ferromagnetic

Low magnetic field : polar/cyclic

$$g_J \mu_B B_c \approx \frac{2\pi\hbar^2 n_0 \left(a_6 - a_4\right)}{m}$$

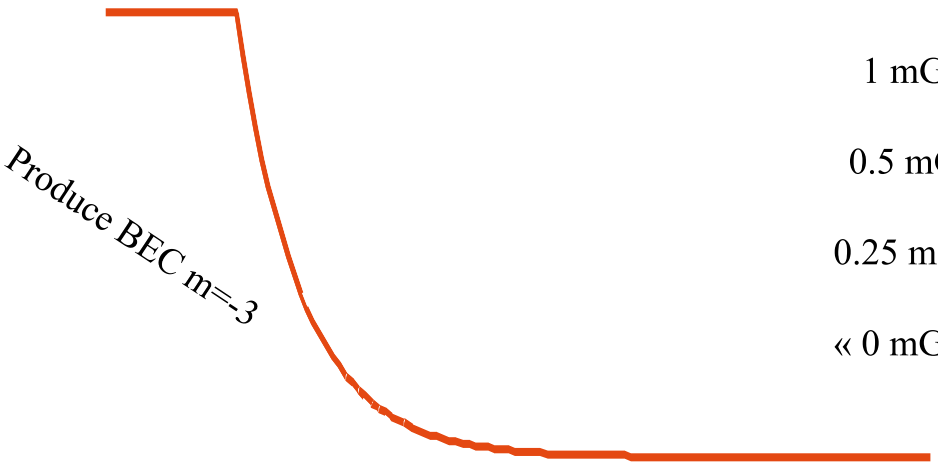


Phases set by contact interactions (a_6, a_4, a_2, a_0)
– differ by total magnetization

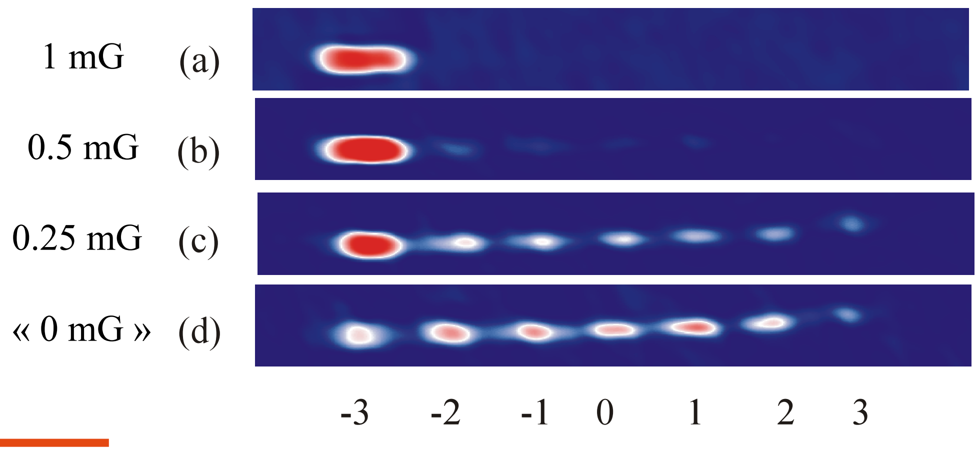
Santos PRL **96**, 190404 (2006)

Ho PRL. **96**, 190405 (2006)

At VERY low magnetic fields, spontaneous depolarization quantum gases



Rapidly lower magnetic field



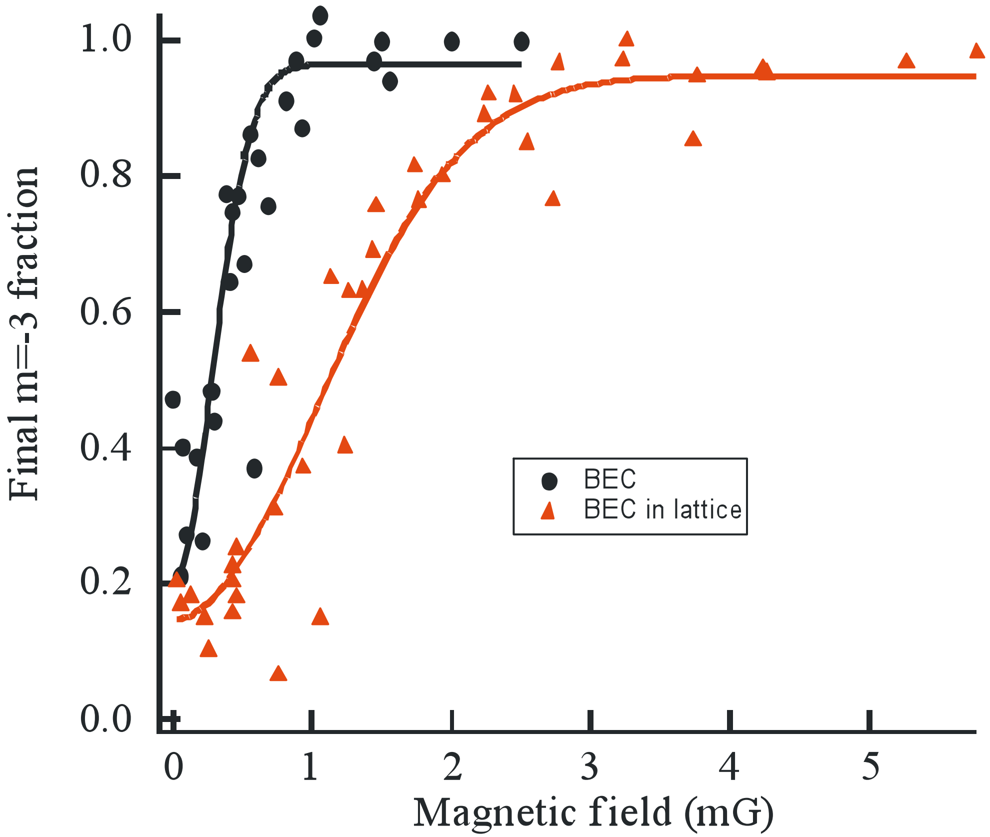
Stern Gerlach experiments

Magnetic field control below .5 mG
(dynamic lock, fluxgate sensors)

(.1mG stability)
(no magnetic shield…)

Phys. Rev. Lett. **106**, 255303 (2011)

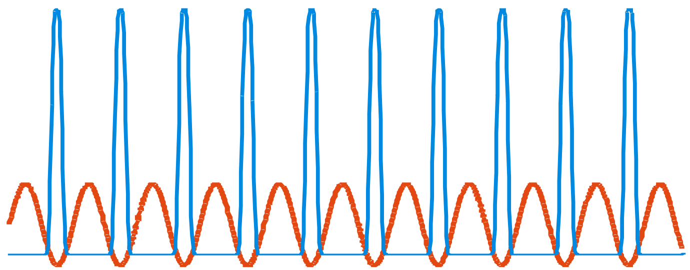
Mean-field effect



$$g_J \mu_B B_c \approx \frac{2\pi\hbar^2 n_0 (a_6 - a_4)}{m}$$

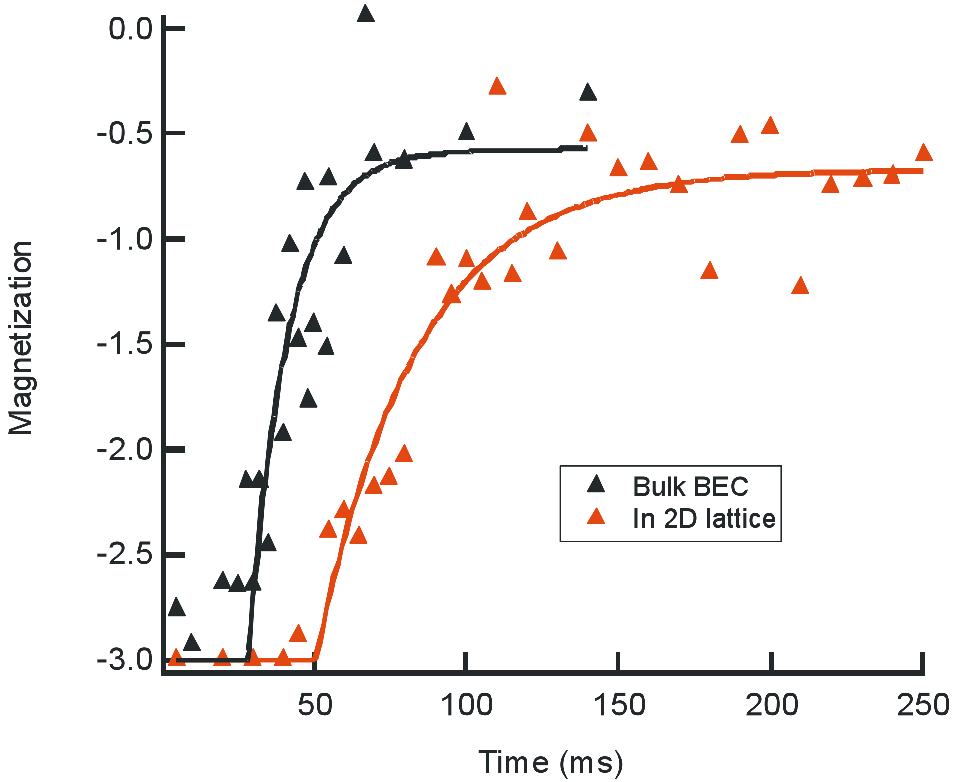
	BEC	Lattice
Critical field	0.26 mG	1.25 mG
1/e fitted	0.4 mG	1.45 mG

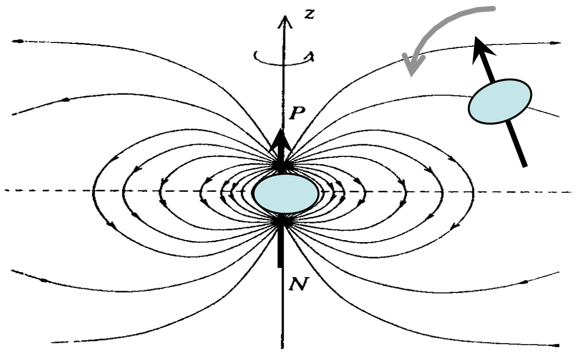
Load into deep 2D optical lattices to boost density.
Field for depolarization depends on density



Phys. Rev. Lett. **106**, 255303 (2011)

Dynamics analysis



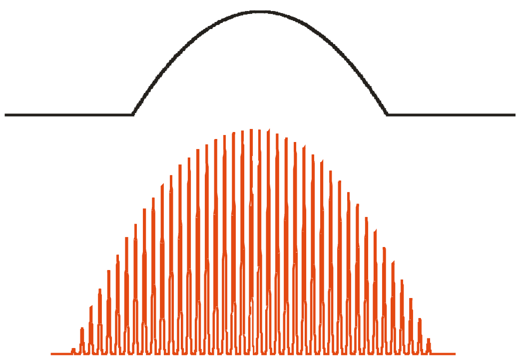


Meanfield picture :
Spin(or) precession (Majorana flips)

PRL **96**, 080405 (2006)
Phys. Rev. A **82**, 053614 (2010)

Natural timescale for depolarization:

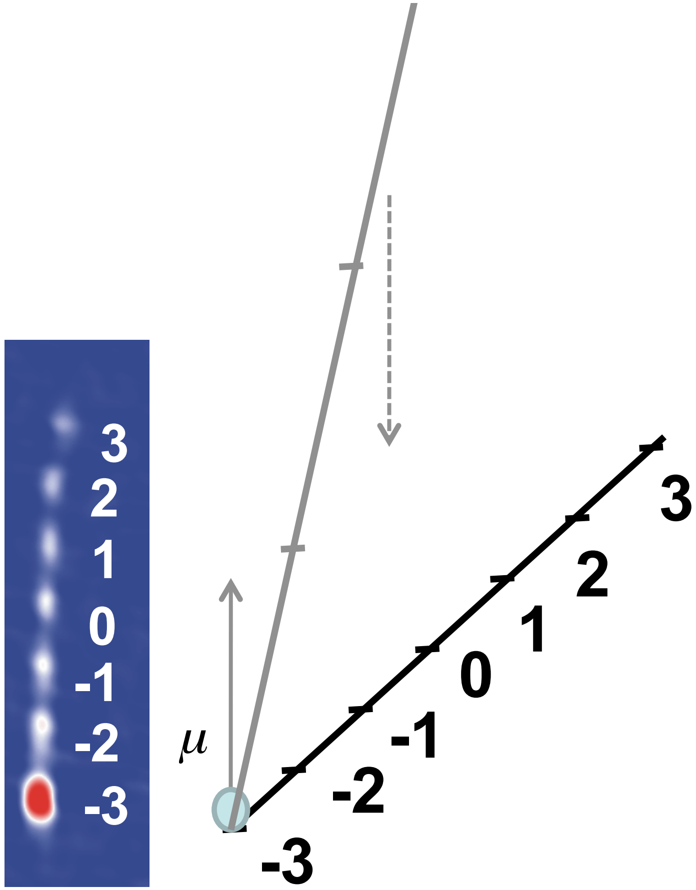
$$V_{dd}\left(r = n^{-1/3}\right) \propto \frac{\mu_0}{4\pi} S^2 \left(g_J \mu_B\right)^2 n$$

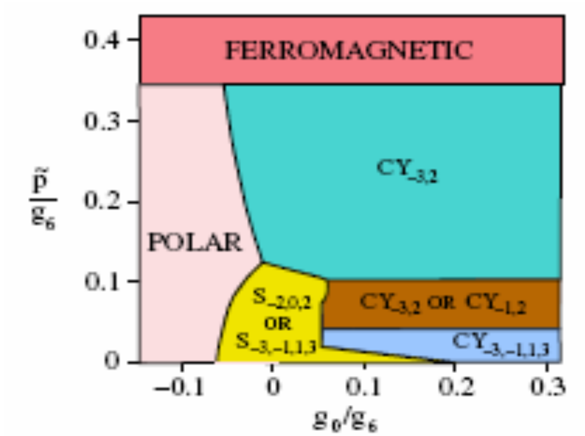


The timescale is slower in the lattice, because the cloud swells when loaded in the lattice (mean-field repulsion), and the dipolar mean-field is long-ranged…

An intersite inelastic effect… PRL **106**, 255303 (2011)

A quench through a zero temperature (quantum) phase transition





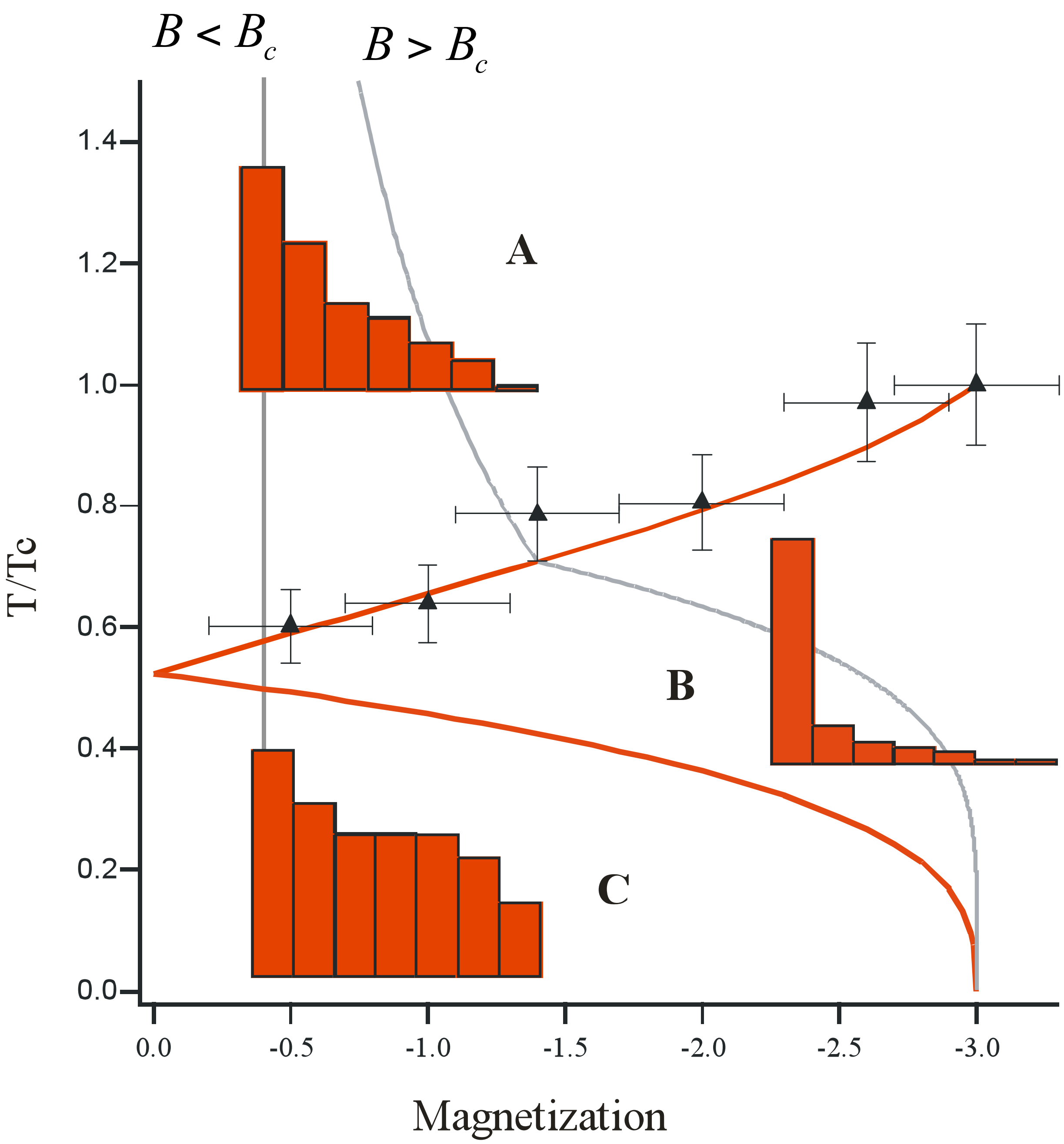
Santos and Pfau
PRL **96**, 190404 (2006)
Diener and Ho
PRL. **96**, 190405 (2006)

- Operate near B=0. Investigate absolute many-body ground-state
- We do not (cannot ?) reach those new ground state phases
- Quench should induce vortices…
- **Role of thermal excitations ?**
- **Metastable state**

Phases set by contact interactions, magnetization dynamics set by dipole-dipole interactions

« quantum magnetism »

Also new physics in 1D:
Polar phase is a singlet-paired phase
Shlyapnikov-Tsvelik NJP, 13, 065012 (2011)



!! Depolarized BEC likely in metastable state !!

Conclusion

Collective excitations – effect of non-local mean-field

Bragg excitations – anisotropic speed of sound

Dipolar relaxation in BEC – new measurement of Cr scattering lengths
non-local correlations

Dipolar relaxation in reduced dimensions - towards Einstein-de-Haas rotation in lattice sites

Spinor thermodynamics with free magnetization
– application to thermometry

Spontaneous demagnetization in a quantum gas
- New phase transition
– first steps towards spinor ground state

G. Bismut (PhD), B. Pasquiou (PhD) , A. de Paz
B. Laburthe, E. Maréchal, L. Vernac,
P. Pedri, M. Efremov (Theory),
O. Gorceix (Group leader)