



The Abdus Salam
International Centre for Theoretical Physics



2253-8

**Workshop on Synergies between Field Theory and Exact Computational
Methods in Strongly Correlated Quantum Matter**

24 - 29 July 2011

Anomalous Spin Excitations in La₂CuO₄ and La_{2-x}Sr_xCuO₄

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University of
BRISTOL



Anomalous Spin Excitations in La_2CuO_4 and $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

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Collaborators

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Toby Perring (ISIS)
Chris Frost (ISIS)

Lipscombe, Vignolle, Perring, Frost, Hayden Phys. Rev. Lett. **102**, 167002 (2009)
Headings, Hayden, Coldea, Perring, Phys. Rev. Lett. **105**, 247001 (2010)

Anomalous Spin Excitations in La_2CuO_4 and $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

1) La_2CuO_4

- Introduction and motivation
- Experimental
- Spin Excitations in La_2CuO_4
- Comparison with theory
- Other Experiments
- Discussion

2) The pseudogap and $\text{La}_{1.91}\text{Sr}_{0.09}\text{CuO}_4$ (UD22)

Mechanism of high temperature superconductivity

1) The ‘holy’ trinity: a 1950’s paradigm

- Local Density Approximation (LDA – DFT) band structure
- Random Phase Approximation (RPA) spin fluctuation superglue
- Eliashberg equations for the superconductivity

Structured excitations e.g. B. Vignolle, Nat. Phys. (2007); S. M. Hayden, et al., Nature (2004), C. Stock, et al., PRB (2005); V. Hinkov et al., Nat. Phys. (2007)

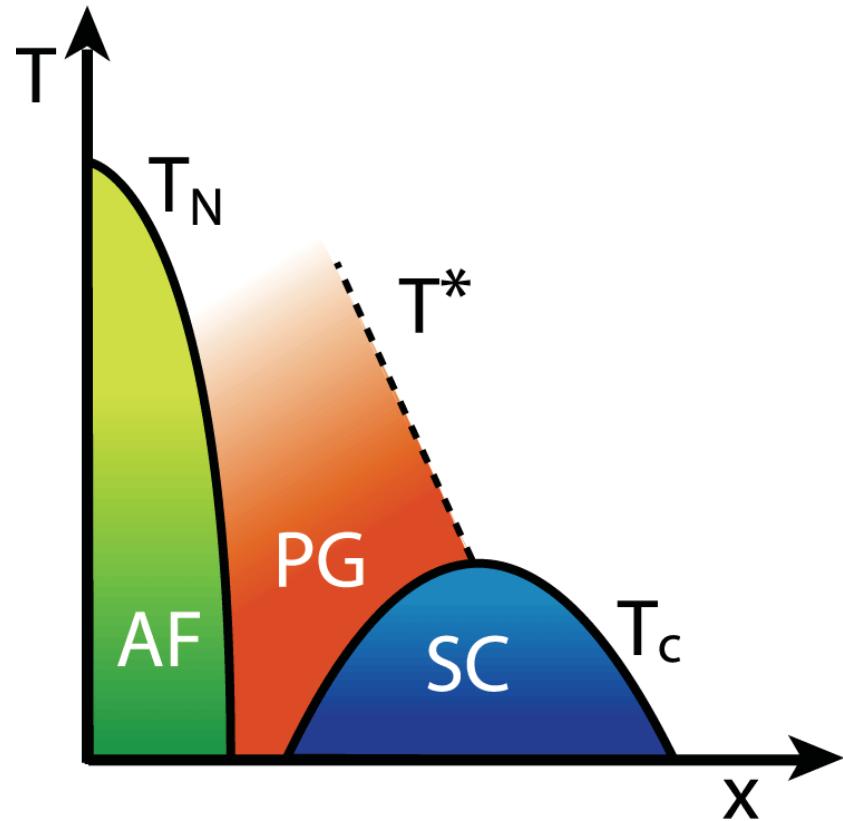
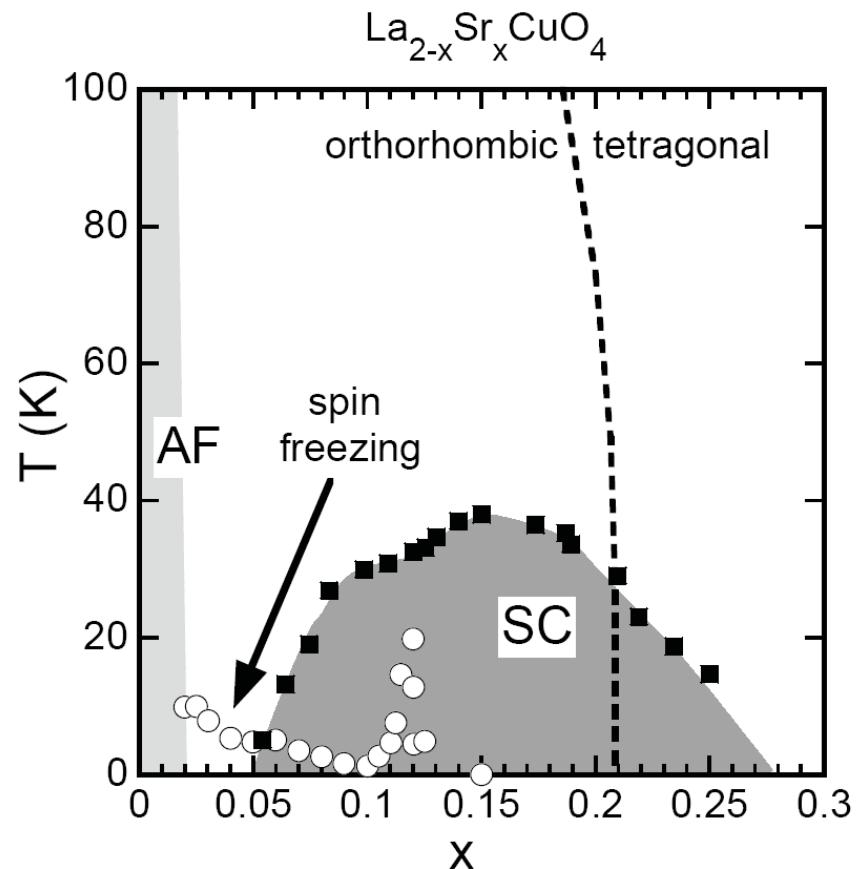
2) The RVB picture: doped Mott insulator

3) Quantum criticality

Competing phase e.g B. Fauque et al., PRL 96, 197001 (2006)

‘A modern, but way too short history of the theory of superconductivity at high temperature’ (Jan Zaanen 2010)

Generic phase diagram of cuprates



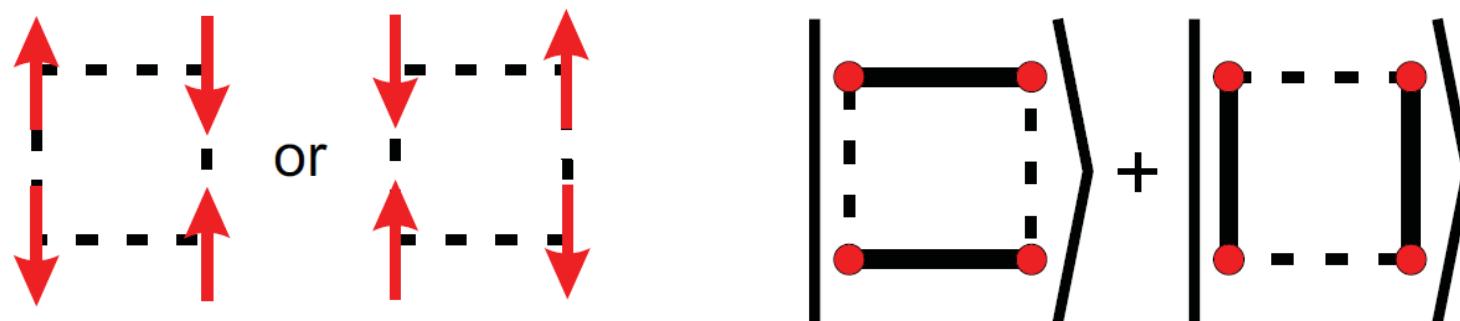
Resonating Valence Bond State in La_2CuO_4

The Resonating Valence Bond State in La_2CuO_4 and Superconductivity

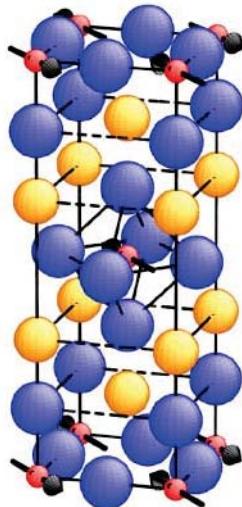
P. W. Anderson,
Science 235, 1196 (1987)

P. W. ANDERSON

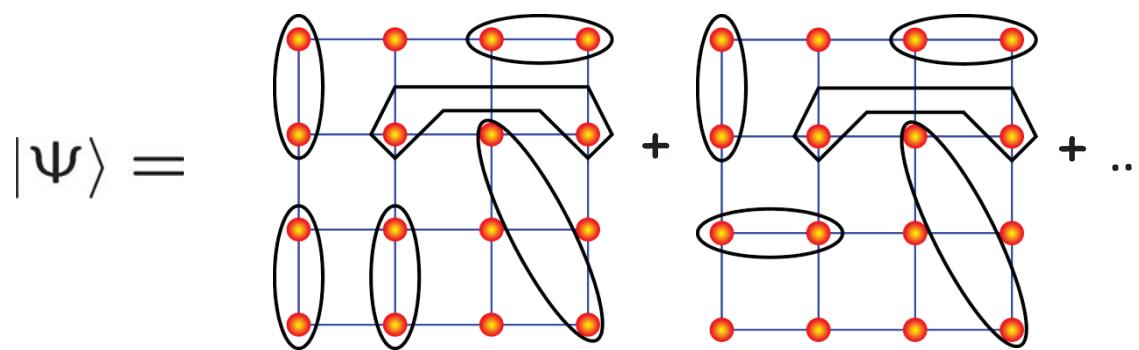
The oxide superconductors, particularly those recently discovered that are based on La_2CuO_4 , have a set of peculiarities that suggest a common, unique mechanism: they tend in every case to occur near a metal-insulator transition into an odd-electron insulator with peculiar magnetic properties. This insulating phase is proposed to be the long-sought “resonating-valence-bond” state or “quantum spin liquid” hypothesized in 1973. This insulating magnetic phase is favored by low spin, low dimensionality, and magnetic frustration. The preexisting magnetic singlet pairs of the insulating state become charged superconducting pairs when the insulator is doped sufficiently strongly. The mechanism for superconductivity is hence predominantly electronic and magnetic, although weak phonon interactions may favor the state. Many unusual properties are predicted, especially of the insulating state.



Resonating valence bond theory in 2D



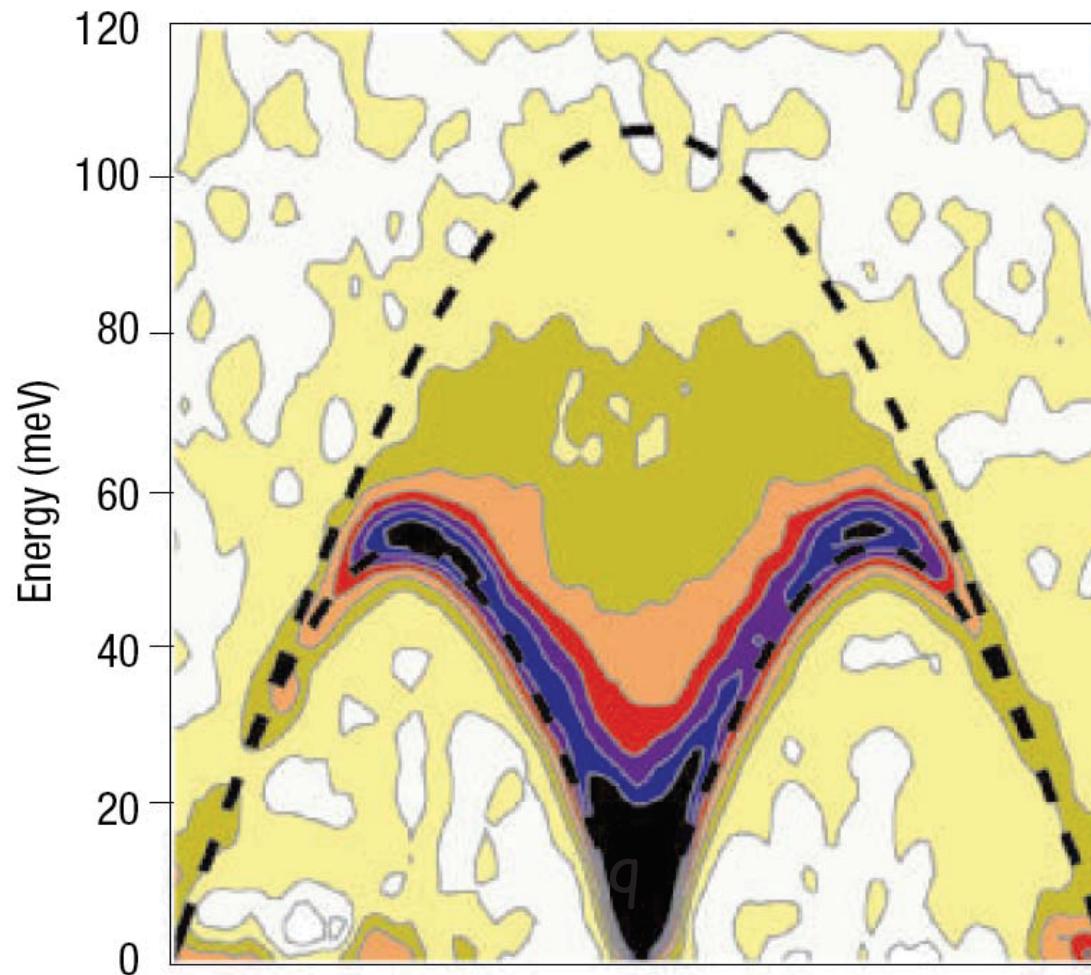
- Spin “liquid” of singlets $\frac{1}{\sqrt{2}} \{ |\uparrow, \downarrow \rangle - |\downarrow, \uparrow \rangle \}$



- Broken singlet “releases” 2 spinons

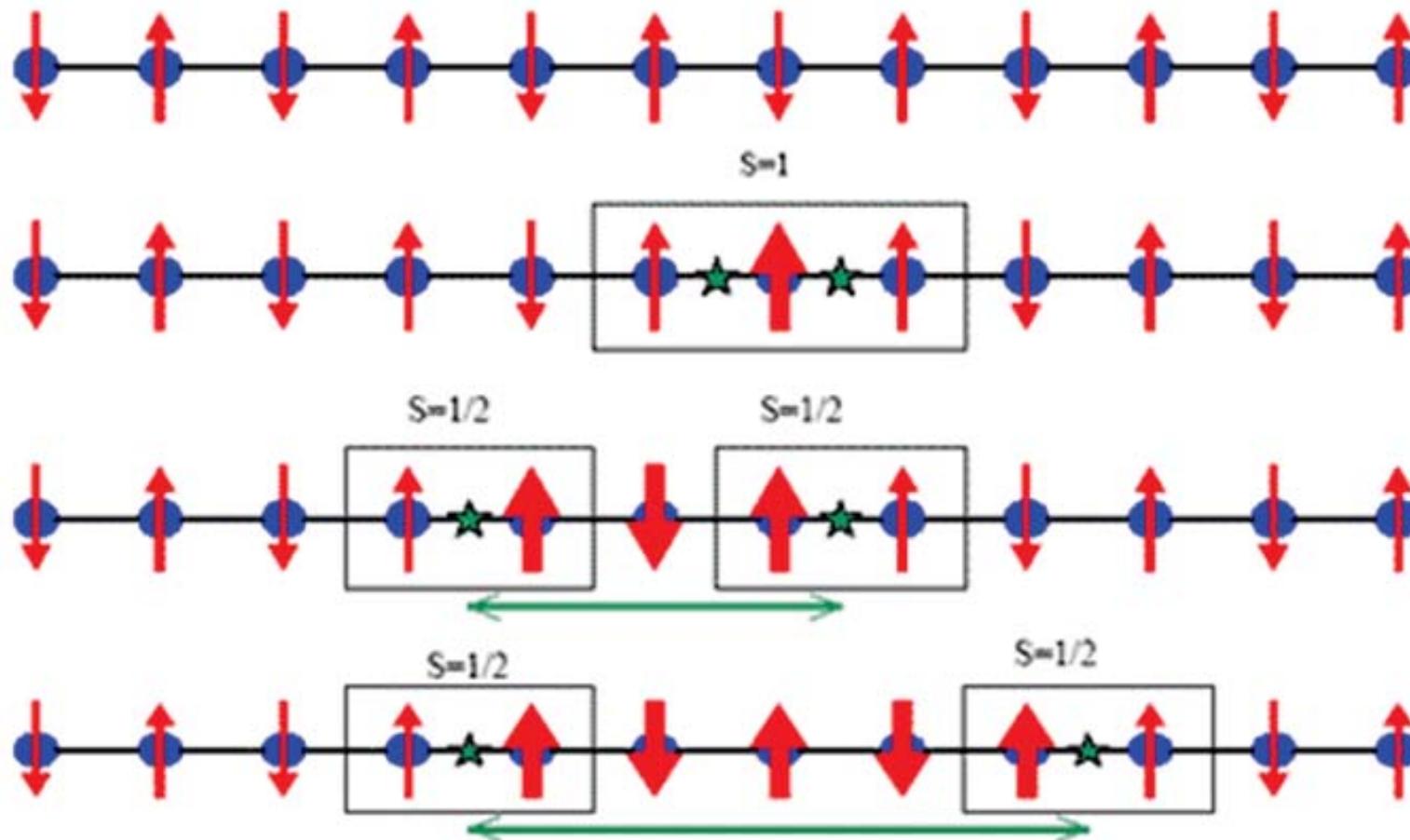
Spinons in 1D antiferromagnets

KCuF_3 $S=1/2$ 1D antiferromagnetic chain



Lake et al., Nature Materials 4, 329 (2005)

Spinons in 1D Heisenberg chain



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ISIS Spallation Source

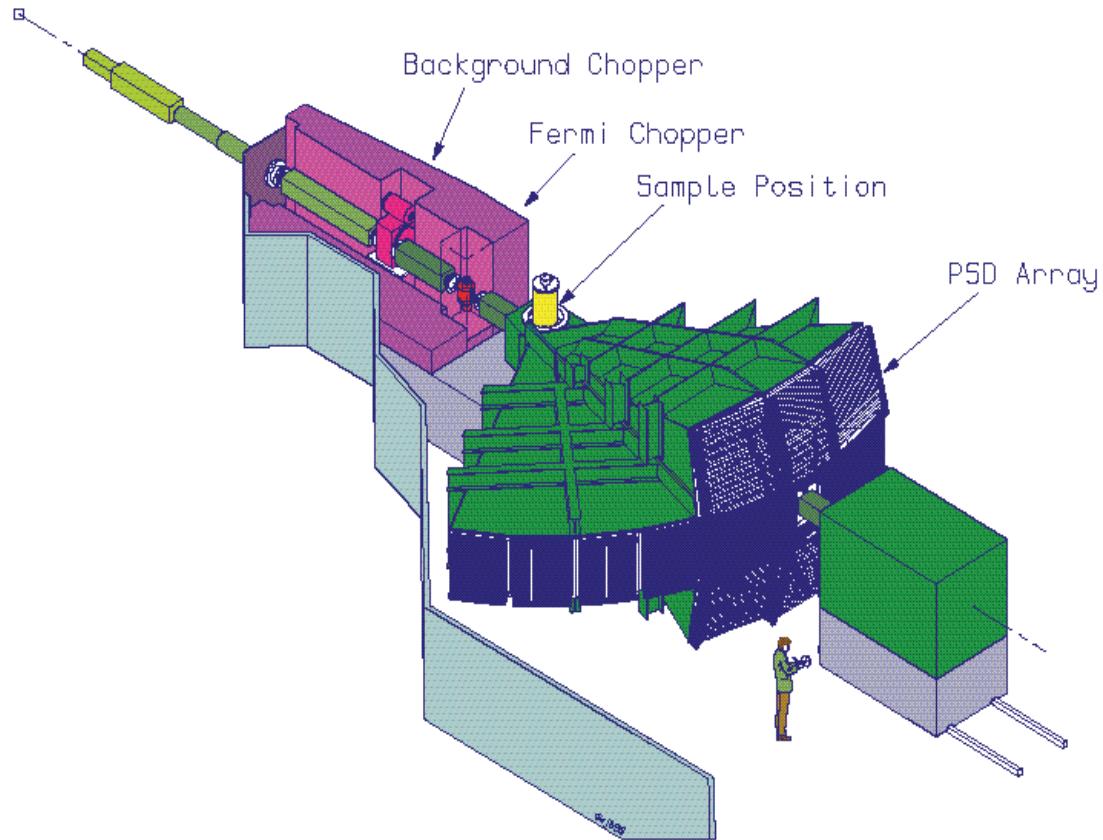


Inelastic neutron scattering (INS)

$$\frac{d^2\sigma}{d\Omega \, dE} = (\gamma r_e)^2 \frac{k_f}{k_i} |F(\mathbf{Q})|^2 \left(\frac{2/\pi g^2 \mu_B^2}{1 - \exp(-\hbar\omega/kT)} \right) \chi''(\mathbf{Q}, \omega).$$



47.8g La_2CuO_4



MAPS spectrometer

Neutron cross section

$$\sigma_{xx}^{\uparrow\downarrow} = \left(\frac{d^2\sigma}{d\Omega dE} \right)_{\mathbf{H}\parallel x}^{\uparrow\rightarrow\downarrow} = \frac{k_f}{k_i} \frac{(\gamma r_e)^2}{g^2 \mu_B^2} \frac{1}{\pi} F^2(\mathbf{Q})$$
$$\times \frac{\chi''_{yy}(\mathbf{q}, \hbar\omega) + \chi''_{zz}(\mathbf{q}, \hbar\omega)}{1 - \exp(-\hbar\omega/kT)}$$

$\mathbf{x} \parallel \mathbf{Q}$

Neutron cross-section well known

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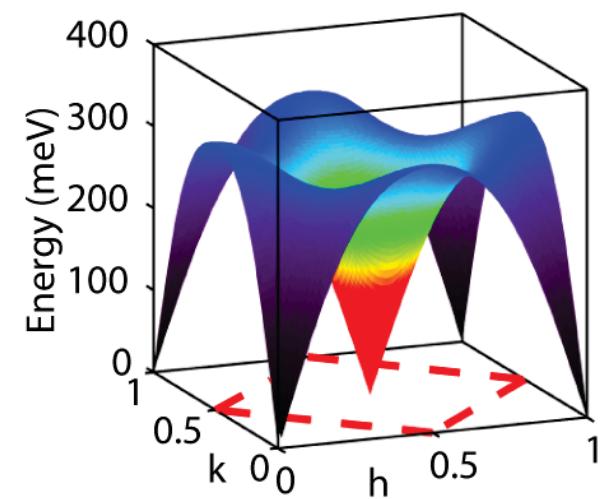
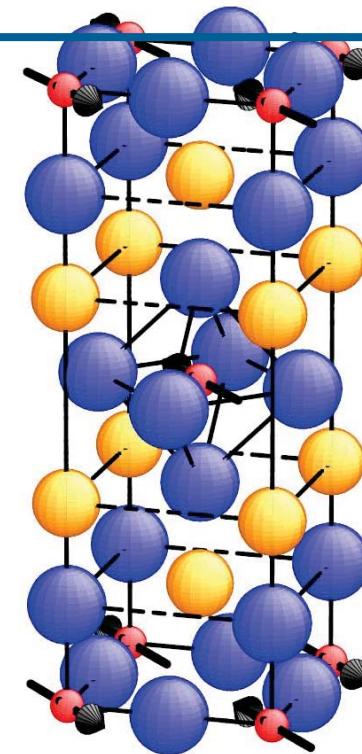
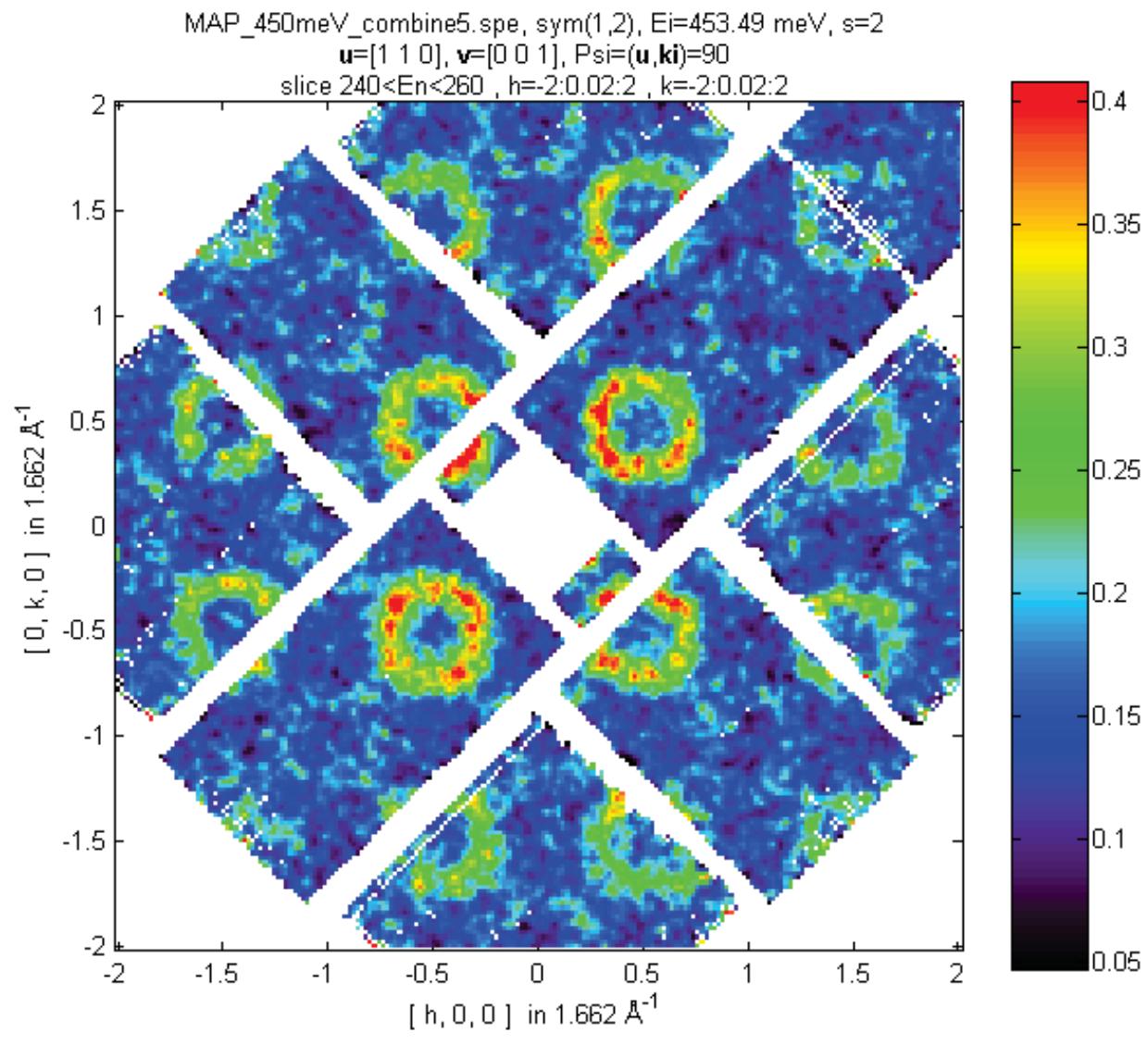
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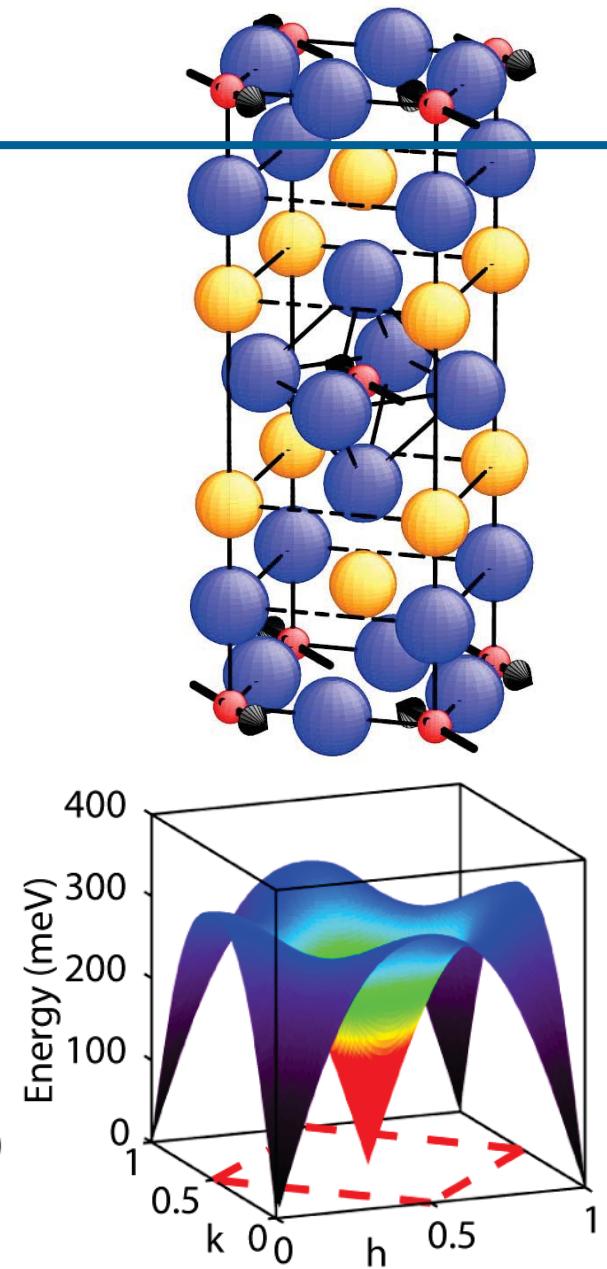
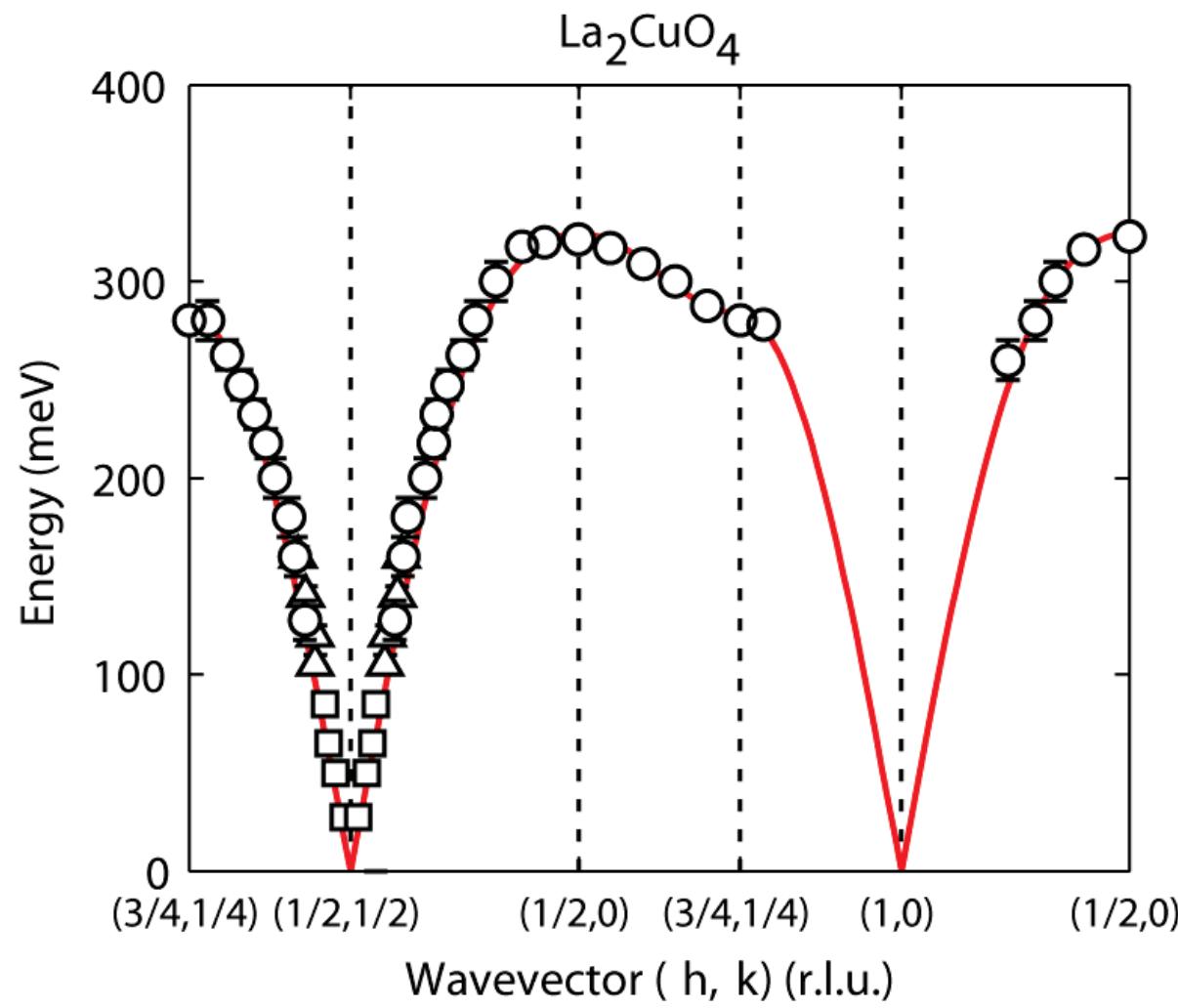
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Spin waves in La_2CuO_4 : Example MAPS data

$E=250 \text{ meV}$



Spin waves in La_2CuO_4



Coldea, Hayden, Aeppli, Perring, Frost, Mason, Cheong, Fisk, PRL **86** 5377 (2001).
Headings, Hayden, Coldea, Perring, Phys. Rev. Lett. **105**, 247001 (2010)

Spin wave dispersion

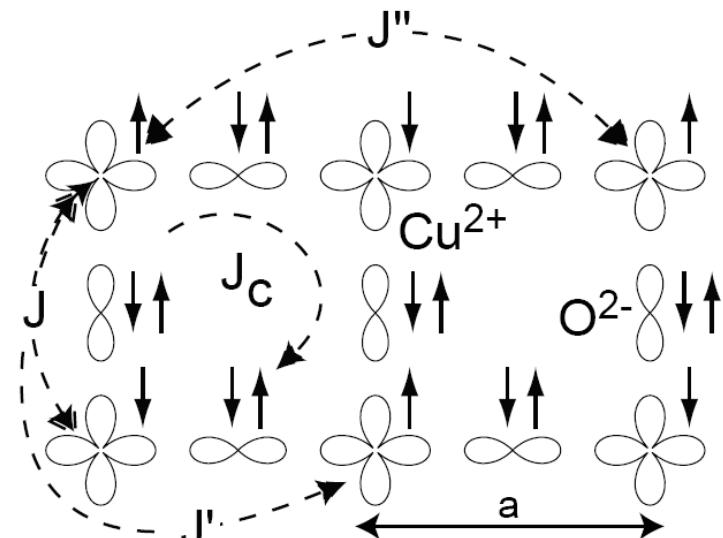
SW dispersion along BZ (3/4,1/4) to (1/2,0) subject to two competing effects):

- 1) Quantum fluctuations (beyond SWT) raise (3/4,1/4) wrt (1/2,0)
- 2) for large t/U the opposite happens due to higher order exchange terms.

$$J = 4t^2/U - 24t^4/U^3$$

$$J_c = 80 t^4/U^3$$

$$J' = J'' = 4t^4/U^3$$

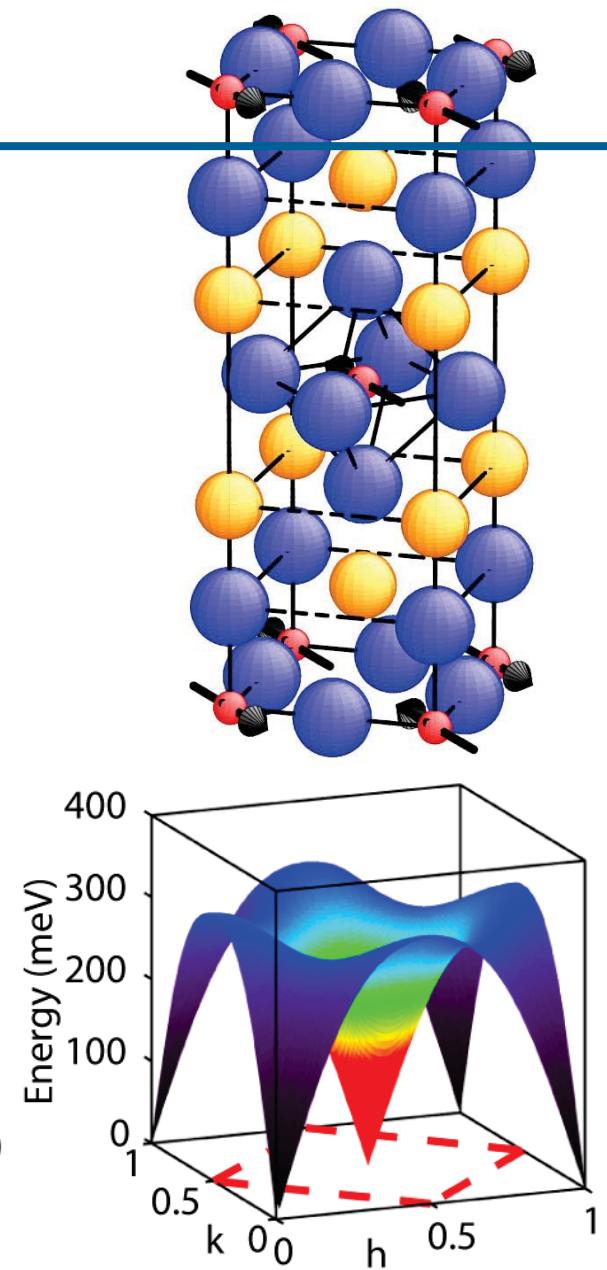
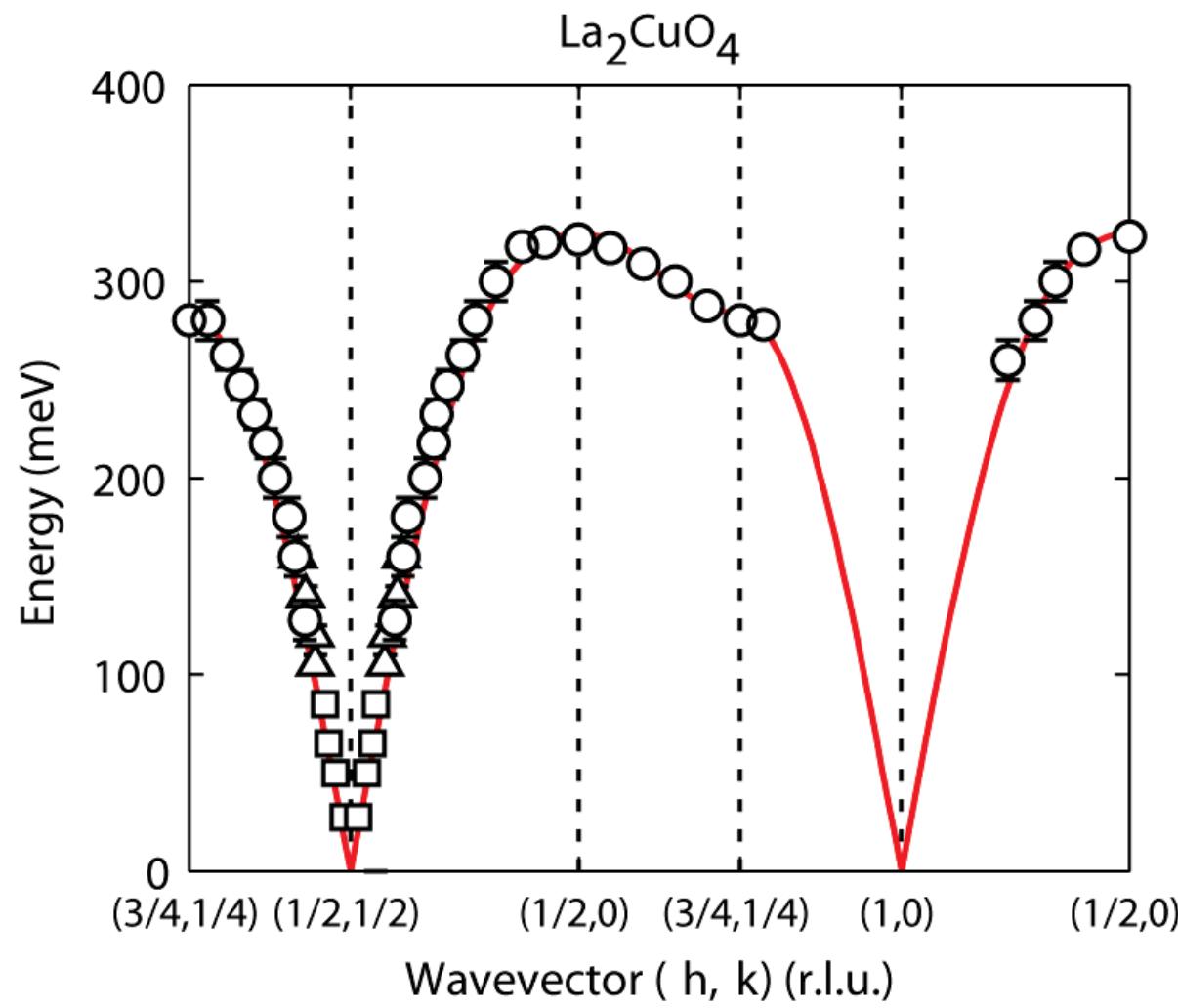


A. H. MacDonald PRB (1990)

$$t = 0.30 \pm 0.02 \text{ eV}, U = 2.2 \pm 0.4 \text{ eV},$$

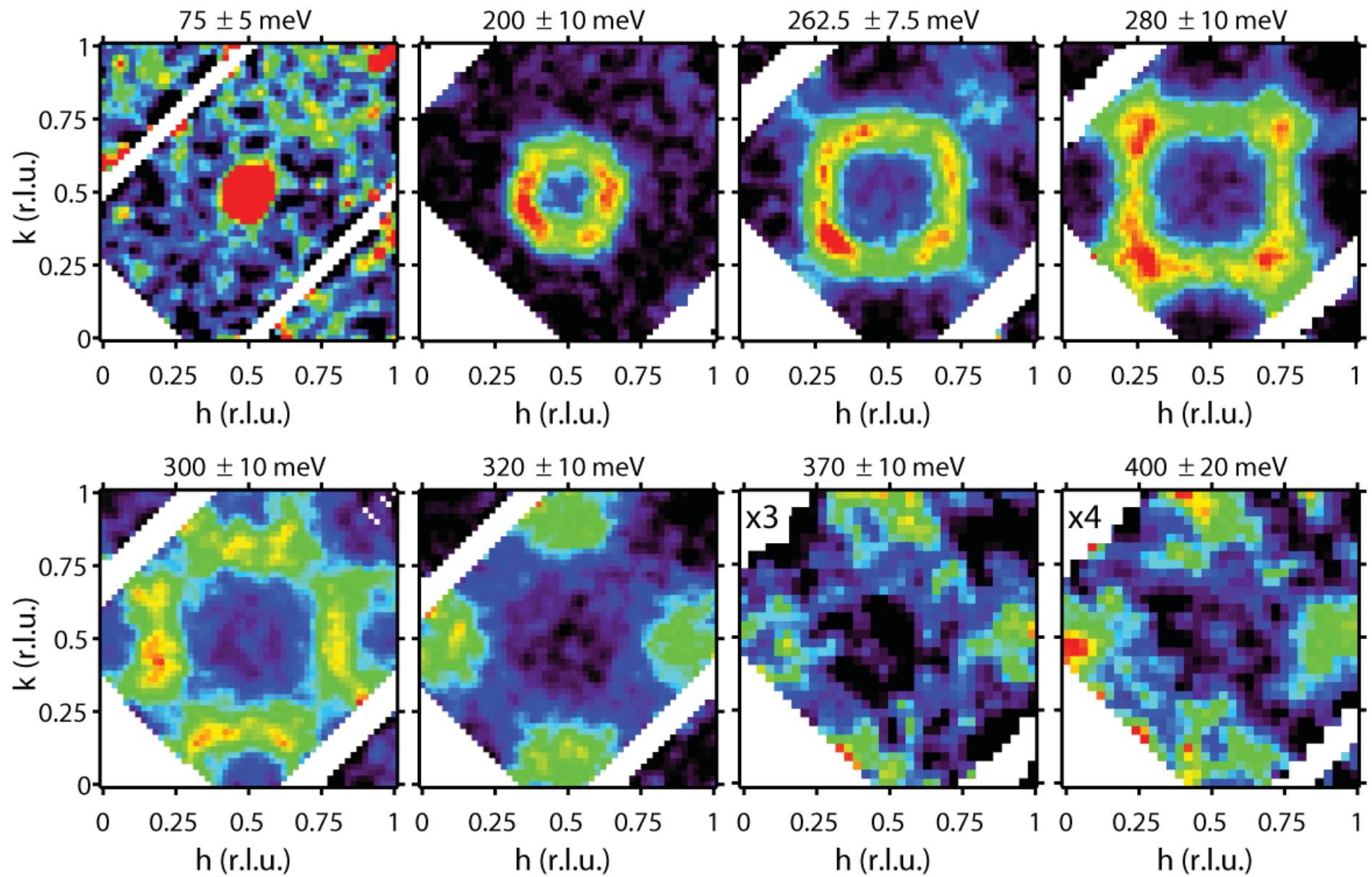
$$\mathcal{J} = 146 \pm 4 \text{ meV}, \text{ and } J_c = 61 \pm 8 \text{ meV}.$$

Spin waves in La_2CuO_4

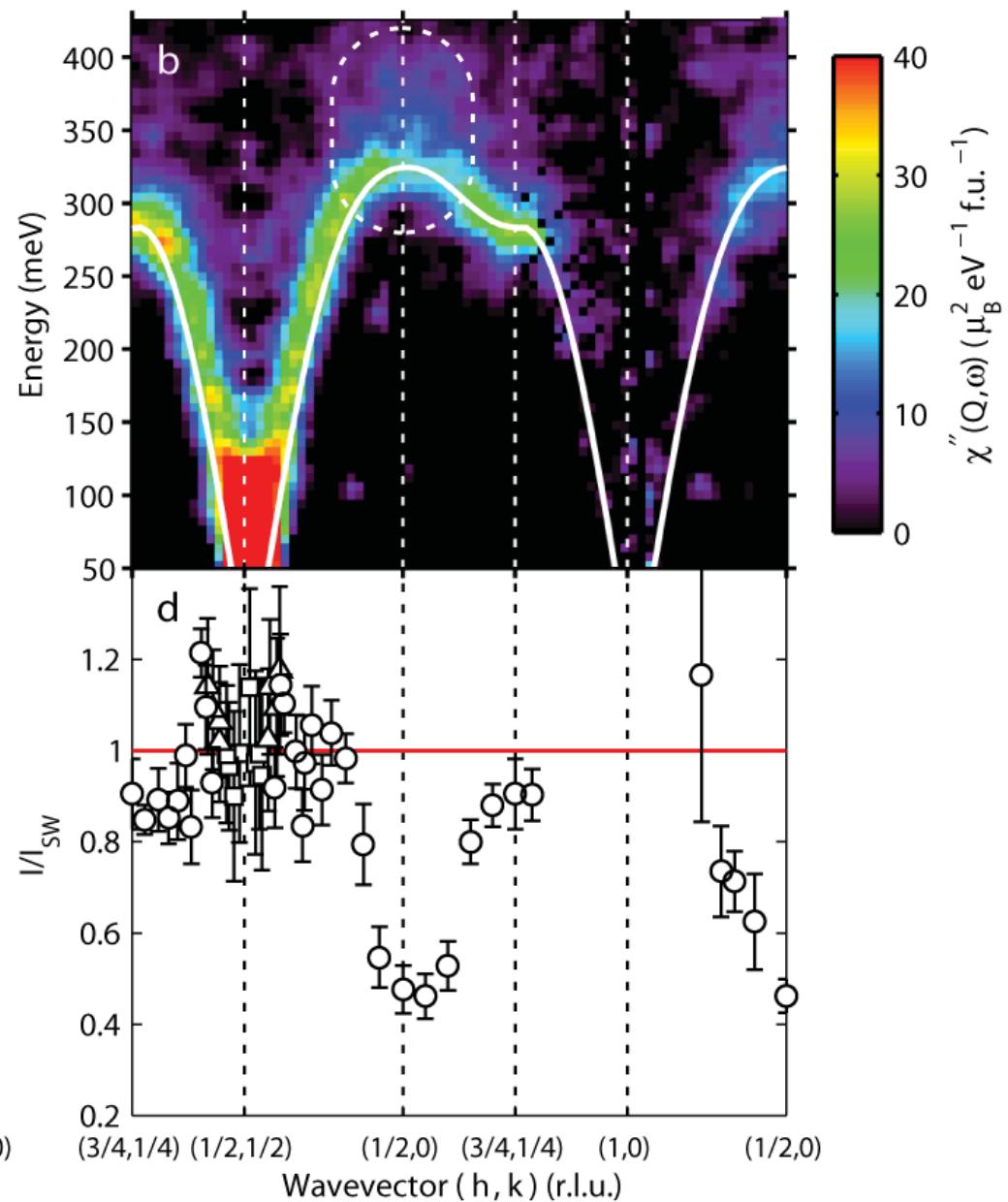
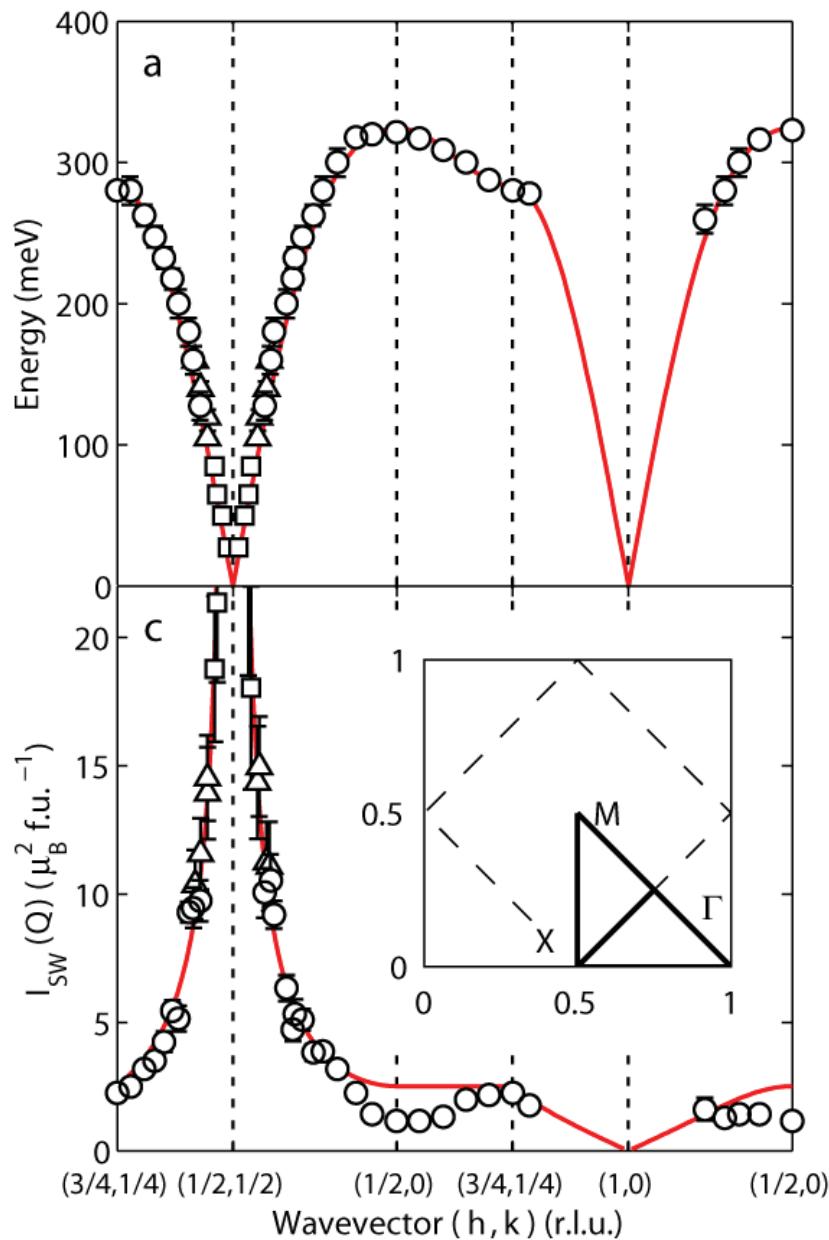


Coldea, Hayden, Aeppli, Perring, Frost, Mason, Cheong, Fisk, PRL **86** 5377 (2001).
Headings, Hayden, Coldea, Perring, Phys. Rev. Lett. **105**, 247001 (2010)

La_2CuO_4 : \mathbf{q} -dependent slices



Magnetic Excitations in La_2CuO_4



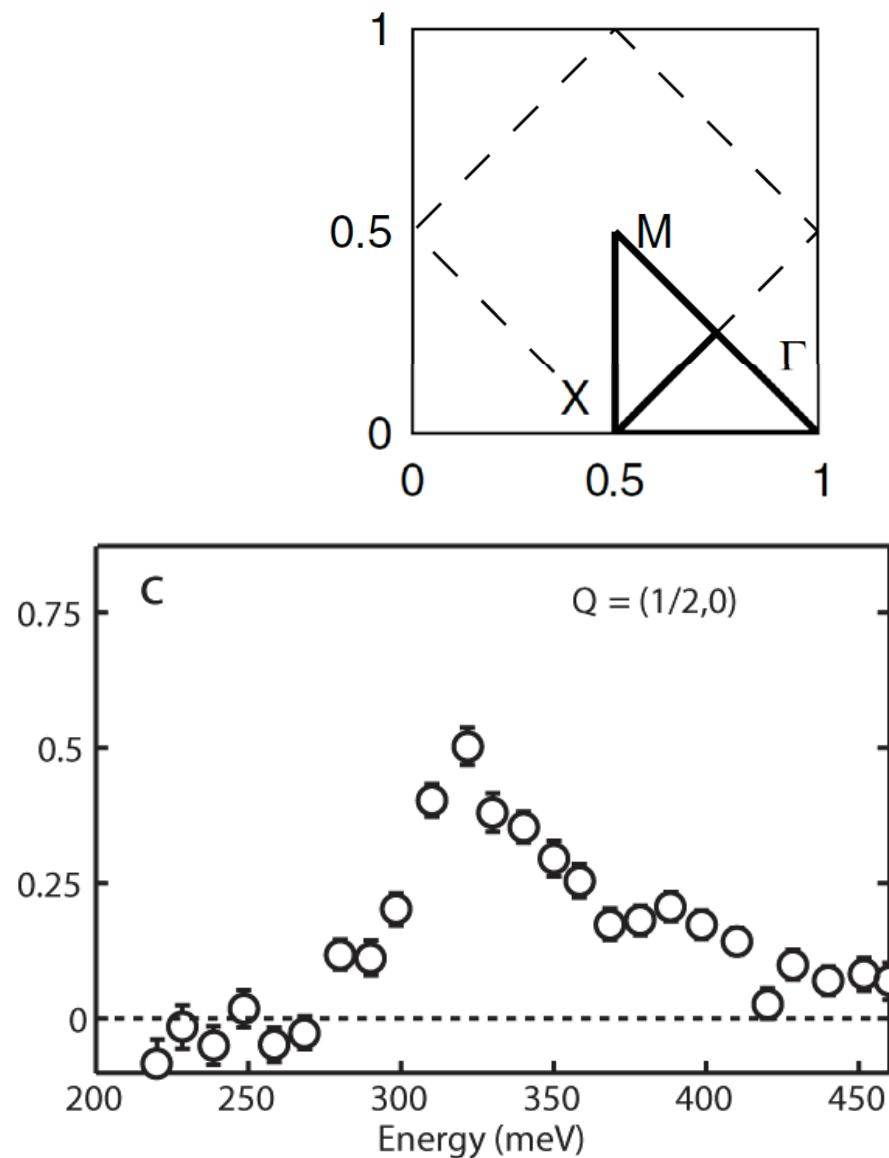
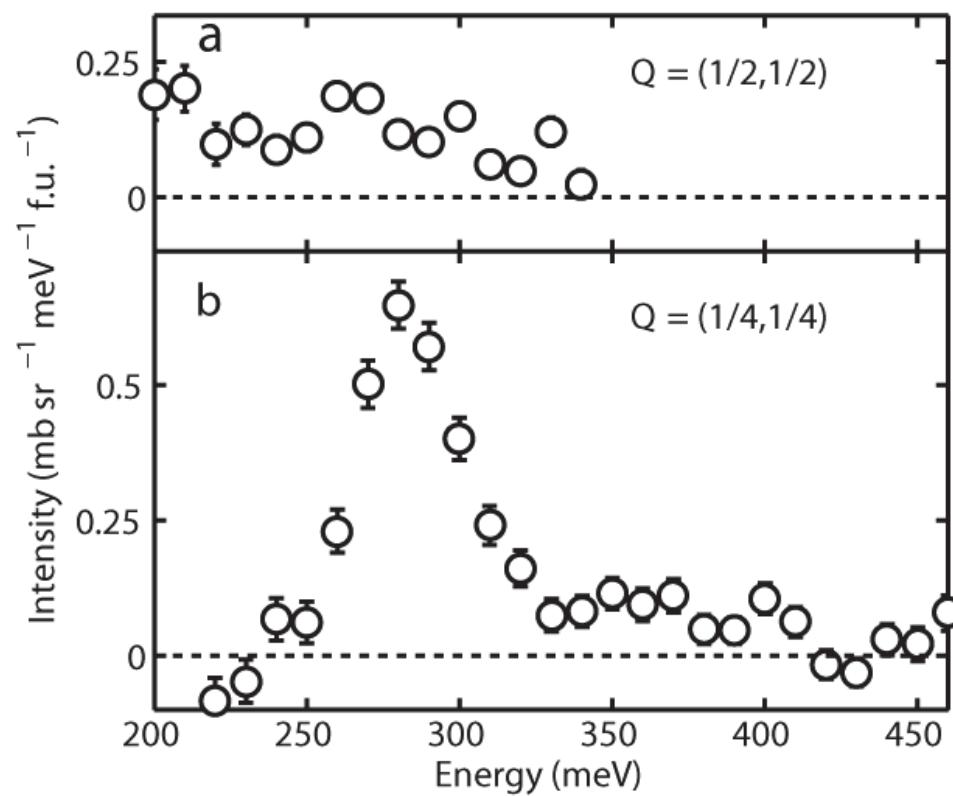
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La_2CuO_4 : High Energy Excitations

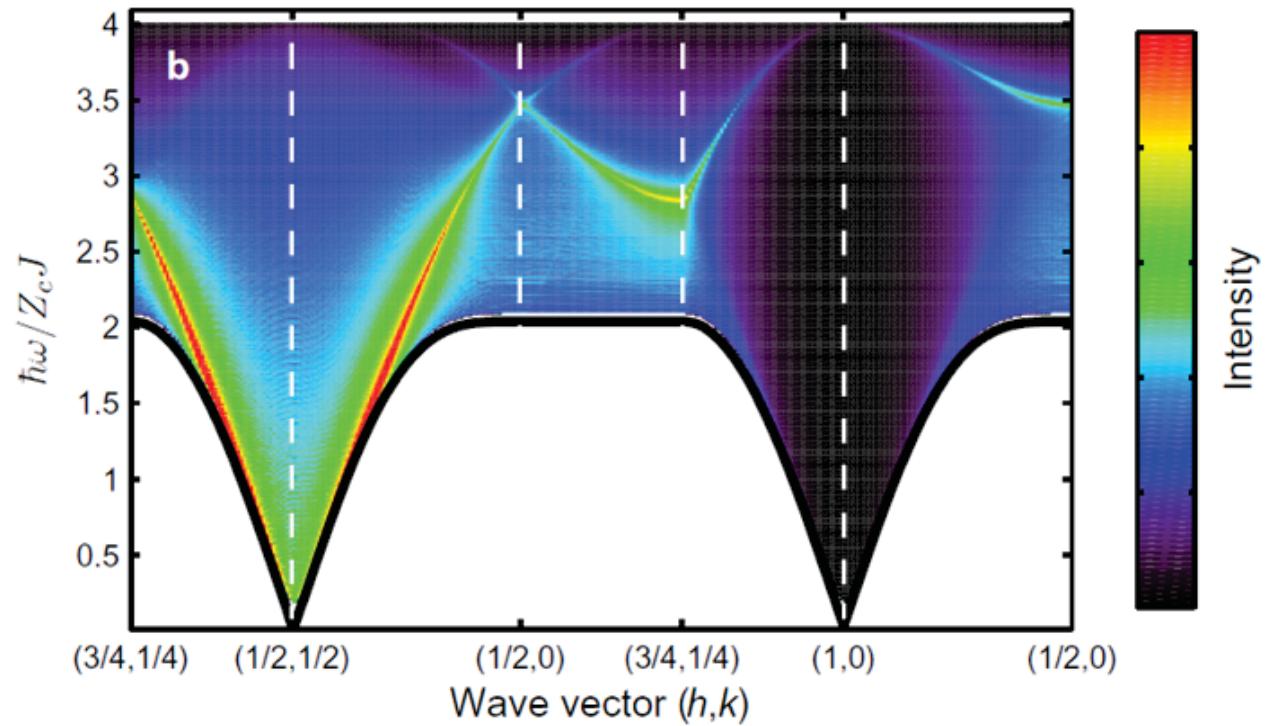
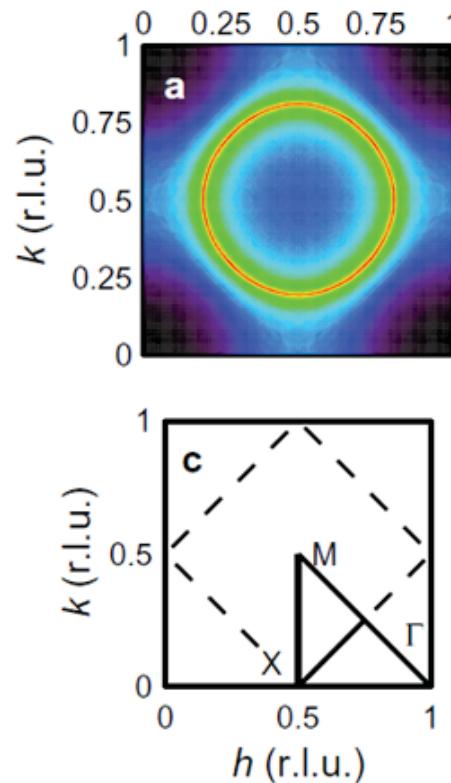


Spin-wave Theory (SWT 1+2 Mag)

$$\chi''_{\perp}(\mathbf{Q}, \hbar\omega) = Z_d(\mathbf{Q}) \frac{\pi}{2} g^2 \mu_B^2 S (u_{\mathbf{Q}} - v_{\mathbf{Q}})^2 \times \delta(\hbar\omega - \hbar\omega_{\mathbf{Q}}) \quad \text{1-mag}$$

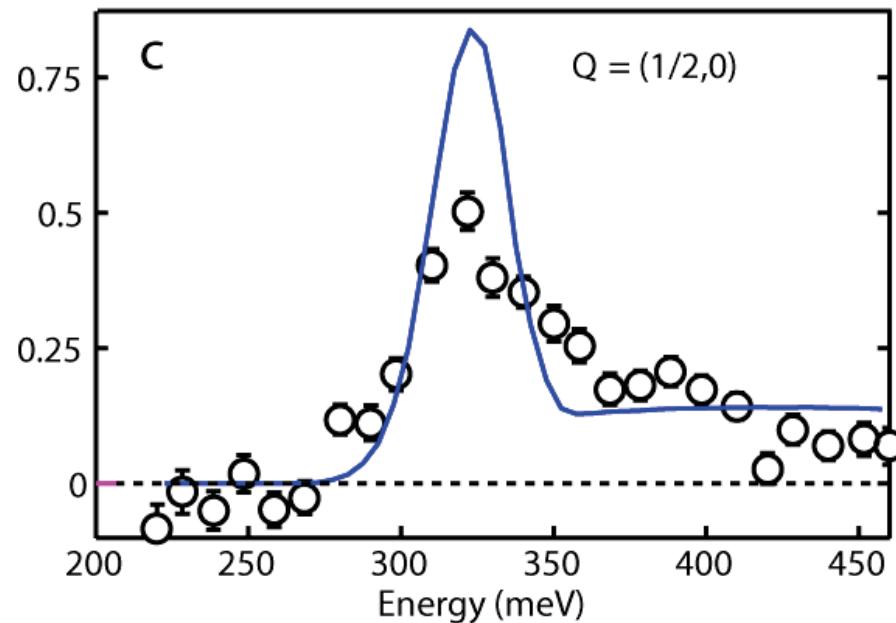
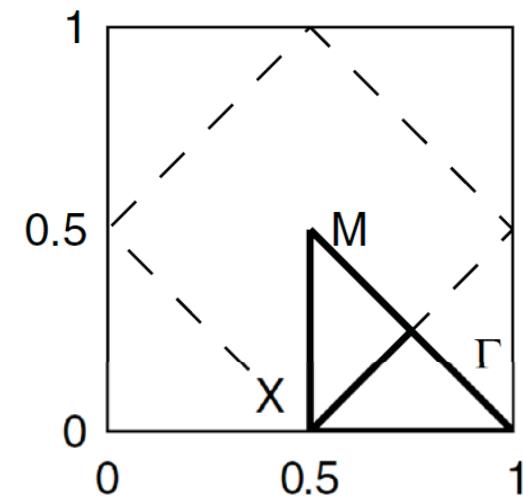
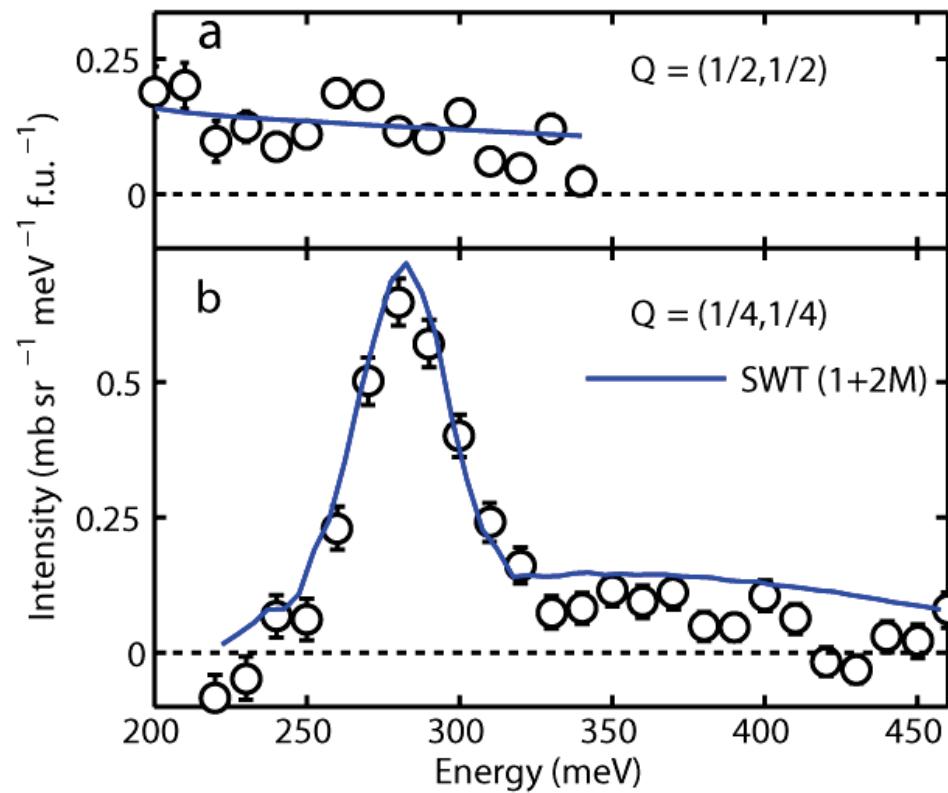
$$\begin{aligned} \chi''_{\parallel}(\mathbf{Q}, \hbar\omega) &= Z_{2M} \frac{\pi g^2 \mu_B^2}{2N} \sum_{\mathbf{Q}_1, \mathbf{Q}_2} f(\mathbf{Q}_1, \mathbf{Q}_2) \\ &\times \delta(\hbar\omega - \hbar\omega_{\mathbf{Q}_1} - \hbar\omega_{\mathbf{Q}_2}) \delta(\mathbf{Q} + \mathbf{Q}_1 - \mathbf{Q}_2) \end{aligned} \quad \text{2-mag}$$

h (r.l.u.)

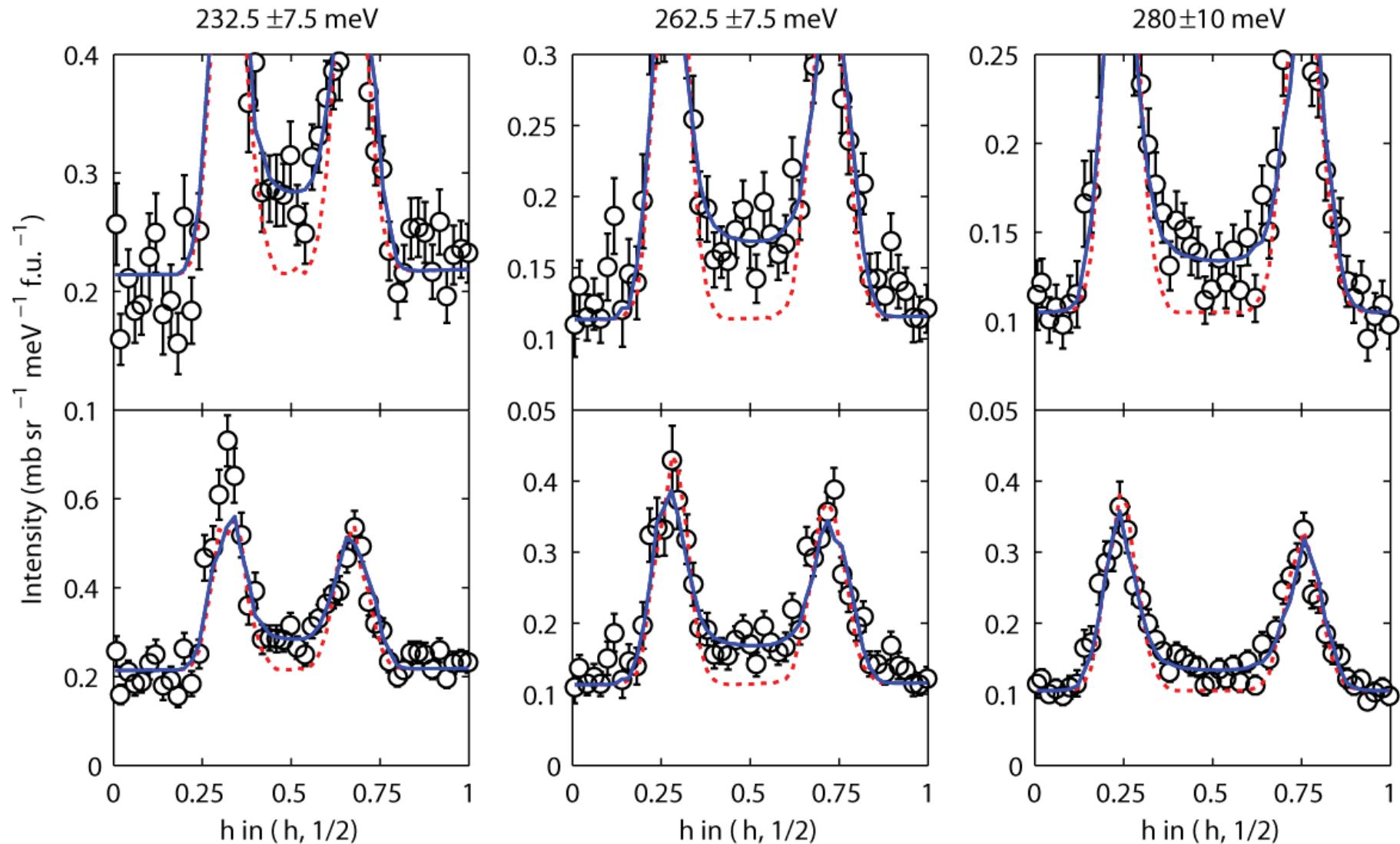


Spin-wave Theory (SWT 1+2 Mag)

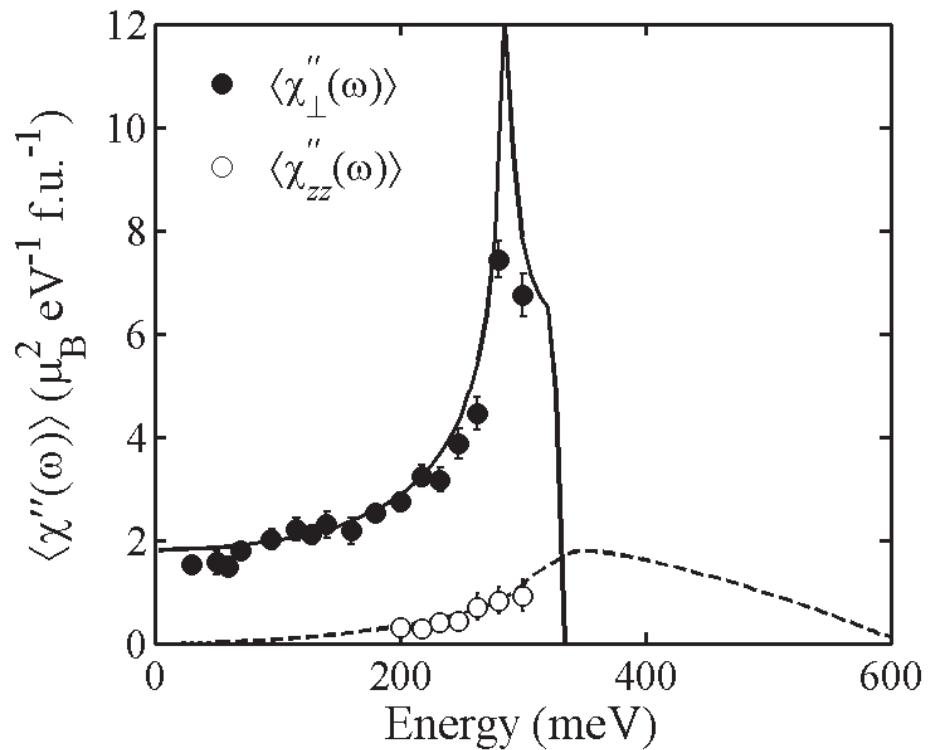
$$Z_d = 0.4 \pm 0.04$$
$$Z_{2M} = 0.47 \pm 0.1$$



Continuum (2-magnon) scattering at lower energies

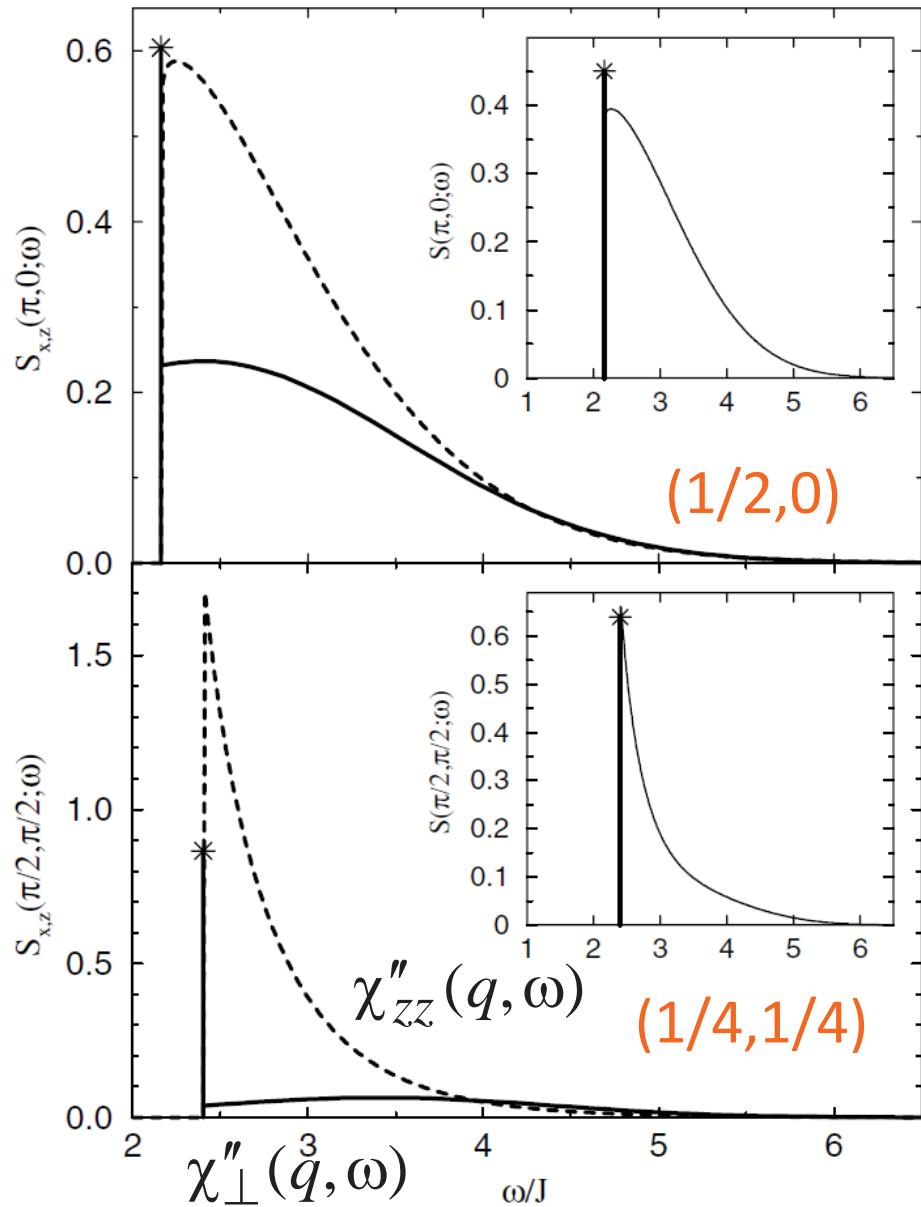


Local susceptibility based on SWT (1+2M)



$$\chi''_{\text{local}}(\omega) = \frac{\int_{BZ} \chi''(\mathbf{Q}, \omega) d\mathbf{Q}}{\int_{BZ} d\mathbf{Q}}$$

Quantum Monte Carlo Simulations



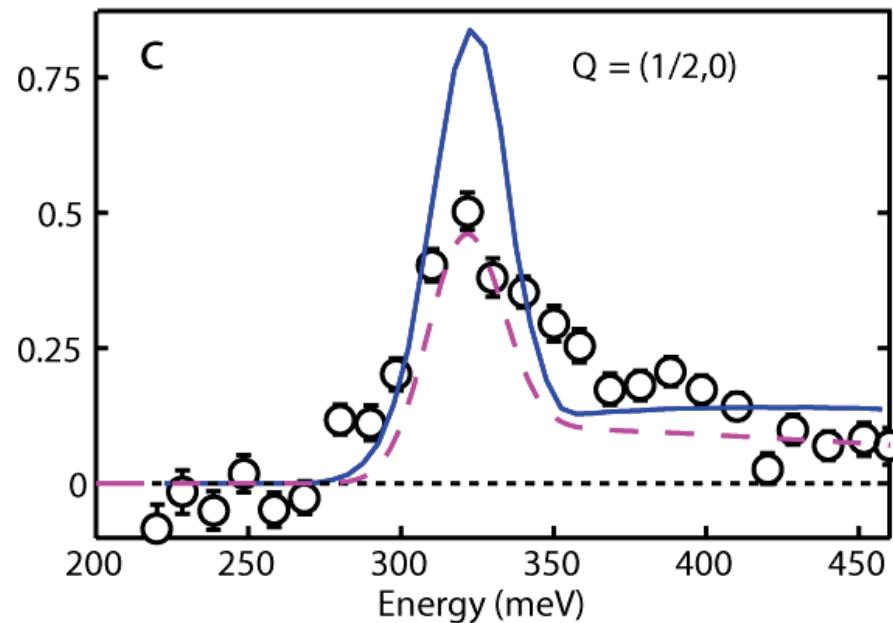
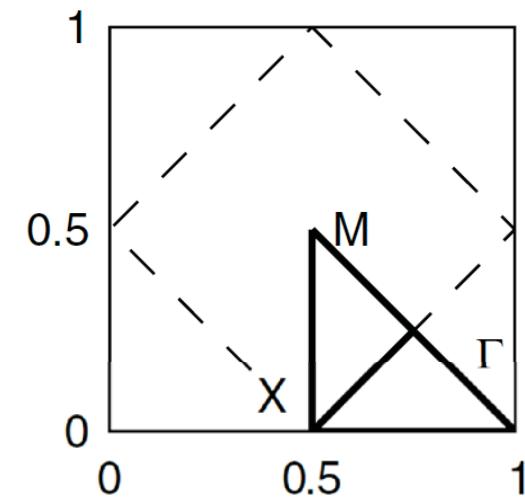
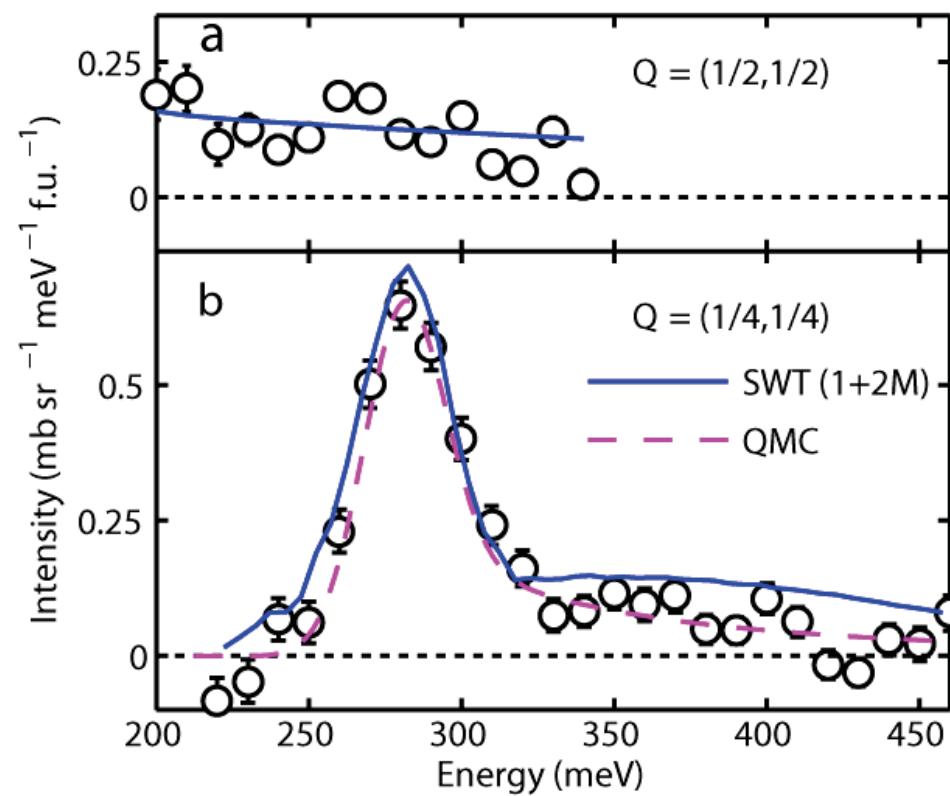
A. W. Sandvik and R. R. P. Singh,
PRL 86, 528 (2001)

Points to note:

- QMC includes only J and assumes lineshapes for continuum responses
- profiles shifted to match the measured SW pole positions
- Experiment measures

$$\chi''_{\perp}(q, \omega) + \chi''_{zz}(q, \omega)$$

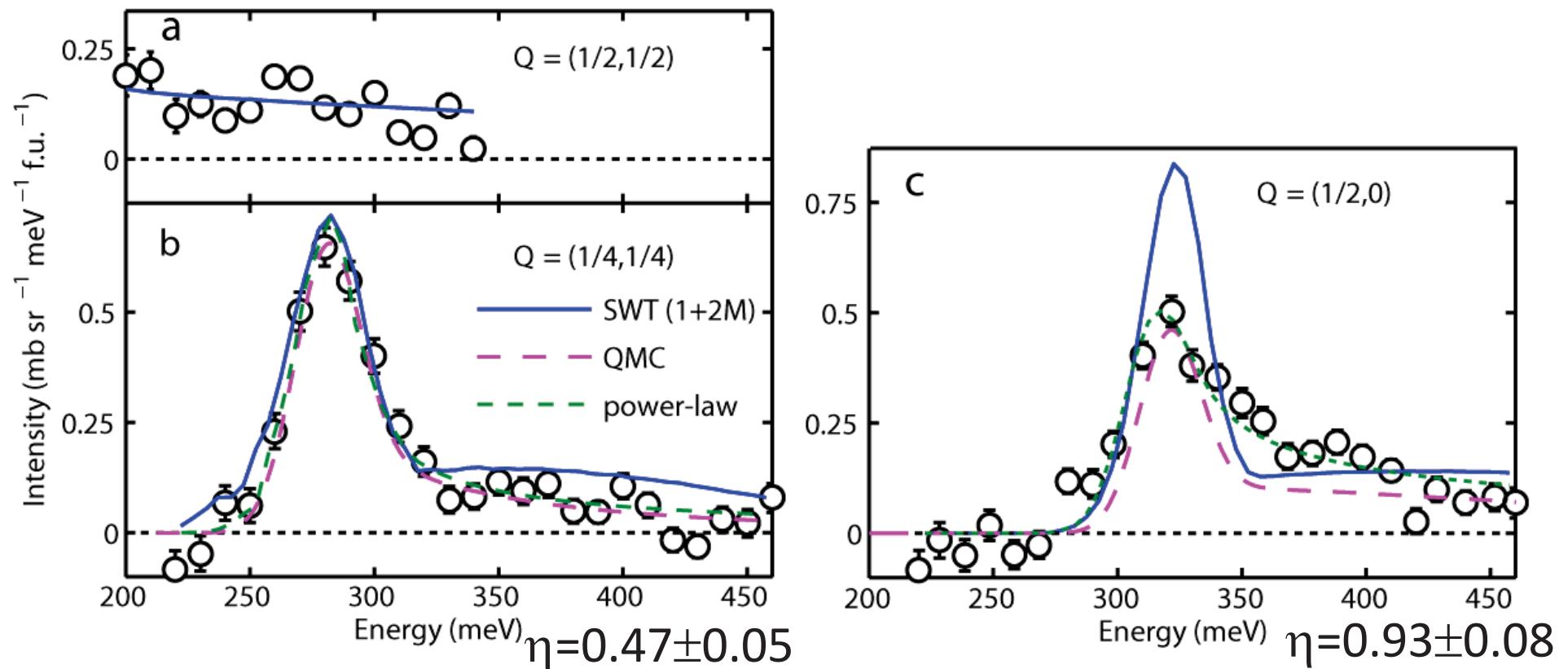
Quantum Monte Carlo Simulations



"Spinon" power law

Generalization of continuum scattering lineshape of the 1D Heisenberg AF chain ($\eta=1$)

$$\chi''(\mathbf{q}, \omega) = A_{\mathbf{q}} \frac{\theta(\omega - \omega_{\mathbf{q}})}{(\omega^2 - \omega_{\mathbf{q}}^2)^{1-\eta/2}}$$



Anomalous Spin Excitations in La_2CuO_4 and $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

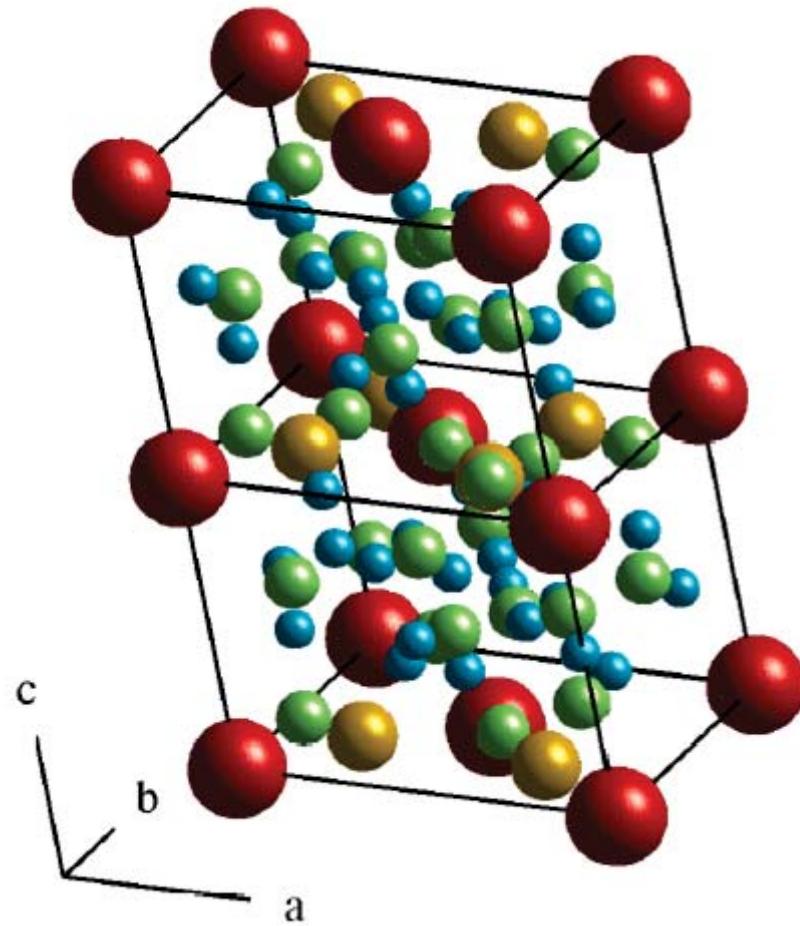
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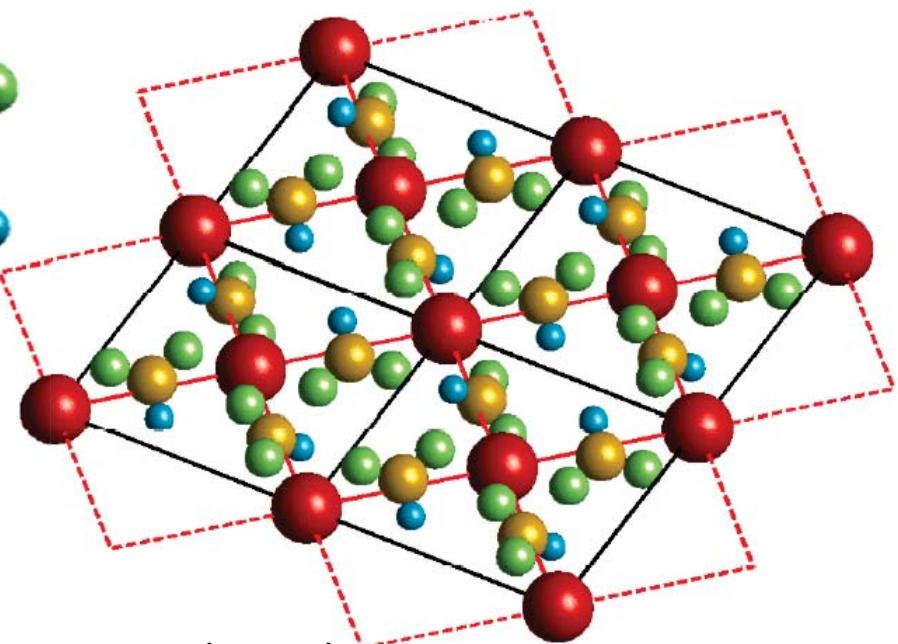
Other S=1/2 Square lattice antiferromagnets

- Reduced (1/2,0) SW intensity and high-energy tail also seen small t/U weak exchange AFs



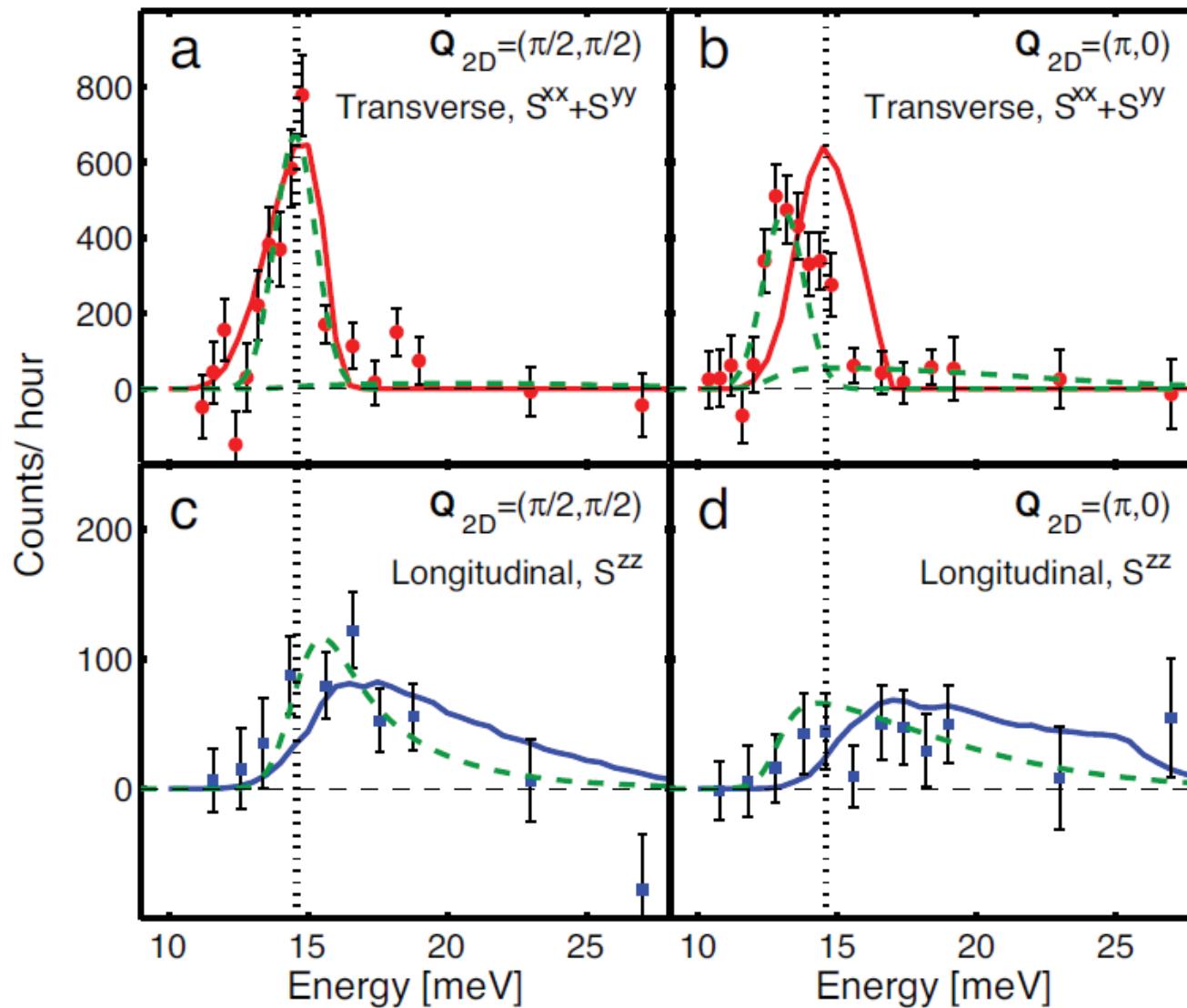
Cu
C
O
D

$\text{Cu}(\text{DCOO})_2 \cdot 4\text{D}_2\text{O}$ (CFTD)
N. B. Christensen et al.,
PNAS 104, 15264 (2007)

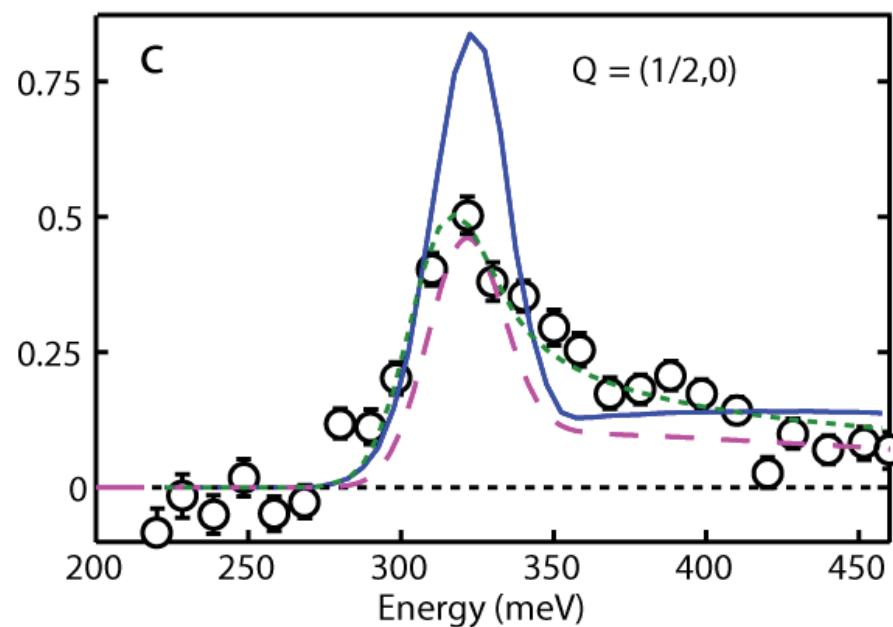
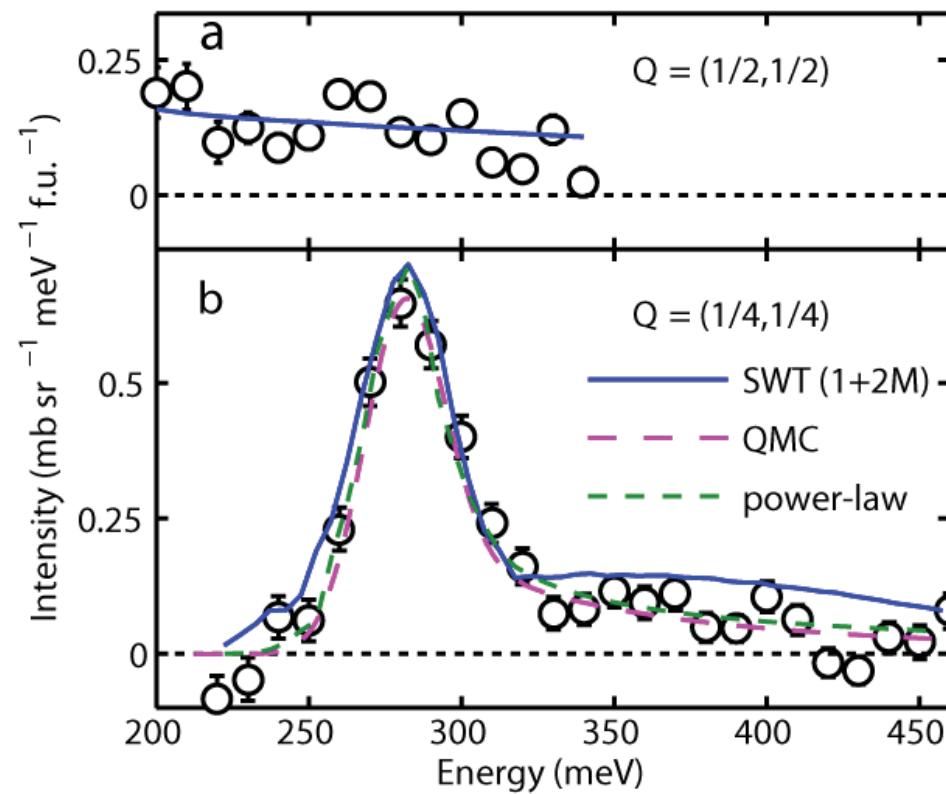


Also $\text{Cu}(\text{pz})_2(\text{ClO}_4)_2$ N. Tsyrulin et al, PRL 102, 197201 (2009)

$\text{Cu}(\text{DCOO})_2 \cdot 4\text{D}_2\text{O}$ (CFTD)

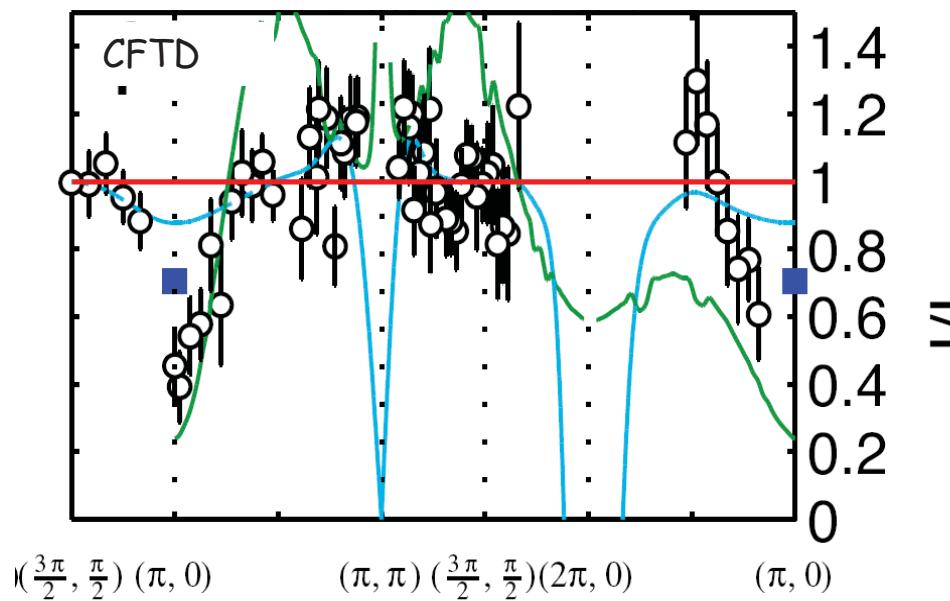


La_2CuO_4



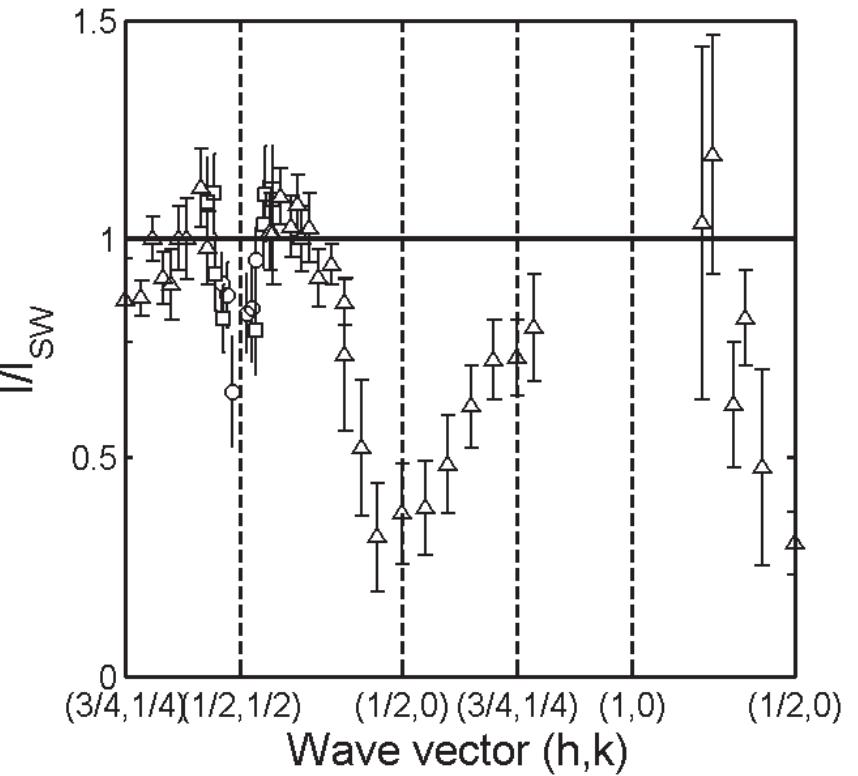
$\text{Cu}(\text{DCOO})_2 \cdot 4\text{D}_2\text{O}$ (CFTD)

SW amplitude dependence



$\text{Cu}(\text{DCOO})_2 \cdot 4\text{D}_2\text{O}$

Christensen et al. PNAS
104 15264 (2007)



La_2CuO_4

Resonant inelastic x-ray scattering (RIXS)

RIXS at the copper L₃ edge, Cu 2p → 3d

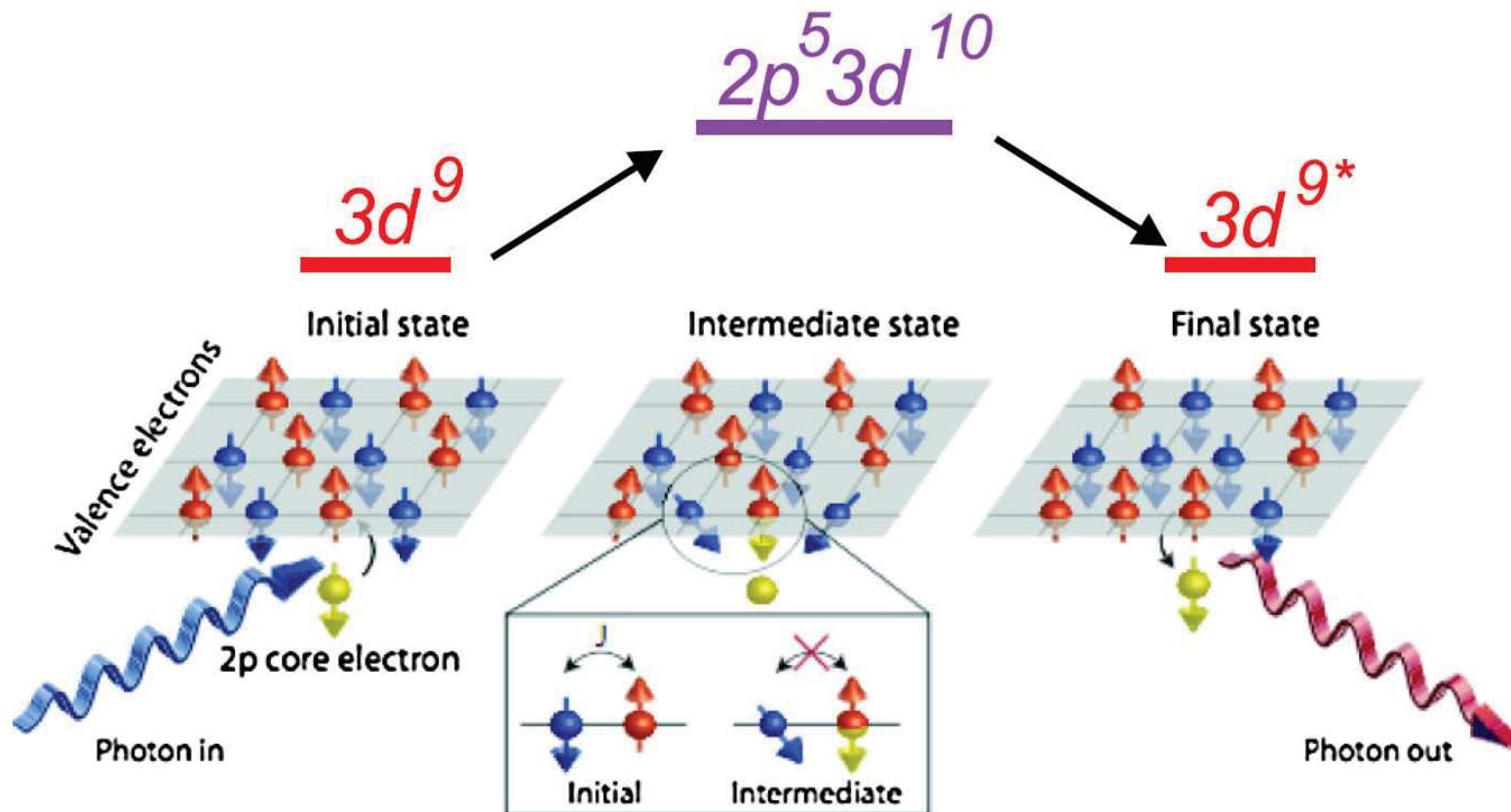
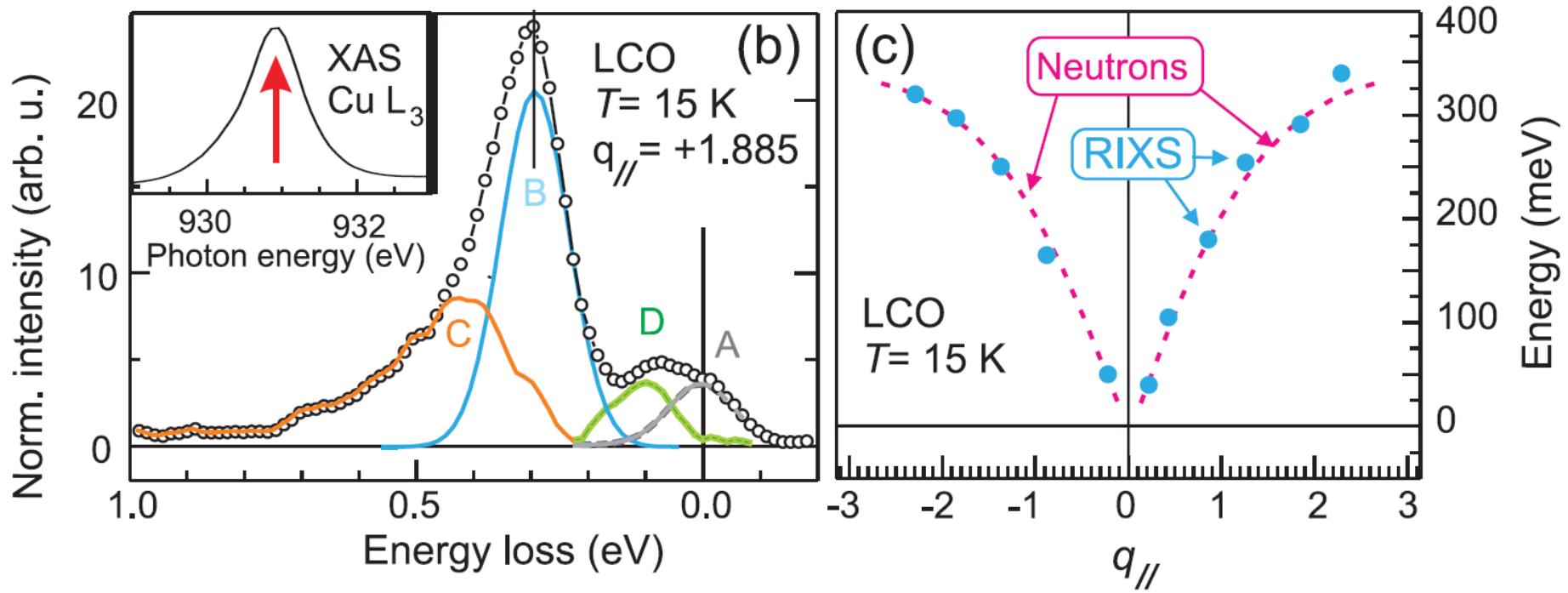
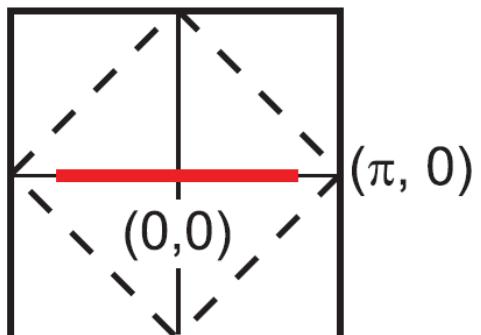


Figure: L. Braicovich et al., PRL 102, 167401 (2009)

RIXS in La_2CuO_4



$(0, \pi)$



RIXS at the copper L_3 edge, $\text{Cu } 2\text{p} \rightarrow 3\text{d}$

L. Braicovich, et al., PRL 104, 077002 (2010)

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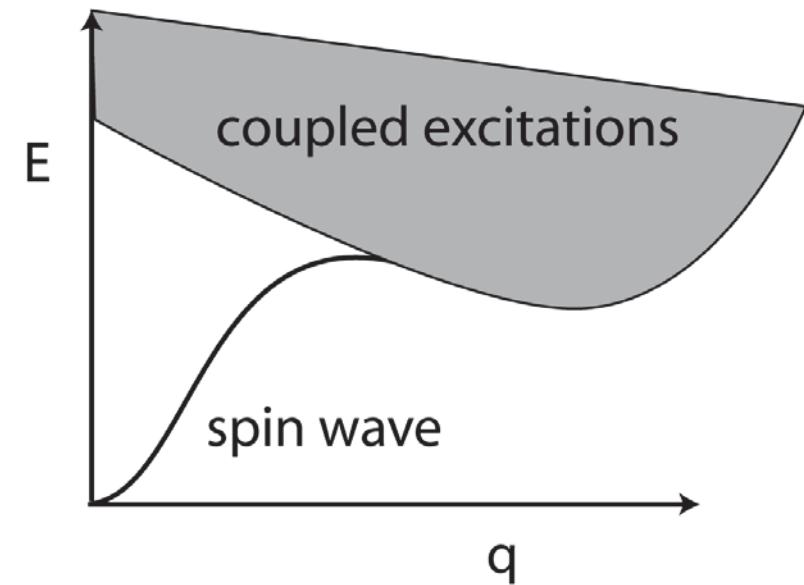
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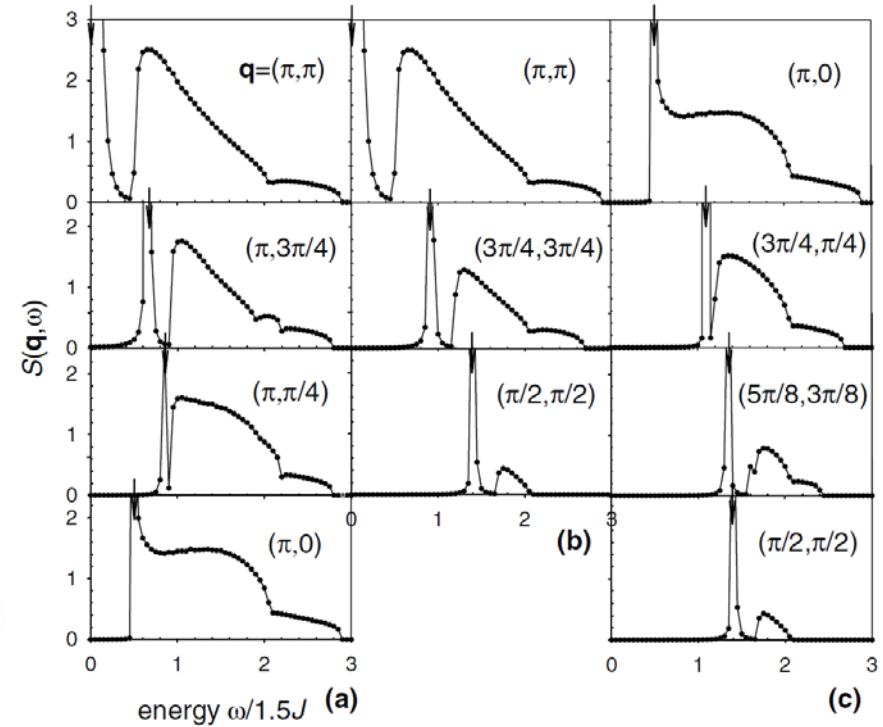
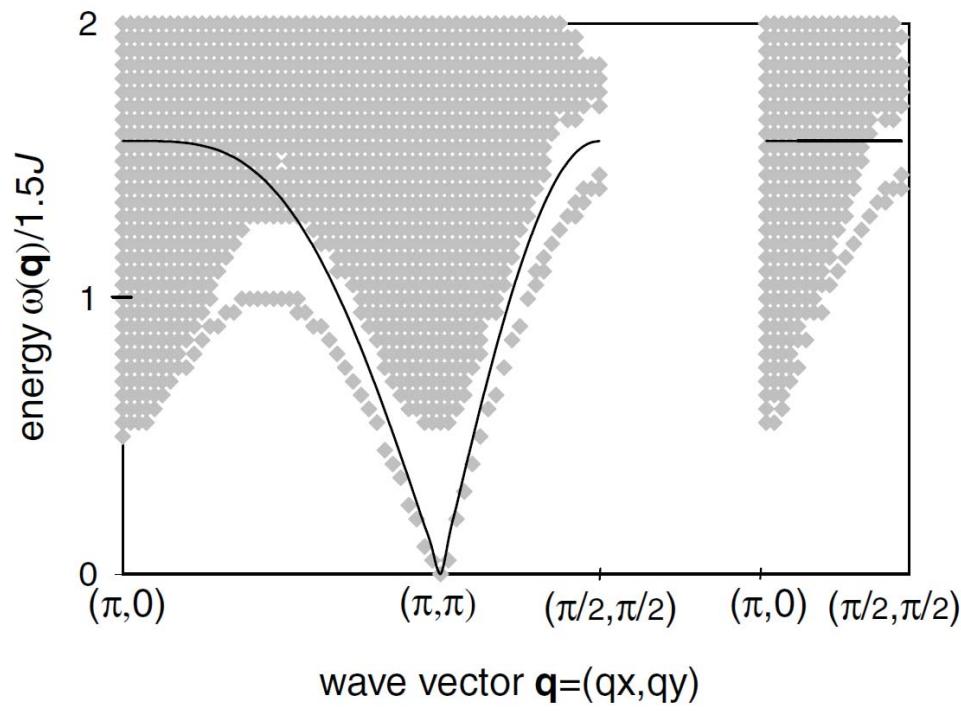
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Interpretation

- Low energies: bound spinons \rightarrow spin waves
 - High energies: Evidence for fractionalization. Spin waves unbind into spinons.
-
- May also be other explanations (cf reduction of multiband Hubbard model to Heisenberg model only works at low energies.)

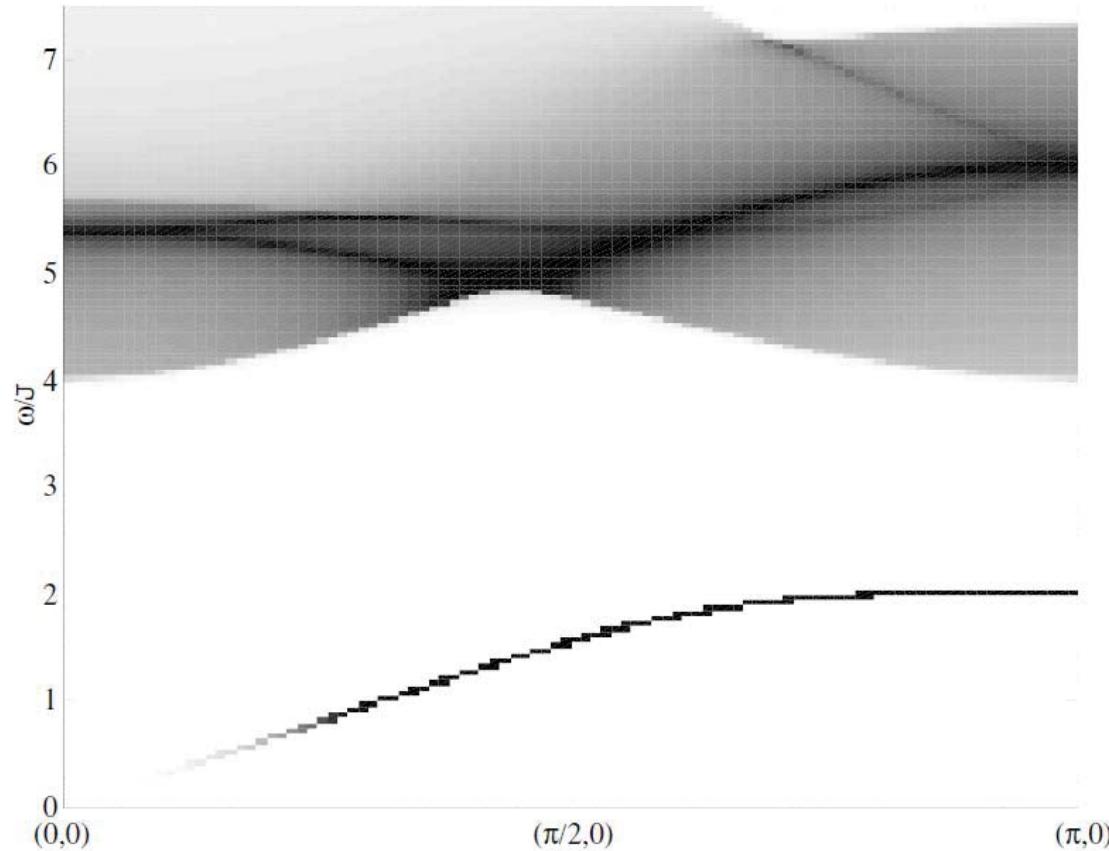


Excitations of RVB state: π -flux phase



C.-M. Ho, V. N. Muthukumar, M. Ogata, and P. W. Anderson,
Phys. Rev. Lett. 86, 1626 (2001).

Fractionalized Antiferromagnet (AF^*)

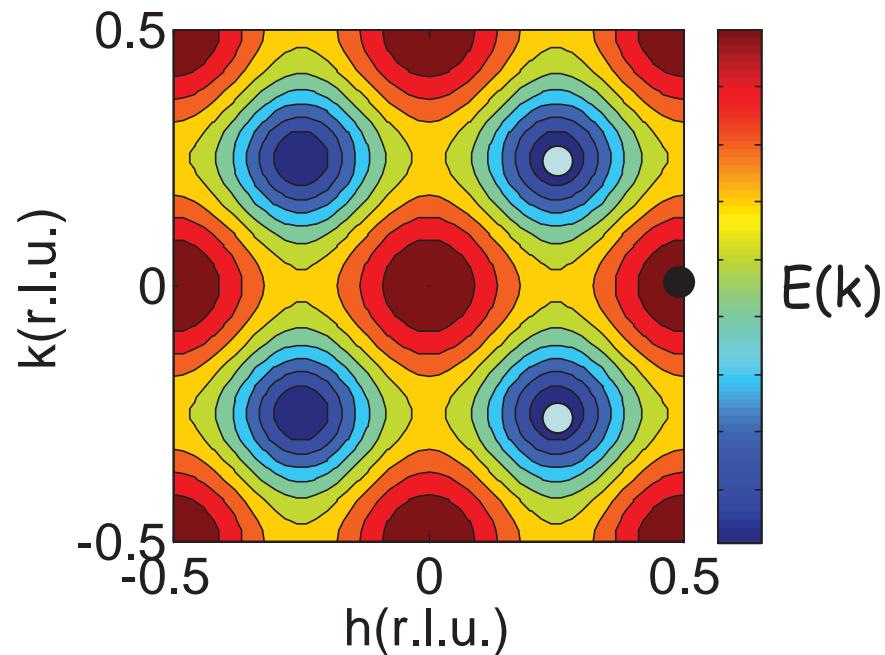


C. Lannert and M. P. A. Fisher, Int. J. Mod. Phys. B 17, 2821 (2003)

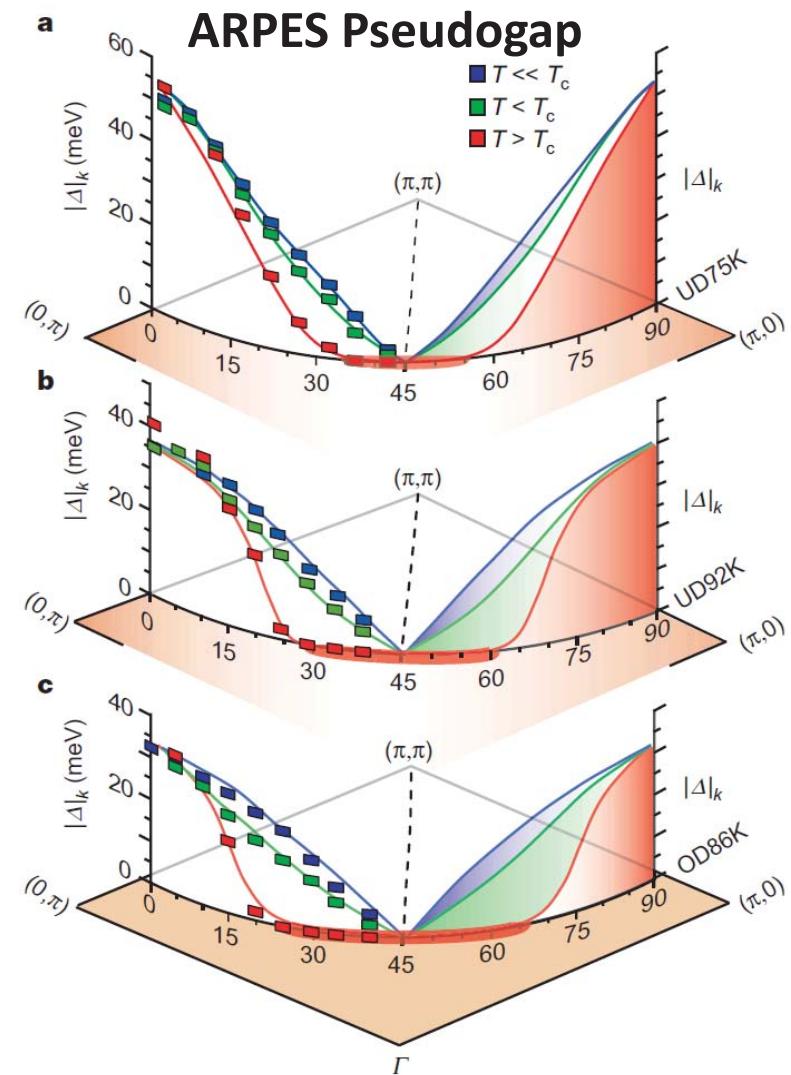
Fermionic Excitations in Cuprates

Spinon Picture

- Spinons are S=1/2 quasiparticles
- $(1/2, 0) \Rightarrow$ two spinons $(1/4, \pm 1/4)$
- spinons model would need a d-wave dispersion with minima at $(\pm 1/4, 1/4)$ and $(1/4, \pm 1/4)$



C-M. Ho et al PRL 2001 π -flux phase



Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$
Lee et al., Nature 450, 81 (2007)

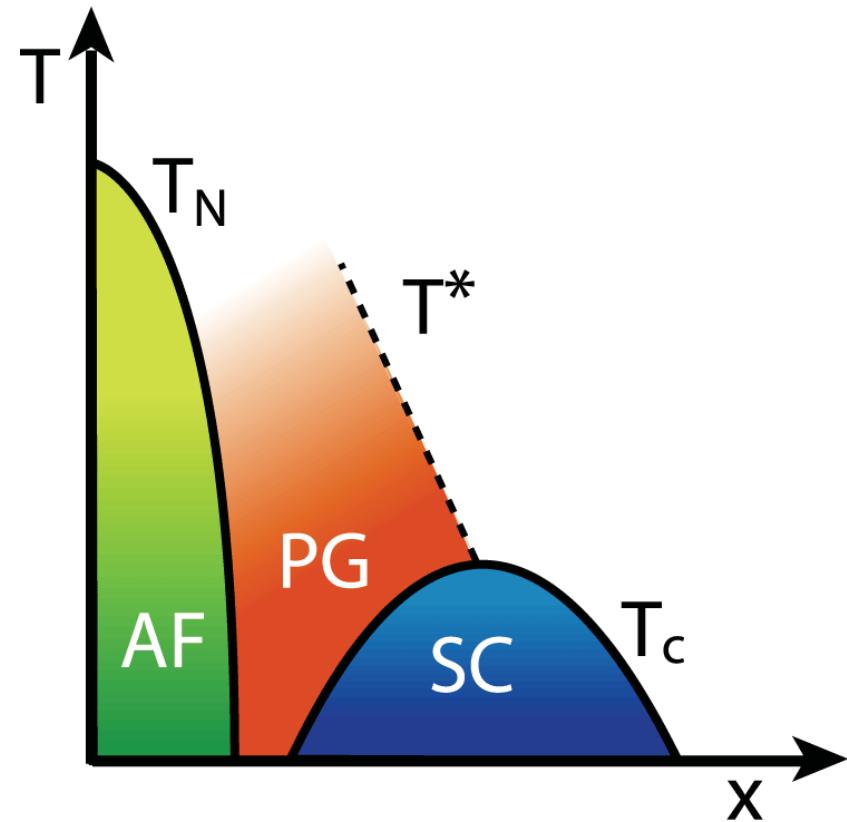
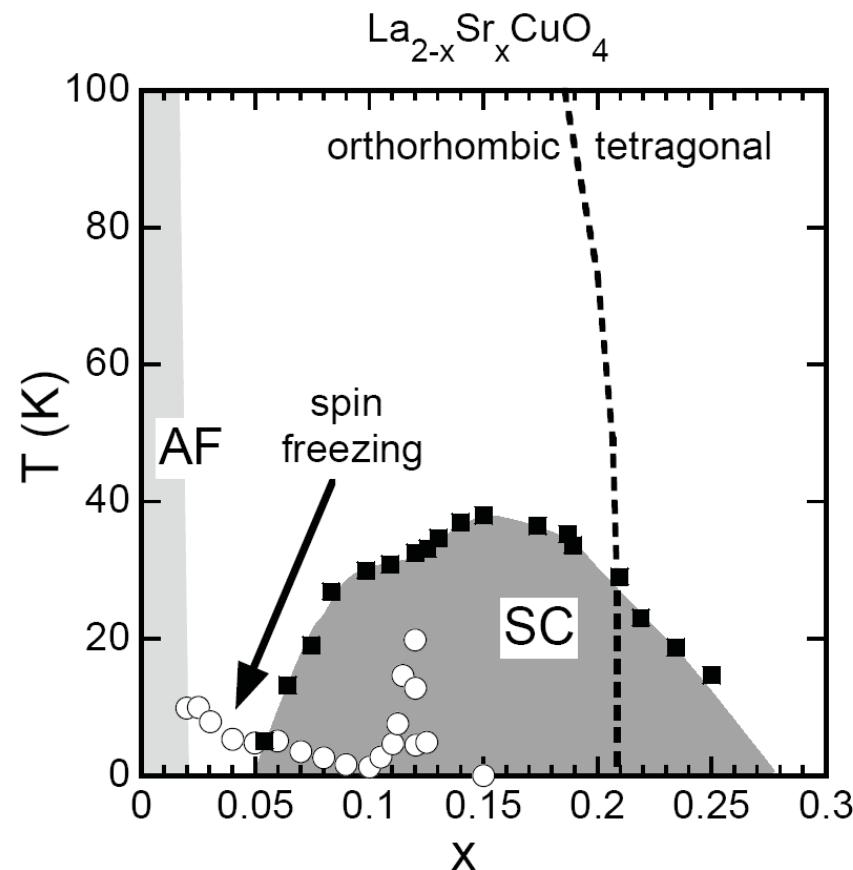
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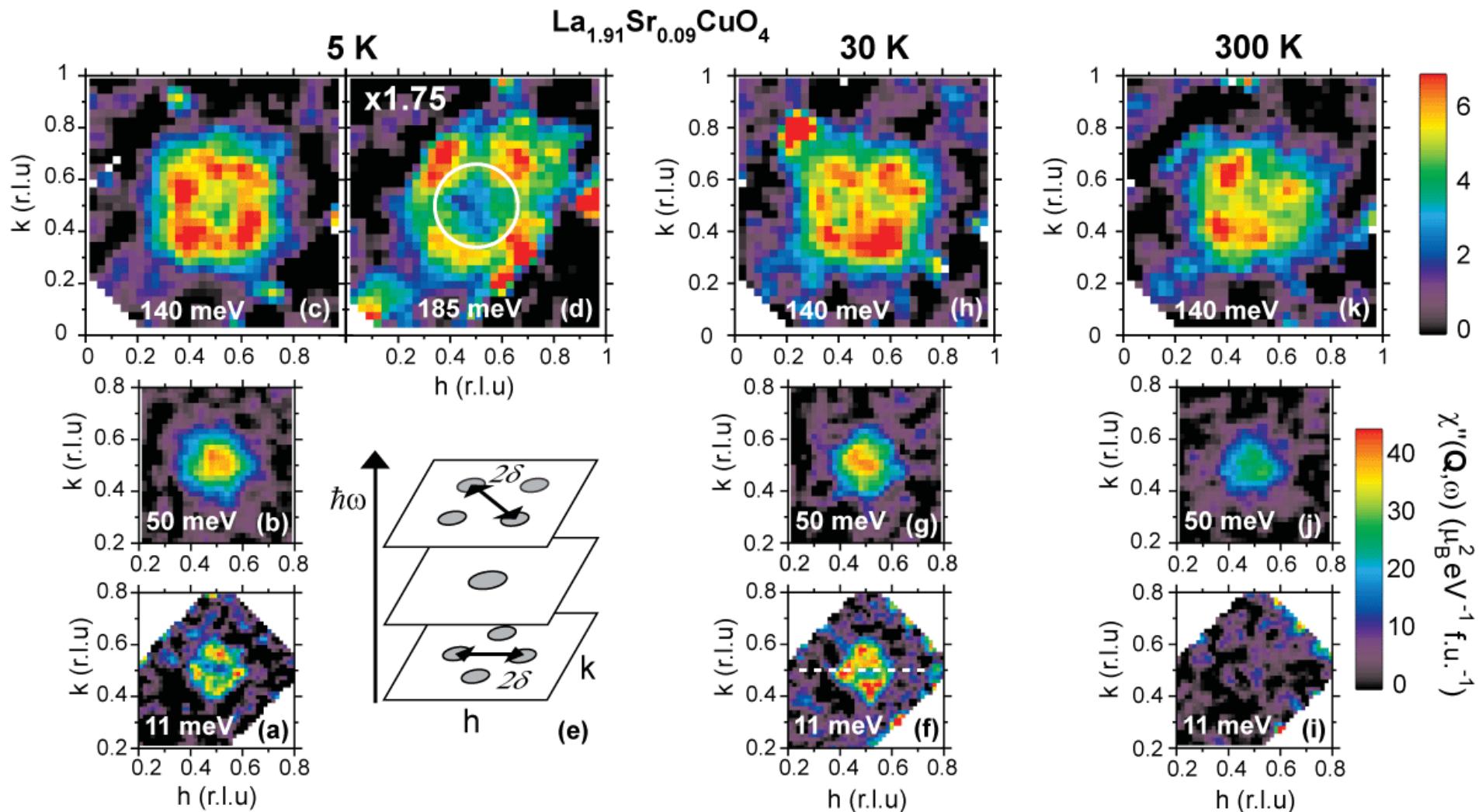
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- Discussion

2) The pseudogap and $\text{La}_{1.91}\text{Sr}_{0.09}\text{CuO}_4$ (UD22)

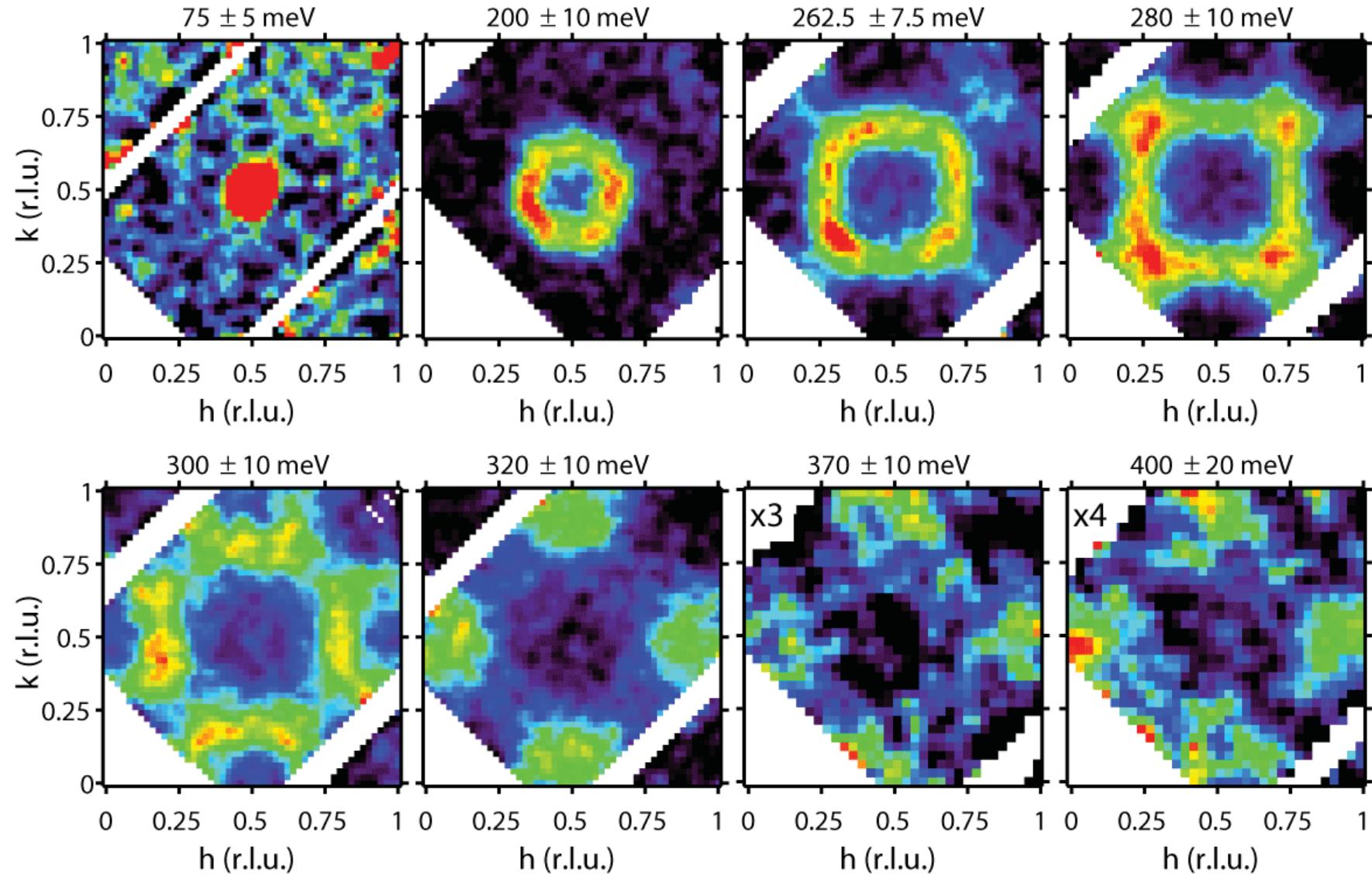
Magnetic excitations in underdoped cuprates



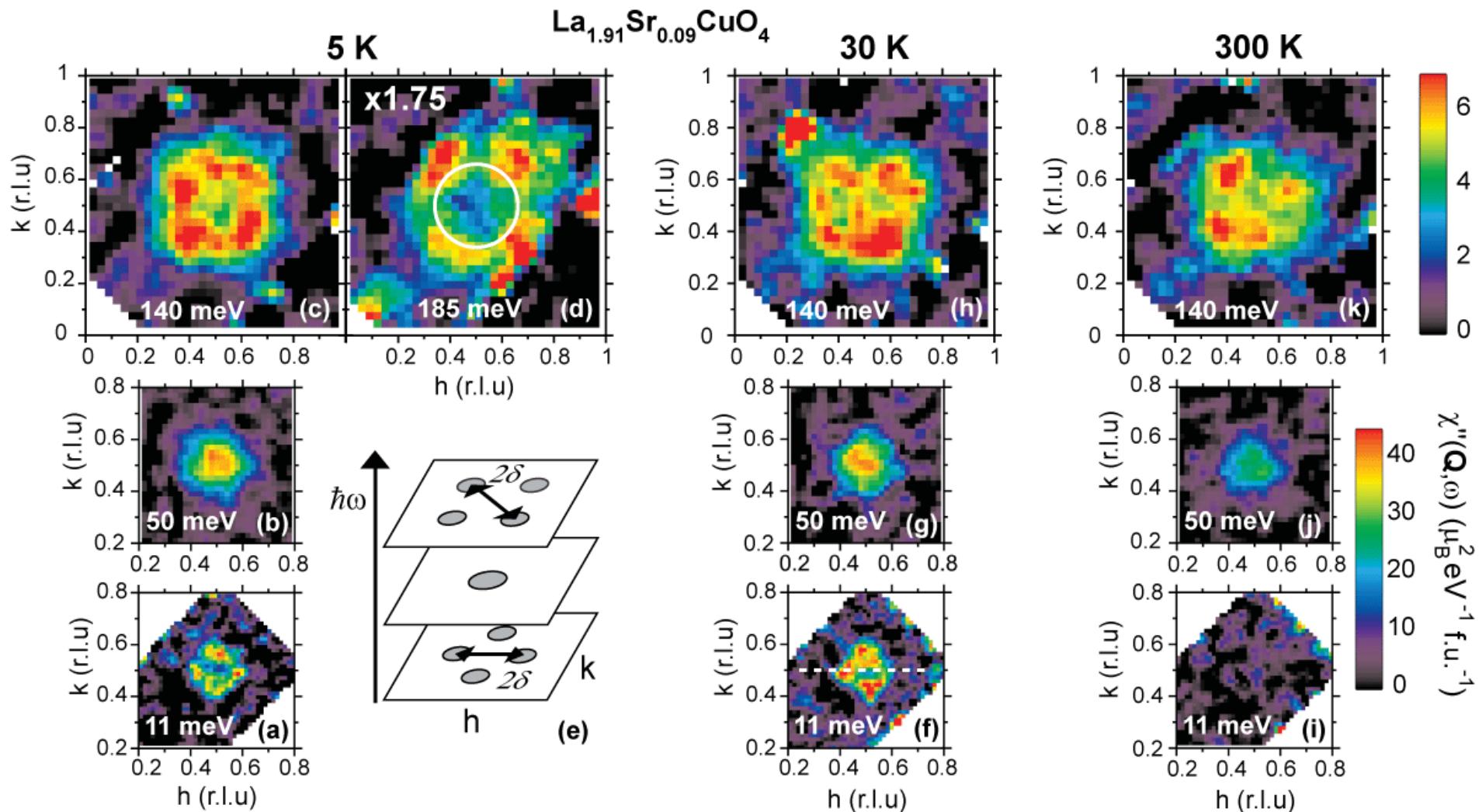
$\text{La}_{1.91}\text{Sr}_{0.09}\text{CuO}_4$ ($T_c=22$ K): q-dependent slices



La_2CuO_4 : \mathbf{q} -dependent slices

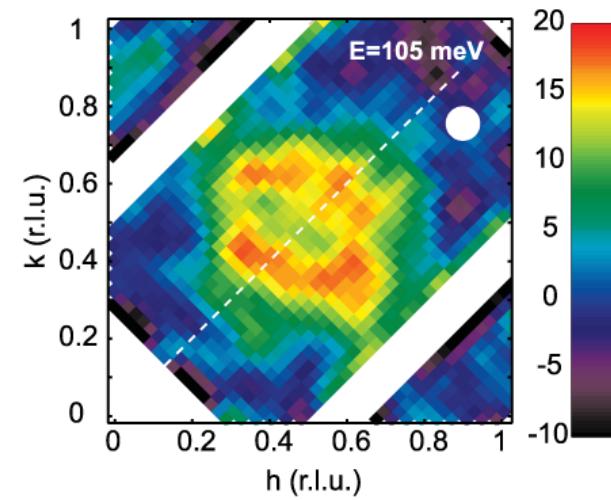
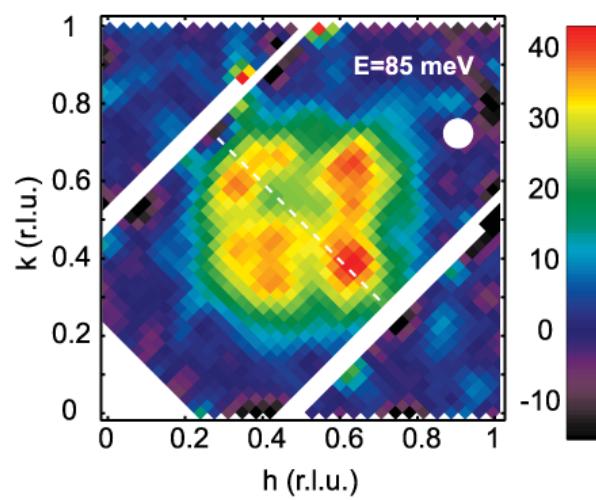
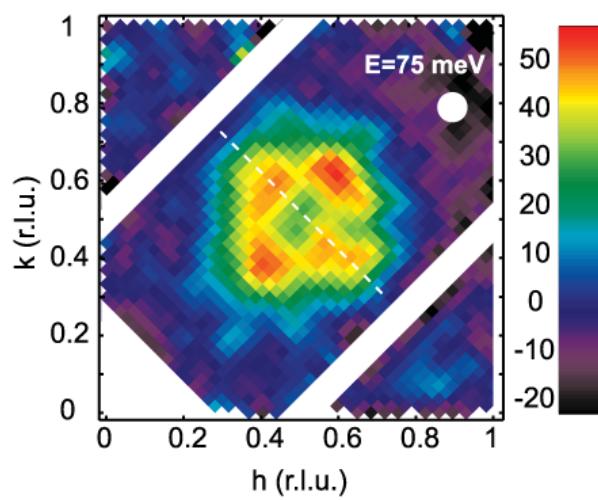
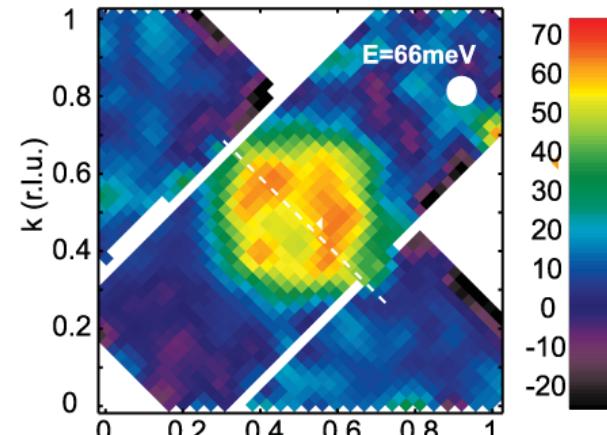
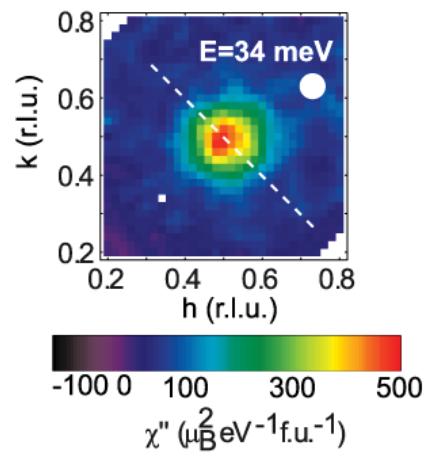
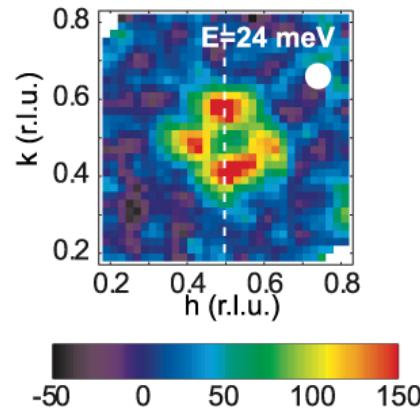


$\text{La}_{1.91}\text{Sr}_{0.09}\text{CuO}_4$ ($T_c=22$ K): \mathbf{q} -dependent slices

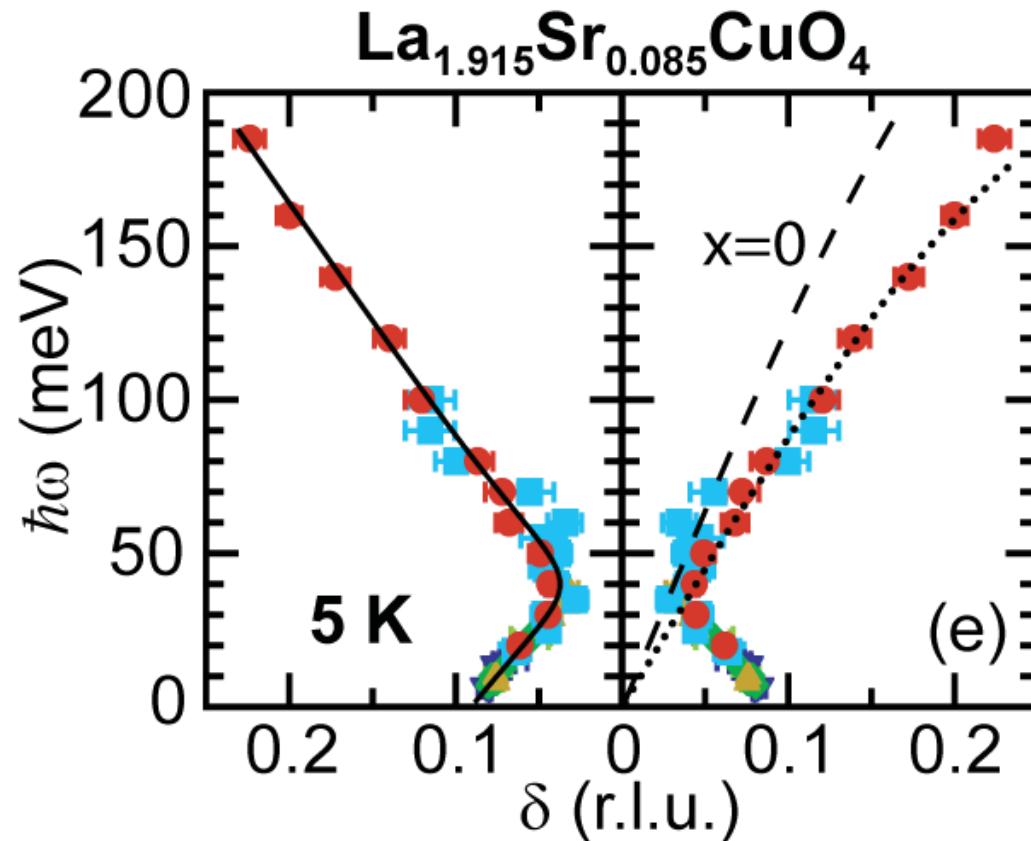


High-energy magnetic excitations in $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$

$\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$ ($T=10$ K)

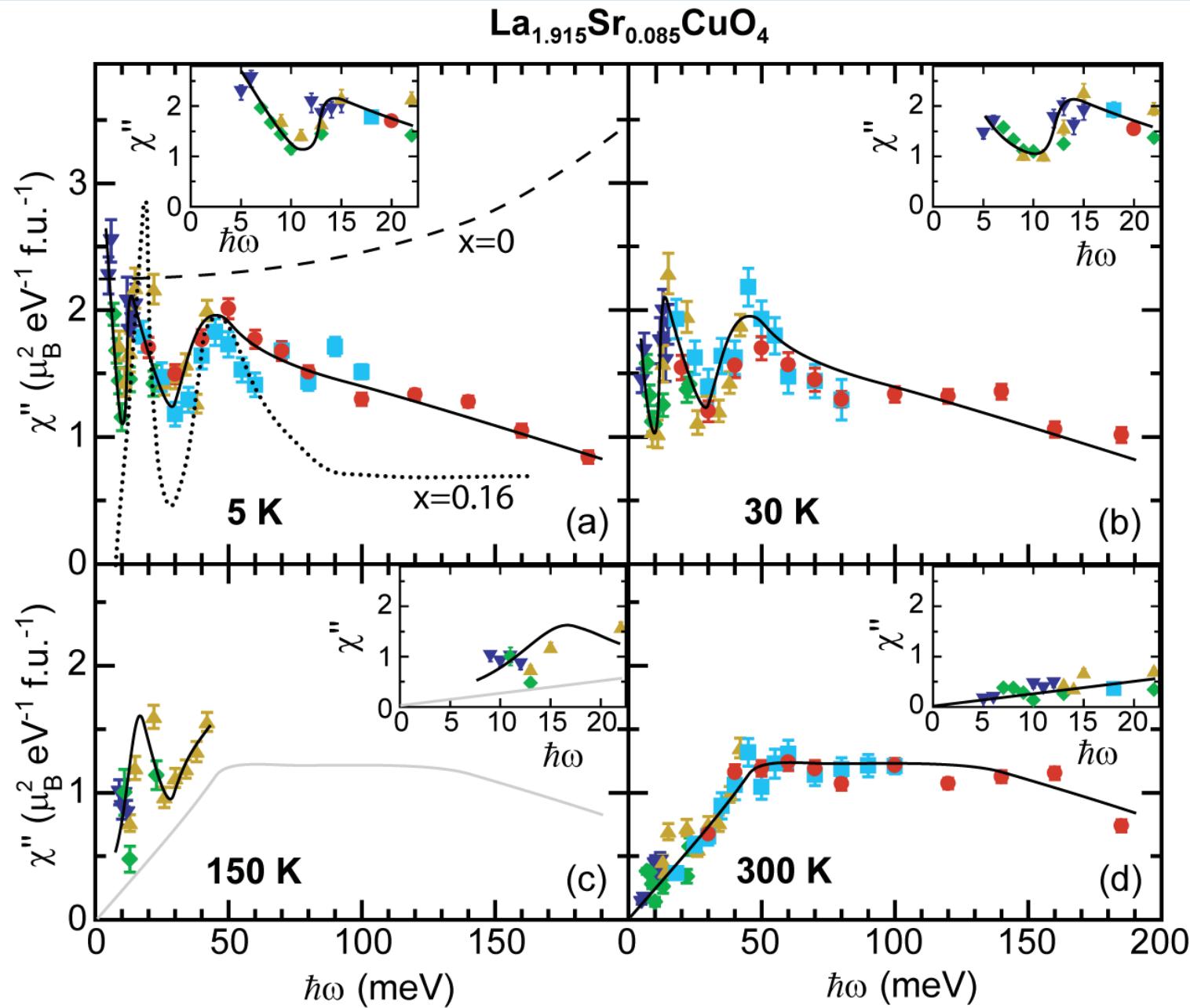


Incommensurability parameter

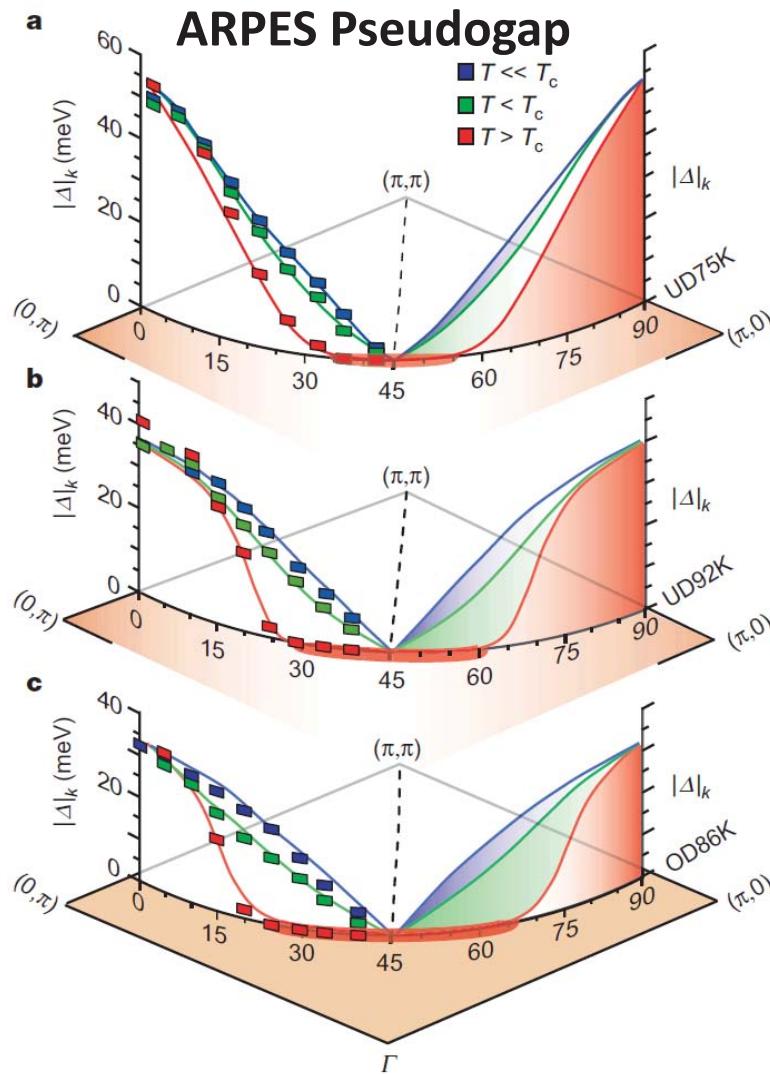


$J=87(4)$ meV [cf $J=146$ meV in La_2CuO_4]

Local Susceptibility

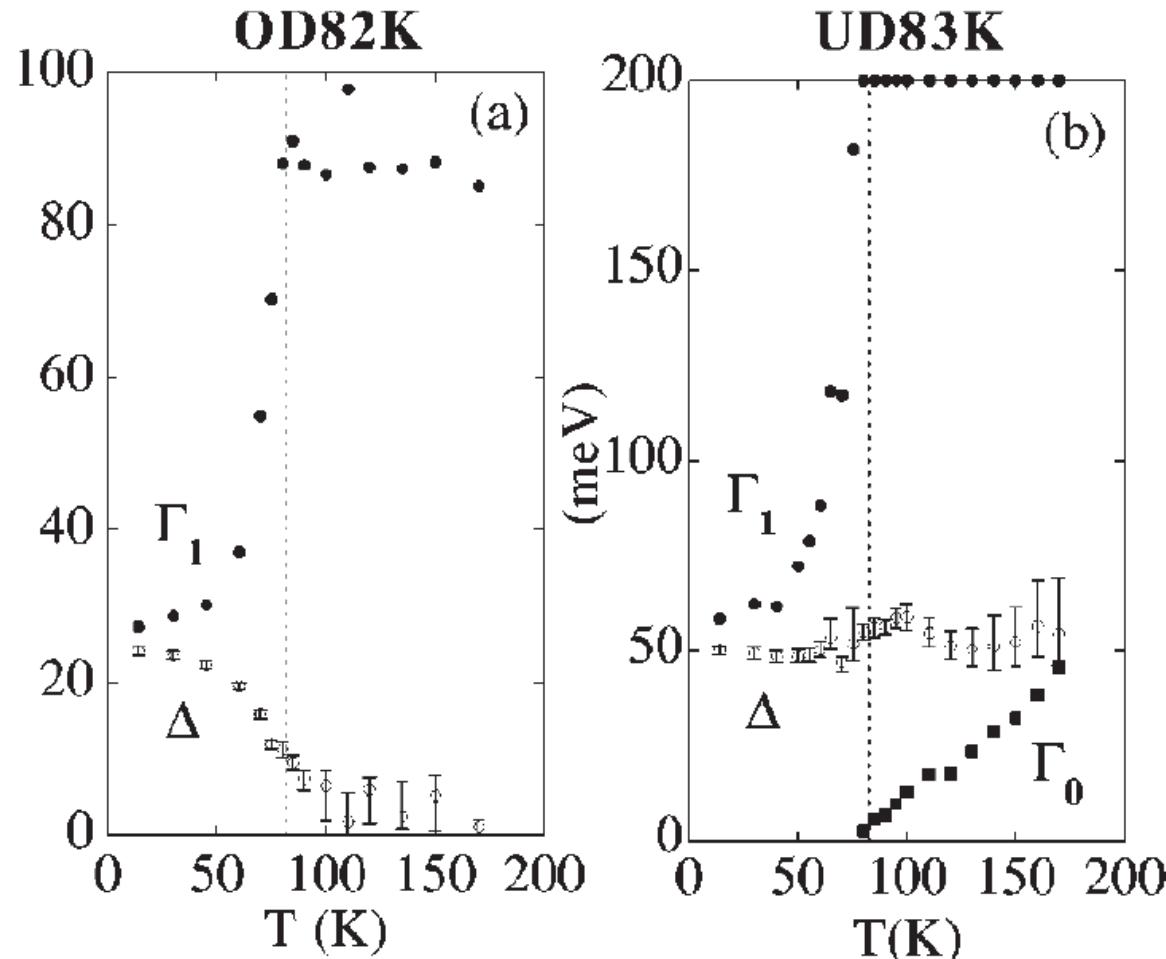


Fermionic Excitations in Cuprates



Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$
Lee et al., Nature 450, 81 (2007)

ARPES pseudogap



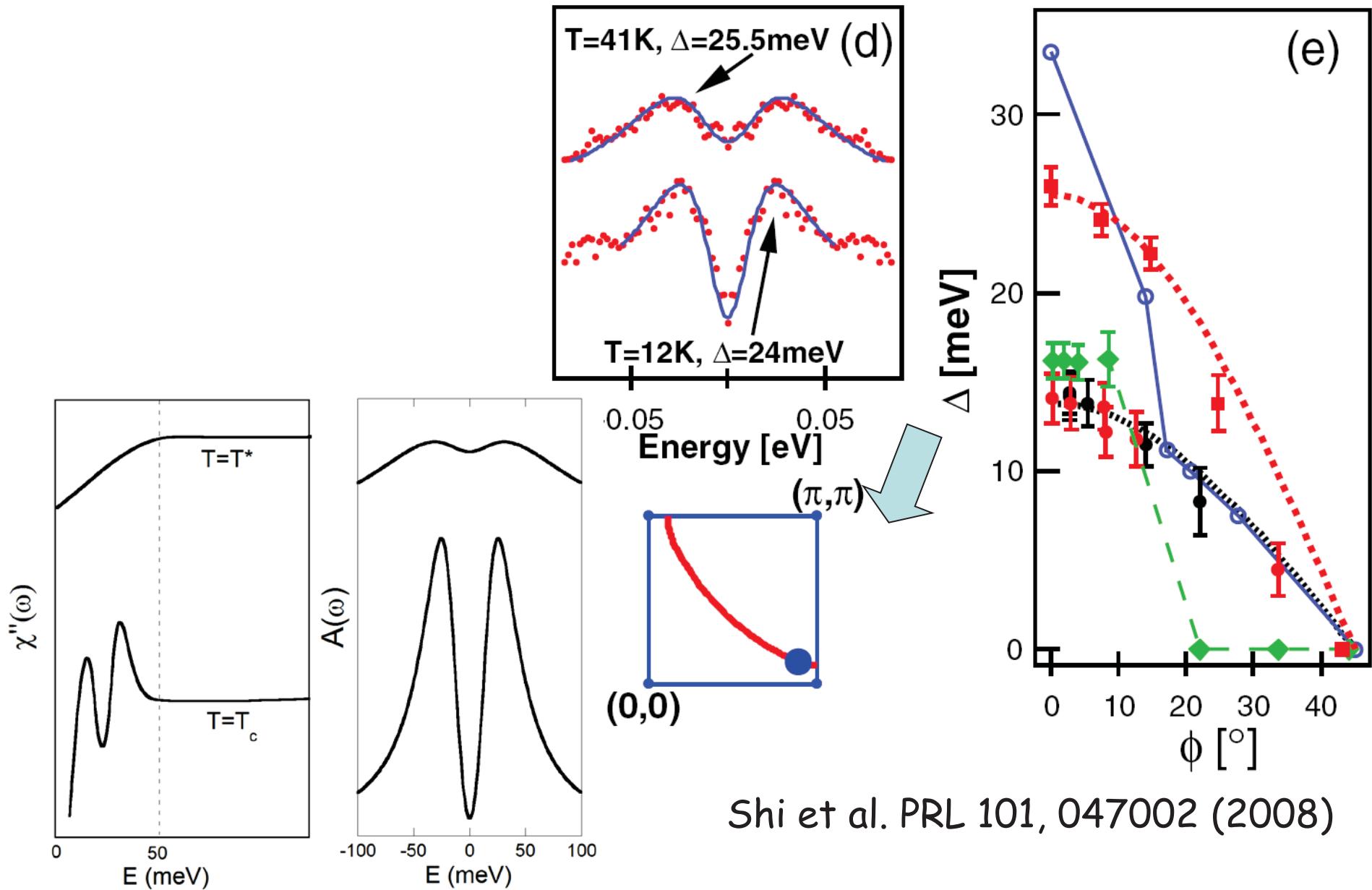
Phenomenological
self energy

$$\Sigma(\mathbf{k}, \omega) = -i\Gamma_1 + \Delta^2 / (\omega + i\Gamma_0)$$

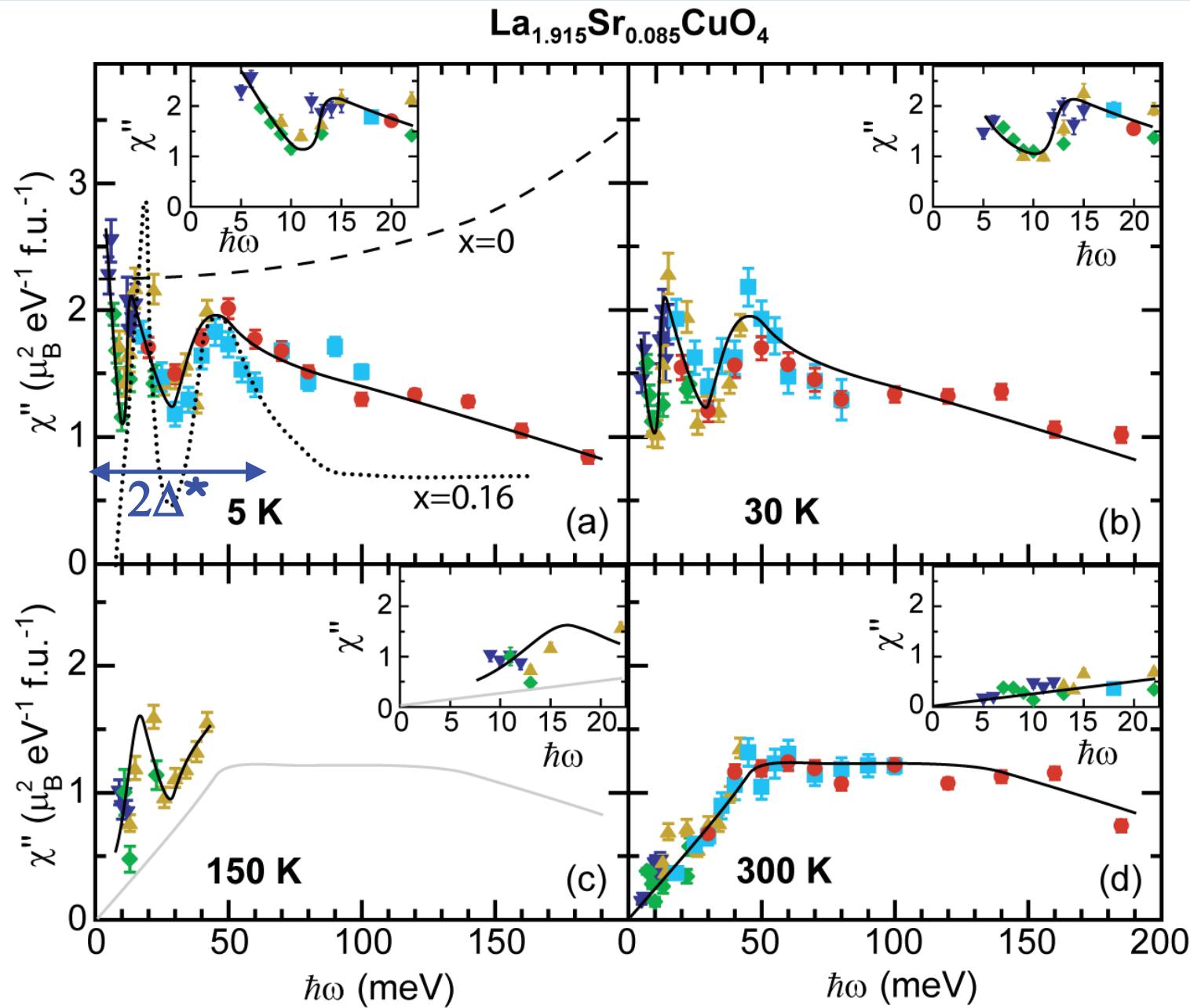
$\text{Bi}_2\text{Sr}_2\text{Ca}\text{Cu}_2\text{O}_{8+\delta}$

M. R. Norman *et al.* PRB 57, R11093 (1998).

Pseudogap and ARPES in $\text{La}_{1.895}\text{Sr}_{0.105}\text{CuO}_4$



Local Susceptibility



Summary

1) La_2CuO_4

- Néel state may be a "starting point" for ground state, but other wavefunctions must be added
- The deviations from the Néel state are strongest at $(1/2, 0)$
- Opportunity for comparison of experiment and numerical techniques
- Open issues: cyclic exchange, difference between systems, relevance to HTC

2) $\text{La}_{1.91}\text{Sr}_{0.09}\text{CuO}_4$ UD22

- Pseudogap manifests itself in the magnetic response as a reduction of damping over wide energy scale.

