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"Bose-metal" type phases in models with ring exchanges

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“Bose-metal”type phases in models with ring exchanges

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Outline

1) "Bose-metals" and "Spin Bose-metals"
   - Motivation and slave particle construction; parton-gauge theory
2) Candidate models with ring exchanges
3) Frustrated hard-core boson J-K model on the square lattice and "d-wave Bose-metal"
4) Ring-only case and connection with "Exciton Bose Liquid"
   - Solvable gauge theory with flat Fermi surfaces of partons
   - Lessons (e.g., failure of the Gutzwiller wave function)
5) Realization of Exciton Bose Liquid in a K_1-K_2 hard-core boson model
6) Conclusions

Introduction and Motivation

Bose-metal:

- Quantum phase of bosons that is *neither* superfluid *nor* insulating; gapless, conducting (but not condensed), ***"metallic" phase of bosons***

Spin Bose-metal: (next talk by Donna Sheng)

- Spin system <-> hard-core bosons, Bose-metal of these
- Any gapless spin liquid phase can be viewed as a B-metal

Motivation:

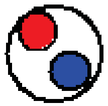
- Spin Bose-metals -- gapless spin liquid materials
- Generic Bose-metals -- new phases of bosons
- Building blocks towards understanding itinerant non-Fermi liquids of electrons (talk by Matthew Fisher)

A route to Bose-metals: “parton”approach

Simplest parton construction for bosons at generic density:

(OIM and M.P.A. Fisher 07; M.Levin and T.Senthil 08)

$$b^\dagger(\mathbf{r}) = d_1^\dagger(\mathbf{r}) d_2^\dagger(\mathbf{r})$$



Constraint:

$$d_1^\dagger d_1 = d_2^\dagger d_2 = b^\dagger b$$

Mean field and Gutzwiller-projected wave function:
treat d_1 and d_2 independently, then “glue”

$$\Psi_{\text{boson}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \Psi_{d1}(\mathbf{r}_1, \ldots, \mathbf{r}_N) \times \Psi_{d2}(\mathbf{r}_1, \ldots, \mathbf{r}_N)$$
$$= det_1[\mathbf{r}_1, \ldots, \mathbf{r}_N] \times det_2[\mathbf{r}_1, \ldots, \mathbf{r}_N]$$

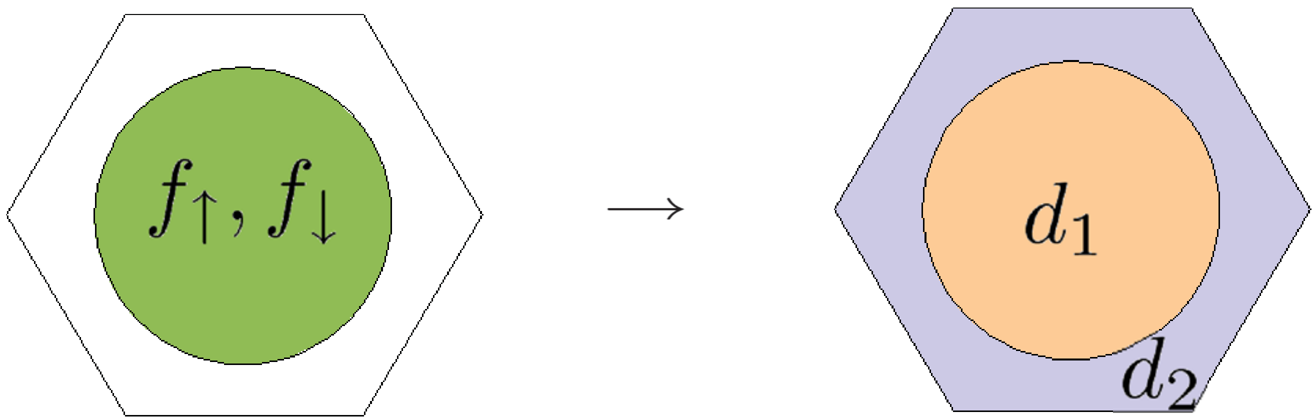
U(1) Bose-metal

Determinantal structure of the Gutzwiller-projected spinon Fermi sea (Spin Bose-Metal) wavefunction

Spins -> hard-core bosons and "parton" construction:

$$S^+ = f_\uparrow^\dagger f_\downarrow \quad \longrightarrow \quad b^\dagger = d_1^\dagger d_2^\dagger$$

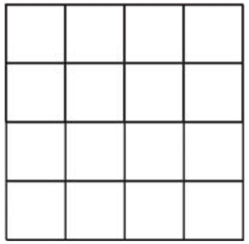
$$f_\uparrow^\dagger f_\uparrow + f_\downarrow^\dagger f_\downarrow = 1 \quad \longrightarrow \quad d_1^\dagger d_1 = d_2^\dagger d_2 = b^\dagger b$$



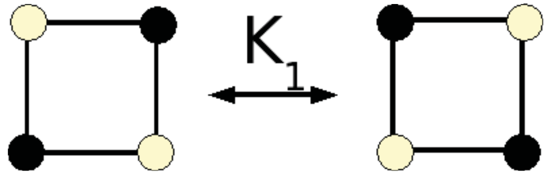
$$\Psi_{\mathrm{boson}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = det_1[\mathbf{r}_1, \ldots, \mathbf{r}_N] \times det_2[\mathbf{r}_1, \ldots, \mathbf{r}_N]$$

Hard-core boson model with ring exchanges

$$\hat{H} = -J \sum_{<ij>} b_i^\dagger b_j + K \sum_{\mathbf{r}} (b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\hat{\mathbf{x}}} b_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^\dagger b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.)$$

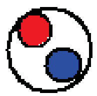


K>0 - frustration

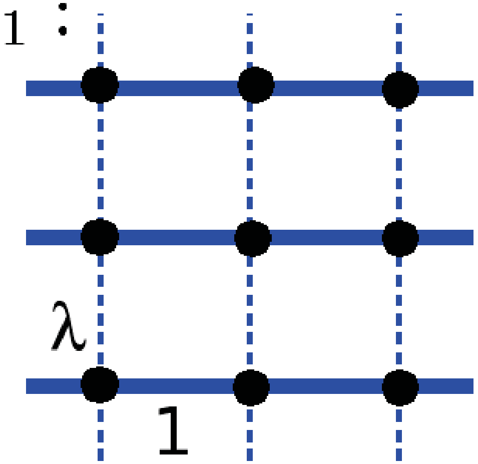


Candidate phase: "d-wave Bose-Liquid"(DBL)

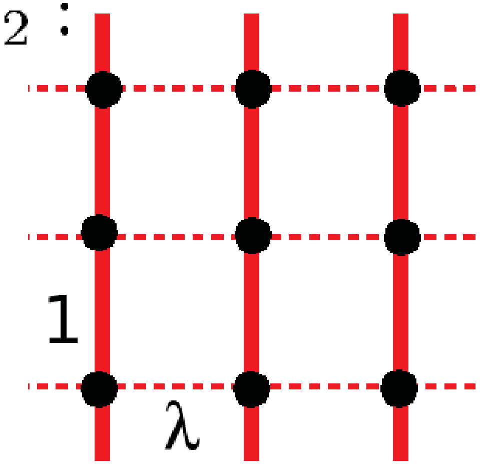
$$b^\dagger(\mathbf{r}) = d_1^\dagger(\mathbf{r}) d_2^\dagger(\mathbf{r})$$

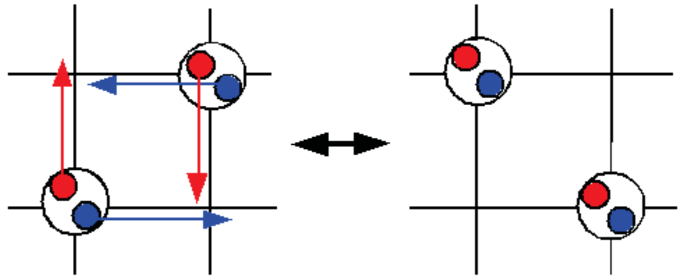


d_1 :



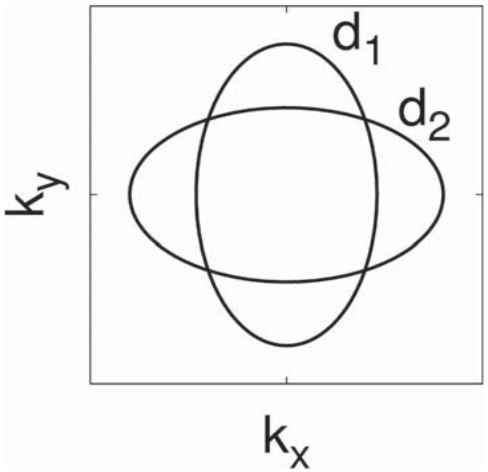
d_2 :



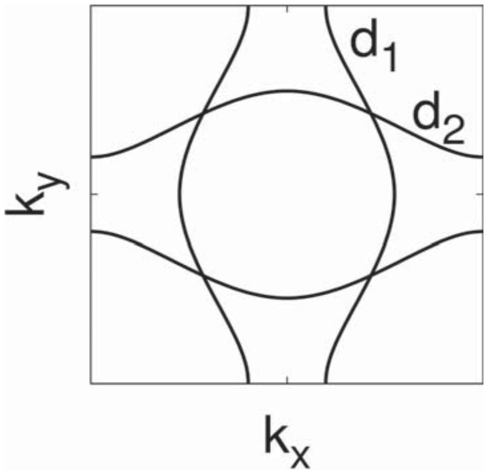


Boson wavefunction $\Psi_{bos} = det_1 \times det_2$ - respects lattice symm.

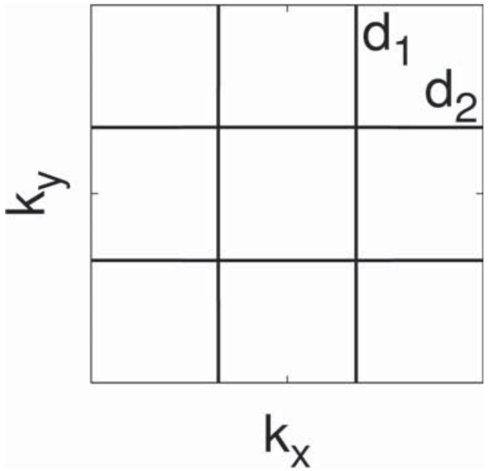
Examples of parton Fermi surfaces



D-wave Bose-Liquid (DBL)

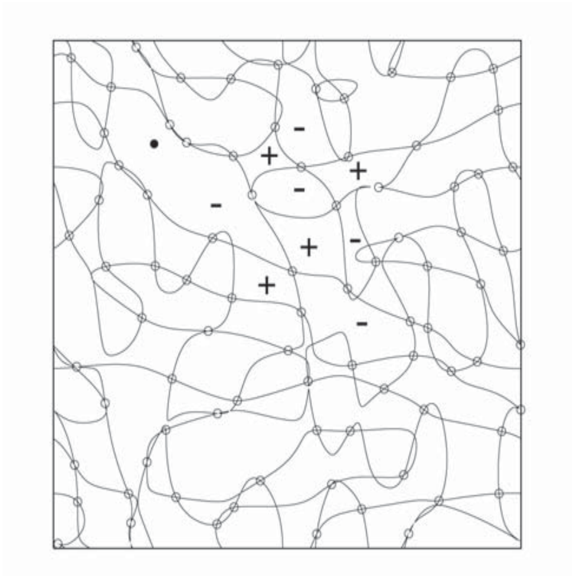


D-wave Localized Bose-Liquid (DLBL)



Extremal DLBL

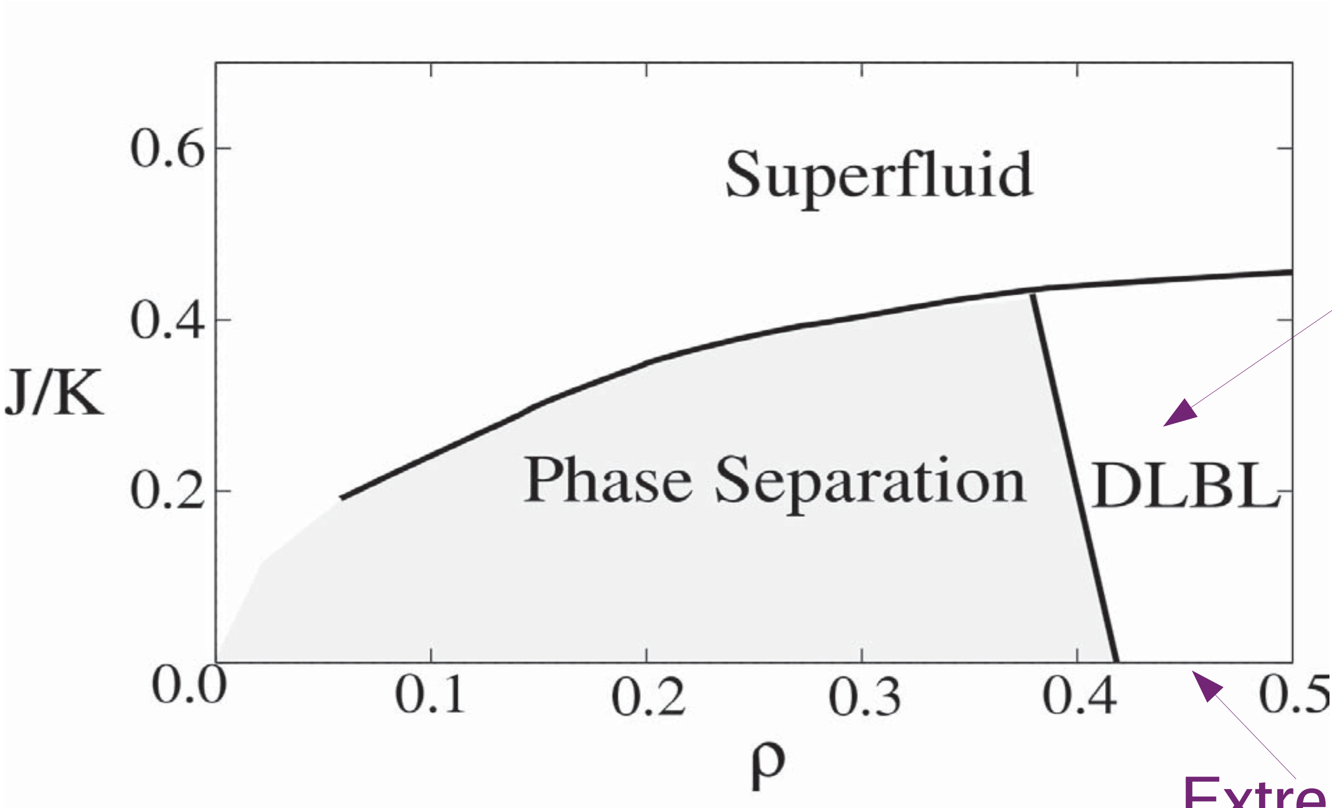
“d-wave”refers to the nodal structure in the wavefunction in real space:



Trial wave function study

$$\hat{H} = -J \sum_{<ij>} b_i^\dagger b_j + K \sum_{\mathbf{r}} (b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\hat{\mathbf{x}}} b_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^\dagger b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.)$$

"Variational Phase diagram"



D-wave Bose-metal

Extremal DLBL
~ Exciton Bose-Liquid

Beyond mean field: parton-gauge theory

U(1) gauge theory Hamiltonian:

$$
\begin{aligned}
H_{\mathrm{U}(1)} &= h\sum_{\mathbf{r},\mu} e^2_{\mathbf{r}\mu} - \kappa \sum_{\mathbf{r}} \cos(\nabla \times \boldsymbol{a})_{\mathbf{r}} \\
&- \sum_{\mathbf{r},\mu}\left[t_{1\mu} e^{i a_{\mathbf{r}\mu}} d^\dagger_{1\mathbf{r}} d_{1,\mathbf{r}+\hat{\mu}} + \mathrm{H.c.}\right] \\
&- \sum_{\mathbf{r},\mu}\left[t_{2\mu} e^{-i a_{\mathbf{r}\mu}} d^\dagger_{2\mathbf{r}} d_{2,\mathbf{r}+\hat{\mu}} + \mathrm{H.c.}\right] , \\
(\nabla \cdot \mathbf{e})_{\mathbf{r}} &= d^\dagger_{1\mathbf{r}} d_{1\mathbf{r}} - d^\dagger_{2\mathbf{r}} d_{2\mathbf{r}}
\end{aligned}
$$

Strong coupling $h \gg \kappa, t_{1,2}$ -- recover constraint $n(\mathbf{r})=n_1(\mathbf{r})=n_2(\mathbf{r})$ and generate hopping + ring Hamiltonian for $b(\mathbf{r})=d_1(\mathbf{r})d_2(\mathbf{r})$

-> $H_{\mathrm{U}(1)}$ and $H_{\mathrm{J\text{-}K}}$ contain qualitatively same physics

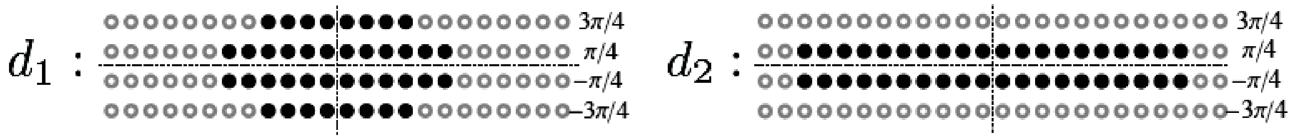
Analytical / numerical challenges

Gauge theory: difficult to treat; *is the U(1) Bose-metal a stable 2d phase?*

Direct study in 2d: very difficult because of the sign problem; *is the DBL realized in the frustrated J-K model?* (the Bose-metals are likely strongly entangled)

One approach: “ladders to rescue” (Donna's and Matthew's talks)

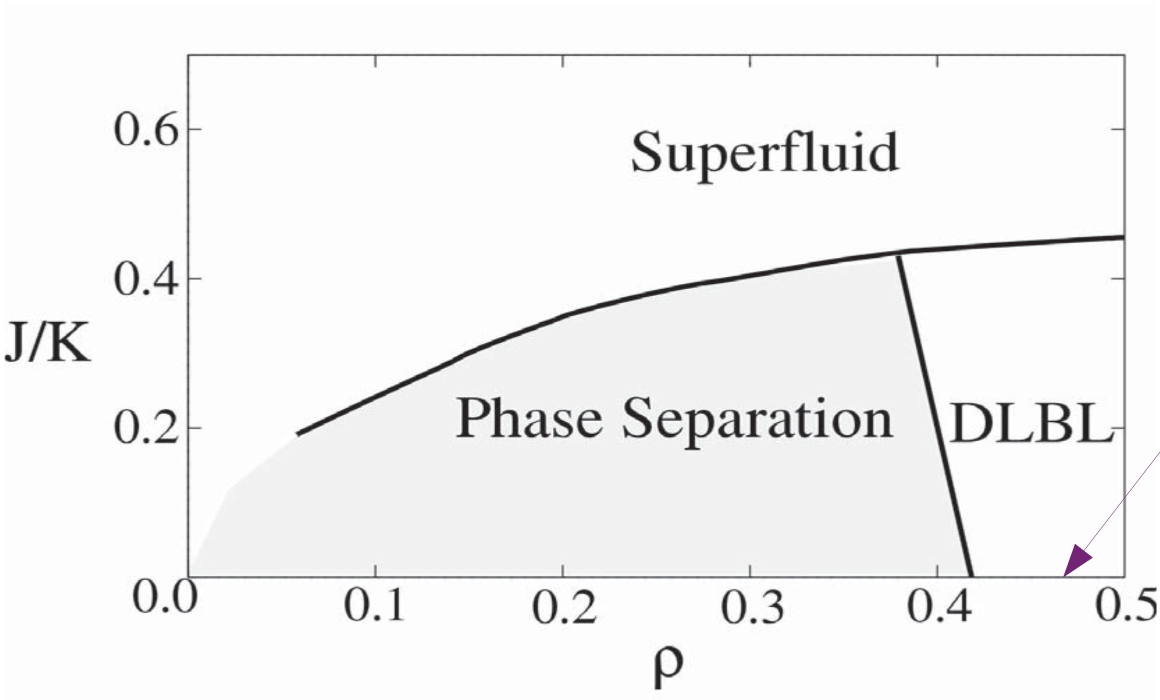
- 2-leg descendant phase DBL[2,1] (D. Sheng et. al. '08)
- 4-leg descendant phase DBL[4,2] (R. Mishmash, M. Block, D. Sheng, et. al. '11)



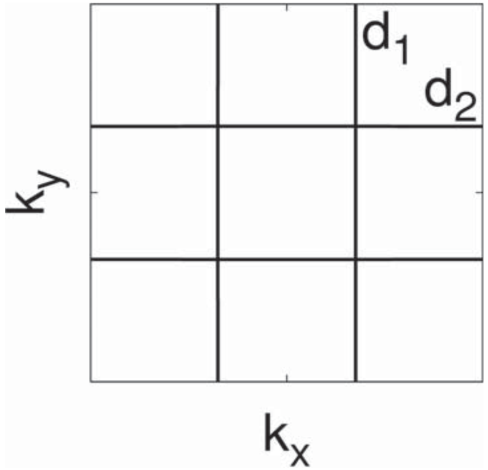
Rest of this talk: ring-only model and connection with “Exciton Bose Liquid”

Ring-only model and connection with the Exciton Bose Liquid (EBL)

$$\hat{H} = K \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.)$$

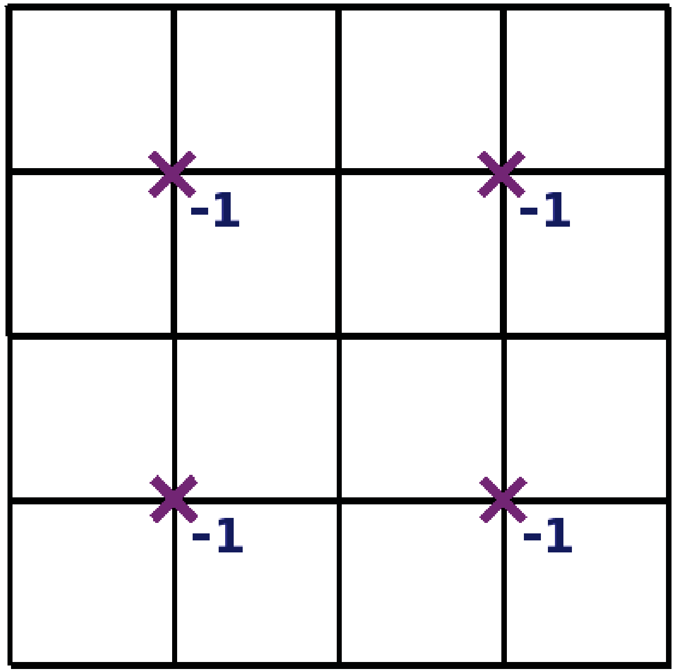


Extremal DLBL



- The parton-gauge theory for the extremal DLBL can be solved by bosonization and leads to EBL-like theory
- EBL can be realized in hard-core boson models

Rest of the talk: unfrustrated ring model



$$b(\mathbf{r}) \to \mathrm{sign}(\mathbf{r}) b(\mathbf{r})$$

- can change the sign of K_1 on all 1x1 plackets

$$\hat{H} = -K_1 \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.)$$

Parton-gauge approach to unfrustrated H_{ring}

Generic unfrustrated ring Hamiltonian (non-negative K_{mn}):

$$H_{\rm ring} = -\sum_{\mathbf{r},m,n} [K_{mn} b^\dagger_{\mathbf{r}}\ b_{\mathbf{r}+m\hat{x}}\ b^\dagger_{\mathbf{r}+m\hat{x}+n\hat{y}}\ b_{\mathbf{r}+n\hat{y}} + \mathrm{H.c.}],$$

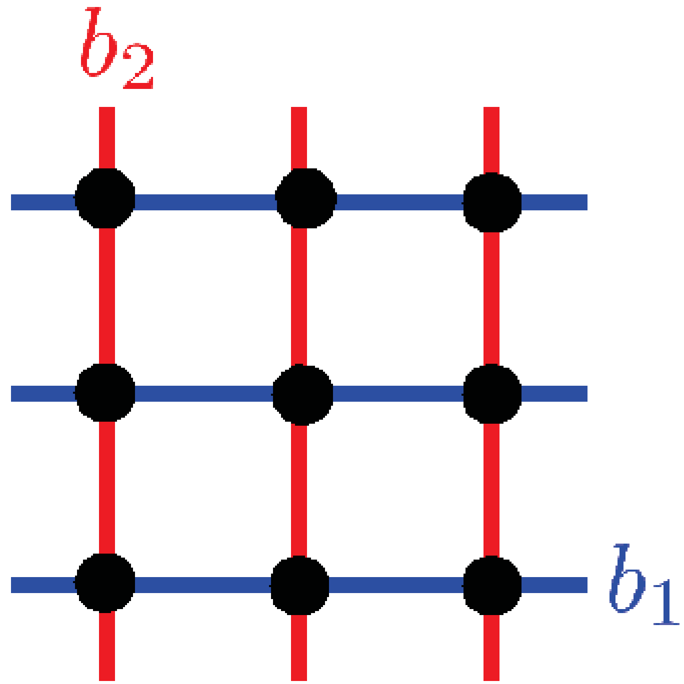
Slave particle approach using bosonic partons: $b(\mathbf{r})=b_1(\mathbf{r})b_2(\mathbf{r})$

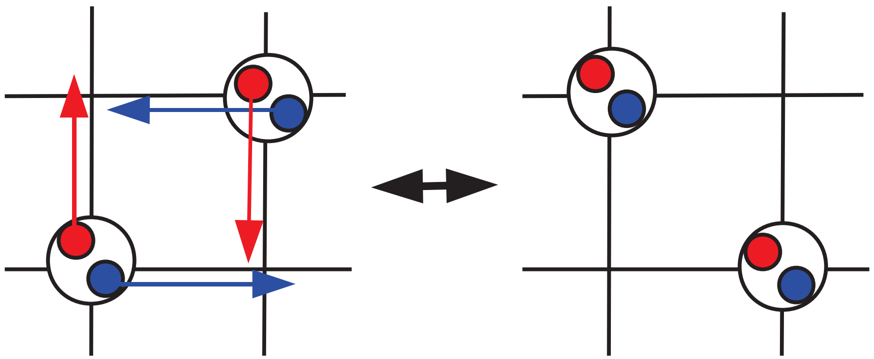
Constraint: $n(\mathbf{r})=n_1(\mathbf{r})=n_2(\mathbf{r})$

Mean field: (OIM and M.P.A. Fisher, '07)

b_1 move only in the x-dir

b_2 move only in the y-dir





Beyond mean field: parton-gauge theory

U(1) gauge theory Hamiltonian:

$$
\begin{aligned}
H_{\mathrm{U}(1)} &= h \sum_{\mathbf{r},\mu} e_{\mathbf{r}\mu}^2 - \kappa \sum_{\mathbf{r}} \cos(\nabla \times \boldsymbol{a})_{\mathbf{r}} \\
&- t \sum_{\mathbf{r}} \left[e^{i a_{\mathbf{r}x}} b_{1\mathbf{r}}^{\dagger} b_{1,\mathbf{r}+\hat{x}} + \mathrm{H.c.} \right] \\
&- t \sum_{\mathbf{r}} \left[e^{-i a_{\mathbf{r}y}} b_{2\mathbf{r}}^{\dagger} b_{2,\mathbf{r}+\hat{y}} + \mathrm{H.c.} \right] , \\
(\nabla \cdot \mathbf{e})_{\mathbf{r}} &= b_{1\mathbf{r}}^{\dagger} b_{1\mathbf{r}} - b_{2\mathbf{r}}^{\dagger} b_{2\mathbf{r}}
\end{aligned}
$$

Strong coupling limit h>>κ,t: recover constraint $n(\mathbf{r})=n_1(\mathbf{r})=n_2(\mathbf{r})$
and generate ring Hamiltonian for $b(\mathbf{r})=b_1(\mathbf{r})b_2(\mathbf{r})$
-> $H_{U(1)}$ and H_{ring} contain qualitatively same physics

Beyond mean field: bosonized continuum theory

$$\mathcal{L}[\varphi_1,\theta_1,\varphi_2,\theta_2,a_x,a_y] = \sum_{\mathbf{r}} \frac{\kappa}{2}(\nabla_x a_y - \nabla_y a_x)^2$$

$$+\sum_{\mathbf{r}} \left[\frac{J}{2}(\nabla_x\varphi_1 - a_x)^2 + \frac{u}{2}\left(\frac{\nabla_x\theta_1}{\pi}\right)^2 + \frac{i}{\pi}\partial_\tau\varphi_1\nabla_x\theta_1\right]$$

$$+\sum_{\mathbf{r}} \left[\frac{J}{2}(\nabla_y\varphi_2 + a_y)^2 + \frac{u}{2}\left(\frac{\nabla_y\theta_2}{\pi}\right)^2 + \frac{i}{\pi}\partial_\tau\varphi_2\nabla_y\theta_2\right].$$

+ constraint $n_1 = \dfrac{\nabla_x\theta_1}{\pi} = n_2 = \dfrac{\nabla_y\theta_2}{\pi} = n = \dfrac{\nabla^2_{xy}\vartheta}{\pi}$

Solve $\theta_1 = \nabla_y\vartheta,\quad \theta_2 = \nabla_x\vartheta$ and integrate out fields:

$$\boxed{\mathcal{L}_{\mathrm{EBL}}[\vartheta] = \sum\left[\frac{u}{\pi^2}(\nabla^2_{xy}\vartheta)^2 + \frac{1}{2\pi^2 J}(\partial_\tau\boldsymbol{\nabla}\vartheta)^2 + \frac{1}{2\pi^2\kappa}(\partial_\tau\vartheta)^2\right]}$$

(less important)

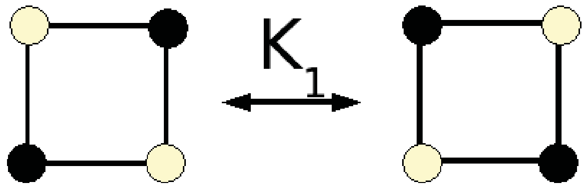
-Dual EBL theory of Paramekanti et al!

$\omega_{\mathrm{EBL}}(\mathbf{k}) \sim |k_x k_y|$

- Partons are strongly coupled; the ultimate description is the EBL
- NOT a sliding or cross-sliding Luttinger liquid, e.g. non-FL specific heat C ~ T log(1/T)

Exciton Bose Liquid (EBL) precis

$$\hat{H}_{\text{ring}} = -K \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.)$$



K>0 –no frustration

Exciton Bose Liquid (EBL) theory (Paramekanti, Balents, and Fisher, '02)

$$\hat{H}_{\text{rotor}} = \frac{U}{2}\sum_{\mathbf{r}}(n_{\mathbf{r}} - \bar{n})^2 - K\sum_{\mathbf{r}} \cos(\phi_{\mathbf{r}} - \phi_{\mathbf{r}+\hat{\mathbf{x}}} + \phi_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} - \phi_{\mathbf{r}+\hat{\mathbf{y}}})$$

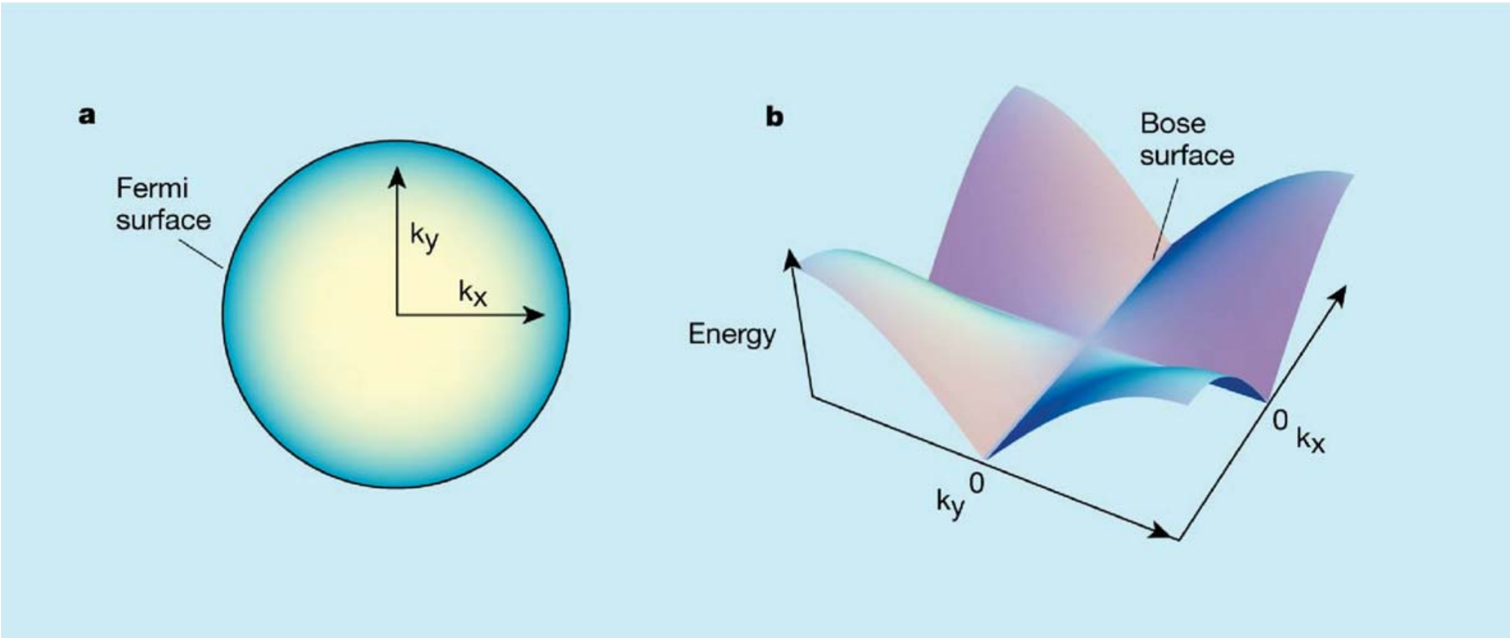
$$\begin{aligned} \hat{H}_{\text{“spinwaves”}} &= \frac{U}{2}\sum_{\mathbf{r}}(\delta n_{\mathbf{r}})^2 + \frac{K}{2}\sum_{\mathbf{r}}(\varphi_{\mathbf{r}} - \varphi_{\mathbf{r}+\hat{\mathbf{x}}} + \varphi_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} - \varphi_{\mathbf{r}+\hat{\mathbf{y}}})^2 \\ &= \frac{U}{2}\sum_{\mathbf{k}}|\delta n_{\mathbf{k}}|^2 + \frac{K}{2}\sum_{\mathbf{k}}|4\sin(k_x/2)\sin(k_y/2)|^2|\varphi_{\mathbf{k}}|^2 \end{aligned}$$

EBL theory: Bose Surfaces

$$\hat{H}_{\text{"spinwaves"}} = \frac{U}{2}\sum_{\mathbf{k}} |\delta n_{\mathbf{k}}|^2 + \frac{K}{2}\sum_{\mathbf{k}} |4\sin(k_x/2)\sin(k_y/2)|^2 |\varphi_{\mathbf{k}}|^2$$

$$\omega_{\mathbf{k}} = \sqrt{KU}|4\sin(k_x/2)\sin(k_y/2)|$$

-- vanishes on lines (k_x=0, k_y) and (k_x, k_y=0) -- **Bose surfaces!**



S. Sachdev, "Scratching Bose Surface" (Nature News and Views, '02)

Stability of EBL - Does the EBL spin wave theory neglecting compactness of angles (= discreteness of boson numbers) correspond to ***a stable phase***?

$$\mathcal{L}_{\text{spinwave}}[\varphi] = \sum \left[\frac{K}{2} (\nabla_{xy}^2 \varphi)^2 + \frac{1}{2U} (\partial_\tau \varphi)^2 \right]$$ + nonlocal topo. defects in φ

Paramekanti et al. - ***YES!*** - by studying dual formulation

$$\mathcal{L}_{\text{dual}}[\vartheta] = \sum \left[\frac{U}{2\pi^2} (\nabla_{xy}^2 \vartheta)^2 + \frac{1}{2\pi^2 K} (\partial_\tau \vartheta)^2 \right]$$ + local cosines in ϑ

$$\delta n = \frac{\nabla_{xy}^2 \vartheta}{\pi}$$

EBL is stable for sufficiently large K/U:

K/U > 9/64 = 0.140625 at ρ=1/2 (Paramekanti, Balents, and Fisher '02)

K/U > 1/16 = 0.0625 at generic ρ<1/2 (Xu and Moore '05, Xu and Fisher '07)

Some technical remarks about parton-gauge approach and EBL theory

1) Compactness of the parton phase variables
-> cosines in the dual θ variables, e.g.

$$-\mathfrak{u}_j \cos[2\theta_1(x,y) + 2\theta_1(x,y+j) + 4\pi\bar{n}x - 2\pi\bar{n}] \quad -\text{umklapp}$$

$$-\mathfrak{h}_j \cos[2\theta_1(x,y) - 2\theta_1(x,y+j)] \quad -\text{non-umklapp}$$

-- can become relevant and destabilize the EBL fixed point

2) Compactness of the gauge field variables
-> cosines in the dual υ variable

$$-\mathfrak{v}_q \cos[q(2\vartheta + 2\pi\bar{n}XY)]$$

(Paramekanti, Balents, and Fisher '02)

-- rendered absent for incommensurate particles density and irrelevant at the EBL fixed point
-- when the EBL fixed point is unstable, these terms are ultimately important to determine the resulting phases

Theory for the parton-Gutzwiller wavefunction

$$\mathcal{L}[\varphi_1, \theta_1, \varphi_2, \theta_2, \cancel{a_x}, \cancel{a_y}] = \sum_{\mathbf{r}} \cancel{\frac{\kappa}{2}(\nabla_x a_y - \nabla_y a_x)^2}$$

$$+\sum_{\mathbf{r}} \left[\frac{J}{2}(\nabla_x \varphi_1 - \cancel{a_x})^2 + \frac{u}{2}\left(\frac{\nabla_x \theta_1}{\pi}\right)^2 + \frac{i}{\pi}\partial_\tau \varphi_1 \nabla_x \theta_1\right]$$

$$+\sum_{\mathbf{r}} \left[\frac{J}{2}(\nabla_y \varphi_2 + \cancel{a_y})^2 + \frac{u}{2}\left(\frac{\nabla_y \theta_2}{\pi}\right)^2 + \frac{i}{\pi}\partial_\tau \varphi_2 \nabla_y \theta_2\right] .$$

+ constraint $\quad n_1 = \dfrac{\nabla_x \theta_1}{\pi} = n_2 = \dfrac{\nabla_y \theta_2}{\pi}$

Solve $\quad \theta_1 = \nabla_y \vartheta, \quad \theta_2 = \nabla_x \vartheta \quad$ and integrate out fields:

$$\boxed{\mathcal{L}_{\text{Gutzw}}[\vartheta] = \sum \left[\frac{u}{\pi^2}(\nabla^2_{xy}\vartheta)^2 + \frac{1}{2\pi^2 J}(\partial_\tau \boldsymbol{\nabla}\vartheta)^2\right]}$$

$\omega(\mathbf{k}) \sim |k_x k_y|/|\mathbf{k}|$ - qualitatively different from the long-wavelength EBL theory!
- similar to cross-sliding Luttinger liquid, e.g., specific heat C ~ T

Failure of the Gutzwiller wavefunction

Dispersion and specific heat:

$$\omega_{\mathrm{EBL}}(\mathbf{k}) \sim |k_x k_y| \qquad \omega_{\mathrm{Gutzw}}(\mathbf{k}) \sim |k_x k_y|/|\mathbf{k}|$$

$$C_{\mathrm{EBL}}(T) \sim T \log(1/T) \qquad C_{\mathrm{Gutzw}}(T) \sim T$$

Density structure factor: $S(\mathbf{k}) = \langle n_{\mathbf{k}} n_{-\mathbf{k}} \rangle$

$$S(k_x \to 0, k_y) = \alpha(k_y)|k_x|$$

$$\alpha_{\mathrm{EBL}}(k_y) \sim |k_y| \qquad \alpha_{\mathrm{Gutzw}}(k_y) \sim \mathrm{const} > 0$$

Exciton propagator: $E_m^\dagger(\mathbf{r}) = b^\dagger(\mathbf{r}) b(\mathbf{r} + m\hat{\mathbf{y}})$

$$\langle E_m^\dagger(0,0) E_m(x,0) \rangle \sim |x|^{-\eta(m)}$$

$$\eta_{\mathrm{EBL}}(m) \sim \log(m) \qquad \eta_{\mathrm{Gutzw}}(m) \sim \mathrm{const}$$

The Gutzwiller wavefunction fails to provide qualitative improvement over the mean field! (Tay and OIM, '11)

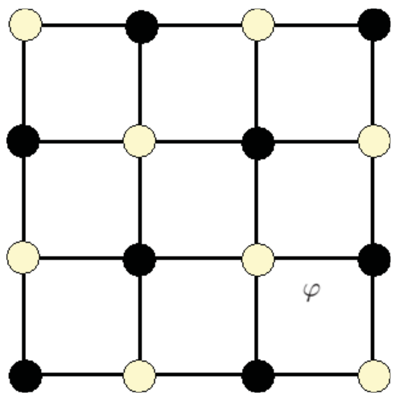
Realizing EBL in a hard-core boson model

Simplest model –no EBL

$$\hat{H}_{\text{ring}} = -K \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.)$$

A. Sandvik et al '02; Melko et al '04;
Rousseau et al '04, '05

At half-filling, ρ=1/2: found to give CDW



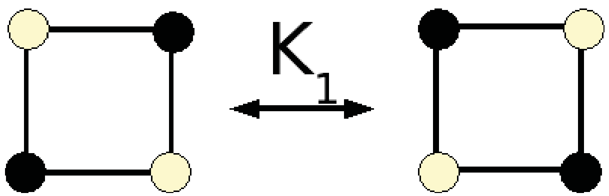
Pure ring model has CDW order
by "kinetic stabilization":
CDW is a very 'hoppable" state

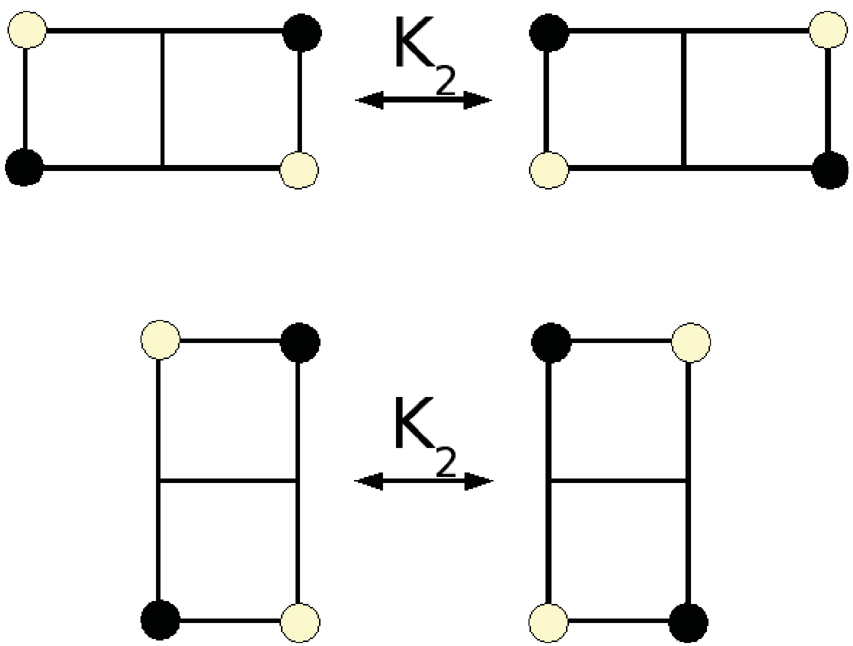
Away from half-filling: quickly phase-separates

Next-simplest ring-only model: K_1-K_2 model

$$P_{\mathbf{r}}^{mn} = b_{\mathbf{r}}^{\dagger}\ b_{\mathbf{r}+m\hat{\mathbf{x}}}\ b_{\mathbf{r}+m\hat{\mathbf{x}}+n\hat{\mathbf{y}}}^{\dagger}\ b_{\mathbf{r}+n\hat{\mathbf{y}}} + \mathrm{H.c.}$$

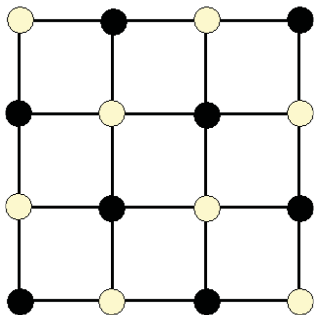
$$\boxed{\hat{H} = -K_1 \sum_{\mathbf{r}} P_{\mathbf{r}}^{11} - K_2 \sum_{\mathbf{r}} \left(P_{\mathbf{r}}^{12} + P_{\mathbf{r}}^{21}\right)}$$





K_1-K_2 model: competing ring exchanges!

Tiamhock Tay and OIM, Phys. Rev. B 83, 205107 (2011)

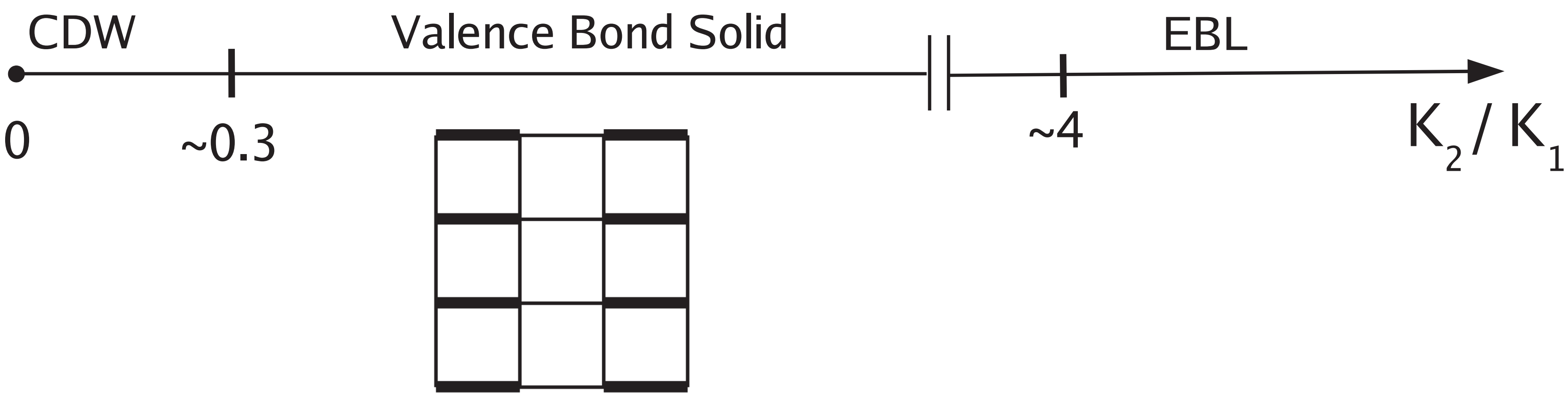


CDW liked by K_1 : all 1x1 are hoppable,

but disliked by K_2 : 2x1 and 1x2 are not hoppable

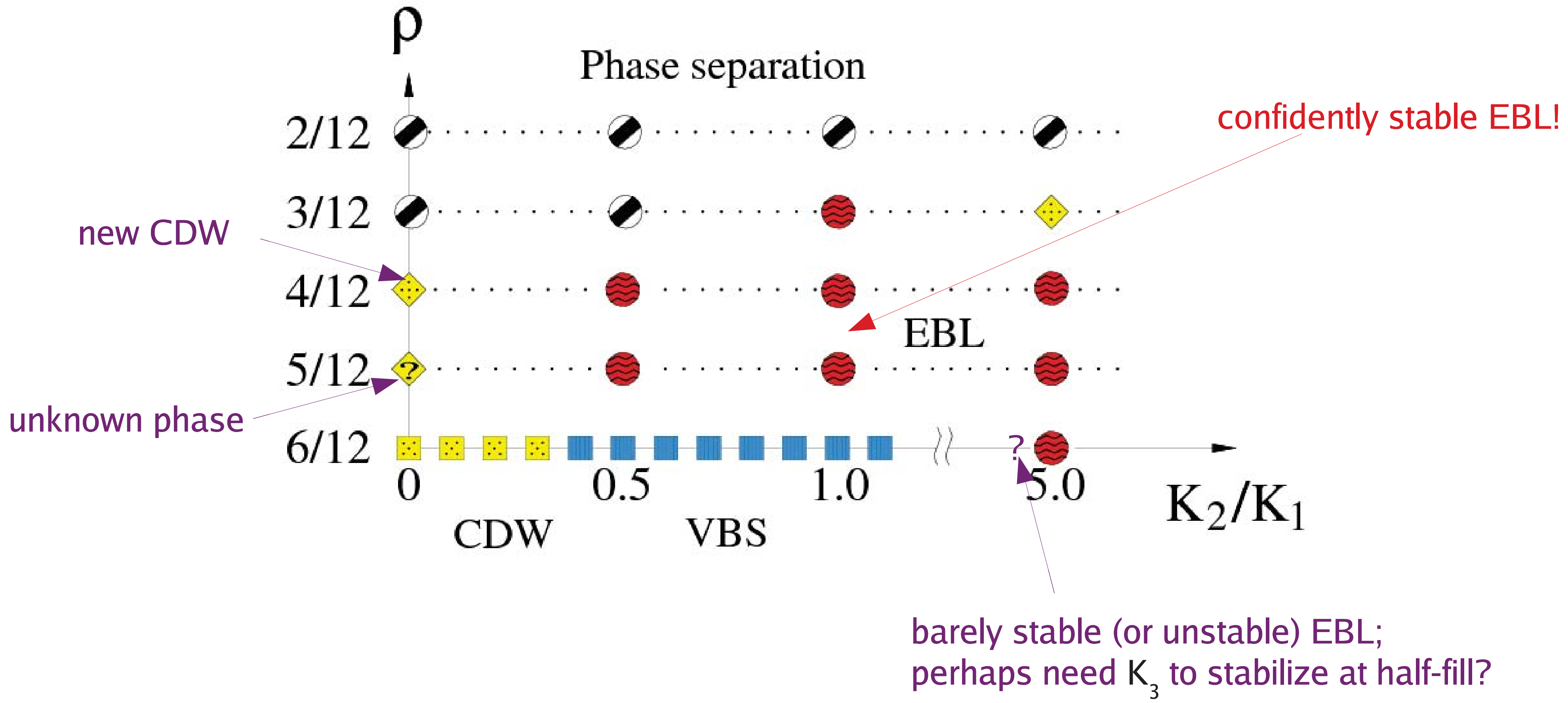
Initial idea: suppress CDW and stabilize EBL (also, more extended rings can suppress phase separation tendencies).

Surprise: indeed, the CDW is suppressed very quickly, but before EBL, first find a different phase –Valence Bond Solid!



GFMC study of K_1-K_2 model for 12x12 system

Tiamhock Tay and OIM, Phys. Rev. B 83, 205107 (2011)



- Many questions about the phase diagram and comparisons with the EBL theory are still open (box correlators; EBL stiffness; stability)
- Challenging phase for QMC because of the gaplessness
- Hard to understand the columnar VBS out of the EBL theory

Summary

- Introduced slave-particle construction of Bose-Metals and parton-gauge theory description
- Described 2d square lattice J-K model as a candidate for realizing d-wave Bose-metal
- Special Bose-metal in ring-only case -> Exciton Bose Liquid
- Some lessons for the parton-gauge theories:
  - Simpler non-gauge theory description is possible
  - Gutzwiller wavefunction does not capture spatial gauge fluctuations
- EBL can be stable in hard-core boson models with extended ring terms
- In the J-K model, the hopping effectively generates further ring terms, so may be helping the Bose-metal phase