The Abdus Salam International Centre for Theoretical Physics

2253-15

Workshop on Synergies between Field Theory and Exact Computational Methods in Strongly Correlated Quantum Matter

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Non-Fermi liquid (NFL) phases of 2d itinerant electrons

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Non-Fermi Liquid (NFL) phases of 2d itinerant electrons

Trieste Workshop on Strongly Correlated Quantum Matter July 28, 2011

MPA Fisher with Hongchen Jiang, Matt Block, Ryan Mishmash, Donna Sheng, Lesik Motrunich (in progress)

Goal: Construct and Analyze non-Fermi liquid phases of strongly interacting 2d ***itinerant*** electrons

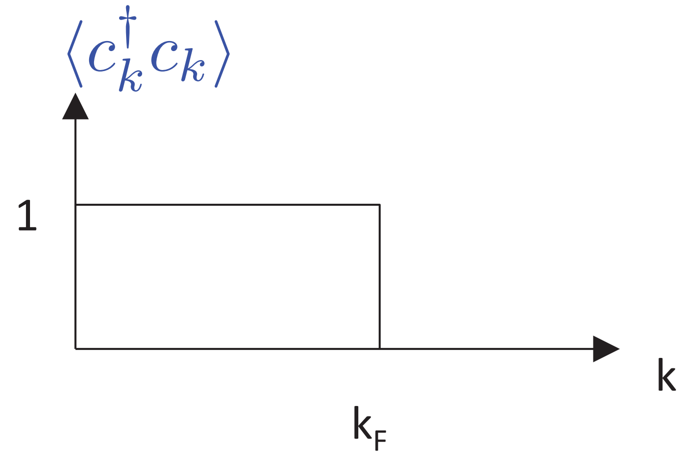
- "Parton" construction for NFL (Gutzwiller wf's)
- Example of a NFL: "D-wave Metal"
- Hamiltonian and energetics (DMRG) for "D-wave Metal"?

What is a “Non-Fermi-liquid metal”?

First: Fermi Liquid Metal

2D Free Fermi Gas

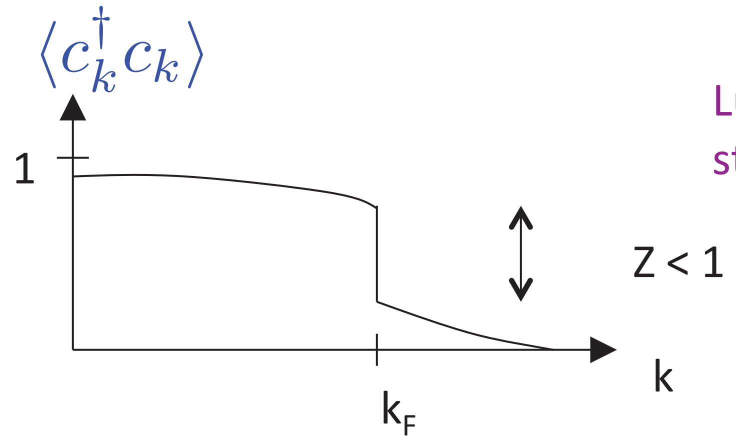
Momentum Distribution Function: $n_k = \langle c_k^\dagger c_k \rangle$



Volume of Fermi sea set by particle density

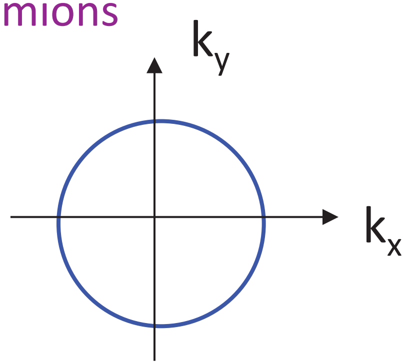
$$\rho = k_F^2/4\pi$$

2D Fermi Liquid Metal



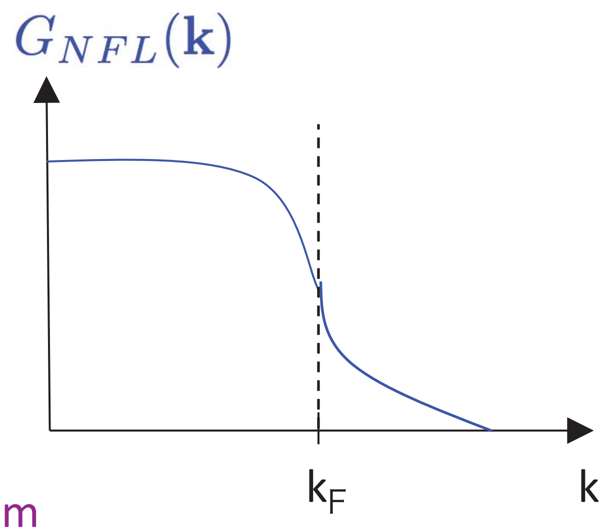
Luttingers Thm: Volume inside Fermi surface still set by total density of fermions

$$\rho = k_F^2/4\pi$$



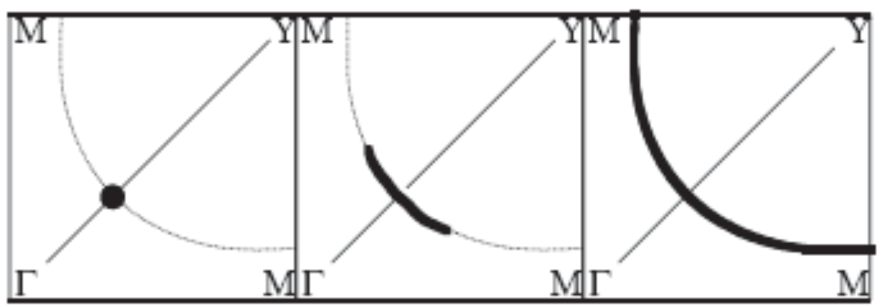
2D Non-Fermi Liquid Metal

Various possibilities:



1) A singular "Fermi surface" that satisfies Luttinger's theorem but without a jump discontinuity in momentum distribution function

2) A singular Fermi surface that violates Luttinger's theorem (eg. volume "x" rather than "1-x")



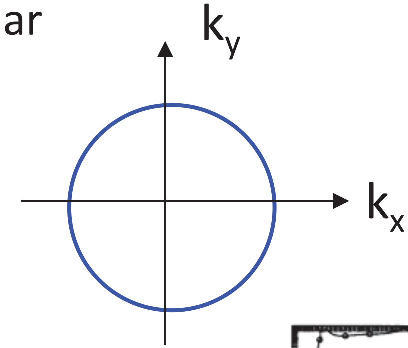
3) A singular "Fermi surface" with ``arc"

4.) Other….

Wavefunction(s) for NFL metals?

Wavefunction for 2D Free Fermi gas

Free Fermion determinant: (eg with 2D circular Fermi surface)

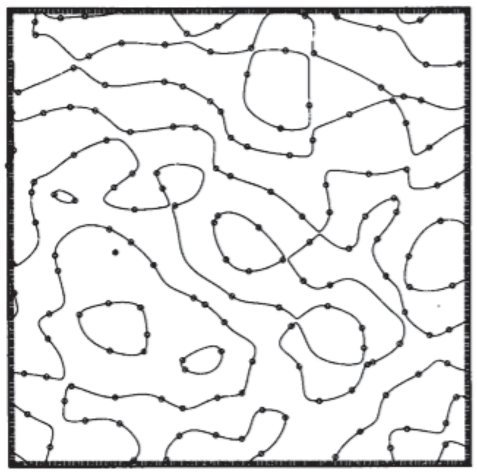


$$\Psi_{FF}(\{\mathbf{r}_i\}) = det[e^{i\mathbf{k}_i \cdot \mathbf{r}_j}]$$

Real space ***"nodal structure"***

Define a ``relative single particle function"

$$\Phi_{\mathbf{r}_2,\ldots,\mathbf{r}_N}(\mathbf{r}) \equiv \Psi(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N)$$



Nodal lines:

Ultraviolet and infrared "locking"

Wavefunction for interacting Fermi liquid?

Retain sign (nodal) structure of free fermions, modify amplitude, eg to keep particles apart.

Common form: Multiply free fermion wf by a Jastrow factor,

$$\Psi_{FL} = e^{-\sum_{i<j} u(\mathbf{r}_i - \mathbf{r}_j)} \psi_{FF}$$

with u(r) a variational function

For Spinful Electrons: Gutzwiller Projection

Slater determinant $$\Psi_{FF}(\mathbf{r_{i\uparrow}}, \mathbf{r_{i\downarrow}}) = \det[\mathbf{e^{ik_i \cdot r_{j\uparrow}}}] \det[\mathbf{e^{ik_i \cdot r_{j\downarrow}}}]$$

$$\Psi_G = \mathcal{P}_G[\Psi_{FF}]$$ Project out doubly occupied sites

Parton approach to NFL wavefunctions

Decompose electron:
spinless charge e boson,
s=1/2 neutral fermionic spinon

$$c_\sigma = b f_\sigma$$

Mean Field Theory

Treat "Spinons" and Bosons as Independent: $\mathcal{H} = \mathcal{H}_f + \mathcal{H}_b$

Wavefunctions $\psi_f(\mathbf{r_{i\uparrow}}, \mathbf{r_{i\downarrow}})$ $\psi_b(\mathbf{R_i})$

(enlarged Hilbert space - twice as many particles)

"Fix-up" Mean Field Theory

"Glue" together Fermion and Boson "partons"

$$\Psi \equiv \psi_f(\mathbf{r_{i\alpha}}) \times \psi_\mathbf{b}(\mathbf{R_i} \rightarrow \mathbf{r_{i\alpha}})$$

Project back into physical Hilbert space

Fermi and Non-Fermi Liquids via partons?

Spinons in a filled Fermi sea $\psi_f = \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}})] \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}})]$

Fermi Liquid: Bosons into Bose condensate

$$\Psi_{FL} = \psi_f^{FF} \times \psi_b^{BEC}$$

$$\psi_b^{BEC} = e^{-\sum_{i<j} u(\mathbf{R_i}-\mathbf{R_j})} \qquad c_\sigma = \langle b \rangle f_\sigma \sim (const) f_\sigma$$

Non-Fermi Liquid: Bosons into *uncondensed* fluid - a "Bose metal"

$$\Psi_{NFL} = \psi_f^{FF} \times \psi_b^{BoseMetal}$$

NFL Metal: Product of Fermi sea and uncondensed "Bose-Metal"

2D Bose-Metal

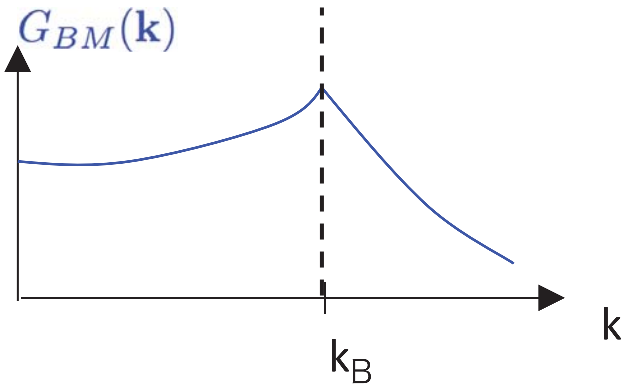
- A ***stable T=0 liquid phase*** of bosons that is not a superfluid

- Green's function has oscillatory power law decay

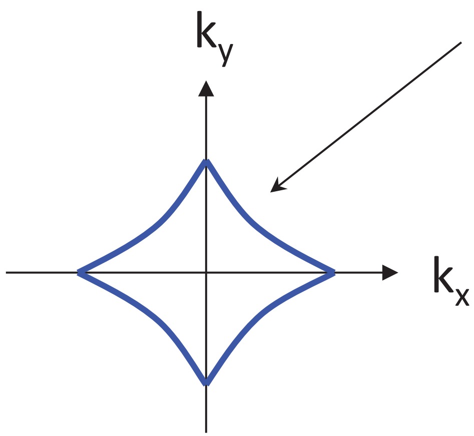
$$G_{BM}(\mathbf{r}) \sim \frac{\cos[\mathbf{k}_B(\hat{\mathbf{r}})\cdot\mathbf{r}]}{|\mathbf{r}|^{\alpha(\hat{\mathbf{r}})}}$$

- Momentum distribution function singular on a ***"Bose surface"***

$$G_b(\mathbf{k}) = \langle b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \rangle$$



$\mathbf{k}_B(\hat{\mathbf{r}})$



Angular dependent anomalous dimension $\alpha(\hat{\mathbf{r}})$

Parton Construction for Bose metal
(following Lesik's talk)

$$b = d_1 d_2 \qquad \psi_b = det_1 \times det_2$$

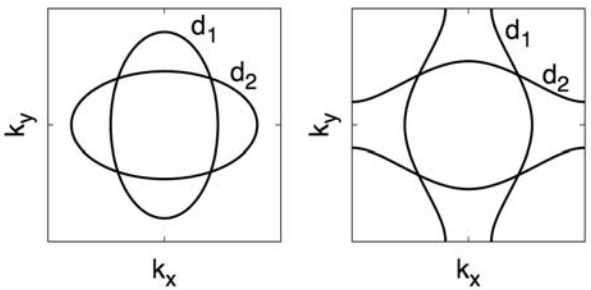
Gutzwiller wavefunction:

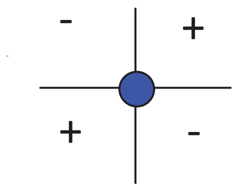
All Fermionic decomposition of electron $\quad c_\alpha = b f_\alpha = d_1 d_2 f_\alpha$

Wf for D-wave Bose-Metal (DBM)

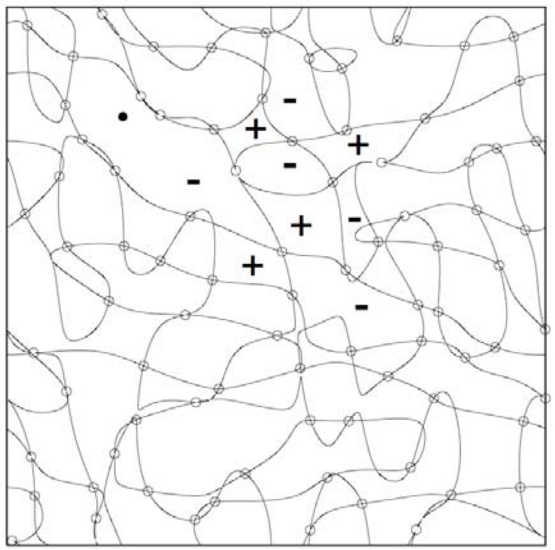
Product of 2 Fermi sea determinants, elongated in the x or y directions

$$\psi_{DBM} = det_x \times det_y$$



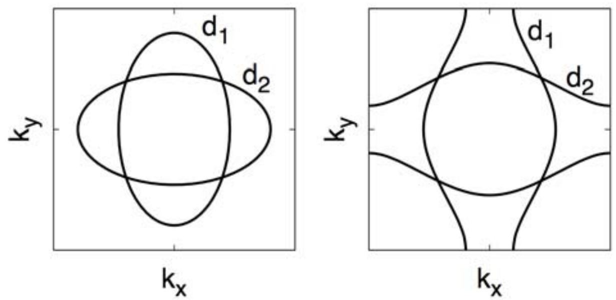


D_{xy} relative
2-particle correlations



Bose Surfaces in D-wave Bose-Metal

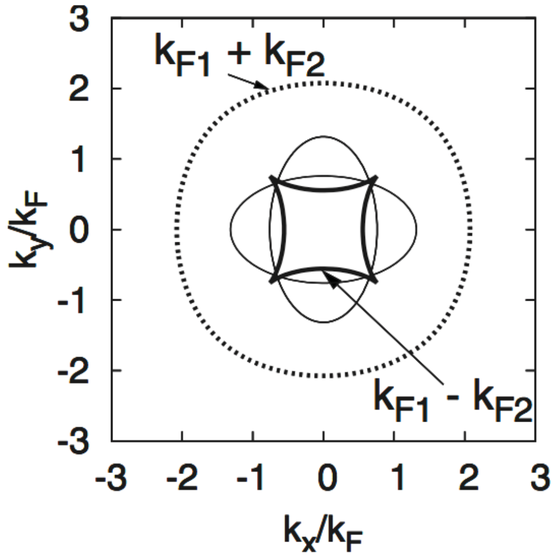
Mean Field Green's functions factorize:



$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau) G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$$



Two singular lines in momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$

"D-Wave Metal"

Itinerant NFL phase of 2d electrons?

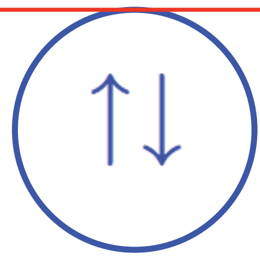
All fermionic Parton construction

$$c^\dagger_\alpha(\mathbf{r}) = b^\dagger(\mathbf{r}) f^\dagger_\alpha(\mathbf{r}) = d^\dagger_x(\mathbf{r}) d^\dagger_y(\mathbf{r}) f^\dagger_\alpha(\mathbf{r})$$

Wavefunction; Product of determinants

$$\{\vec{R}_i\} = \{\vec{r}_{i\uparrow}, \vec{r}_{j\downarrow}\}$$

$$\Psi^{Metal}_{d_{xy}} = \det_x[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \cdot \det_y[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \times \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}}] \cdot \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}}]$$



Filled Fermi sea

Can use Variational Monte Carlo to extract equal time correlation functions from wf

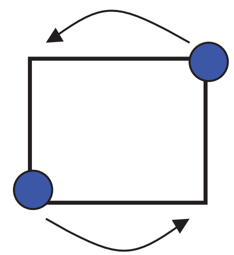
But what about energetics???

Hamiltonians with Bose-metal or NFL metal ground states??

Ring Hamiltonian for D-wave Bose-Metal?

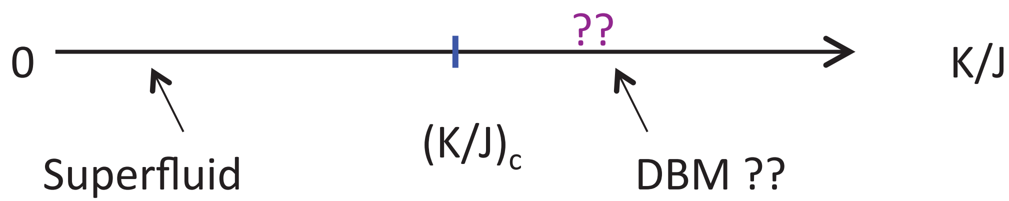
Strong coupling limit of parton gauge theory:

"Ring exchange"



$$
\begin{aligned}
H &= H_J + H_4 \ , \\
H_J &= -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mu}} + h.c.) \ , \\
H_4 &= K_4 \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}} b_{\mathbf{r}+\hat{\mathbf{x}}} b^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,
\end{aligned}
$$

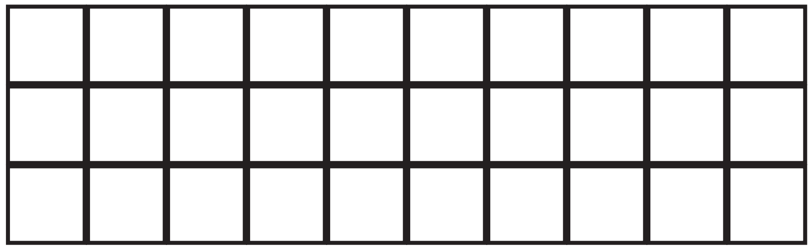
Phase diagram: K/J and density of bosons



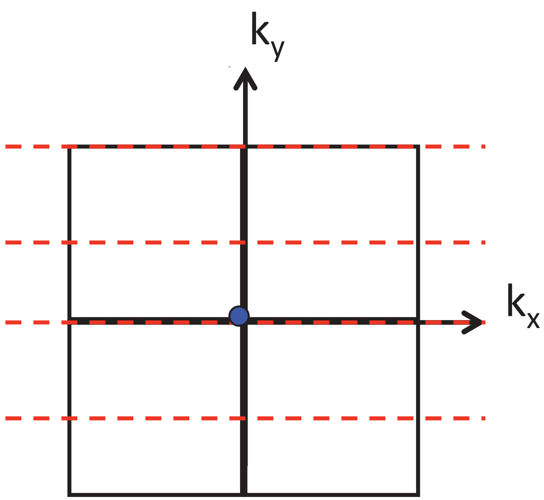
J-K Model has a sign problem for non-zero J and K

Ladders

Transverse y-components of
momentum become quantized

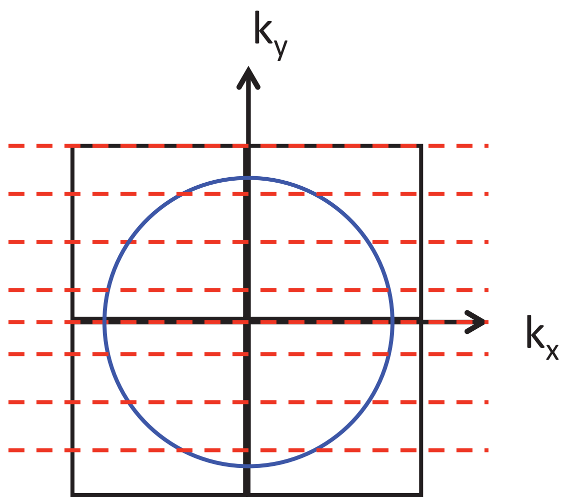


Put Bose superfluid on n-leg ladder



Single gapless 1d mode

Put D-wave Bose metal on n-leg ladder



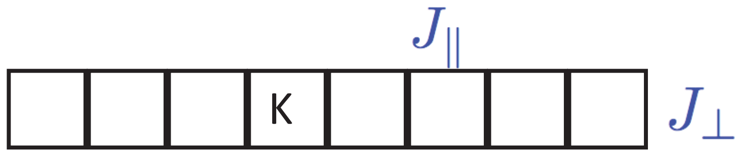
Many gapless 1d modes, one for each “Bose” point

Signature of 2d Bose surface present on ladders

Expectation: Signature of Bose surface in Bose-Metal on n-leg ladders

Boson ring model on the 2-Leg Ladder

- Exact Diag.
- Variational Monte Carlo
- DMRG
- Bosonization of quasi-1d gauge theory



E. Gull, D. Sheng, S. Trebst,
O. Motrunich and MPAF,
PRB 78, 54520 (2008)

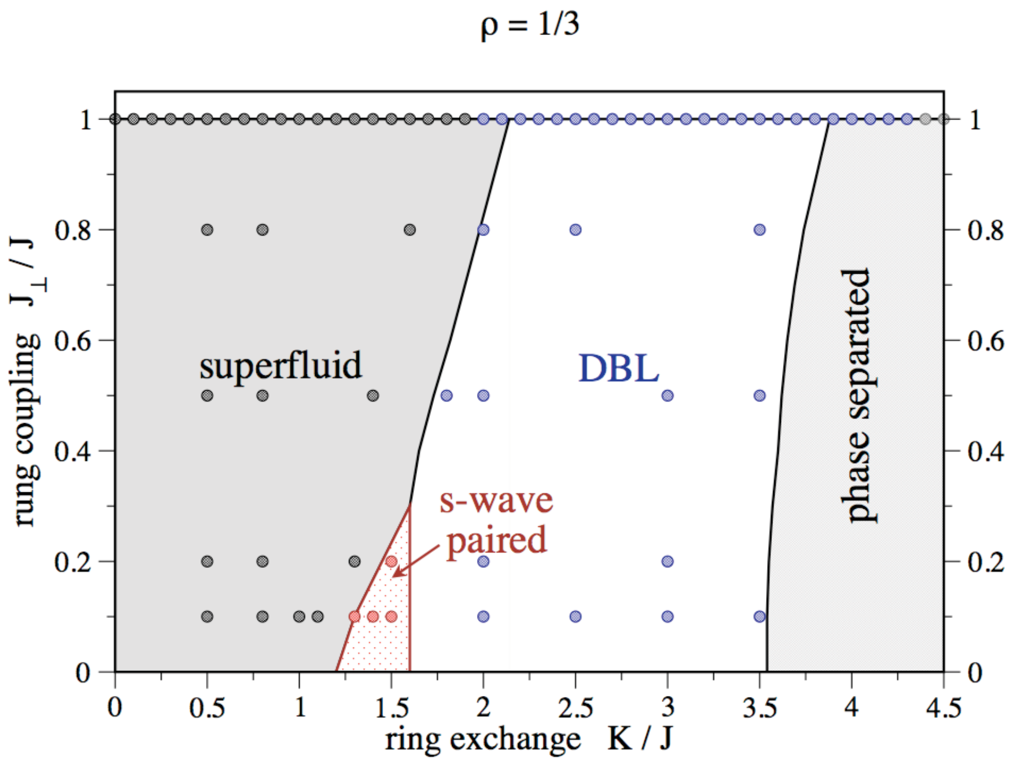
$$H = H_J + H_4 ,$$

$$H_J = -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b_{\mathbf{r}}^{\dagger} b_{\mathbf{r}+\hat{\mu}} + h.c.) ,$$

$$H_4 = K_4 \sum_{\mathbf{r}} (b_{\mathbf{r}}^{\dagger} b_{\mathbf{r}+\hat{\mathbf{x}}} b_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^{\dagger} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) ,$$

Ladder descendant of 2D Bose-metal??

Phase Diagram for 2-leg ladder

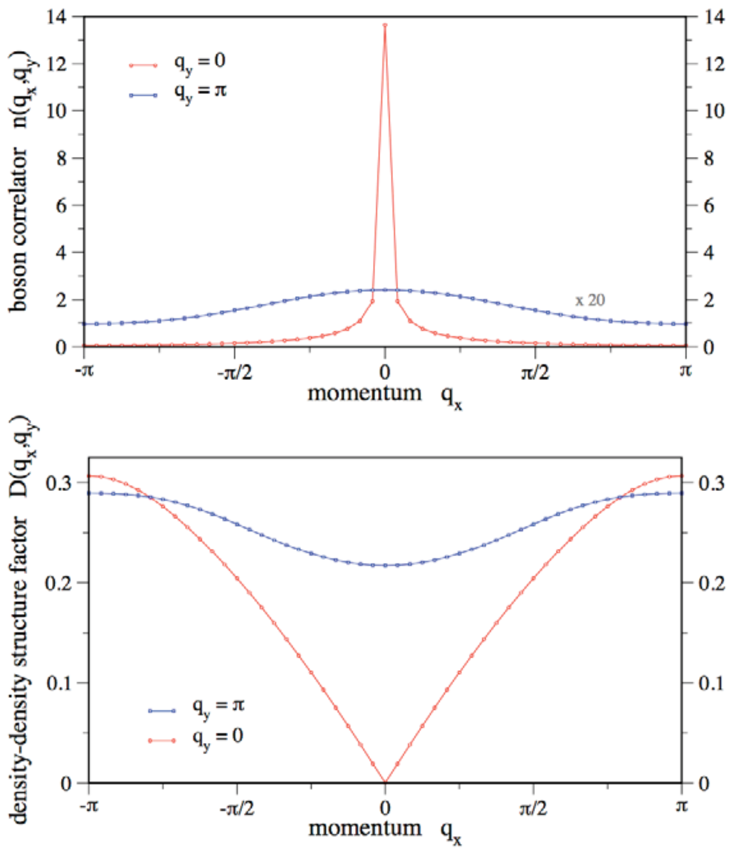


Phases:

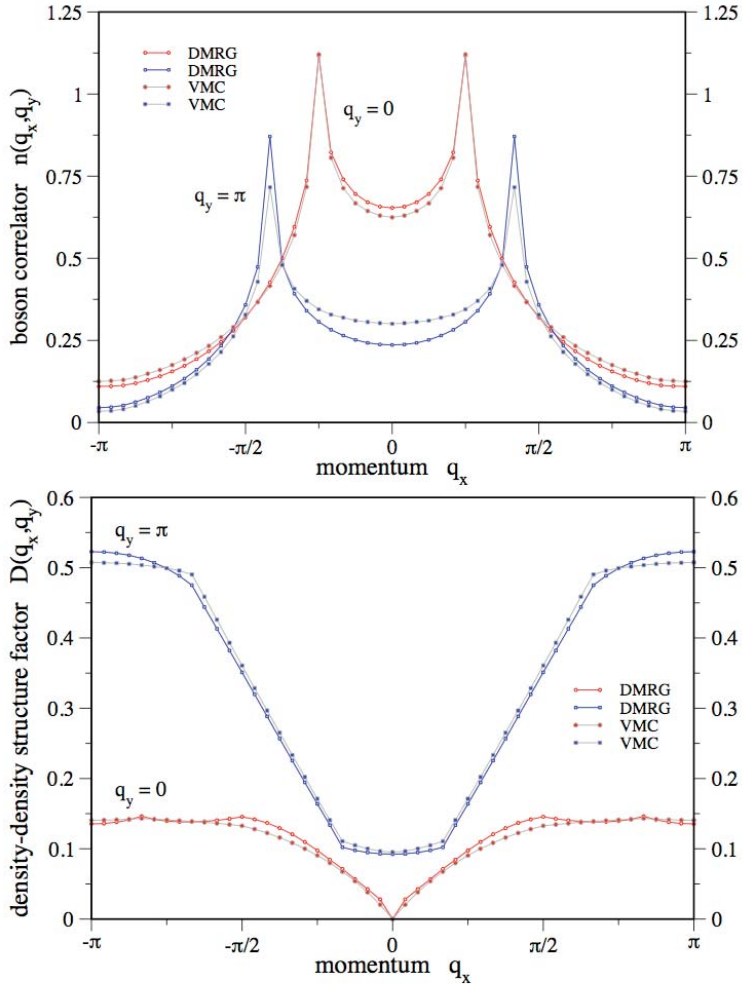
1) Superfluid – "Bose condensate"
2) D-Wave Bose Metal - DBL
3) s-wave Pair-Boson "condensate"

D-wave Bose-Metal occupies large region of phase diagram

Superfluid versus D-wave Bose-Metal



Superfluid - “condensed”
at zero momentum



D-wave Bose-Metal; Singular
“Bose points” at $q_y = 0, \pi$

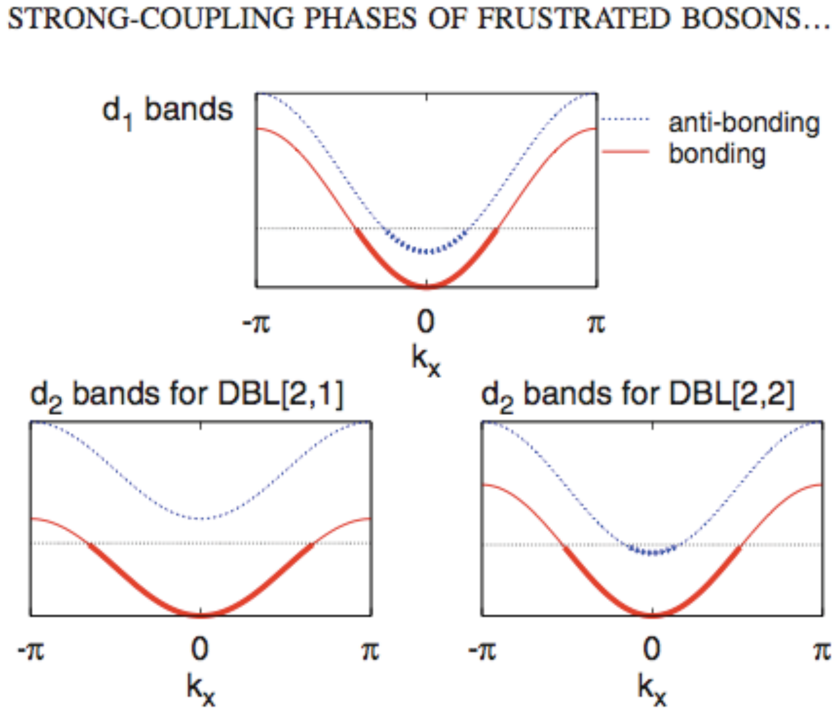
Variational Wf for D-wave Bose-metal on 2-leg ladder

In DBM:

Bonding/Antibonding occupied
For d_1 Fermion

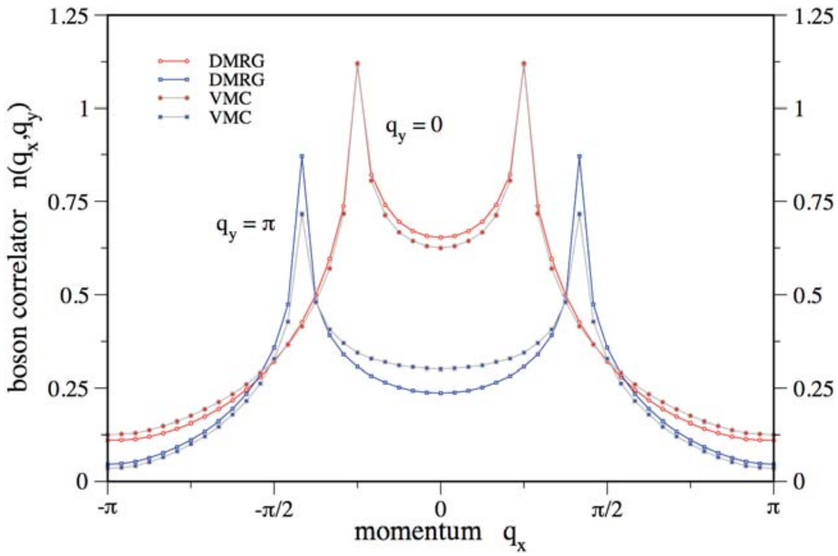
Just Bonding occupied
For d_2 Fermion

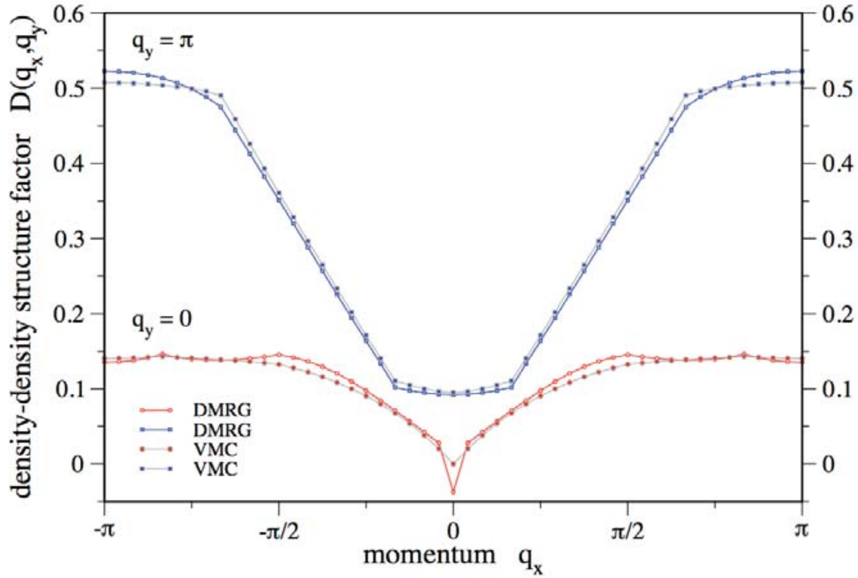
Variational parameter:
Fermi wavevectors in d_1 bands



$$\Psi_{\text{bos}}(r_1, r_2, \ldots) = \Psi_{d_1}(r_1, r_2, \ldots) \cdot \Psi_{d_2}(r_1, r_2, \ldots).$$

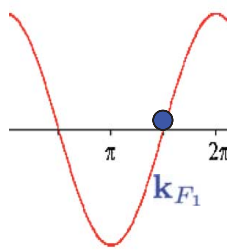
DBM: How good is ladder variational wavefunction?



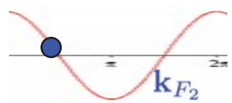


Gauge mean field theory predicts singularities
in momentum distribution function at: $\mathbf{k}_{F_1} \pm \mathbf{k}_{F_2}$

Both DMRG and $\det_1$ x $\det_2$ Wavefunction
show singular cusps *only* at $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$
(Ampere's law)



d_1



d_2

Hamiltonian for D-wave Metal?

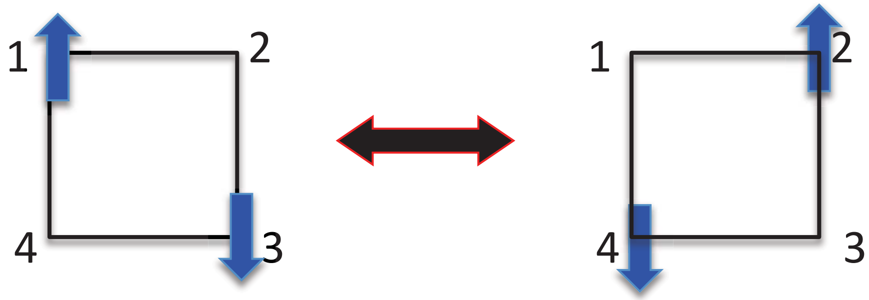
Strong coupling limit of parton gauge theory $c_\alpha = f_\alpha d_x d_y$

t-K "Ring" Hamiltonian (no double occupancy constraint)

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c^\dagger_{i\alpha} c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}^\dagger_{13} \mathcal{S}_{24} + h.c.]$$

$$\mathcal{S}^\dagger_{ij} = \frac{1}{\sqrt{2}} [c^\dagger_{i\uparrow} c^\dagger_{j\downarrow} - c^\dagger_{i\downarrow} c^\dagger_{j\uparrow}]$$

Electron singlet pair
"rotation" term



Phase diagram of electron t-K Hamiltonian?

$$\mathcal{H}_{tK} = -t\sum_{\langle ij\rangle}[c^{\dagger}_{i\alpha}c_{j\alpha} + h.c.] + K\sum_{\langle 1234\rangle}[\mathcal{S}^{\dagger}_{13}\mathcal{S}_{24} + h.c.]$$

(Density and K/t)

Severe sign problem - intractable

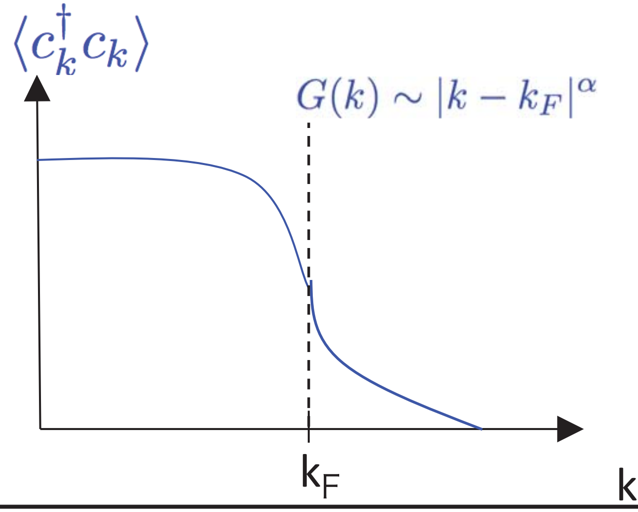
Once again: Analyze t-K electron ring Hamiltonian on ***2-leg ladder***

Possible to identify a NFL on a 2-leg ladder?

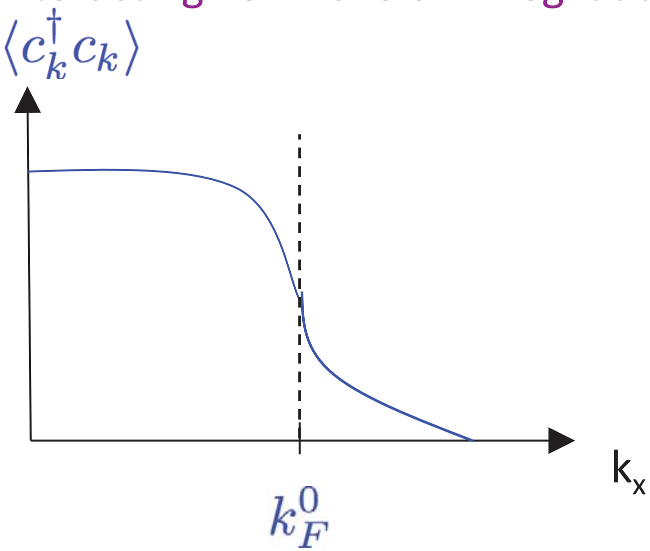
Interacting Fermions in 1d: A Luttinger liquid

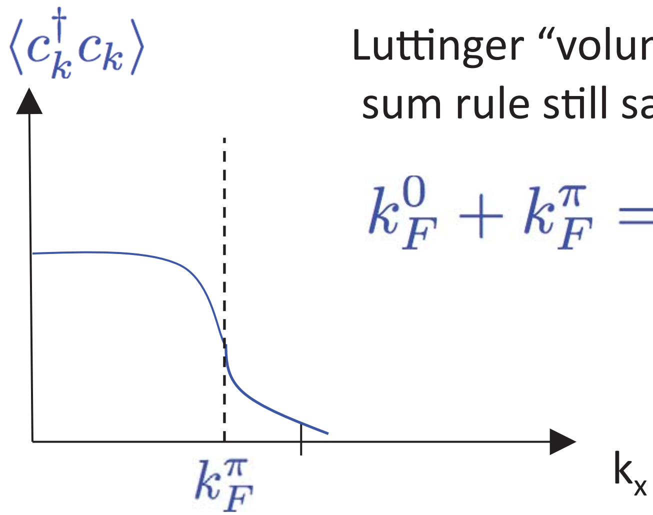
$G(x) \sim \sin(k_F x)/x^{1+\alpha}$ Luttinger liquid exponent: α

Momentum distribution function has (dominant) singularity at $k=k_F$ satisfying Luttinger sum rule



Interacting Fermions on 2-leg ladder: 2-bands





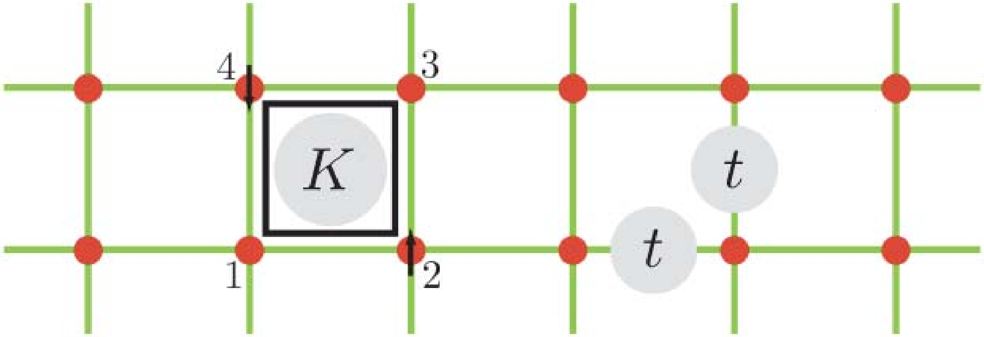
Luttinger "volume" sum rule still satisfied:

$$k_F^0 + k_F^\pi = \pi n_{1d}$$

Searching for a "non-Luttinger liquid" (ie. a Luttinger-liquid violating Luttinger's sum rule)

Electron t-K model on 2-leg ladder

$$\mathcal{H}_{tK} = -t\sum_{\langle ij\rangle}[c^\dagger_{i\alpha}c_{j\alpha} + h.c.] + K\sum_{\langle 1234\rangle}[\mathcal{S}^\dagger_{13}\mathcal{S}_{24} + h.c.]$$

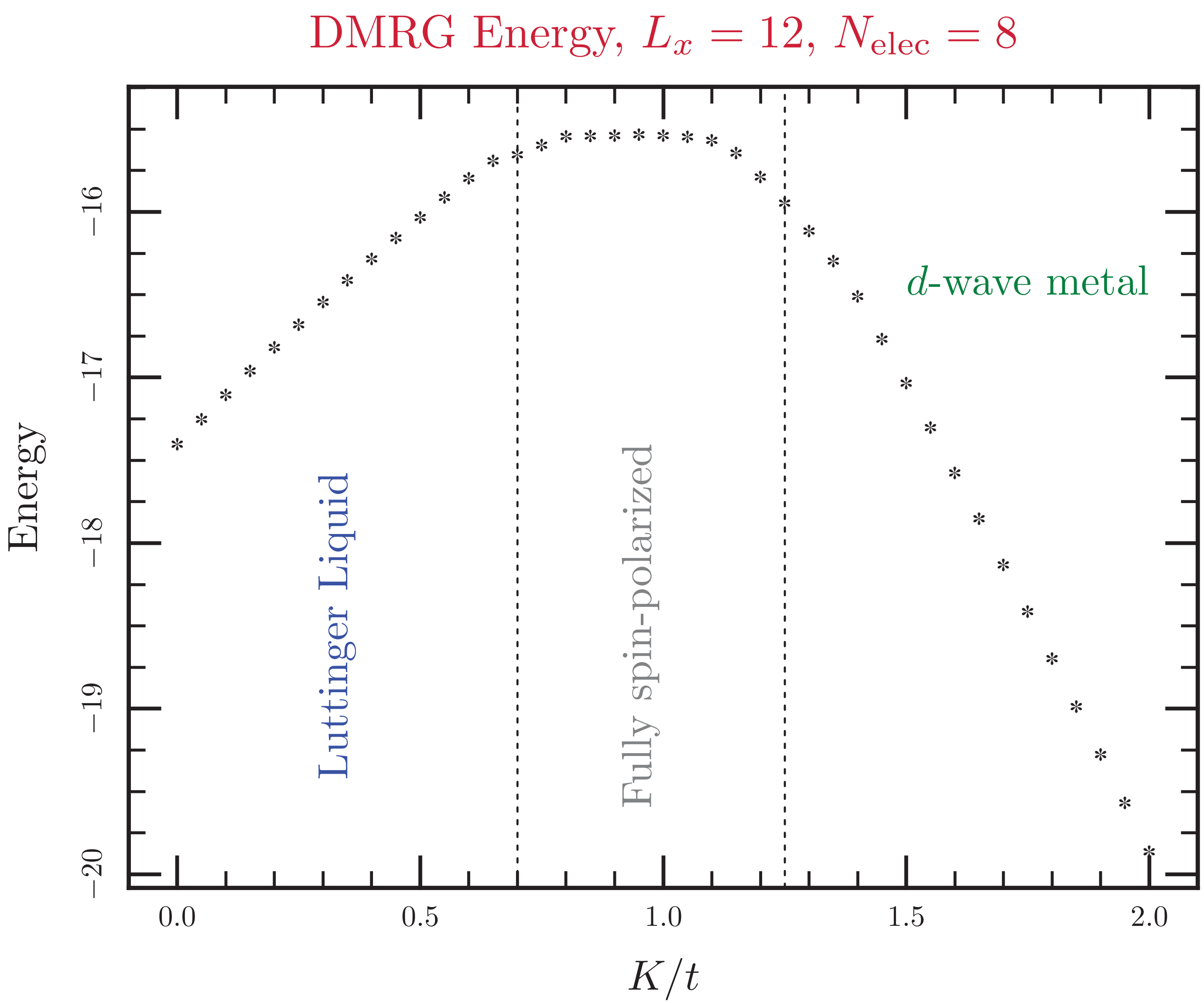


Two dimensionless parameters:
K/t and density n
(**n=1/3 henceforth**)
No double occupancy

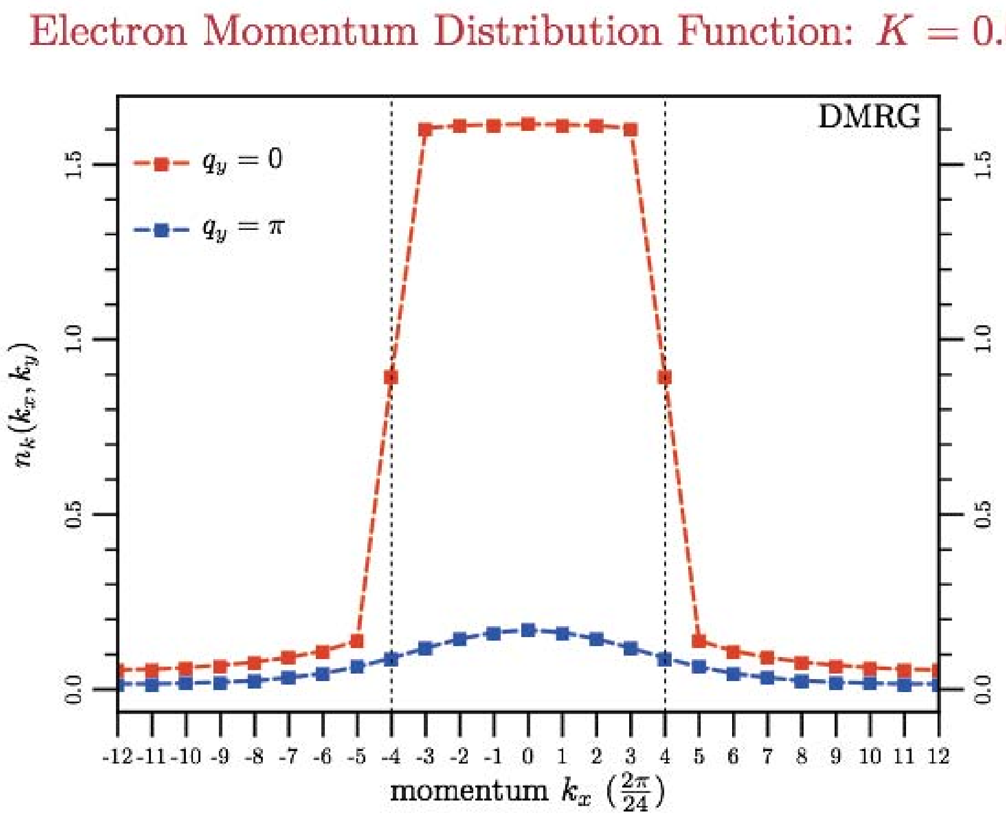
Hongchen Jiang, Matt Block, Ryan Mishmash, Donna Sheng, Lesik Motrunich and MPAF (in progress)

ED
DMRG
VMC
Bosonization of Quasi-1d U(1) gauge theory

Ground State energy: DMRG



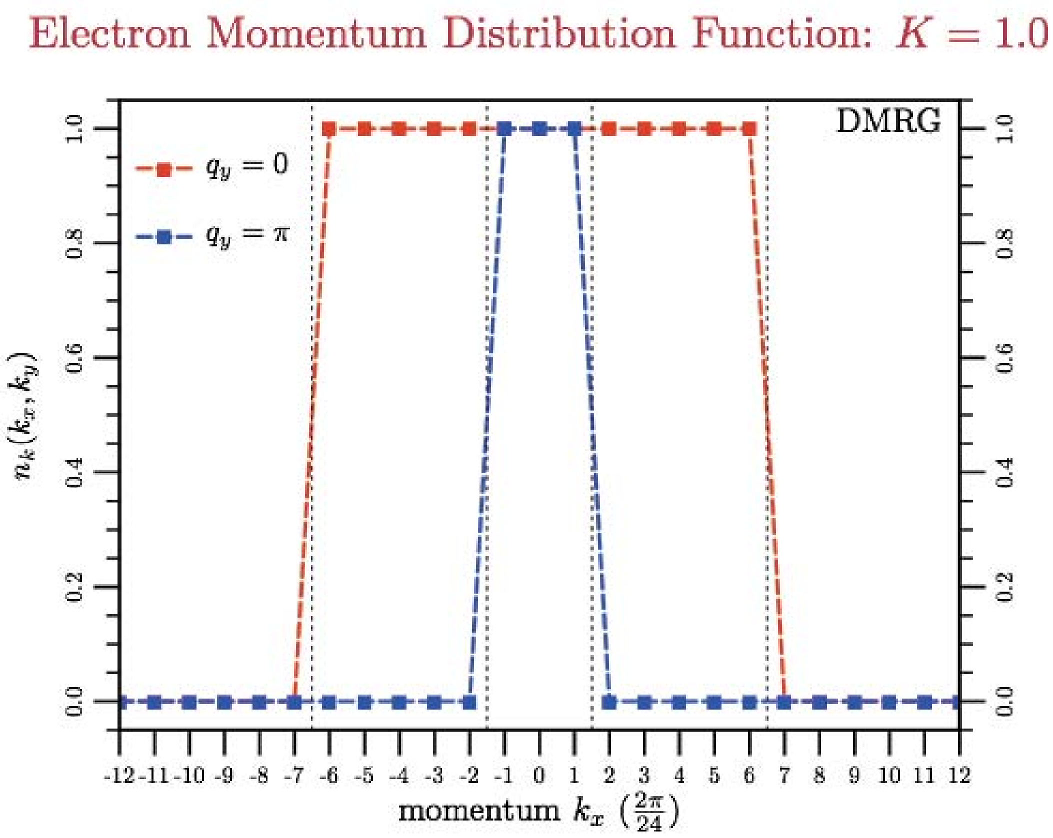
K/t <0.7 Luttinger Liquid



Satisfies Luttinger's Theorem: the volume enclosed by the "Fermi surface" yields the particle density. (16 particles, singlet, 8 up and 8 down)

A canonical (single band) Luttinger liquid

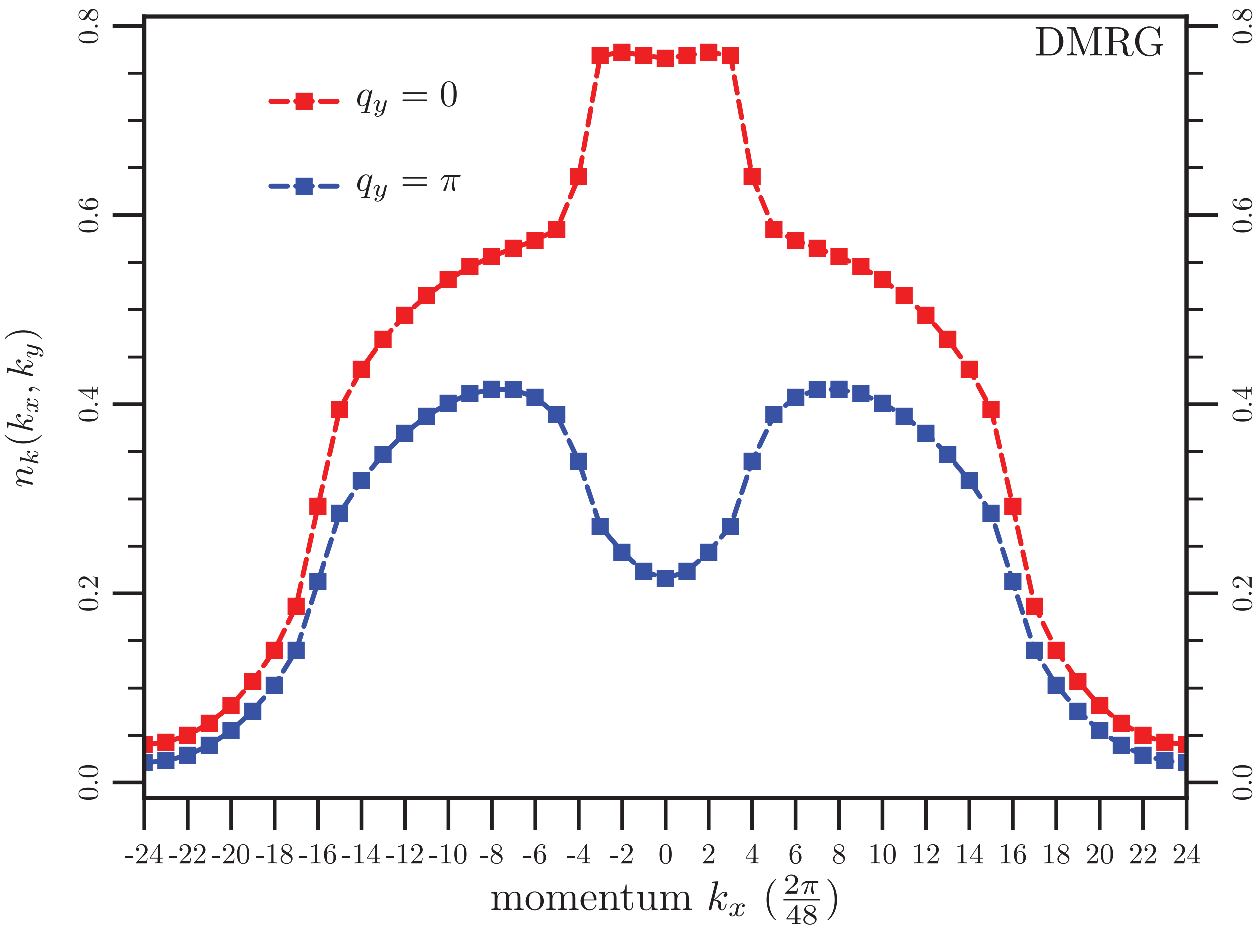
0.7< K/t <1.25 : Spin Polarized



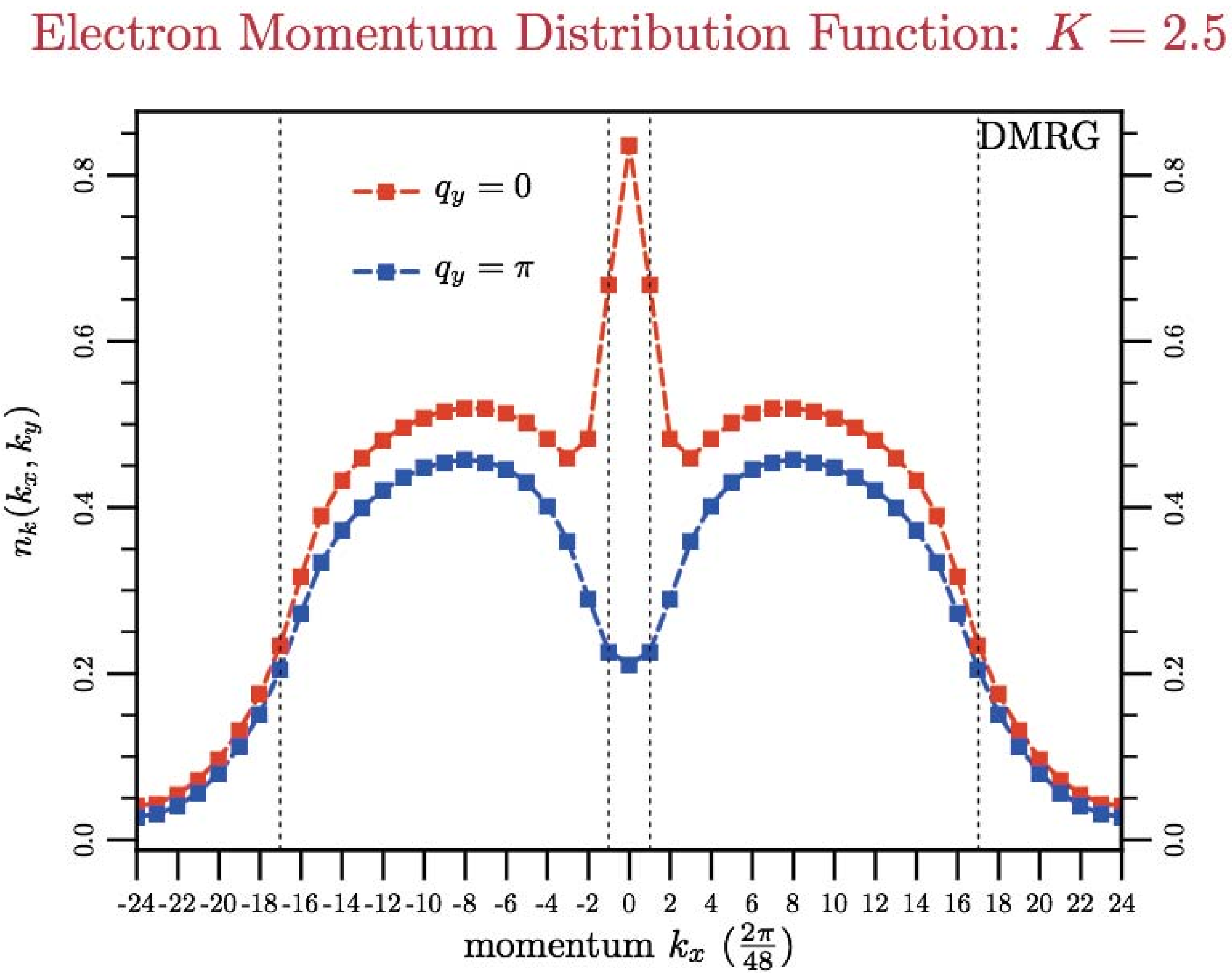
Non-interacting spin polarized Fermi sea is exact ground state here.
Luttinger theorem satisfied

K/t>1.25: Non-LL Phase

Electron Momentum Distribution Function: $K = 2.0$



K/t > 1.25: Non-LL Phase



Non-monotonic momentum distribution function; No sign of Luttingers volume

Non-Luttinger-Liquid phase for K>1.25?

Electron momentum distribution function: Singular features,
but at momenta which do not satisfy Luttinger's volume theorem

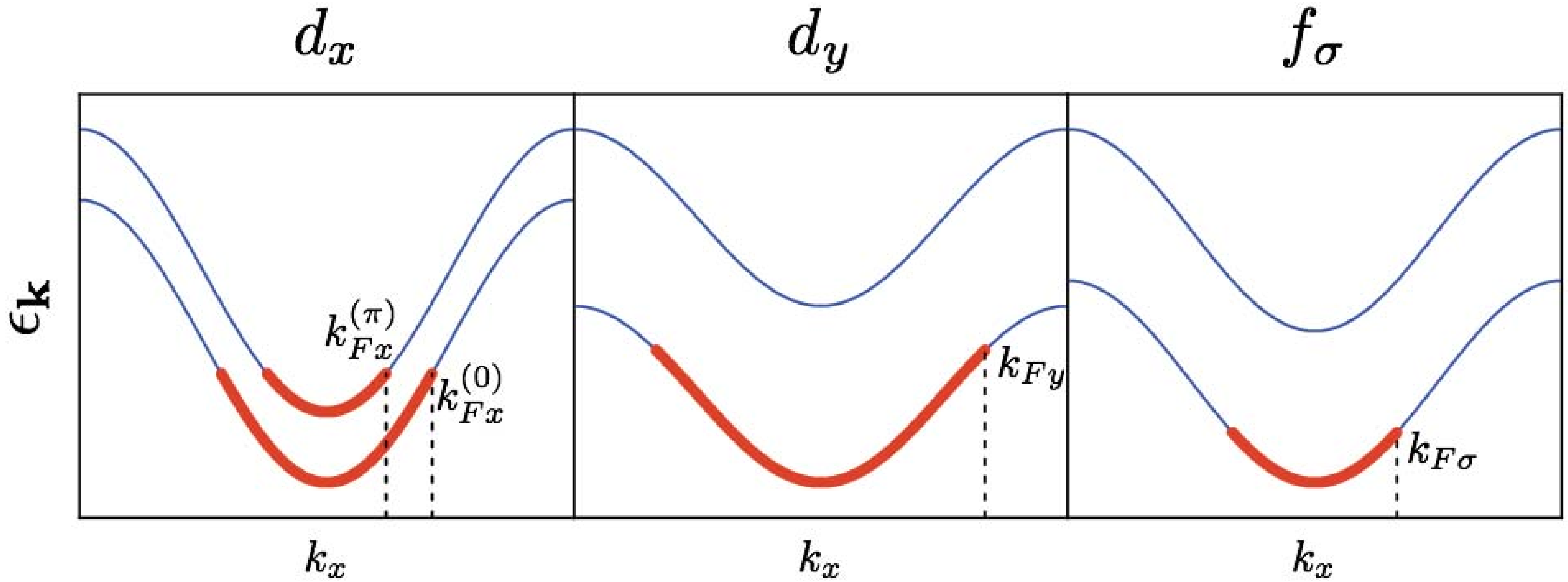
$$n_{k\sigma}(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

Can we understand in terms of
D-wave Metal on 2-leg ladder??

Employ parton construction, gauge theory and VMC $c_\alpha = f_\alpha d_x d_y$

The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$



$$k_{Fx}^{(0)} + k_{Fx}^{(\pi)} = k_{Fy} = 2k_{F\sigma} = 2\pi\rho$$

Gauge theory: Projects down to physical Hilbert space

Number of 1d modes = (number in MFT) – (gauge constraints) = (2+1+2)-(2) = 3

Central charge c=3, strongly entangled

Electron momentum distribution function

Mean Field Theory: electron momentum distribution, convolution of partons

$$n_c^{MFT}(k) = n_{d_x}(k) \otimes n_{d_y}(k) \otimes n_f(k) \qquad c_\sigma = d_x d_y f_\sigma$$

Gauge theory - certain wavevectors enhanced

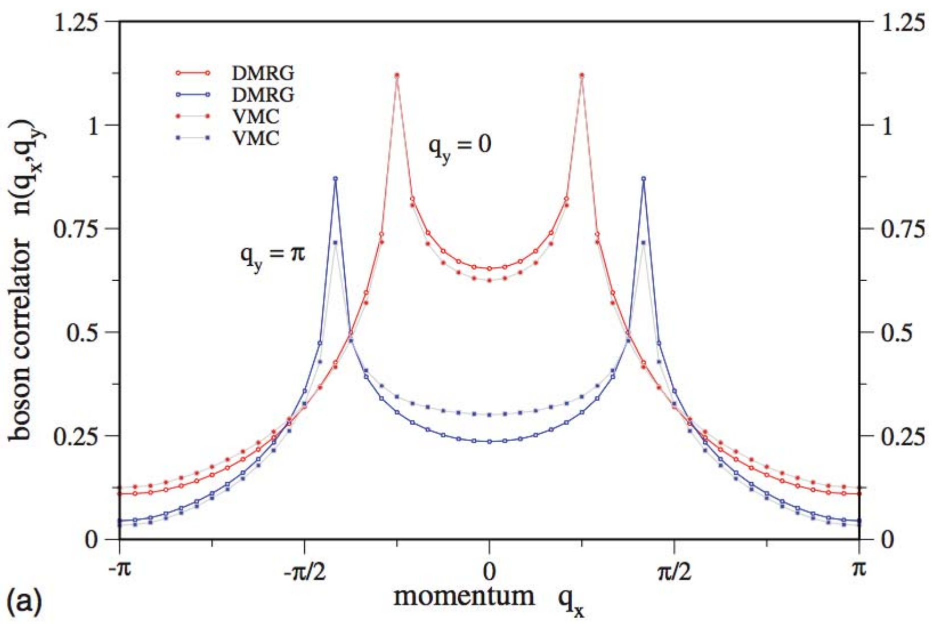
Illustrate with Boson ring model (MFT)

$$n_b^{MFT}(k) = n_{d_x}(k) \otimes n_{d_y}(k)$$

$$b = d_x d_y$$

Very sharp peaks in the **exact** boson momentum distribution function! (from DMRG)

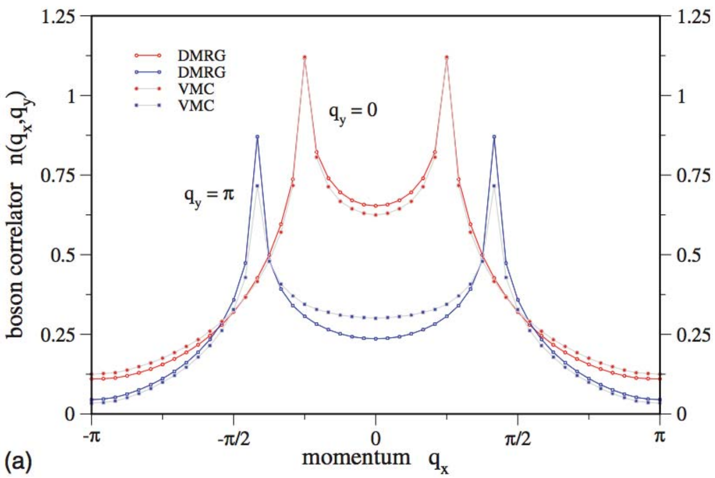
$$n_b(k)$$



(a)

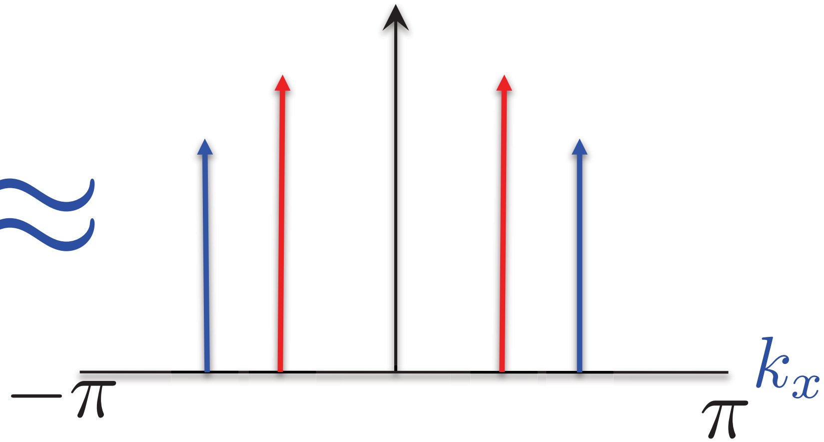
Momentum distribution function in the d-wave metal?

$$n_b(k)$$



$$\tilde{n}_b(k)$$

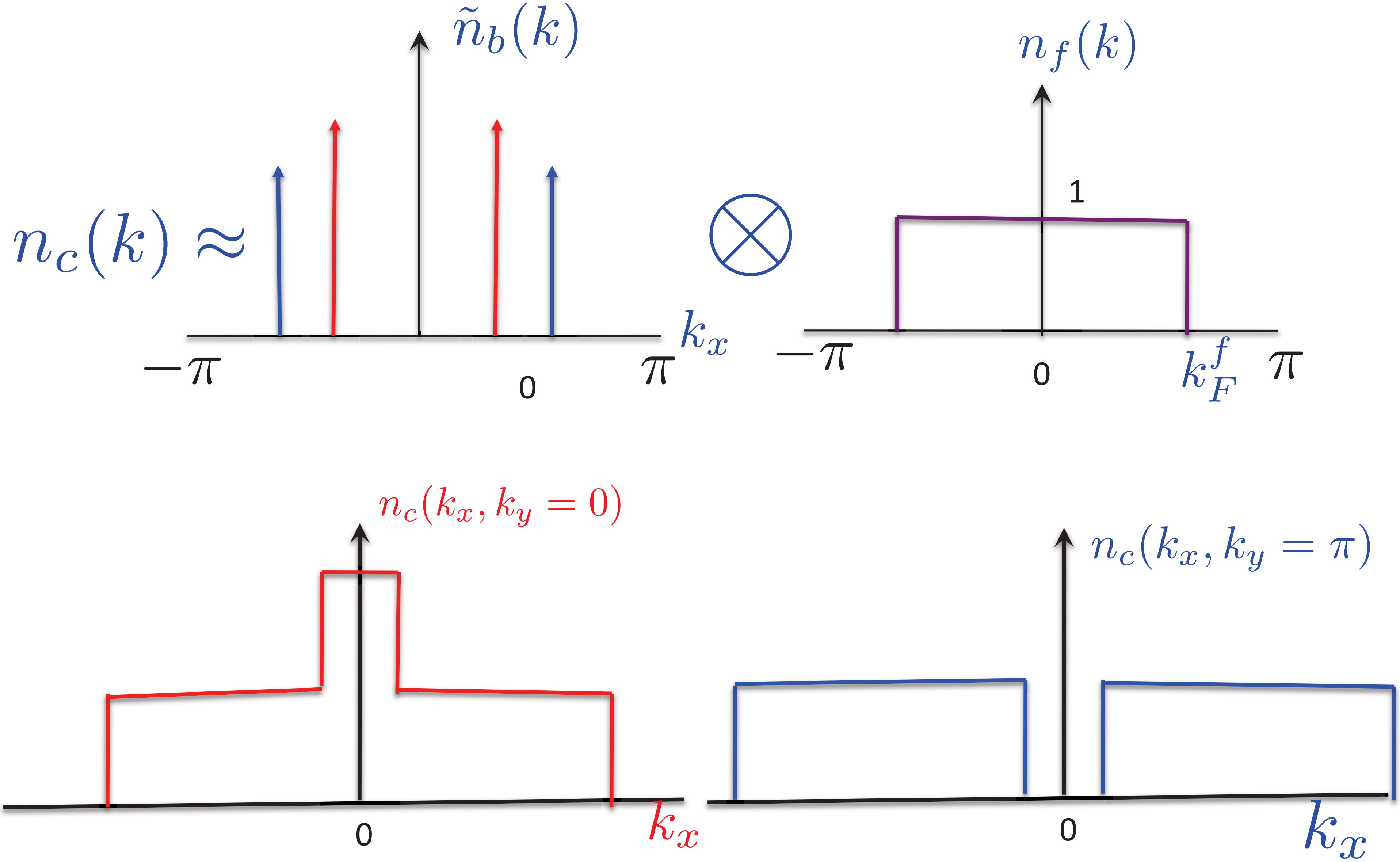
$$\approx$$



$$n_c(k) \overset{?}{\approx} \tilde{n}_b(k) \otimes n_f(k)$$

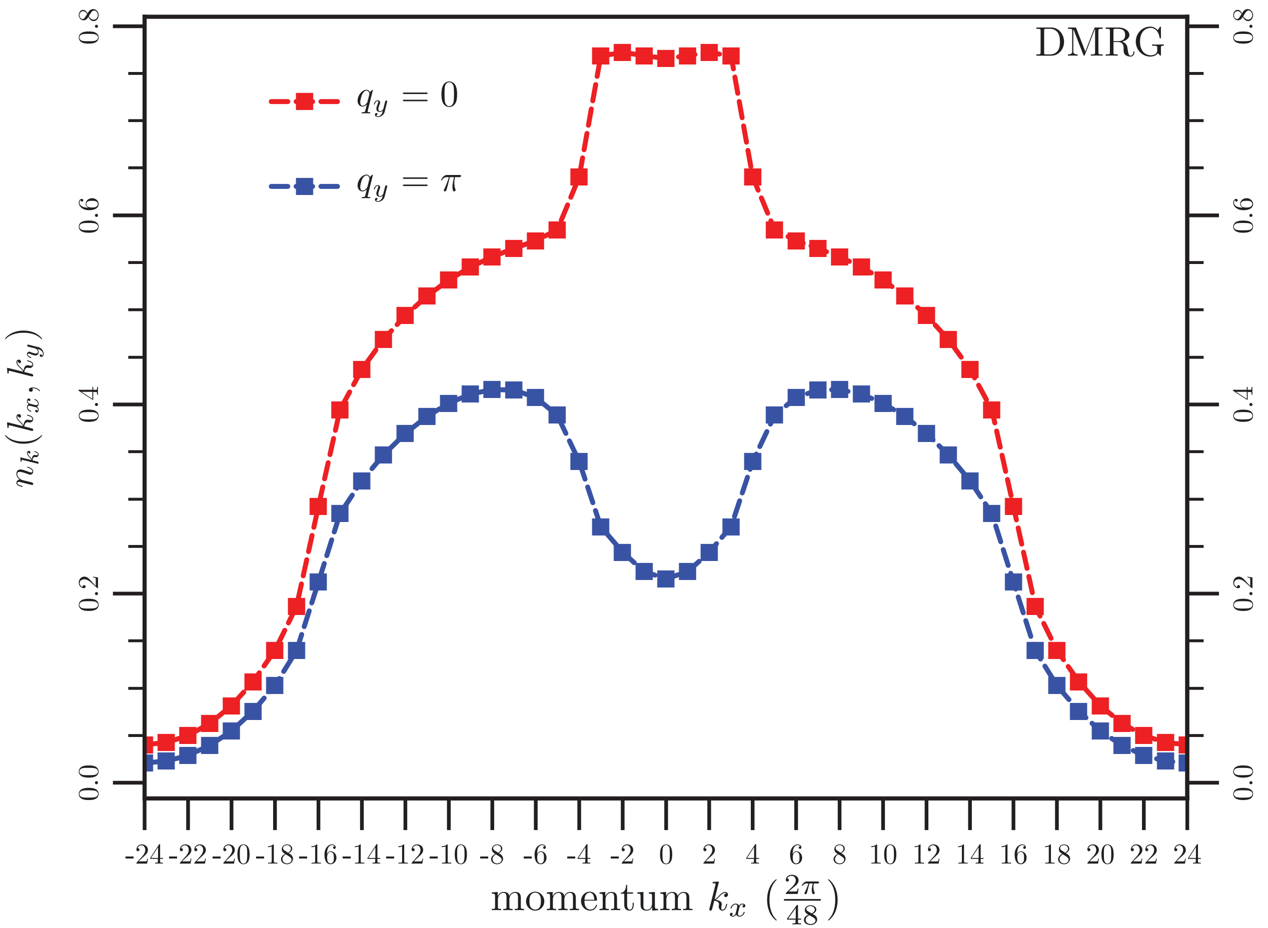
$n_f(k) = \Theta(K_F^f - |k|)$ (Free spinon sea)

Convolution: c = b f

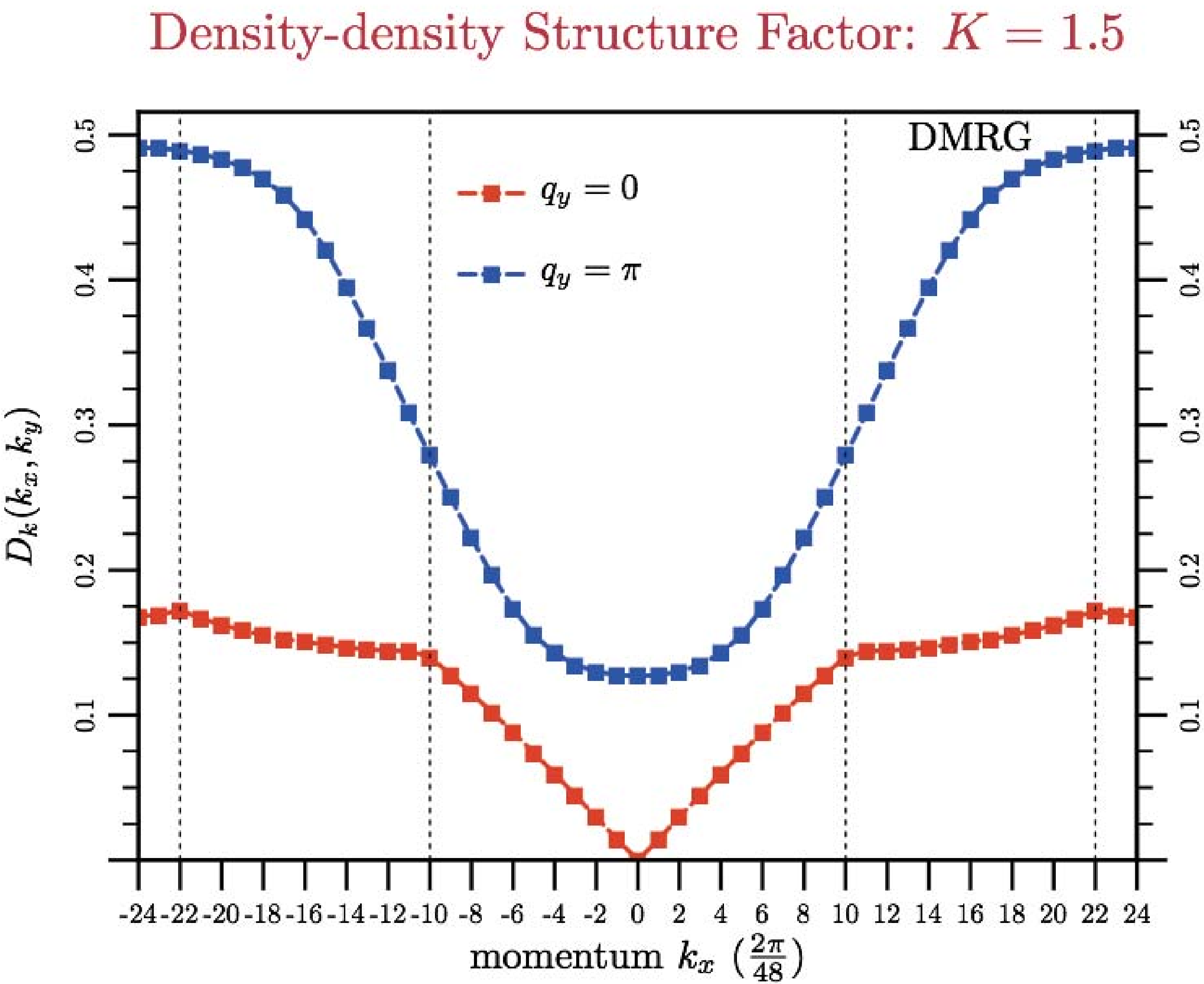


K/t>1.25: Non-LL Phase

Electron Momentum Distribution Function: $K = 2.0$

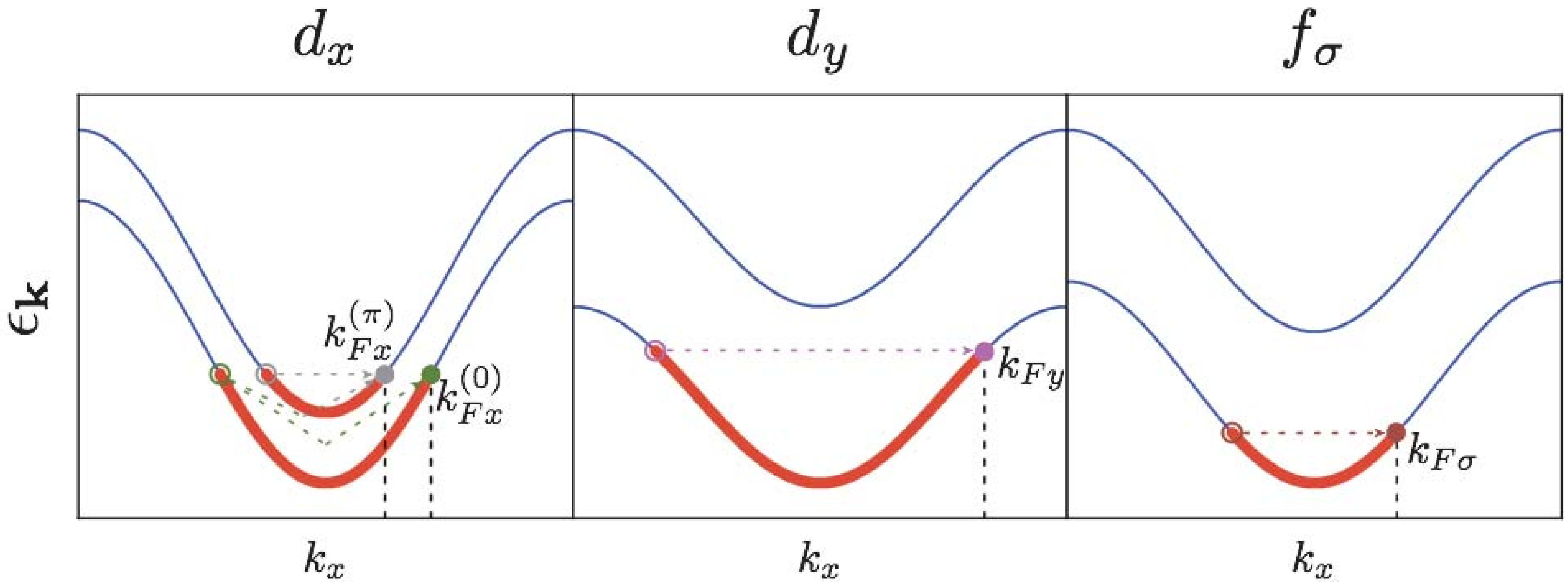


Density-density structure factor: DMRG



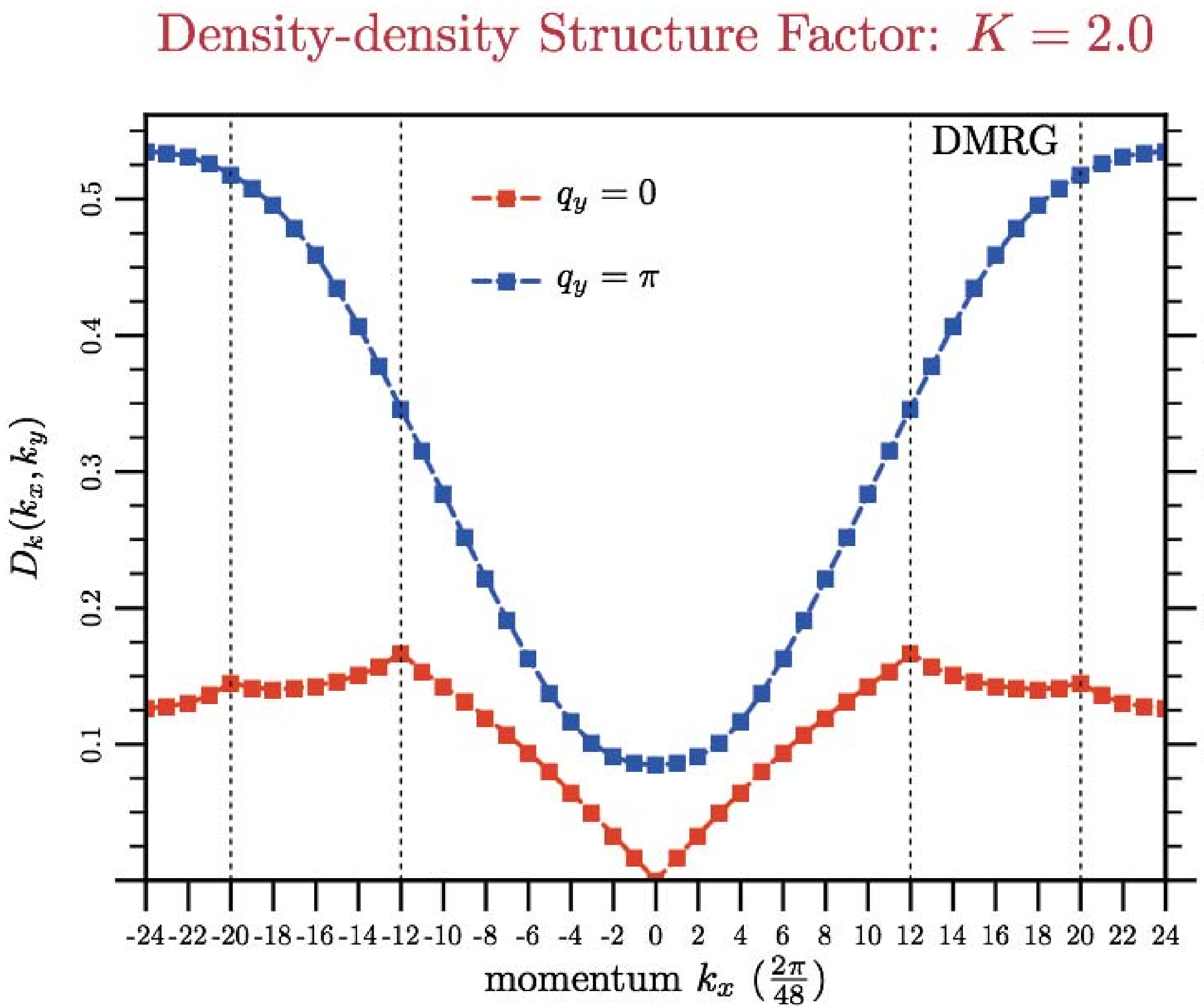
The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$

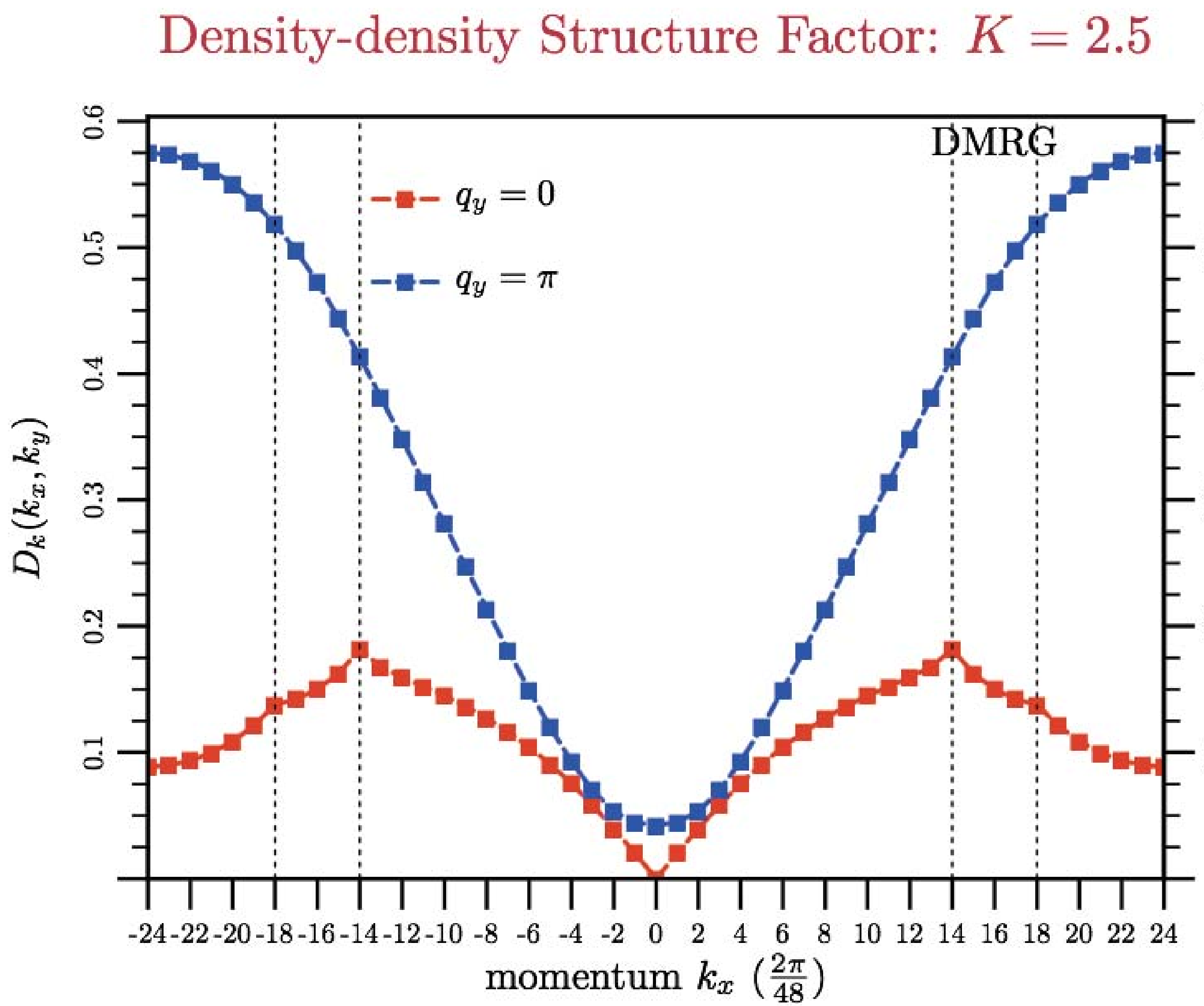


In D_k, enhanced singularities are predicted by the gauge theory at various "$2k_F$" wavevectors.

Evolution of Peak Locations

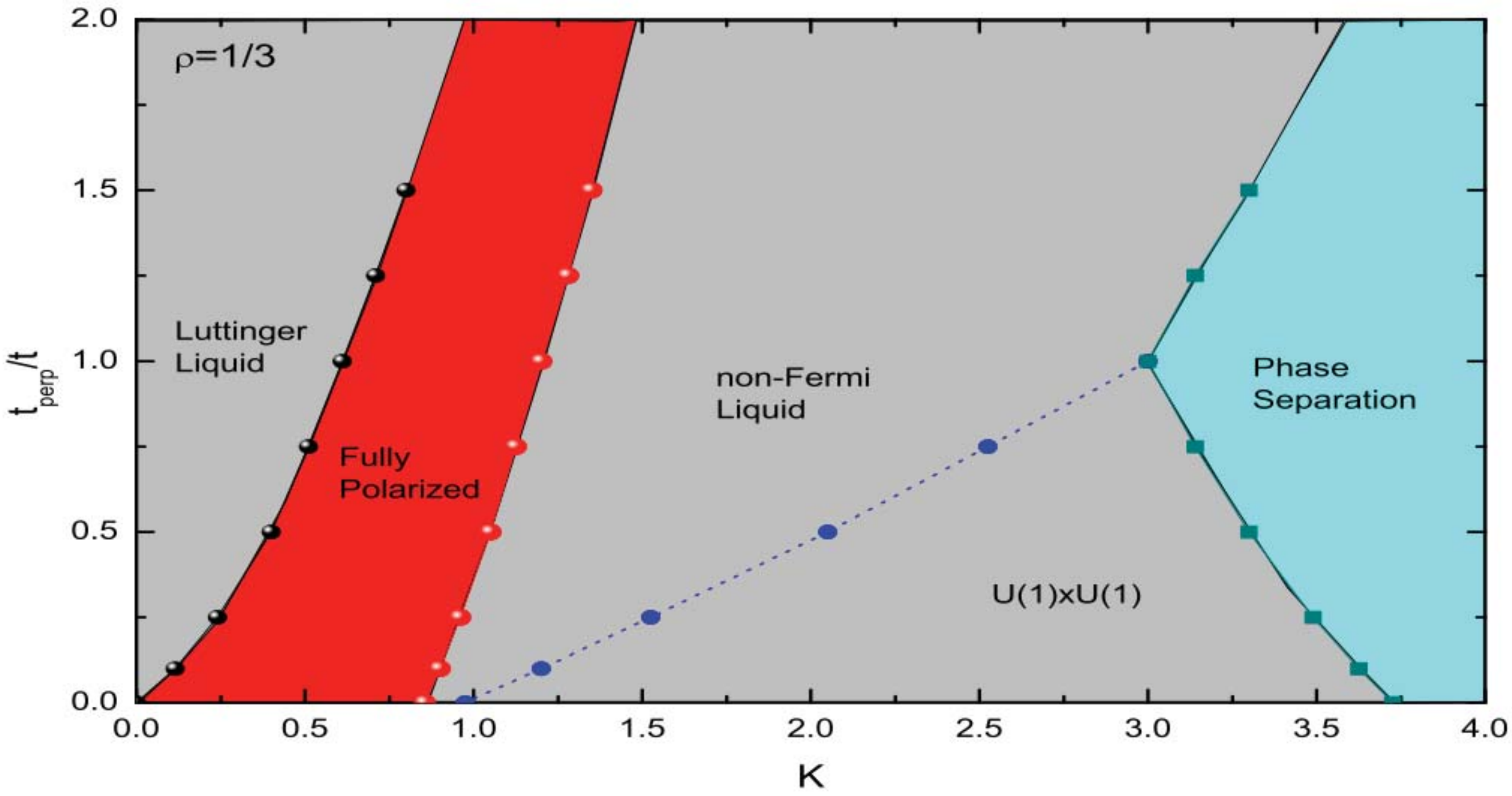


Evolution of Peak Locations



DMRG Phase diagram varying transverse electron hopping, t_{perp}

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c^\dagger_{i\alpha} c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}^\dagger_{13} \mathcal{S}_{24} + h.c.]$$



Variational Monte Carlo (VMC)

D-wave Metal: Product of Slater determinants

$$\Psi_{d_{xy}}^{Metal} = \det_x[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \cdot \det_y[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \times \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}}] \cdot \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}}]$$

Variational Parameters:

Distribution of d_x partons between bonding/anti-bonding bands (f-spinons and d_y partons only in bonding band)

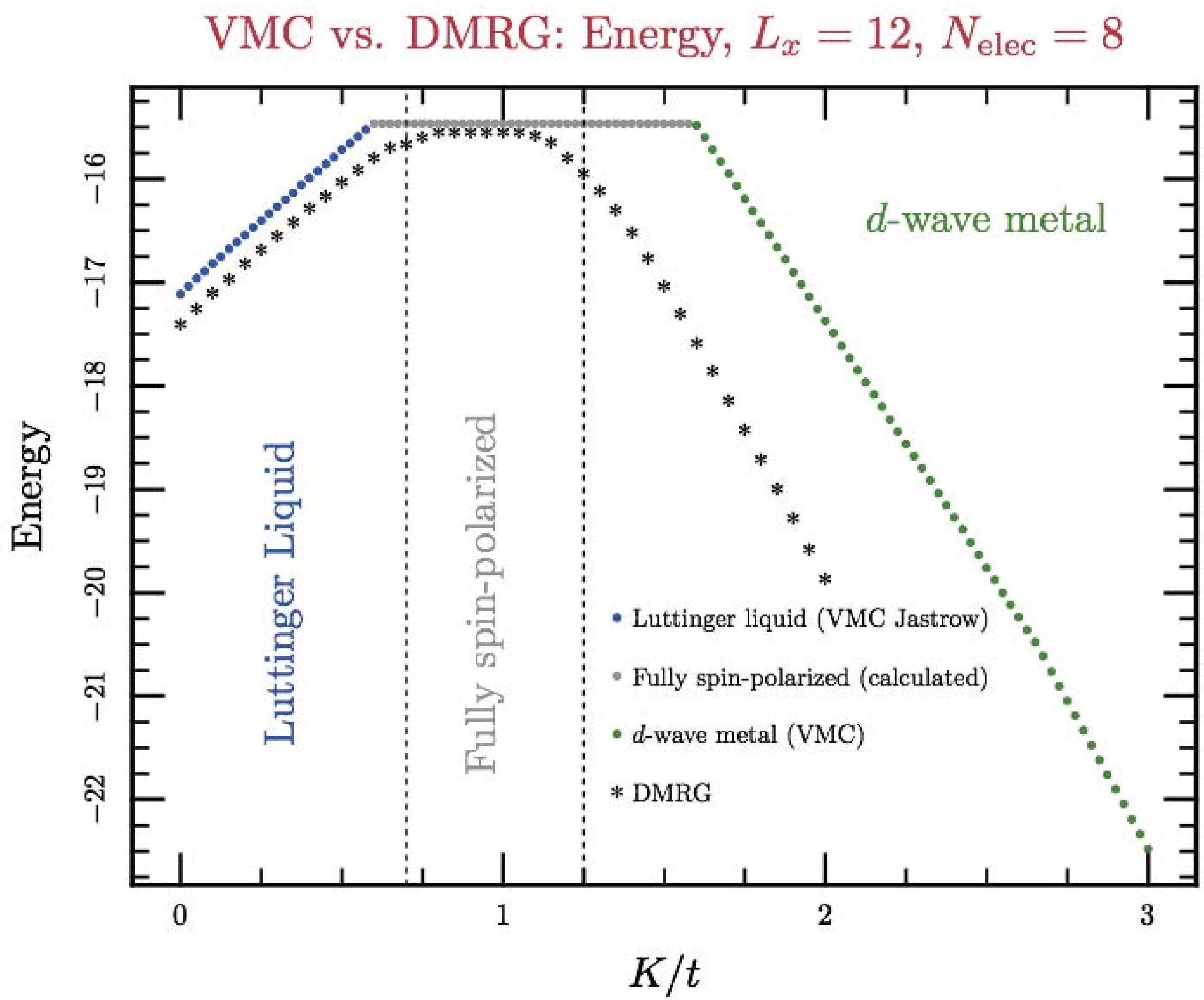
2 parameters to tune the Luttinger exponents

$$det_x = |det_x| sgn(det_x) \rightarrow |det_x|^{\gamma_x} sgn(det_x)$$

$$det_y = |det_y| sgn(det_y) \rightarrow |det_y|^{\gamma_y} sgn(det_y)$$

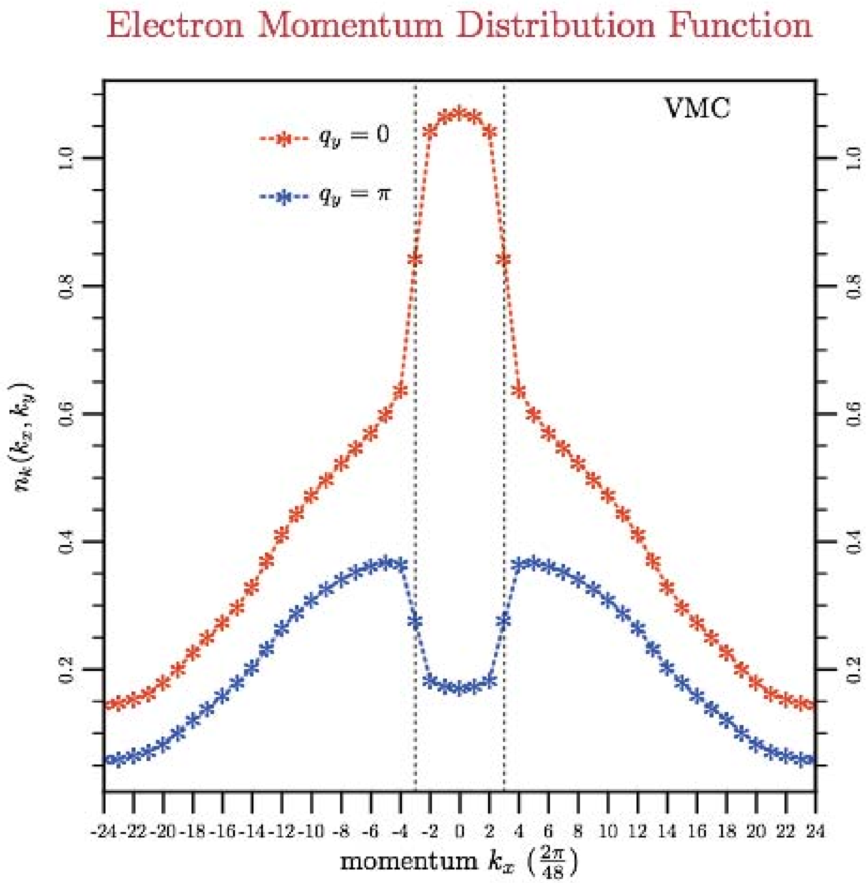
(Luttinger liquid phase: Jastrow factor multiplying filled Fermi sea)

Ground State energy: DMRG vs VMC



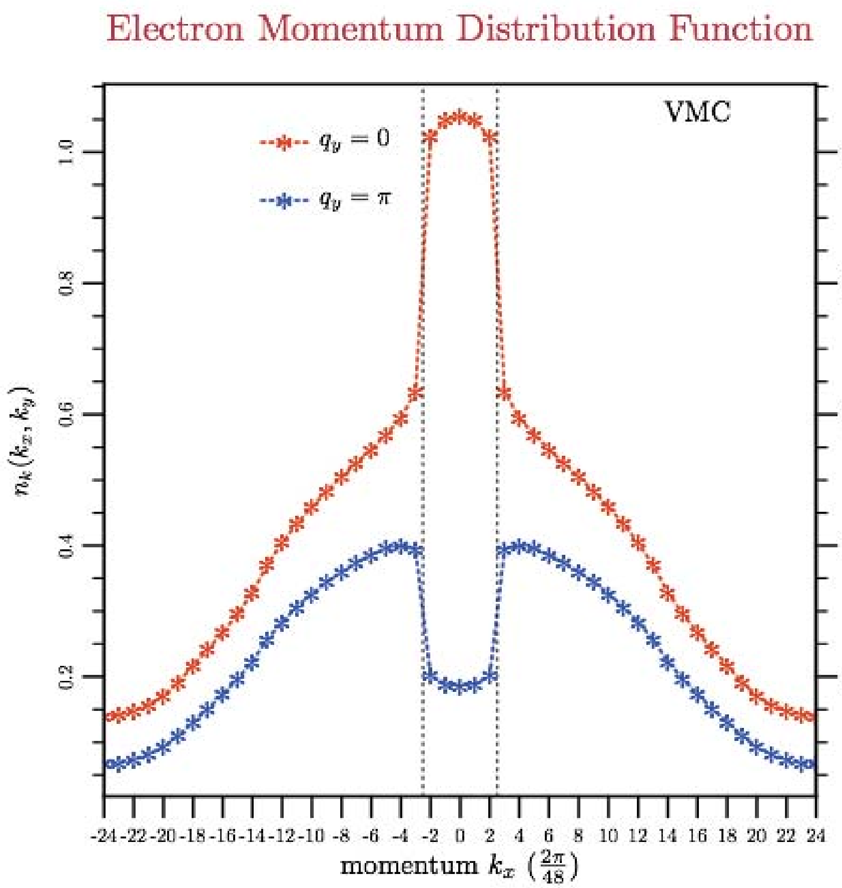
Evolution of VMC States

$$d_x : N_0 = 22, N_\pi = 10$$



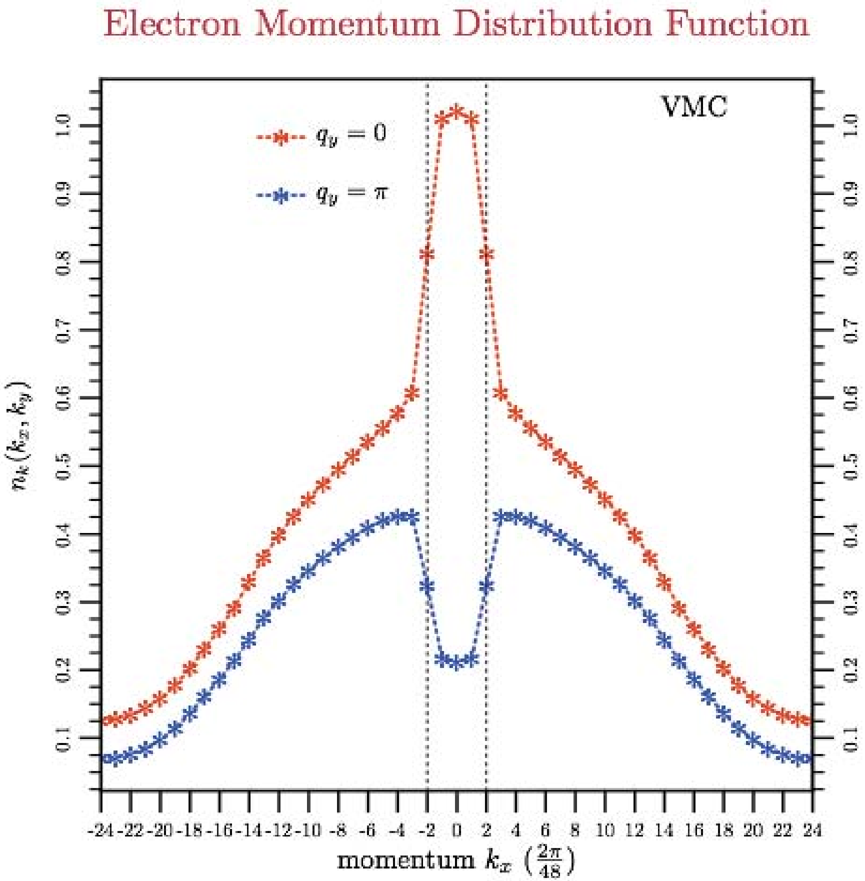
Evolution of VMC States

$$d_x : N_0 = 21, N_\pi = 11$$



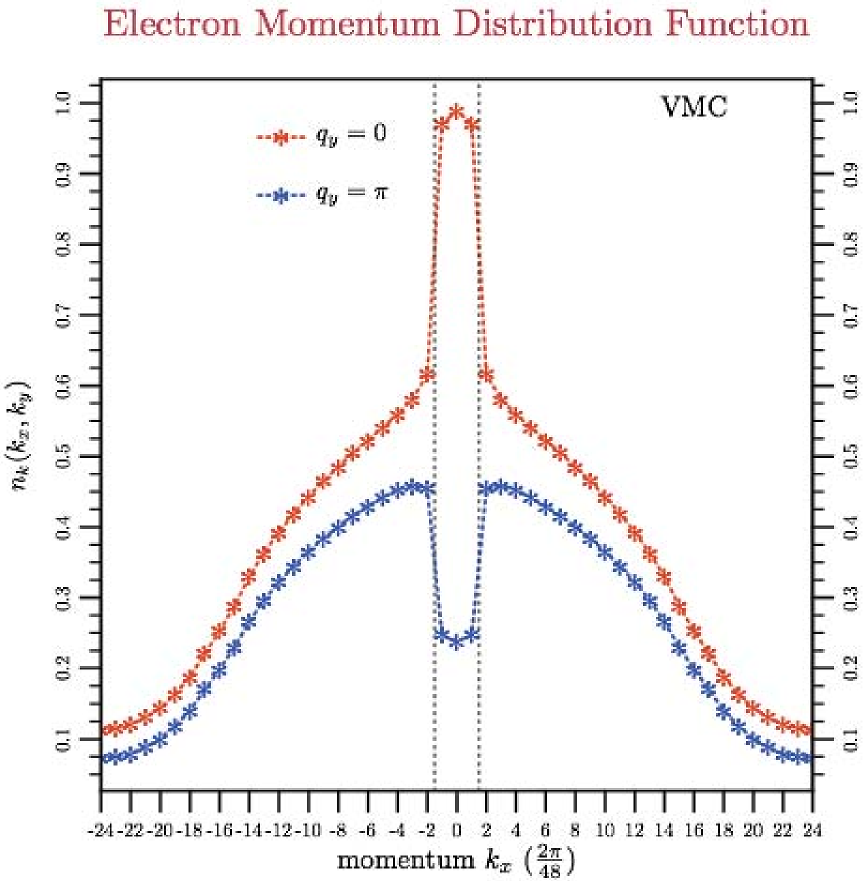
Evolution of VMC States

$$d_x : N_0 = 20, N_\pi = 12$$



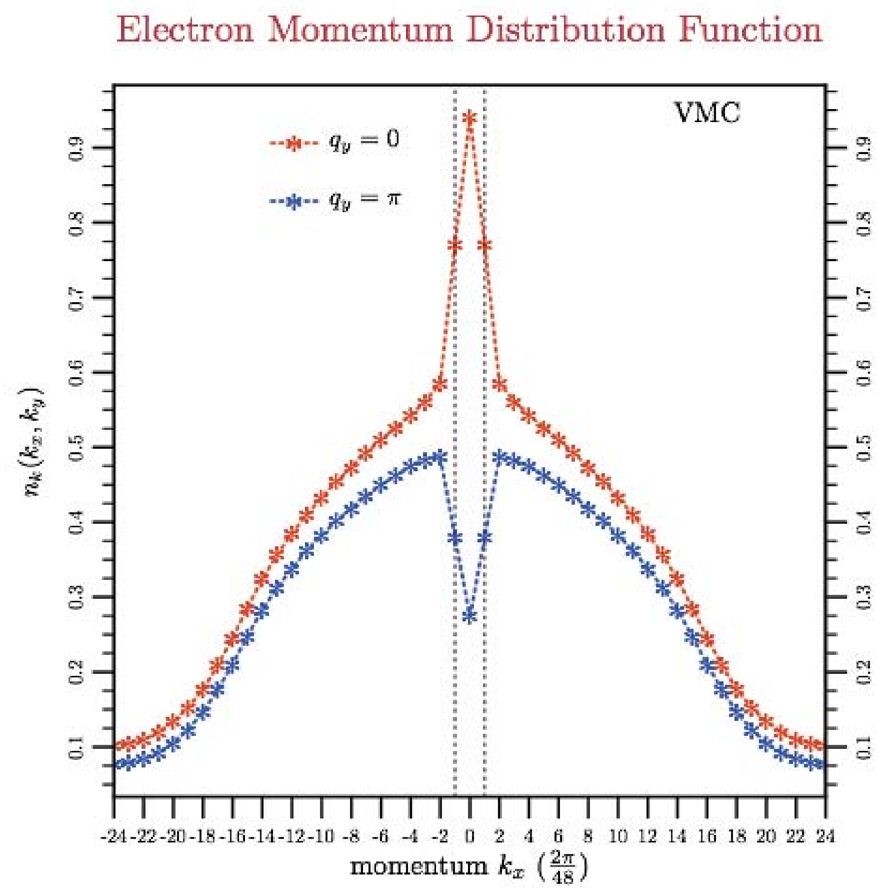
Evolution of VMC States

$$d_x : N_0 = 19, N_\pi = 13$$



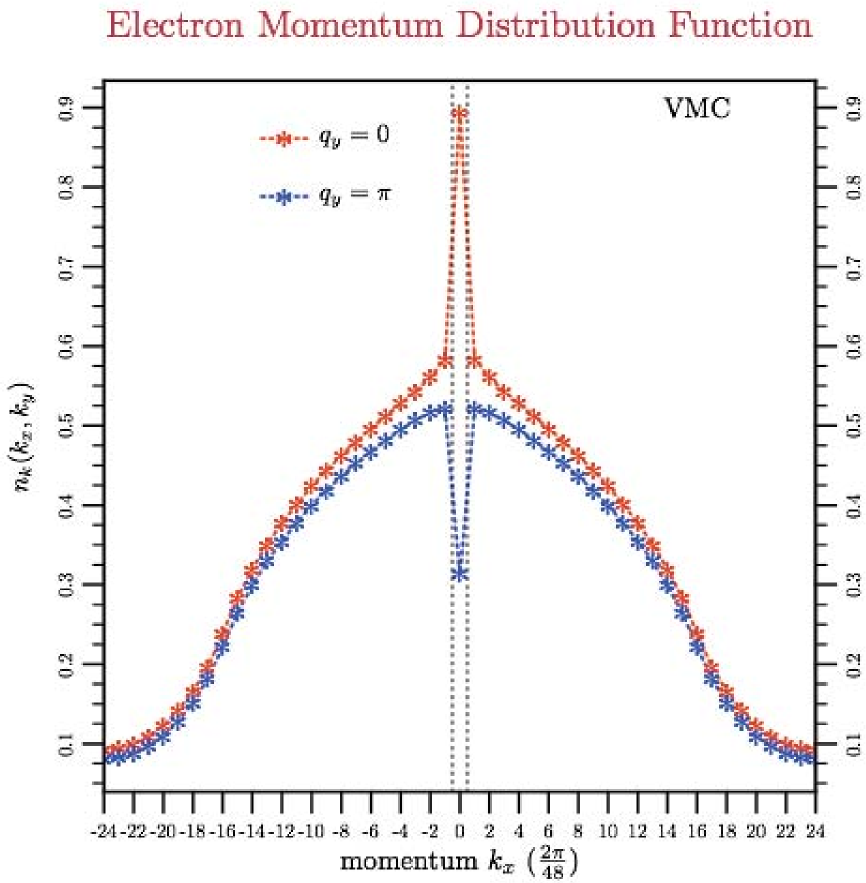
Evolution of VMC States

$$d_x : N_0 = 18, N_\pi = 14$$



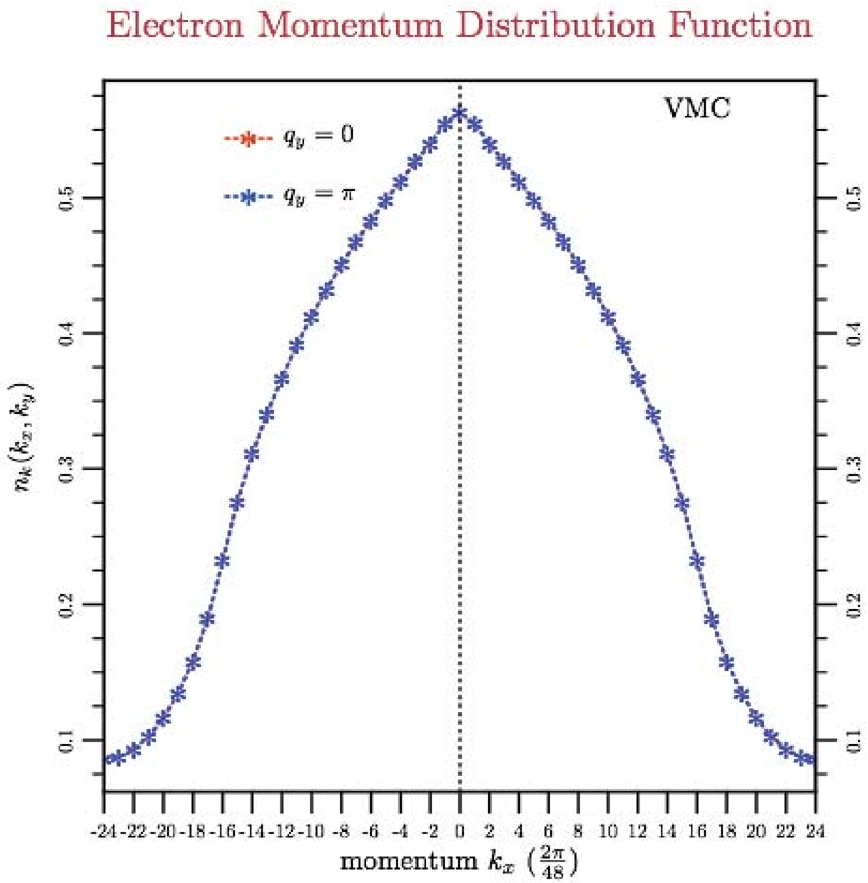
Evolution of VMC States

$$d_x : N_0 = 17, N_\pi = 15$$



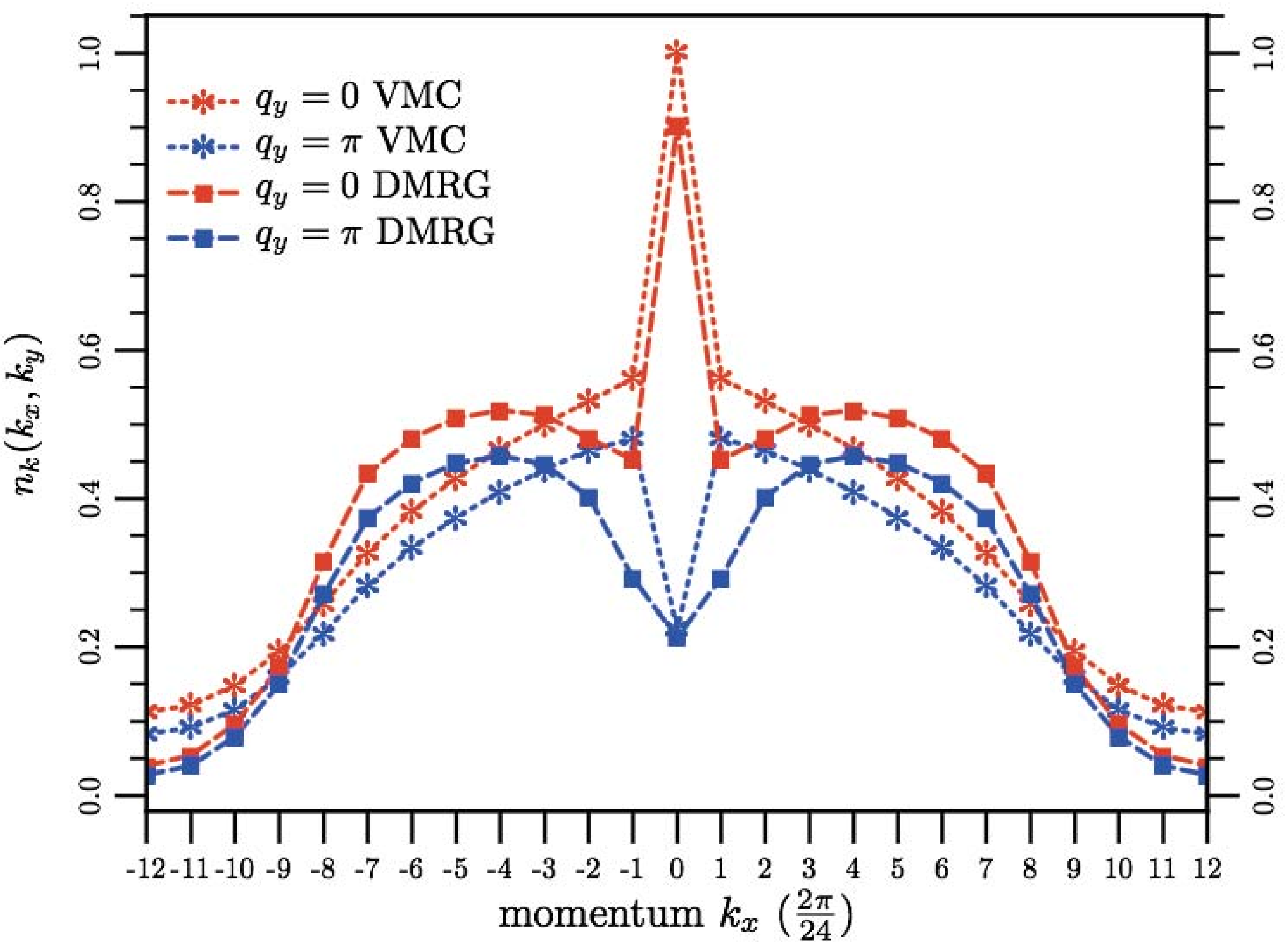
Evolution of VMC States

$$d_x : N_0 = 16, N_\pi = 16$$

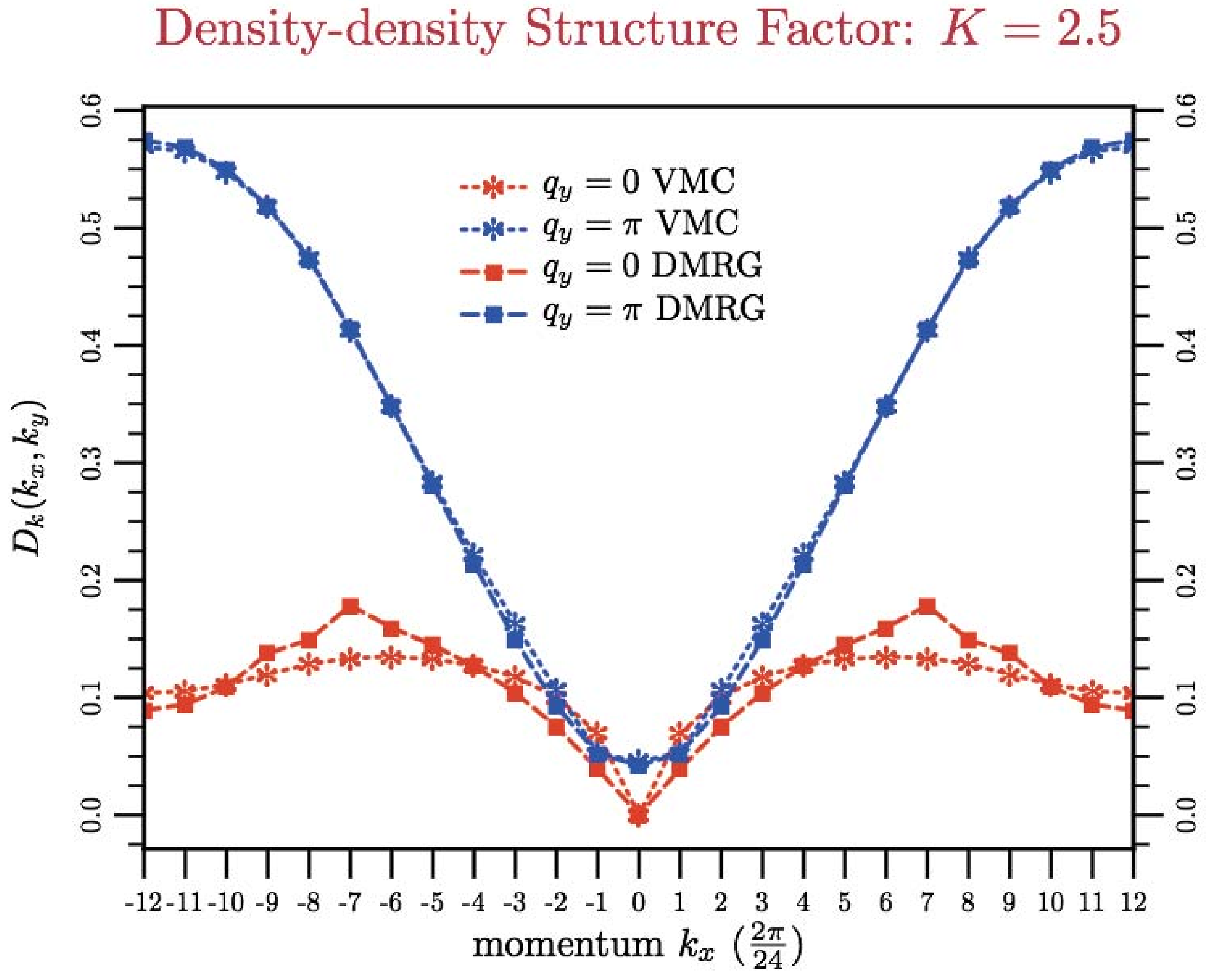


VMC vs. DMRG

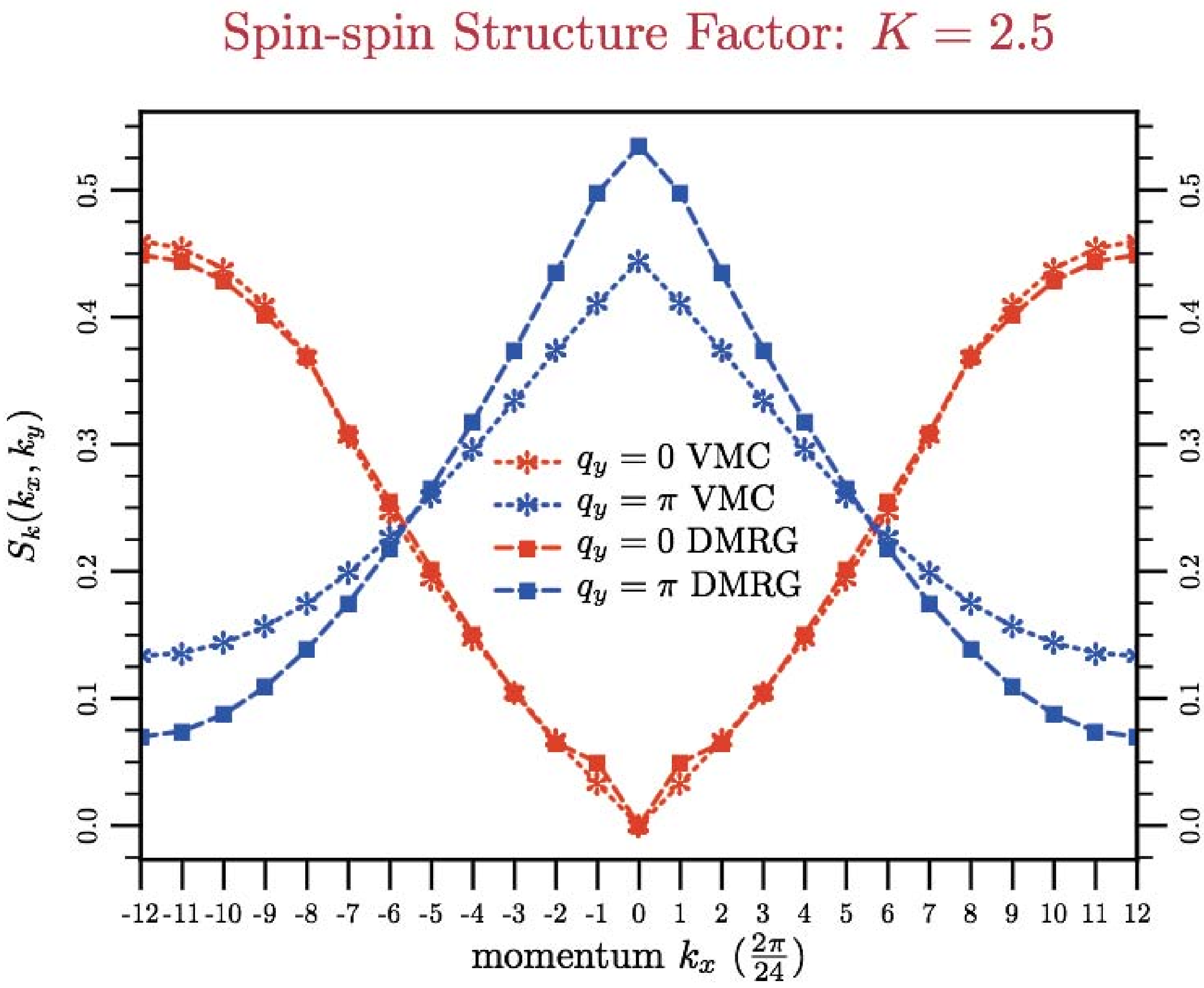
Electron Momentum Distribution Function: $K = 2.5$



VMC vs. DMRG


Density-density Structure Factor: $K = 2.5$
$q_y = 0$ VMC
$q_y = \pi$ VMC
$q_y = 0$ DMRG
$q_y = \pi$ DMRG
$D_k(k_x, k_y)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
momentum k_x $(\frac{2\pi}{24})$

VMC vs. DMRG



Spin-spin Structure Factor: $K = 2.5$
$S_k(k_x, k_y)$
$q_y = 0$ VMC
$q_y = \pi$ VMC
$q_y = 0$ DMRG
$q_y = \pi$ DMRG
0.0
0.1
0.2
0.3
0.4
0.5
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
momentum k_x $(\frac{2\pi}{24})$

Conclusions

- NFL phases of 2d itinerant electrons are extremely challenging
- Example NFL: "D-wave Metal"
- Electron Ring model on 2-leg ladder has "non-Luttinger liquid" phase
- DMRG/VMC establish non-LL is a ladder descendant of the 2d D-wave Metal.

Open Issues

- D-wave Metal on 2-legs; Dynamics, other filling factors
- Multi-leg ladders towards 2d
- t-J-K Hamiltonians with D-wave metal ground states?
- Electron ring-only model (K-model) has ***no sign problem*** – QMC?
- VMC energetics on 2d ring Hamiltonian (FL, D-wave BCS, D-wave Metal,…)
- Other wfs/Hamiltonians for 2d NFL phases??

Correlators and Structure Factors

Electron Momentum Distribution Function:

$$n_{k\sigma}(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \qquad n_k = n_{k\uparrow} + n_{k\downarrow}$$

Density-density Structure Factor:

$$D_k(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \left[\langle \rho(\mathbf{r}_i)\, \rho(\mathbf{r}_j) \rangle - \langle \rho(\mathbf{r}_i) \rangle \langle \rho(\mathbf{r}_j) \rangle \right] e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

$$\rho(\mathbf{r}_i) = c_{i\uparrow}^{\dagger} c_{i\uparrow} + c_{i\downarrow}^{\dagger} c_{i\downarrow}$$

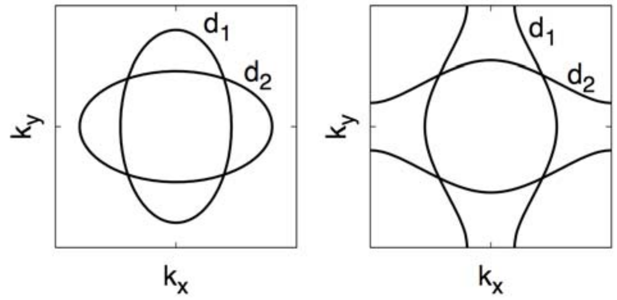
Spin-spin Structure Factor:

$$S_k(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

Bose Surfaces in D-wave Bose-Metal

Mean Field Green's functions factorize:

$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau) G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$



$$G_{d_\alpha}^{MF}(\mathbf{r}) \approx \frac{1}{2^{1/2}\pi^{3/2}} \frac{\cos(\mathbf{k}_{F_\alpha} \cdot \mathbf{r} - 3\pi/4)}{c_\alpha^{1/2} |\mathbf{r}|^{3/2}}$$

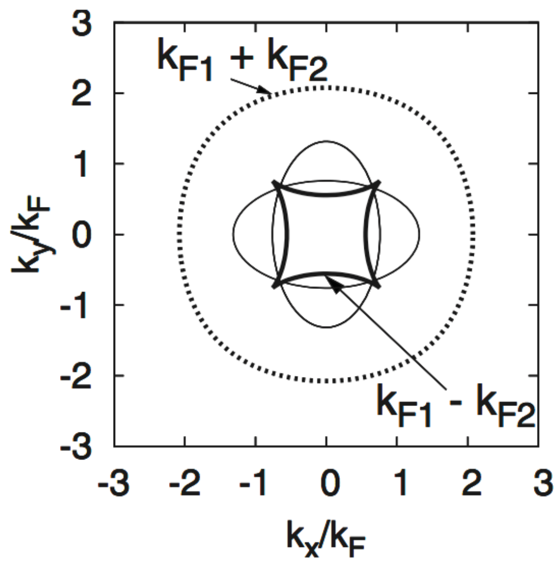
$$(\partial \epsilon_\alpha / \partial \mathbf{k})_{\mathbf{k}_{F\alpha}(\hat{\mathbf{r}})} = (const)\hat{\mathbf{r}}$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$$

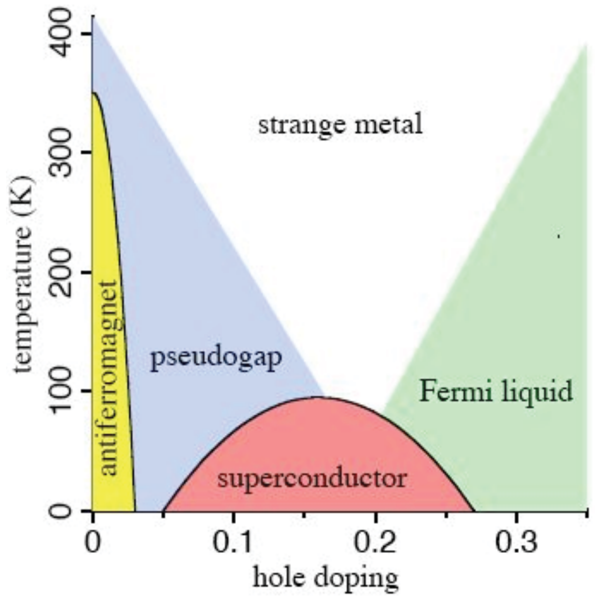
Two singular lines in momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$



Motivation for Non-Fermi-Liquid Metal: "Abnormal" state of High T_c Superconductors

Phase Diagram

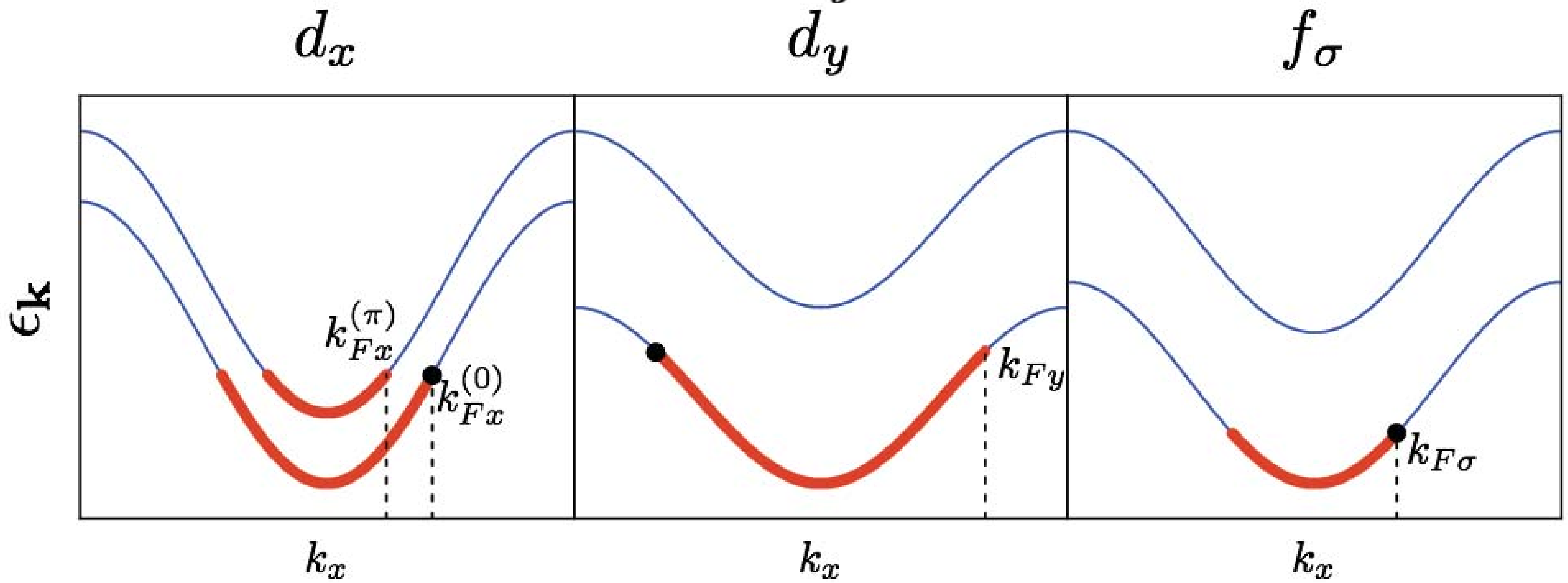


Strange metal: "Fermi surface" but quasiparticles are not "sharp"

Spectral function measured with ARPES suggests Z=0

The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$



In n_k, an enhanced singularity is predicted by the gauge theory at $k_{Fx}^{(k_y)} - k_{Fy} + k_{F\sigma}$

But what is a “Bose-Metal”?

First - A conventional interacting superfluid:

Boson Green’s function $$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

Off-diagonal long-ranged order $$G_b(\mathbf{r} \to \infty) = \rho_c = Z\rho; \qquad Z < 1$$

Momentum distribution function