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Novel phases of fermionic polar molecules

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Novel phases of fermionic polar molecules

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Outline

- Experiments with ultracold polar molecules
- RF-dressed fermionic polar molecules in 2D
- Topological $p_x + ip_y$ phase
- p-wave and d-wave pairing in bilayered systems
- Conclusions and outlook

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Trieste, June 18, 2011

Experiments with atomic fermions

Remarkable experiments with two-component s-wave interacting Fermi gases

- s-wave BCS-BEC crossover. Strongly interacting regime
- Remarkably stable weakly bound molecules
- Vortices in the strongly interacting Fermi gas
- Thermodynamics of strongly interacting Fermi gases

What about p-wave interacting fermions?

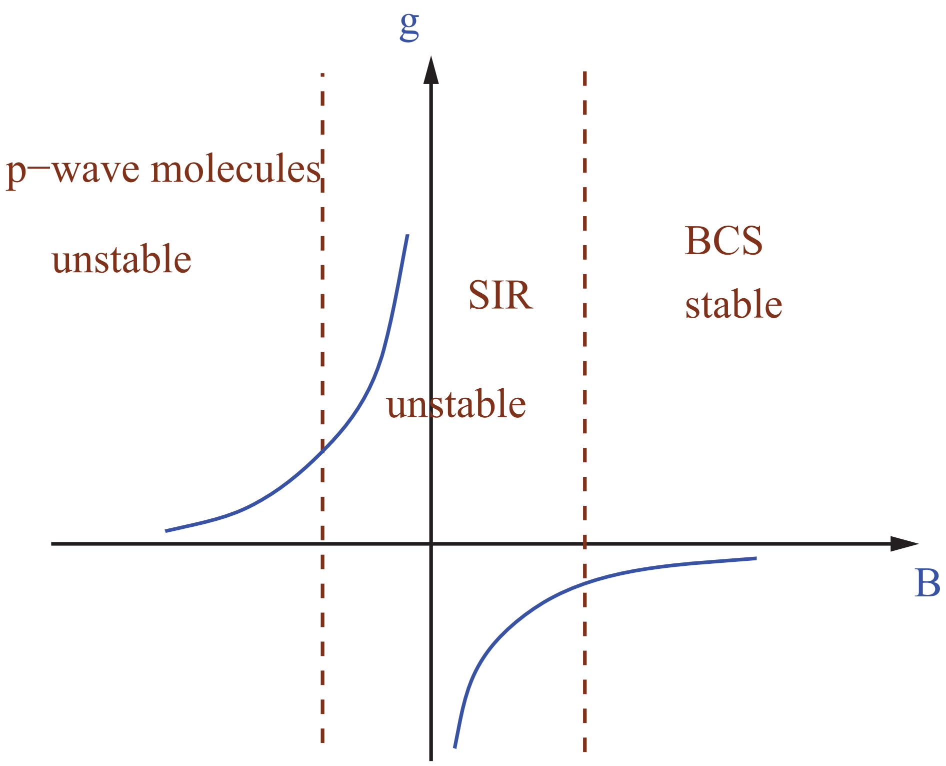
p-wave resonance for fermionic atoms

p-wave resonance   Experiments at JILA, ENS, Melbourne, Tokyo, elsewhere

BCS $\Rightarrow$ $T_c \sim \exp\left(-\frac{1}{(k_F b)^2}\right)$ practically zero

Molecular and strongly interacting regimes $\Rightarrow$ rather high T_c, but collisional instability

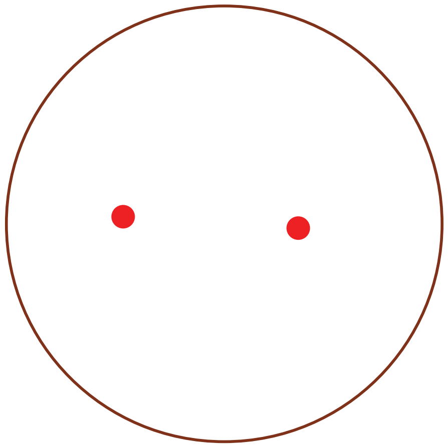
Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio

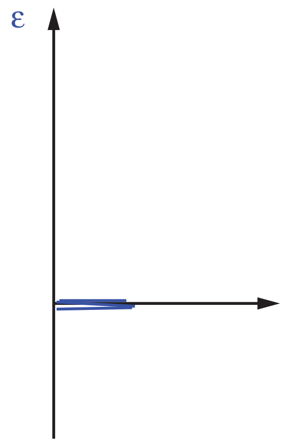


Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)





The number of zero-energy states exponentially grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics $\Rightarrow$ Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

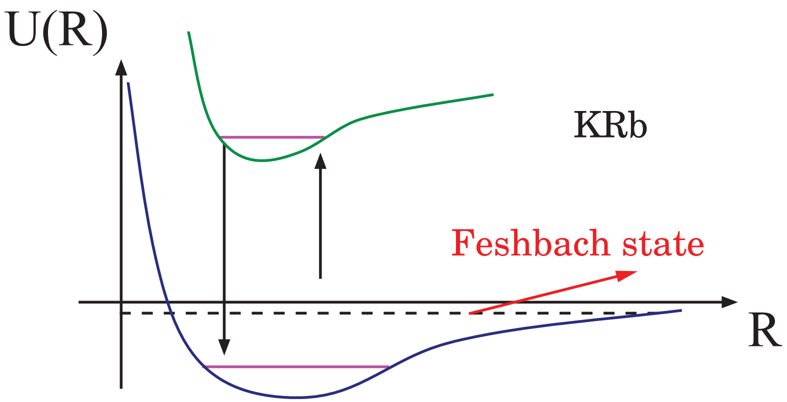
Topologically protected state for quantum information processing

Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb molecules to the ground rovibrational state

JILA, D. Jin, J. Ye groups



$$n \sim 10^{12} - 10^{13}\ \text{cm}^{-3}$$
$$T \approx 200nK \ \sim E_F$$

Ground-state LiCs molecules at Heidelberg at low density

Ultracold chemical reactions $\mathrm{KRb} + \mathrm{KRb} \Rightarrow \mathrm{K_2} + \mathrm{Rb_2}$

New trends in ultracold chemistry

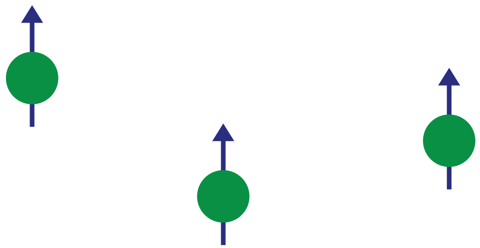
Suppress instability $\rightarrow$ induce intermolecular repulsion

For example, 2D geometry with dipoles perpendicular to the plane

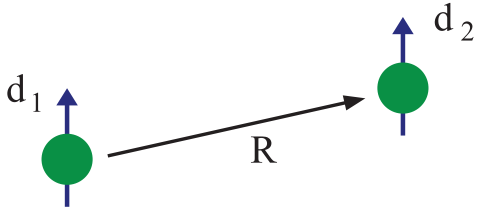
Select non-reactive molecules, like NaK

What are prospects for novel physics ?

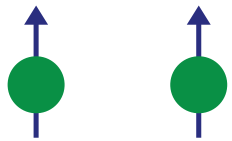
Polar molecules. New object - Dipolar gas



Dipole-dipole interaction $V_d = \frac{\vec{d}_1 \vec{d}_2 R^2 - 3(\vec{d}_1 \vec{R})(\vec{d}_2 \vec{R})}{R^5} \sim \frac{1}{R^3}$



long-range, anisotropic



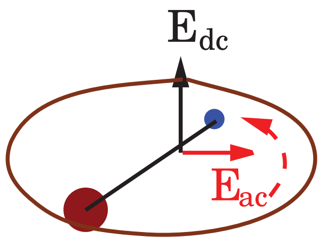
repulsion

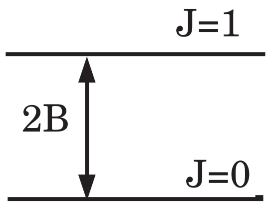
attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules d from $0.6\,D$ for KRb to $5.5\,D$ for LiCs

RF-dressed polar molecules in 2D;Gorshkov et al (2008)





σ^+ polarized RF E_{ac}

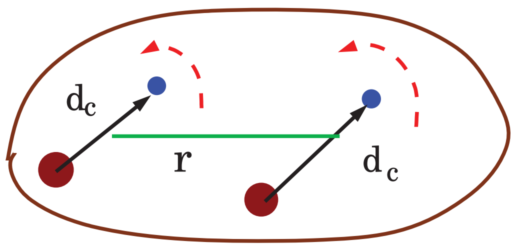
+ constant $E_{dc} \rightarrow 0$

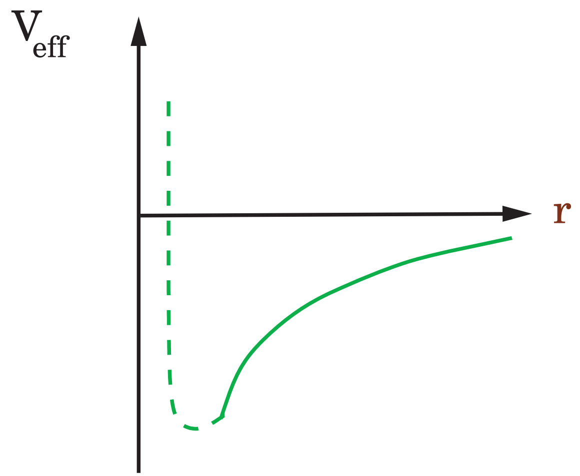
Dressed states $|+\rangle = \alpha|0,0\rangle + \beta|1,1\rangle; \quad ,|-\rangle = \beta|0,0\rangle - \alpha|1,1\rangle$

$$\alpha = -\frac{A}{\sqrt{A^2+\Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2+\Omega^2}}; \quad A = \frac{1}{2}(\delta + \sqrt{\delta^2 + 4\Omega^2})$$

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency

$d_c = 2^{1/2} \alpha \beta d$





Large $r \rightarrow \quad V_{eff} = \langle (1 - 3\cos^2\phi) \rangle \frac{d_c^2}{r^3} = -\frac{d_c^2}{2r^3}; \quad r_* = m d_c^2 / 2\hbar^2$

Fermionic RFD molecules. Superfluid transition; Cooper/G.S. (2009)

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the p-wave scattering ($l = \pm 1$)

$$\hat{H} = \int d^2r\, \hat{\Psi}^\dagger(\mathbf{r})\{-(\hbar^2/2m)\Delta + \int d^2r' \hat{\Psi}^\dagger(\mathbf{r}')V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}') - \mu\}\hat{\Psi}(\mathbf{r})$$

$$\Delta(\mathbf{r}-\mathbf{r}') = \langle V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

Gap equation $\Delta(\mathbf{k}) = -\int \frac{d^2k}{(2\pi)^2} V_{eff}(\mathbf{k}-\mathbf{k}')\Delta(\mathbf{k}')\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}$

$$\epsilon(k) = \sqrt{(\hbar^2k^2/2m - \mu)^2 + |\Delta(k)|^2}; \quad \mu \approx E_F$$

$$T_c \approx E_F \exp(-3\pi/4k_F r_*)$$

$\Delta(\mathbf{k}) = \Delta \exp(i\phi_k)$ $\quad p_x + ip_y$ state ($l = \pm 1$)

Superfluid transition. Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$
This is the case for the atoms

Anomalous scattering in $1/r^3$ potential $\rightarrow$ Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m}(kr_*); \qquad |k| = |k'|$$

$$T_c \propto \exp\left(-\frac{1}{\nu(k_F)|V_{eff}(k_F)|}\right); \qquad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto \exp\left(-\frac{3\pi}{4k_F r_*}\right)$$

Transition temperature

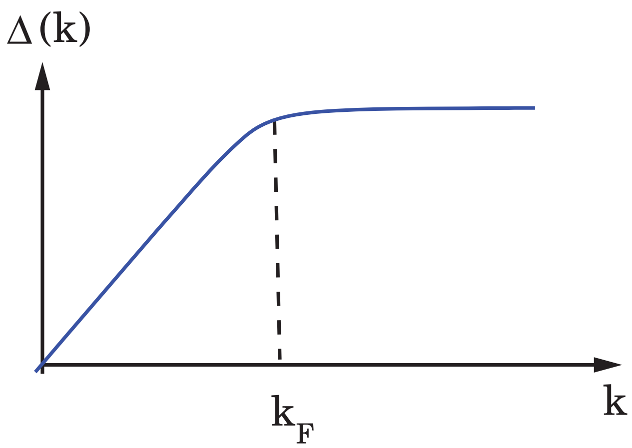
Do better than simple BCS. Reveal the role of short-range physics

Renormalized gap equation

$$\Delta(\mathbf{k}') = -\int f(\mathbf{k}', \mathbf{k})\Delta(\mathbf{k}) \left\{ \frac{\tanh[\epsilon(k)/2T]}{2\epsilon(k)} - \frac{1}{(E_k - E_{k'} - i0)} \right\} \frac{d^2k}{(2\pi)^2}$$

$\Delta(\mathbf{k}) = \Delta(k)\exp(i\phi_k)$; $f(\mathbf{k}', \mathbf{k}) = f(k', k)\exp[i(\phi_k - \phi_{k'})]$ scattering amplitude

$$\Delta(k) = \Delta(k_F) f(k, k_F)/f(k_F, k_F)$$



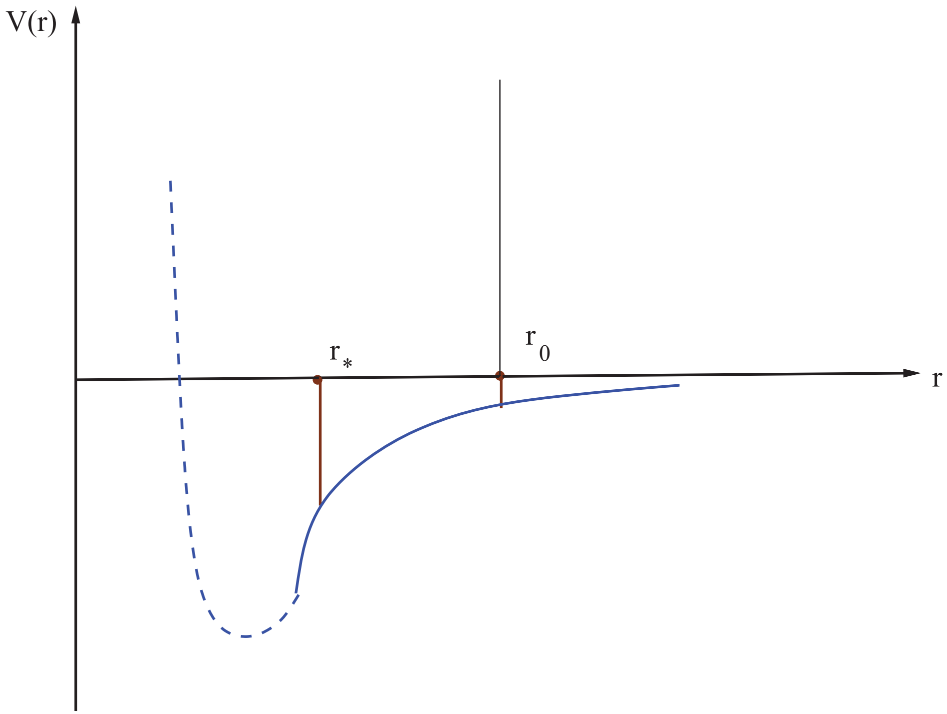
2D scattering in the potential with a $1/r^3$ tail

Scattering amplitude. No transparent exact solution for a finite k

Asymptotic method for slow scattering ($kr_* \ll 1$)

Divide the range of distances into two parts, $r < r_0$ and $r > r_0$

The distance r_0 is such that $r_0 \gg r_*$, but $kr_0 \ll 1$



$r < r_0$ Match exact zero-energy with free finite-k solution at $r = r_0$: $f \Rightarrow (\pi/2)d^2 r_* k^2 \ln k$

$r > r_0$ interaction as perturbation: $f = -(8\pi/3)d^2 k + (\pi/2)d^2 r_* k^2 \ln k$

Related results for the off-shell scattering amplitude

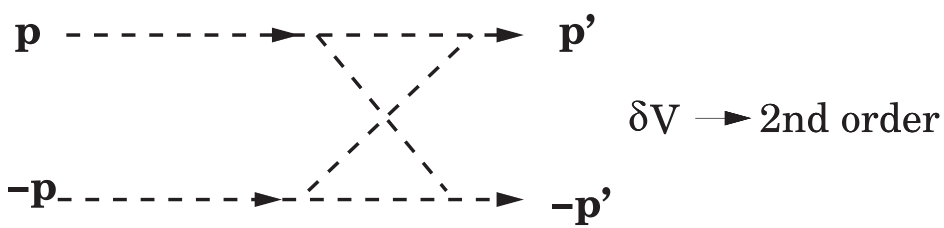
Manipulate T_c?

$$f(k',k) = -\pi d^2 k F\left(\frac{1}{2}, -\frac{1}{2}, 2, \frac{k^2}{k'^2}\right); \quad k \le k'; \;\; k r_* \ll 1$$

Include k^2-term $f = \frac{1}{2}\pi d^2 r_* k^2 \ln[k r_* u]$

$$T_c = \frac{2e^C}{\pi} E_F \exp\left\{-\frac{3\pi}{4k_F r_*} - \frac{9\pi^2}{64}\ln[k_F r_* u]\right\}$$

Take into account second-order Gor'kov-Melik-Barkhudarov processes



$$\Delta(\mathbf{k}) = -\int \frac{d^2k'}{(2\pi)^2} f(\mathbf{k},\mathbf{k}')\left\{\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')} - \frac{1}{2(E_{k'} - E_k)}\right\}\Delta(\mathbf{k}')$$
$$- \int \frac{d^2k'}{(2\pi)^2}\delta V(\mathbf{k},\mathbf{k}')\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}\Delta(\mathbf{k}')$$

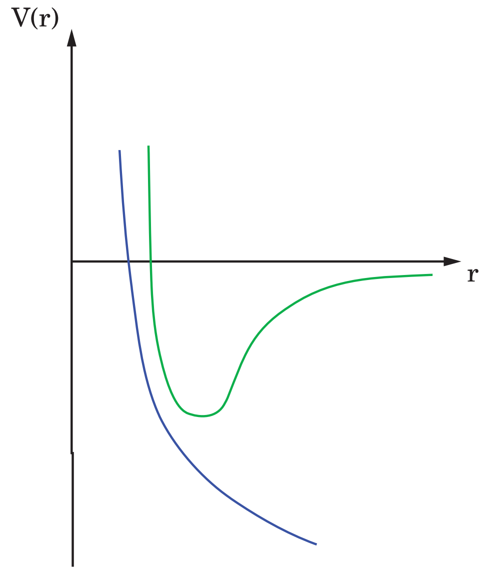
$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp\left\{-\frac{3\pi}{4k_F r_*}\right\}; \quad E_* = \frac{\hbar^2}{2mr_*^2} \gg E_F$$

κ depends on short-range physics and can be varied within 2 orders of magnitude

Collisional stability and T_c

p-wave atomic superfluids: $\mathrm{BCS} \Rightarrow T_c \to 0$ Resonance $\Rightarrow$ collisional instability

Polar molecules $\Rightarrow$ sufficiently large T_c and collisional stability

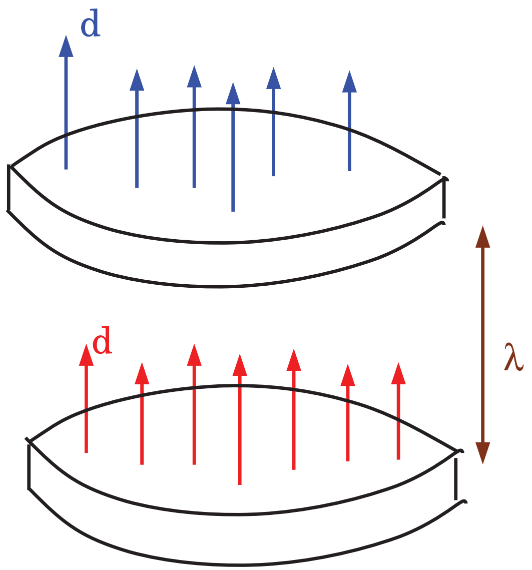


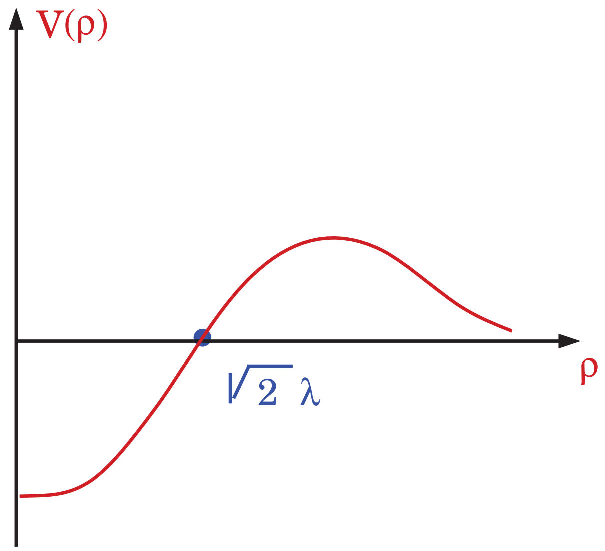
$\alpha_{in} = A \frac{\hbar}{m} (k r_*)^2; \qquad A \Rightarrow \; 10^{-3} - 10^{-4} \qquad \alpha_{in} \to \; (10^{-8} - 10^{-9})$ cm^2/s

LiK molecules $\to$ $d \simeq 3.5$ D $\quad r_* \approx 4000 a_0$

$n = 2 \times 10^8$ cm^{-2} $\Rightarrow$ $E_F = 2\pi\hbar^2 n/m = 120$ nK $\quad T_c \approx 10$ nK; $\quad \tau \sim 2$s

Bilayered dipolar fermionic systems. BCS-BEC crossover





$$V(\rho) = d^2 \left\{ \frac{1}{(\rho^2+\lambda^2)^{3/2}} - \frac{3\lambda^2}{(\rho^2+\lambda^2)^{5/2}} \right\}$$ Always a bound state of $\uparrow$ and $\uparrow$ dipoles

Dipole-dipole length $r_* = md^2/\hbar^2$ Dipole-dipole strength $\beta = r_*/\lambda$.

$$r_* \leq b \Rightarrow \quad \epsilon_b \simeq \frac{\hbar^2}{4mb^2} \exp\left[-\frac{8\lambda^2}{r_*^2}\left(1-\frac{r_*}{\lambda}\right) - (5+2\gamma) \right]$$

Interlayer superfluids

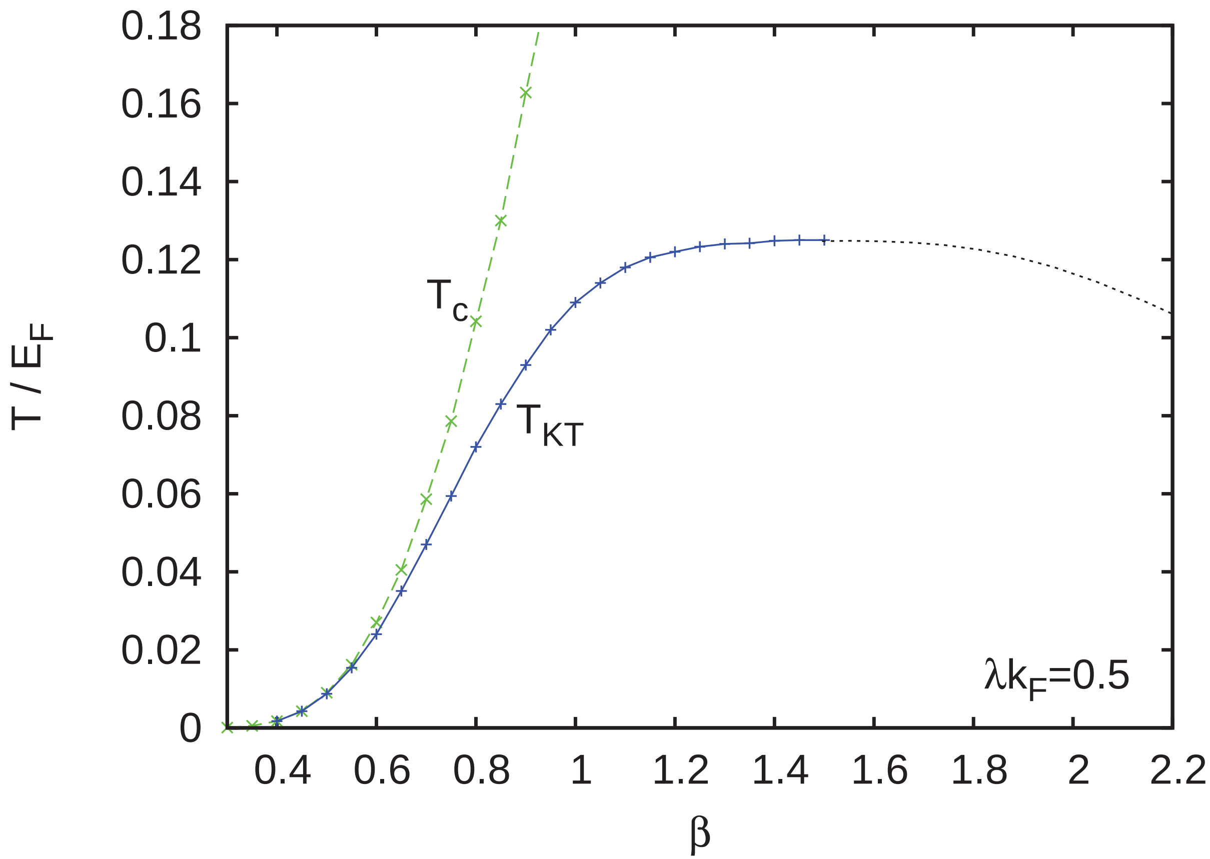
$\epsilon_b \ll E_F \Rightarrow f < 0 \rightarrow$ s-wave BCS pairing

$\epsilon_b \gg E_F \Rightarrow$ Molecules of $\uparrow$ and $\uparrow$ dipoles. Molecular BEC

Transition temperature

New BCS-BEC crossover (Pikovski, Klawunn, Santos, GS)

Innsruck group of P. Zoller



LiCs and KRb molecules $\lambda =\simeq 250$ nm, $n \simeq 5\,10^8$ cm^{-2}, $k_F\lambda \simeq 2$, $E_F \simeq 110$ nk

$\Rightarrow T_{KT}$ of a few nanokelvin

Interlayer superfluids

Multilayer system $\rightarrow$ Harvard group of E. Demler

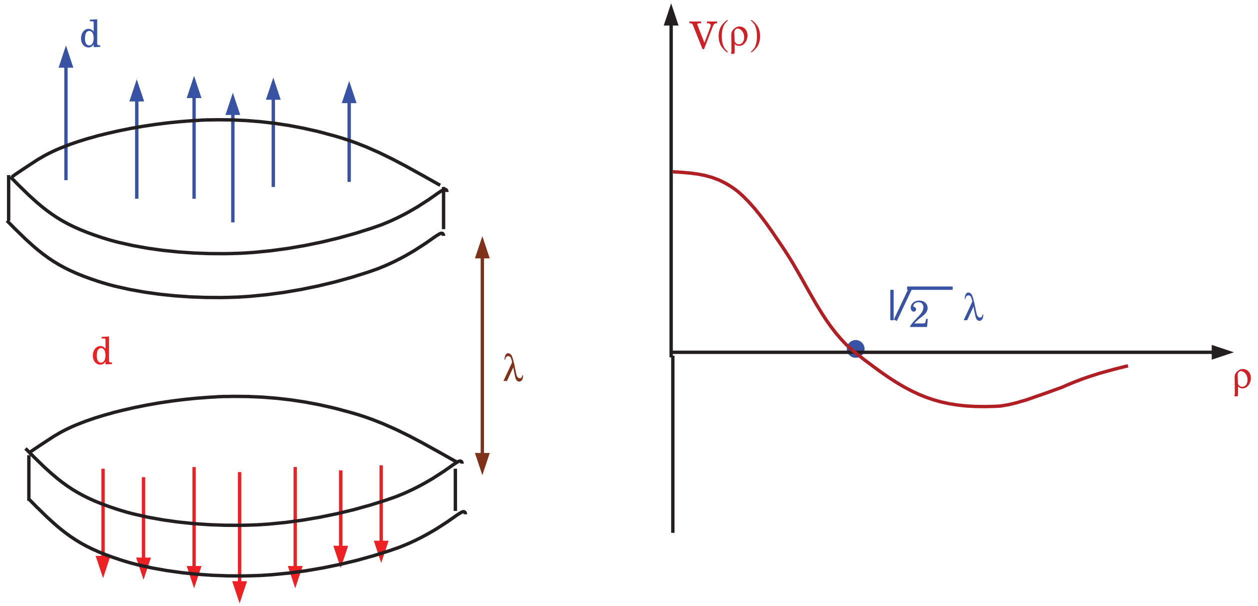
Bilayer systems of $\uparrow$ and $\downarrow$ dipoles

Bilayer system of ↑ and ↓ dipoles

Put $J = 0$ molecules in one layer and $J = 1$ in the other

Apply an electric field perpendicularly to the layers

Slightly non-uniform to prevent resonant dipolar flips leading to a rapid decay



Always a bound state of ↑ and ↓ dipoles

$$\beta \lesssim 1 \Rightarrow \quad \epsilon_b \simeq \frac{\hbar^2}{m\lambda^2} \exp[-8/\beta^2 - 8/\beta - (5 + 2C - 2\ln 2)]$$

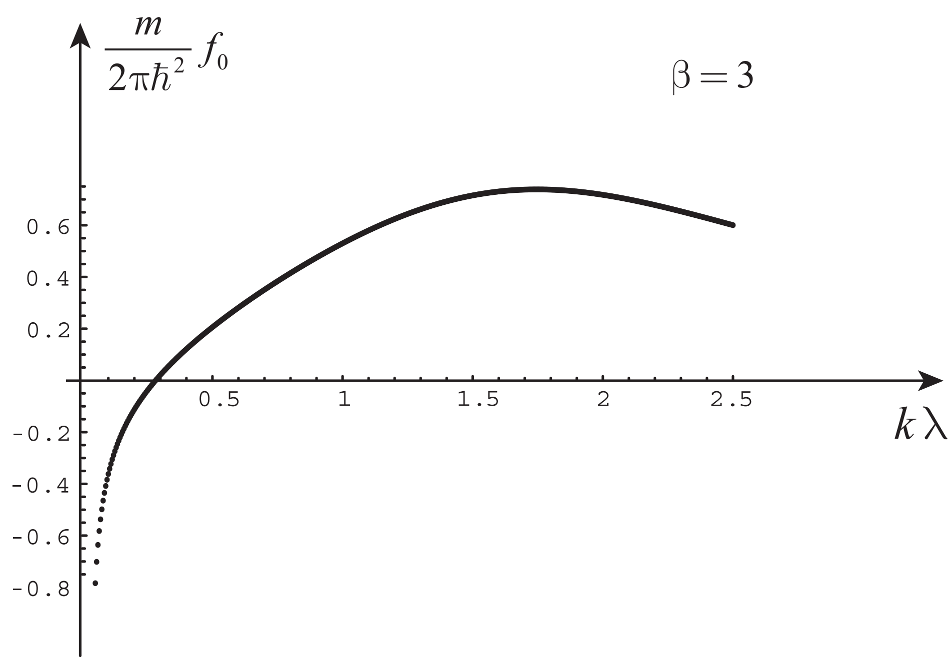
$$\beta = r_*/\lambda$$

Interlayer interaction. Scattering amplitudes

s-wave amplitude $k \to 0$ $\quad f_0(k) = \dfrac{4\pi\hbar^2}{m\ln(\epsilon_b/\epsilon)} + \dfrac{8\hbar^2}{m} k r_*$

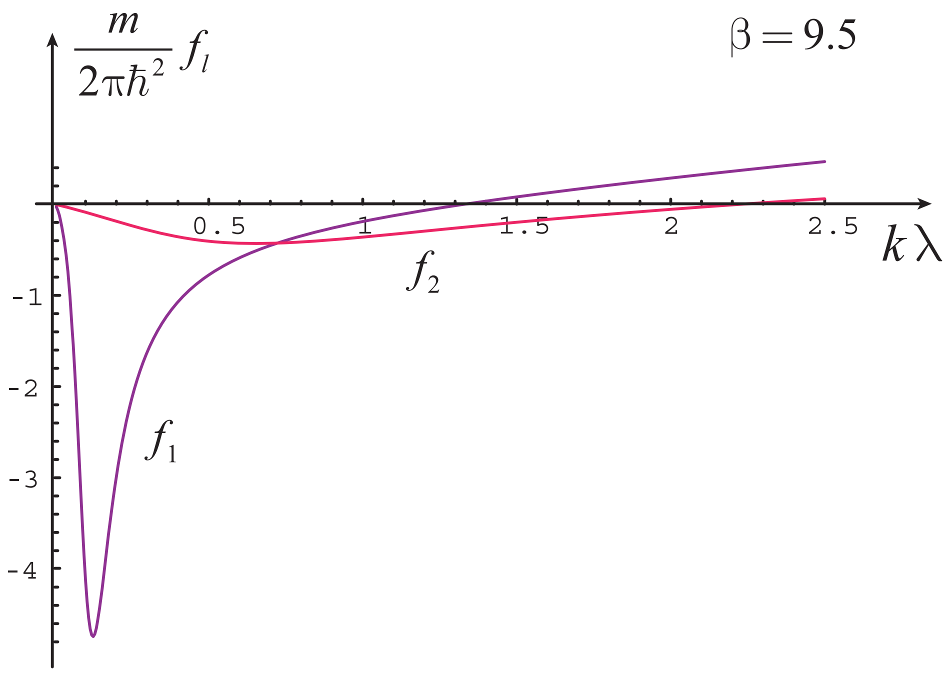
$$\epsilon = \hbar^2 k^2/m \quad r_* = md^2/\hbar^2$$

$f_0 > 0$ for reasonable k. No interlayer superfluid pairing



Interlayer interaction. Scatering amplitudes

p-wave and d-wave amplitudes are < 0



Interlayer p-wave and d-wave pairing

For $k_F r_* \gtrsim 1$ the effective mass significantly decreases

Transition temperature $T_c \sim E_F^* \exp\left(\frac{2\pi\hbar^2}{m_*|f(k_F)|}\right)$

The quasiparticle Fermi energy increases

Compensate the decrease of m_* in the exponent by increasing d^2 and, hence, f

p-wave interlayer superfluid with $T_c \sim$ tens of nK

d-wave superfluids with $T_c \sim$ nK. Analogy with high-temperature superconductors

LiCs with $n > 10^9$ cm^{-2}

Conclusions

Creation of ultracold polar molecules opens

wide avenues to make new quantum states

- $p_x + ip_y$ topological state for identical fermions
- p-wave and d-wave interlayer superfluids in bilayered fermionic dipolar systems