



**The Abdus Salam  
International Centre for Theoretical Physics**



**2254-12**

## **Workshop on Sphere Packing and Amorphous Materials**

*25 - 29 July 2011*

### **Microstructure and Stability of Granular Piles**

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# Microstructure and Stability of Granular Piles

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## What sandpiles look like



*Great Sand Dunes National Park, Colorado, USA*

A jammed state

Note similar angles of piles

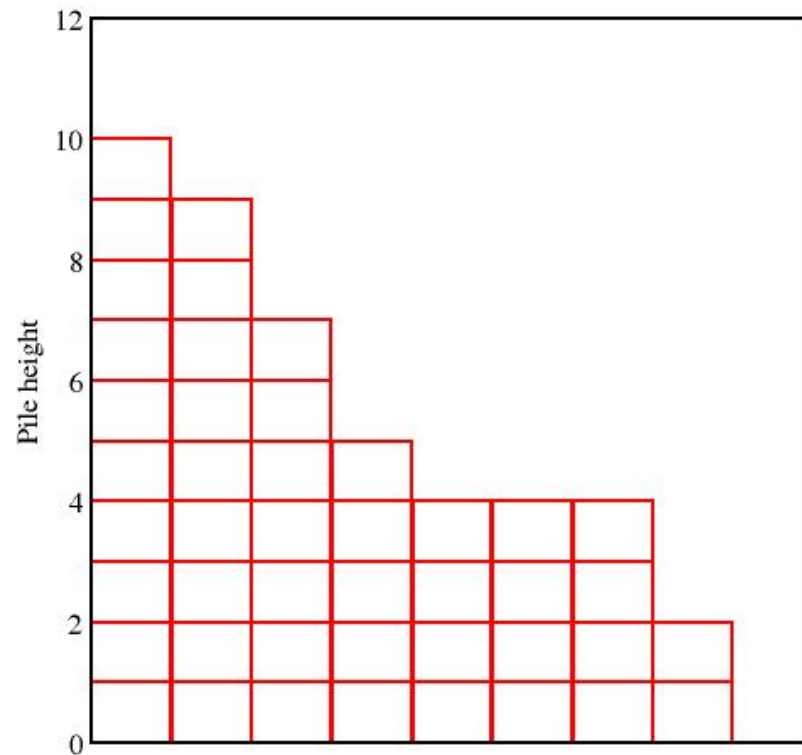
## Self-organized criticality

Feedback keeps system at a critical angle

Characteristic power laws (e.g., in avalanche size)

Length scales extend to cover entire pile in critical state

### A simple “sandpile” model



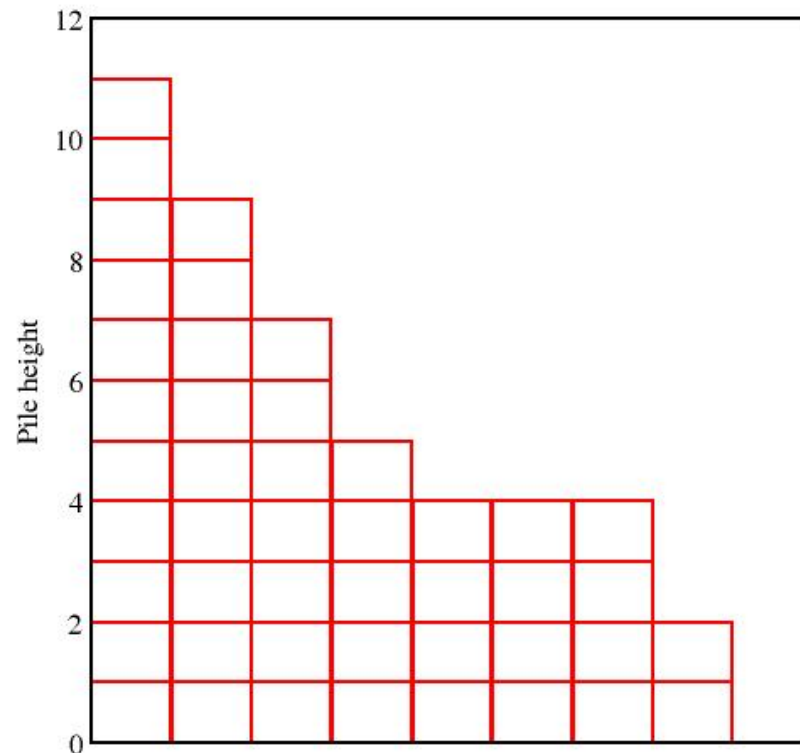
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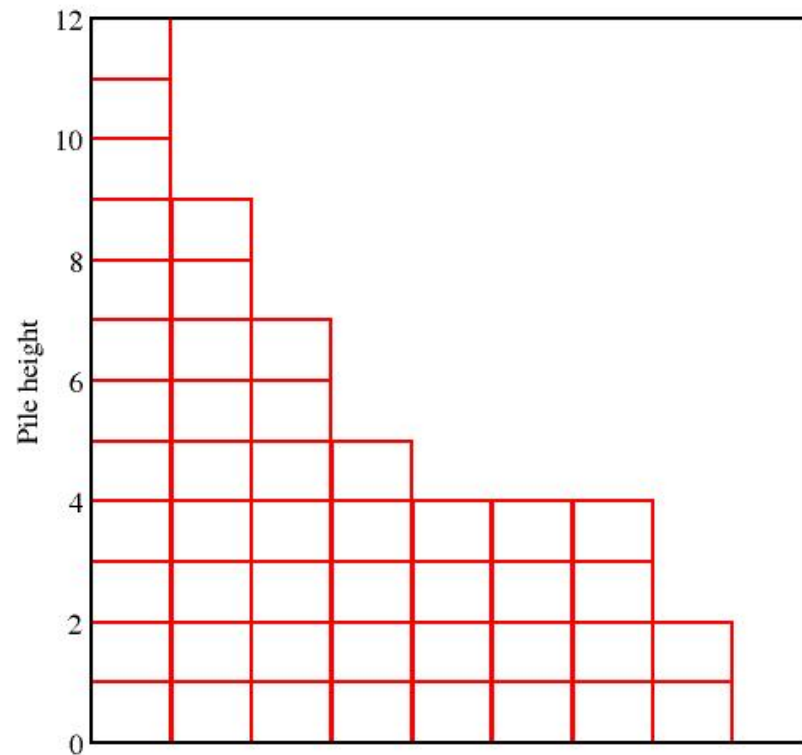
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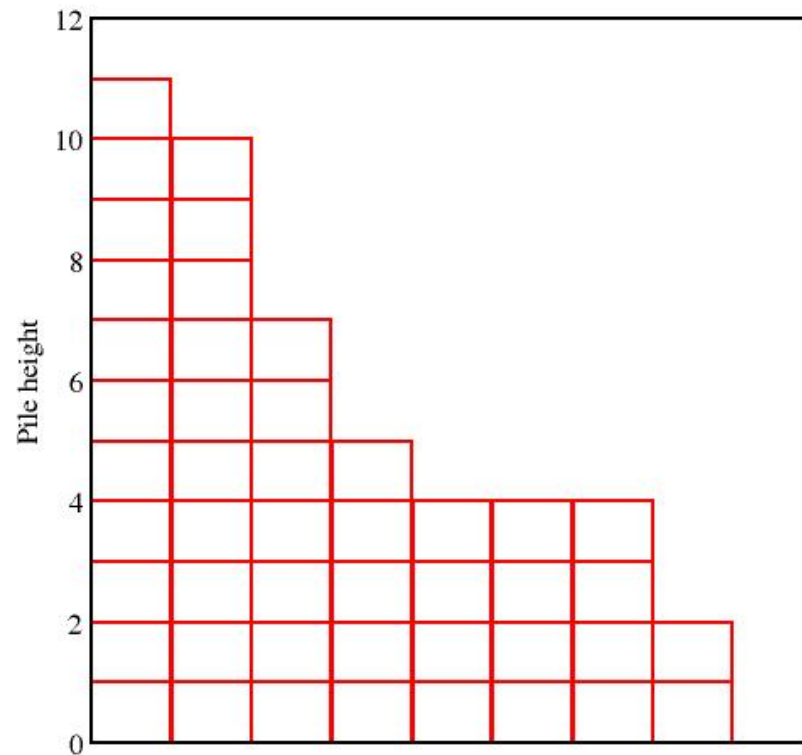
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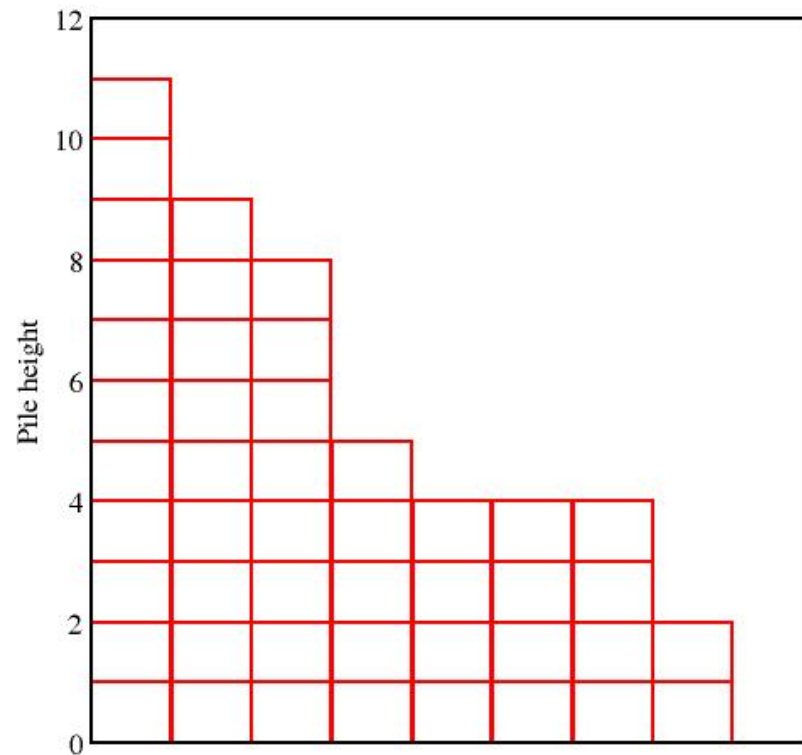
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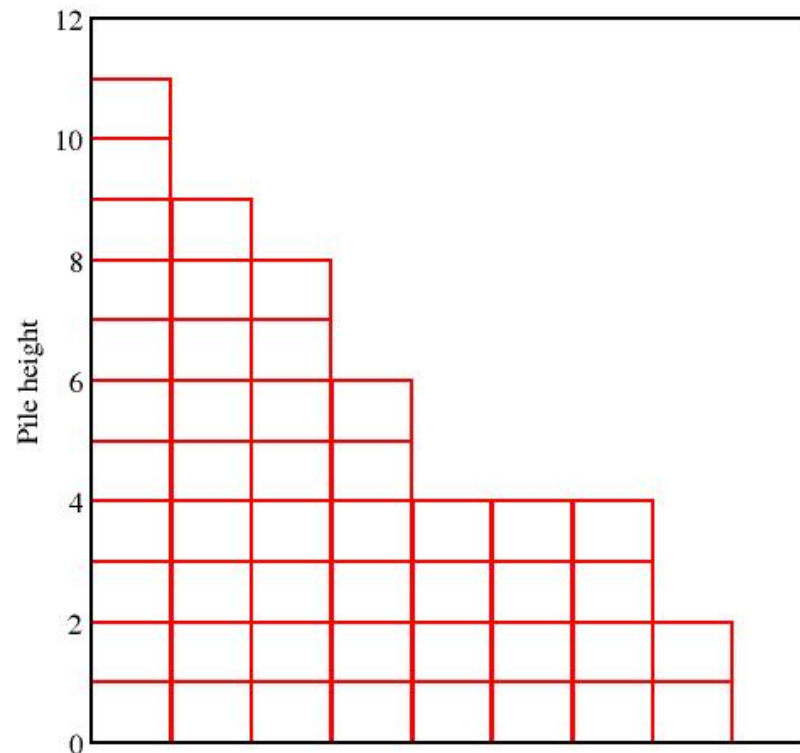
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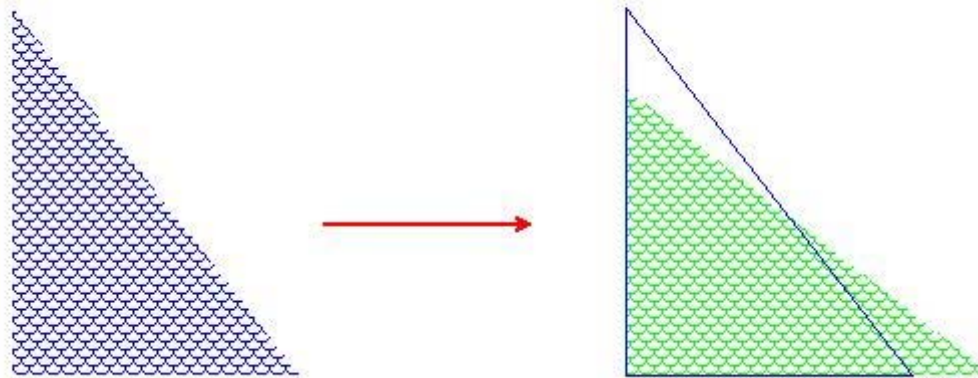


...but real sandpiles are more complicated

TWO angles:

Angle of stability – max angle before avalanche

Angle of repose – angle after avalanche



## A jamming transition

Fixed vs. moving grains

SOC model: second-order transition (critical behavior)

Real sandpiles: first-order transition (with hysteresis)

Dilatation – pile expands so grains can move

Inertia – avalanches can't stop cold

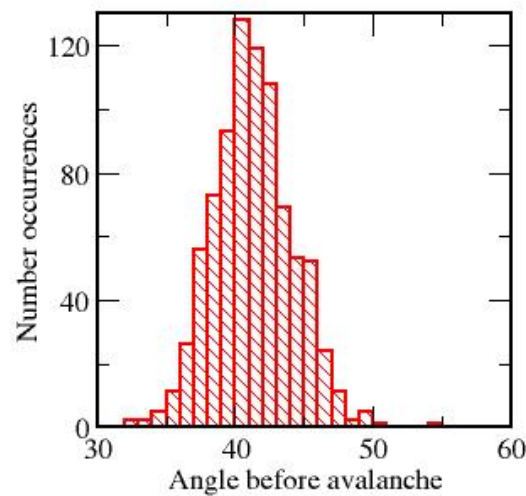
Different grains move qualitatively differently: tumbling, sliding, moving in clumps

## Another complication

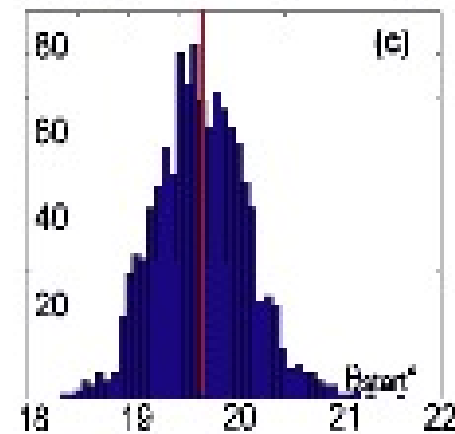
There is no single stability angle; angles just before avalanches have Gaussian distribution of angles.

Spread in angle can be sizable.

Deboeuf et al.,  
EPJB 36 (2003)



2D

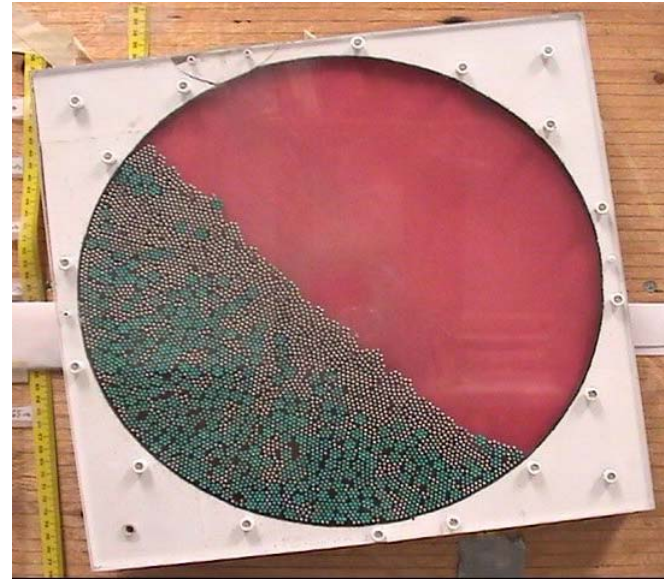


3D

# The tumbler

Drum diameter 14"  
(112 times ball diameter)

Balls confined to single layer  
by Plexiglas sheets



Slow rotation, about 1 revolution in 30 minutes (about 1 avalanche per minute)

Record with videocamera

Capture frames immediately before and after each avalanche

Our goal: understand how variation in avalanche angles comes about from the precise arrangement of grains

# The grains



single



dimer (double)



hexagon

Ball bearings, 1/8" diameter

Make non-spherical shapes by welding together steel balls

Here: mainly hexagons, dimers, singles

*(Have also used trimers, triangles, diamonds, trapezoids)*

# Advantages

All shapes based on spheres:

- same maximum packing fraction
- only point contacts between shapes
- can keep track of overlaps in simulations

Two dimensions: can visualize entire arrangement

Metal: avoids charging problems, humidity control

Large, heavy grains: gravity dominates

## Disadvantages

Small aspect ratio between container and grains

- comparable to that in many simulations
- about 1000 grains here

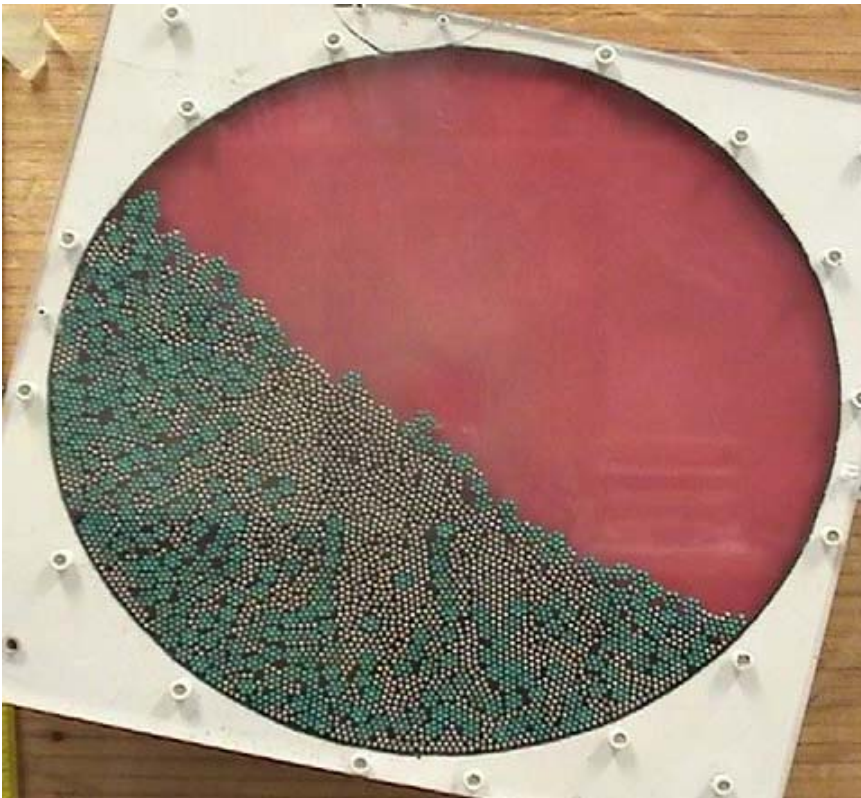
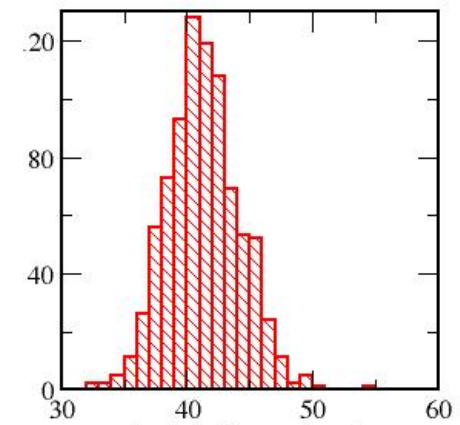
Grains can become magnetized

- check for magnetization
- demagnetize grains regularly

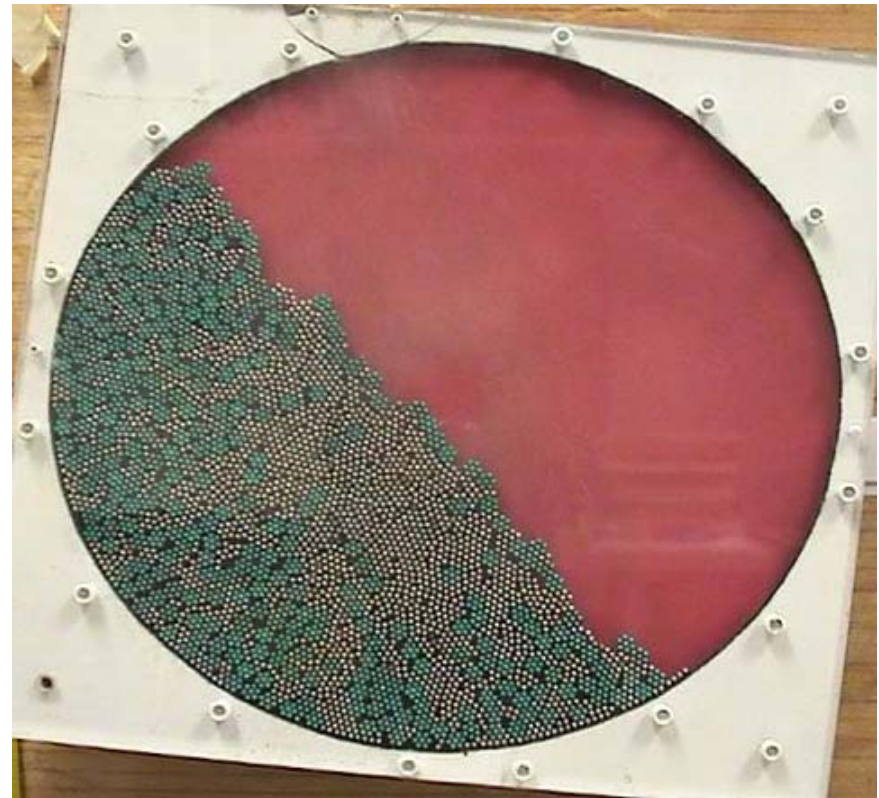
Two dimensions: our world is three-dimensional



Difference in angles easily visible!

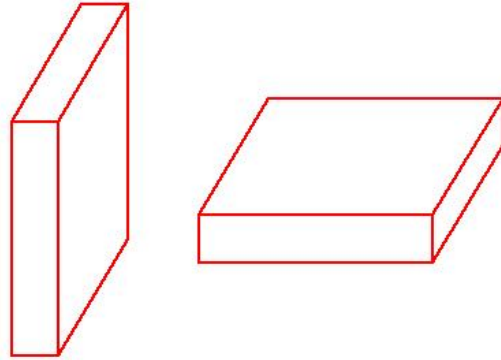


**37.7°**



**49.6°**

## Simulations with configuration effects



[Mehta & Barker, EPL **56** (2001) ]

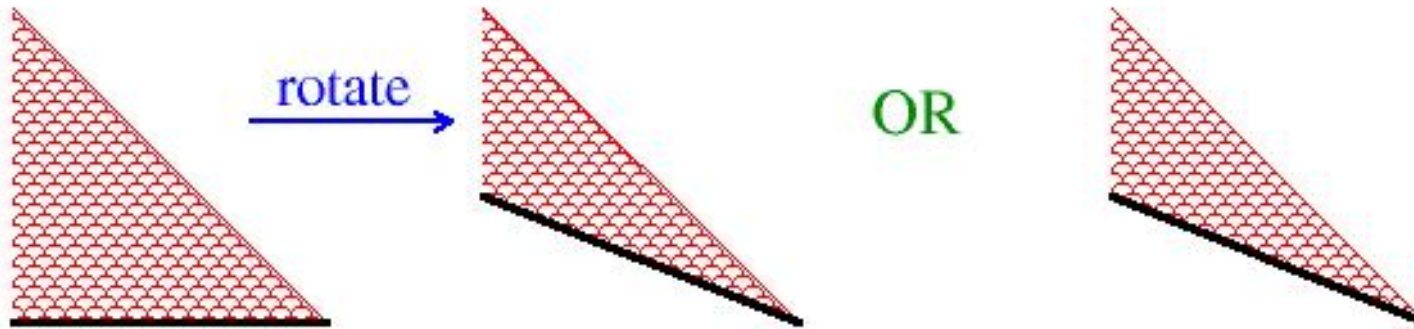
Can model density changes with non-spherical grains

Designed so density increases with depth (agrees with experiment)

Control slope of free surface through condition on when grains move

## Two ways to construct pile

Construct pile directly (right) or construct larger pile (left) and then reduce the angle criterion to cause an abrupt avalanche.



Result: sudden avalanches improve stability over gradual pile formation.

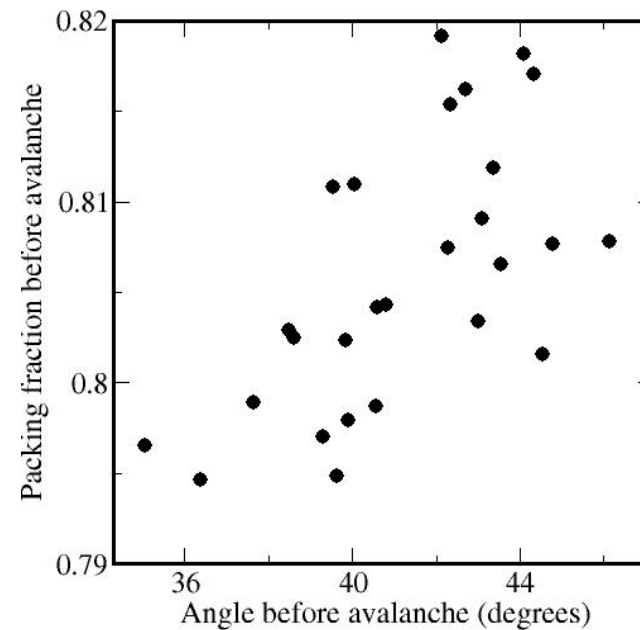
Why? An avalanche brings deeper, better-packed grains to the surface. (Consistent with our direct observations.)

# Packing fraction (filling factor, density)

Not truly microstructure

Positive correlation between packing fraction and stability

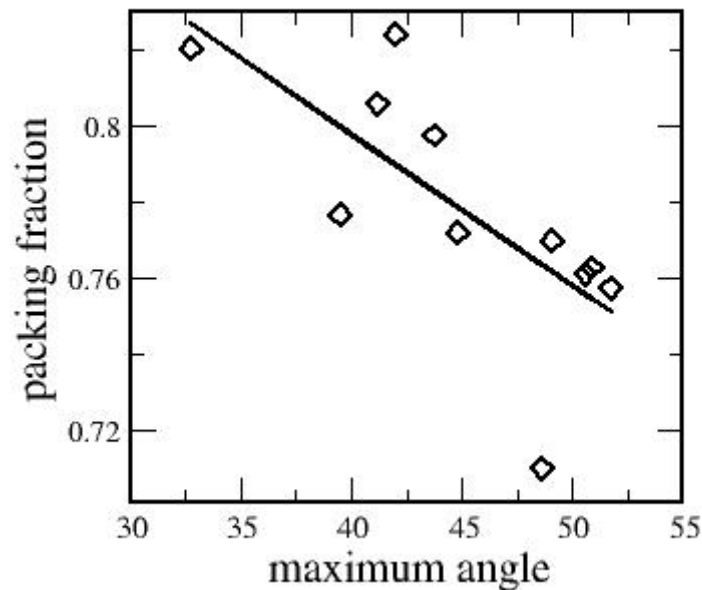
3-ball triangles  
(other shapes similar)



[an aside]

## Opposite result across shapes

*Negative* correlation between density and stability!



each point represents  
a different grain shape

Our piles are only those resulting from prior avalanches

Emphasizes the importance of the formation mechanism

## Statistical information

Positive correlation (0.47) between packing fraction and avalanche angle

Positive correlation (0.6) between packing fraction for successive avalanches

- hardly surprising; much of the packing remains unchanged

No correlation between angles of successive avalanches

Implication: the top of pile (where packing fraction changes) is responsible for triggering the avalanches

*What else can we see about the start of the avalanches?*

# Binary mixtures

Larger spread of angles

Wider range of configurations

- segregation patterns
- loading grains far from steady-state patterns

Brass singles/steel hexes

OR

Steel dimers/steel hexes

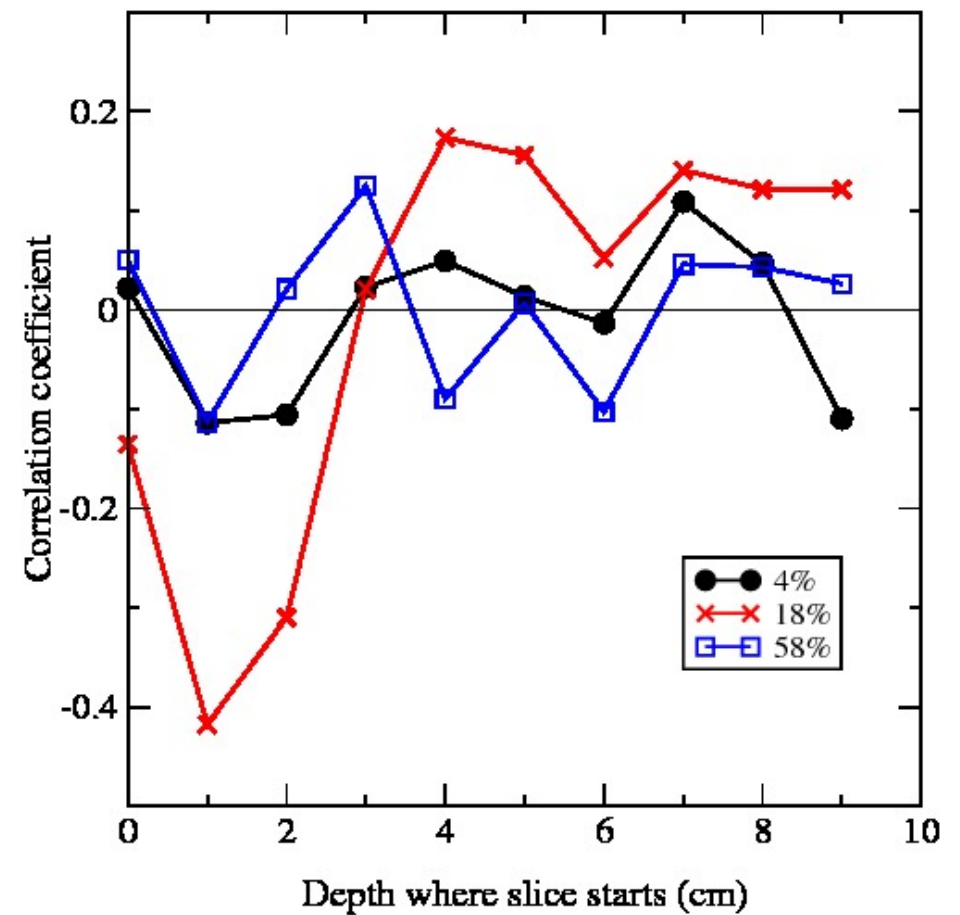
singles/hexes  
(artificial green)



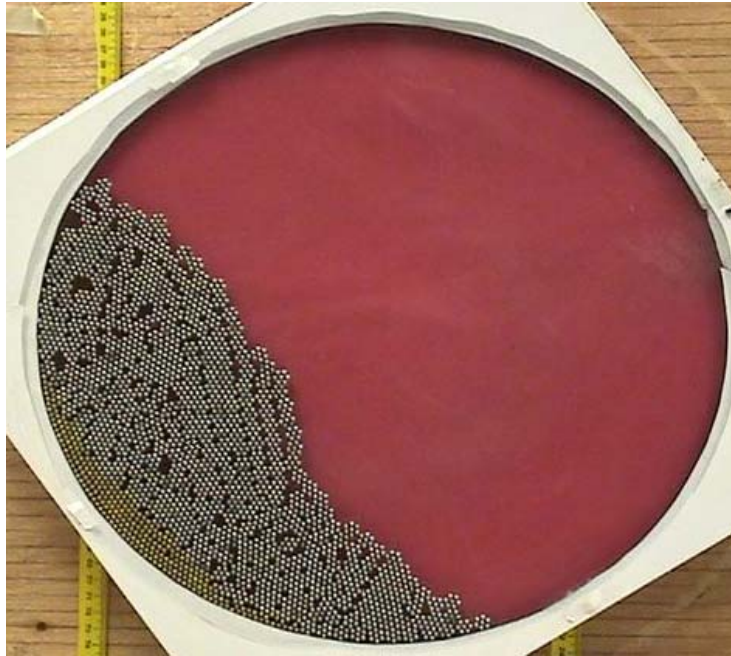


Divide pile into strips

Correlate number singles in each strip to avalanche angle



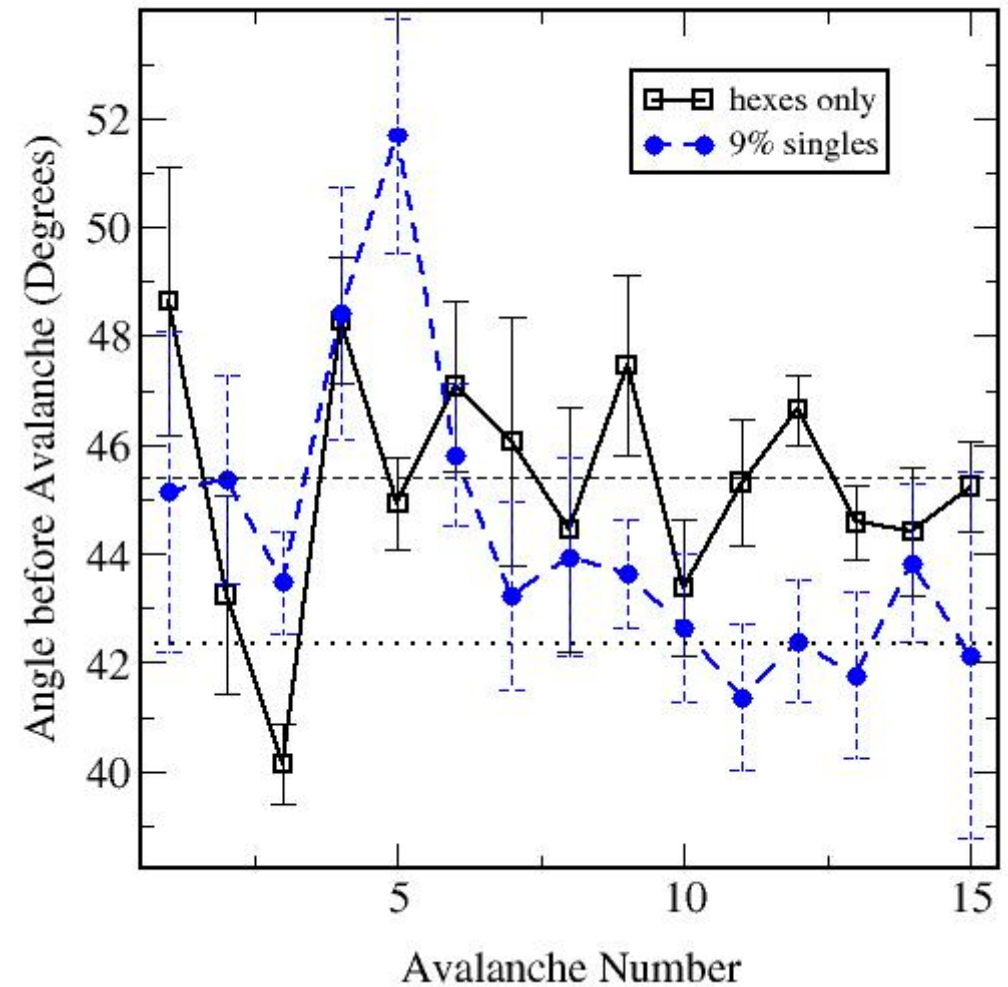




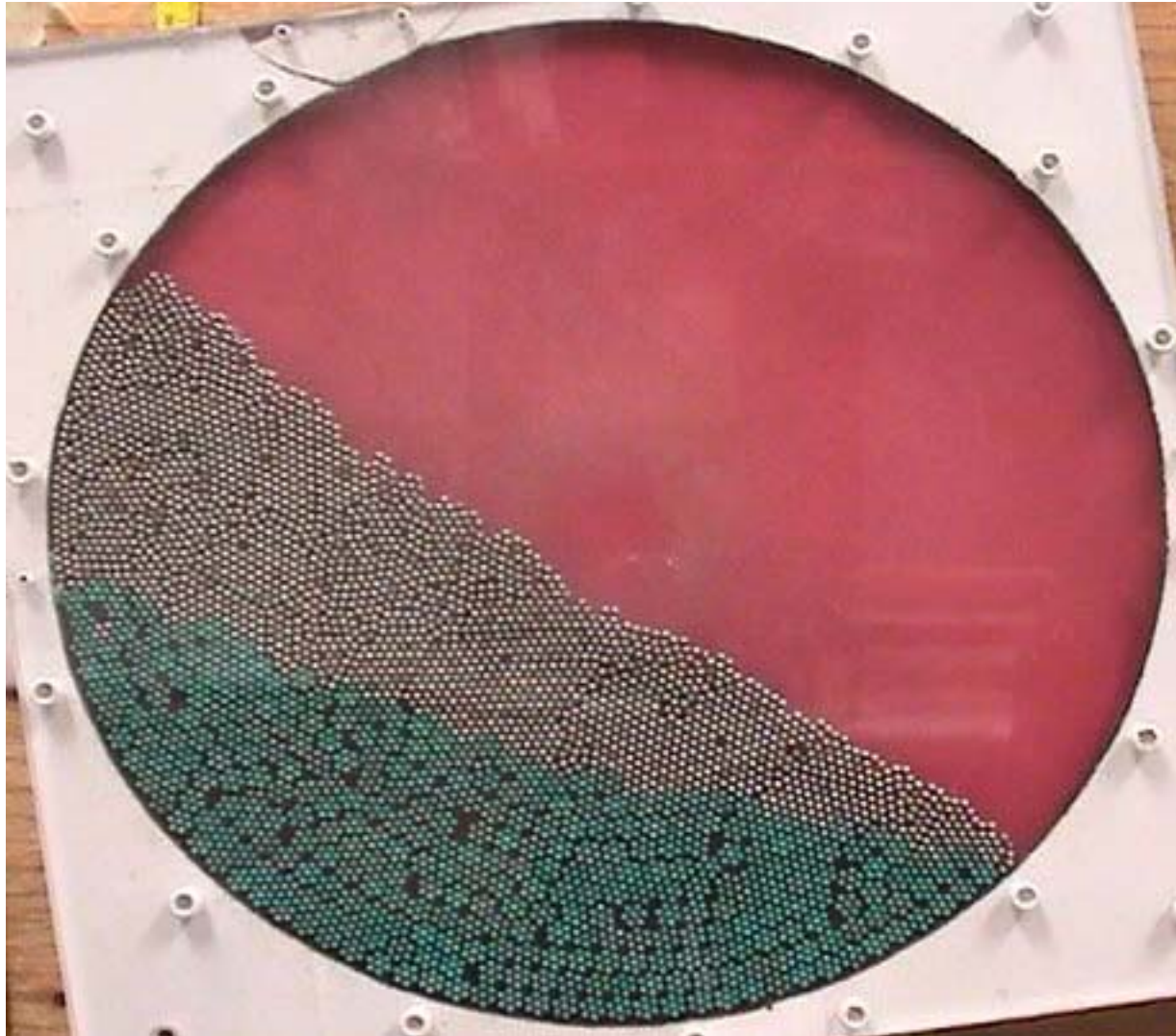
singles (yellow) along bottom

First few avalanches at higher angles

Segregation develops within 10 avalanches

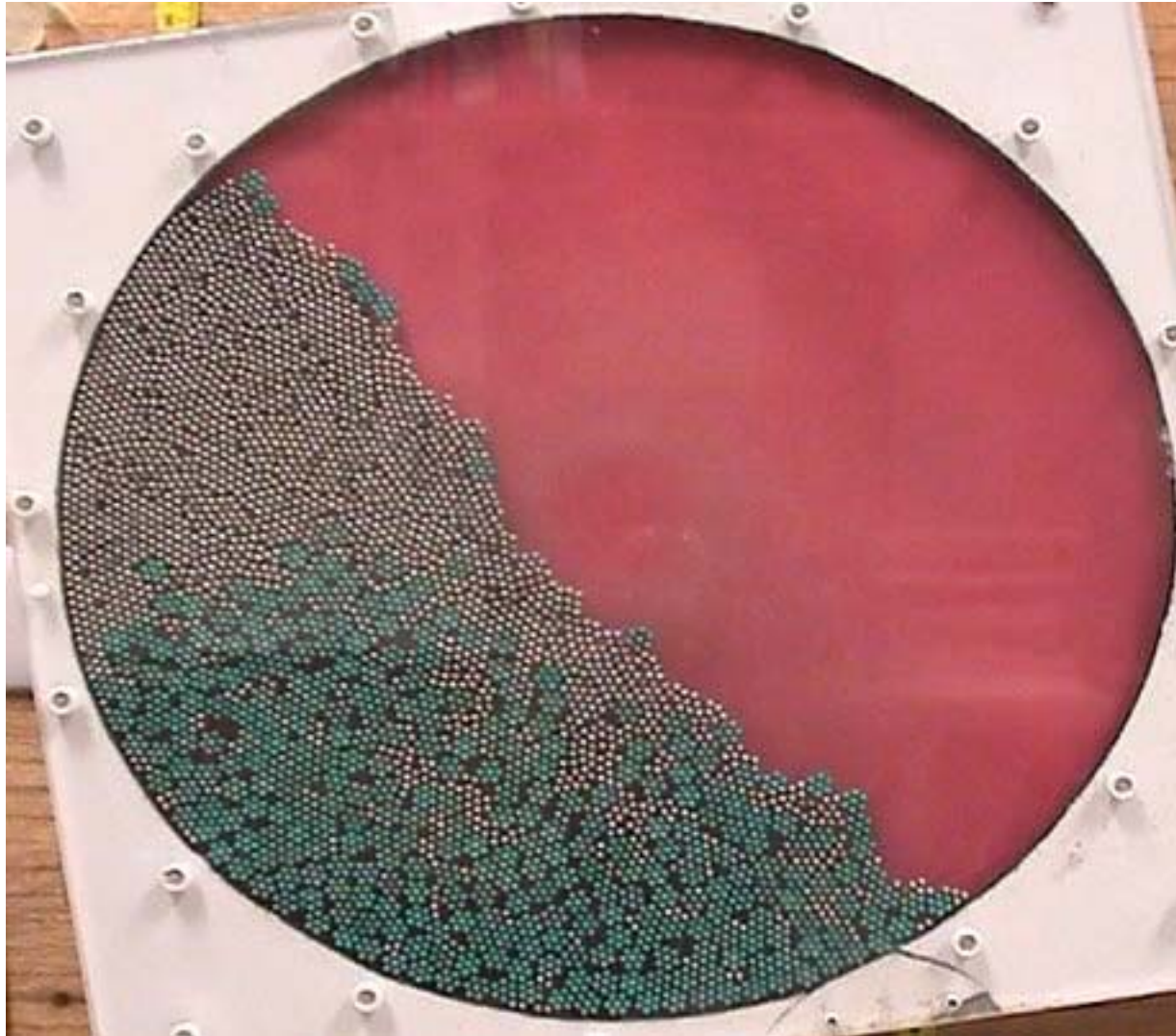


Dimers-hexes: more than 100 avalanches before reaching steady-state patterns



avalanche 1

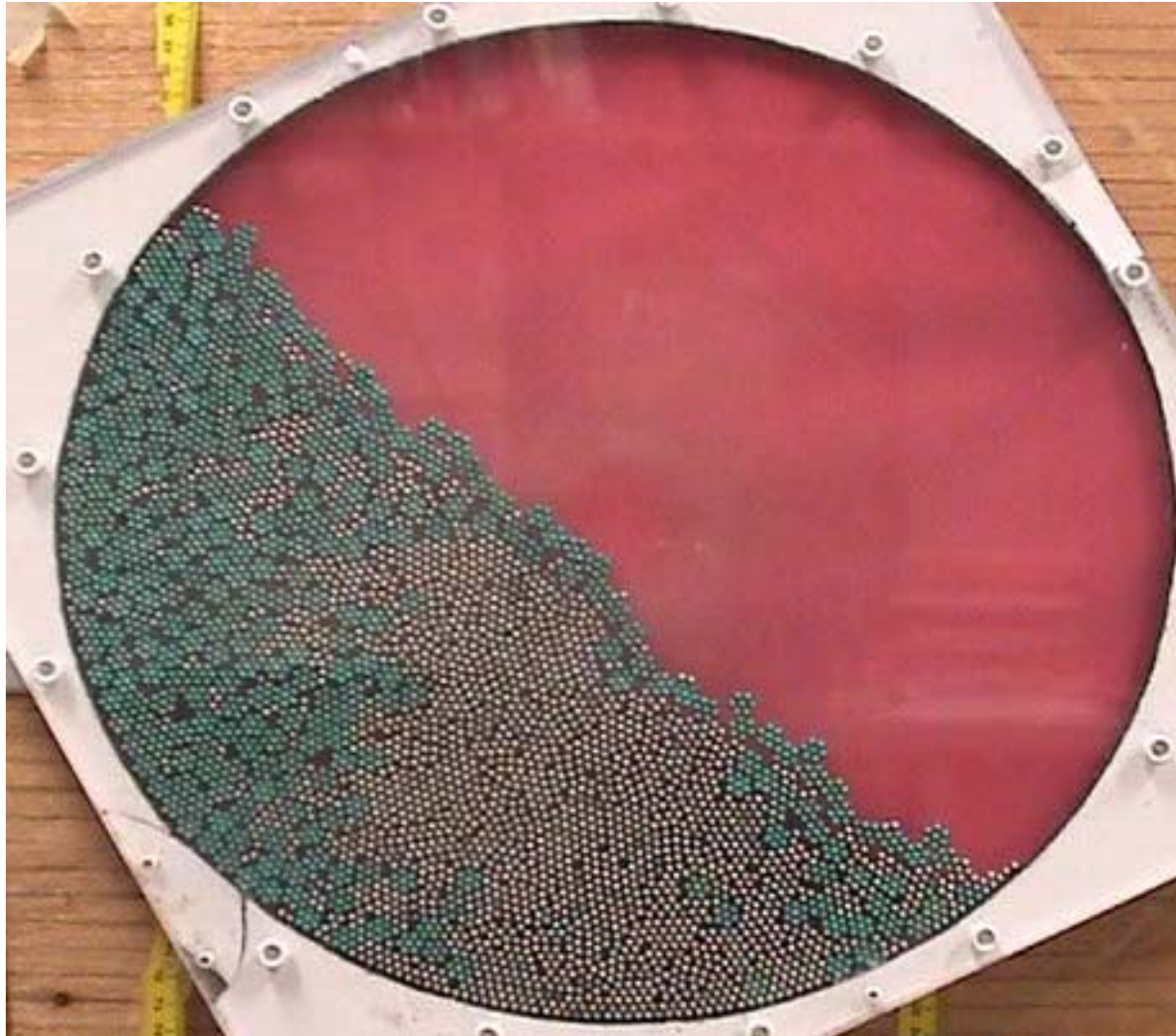
Dimers-hexes: more than 100 avalanches before reaching steady-state patterns



avalanche 18

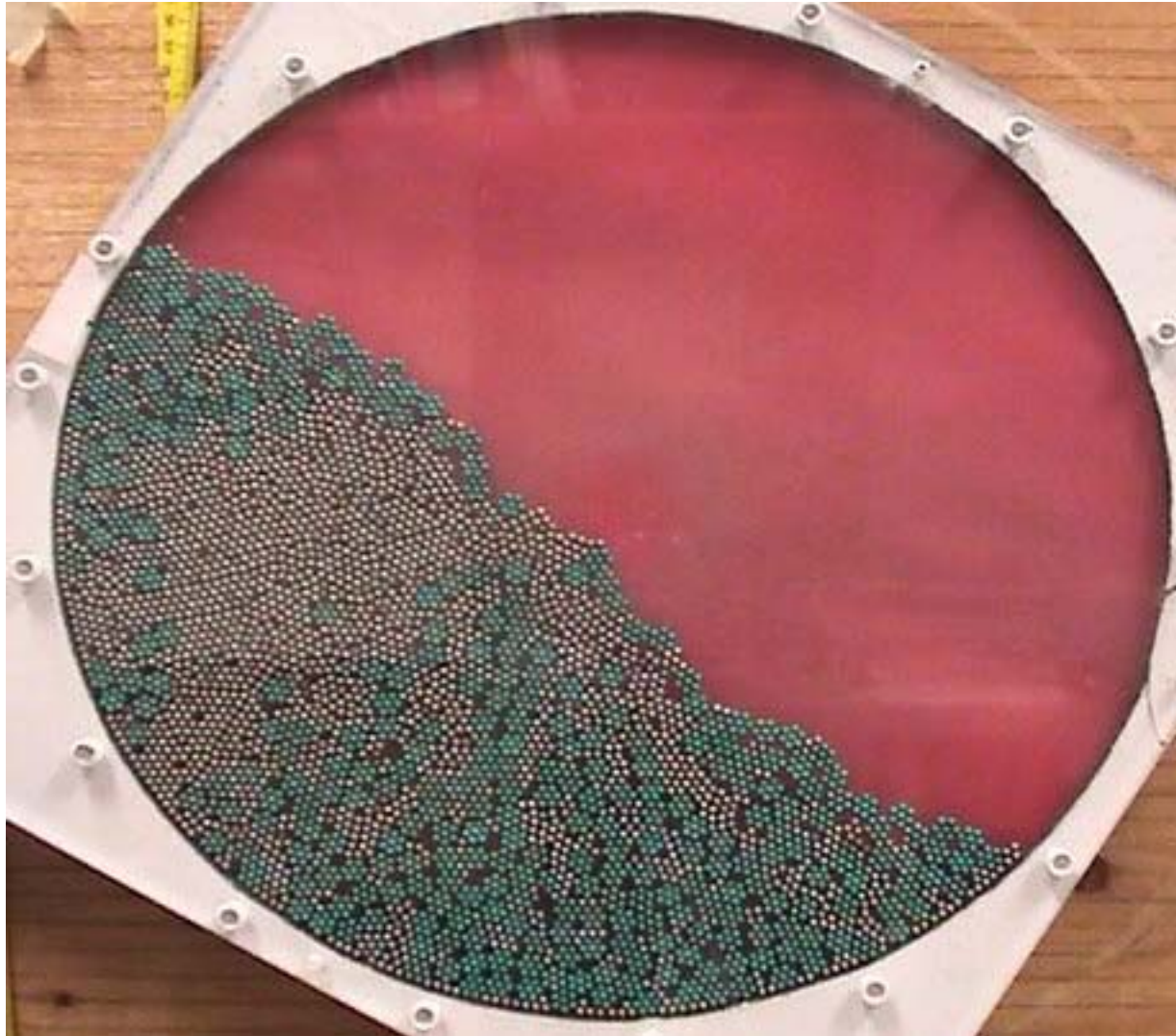


Dimers-hexes: more than 100 avalanches before reaching steady-state patterns



avalanche 25

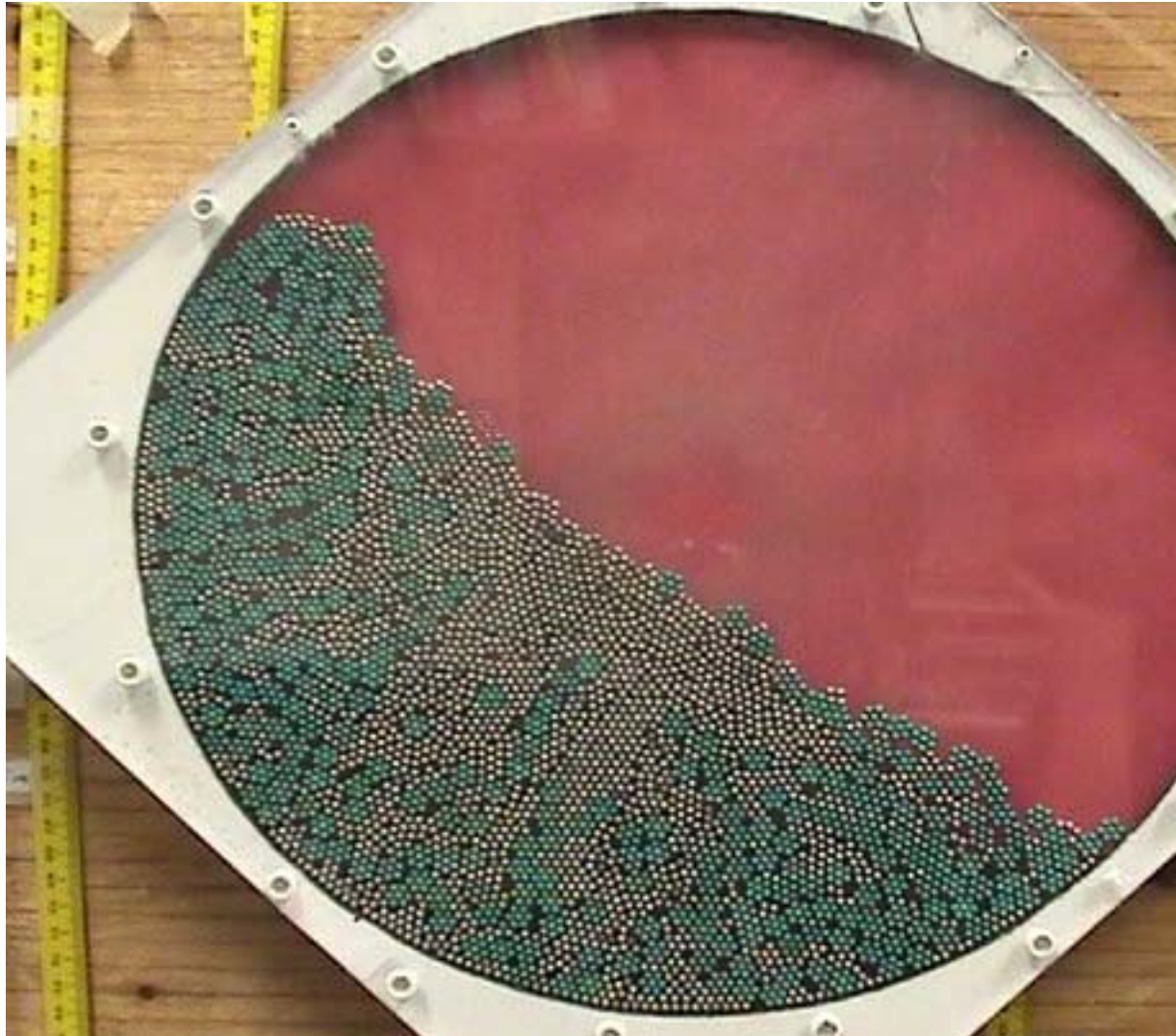
Dimers-hexes: more than 100 avalanches before reaching steady-state patterns



avalanche 71



Dimers-hexes: more than 100 avalanches before reaching steady-state patterns

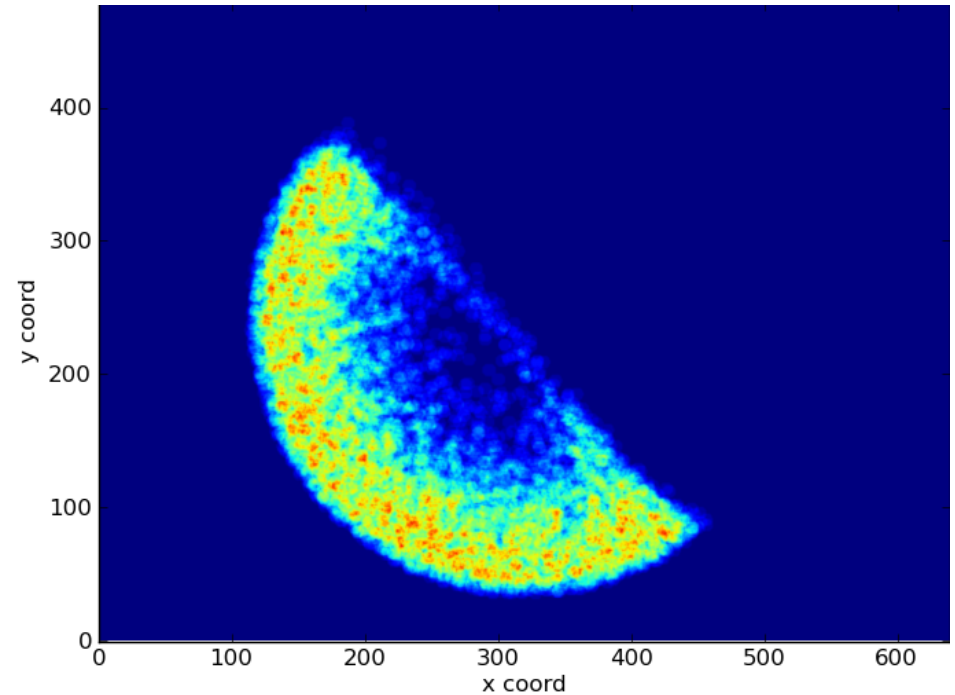
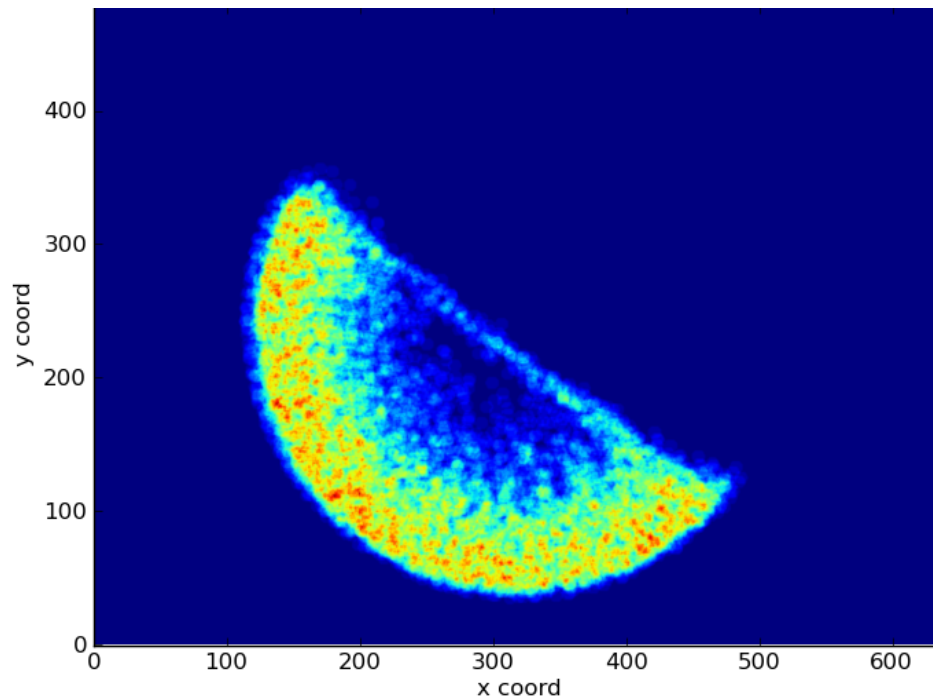


avalanche 150

# Heat maps

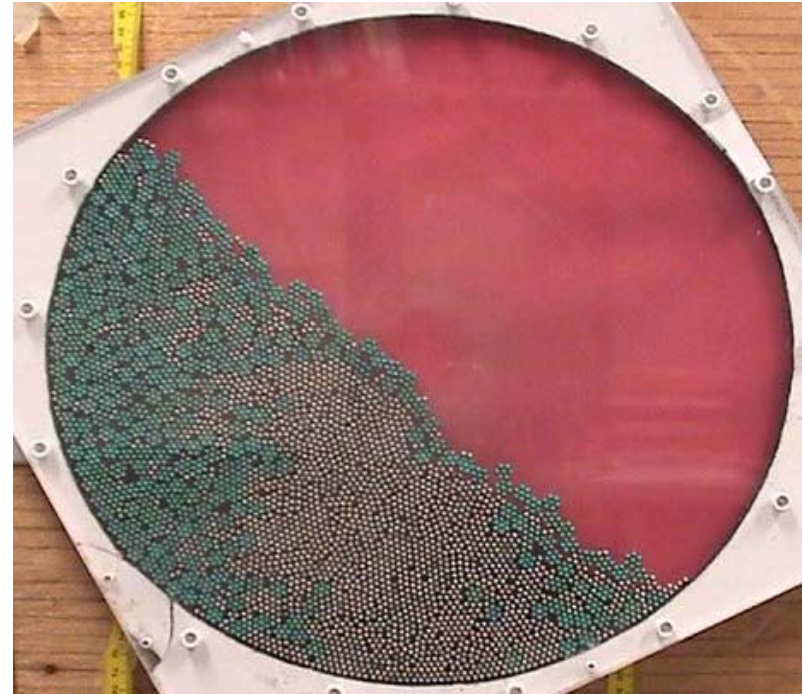
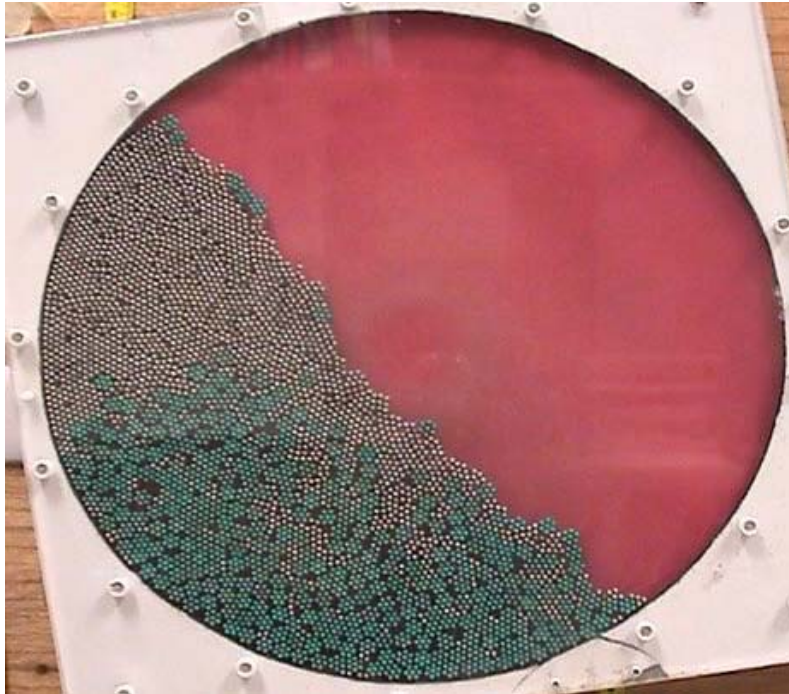
- identify centers of hexagons
- sort images by avalanche angle
- make heat maps of hex locations

blue – few hexes  
red – many hexes



*no apparent configuration differences as angle changes*

Also sorted images visually, found distribution of angles



*no statistically significant differences*

[anything dramatic should have appeared in heat maps]



# Conclusions

## One-shape piles:

- Density/angle correlation within a shape
- Density/angle inverse correlation across shapes
- Statistical evidence that stability depends only on top layers

## Single-hex mixes:

- Some effects of exact configuration found
- Presence of singles near surface triggers avalanches

## Double-hex mixes:

- No effects of configuration on angle found
- Clearly different patterns have same range of avalanche angles
- Avalanches may have different mechanism for starting than in single-hex system