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Conference on Cold Materials, Hot Nuclei, and Black Holes: Applied Gauge/Gravity Duality

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Thermalization of fluctuations in strongly coupled plasmas

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Thermalization of fluctuations in strongly coupled plasmas

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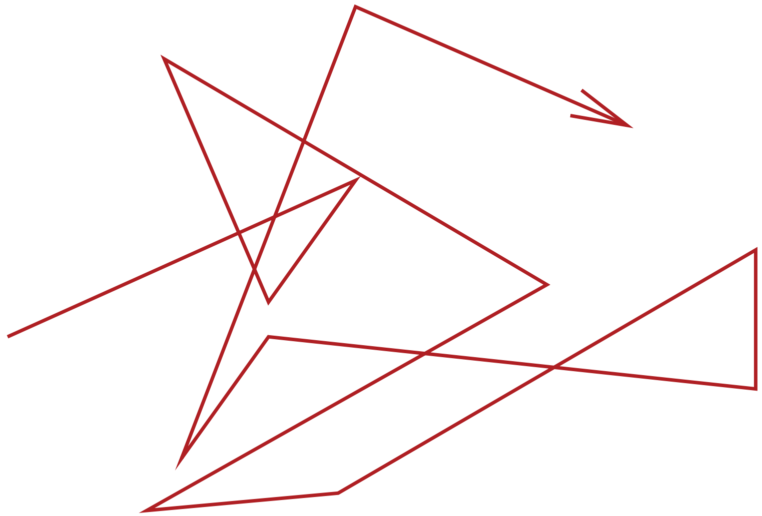
- Dam T. Son, DT; JHEP. arXiv:0901.2338
- Simon Caron-Huot, DT, Paul Chesler; PRD, arXiv:1102.1073

Related works by others

- J. de Boer, V. E. Hubeny, M. Rangamani and M. Shigemori, JHEP , arXiv:0812.5112.
- V. Balasubramanian, A. Bernamonti, J. de Boer *et al.*, PRL, arXiv:1012.4753 [hep-th].

Heavy Quarks in equilibrium Quantum Field Theory

$$M \frac{d^2\mathbf{x}}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{\mathbf{x}} + \underbrace{\xi}_{\text{Noise}}$$



← "Artist's" conception of Brownian Motion

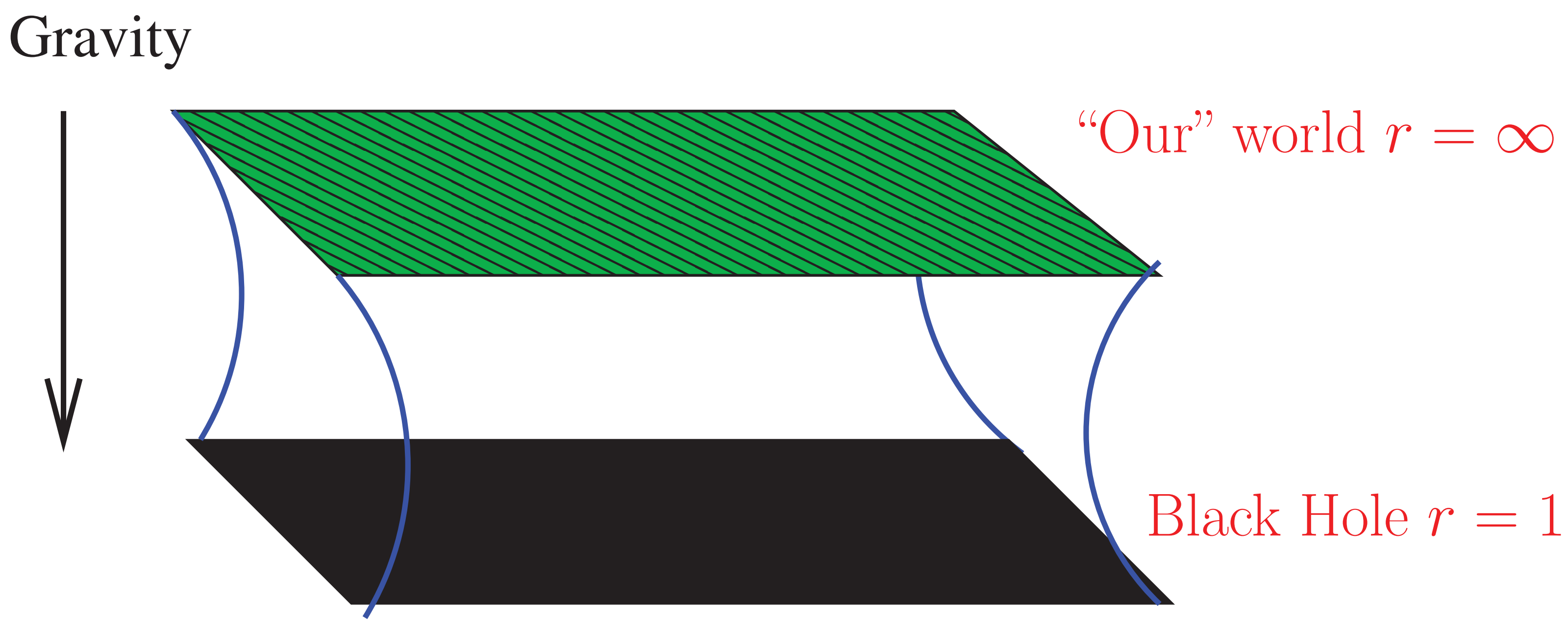
1. In equilibrium the drag and noise are balanced

$$\langle \xi(t)\xi(t') \rangle = 2T\eta\,\delta(t-t') \Leftarrow \text{Fluctuation Dissipation Theorem}$$

AdS/CFT

- Classical solutions in curved spacetime = CFT for nonzero temperature

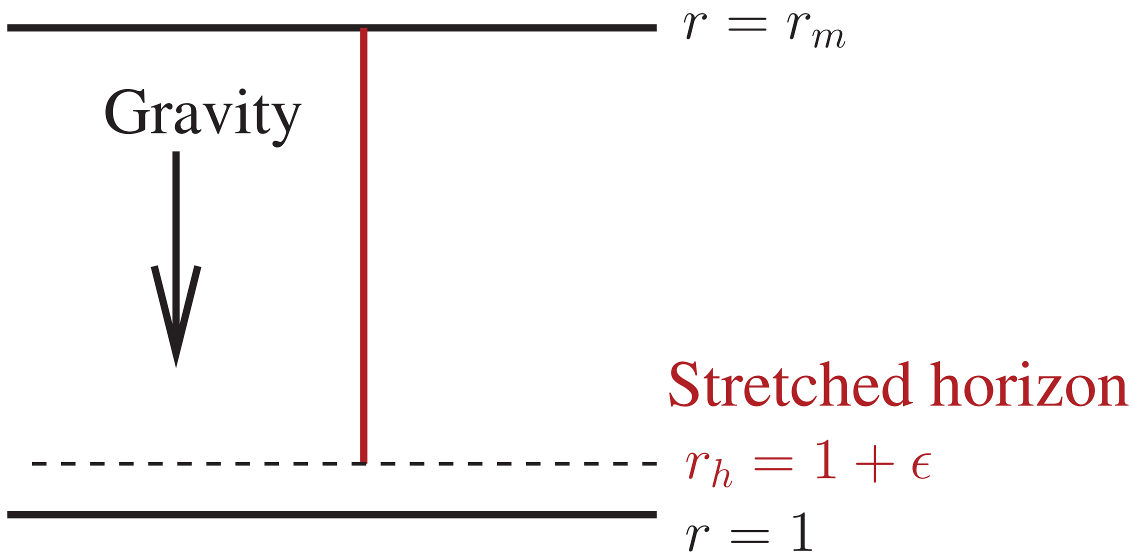
$$ds^2 = (\pi T)^2 r^2 \left[-f(r)\mathrm{d}t^2 + \mathrm{d}x^2\right] + \frac{\mathrm{d}r^2}{r^2 f(r)} \qquad f(r) = 1 - \frac{1}{r^4}$$



How can a static metric be dual to equilibrium=constant fluctuations ?

Heavy Quarks in equilibrium AdS

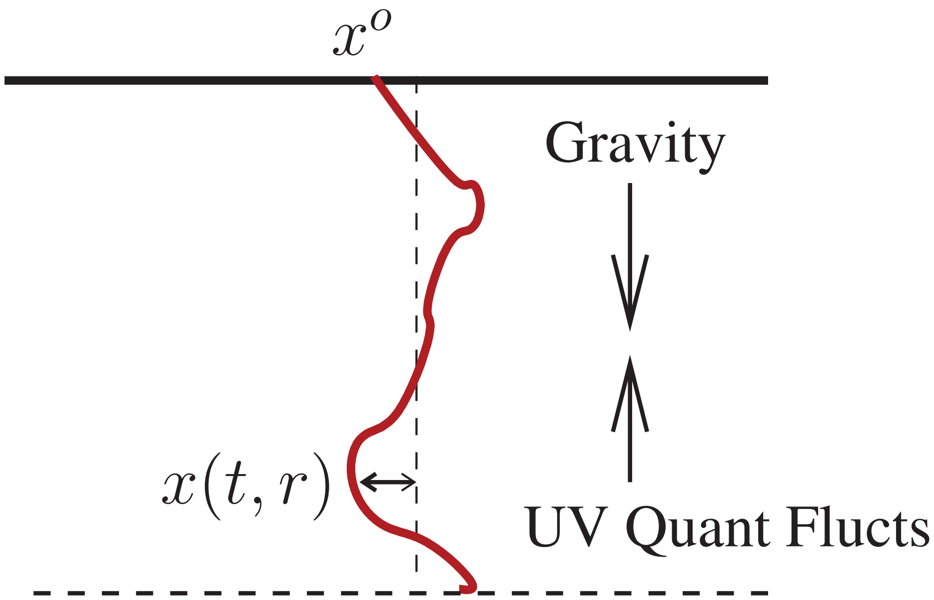
- Heavy quarks are classical strings in the 5d equilibrium AdS black hole geometry
- Solve classical string EOM and find:



Not the dual of an equilibrated quark!

Detailed Balance and Hawking Radiation:

$$M\frac{d^2x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x}^o + \underbrace{\xi}_{\text{Noise}}$$



Evolves to Classical

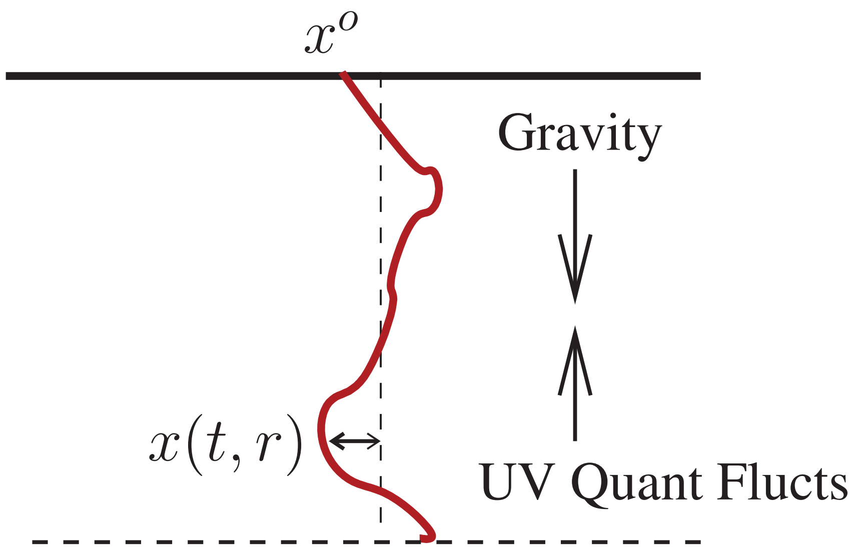
Prob Dist: (Son,DT;deBoer et al)

$$P[x,\pi_x] \propto e^{-\beta H[x,\pi_x]}$$

Goals:

1. Will show that Hawking Radiation is balanced by gravity
2. Generalize to non-equilibrium

Detailed Balance and Hawking Radiation (Technical Discussion)



1. Fluctuations:

$$G_{rr} \equiv \frac{1}{2} \langle \{ \hat{x}(t_1, r_1), \hat{x}(t_2, r_2) \} \rangle \ ,$$

2. Dissipation (Spectral Density)

$$\rho_{ra-ar} \equiv \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle \ .$$

- Equilibrium $\equiv$ Fluctuation Dissipation Theorem

$$G_{rr}(\omega, r_1, r_2) = \left(\frac{1}{2} + n_B(\omega) \right) \rho_{ra-ar}(\omega, r_1, r_2) \qquad n(\omega) \equiv \frac{1}{e^{\omega/T} - 1}$$

Formulas

- Action for string fluctuations, $h^{\mu\nu}$ = string metric

$$S_1 - S_2 = \frac{\sqrt{\lambda}}{2\pi} \int \mathrm{d}t\mathrm{d}r \, g_{xx} \left[-\sqrt{h} h^{\mu\nu} \partial_\mu x_r \partial_\nu x_a \right] \,,$$

- $h^{\mu\nu}$ is the string metric

$$h_{\mu\nu} \mathrm{d}\sigma^\mu \mathrm{d}\sigma^\nu = -(\pi T)^2 r^2 f(r) dt^2 + \frac{dr^2}{f(r) r^2} \,,$$

- Retarded Green Function

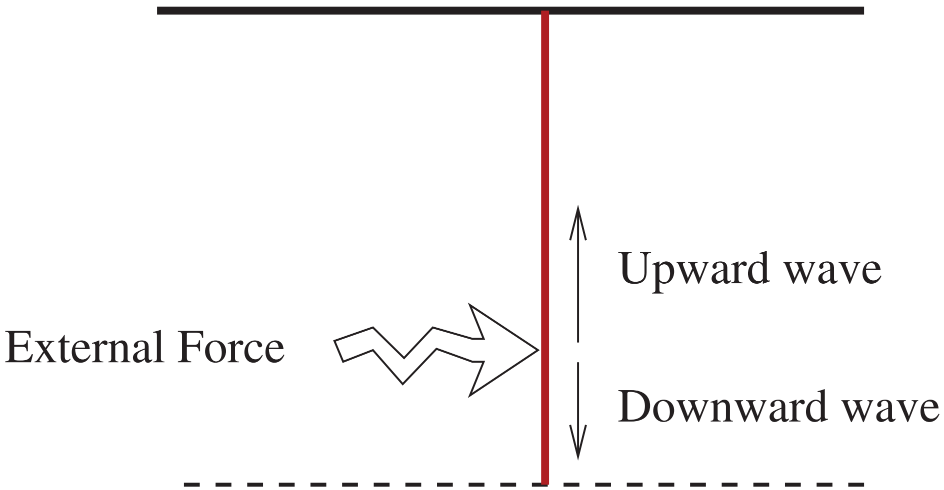
$$iG_{ra}(t_1 r_1 | t_2 r_2) \equiv \theta(t - t') \left\langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \right\rangle \,,$$

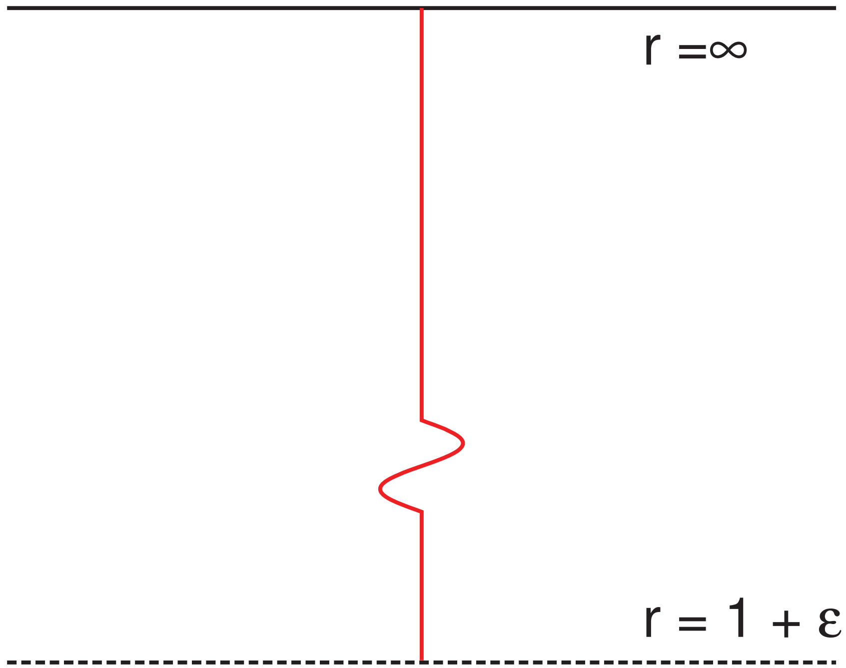
$G_{ra}(t_1 r_1 | t_2 r_2)$ is the classical response to a force at $t_2 r_2$

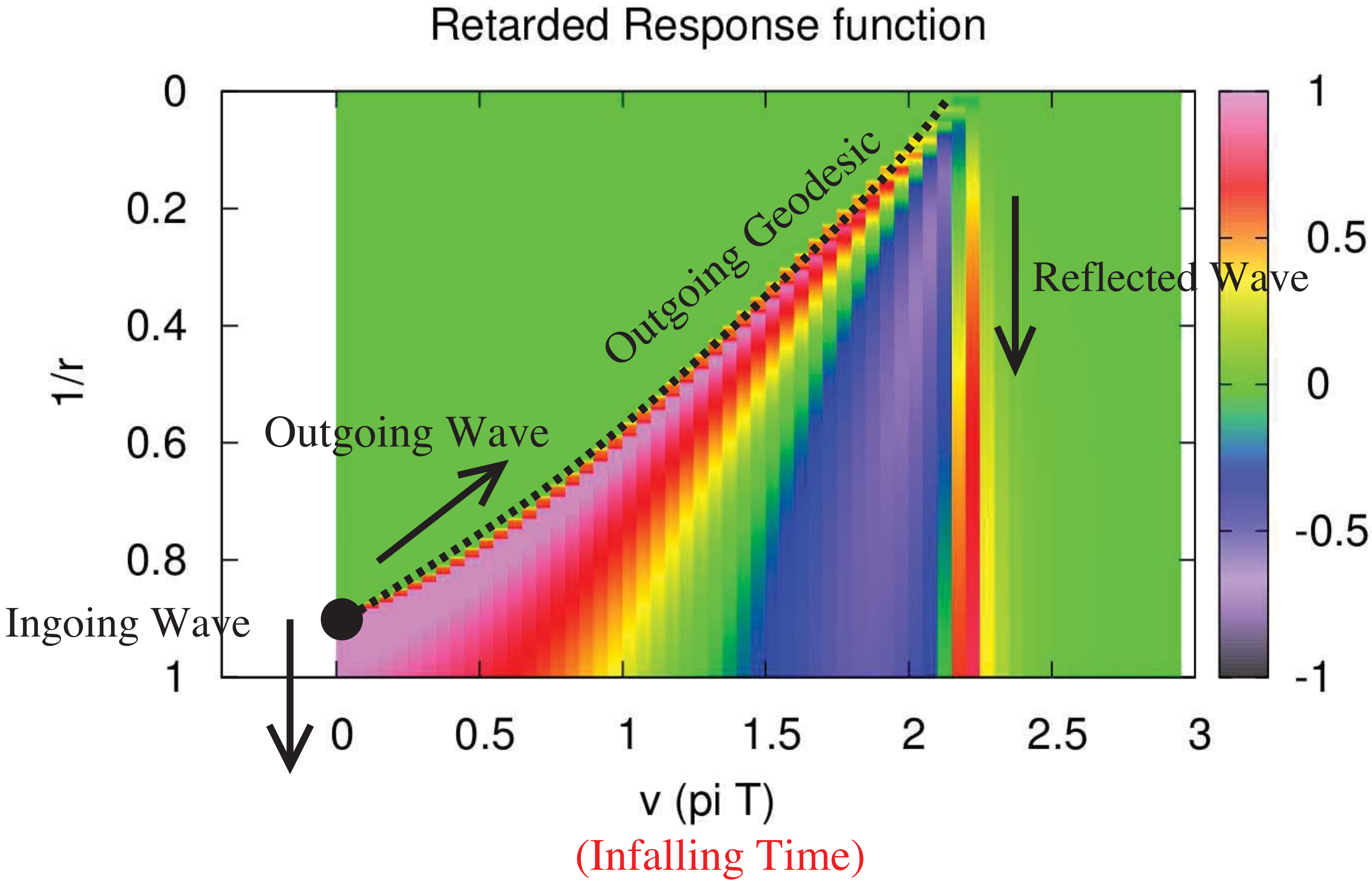
$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] G_{ra}(t_1 r_1 | t_2 r_2) = \delta(t_1 - t_2) \delta(r_1 - r_2) \,,$$

The classical Green Function or response to a force:

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right]G=\mathcal{F}\,\delta(t_1-t_2)\delta(r_1-r_2)\,,$$

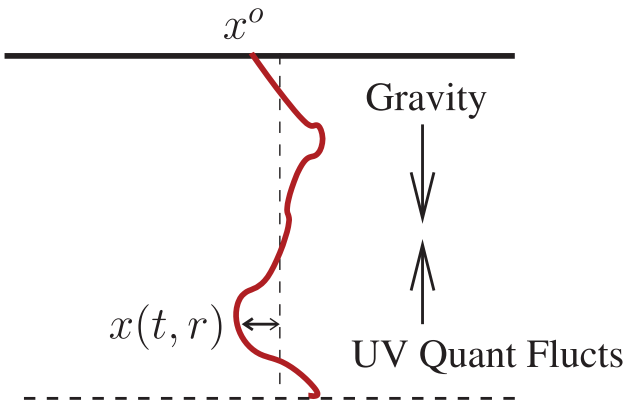






$$v = t - \frac{1}{2\pi T}\left[\tan^{-1}(r) + \tanh^{-1}(r)\right] \qquad v = \text{Eddington time}$$

Statistical Fluctuations



$$G_{rr} = \frac{1}{2}\langle\{x(t_1,r_1), x(t_2,r_2)\}\rangle$$

- The statistical correlator obeys the homogeneous EOM

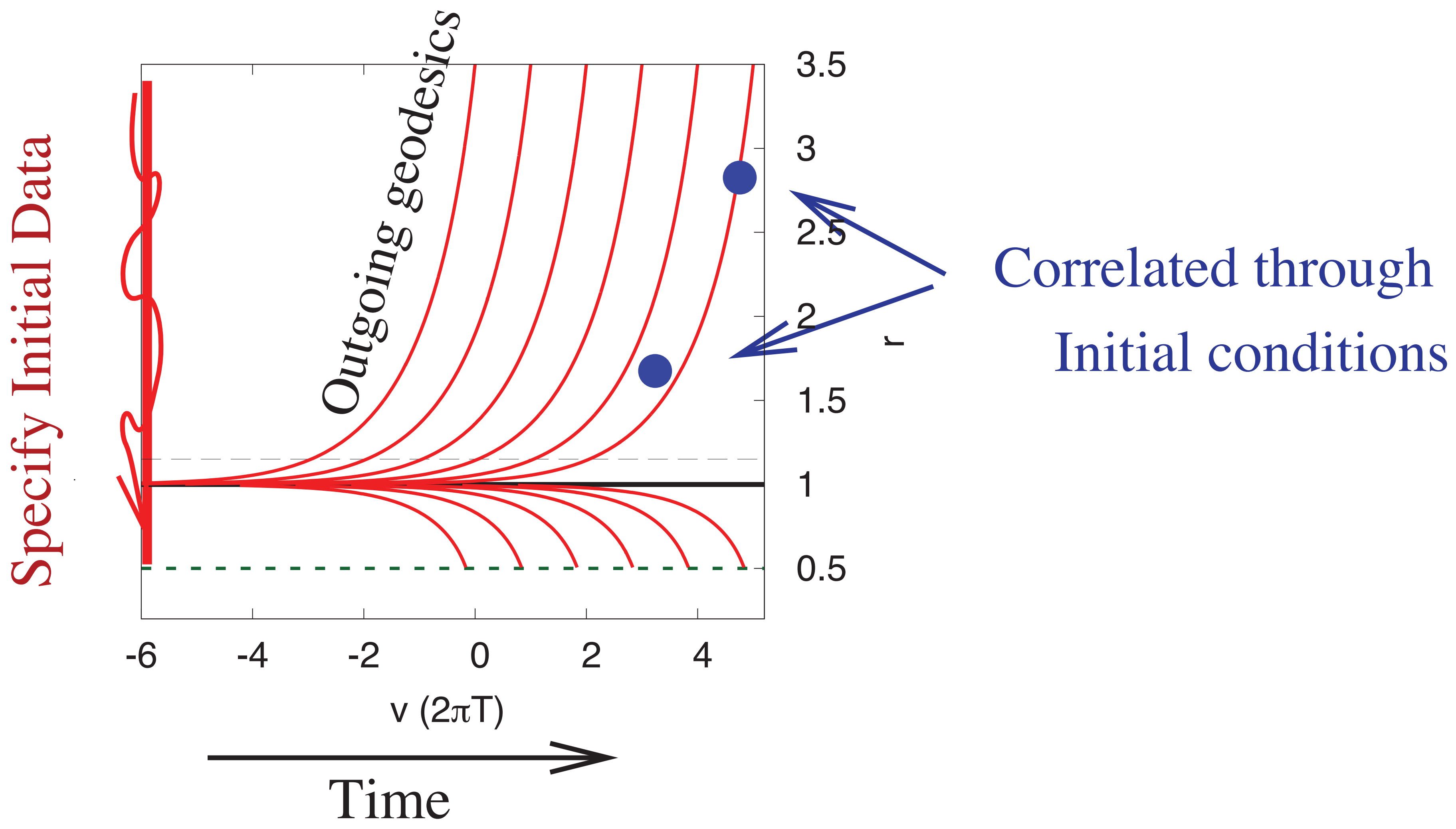
$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G_{rr}(t_1 r_1|t_2 r_2) = 0$$

- So:

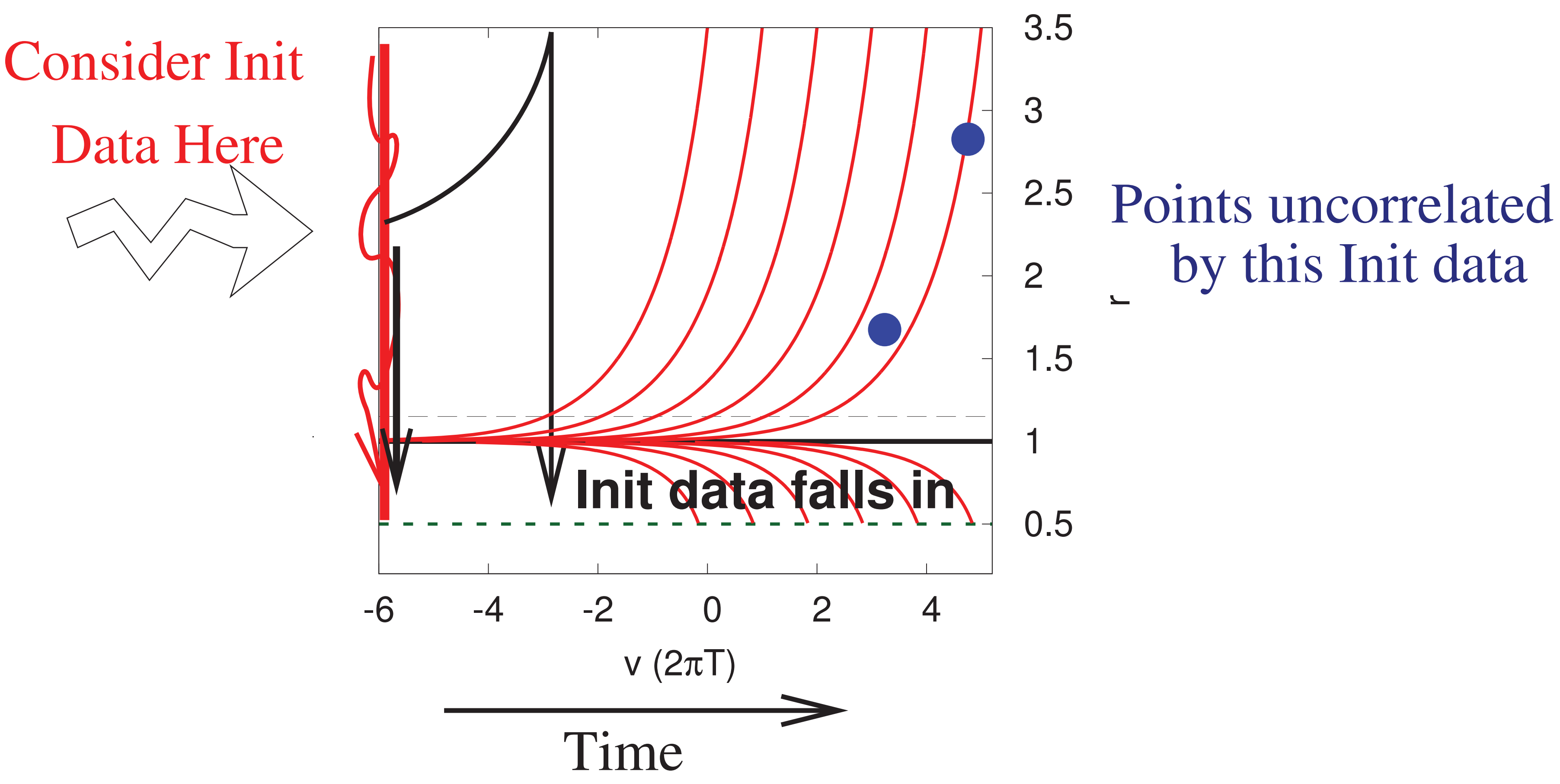
1. Specify the correlations (or density matrix) in the past

2. Final state fluctuations are correlated only through initial conditions

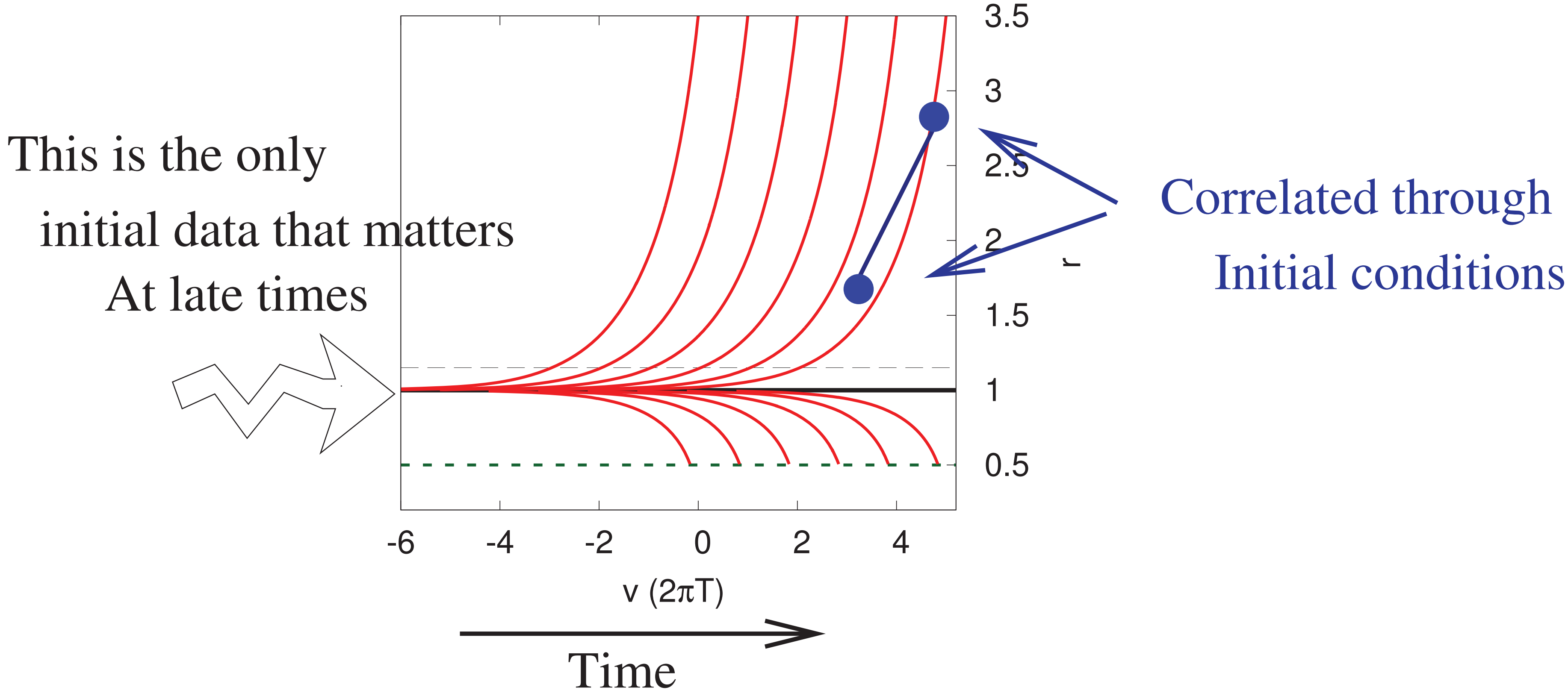
Correlations through Initial conditions



Correlations through Initial conditions



Correlations through Initial conditions



1. Final correlation come from correlated initial data <u>very</u> near horizon
   - Short Wavelength
2. Initial data is inflated by near horizon geometry

Initial Data from Quantum Fluctuations

1. Initial data is determined at short distance = Flat Space Physics
2. Scalar Field in 1+1D vacuum flat space

$$\frac{1}{2}\langle\{\phi(X_1),\phi(X_2)\}\rangle = -\frac{1}{4\pi K}\log|\mu\,\eta_{\mu\nu}\Delta X^{\mu}\Delta X^{\nu}|$$

K=norm of action

3. String flucts in near horizon geometry

$$S^{\text{near-horizon}} = \eta\int \mathrm{d}t\mathrm{d}r\left[-\frac{1}{2}\sqrt{h}h^{\mu\nu}\partial_{\mu}x\partial_{\nu}x\right]$$

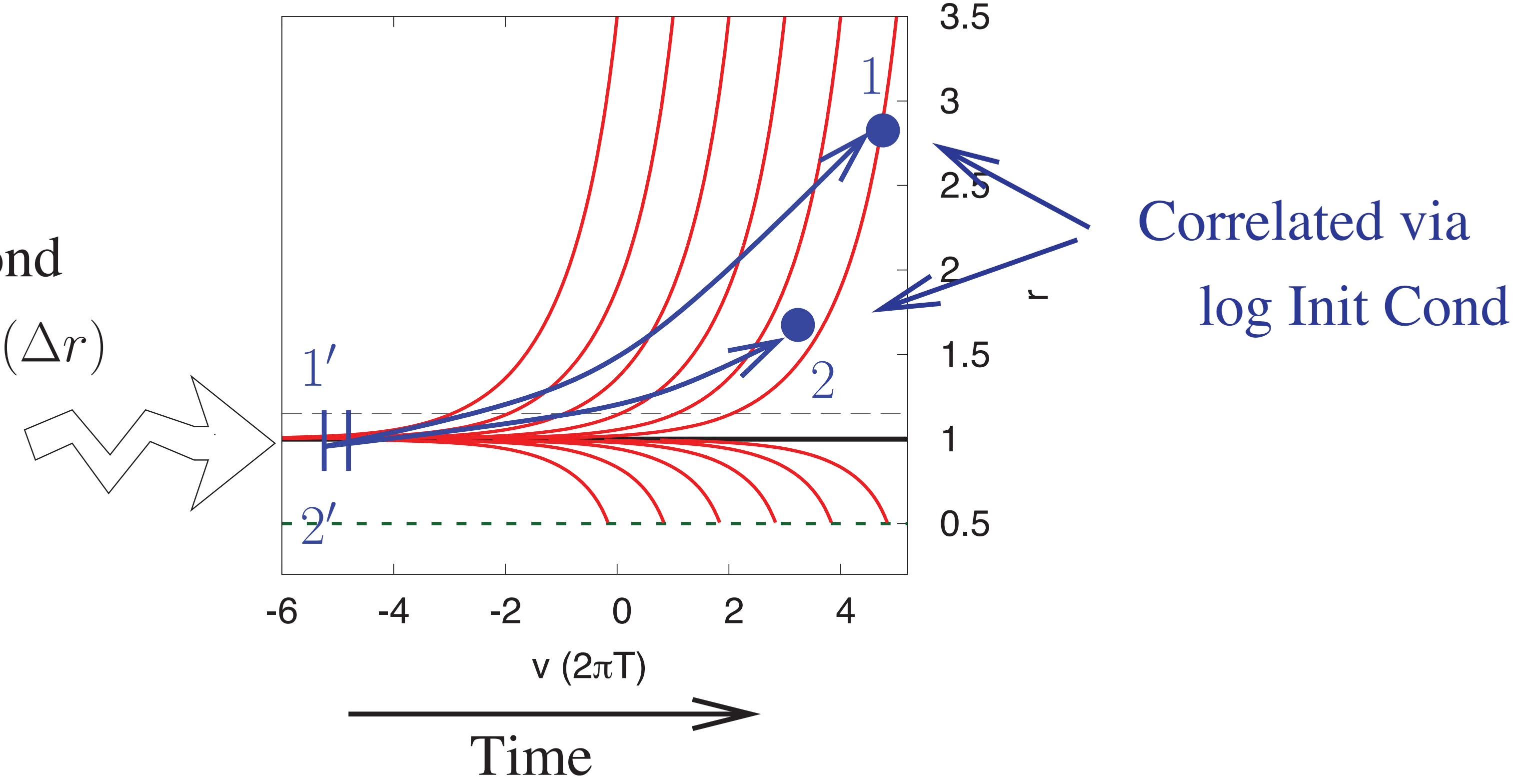
$\eta =$ Drag Coefficient

The near horizon initial condition is:

$$G_{rr}(v_1r_1|v_2r_2) \to -\frac{1}{4\pi\eta}\log\left|\mu\,\overbrace{2\Delta v\,\Delta r}^{\text{local }\Delta s^2}\right|$$

Summary: Specify IC and Solve Equations of Motion

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G_{rr}(t_1 r_1|t_2 r_2) = 0$$

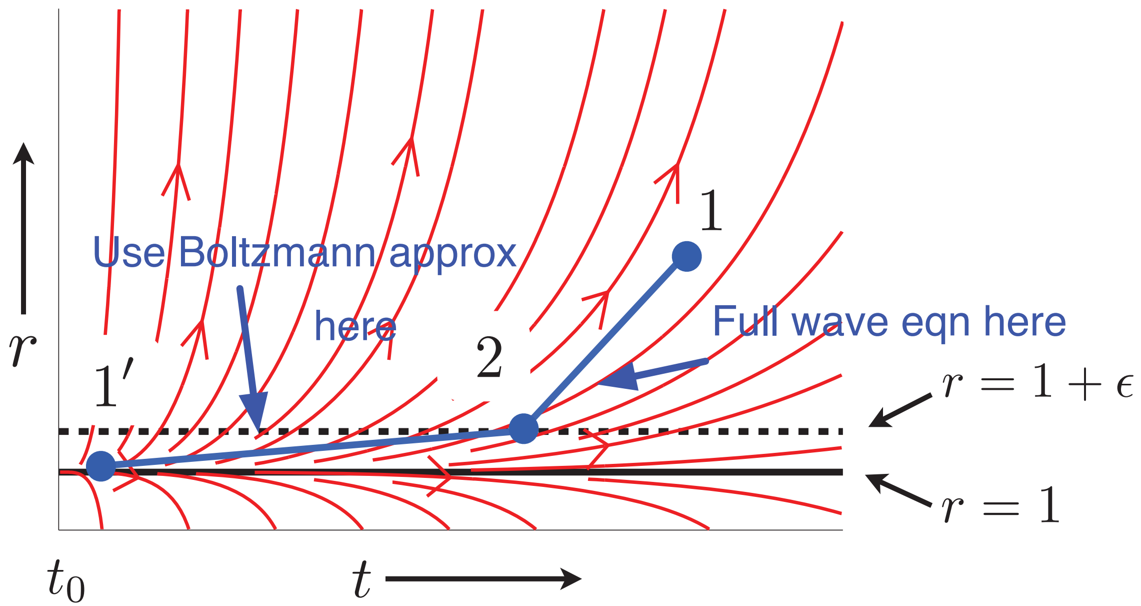


$$G_{rr}(1|2) = \left[\frac{\sqrt{\lambda}}{2\pi}\int \mathrm{d}r'_1\; g_{xx}\sqrt{h}h^{tt}(r'_1)\, G_{ra}(1|1')\overleftrightarrow{\partial}_{t'_1}\right]$$

Like Harmonic Osclator

$$\times\left[\frac{\sqrt{\lambda}}{2\pi}\int \mathrm{d}r'_2\; g_{xx}\sqrt{h}h^{tt}(r'_2)\, G_{ra}(2|2')\overleftrightarrow{\partial}_{t'_2}\right] G_{rr}(1'|2')\,,$$

From initial data to final correlations in two steps:



$$G_{ra}(1|1') = \int \mathrm{d}t_2\, G_{ra}(1|2) \left[\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2}\right]_{r_2=1+\epsilon} g_{ra}(2|1')$$

(a) From horizon to stretched horizon – Waves are very short wavelength

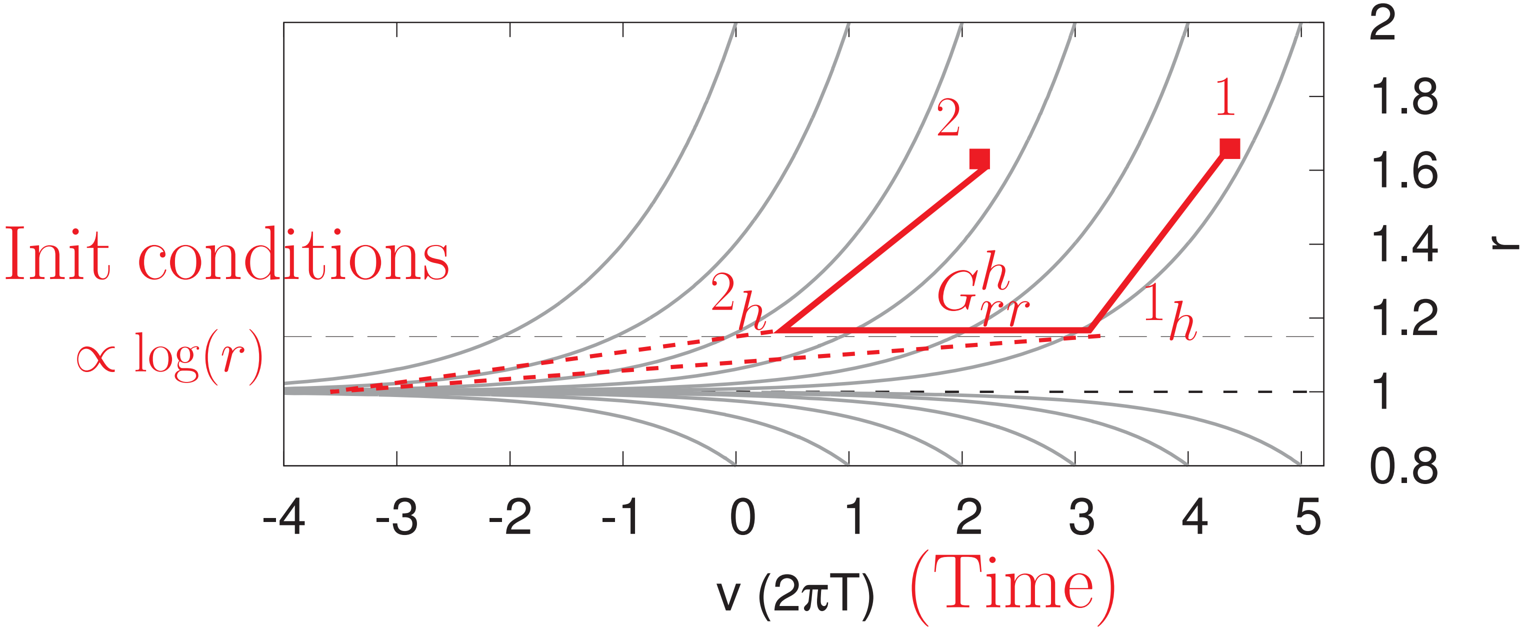
- Will use free Boltzmann approximation (geodesic/WKB/eikonal approx)

(b) The stretched horizon to boundary – Waves are longer wavelength

- Will use full wave equation

Fluctuations from Equations of Motion

$$\underbrace{G_{rr}(1|2)}_{\text{bulk flucts}} = \int \mathrm{d}t_{1h}\mathrm{d}t_{2h}\ \underbrace{G_R(1|1_h)\ G_R(2|2_h)}_{\text{outgoing Green fcns}}\ \underbrace{G^h_{rr}(1_h|2_h)}_{\text{horizon flucts}}\,,$$



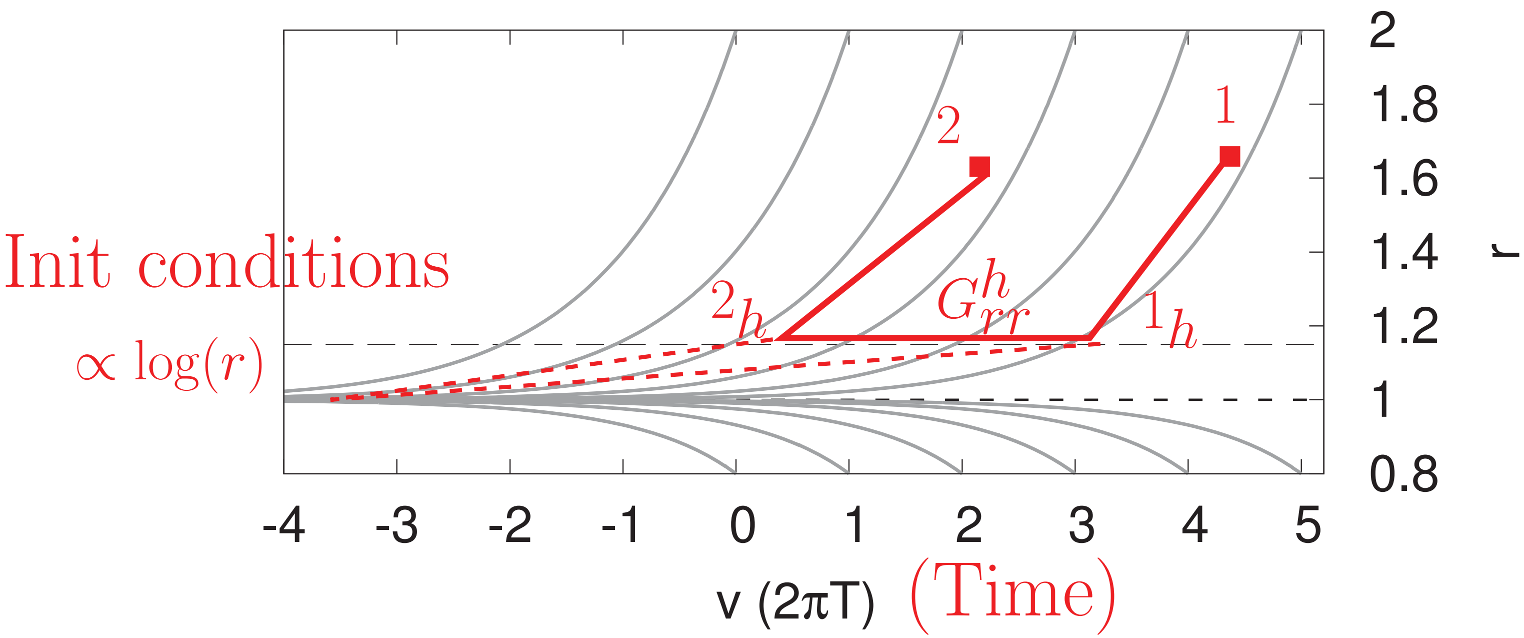
The horizon correlator is found from the horizon limit of correlation fcn in strip

$$G^h_{rr}(t_1|t_2) = \lim_{r_1 r_2 \to r_h} \left[-\eta\sqrt{h}h^{rr}(r_1)\partial_{r_1}\right]\left[-\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2}\right] g_{rr}(t_1 r_1|t_2 r_2)$$

Bulk to bulk symmetrized correlation function ***in strip*** (with reflective bc)

Fluctuations from Equations of Motion

$$\underbrace{G_{rr}(1|2)}_{\text{bulk flucts}} = \int \mathrm{d}t_{1h}\mathrm{d}t_{2h}\ \underbrace{G_R(1|1_h)\ G_R(2|2_h)}_{\text{outgoing Green fcns}}\ \underbrace{G^h_{rr}(1_h|2_h)}_{\text{horizon flucts}}\,,$$

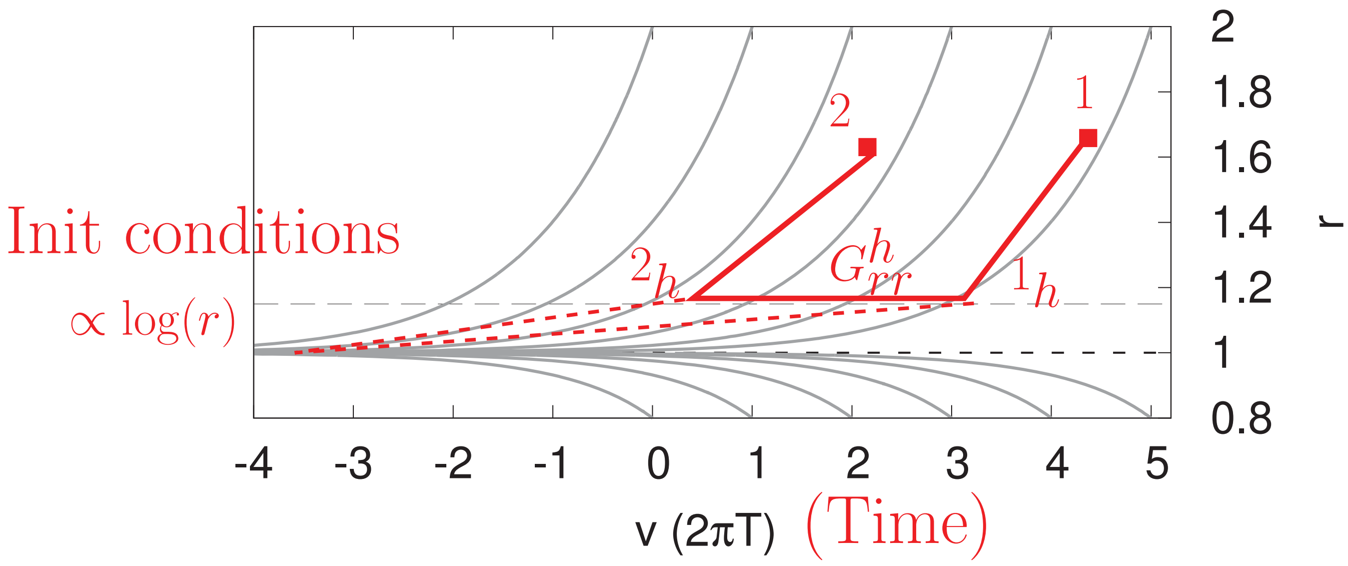


The horizon correlator is found from the horizon limit of correlation fcn in strip

$$G^h_{rr}(t_1|t_2) = \text{Blow-up of initial data} \propto \log(r)$$

$$= -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2}\log\left|1 - e^{-2\pi T(t_1-t_2)}\right|\,.$$

The horizon fluctuations and the Lyapunov exponent



1. Thermal looking:

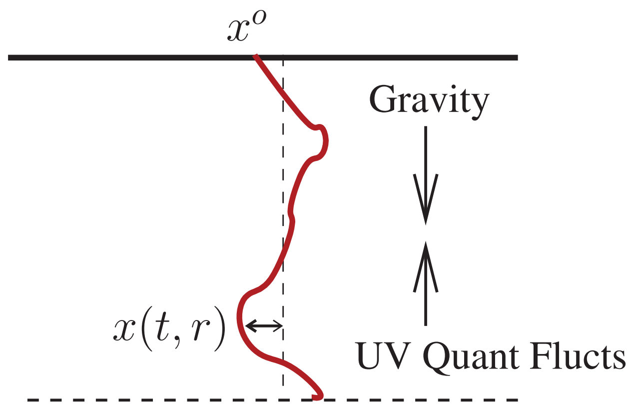
$$G^h_{rr}(\omega) = \text{Fourier-Trans of } -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2}\log|1-e^{-2\pi T(t_1-t_2)}|$$

$$= \left(\tfrac{1}{2}+n(\omega)\right)2\omega\eta \qquad\qquad n(\omega)\equiv\frac{1}{e^{\omega/T}-1}$$

2. Temperature $\propto$ inflation rate

$$2\pi T = \text{Lyapunov exponent of diverging geodesics}$$

Dissipation - Spectral Density



$$\rho_{ra-ar} = \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle$$

- The spectral density also obeys the EOM

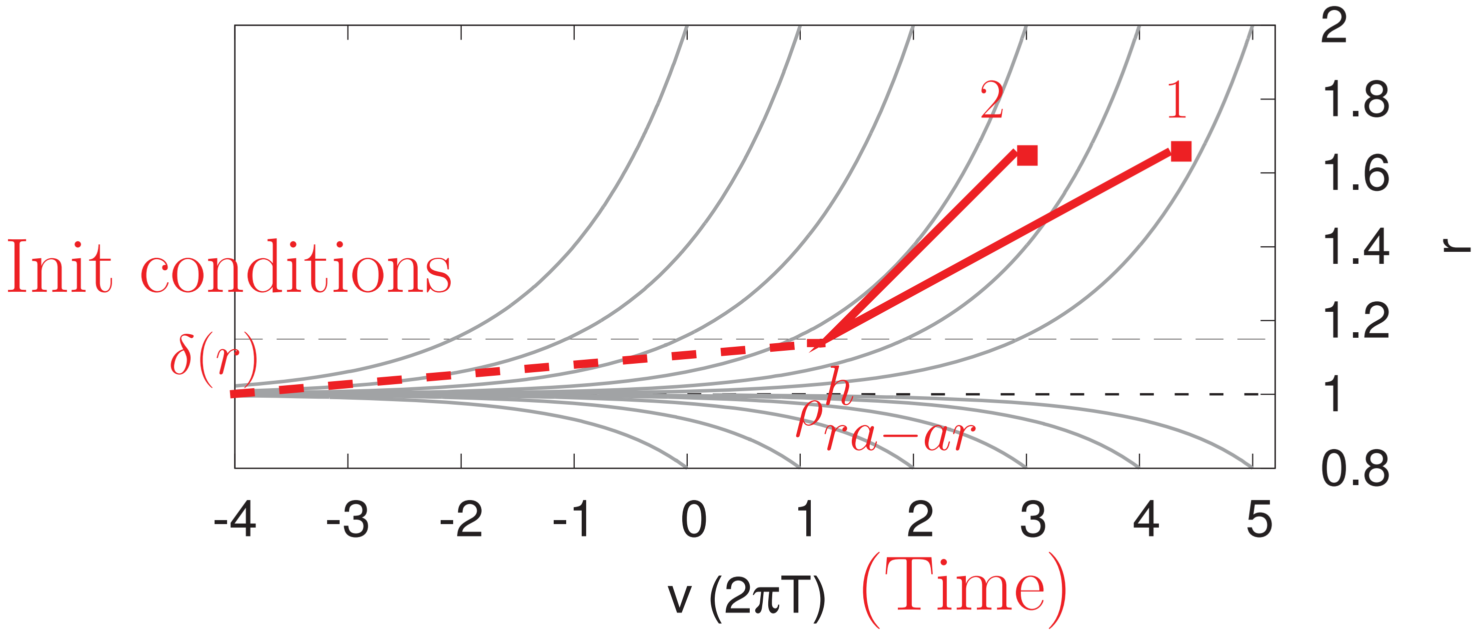
$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] \rho_{ra-ar}(t_1 r_1 | t_2 r_2) = 0$$

- But initial conditions are given by the canonical commutation relations

$$\eta \sqrt{h} h^{tt}(r_1) \lim_{\mathrm{t}_2 \to \mathrm{t}_1} \partial_{t_1} \rho_{ra-ar}(t_1 r_1 | t_2 r_2) = i\delta(r_1 - r_2) \, .$$

Spectral Density

$$\underbrace{\rho_{ra-ar}(1|2)}_{\text{bulk spectral fcn}} = \int \mathrm{d}t_{1h}\mathrm{d}t_{2h}\; \underbrace{G_R(1|1_h)\; G_R(2|2_h)}_{\text{outgoing Green fcns}} \;\; \underbrace{\rho^h_{ra-ar}(1_h|2_h)}_{\text{horizon spectral fcn}} \;,$$



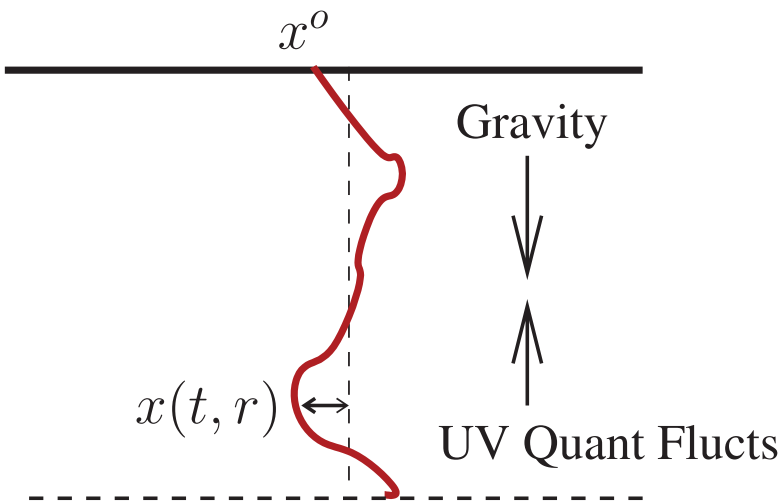
Where the horizon spectral density

$$\rho^h_{ra-ar}(t_1, t_2) = \text{local due to canonical commutation relations}$$

$$= 2\eta\left[-i\delta'(t_1 - t_2)\right] \qquad (2\omega\eta \text{ in Fourier space})$$

Detailed Balance

$$G_{rr}(\omega, r_1, r_2) = \left(\tfrac{1}{2} + n(\omega)\right) \rho(\omega, r_1, r_2)$$



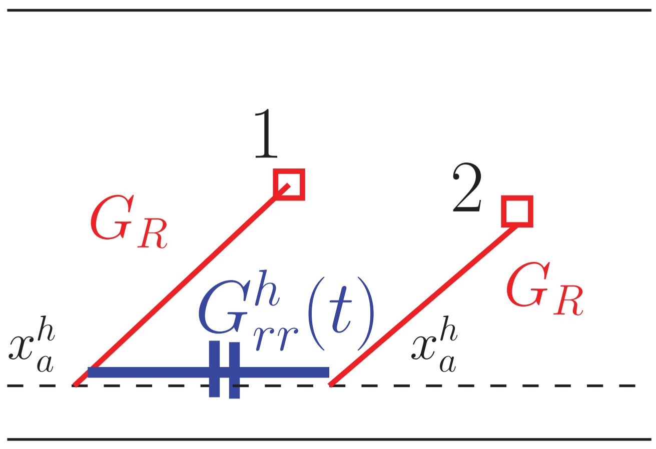
1. Fluctuations (Anti-commutator)

$$\underbrace{G_{rr}(\omega, r_1, r_2)}_{\text{bulk flucts}} = \underbrace{G_R(\omega, r_1|r_h)\; G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \; \underbrace{\left(\tfrac{1}{2} + n(\omega)\right) 2\omega\eta}_{\text{Horizon-flucts}}$$

2. Dissipation: (Commutator)

$$\underbrace{\rho_{ra-ar}(\omega, r_1, r_2)}_{\text{bulk spec dense}} = \underbrace{G_R(\omega, r_1|r_h)\; G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \qquad \underbrace{2\omega\eta}_{\text{Horizon spec dense}}$$

Horizon Effective Action



1. G^h_{rr} and G^h_{ra} record correlated *boundary* conditions of x^h_r and x^h_a on r_h

$$G^h_{rr}(t_1|t_2) = \lim_{r_1 r_2 \to r_h} \left[-\eta\sqrt{h}h^{rr}(r_1)\partial_{r_1}\right]\left[-\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2}\right] g_{rr}(t_1 r_1|t_2 r_2)$$

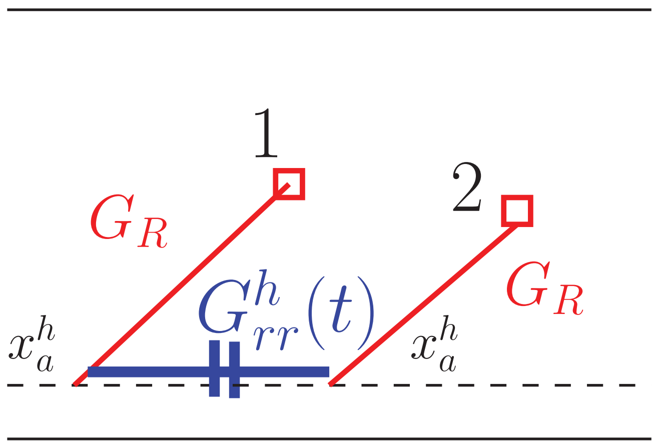
$$G^h_{ra}(t_1|t_2) = \lim_{r_1 r_2 \to r_h} \left[-\eta\sqrt{h}h^{rr}(r_1)\partial_{r_1}\right]\left[-\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2}\right] g_{ra}(t_1 r_1|t_2 r_2)$$

2. These feynman graphs are reproduced by a membrane Effective Action

$$iS^h_{\rm eff} = \int_\omega x^h_a \left[iG^h_{ra}(\omega)\right] x^h_r - \frac{1}{2}\int_\omega x^h_a \left[G_{rr}(\omega)\right] x^h_a + \ldots$$

The horizon effective action provides a horizon boundary condition

Horizon Effective Action



1. G^h_{rr} and G^h_{ra} record correlated *boundary* conditions of x^h_r and x^h_a on r_h

$$G^h_{rr}(t_1|t_2) = \lim_{r_1 r_2 \to r_h} \left[-\eta\sqrt{h}h^{rr}(r_1)\partial_{r_1} \right] \left[-\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2} \right] g_{rr}(t_1 r_1 | t_2 r_2)$$

$$G^h_{ra}(t_1|t_2) = \lim_{r_1 r_2 \to r_h} \left[-\eta\sqrt{h}h^{rr}(r_1)\partial_{r_1} \right] \left[-\eta\sqrt{h}h^{rr}(r_2)\partial_{r_2} \right] g_{ra}(t_1 r_1 | t_2 r_2)$$

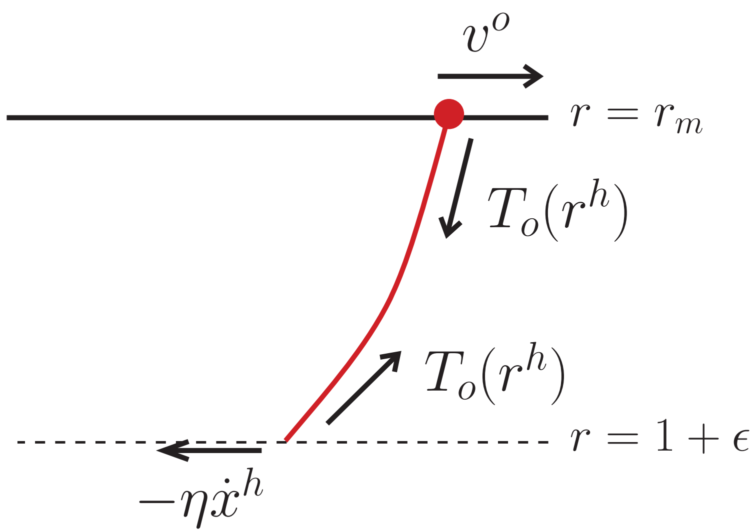
2. These feynman graphs are reproduced by a <u>membrane Effective Action</u>

$$iS^h_{\rm eff} = i\int_\omega x^h_a \Big[\underbrace{-i\omega\eta}_{\text{Horizon diss.}} \Big] x^h_r - \frac{1}{2}\int_\omega x^h_a \Big[\underbrace{(1+2n)\omega\eta}_{\text{Horizon flcts}} \Big] x^h_a + \ldots$$

The horizon effective action provides all horizon boundary condition

Classical membrane paradigm (ra part of effective action)

$$S_1 - S_2 + S^h_{\rm eff} = \frac{\sqrt{\lambda}}{2\pi}\int {\rm d}t{\rm d}r\, g_{xx}\left[-\sqrt{h}h^{\mu\nu}\partial_\mu x_r \partial_\nu x_a\right] + \int_\omega x^h_a \Big[\underbrace{-i\omega\eta}_{\text{Horizon diss.}} \Big] x^h_r$$



- Classical Boundary condition from membrane paradigm part of $S^h_{\rm eff}$

$$\overbrace{\frac{\sqrt{\lambda}}{2\pi} g_{xx}\sqrt{h}h^{rr}\partial_r x(t,r_h)}^{\text{Tension}} = \overbrace{\eta\,\dot{x}^h(t)}^{\text{Drag}}$$

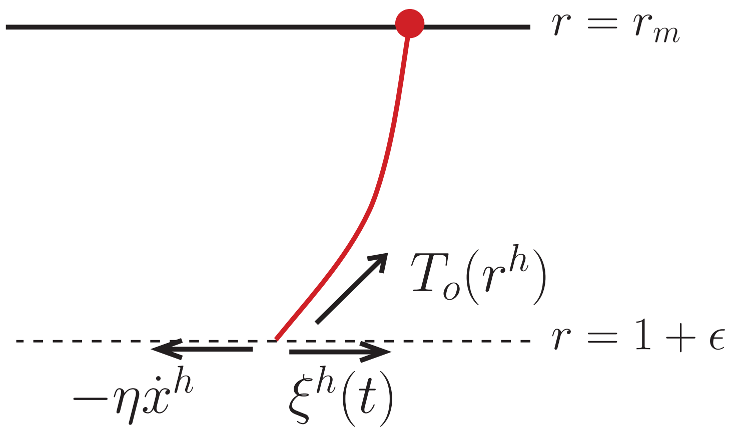
Boundary conditions from horizon effective action with quantum part

$$\text{Action} = iS_1 - iS_2 + i\int_\omega x_a^h [\underbrace{-i\omega\eta}_{\text{Classical}}] x_r^h - \frac{1}{2}\int_\omega x_a^h [\underbrace{(1+2n)\omega\eta}_{\text{Quant Gaussian Part}}] x_a^h$$

- Still overdamped horizon motion with a random horizon force

$$\overbrace{T_o(r_h)\partial_r x(t, r_h)}^{\text{Tension}} + \overbrace{\xi^h(t)}^{\text{Random force}} = \overbrace{\eta\,\dot{x}^h(t)}^{\text{Drag}}$$

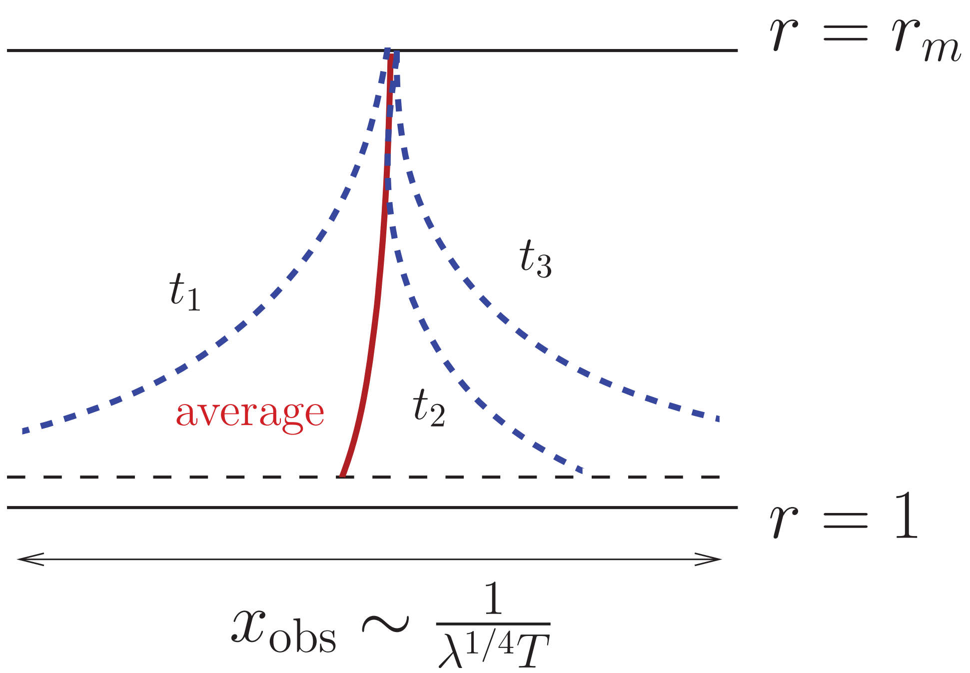
- Picture



Find a Horizon Fluct-Diss Thrm:

$$\langle \xi^h(t)\xi^h(t') \rangle = 2T\eta\delta(t - t')$$

Fluctuation dissipation and stochastic dynamics



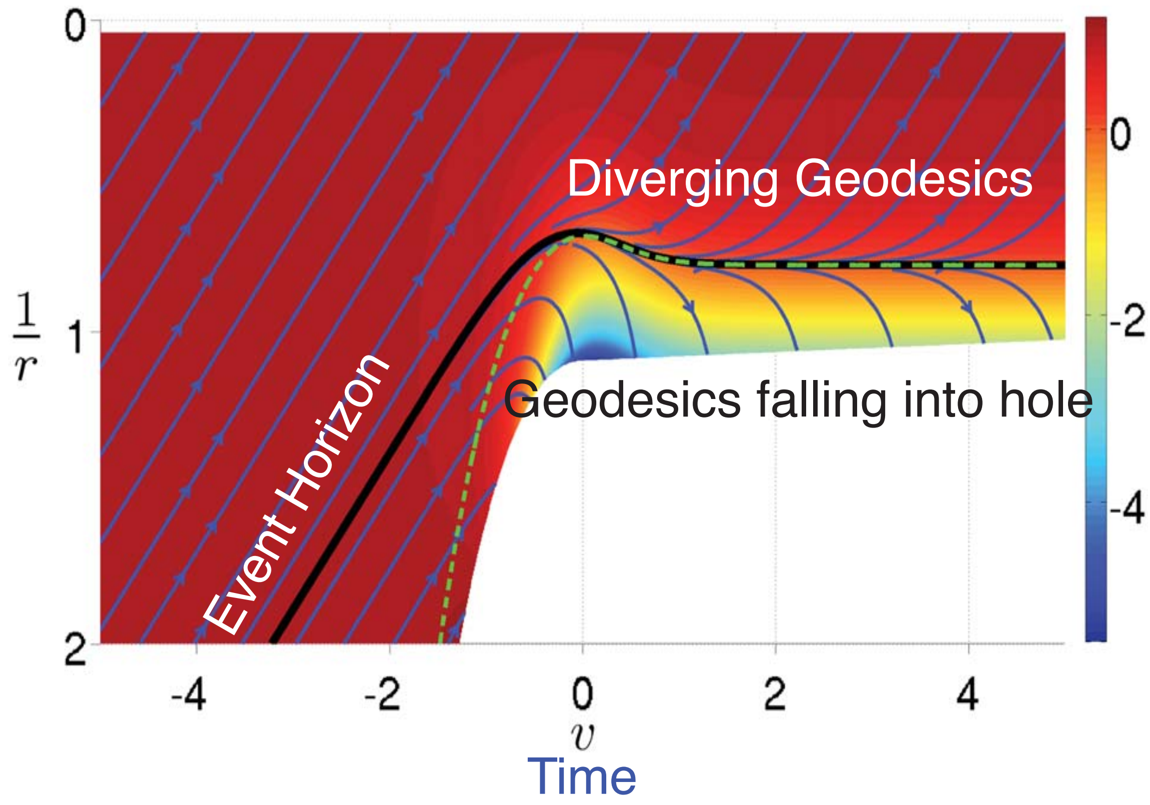
1. Every step t_1, t_2, t_3 fluctuates to a new trailing string – $\longrightarrow$ random force
2. The average of the trailing strings gives the drag – average string $\longrightarrow$ drag

Non-equilibrium

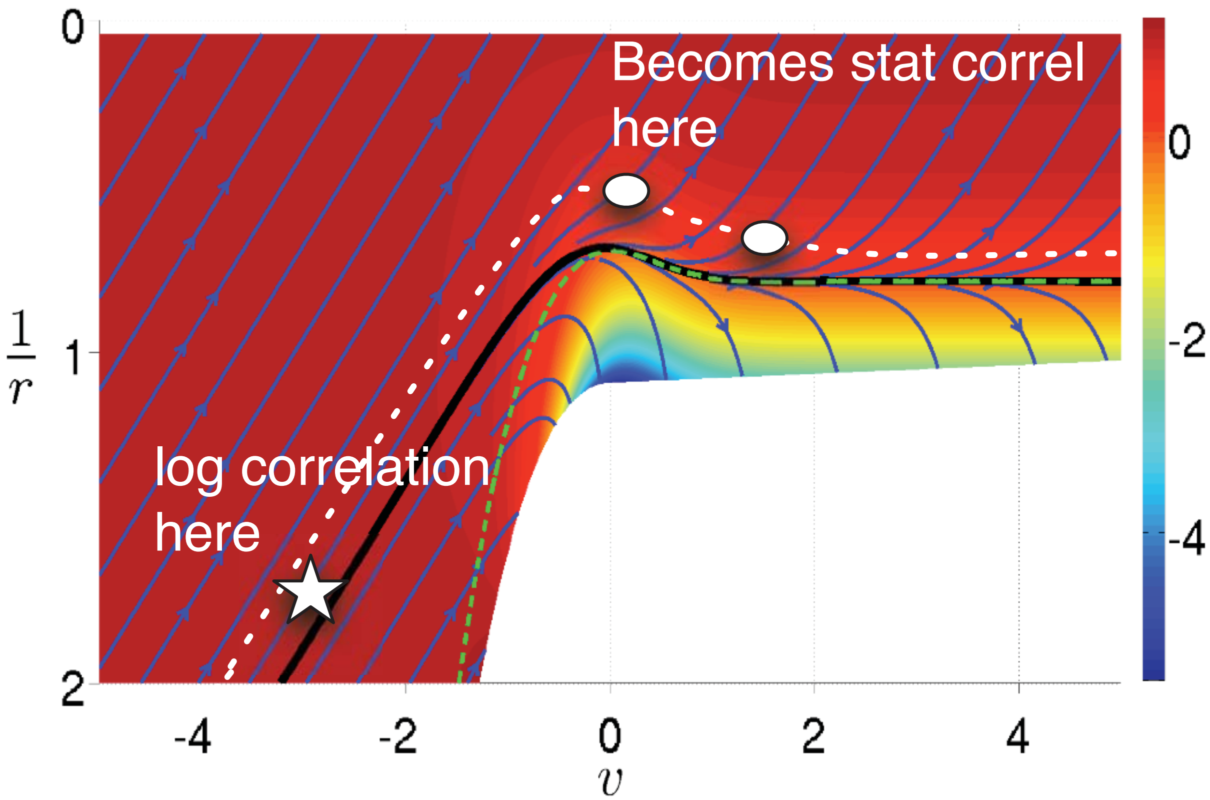
Non-equilibrium setup Chesler-Yaffe

1. Chesler&Yaffe create QGP by turning a gravitational pulse in vacuum

2. Corresponds to non-equilbrium geometry with BH formation in AdS_5



Fluctuations in non-equilibrium



- Surface Properties – on event horizon

$$\underbrace{2\pi T_{\text{eff}}(v)}_{\text{Lyapunov exponent}} \equiv \overbrace{\frac{1}{2}\frac{\partial A(r,v)}{\partial r}}^{\text{Metric-coeff}}\Bigg|_{r=r_h(v)} \propto \text{extrinsic curvature}$$

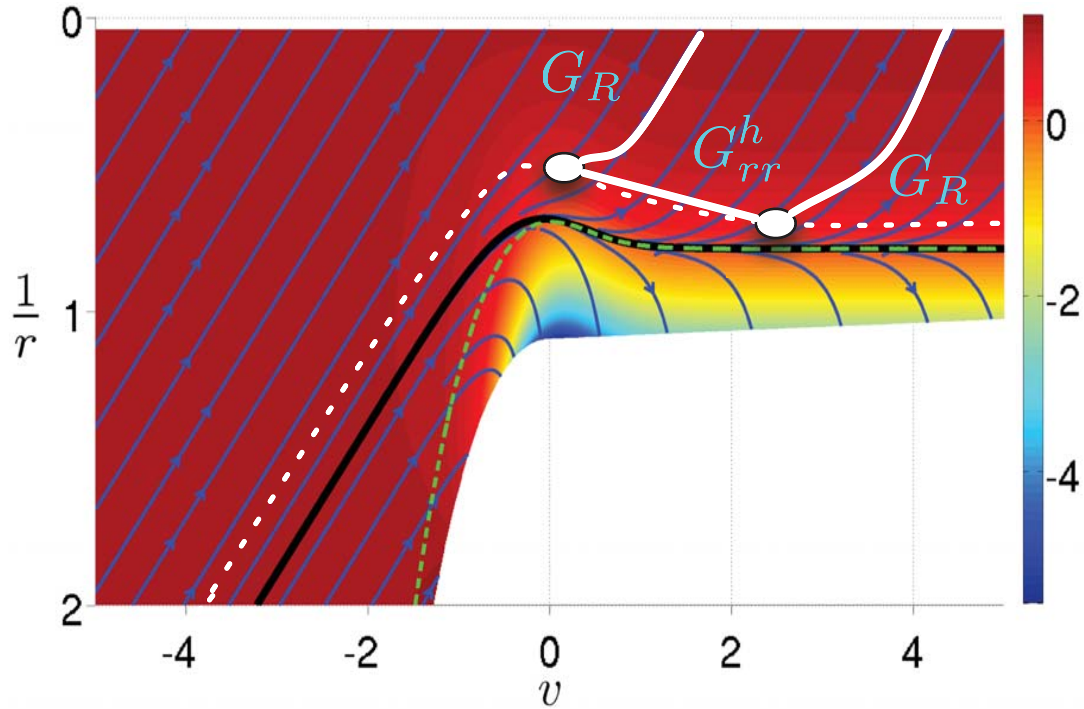
Result:

- General form of near horizon fluctuations in non-equilibrium

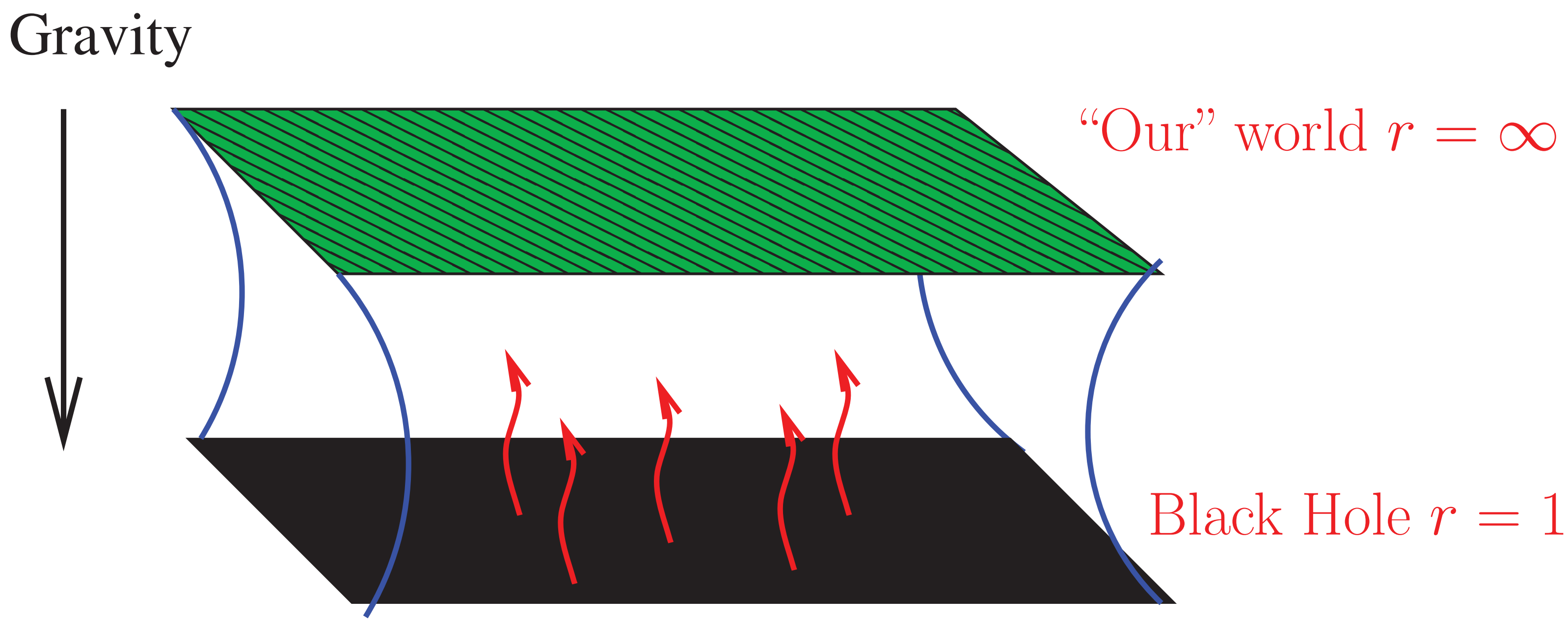
$$G^h_{rr}(v_1|v_2) = -\frac{\sqrt{\eta(v_1)\eta(v_2)}}{\pi}\partial_{v_1}\partial_{v_2}\log\left|1 - e^{-\int_{v_1}^{v_2} 2\pi T_{\rm eff}(v')dv'}\right|.$$

- Can map the near horizon fluctuations up to boundary (numerics in progress)

$$G_{rr}(1|2) = \int dv_{1h}dv_{2h}\, \underbrace{G_R(1|1_h)\; G_R(2|2_h)}_{\text{outgoing Green Fcns}}\; \underbrace{G^h_{rr}(1_h|2_h)}_{\text{horizon flucts}}\, .$$



Not conclusions, but picture:



Gravity pulls down, but quantum fields fluctuate, reaching equilibrium