The Abdus Salam
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Educational, Scientific and
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Workshop on Sphere Packing and Amorphous Materials

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Fast and Slow Lengths in Amorphous Solids

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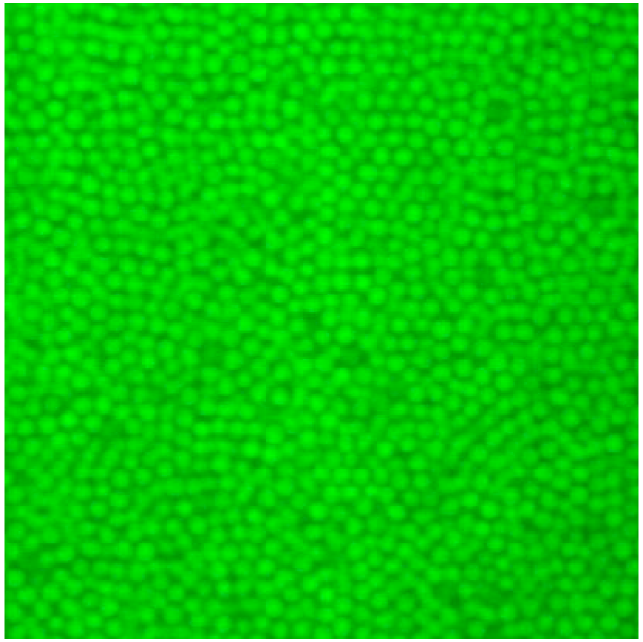
Fast and slow lengths in amorphous solids

PMMH-ESPCI, Paris

Trieste 2011

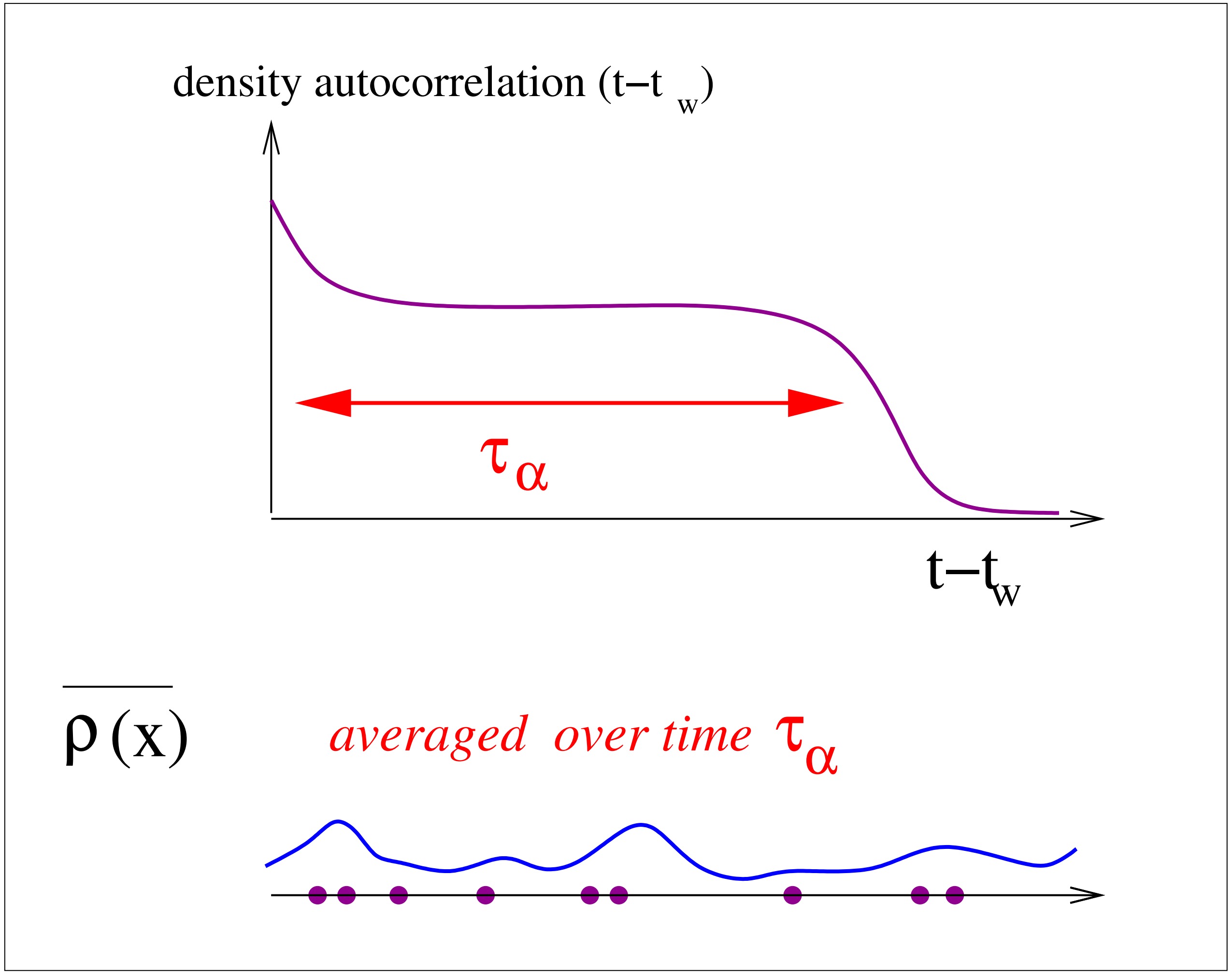
Amorphous solids



- **Amorphous:** ***there doesn't seem to be a rule to construct the density profile***

- **Solid:** ***spatial density modulations not erased by thermal fluctuations***

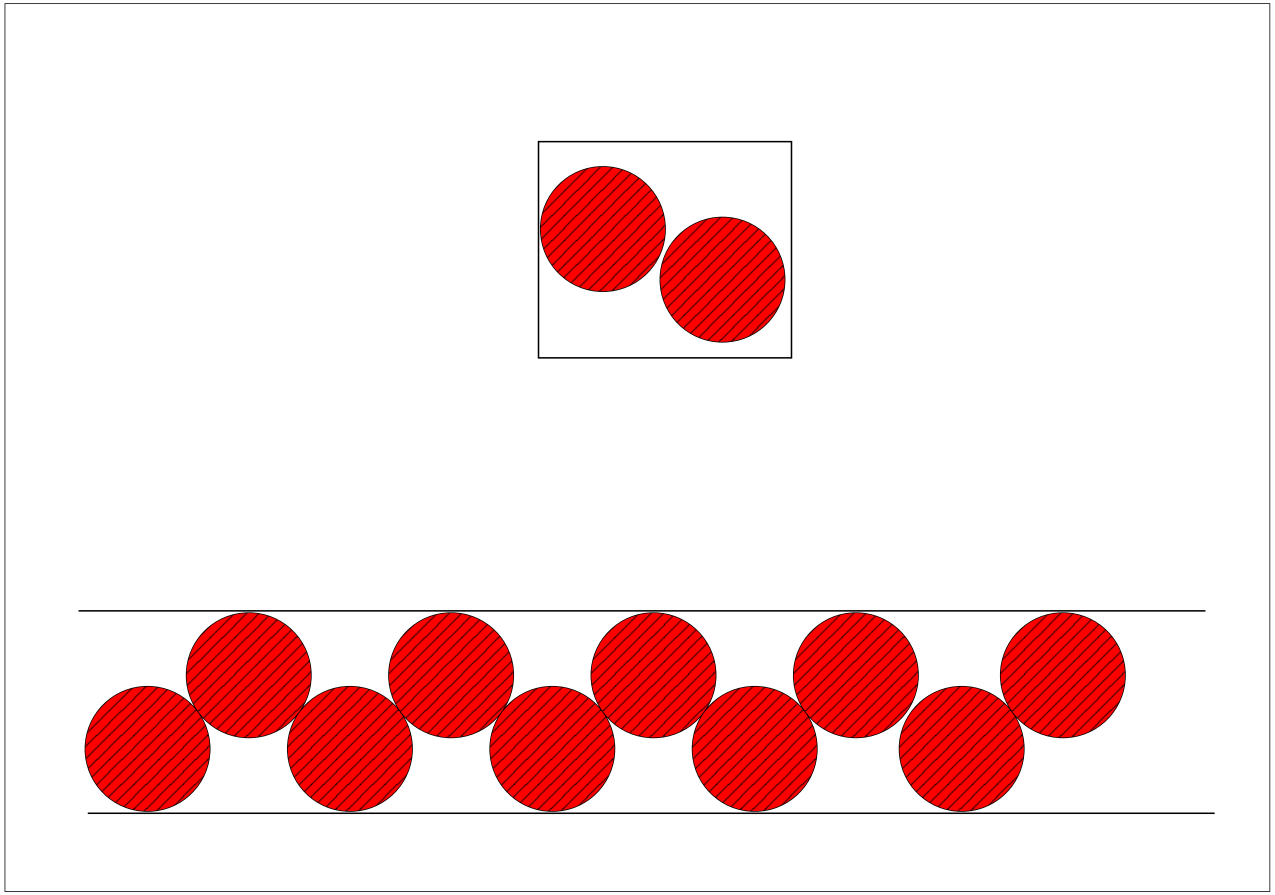
Particle systems: supercooled liquid



i) Jammed non-thermodynamc solidity: stable packings

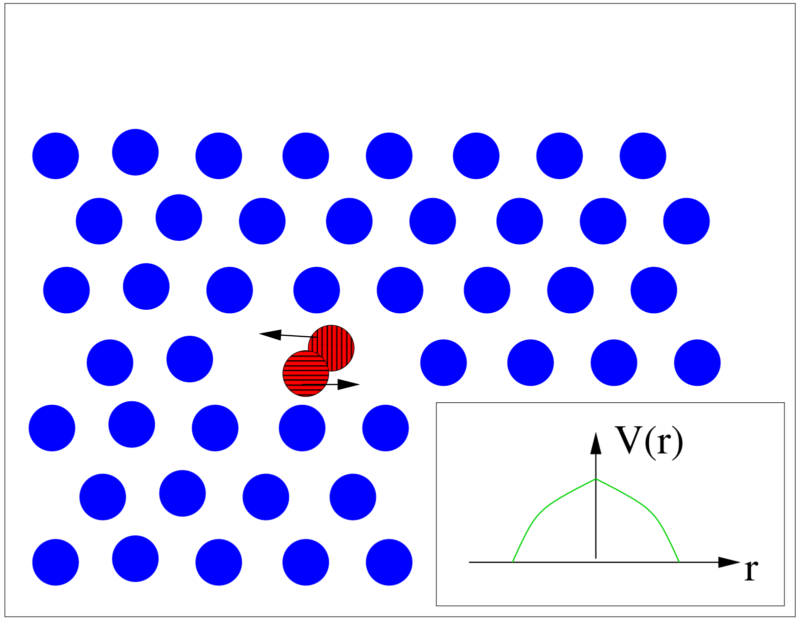
ii) Thermodynamc solidity

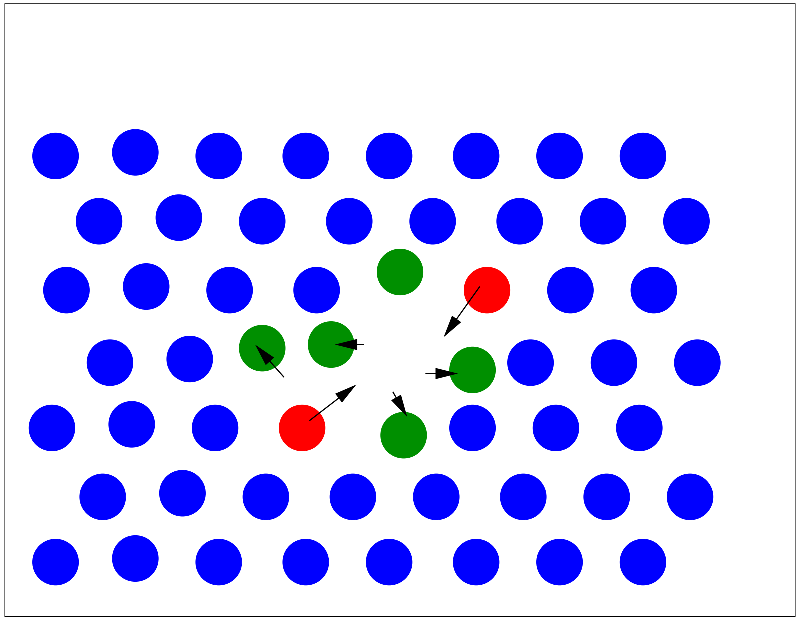
jammed non-thermodynamc solidity



Thermodynamic solidity

Soft (and even hard) particles may rearrange

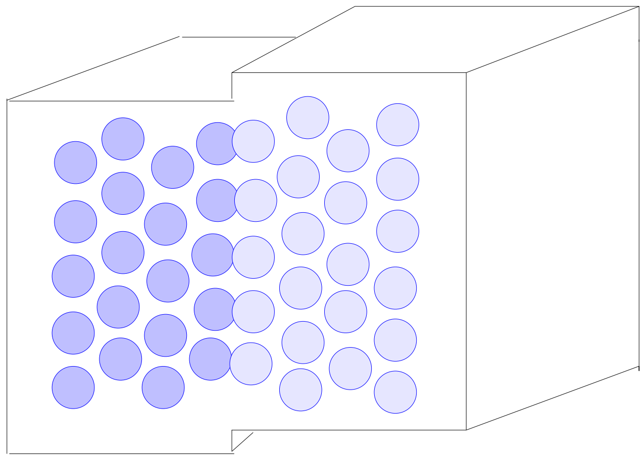


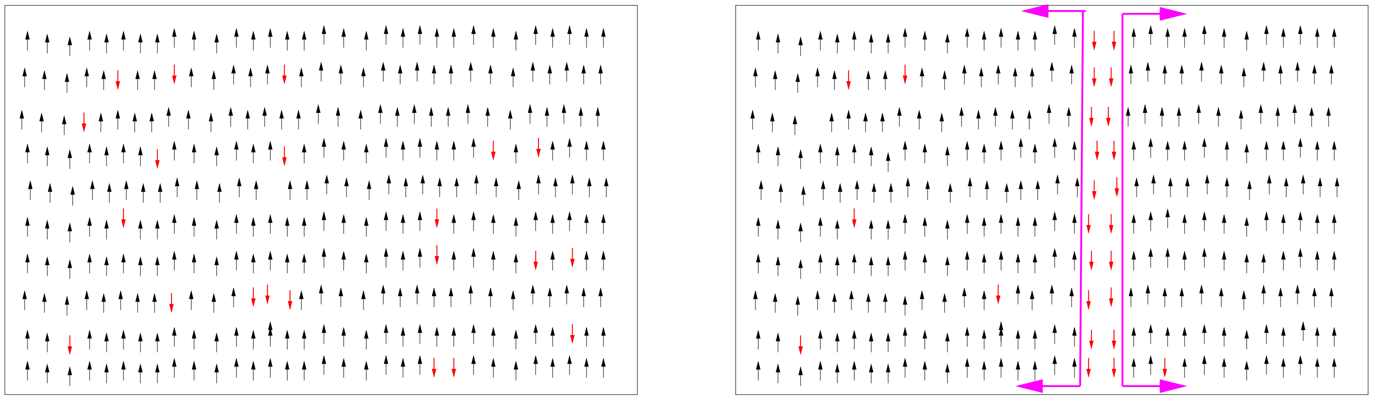


It is a conspiracy of soft constituents that makes a solid

a hard building made with soft bricks

energetic





entropic

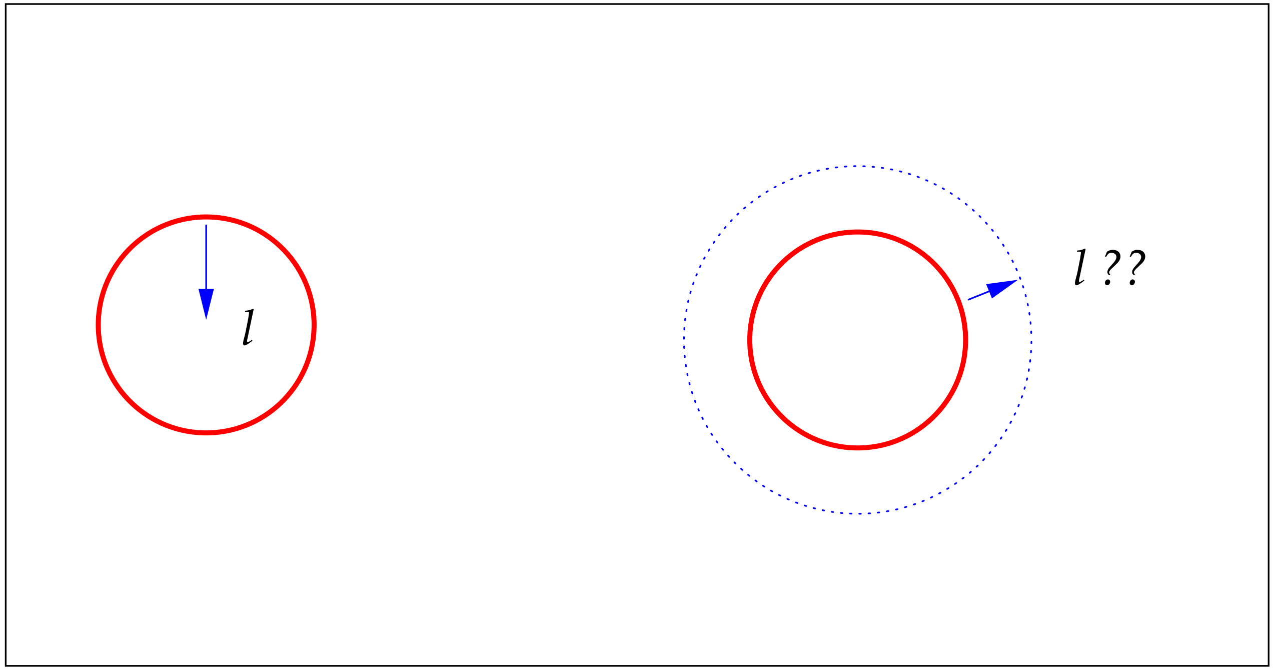
Two lengths that should diverge in a

thermodynamic solid

A Theorem for point-to-set correlations Montanari-Semerjian

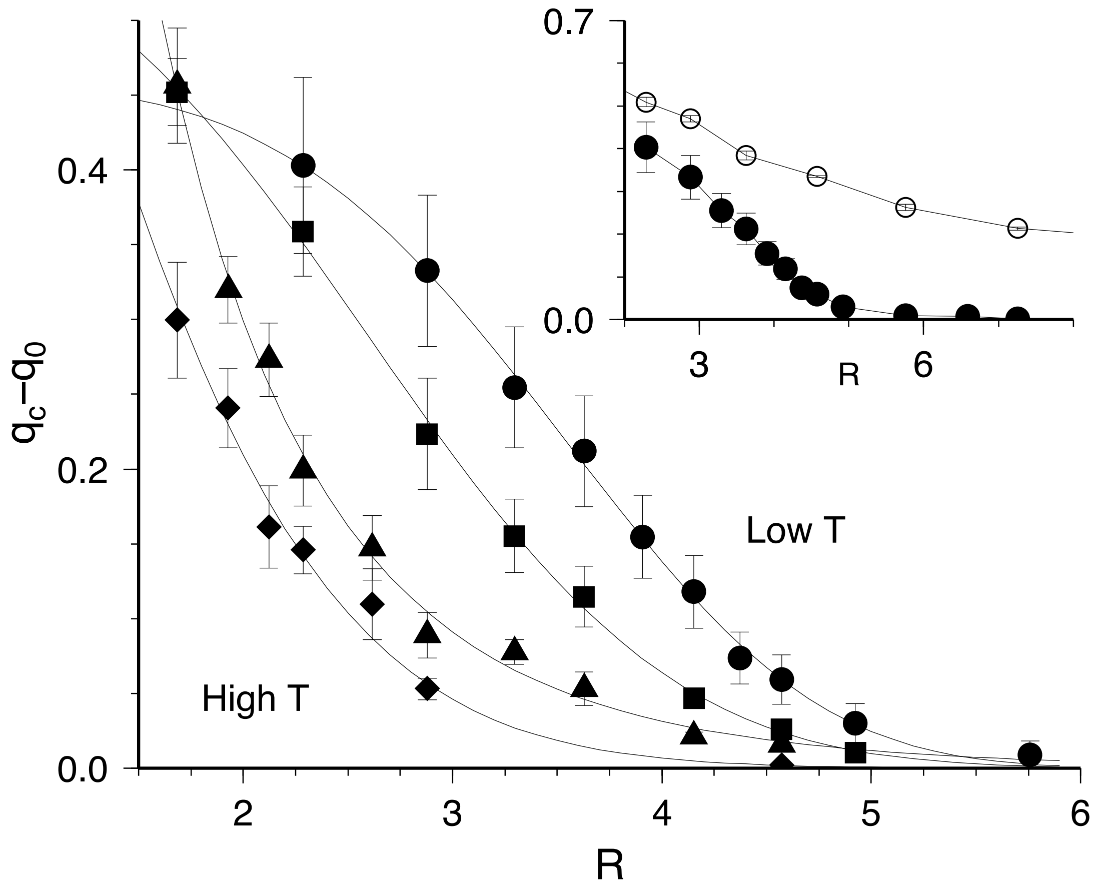
boundary determines interior for $\ell < \ell(T)$

$\ell \to \infty$ **if** ($timescale \quad \to \infty$)



Point-to set correlation for a supercooled liquid

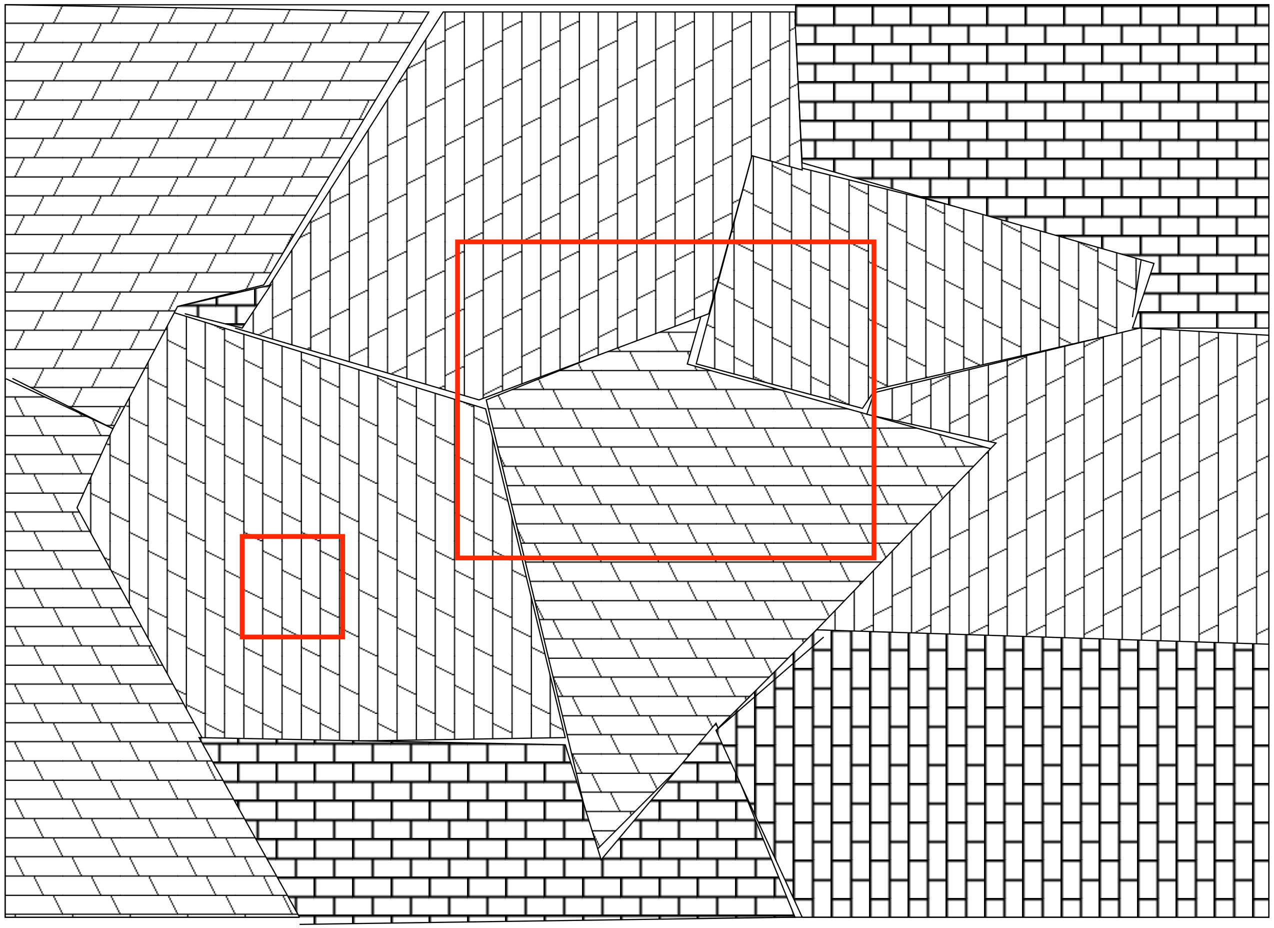
Biroli, Bouchaud, Cavagna and Grigera



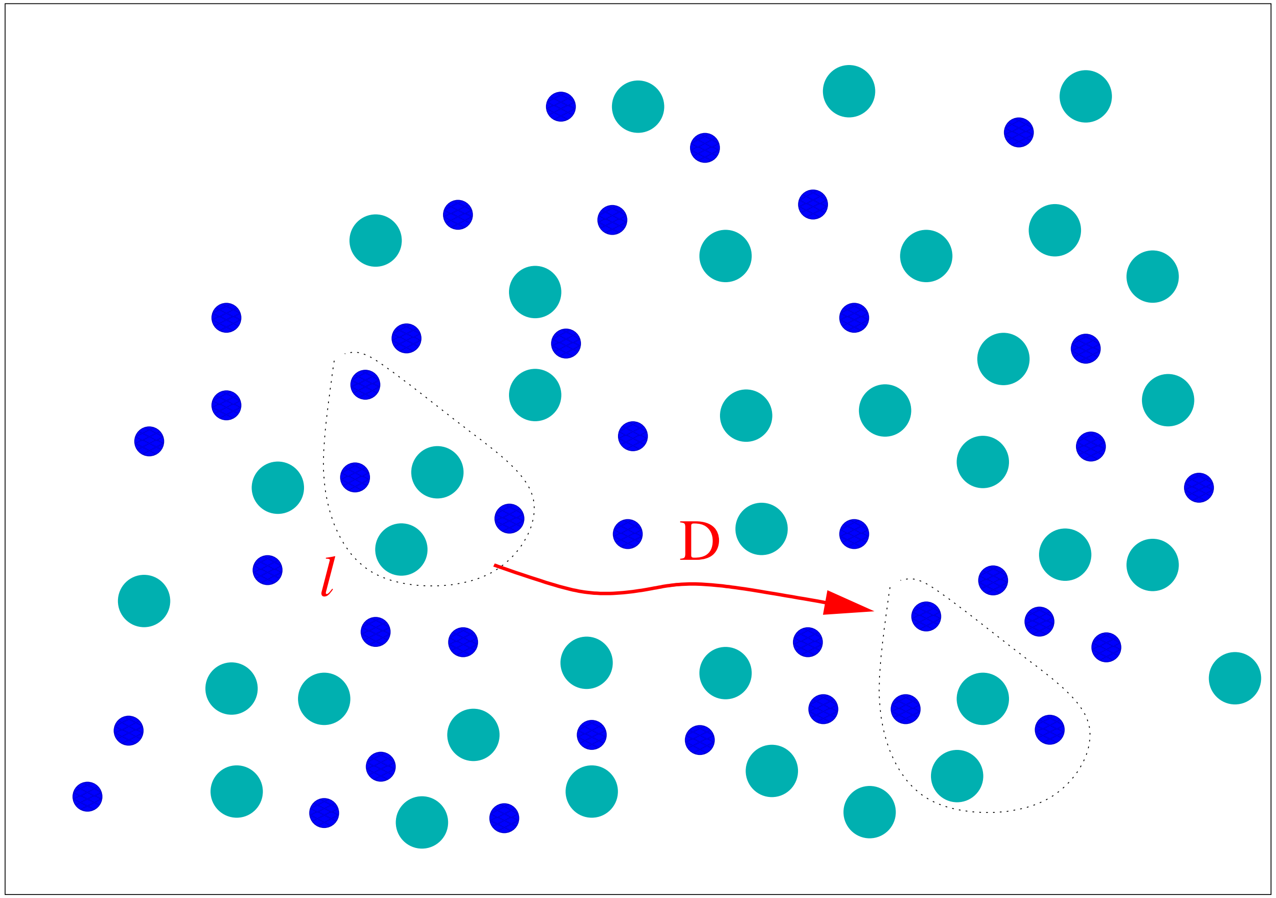
Patch - recurrence length $D(l)$ crossover l_o

J.K. , Levine

detects crystallite length.



Generalize this to general systems



Three levels of order

0101010101010101010101010101

Periodic, *Fourier transform gives deltas.*

1011010110110101101011011010110110

Fibonacci sequence *Quasiperiodic, Fourier transfom → dense set of δ-functions*

01101001100101101001011001101001

Thue-Morse sequence *'Non-Pisot" Fourier transform has no δ functions.*

Inflation rules

$$\textbf{Rule 1}: \qquad 1 \to 10 \qquad 0 \to 10 \tag{1}$$

$$\textbf{Rule 2}: \qquad 1 \to 10 \qquad 0 \to 1 \tag{2}$$

$$\textbf{Rule 3}: \qquad 1 \to 10 \qquad 0 \to 01 \tag{3}$$

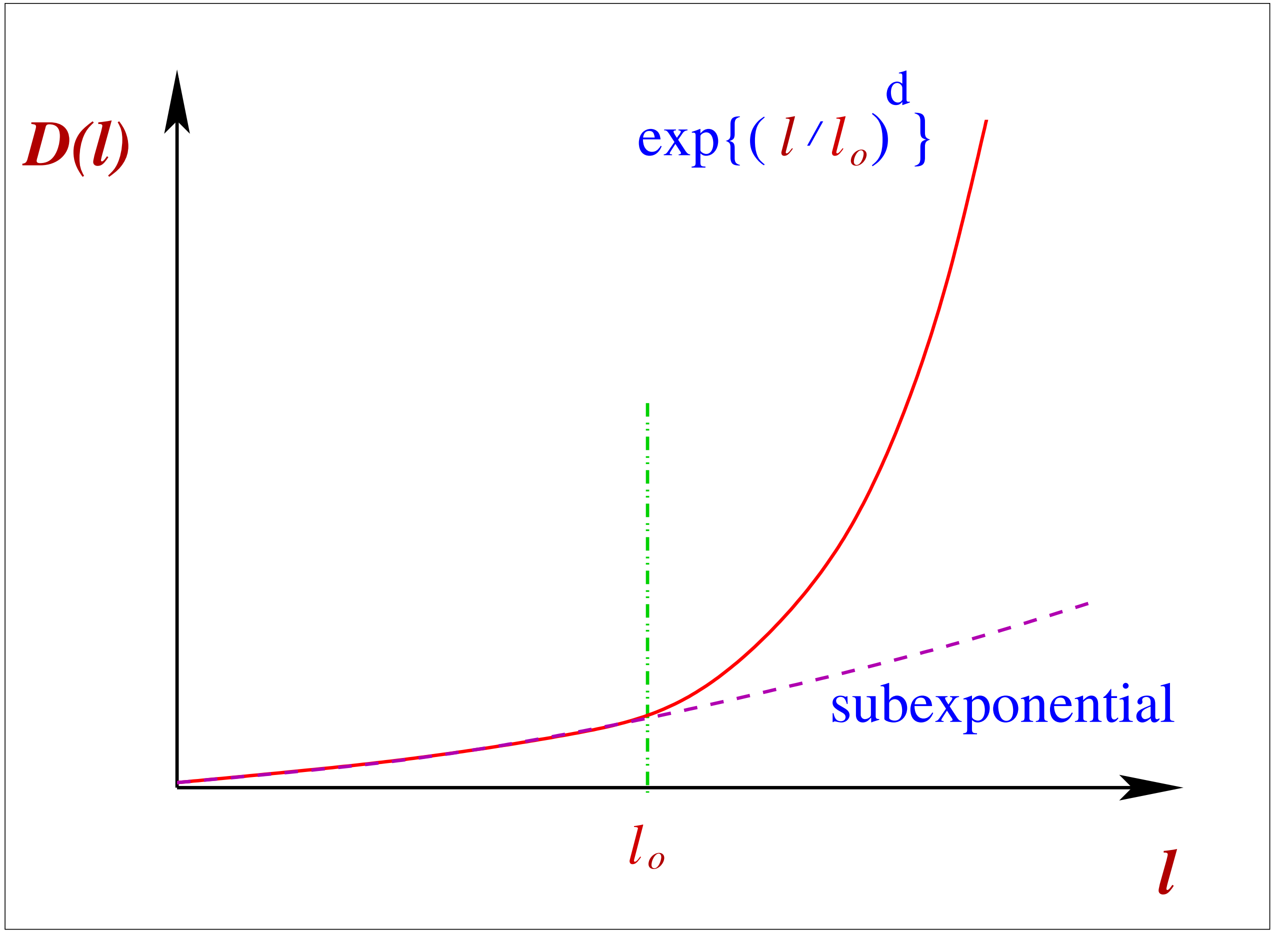
Patch repetition is a matter of entropy

subextensive entropy $\rightarrow$ infinite length

independent pieces will always yield extensive entropy

Related idea: ***weak periodicity*** **S. Aubry**

Patch - recurrence length $D(l)$ crossover l_o

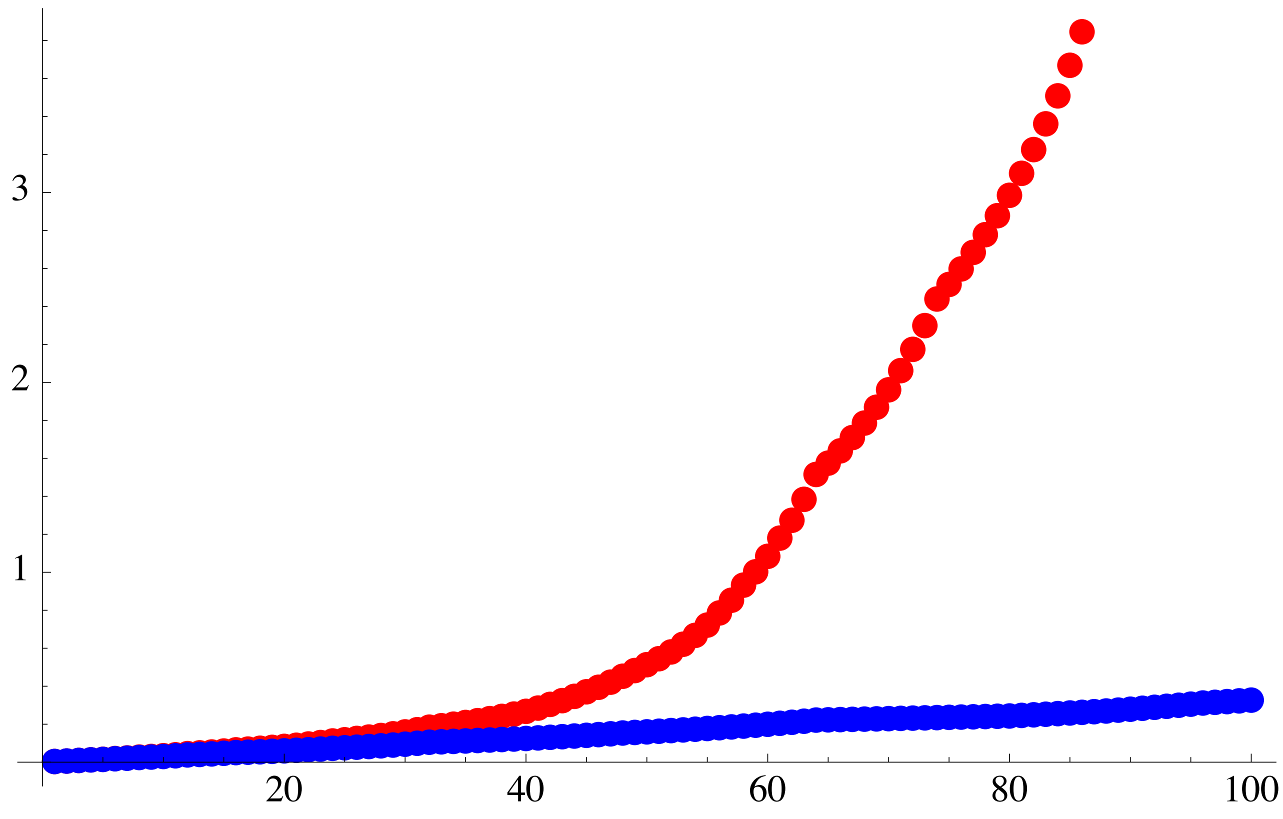


Finite correlation lengths in imperfect sequences

$D(l)$ vs. l

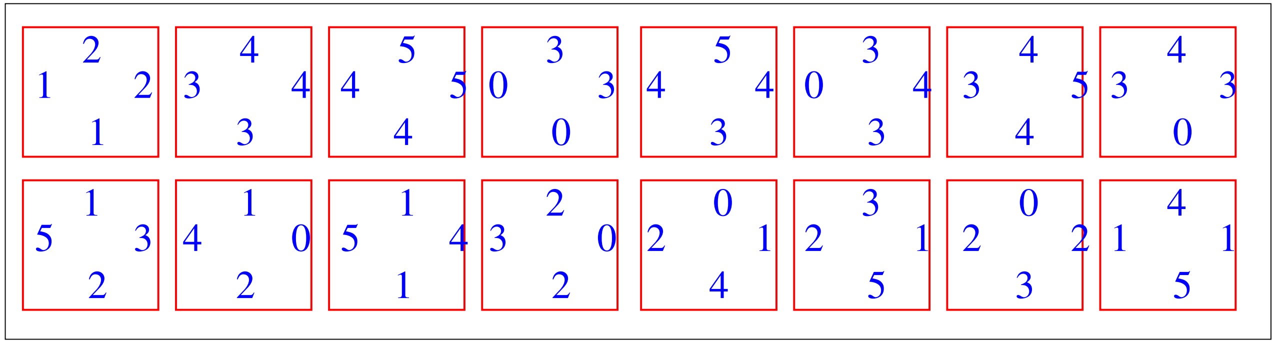
11001011010010110010110011010000101101001011101101001100

1100101101001011001011001101000010110100101110110100 1100



More examples: higher dimensionalities

Wang Tiles

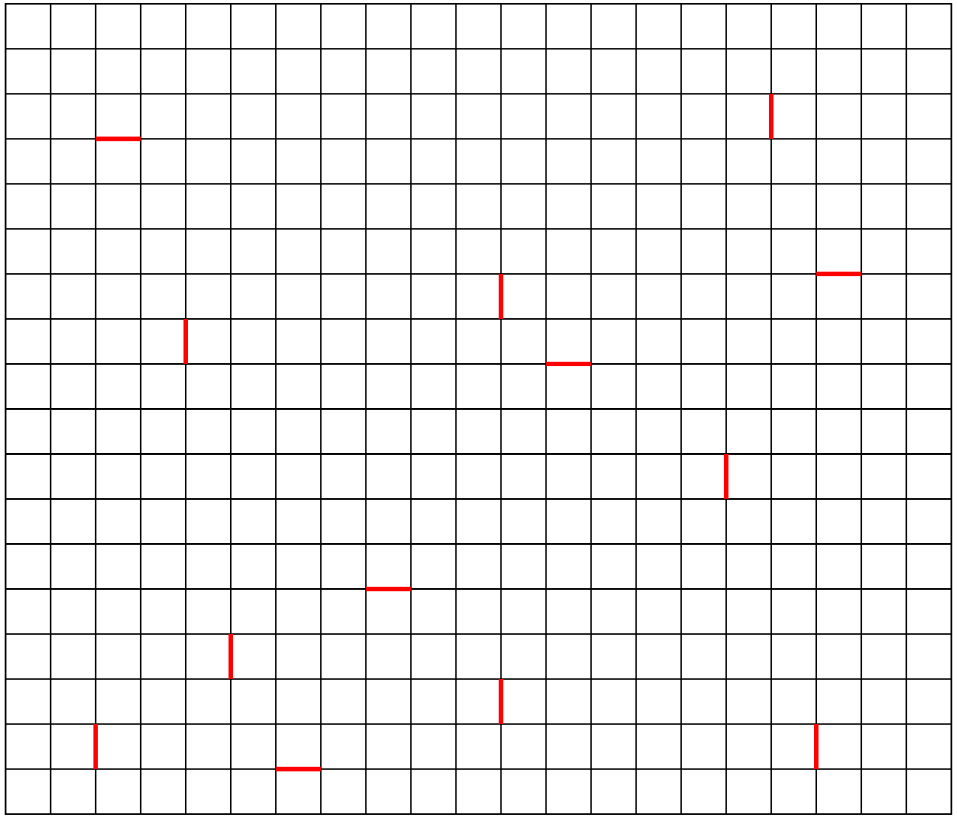


Quasiperiodic ground states

can be seen as a 12-state spin model (Leuzzi and Parisi)

Monte Carlo dynamics is slow

annealing to zero temperature leads to a system with point defects



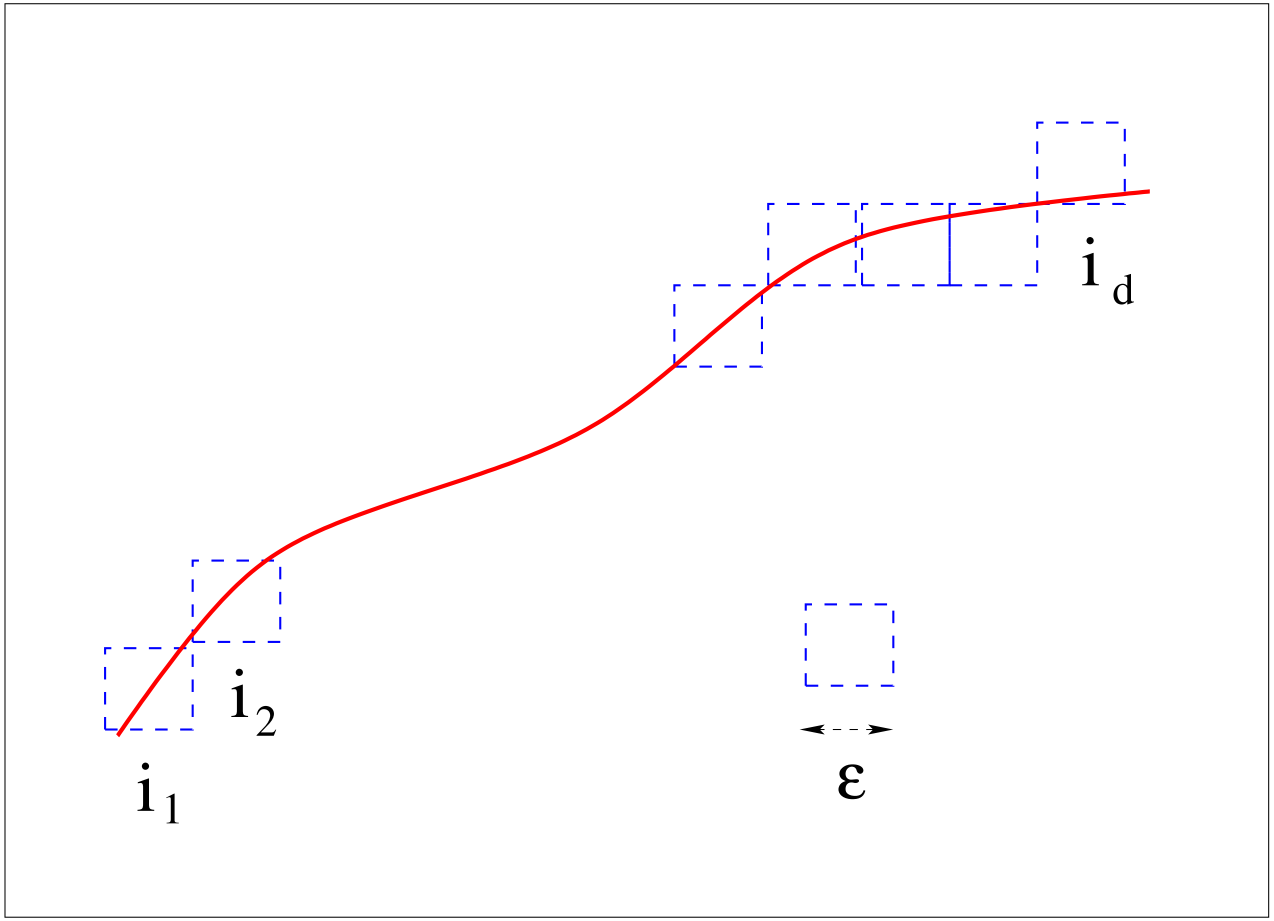
coherence length = inter-defect lenght!!!

if it is $> O(1)$, then infinite lifetime is possible

Note that coherence is lost through energetically pointlike defects

Particle systems

we need to count profiles $\leftrightarrow$ identify patches



inspiration from dynamic systems

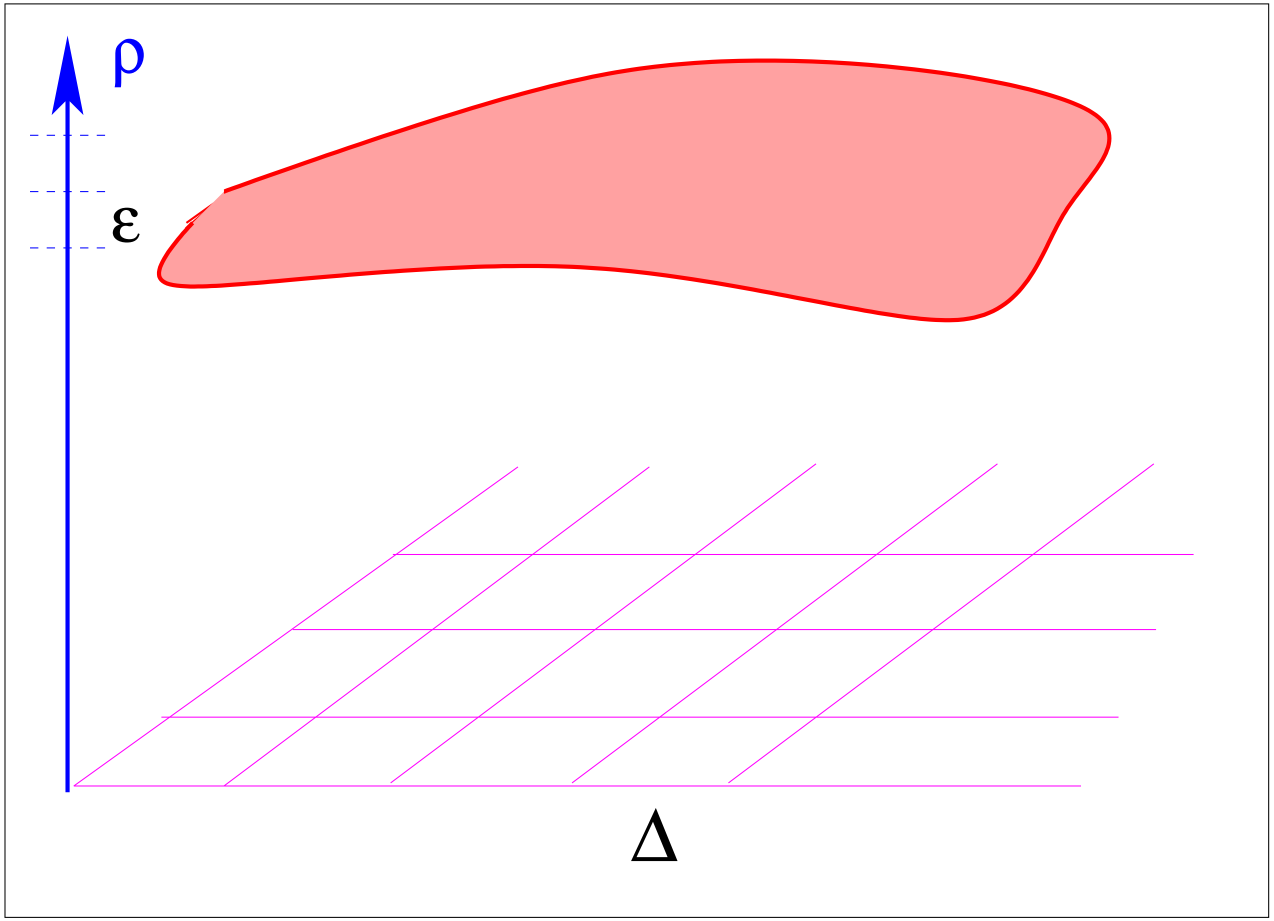
The limit is well-defined:

$$K_1 \sim - \lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau d} \sum_{i_1,\ldots,i_d} P_\epsilon(i_1, \ldots, i_d) \ln P_\epsilon(i_1, \ldots, i_d)$$

Renyi: a measure of 'rare' patches (very frequent or very unfrequent):

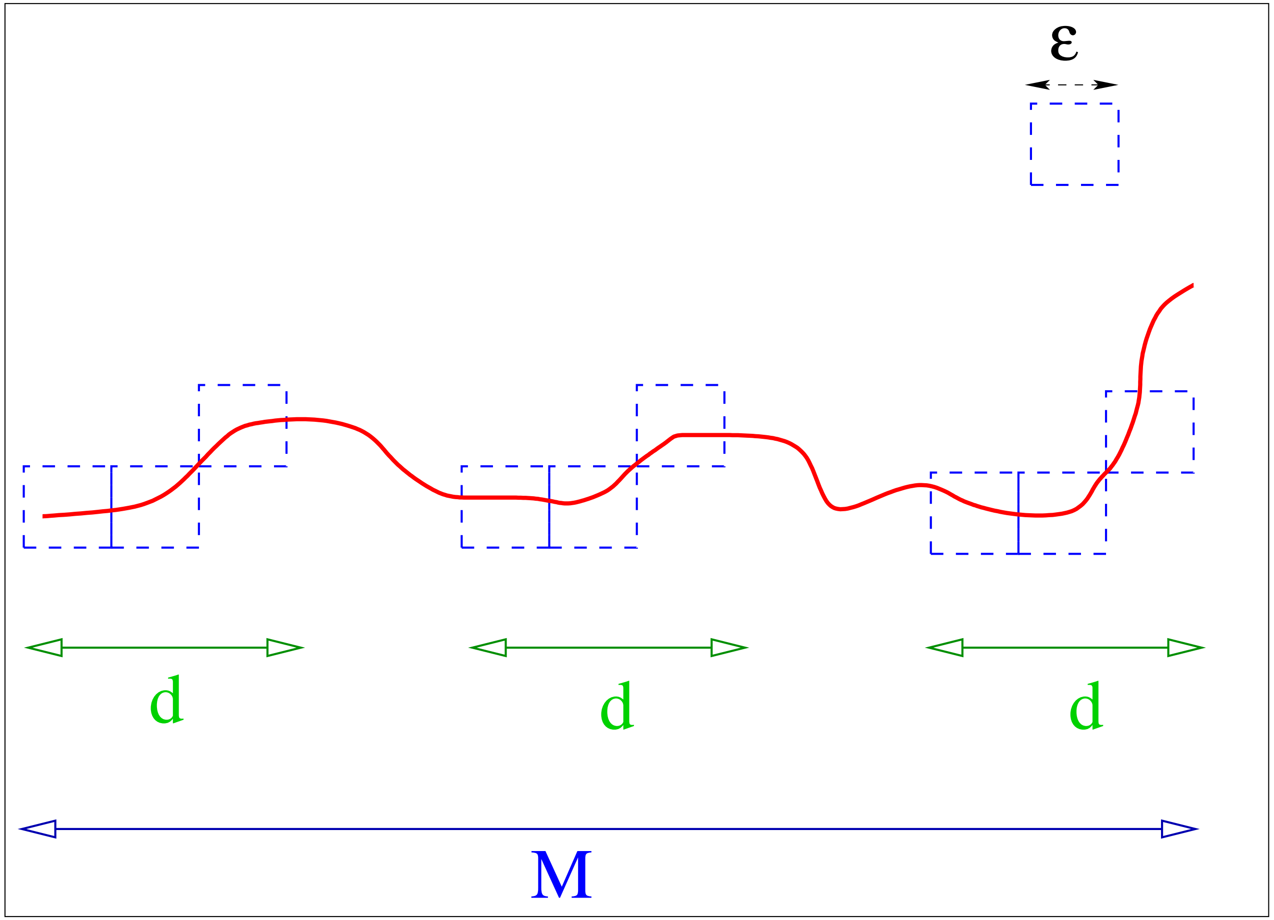
$$K_q \sim - \lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau d(q-1)} \ln \left(\sum_{i_1,\ldots,i_d} P_\epsilon(i_1, \ldots, i_d)^q \right)$$

$\ldots \to \mathcal{P}[P_\epsilon]$ **by Legendre transform.**



$$t \to \vec{r} \qquad x \to \rho$$

Grassberger-Procaccia:



count the number of repetitions n_i of a patch of size d within a large box M and average over patches

$$P_\epsilon(i_1, ..., i_d)^q \sim \frac{1}{M} \sum_i [n_i^d(\epsilon)]^{q-1} \sim \epsilon^\phi \; e^{\tau(q-1)d\, K_q}$$

So that:

$$K_d \sim \lim_{\tau\to 0} \lim_{\epsilon\to\infty} \lim_{d\to\infty} \frac{1}{\tau(q-1)} \frac{\delta}{\delta d} \; \ln \left[\sum_i [n_i^d(\epsilon)]^{q-1} \right]$$

for K_1 we use $[\sum_i \ln[n_i^d(\epsilon)]]$

practical because we work at finite precision

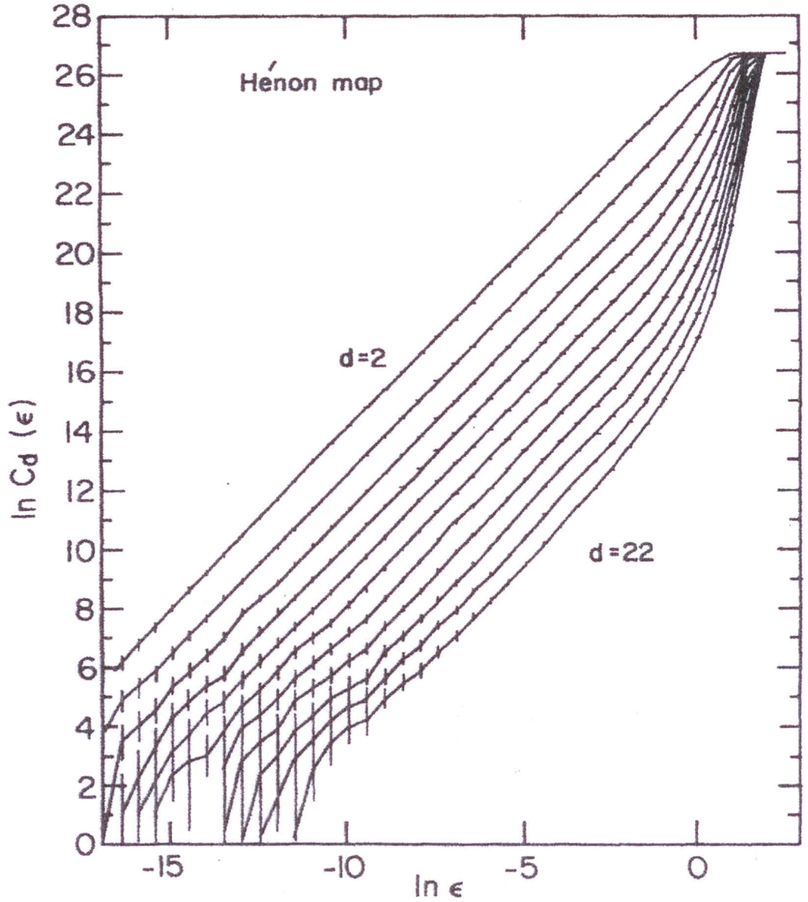


FIG. 3. Same as Fig. 1, but for the Hénon map. The values of d are $d=2$ (top curve), 4,6,8, . . . ,22 (bottom curve).

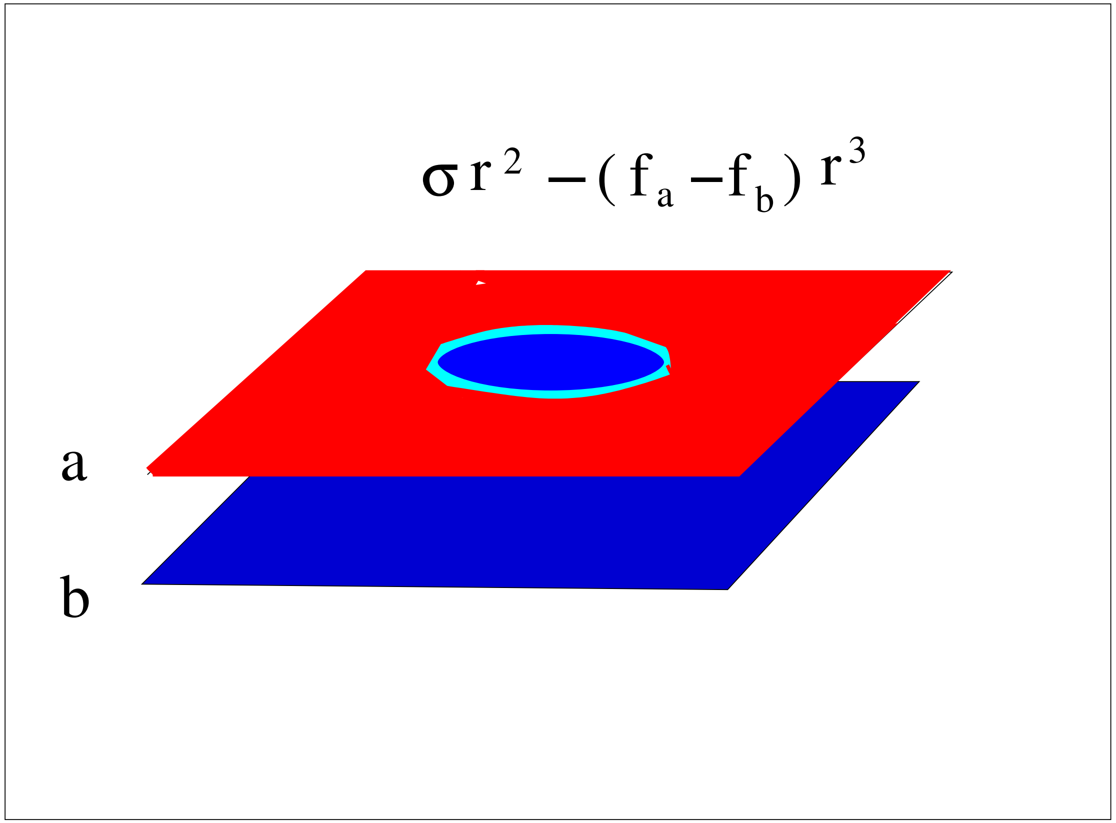
Now, let us argue that if the timescale goes to infinity

in any super-Arrhenius manner

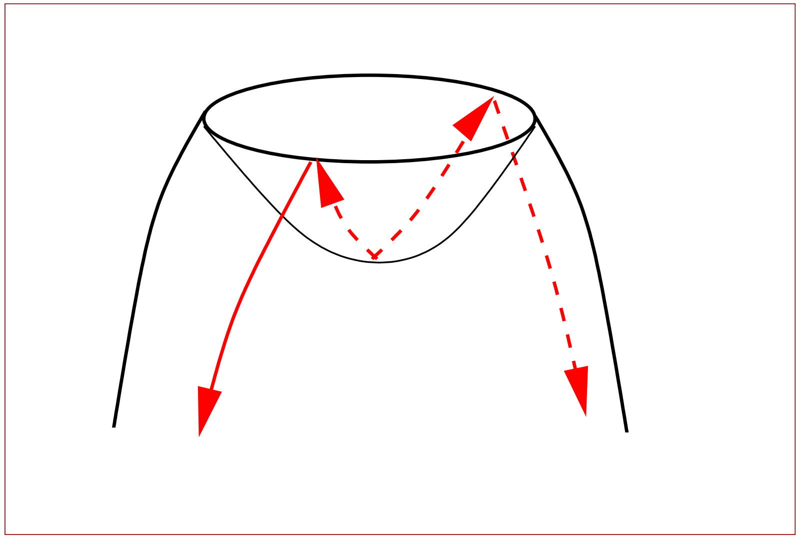
complexity is necessarily subextensive

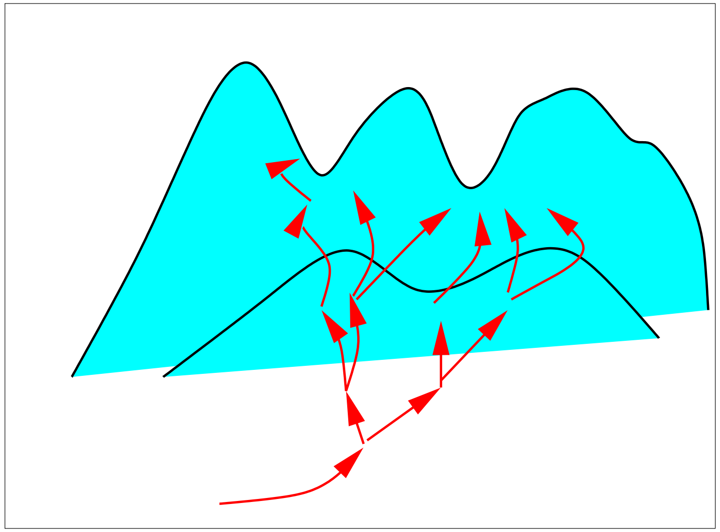
- **and lengthscale goes to infinity**

Usual nucleation argument



An entropic nucleation argument





$$V_{eff} = V(r) - (d-1)T \ \ln r = V(r) - TS(r)$$

Both for point-to-set and patch-repetition

Order appears as a *necessary* consequence

of super-Arrhenius timescales,

equilibrium and the thermodynamic limit.

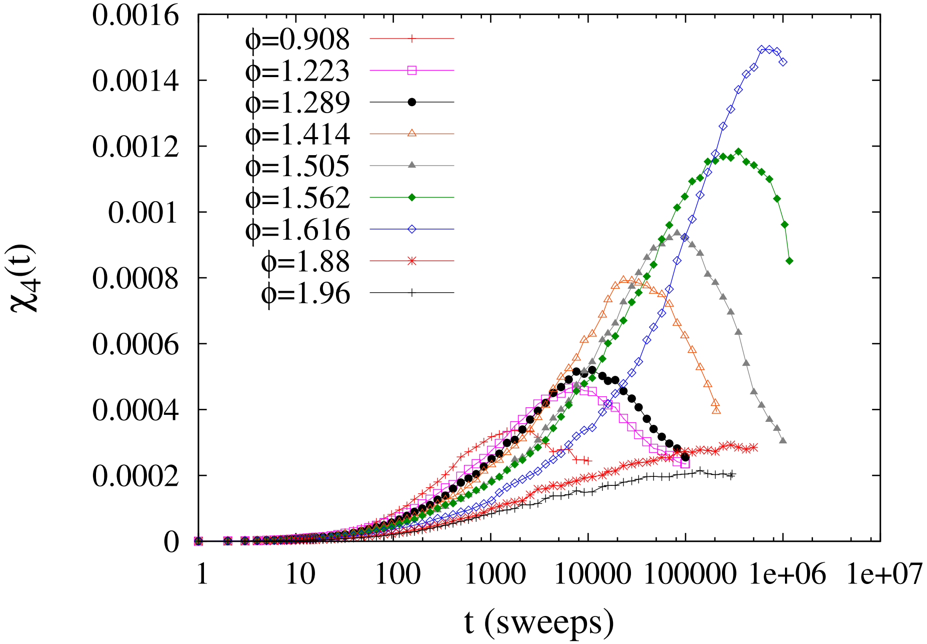
For a supercooled liquid it grows painfully slowly

Fast, non-thermodynamic lengths

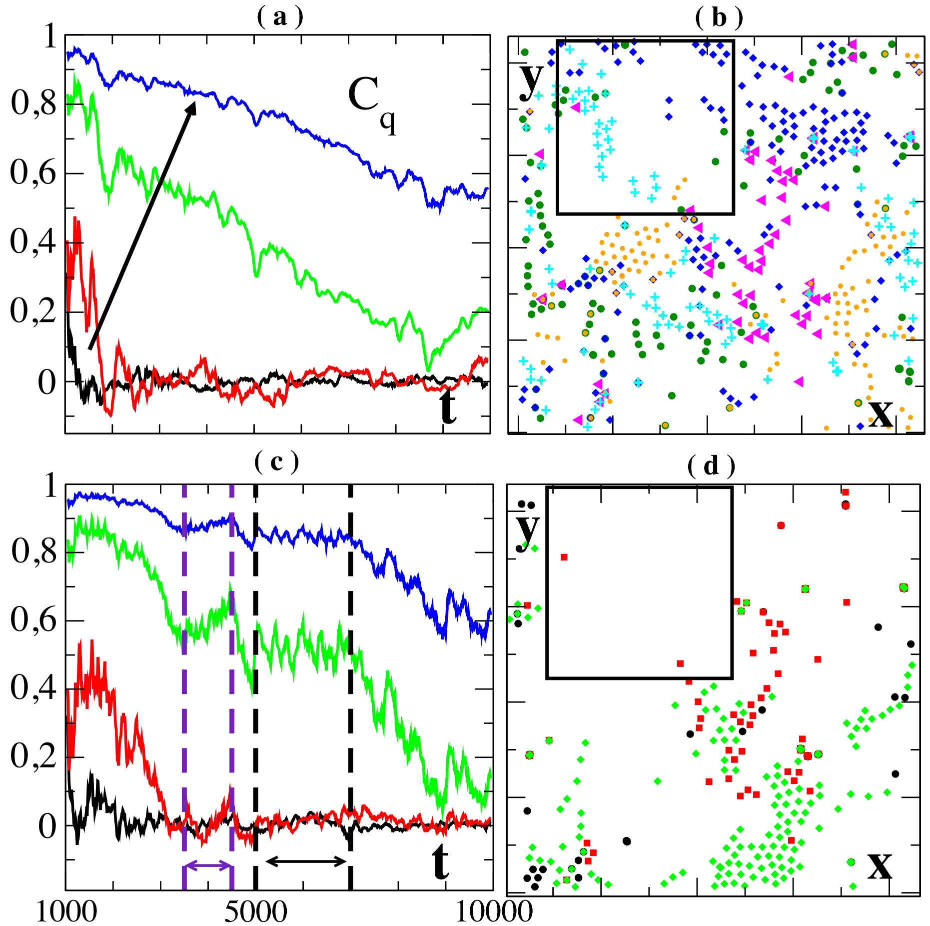
Dynamic heterogeneities

$$C(q,t) = \frac{1}{3N}\sum_{i=1}^{N}\sum_{\alpha=1}^{d}\cos\left(q(\mathbf{x}_i^\alpha(0) - \mathbf{x}_i^\alpha(t))\right)$$

$$\chi_4(t) = N(< C(q,t)^2 > - < C(q,t) >^2) \propto \sum \langle cos(\Delta x_i(t)) \cos(\Delta x_j(t)) \rangle_c$$



$$\chi_4(t) = N(< C(q,t)^2 > - < C(q,t) >^2)$$



how can they grow so fast?

and also note that :

χ_4 **diverges also for acoustic correlations!**

J-Point

Procedure: increase the radius of the spheres gradually, infinitesimal overlaps are removed through repulsion. Continue until the pressure is infinite: this is the J-Point.

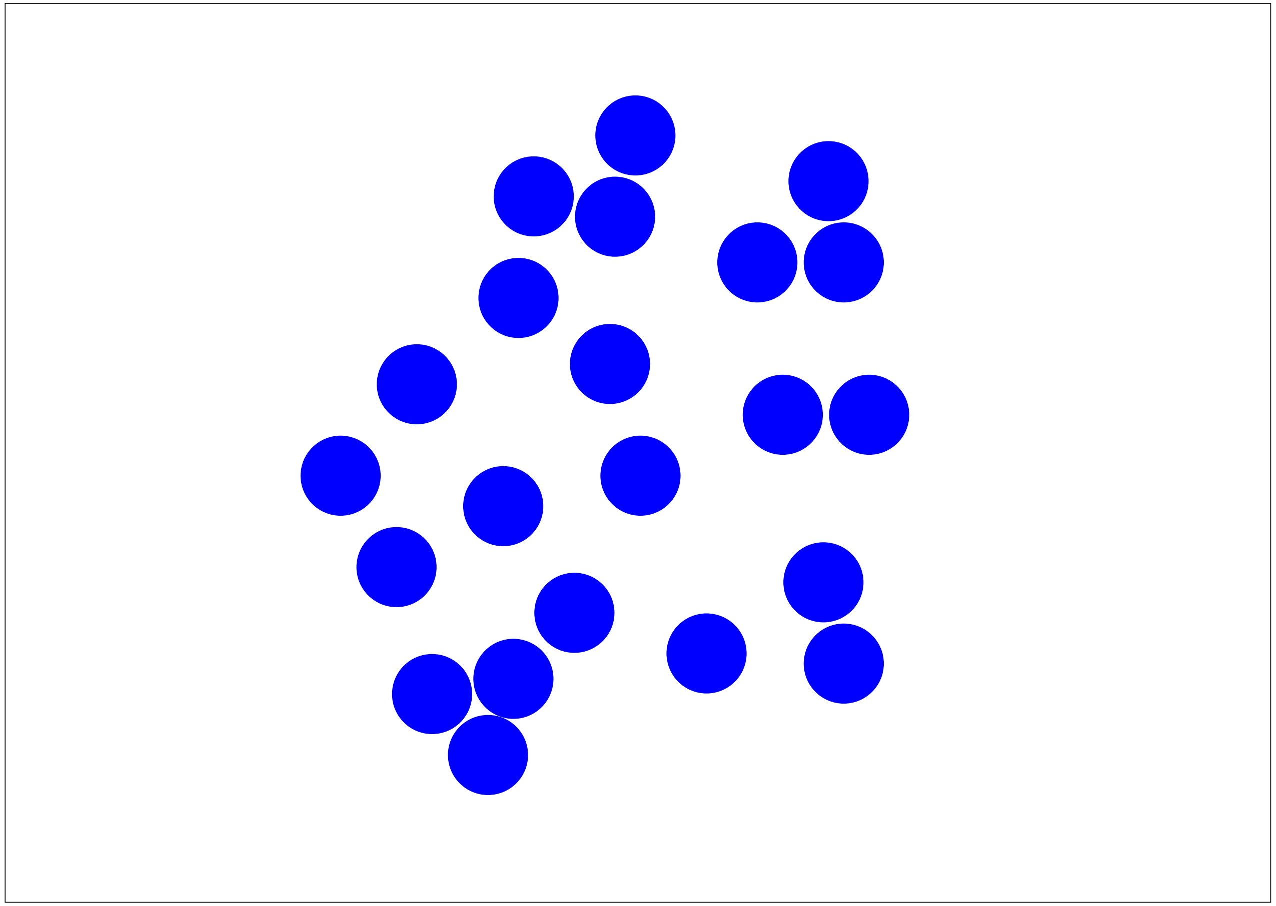
(O'Hern et al., Lubachevsky-Stillinger,...)

The actual volume fraction reached is very close to the one quoted as Random Close Packing

The J-Point so defined has criticality properties (soft modes, diverging lengths and susceptibilities, isostaticity).

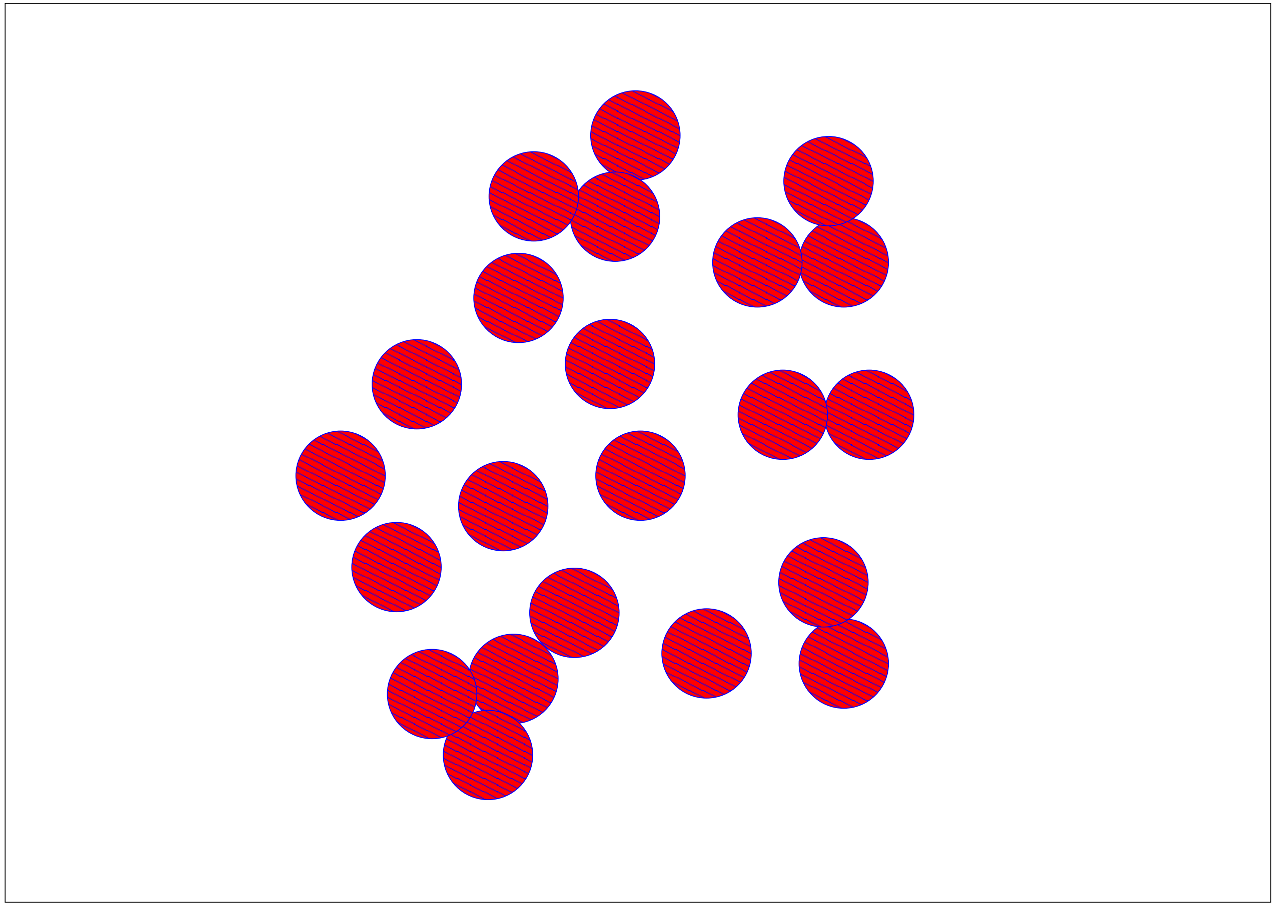
PACKING

A given configuration

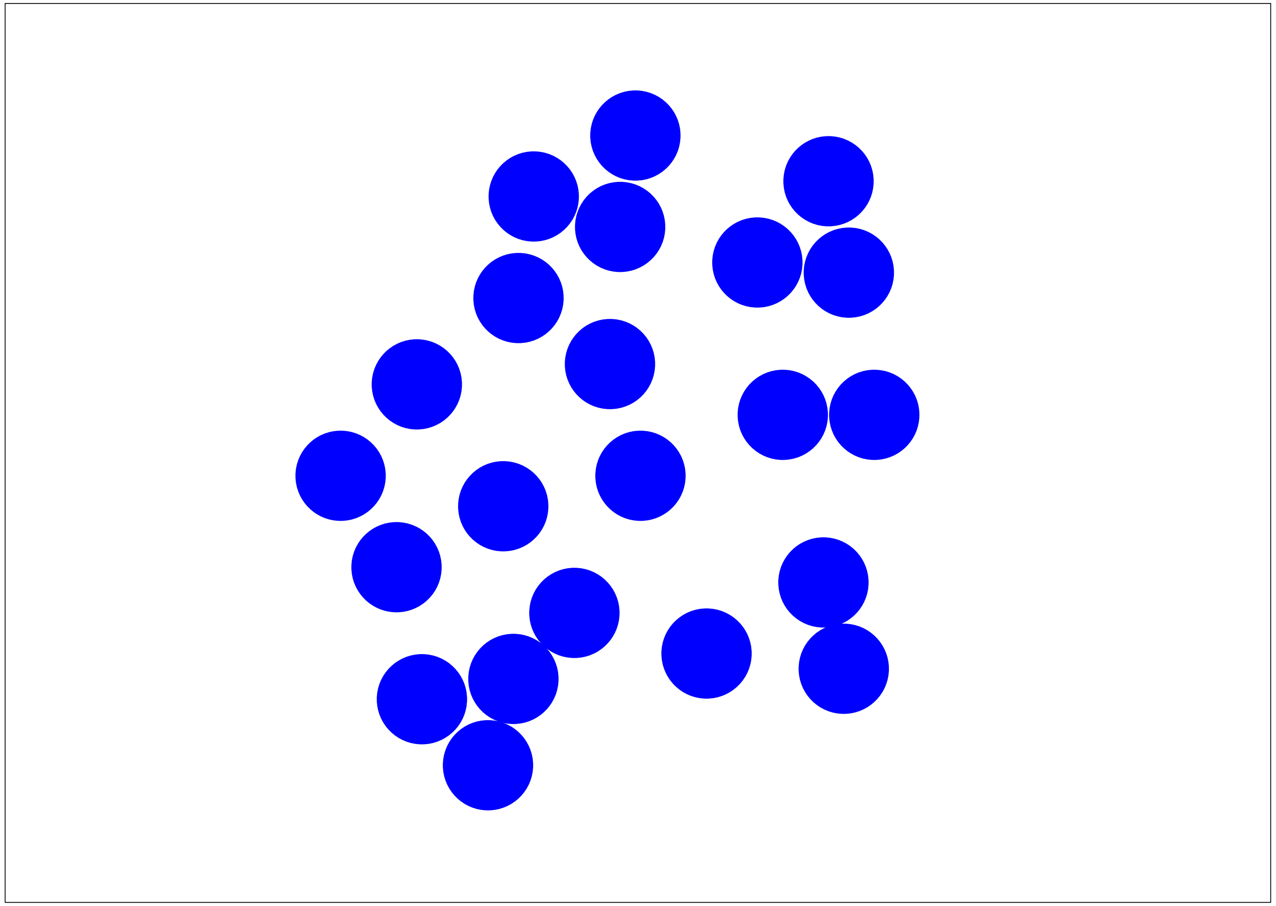


PACKING

Inflate slightly



Displace particles to resatisfy



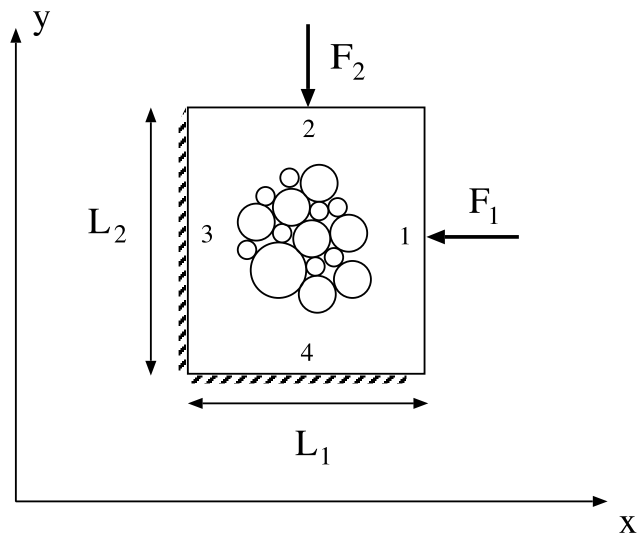
Unless something is finely tuned, a system of spherical particles evolves until reaching an isostatic situation...

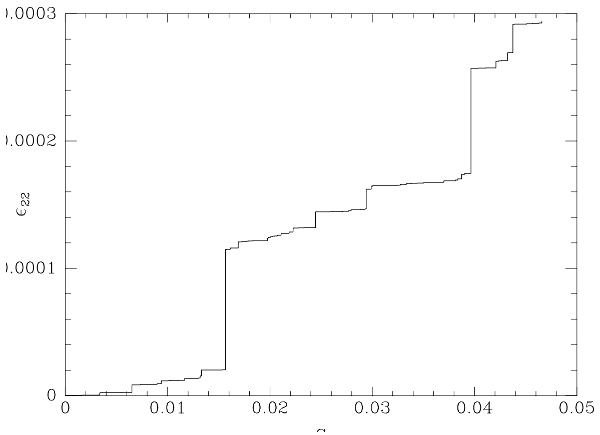
...just as a table on a rough floor generically has three legs touching.

But a globally isostatic system is very sensitive to perturbations.
Breaking one contact already makes it unstable.

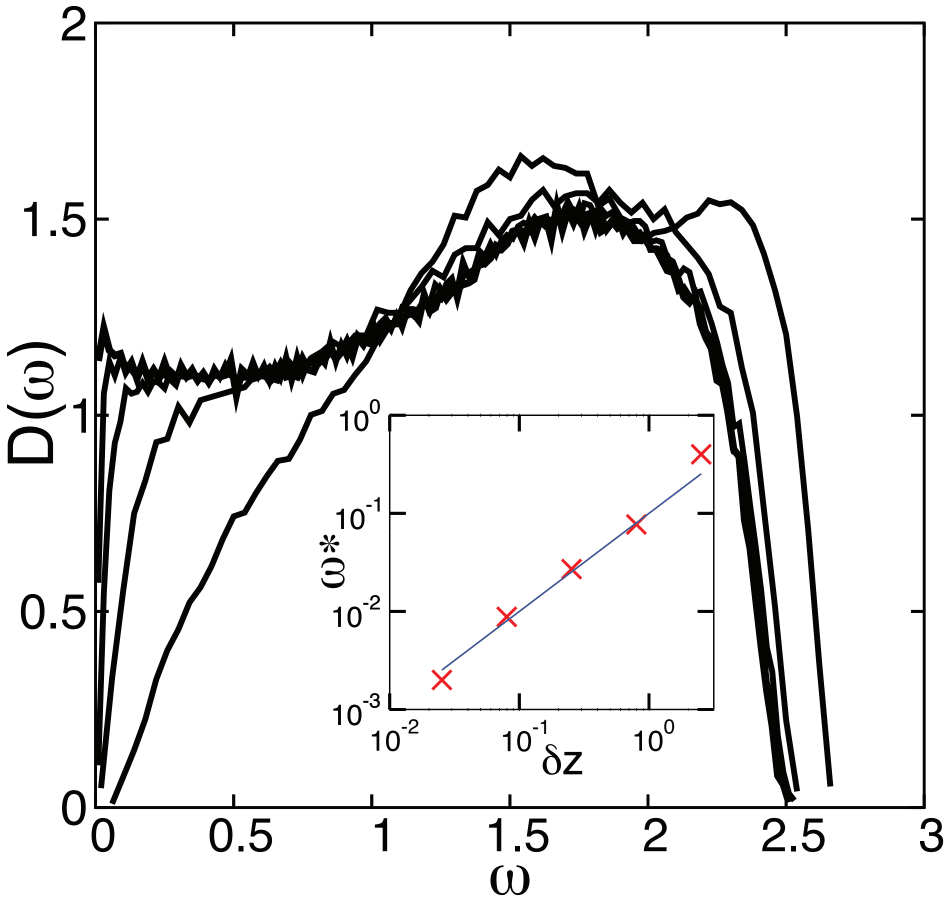
Has zero modes, diverging susceptibilities and lengths.

Fragility of an isostatic system (Combe and Roux)



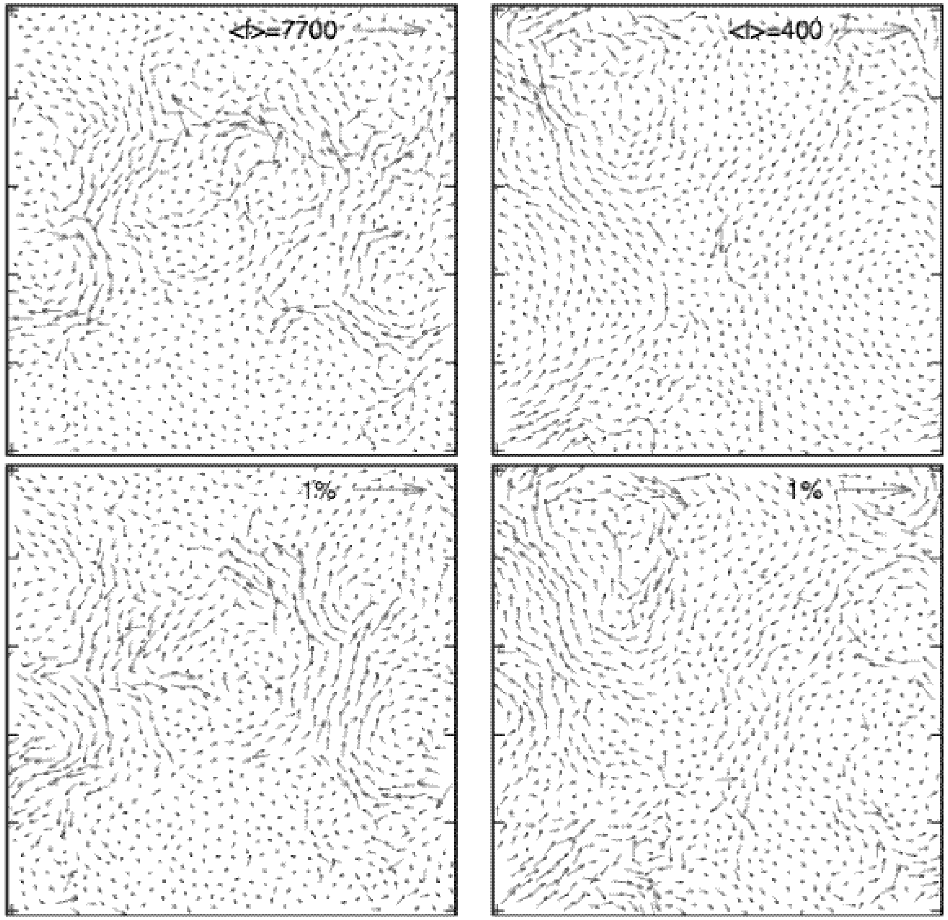


response is non-smooth and nonlinear


2
1.5
1
0.5
0
D(ω)
0
0.5
1
1.5
2
2.5
3
ω
10^0
10^{-1}
10^{-2}
10^{-3}
ω*
10^{-2}
10^{-1}
10^0
δz

evolution happens through instantaneous flat modes

which implies a diverging length at ϕ_c

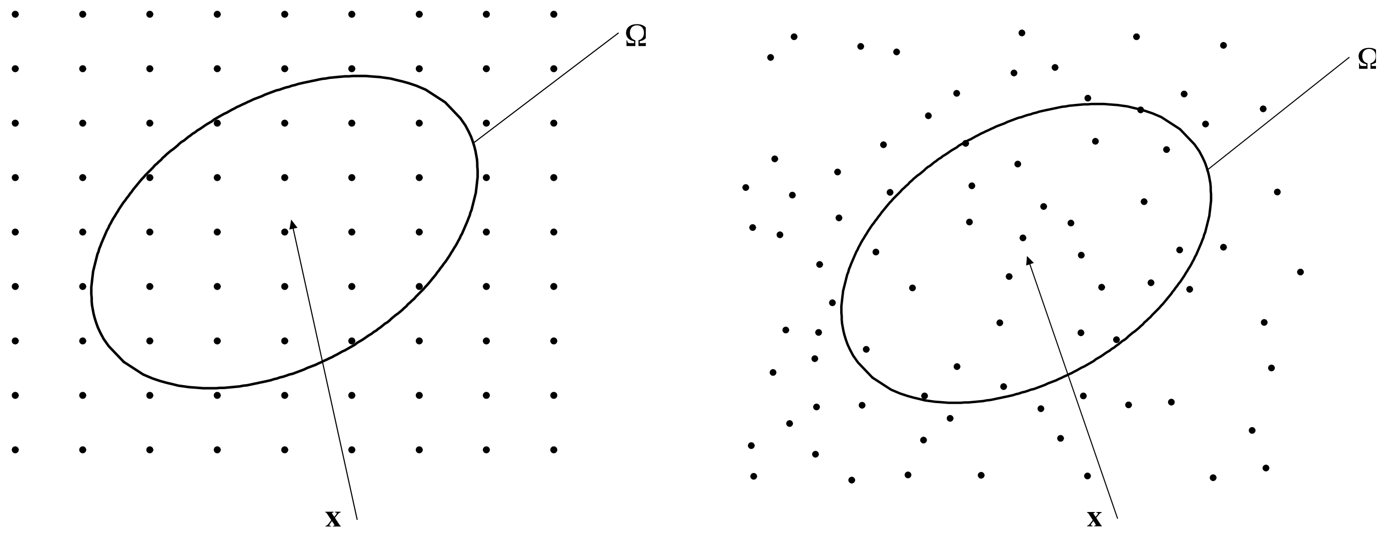


Brito and Wyart

shear and compression moduli go to zero at ϕ_c with different exponents

close to ϕ_c the system becomes like an incompressible fluid

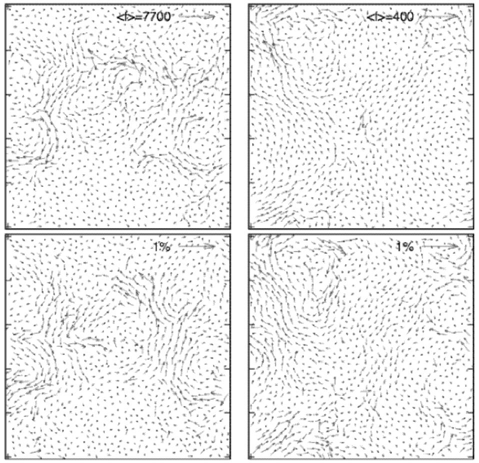
Volume occupation statistics Torquato, Stillinger,...



$\Delta[V\phi] \ll O(V^{1/2})$ $\rightarrow$ **quasi-long range correlations**

Very Tentative: mixing an incompressible fluid

push a particle and generate rearrangements via soft modes



the mass displacement field is orthogonal to the compressions

hence the procedure "mixes" in a mass-conserving manner
and an argument as in ergodic theory says that uniform distribution is the attractor!

Redefining the J-point as the one obtained by randomly ’mixing’ particles :

using vanishingly (as $\phi \to \phi_c$) small force

perhaps one would prove that hyperuniformity is a consequence of isostaticity + mixing

Hyperuniformity and fragility require *global* mixing and global isostaticity

these are (relatively) rapid processes.