"Sphere Packings, Density Fluctuations, Coverings and Quantizers"

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I briefly review the sphere-packing, number variance, coveringand quantizer problems. It is known that the sphere-packing problem and the number-variance problem (closely related to an optimizationproblem in number theory) can be posed as energy minimizations associated point particles in d-dimensional Euclidean space interacting via certain repulsive pair potentials. Recently, I have reformulatedthe covering and quantizer problems, well-known indiscrete geometry, as the determination of the ground states ofinteracting particles in d-dimensional Euclidean space that generallyinvolve single-body, two-body, three-body, and higher-body interactions[1]. This is done by linking the covering and quantizer problems tocertain optimization problems involving the "void" nearest-neighborfunctions that arise in the theory of random media and statisticalmechanics. These reformulations again exemplify the deepinterplay between geometry and physics. The covering and quantizer problems have relevance in numerous applications, including wireless communication network layouts, the search of high-dimensional data parameter spaces, stereotactic radiation therapy,data compression, digital communications, meshing of space for numericalanalysis, and coding and cryptography, among other examples. Theconnections between the covering and quantizer problems and thesphere-packing and number-variance problems (related to problems in numbertheory) are discussed. I also show that disordered saturated spherepackings provide relatively thin (economical) coverings and may yieldthinner coverings than the best known lattice coverings in sufficientlylarge dimensions. I derive improved upper bounds on the quantizer errorusing sphere-packing solutions, which are generally substantially sharperthan an existing upper bound in low to moderately large dimensions. Idemonstrate that disordered saturated sphere packings yield relativelygood quantizers. Finally, I remark on possible applications of the resultsto the detection of gravitational waves.

[1] S. Torquato, Phys. Rev. E, 82, 056109 (2010).