The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA
International Atomic Energy Agency

2253-3

Workshop on Synergies between Field Theory and Exact Computational Methods in Strongly Correlated Quantum Matter

24 - 29 July 2011

Confinement, duality, and topology of gauge fields in CP_{N-1} and QED models in 2+1 dimensions

F.S. Nogueira
Institut fuer Theoretische Physik III
Ruhr-Universitaet Bochum
Bochum
Germany

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

Confinement, duality, and topology of gauge fields in CP^{N-1} and QED models in 2+1 dimensions

Flavio S. Nogueira

Institut für Theoretische Physik III, Ruhr-Universität Bochum, Universitätsstraße 150, 44801 Bochum, Germany

Trieste, 25 July 2011

Quantum antiferromagnets

Magnetic Mott insulator:

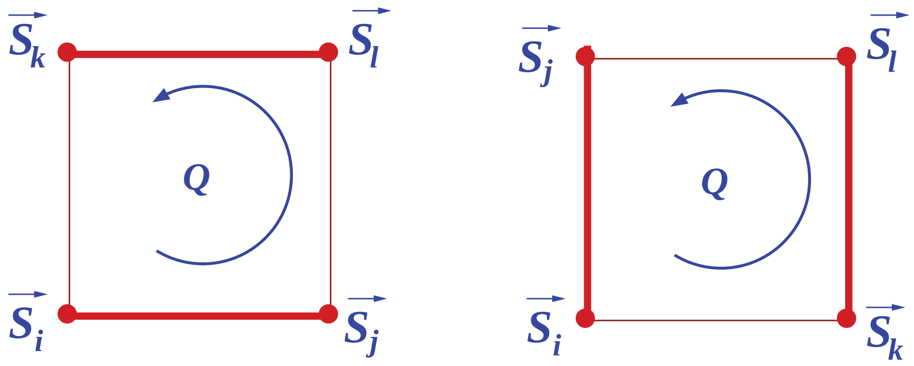
$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + [\mathrm{SU(2)} \quad \mathrm{symmetric} \quad \mathrm{interactions}]$$

Competing interactions lead to competing orders

Example: The $J - Q$ model:

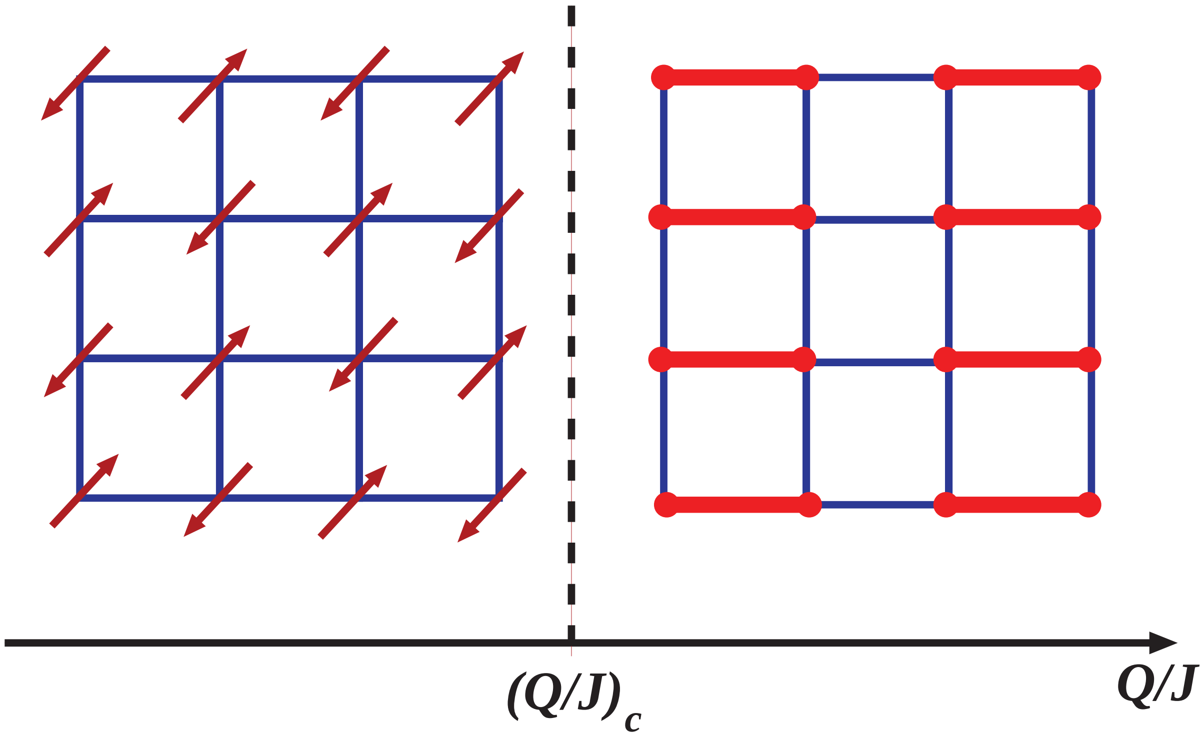
$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - Q \sum_{\langle ijkl \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \right) \left(\vec{S}_k \cdot \vec{S}_l - \frac{1}{4} \right)$$

[A. W. Sandvik, PRL **98**, 227202 (2007)]



Competing orders in Mott insulators

Phases of the $J - Q$ model:



$$|\bullet\!\!-\!\!\bullet\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

CP^1 models in 2+1 dimensions

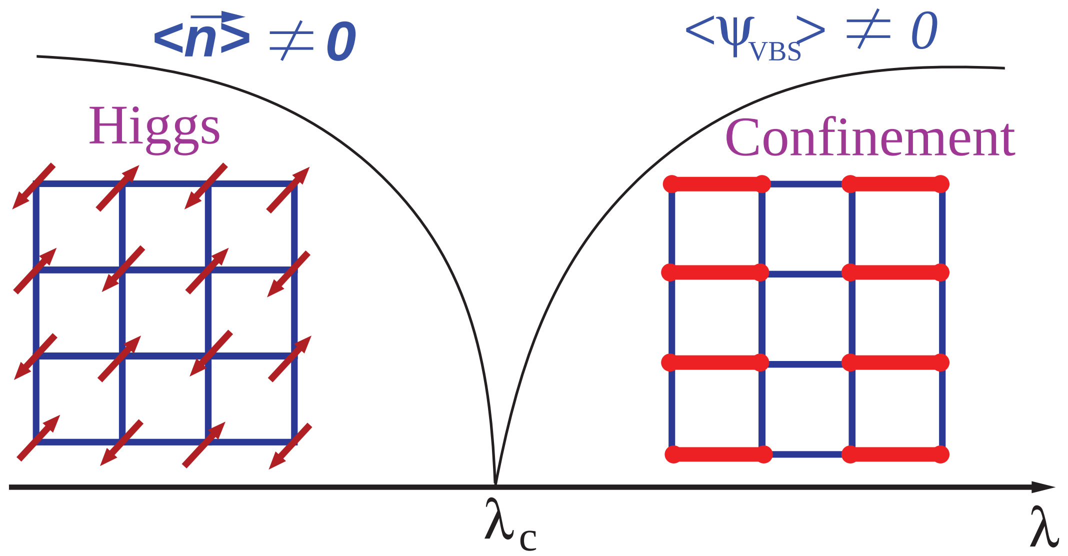
CP^1 map:

$$\vec{n} = z_a^* \vec{\sigma}_{ab} z_b \Longrightarrow n_1^2 + n_2^2 + n_3^2 = 1 \quad \Longrightarrow \quad |z_1|^2 + |z_2|^2 = 1$$

$$\mathcal{L} = \frac{1}{2g}(\partial_\mu \vec{n})^2 \quad \Longrightarrow \quad \mathcal{L} = \frac{2}{g}|(\partial_\mu - iA_\mu) z_a|^2$$

- From the equations of motion: $A_\mu = \frac{i}{2}(z_\alpha^* \partial_\mu z_\alpha - z_\alpha \partial_\mu z_\alpha^*)$
- Topological charge: $\frac{1}{2\pi} \oint_{S_2} dS_\mu \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda = Q \in \mathbb{Z}$
- In terms of $\vec{n}$: $Q = \frac{1}{8\pi} \oint_{S_2} dS_\mu \epsilon_{\mu\nu\lambda} \vec{n} \cdot \partial_\nu \vec{n} \times \partial_\lambda \vec{n}$
- Second-order phase transition $\Longrightarrow$ Quantum critical point

CP^1 models in 2+1 dimensions



($\lambda = Q/J$, for example)

- The spinons z_α constitute both the magnetic and VBS order parameters
- Antiferromagnetic phase (Higgs phase) $\Longrightarrow$ Photon gapped
- VBS phase (Confinement phase) $\Longrightarrow$ Dual photon gapped (i.e., it is $B_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$ that gets gapped)
- If there is a QCP, the anomalous dimension is large.
- $O(3)$ universality class $\Longrightarrow$ $\eta \approx 0.03$

  AF-VBS transition in the $J - Q$ model $\Longrightarrow$ $\eta \approx 0.35$ [Melko and Kaul, PRL **100**, 017203 (2008)]

CP^1 models in 2+1 dimensions

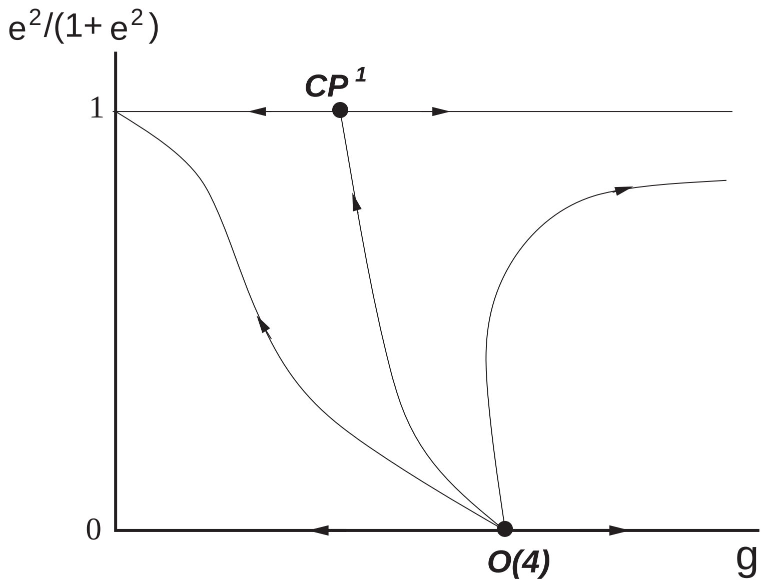
CP^1 + Maxwell (non-compact) [Senthil, Vishwanath, Balents, Sachdev, Fisher, Science, **303**, 1490 (2004)]:

$$\mathcal{L} = \frac{2}{g}|(\partial_\mu - iA_\mu)z_a|^2 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$

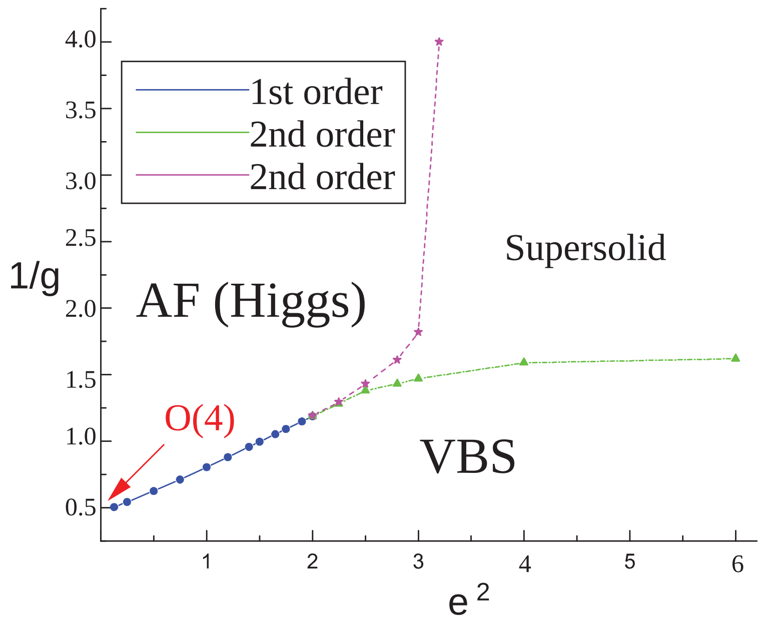
Topological current is not any longer given by $K_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

It can only be written in the form $K_\mu = i\epsilon_{\mu\nu\lambda}\partial_\nu(z_a^*\partial_\lambda z_a)$

RG flow diagram



Numerical phase diagram

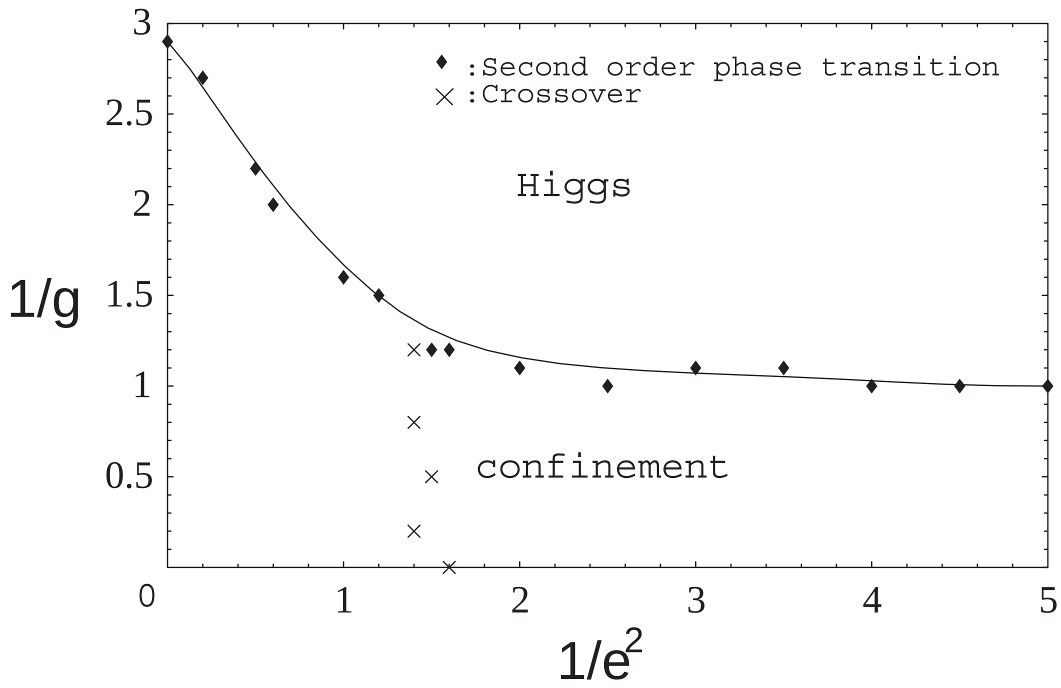


Kuklov et al., PRL **101**, 050405 (2008)

CP^1 models in 2+1 dimensions

CP^1 + Maxwell (compact):

$$S = -\frac{1}{g}\sum_{j,\mu}\sum_{a} z^*_{aj} e^{-iA_{j\mu}} z_{aj+\hat{\mu}} + \text{h.c.} - \frac{1}{e^2}\sum_{j,\mu,\nu,\lambda}\cos(\epsilon_{\mu\nu\lambda}\Delta_\nu A_{j\lambda})$$



[Takashima et al., PRB **72**, 075112 (2005)]

CP^1 models in 2+1 dimensions

CP^1 + Maxwell (compact) + Berry phase:

$$
\begin{aligned}
S &= -\frac{1}{g}\sum_{j,\mu}\sum_{a} z^*_{aj} e^{-iA_{j\mu}} z_{aj+\hat{\mu}} + \text{h.c.} - \frac{1}{e^2}\sum_{j,\mu,\nu,\lambda} \cos(\epsilon_{\mu\nu\lambda}\Delta_\nu A_{j\lambda}) \\
&+ i\sum_j (-1)^j A_{j\tau}
\end{aligned}
$$

[Sachdev and Jalabert, MPL B **4**, 1043 (1990)]

CP^1 + Maxwell (compact) + Berry phase $\Longleftrightarrow$ CP^1 + Maxwell (non-compact)

[T. Senthil et al., Science, **303**, 1490 (2004)]

CP^1 + Maxwell (compact) + Berry phase $\Longrightarrow$ First-order phase transition

CP^1 models in 2+1 dimensions

CP^1 + Maxwell (compact) + Berry phase: the easy-plane case
Set constraint $|z_1|^2 = 1/2$ and $|z_2|^2 = 1/2$

- MC simulation of compact model with Berry phase: Kragset, Smørgrav, Hove, Nogueira, Sudbø, PRL **97**, 247201 (2006) $\Longrightarrow$ *Numerical demonstration of the cancelation between monopoles and Berry phases*

  Numerical demonstration of equivalence CP^1 + Maxwell (compact) + Berry phase $\Longleftrightarrow$ CP^1 + Maxwell (non-compact)

  $\Longrightarrow$ First-order phase transition

- MC simulation of non-compact model via current-loop models (no Berry phase): Kuklov, Prokof'ev, Svistunov, Troyer, AP (N.Y.) 321, 1602 (2006) $\Longrightarrow$ First-order phase transition

- All non-compact CP^1 + Maxwell models in 2+1 dimensions undergo a first-order phase transition

- Numerical evidence for second-order phase transition in compact CP^1 + Maxwell in 2+1 dimensions

Compact Maxwell theory in 2+1 dimensions

$U(1)$ lattice Maxwell theory in $d = 2+1$ and $d = 3+1$ [Banks, Myerson, and Kogut, NPB 129, 493 (1977); Polyakov, NPB 120, 429 (1977); Peskin, AoP 113, 122 (1978)]

$$S = -\frac{1}{e^2} \sum_{i,\mu,\nu} \cos(F_{i\mu\nu})$$

- Dual theory: field theory for the topological excitations instead for particles
- Topological mechanism for mass generation
- Dual theory in 2+1 dimensions (permanently confining, no phase transition):

$$\mathcal{L} = \frac{e^2}{8\pi^2} (\partial_\mu \varphi)^2 - z \cos \varphi$$

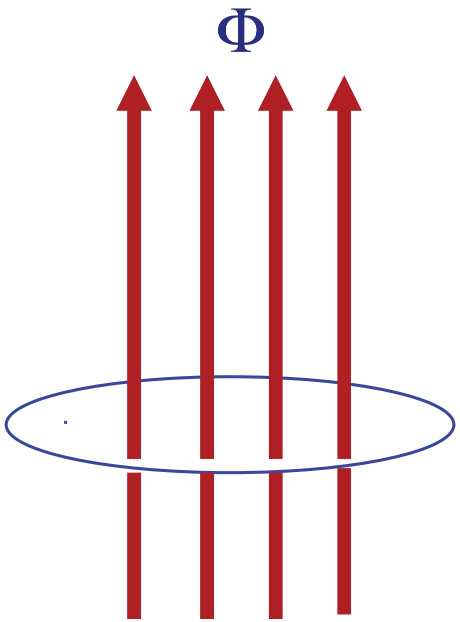
- Dual theory in 3+1 dimensions (deconfinement first order phase transition): non-compact Abelian Higgs model

Compact Maxwell theory in 2+1 dimensions

What does compact $U(1)$ in continuum limit means?

Flux quantization (open surface)

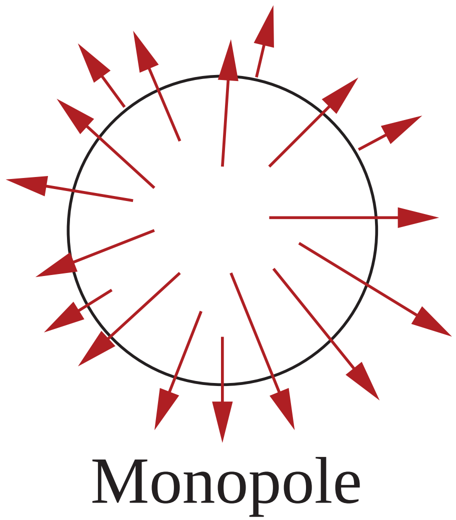
$$\int_S d\vec{S} \cdot (\nabla \times \vec{A}) = 2\pi n/e$$



Example: Superconductor, Abrikosov vortices

Flux quantization (closed surface)

$$\oint_S d\vec{S} \cdot (\nabla \times \vec{A}) = 2\pi n/e$$



Example: Magnetic monopoles

Compact Maxwell theory in 2+1 dimensions

[Nogueira and Sudbø, 07/2011]

- A continuum version of compact QED:

$$\mathcal{L}_M = \frac{1}{2e^2}\left[\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda - \frac{1}{k}\epsilon_{\mu\nu\lambda}\vec{n}\cdot(\partial_\nu\vec{n}\times\partial_\lambda\vec{n})\right]^2$$

a_μ non-compact, k is an integer (typically 1,2, or 4)

- Can be dualized directly in the continuum. Dual model:

$$\tilde{\mathcal{L}} = \frac{e^2}{8\pi^2}(\partial_\mu\varphi)^2 - z\cos(4\varphi/k)$$

$\Longrightarrow k = 1$ is relevant case for VBS phase

- Incorporating AF quantum fluctuations:

$$\mathcal{L} = \mathcal{L}_M|_{k=1} + \frac{1}{2g}(\partial_\mu\vec{n})^2$$

A compact CP^1 + Maxwell model

[Nogueira and Sudbø, 07/2011]

- CP^1 map and gauging away a_μ:

$$\mathcal{L} = \frac{2}{g}|(\partial_\mu - iA_\mu)z_a|^2 - \frac{2}{e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu z_a^*\partial_\lambda z_a)^2$$

- Rewriting back in terms of $\vec{n}$:

$$\mathcal{L} = \frac{1}{2g}(\partial_\mu\vec{n})^2 + \frac{1}{2e^2}[(\partial_\mu\vec{n})^2(\partial_\nu\vec{n})^2 - (\partial_\mu\vec{n}\cdot\partial_\nu\vec{n})^2]$$

$\Longrightarrow$ Faddeev-Skyrme model

- Continuum version of $J - Q$ model
- Conserved current:

$$\mathbf{J}_\mu = \mathbf{n}\times\left[\frac{1}{g}\partial_\mu\mathbf{n} + \frac{2}{e^2}(\partial_\alpha\mathbf{n}\cdot\partial_\nu\mathbf{n})(\delta_{\alpha\nu}\partial_\mu\mathbf{n} - \delta_{\alpha\mu}\partial_\nu\mathbf{n})\right]$$

A compact CP^1 + Maxwell model

[Nogueira and Sudbø, 07/2011]
Spin stiffness: $\rho_s \sim \int d^d x \langle \mathbf{J}_\mu(x) \cdot \mathbf{J}_\mu(0) \rangle$
$d = D + 1$

- $ge^2 \sim m^{6-2d}$, $m = \xi^{-1} \Longrightarrow ge^2 \sim 1/\ln(m/\Lambda)$ for $d = 2 + 1$
- Spin stiffness:

$$\rho_s \approx c_1 m^{d-2} + \frac{c_2}{2d-6} \left[\left(\frac{m}{\Lambda} \right)^{2d-6} - 1 \right] m^{4-d}$$

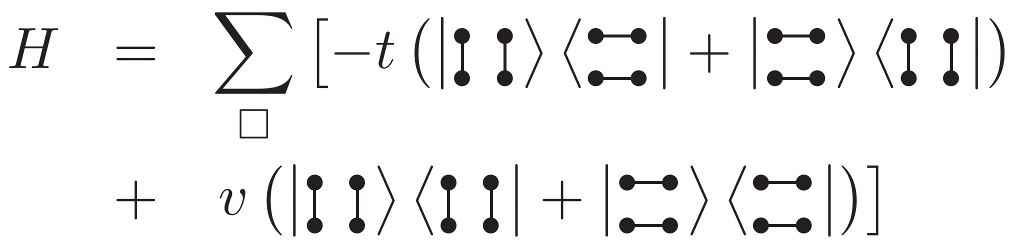
- For $d = 2 + 1$:

$$\rho_s \sim m \ln(m/\Lambda)$$

- Log scaling behavior in the $J - Q$ model: Sandvik, PRL **104**, 177201 (2010); Banerjee, Damle, and Alet, PRB **82**, 155139 (2010); Sandvik, Kotov, Sushkov, PRL **106**, 207203 (2011)

Gauge theory of quantum dimer model

- Hamiltonian:



- Lattice gauge theory action (valid both in $d = 2+1$ and $d = 3+1$):

$$S = -\sum_{i,\tau,n} \ln\left[\cos(s\nabla_\tau A_{in})\right] + \sum_{i,\tau,m,n}\left[-\frac{t}{2}\cos(F_{imn}) + \frac{v}{8}\cos(2F_{imn})\right].$$

($F_{imn} = \nabla_m A_{in} - \nabla_n A_{im}$)
[F. S. Nogueira and Z. Nussinov, PRB **80**, 104413 (2009)]

Gauge theory of quantum dimer model

- Dual model ($d = 2 + 1$, $\rho = t - v$):

$$\mathcal{L} = \frac{c}{2}(\partial_\tau h)^2 + \frac{\rho}{2}(\nabla h)^2 + \frac{1}{2K}(\nabla^2 h)^2 - z\cos(2\pi h).$$

  Kosterlitz-Thouless-like phase transition at $T = 0$ and for $\rho = 0$ ($t = v$); no transition for $T > 0$ and $\rho = 0$. VBS state for $\rho > 0$ and staggered VBS for $\rho < 0$; KT transition for $T > 0$ and $\rho > 0$

- Dual model ($d = 3 + 1$) at the Rokhsar-Kivelson ($t = v$) point:

$$\tilde{\mathcal{L}} = \frac{K}{2}(\partial_\tau \mathbf{a})^2 + \frac{1}{2c}(\nabla \times \nabla \times \mathbf{a})^2 + |(\nabla - 2\pi i \mathbf{a})\psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4$$

  First-order phase transition at $T = 0$; Second-order transition for $T > 0$

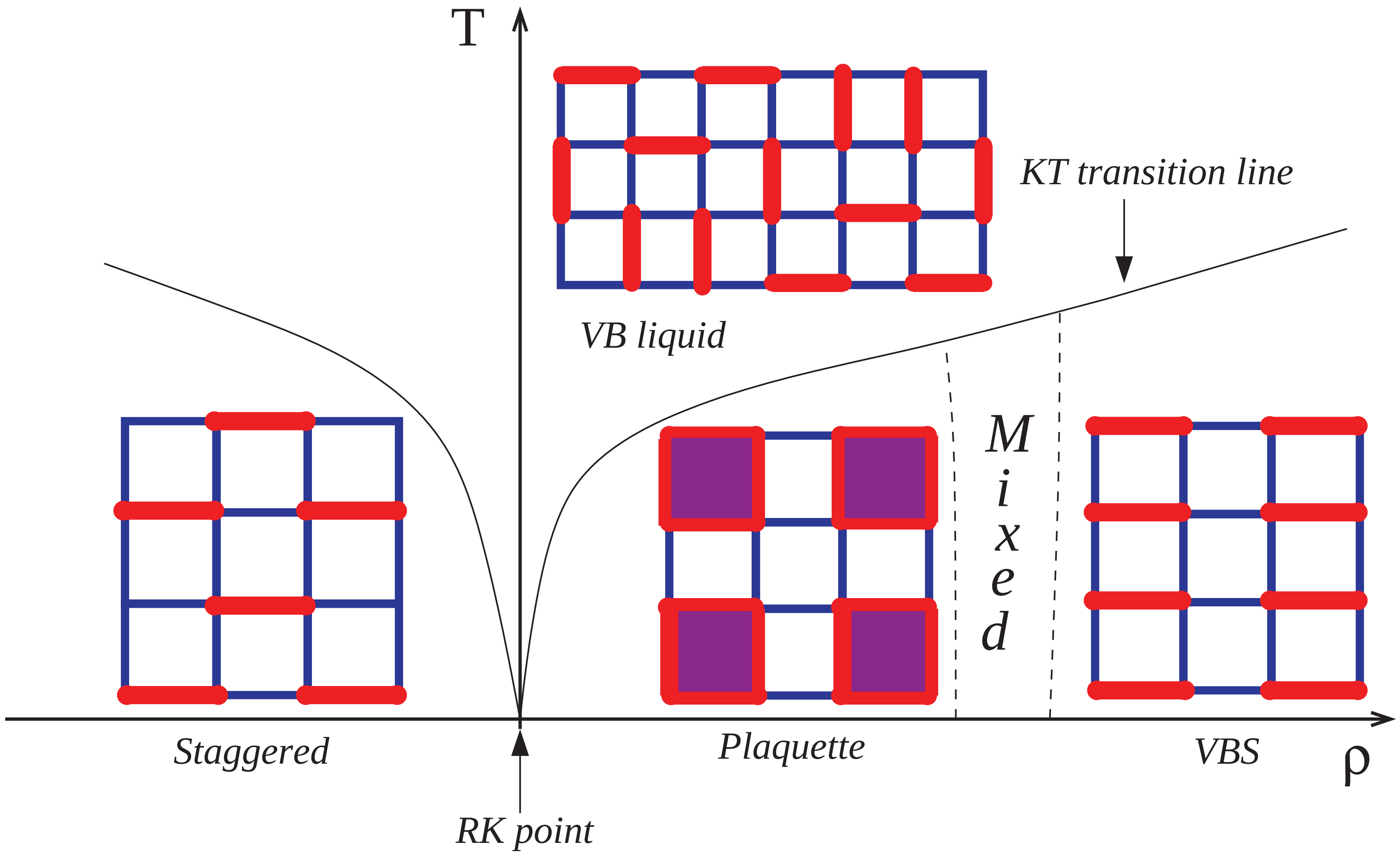
- Dual model ($d = 3 + 1$) above the RK point ($t > v$):

$$\tilde{\mathcal{L}}_{\rho>0} = \frac{1}{2}(\partial_\tau \mathbf{a})^2 + \frac{\rho}{2c}(\nabla \times \mathbf{a})^2 + |(\nabla - 2\pi i\sqrt{\rho}\,\mathbf{a})\psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4$$

  $T = 0$: First-order transition; $T > 0$: Second-order transition

Gauge theory of quantum dimer model

Phase diagram for $d = 2 + 1$ ($\rho = t - v$):



Conclusions

- Exotic Mott insulating states are sometimes well described by lattice gauge theories featuring confining and Higgs phases
- Mott insulators featuring AF and VBS order parameters belong to a completely new universality class featuring large anomalous dimension and logarithmic deviations of scaling
- We have shown a continuum version of compact CP^1 model which is equivalent to Faddeev-Skyrme model: field theory for the $J - Q$ model
- It would be interesting to perform MC simulations of the Faddeev-Skyrme model and to compare the results with those from the $J - Q$ model