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Workshop on Synergies between Field Theory and Exact Computational Methods in Strongly Correlated Quantum Matter

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Anomalous elasticity in disordered superfluids, superconductors, and magnets

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Anomalous elasticity in disordered superfluids, superconductors, and magnets

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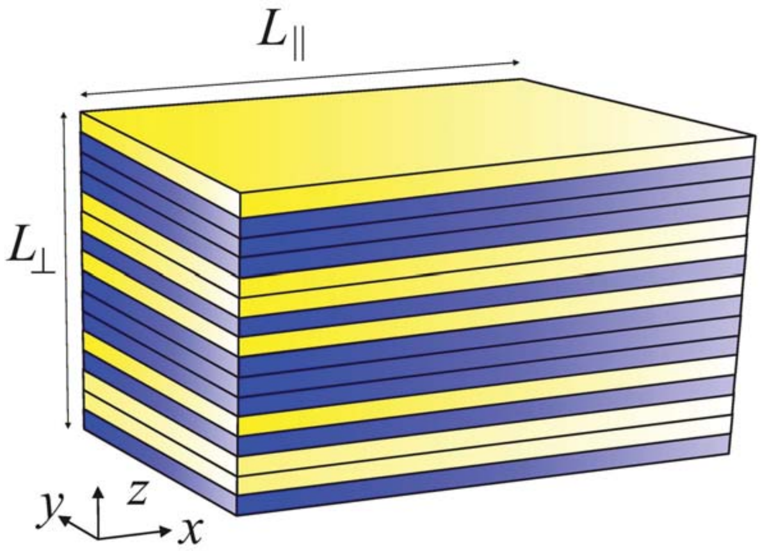
Outline

- Motivation
- Weakly disordered phase transitions
- Randomly layered superfluids, superconductivity, and XY magnets
- Randomly layered Heisenberg magnets
- Monte-Carlo simulations

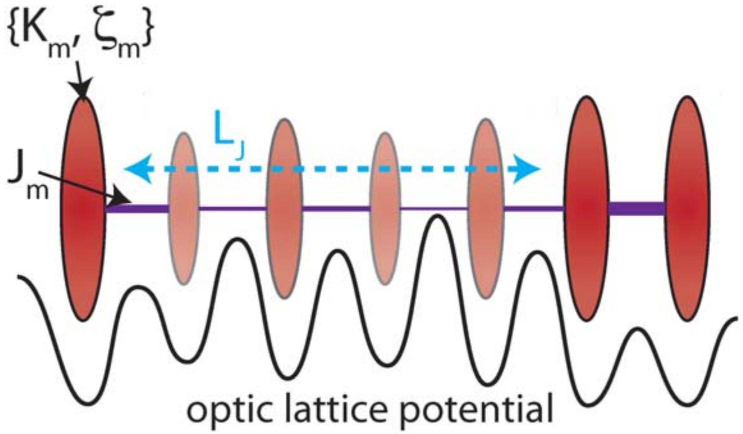
Theory: Phys. Rev. Lett. **105**, 085301 (2010), Phys. Rev. B **81**, 144407 (2010)
Preliminary simulation results: J. Phys. Conf. Series, **273**, 012004 (2011)

Randomly layered superfluids, and superconductors, and magnets



material consists of **random sequence of layers** of two materials, for example

- two different ferromagnets with different Curie temperatures
- superconducting layers of varying thickness, separated by thin insulating layers



(Pekker et al. 2010)

system can also be realized using **ultracold atoms**

- Bose-Einstein condensate in one-dimensional random optical lattice

$\Rightarrow$ two-dimensional condensate “puddles” separated by potential barriers

Question: How is the order-disorder phase transition in these systems affected by the **two-dimensional correlations** of the randomness?

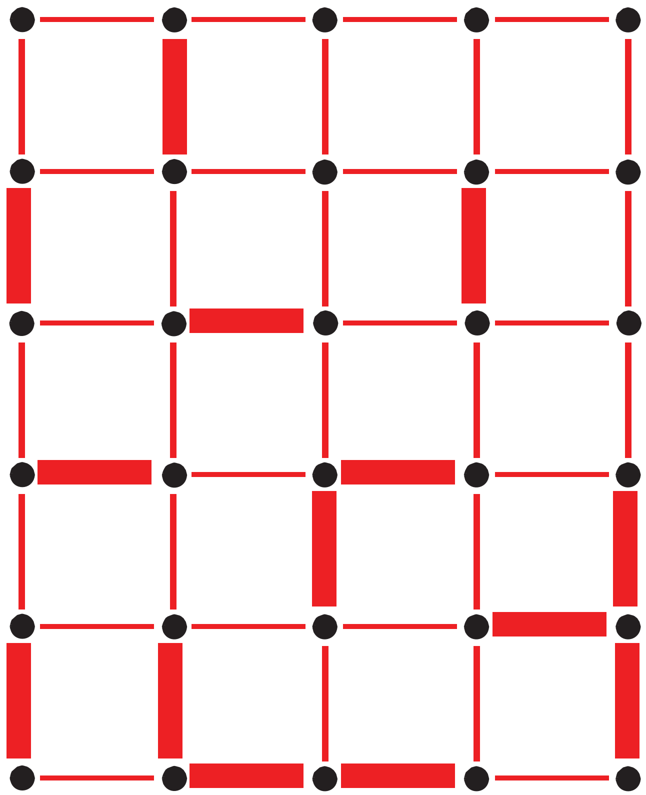
- Motivation
- **Weakly disordered phase transitions**
- Randomly layered superfluids, superconductivity, and XY magnets
- Randomly layered Heisenberg magnets
- Monte-Carlo simulations

Phase transitions and (weak) disorder

Real systems always contain
impurities and other **imperfections**

Weak (random-T_c) disorder:

spatial variation of coupling strength but
no change in character of the ordered phase



Will the phase transition remain sharp or become smeared?

Will the transition be of first order or continuous?

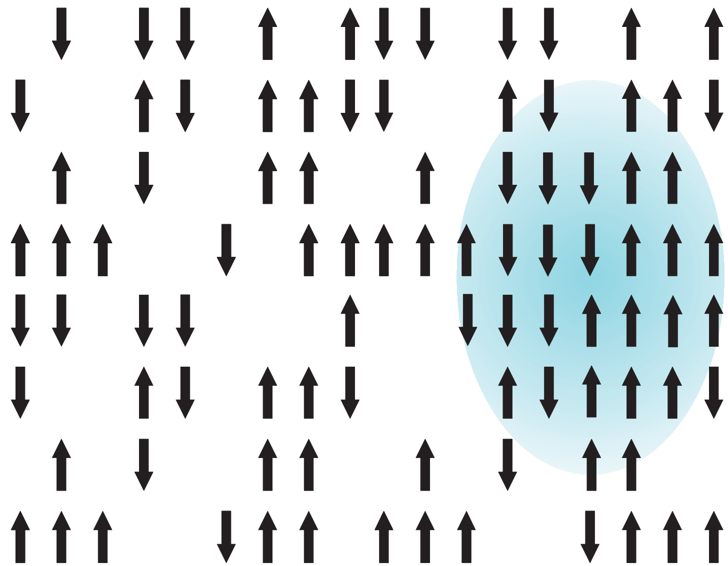
Will the critical behavior change? (Harris criterion!)

Importance of rare regions

Example: classical dilute ferromagnet

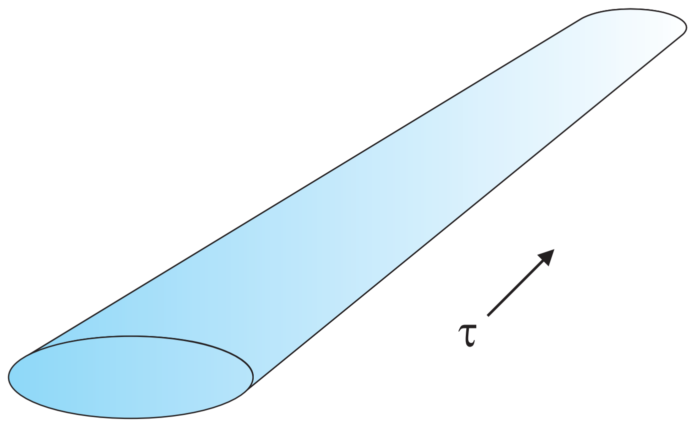
- critical temperature T_c is reduced compared to clean value T_{c0}
- for $T_c < T < T_{c0}$: no global order but local order on **rare regions** devoid of impurities
- each rare region acts as large **superspin**
- each rare region makes **large** contribution to thermodynamics

$\Rightarrow$ **Griffiths singularities** in the free energy



Disorder correlations:

- rare regions are “infinitely” large in correlated directions
- Griffiths singularities are **strongly enhanced**



Classification of phase transitions in weakly disordered systems

- order-disorder transitions in random systems can be classified by **dimensionality** d_{RR} of defects/rare regions (including imaginary time for QPTs)
- applies to transitions governed by LGW order-parameter field theories (thermal phase transitions + **some** quantum phase transitions)

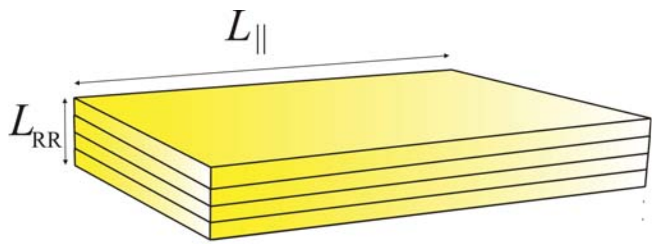
Dimension	Griffiths effects	Dirty critical point	Examples
$d_{RR} < d_c^-$	RR do not order weak essential singularity	conventional	class. magnet with point defects dilute bilayer Heisenberg model
$d_{RR} = d_c^-$	RR marginal power-law singularity	exotic (infinite randomness)	Ising model with linear defects random quantum Ising model
$d_{RR} > d_c^-$	RR order independently	smeared transition	Ising model with planar defects itinerant quantum Ising magnet

J. Phys. A **39**, R143 (2006), J. Low Temp. Phys. **161**, 299 (2010)

Randomly layered superfluids, and superconductors, and magnets

In our case:

- rare regions are stacks consisting of strongly coupled layers only
- rare regions are two-dimensional, $d_{RR} = 2$



Heisenberg symmetry:

- rare regions are exactly at d_c^- $\Rightarrow$ **exotic critical point** expected

XY symmetry:

- rare regions do not show long-range order but independently undergo **Kosterlitz-Thouless** transition

$\Rightarrow$ Question: fate of global phase transition in this special case??

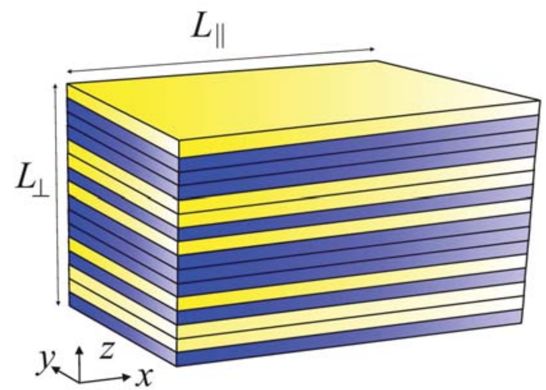
- Motivation
- Weakly disordered phase transitions
- **Randomly layered superfluids, superconductivity, and XY magnets**
- Randomly layered Heisenberg magnets
- Monte-Carlo simulations

XY model with plane defects

classical XY model on cubic lattice (use "magnetic language")

$$H = -\sum_{\mathbf{r}} J_z^{\parallel} \left(\mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+\hat{\mathbf{x}}} + \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+\hat{\mathbf{y}}} \right) - \sum_{\mathbf{r}} J_z^{\perp} \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+\hat{\mathbf{z}}}.$$

$J_z^{\parallel}$: exchange interactions within the layers
$J_z^{\perp}$: exchange interactions between the layers



$J_z^{\parallel}$ and $J_z^{\perp}$ are random functions of vertical position z

- $J_z^{\perp} \equiv J^{\perp}$ for simplicity:
- $J_z^{\parallel}$ binary distributed:

$$P(J^{\parallel}) = (1-c)\,\delta(J^{\parallel} - J_u) + c\,\delta(J^{\parallel} - J_l)$$

Overview over phase diagram

SD: strongly disordered phase at high temperatures, all layers in nonmagnetic phase

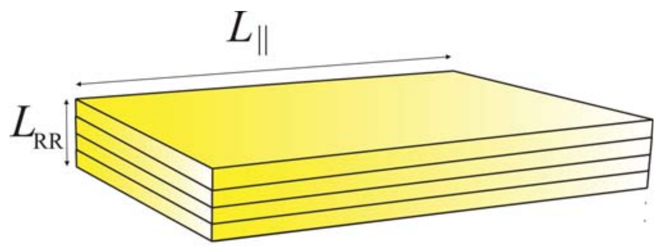
SO: strongly ordered phase at low temperatures, all layers in magnetic phase

G: Griffiths phase, locally magnetic layers coexist with locally nonmagnetic layers, the phase transition temperature, if any, must be in this region

T_u, T_l: upper and lower Griffiths temperatures, transition temperatures of clean systems having only strong or only weak bonds, respectively

Optimal fluctuation theory

crucial role is played by **rare regions**, i.e., stacks consisting of strong layers only



- probability for rare region of thickness L_{RR}: $w(L_{RR}) \sim (1-c)^{L_{RR}} = e^{-\tilde{c}L_{RR}}$
- each rare region can undergo **Kosterlitz-Thouless** transition by itself
  from finite-size scaling: $(T_u - T_{KT}(L_{RR})) \sim L_{RR}^{-1/\nu}$ with $\nu = 0.6717$ (3D XY)

$\Rightarrow$ cut-off thickness $L_c(T) \sim (T_u - T)^{-\nu}$
if $L_{RR} > L_c(T)$, RR is in **KT phase**; if $L_{RR} < L_c(T)$, RR is in disordered phase

- rare regions in KT phase have **long-range** correlations: $C(\mathbf{x}) \sim |\mathbf{x}|^{-\eta}$
  $\eta \approx \frac{1}{4} L_c(T)/L_{RR}$
- rare regions in KT phase have **infinite susceptibility**: $m \sim H^{\eta/(4-\eta)}$

Results: Magnetization

- combine KT physics within the rare regions with exponential size distribution
- close to T_u, rare regions are essentially decoupled

magnetization-field curve: $M \sim \int_{L_c(T)}^{\infty} dL_{RR}\, w(L_{RR}) H^{\eta(L_{RR})/[4-\eta(L_{RR})]}$

$\Rightarrow$ magnetization vanishes more slowly than any power with $H \to 0$

$$M \sim \exp\left(-A\sqrt{|\ln(H)|(T_u - T)^{-\nu}}\right)$$

spontaneous magnetization: take weak coupling between RRs into account

$\Rightarrow$ infinite susceptibility of RRs leads to nonzero spontaneous M for all $T < T_u$

$$\ln(M) \sim -\exp[B(T_u - T)^{-\nu}] \qquad (T \to T_u^-)$$

Results: Spin-wave stiffness

- twist the spins of two opposite boundaries by a relative angle Θ
- spin-wave stiffness ρ_s defined by free-energy difference $f(\Theta) - f(0) = \frac{1}{2}\rho_s(\Theta/L)^2$

in-plane (parallel) stiffness:

- all layers have the **same twisted BC**: $\rho_{s,\|} \sim \int_{L_c(T)}^{\infty} dL_{RR}\, w(L_{RR})\, \rho_{s,RR}(L_{RR})$
- nonzero $\rho_{s,\|}$ **appears already at** T_u: $\rho_{s,\|} \sim \exp[-C(T_u - T)^{-\nu}]$ $(T \to T_u^-)$

perpendicular stiffness:

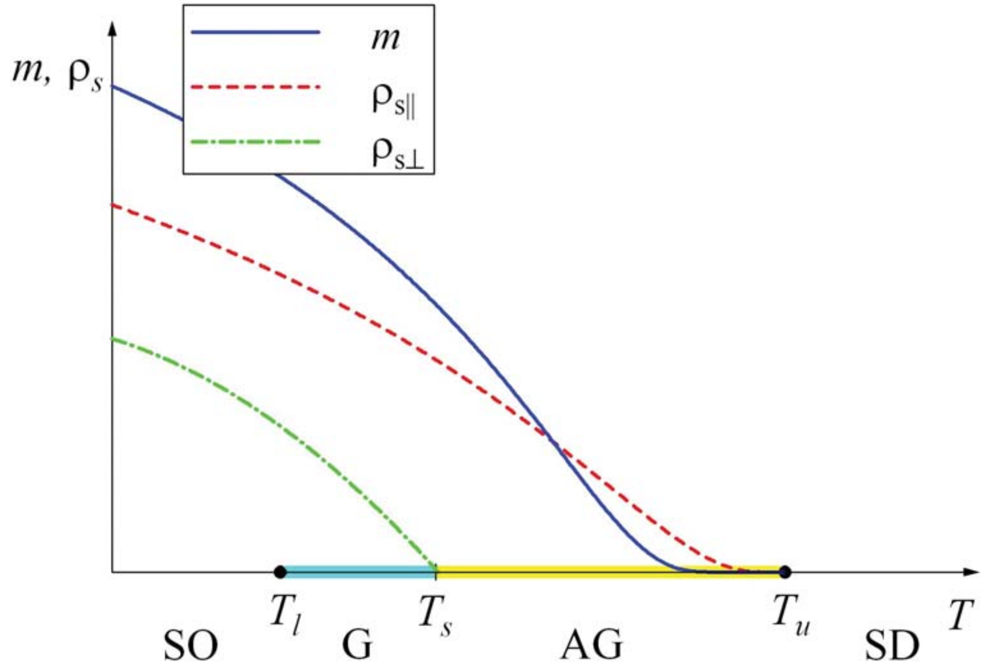
- local twists **vary from layer to layer**, occur mostly in disordered bulk
- $\rho_{s,\perp}$ is nonzero **only below** $T_s < T_u$

Anomalously elastic intermediate phase

- spontaneous magnetization and parallel stiffness appear already at upper Griffiths temperature T_u
- perpendicular stiffness appears only at a lower temperature T_s
- for $T_u > T > T_s$ system shows **anomalous elasticity,**

$$f(\Theta) - f(0) \sim \Theta^2 L_\perp^{-(1+z)}$$

with non-universal exponent $z > 1$
($z \to \infty$ at T_u and $z \to 1$ at T_s)



⇒ interplay between randomness and Kosterlitz-Thouless physics in the layers leads to **hybrid between smeared and sharp** phase transition

Alternative strong-disorder RG approach by Pekker et al. (2010)

- Motivation
- Weakly disordered phase transitions
- Randomly layered superfluids, superconductivity, and XY magnets
- **Randomly layered Heisenberg magnets**
- Monte-Carlo simulations

Heisenberg model with plane defects

classical Heisenberg model on cubic lattice

$$H = -\sum_{\mathbf{r}} J_z^{\parallel} \left(\mathbf{S_r} \cdot \mathbf{S_{r+\hat{x}}} + \mathbf{S_r} \cdot \mathbf{S_{r+\hat{y}}} \right) - \sum_{\mathbf{r}} J_z^{\perp} \, \mathbf{S_r} \cdot \mathbf{S_{r+\hat{z}}}.$$

$J_z^{\parallel}$: exchange interactions within the layers
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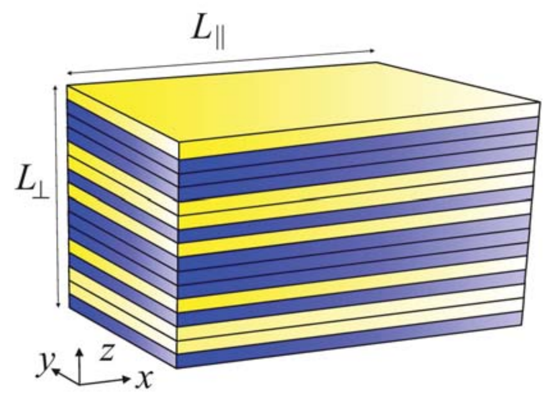


$J_z^{\parallel}$ and $J_z^{\perp}$ are random functions of vertical position z

for simplicity: $J_z^{\perp} \equiv J^{\perp}$, binary distribution of $J_{\parallel}$

$$P(J^{\parallel}) = (1-c)\,\delta(J^{\parallel} - J_u) + c\,\delta(J^{\parallel} - J_l)\ ,$$

Large-N order parameter field theory

- N-component real order parameter field $\phi_{x,y,z}$
- space is continuous in the in-plane (x, y) directions but discrete in perpendicular (z) direction
- large-N limit of an infinite number of order parameter components

Action:

$$S = \sum_{z,\mathbf{q}} \left(r_z + \lambda_z + \gamma_z^2 \mathbf{q}^2\right) |\phi_z(\mathbf{q})|^2 - \sum_{z,\mathbf{q}} J_z \, \phi_z(-\mathbf{q}) \, \phi_{z+1}(\mathbf{q})$$

r_z, $\gamma_z > 0$, $J_z > 0$: random functions of perpendicular position z

λ_z: Lagrange multiplier enforcing large-N constraint $\langle \phi^2_{x,y,z} \rangle = 1$

$\epsilon_z = r_z + \lambda_z$: renormalized (local) distance from criticality

Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- asymptotically exact if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

in our system

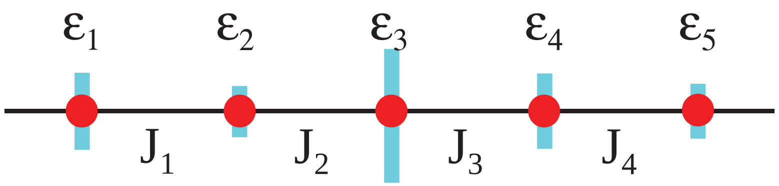
$$S = \sum_{z,\mathbf{q}} \left(\epsilon_z + \gamma_z^2 \mathbf{q}^2\right) |\phi_z(\mathbf{q})|^2 - \sum_{z,\mathbf{q}} J_z \, \phi_z(-\mathbf{q}) \, \phi_{z+1}(\mathbf{q})$$

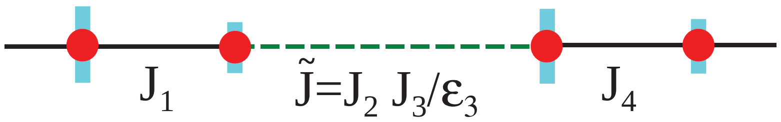
the competing local energies are:

- interactions (bonds) J_z favoring the ordered phase
- local "gaps" ϵ_z favoring the disordered phase

$\Rightarrow$ in each RG step, integrate out largest among all J_z and ϵ_z

Recursion relations

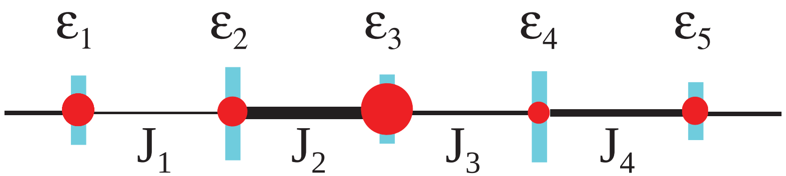


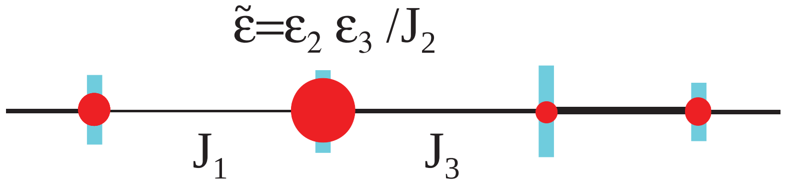


if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$:

- layer 3 is removed from the system
- coupling to neighbors is treated in 2nd order perturbation theory

new renormalized bond $\tilde{J} = J_2 J_3 / \epsilon_3$





if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$:

- spins of layers 2 and 3 are parallel
- can be replaced by single layer with moment $\tilde{\mu} = \mu_2 + \mu_3$

renormalized gap $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$

Renormalization-group flow equations

- RG step is iterated gradually reducing maximum energy Ω

$\Rightarrow$ **flow equations** for the probability distributions $P(J)$ and $R(\epsilon)$

$$-\frac{\partial P}{\partial \Omega} = [P(\Omega) - R(\Omega)]\, P + R(\Omega) \int dJ_1 dJ_2\, P(J_1)P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right)$$

$$-\frac{\partial R}{\partial \Omega} = [R(\Omega) - P(\Omega)]\, R + P(\Omega) \int d\epsilon_1 d\epsilon_2\, R(\epsilon_1)R(\epsilon_2)\, \delta\left(\epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega}\right)$$

Flow equations are identical to those of the **random transverse-field Ising chain**

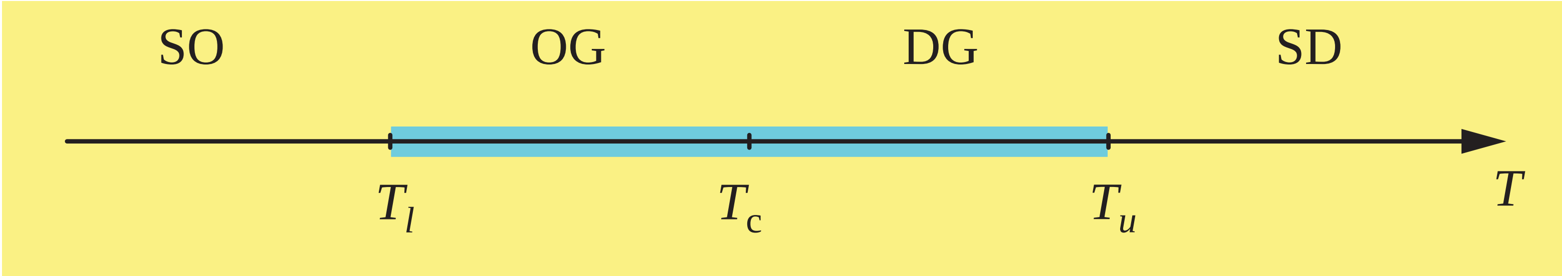
$\Rightarrow$ exotic infinite-randomness critical point

$\Rightarrow$ activated (exponential) scaling $\ln(\xi_\parallel / a) \sim \xi_\perp^\psi$ with $\psi = 1/2$

$\Rightarrow$ accompanied by power-law "quantum" Griffiths singularities

Classical transition of the 3D randomly layered Heisenberg magnet is in the same universality class as the quantum phase transition of the 1D transverse-field Ising model.

Schematic phase diagram



Phases:

SD: Strongly Disordered (conventional) paramagnetic phase

DG: Disordered Griffiths phase (rare locally ordered slabs in paramagnetic bulk)

OG: Ordered Griffiths phase (rare disordered slabs in ferromagnetic bulk)

SO: Strongly Ordered (conventional) ferromagnetic phase

T_u, T_l**:** upper and lower Griffiths temperatures (transition temperatures of hypothetical systems having only strong or only weak bonds, respectively)

Results: Magnetization

- critical behavior exactly known (very rare for phase transition in 3D)

Spontaneous magnetization:

$$m \sim (T_c - T)^{\nu(1-\phi\psi)} \qquad \text{with } \nu = 2, \psi = 1/2, \phi = (\sqrt{5}+1)/2$$

Magnetization-field curve:

$$\begin{array}{lcll} m(h) - m(0) & \sim & h^{1/(1+z)} & \text{ordered Griffiths phase} \\ m(h) & \sim & [\ln(1/h)]^{\phi - 1/\psi} & \text{at criticality} \\ m(h) & \sim & h^{1/z} & \text{disordered Griffiths phase} \end{array}$$

z is non-universal dynamical exponent of the Griffiths phase, z diverges at T_c

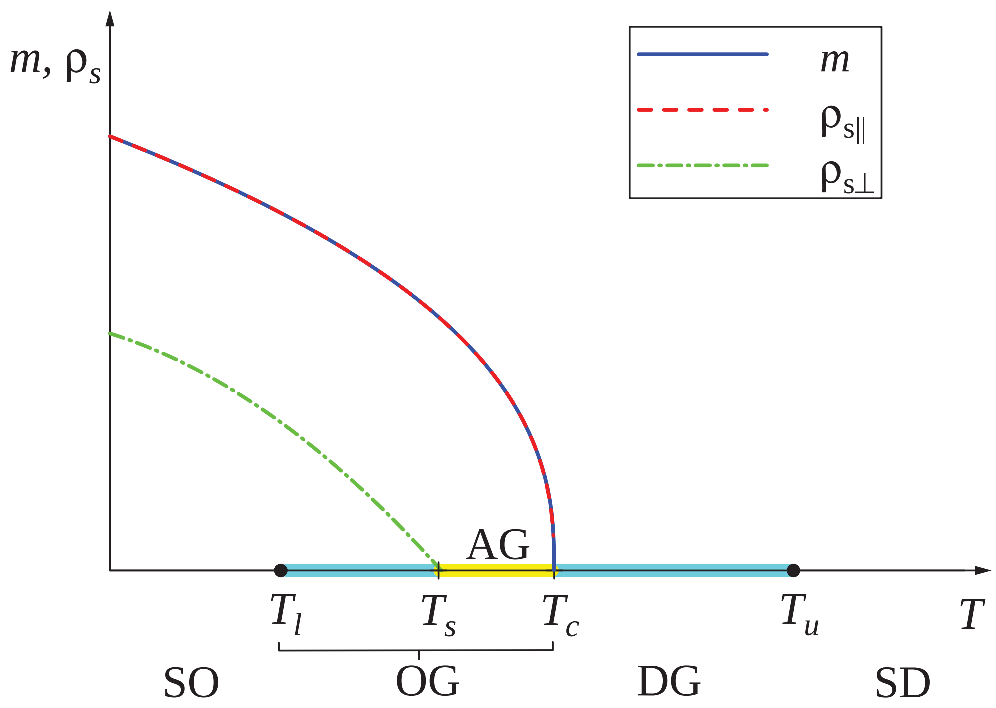
Magnetic susceptibility:

- diverges not just at critical point but in finite temperature range around T_c

Results: Spin-wave stiffness

Spin-wave stiffness:

- parallel stiffness $\rho_{\|}$ (twist in B.C. in x or y direction) scales like magnetization, $\rho_{\|} \sim m \sim (T_c - T)^{\nu(1-\phi\psi)}$
- perpendicular stiffness $\rho_{\perp}$ (twist in B.C. in z direction) nonzero only below $T_s < T_c$
- **anomalous elasticity** in part of the ordered Griffiths phase



- Motivation
- Weakly disordered phase transitions
- Randomly layered superfluids, superconductivity, and XY magnets
- Randomly layered Heisenberg magnets
- **Monte-Carlo simulations**

Why (unbiased) numerical methods?

- strong-disorder methods can at best identify possible fixed points and sometimes verify their asymptotic stability
- basins of attraction of these fixed points cannot be worked out analytically

Questions:

- Are the strong-disorder phenomena **accessible at all** in a realistic bare system?
- Is the the strong-disorder physics dominating the phase transition for **any** bare disorder strength?
- Is there a **critical disorder strength** that separates conventional from strong-disorder behavior?

Monte-Carlo simulations do not only allow us to verify or falsify the theoretically predicted strong-disorder phenomena, they also help us clarifying the fate of weakly or moderately disordered systems.

Monte-Carlo simulations of randomly layered Heisenberg model

- large-scale Monte Carlo simulations of three-dimensional Heisenberg (and XY) models with planar defects
- run in parallel on up to 300 CPUs on the Pegasus Cluster at Missouri S&T
- Wolff cluster algorithm
- finite-size scaling using system sizes up to $L_\perp = 800, L_\| = 400$
- averages over several hundred disorder configurations

Finite-size scaling of the susceptibility

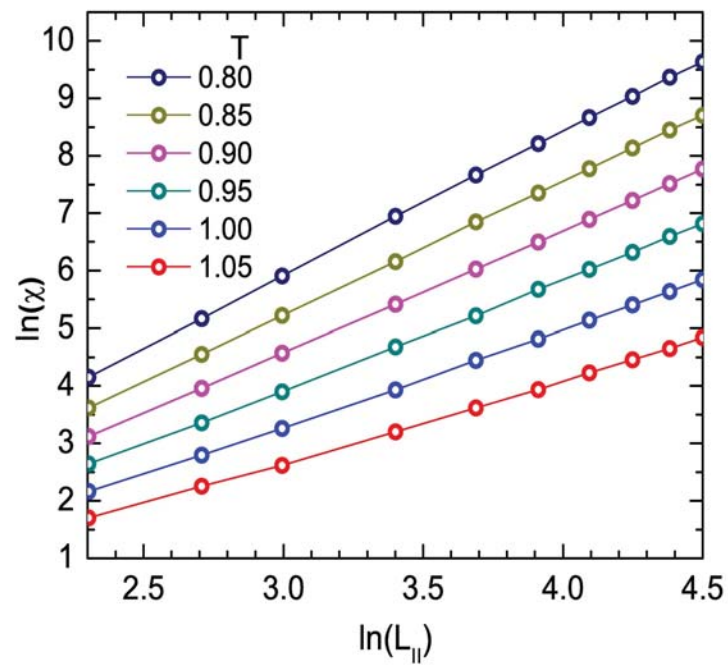
Strong-disorder RG prediction:

- finite in-plane size $L_\parallel$ cuts off singularity in local "gap" distribution because $\epsilon \gtrsim 1/L_\parallel^2$
- to find susceptibility, run RG to scale $\Omega = 1/L_\parallel^2$ and treat remaining layers as independent

$$\chi \sim L_\parallel^2 \left[\ln(L_\parallel/a)\right]^{2\phi-1/\psi} \quad \text{at criticality}$$

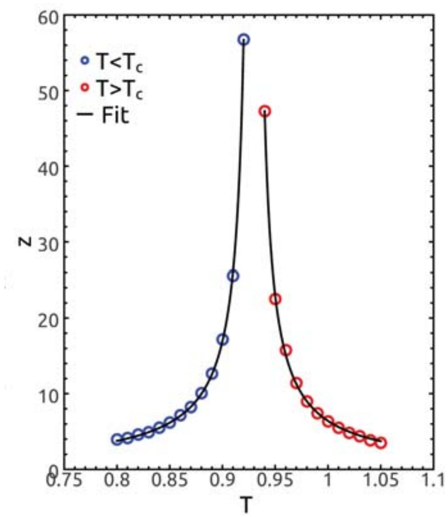
$$\chi \sim L_\parallel^{2-2/z} \quad \text{disordered Griffiths phase}$$

$$\chi \sim L_\parallel^{2+2/z} \quad \text{ordered Griffiths phase}$$



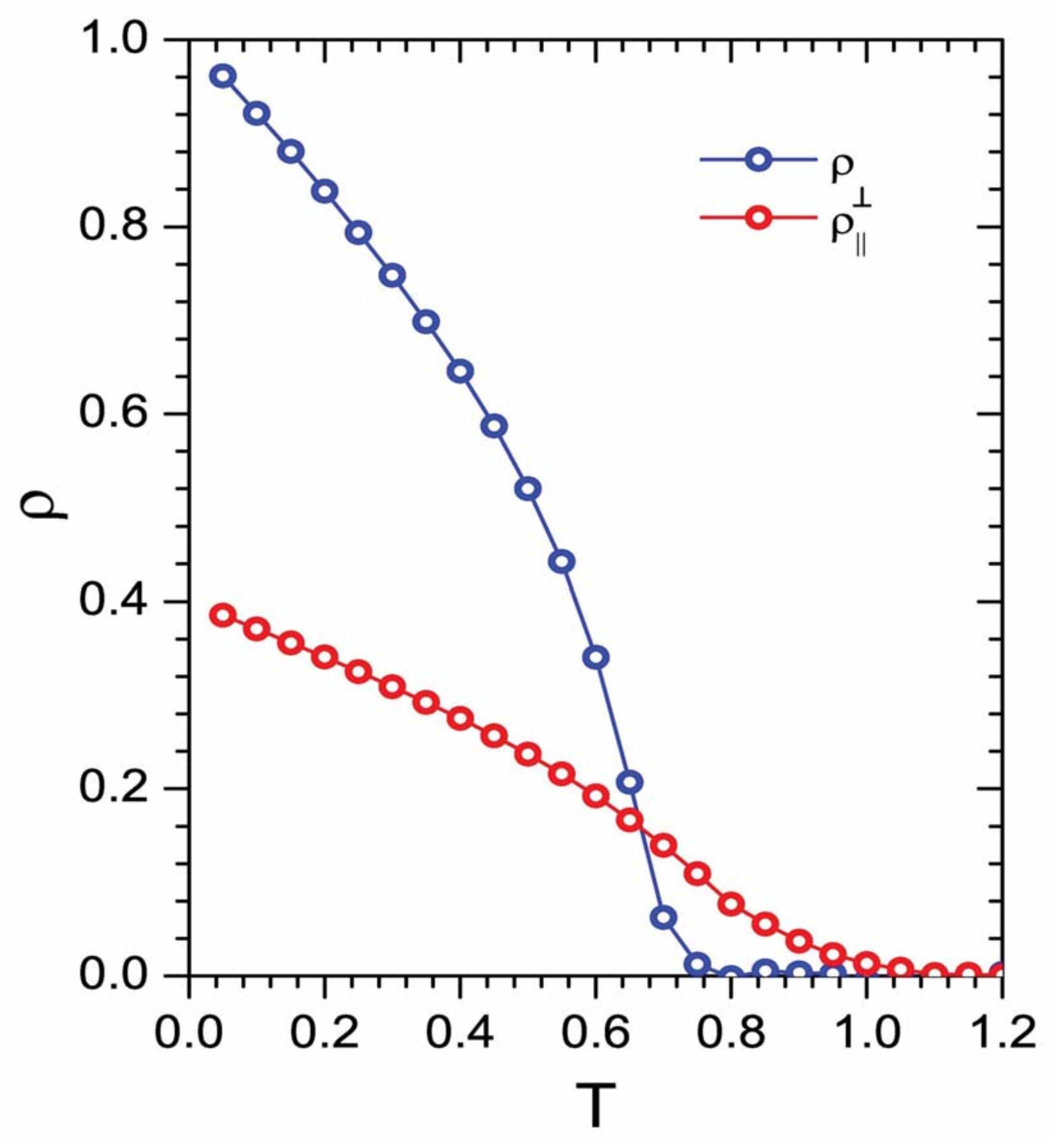
Simulations:

- Monte-Carlo data indeed show nonuniversal power law in Griffiths phase
- exponent varies in agreement with theoretical prediction $z \sim 1/(T - T_c)$



Spin-wave stiffness

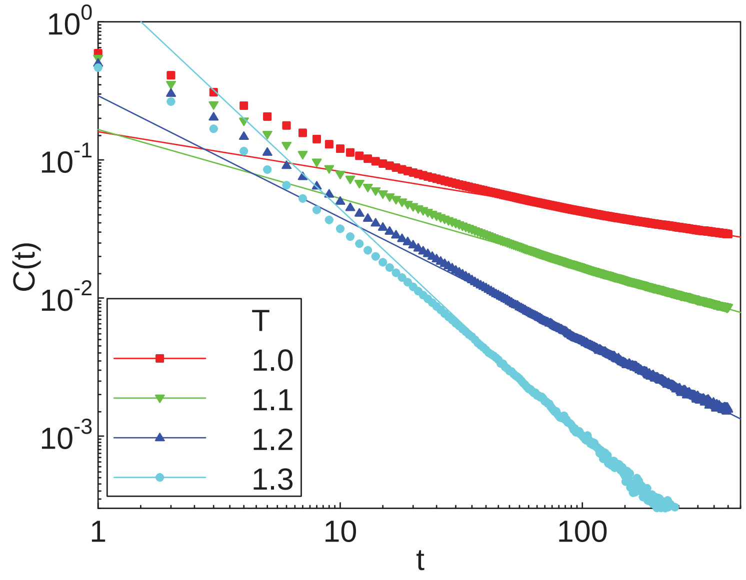
- parallel stiffness $\rho_s^{\|}$ appears at $T \approx 0.95 \approx T_c$
- perpendicular stiffness appears at lower temperature, $T \approx 0.7$
- between these temperatures: **anomalous elasticity**



Critical dynamics

Time autocorrelation function:

$$C(t) = \frac{1}{L_\perp L_\parallel^2} \int d^3r \langle \phi(\mathbf{r},t)\phi(\mathbf{r},0)\rangle$$



Strong-disorder RG prediction:

$$C(t) \sim [\ln(t/t_0)]^{\phi-1/\psi} \quad \text{at criticality}$$
$$C(t) \sim t^{-1/z} \quad \text{Griffiths phase}$$

Simulations:

- critical temperature identified as $T_c \approx 0.9$, in agreement with value from χ
- autocorrelation function indeed shows nonuniversal power-laws in Griffiths phase

Conclusions

- randomly layered superfluids, superconductors, and magnets display exotic finite-temperature behavior analogous to that found at certain disordered quantum phase transitions
- Heisenberg [O(3)] symmetry: **infinite-randomness** critical point in the same universality class as the QCP of the random transverse-field Ising chain
- XY symmetry: interplay between randomness and Kosterlitz-Thouless physics in the layers leads to **hybrid between smeared and sharp** phase transition
- in both cases: **anomalous elasticity** appears in part of the Griffiths phase, excess free energy due to twisted BC scales with nonuniversal power of system size
- can be probed in nanostructured magnets and superconductors as well as ultracold atomic gases