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Lattice models and marginal Fermi liquids

Shamit Kachru

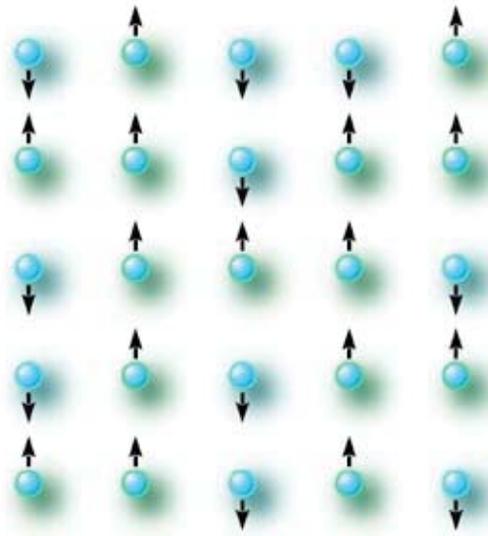
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Lattice models and Marginal Fermi Liquids

“Cold Materials, Hot Nuclei and Black Holes”

ICTP Trieste, August 2011

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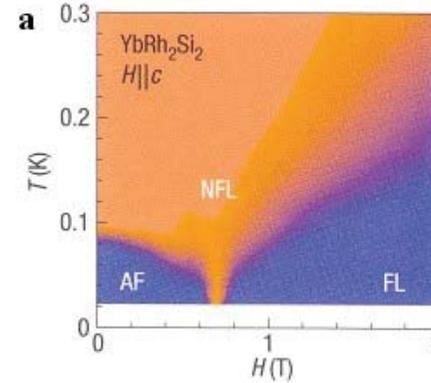
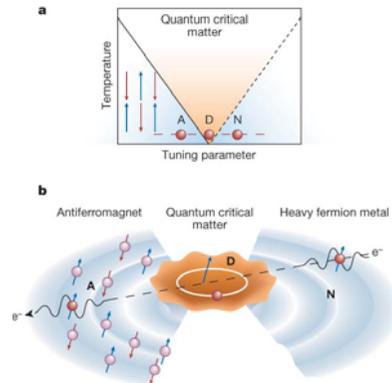
Based in part on papers written with:
Harrison, Torroba (to appear)
Jensen, Karch, Polchinski, Silverstein (arXiv:1105.1772)
Karch, Yaida (arXiv:1009.3268)
Karch, Yaida (arXiv:9909.2639)

I. Introduction

In the past four years there have been many interesting developments at the interface of holography and “condensed matter theory.” These include developments studying transport in strongly coupled theories, novel superconductors, precise measures of entanglement entropy, and many other subjects.

I will focus on just one small piece of these developments: the important role played by theories with very large dynamical critical exponent. Theories with $z = \infty$, often called “locally critical” quantum field theories, can naturally give rise to interesting non-Fermi liquids in a large N limit.

Before jumping in, let me warn you where I am heading. One of the classes of materials which exhibits linear resistivity, but is perhaps better understood or at least more experimentally accessible than the cuprates, is the heavy fermion metals:



These materials describe lattices of localized Kondo spins, interacting with bands of “itinerant” (conduction) electrons.

Much of my talk will involve describing the simplest holographic avatars of such Kondo lattice models, and studying if and how they admit locally quantum critical phases.

II. Local quantum criticality and the marginal Fermi liquid from holography

One class of behaviors that is seen in a variety of systems, most notably in the strange metallic phase of the cuprates and in the heavy fermion metals, can be conjecturally explained by “local quantum criticality.”

A locally critical theory has a scaling symmetry under which energies are rescaled, but momenta are not. In a general theory with dynamical critical exponent z , time and space scale as:

$$t \rightarrow \lambda^z t, \quad x \rightarrow \lambda x$$

* $z=1$ corresponds to a theory which can have Lorentz symmetry; typically $z=1$ theories are CFTs and can be dual to gravity in AdS space.

Maldacena;
GKPW

* Other values of z have gravity duals characterized by the so-called “Lifshitz” space-times:

$$ds^2 = -r^{2z} dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{r^2}$$

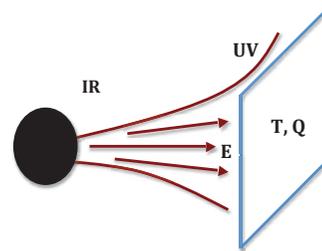
SK, Liu,
Mulligan;
c.f. Gauntlett et al
.....

* The extreme limit as $z \rightarrow \infty$ is captured by the metric:

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + dx^2 + dy^2$$

which is nothing but $AdS_2 \times R^2$.

In fact, this is the geometry that emerges in the near-horizon limit of the extremal charged black brane:



$$ds^2 \equiv g_{MN} dx^M dx^N = \frac{r^2}{R^2} (-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} \frac{dr^2}{f}$$

$$f = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \quad A_t = \mu \left(1 - \frac{r_0^{d-2}}{r^{d-2}} \right).$$

Chamblin,
Empanan,
Johnson,
Myers

In fascinating work of S-S Lee; Cubrovic, Schalm and Zaanen; and especially Liu, McGreevy and various collaborators (Faulkner, Iqbal, Vegh), it has been shown that fermion probes in such black brane geometries (anti)-holographically realize non-Fermi liquid behavior.

* The original papers uncover the behavior by studying the retarded fermion propagators in a complicated bulk geometry, using matched asymptotic expansions.

* A simpler way of thinking about the emergence of the non-Fermi liquid has been emphasized in the “semi-holographic” approach of Faulkner and Polchinski, which I’ll review now.

Consider a quantum field theory whose action takes the schematic form:

$$S = S_{\text{strong}} + \sum_{J,J'} \int dt \left[c_J^\dagger (i\delta_{J,J'} \partial_t + \mu\delta_{J,J'} + t_{J,J'}) c_{J'} \right] \\ + g \sum_J \int dt \left[c_J^\dagger \mathcal{O}_J^F + (\text{Hermitian conjugate}) \right].$$

- * There is a strongly coupled sector which we'll assume is a large N theory that we can describe using gravity.
- * There is a free (lattice) fermion with a Fermi surface.
- * We deform these two theories by coupling them with coupling constant “g”; c should couple to the lowest dimension fermionic operator in the strong sector that is permitted by symmetries.

In perturbation theory in g , we can turn on the interactions between the free fermion (with its Fermi liquid behaviour) and the strongly interacting sector. For instance, there are a set of tree graphs that renormalize the c propagator:

$$\text{---} + \text{---} \cdots \text{---} + \text{---} \cdots \text{---} \cdots \text{---} + \dots$$

Normally, we would have to include interactions coming off the strong \mathcal{O} lines. But in a large N strong sector, such interactions are **suppressed by powers of $1/N!$**

Then, the resulting “dressed” c propagator can be written purely in terms of the two-point function of \mathcal{O} in the strongly coupled sector:

$$G_g(\mathbf{k}, \omega) \sim \frac{1}{\omega - v|\mathbf{k} - \mathbf{k}_F(\mathbf{k})| - g^2\mathcal{G}(\mathbf{k}, \omega)} .$$

$$\mathcal{G}(\omega) = \int dt e^{i\omega t} \langle \mathcal{O}_J^F(t) \mathcal{O}_J^{F\dagger}(0) \rangle .$$

If we make the **strong dynamical assumption** that the strongly coupled sector exhibits local quantum criticality, then the two-point function is constrained:

$$\mathcal{G}(\omega) = c_{\Delta} \omega^{2\Delta-1}$$

The unitarity bound on the dimension is 1/2; for any value less than 1, one obtains a non-Fermi liquid, and if Delta is precisely 1, one has a marginal Fermi liquid with

$$\mathcal{G}(\omega) \sim c \omega \log(\omega)$$

Varma et al
(1989)

This is the essential physics of the MIT/Leiden model of non-Fermi liquids. This is an advance; it is one of the few ways known of quantitatively studying non-Fermi liquids in a controllable way. But it leaves a few natural questions.

Most obviously:

* The value of Δ , and hence the non-Fermi liquid behaviour, is parametrized but not explained. Can we write down full microscopic string solutions where we can predict Δ and see if/why it would be 1?

* Local criticality is a very surprising feature in a real quantum field theory, implying decoupling of spatially distinct points. Can we see real field theories where this happens? Can it be robust to finite N corrections or is it just a peculiarity of the unphysical gravity limit?

III. Lattice models vs AdS/RN black branes

To start on answering these questions, I am first going to explain a different method of obtaining AdS2 geometries in string theory. This is motivated by two sets of facts:

I. The AdS/RN black brane has various instabilities:

a) in the presence of charged scalars, one can get a holographic superconductor;

Gubser; Hartnoll,
Horowitz, Herzog

b) in the presence of neutral scalars, one can get an emergent Lifshitz geometry;

Taylor; Goldstein, SK,
Prakash, Trivedi

c) in the presence of bulk charged fermions, one can develop “Fermi sea-sickness”;

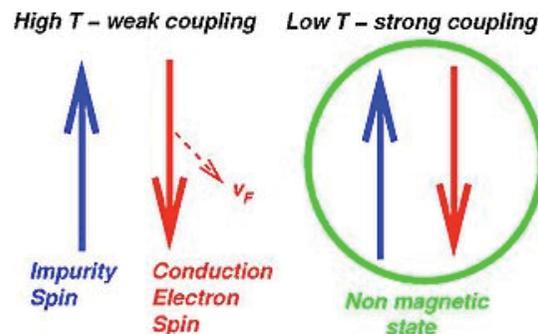
Hartnoll, Polchinski,
Silverstein, Tong

[Note however that the instabilities may be a feature and not a bug for various reasons, **as long as they occur in the deep IR.**]

II. Many of the models which exhibit non-Fermi liquid behavior, are thought to essentially be Kondo lattice models.

c.f. Sachdev (2010)

The essential physics of these materials is as follows. There is a gas of itinerant or conduction electrons, interacting with localized spins:



The dominant effects are thought to be:

- i) Kondo effect, which favors hybridization of impurities with itinerant electrons (& renormalized Fermi liquid)

- ii) RKKY interaction between Kondo spins, via the conduction electrons, favoring magnetic ordering

The resulting competition results in rich phase diagrams which exhibit Fermi and non-Fermi liquid phases.

In the remainder of this lecture, I will build the simplest analogue models in string theory. We'll find concrete gauge theory + defect systems which can realize MFL behaviour at large N ; discuss finite N corrections; and begin a discussion of Fermi/non-Fermi liquid transitions.

I'll briefly present the stringy brane constructions that are relevant, but try to constantly move back and forth to the dual field theory Lagrangians. The advantage of using full stringy constructions here, is that **we actually do know the concrete dual field theories.**

A) The first system we'll discuss, arises in M-theory. It involves a system of N M2-branes intersecting M2' branes at points; the M2' branes form a lattice in the M2 brane field theory dimensions:

	0	1	2		3	4	5	6	7	8	9	10
M2	x	x	x									
M2'	x	::	::		x	x						

In fact, to get a two-parameter family of theories, we'll consider both this setup, and its “orbifolds” by a finite group, where the M2 branes sit at a conical singularity:

$$g_k : \quad z_i = x_{2i+1} + ix_{2i+2}, \quad z_i \rightarrow e^{\frac{i2\pi}{k}} z_i, \quad i = 1 \dots 4 .$$



Following the work of ABJM, it has become clear that the M2-brane theories which arise in this way are supersymmetric Chern-Simons theories:

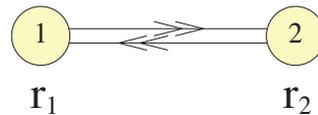
$$S = \int d^3x \frac{k}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + D_\mu \bar{\phi}_i D^\mu \phi_i + i \bar{\psi}_i \gamma^\mu D_\mu \psi_i$$

$$- \frac{16\pi^2}{k^2} (\bar{\phi}_i T_{R_i}^a \phi_i) (\bar{\phi}_j T_{R_j}^b \phi_j) (\bar{\phi}_k T_{R_k}^a T_{R_k}^b \phi_k)$$

$$- \frac{4\pi}{k} (\bar{\phi}_i T_{R_i}^a \phi_i) (\bar{\psi}_j T_{R_j}^a \psi_j) - \frac{8\pi}{k} (\bar{\psi}_i T_{R_i}^a \phi_i) (\bar{\phi}_j T_{R_j}^a \psi_j) .$$

Gaiotto, Yin;
many earlier

The gauge group is $SU(N) \times SU(N)$, and there are bifundamental matter fields connecting the two group factors, and a superpotential in the Lagrangian:



$$W = \frac{2\pi}{k} \epsilon^{ab} \epsilon^{\dot{a}\dot{b}} \text{Tr}(A_a B_{\dot{a}} A_b B_{\dot{b}}) .$$

The effect of the M2' defects intersecting the M2s, is as follows. They add localized matter multiplets at the intersections. There are charged bosons:

$$Q_1 (N, 1), \quad Q_2(1, N)$$

$$\tilde{Q}_1 (\bar{N}, 1), \quad \tilde{Q}_2(1, \bar{N})$$

as well as their Fermi partners $\chi_{1,2}, \tilde{\chi}_{1,2}$ with the same gauge quantum numbers.

The defect fields interact with the bulk A,B chiral fields via a schematic action:

$$\Delta S = \int dt \sum_i |(A_1 B_1 - A_2 B_2) Q_i|^2 + |(A_1 B_2 - A_2 B_1) Q_i|^2 + |(A_1 B_2 + A_2 B_1) Q_i|^2 \quad (6)$$

(+ fermionic terms).

We will momentarily see that in the gravity regime, this class of theories predicts marginal Fermi liquid behavior. But to remedy some of its clearest shortcomings, we also mention another class of stringy defect models.

B) Our second class of models arises in type IIB string theory. To those of you for whom this is helpful, the brane configuration is:

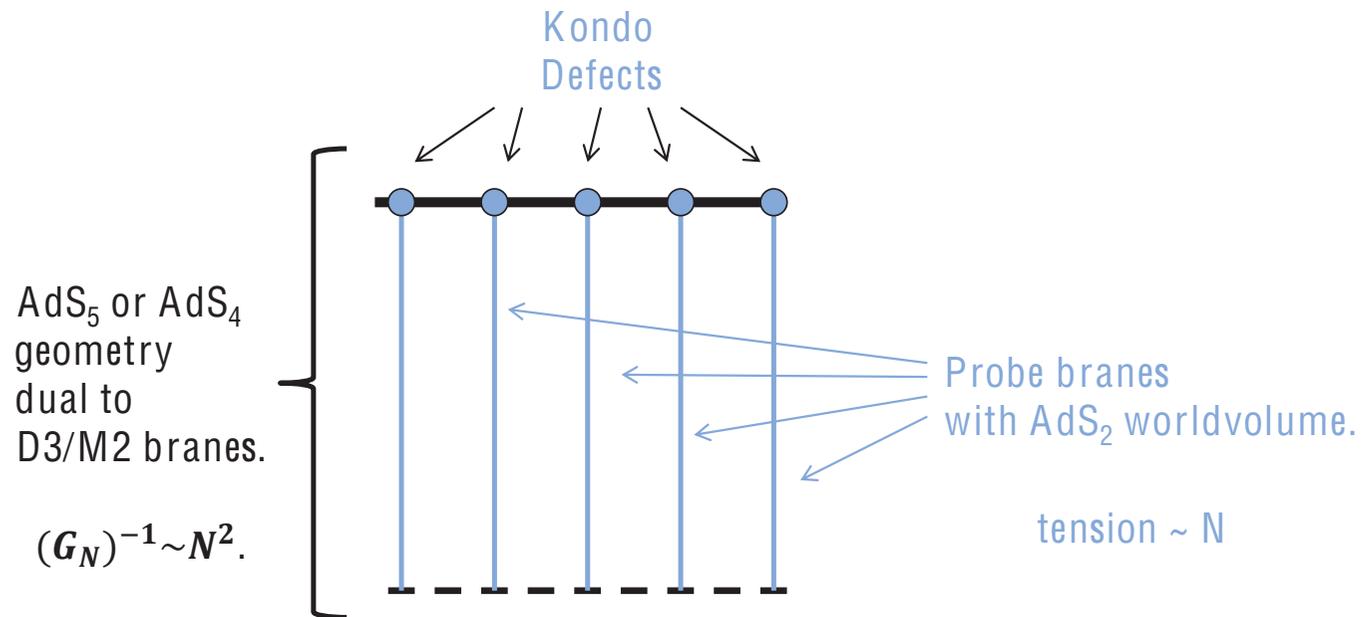
	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D5($\bar{5}$)	x	::	::	::	x	x	x	x	x	x

The N D3 branes give rise to a large N maximally supersymmetric 4D gauge theory. The D5 branes add fermionic defects localized at a lattice of points in this field theory. The action of this theory is:

$$S_{\text{field theory}} = S_{\mathcal{N}=4} + \int dt \left[i\chi_b^\dagger \partial_t \chi^b + \chi_b^\dagger \left\{ (A_0(t, \vec{0}))_c^b + v^I (\phi_I(t, \vec{0}))_c^b \right\} \chi^c \right],$$

The main drawback of construction “A” that is remedied here, is that the defects are now **purely fermionic**.

A figure summarising both of our classes of models:



Note that in such models, the AdS₂ geometries arise as the defect worldvolumes in AdS space; no RN black brane.

In these classes of models, it is very easy to answer the questions left open by the MIT/Leiden group papers:

* The value of the parameter Δ governing non-Fermi liquid behaviour, is determined by the scaling dimension of the lowest fermionic operator involving defect fields in the dual field theory (or equivalently, the mass of the lightest defect-localized fermion in the bulk geometry)

* The fate of local criticality is determined by the RG running of our field theories, or by whether or not an AdS₂ factor survives in the backreacted bulk geometry.

IV. Local criticality and marginal Fermi liquids from the M2/M2' system

Let us begin by studying my first candidate lattice model, intersecting M2 and M2' branes in M-theory.

What would we expect the dimension of the lowest fermionic operator \mathcal{O}_F , to which we could couple our semi-holographic “c” fermion, to be?

* The M2-brane bulk theory of course has fermionic operators. In the ABJM theory, the fermions in the A,B chiral supermultiplets are bi-fundamentals. Naively they would have dimension $\Delta = 1$, but they aren't gauge invariant. A bulk gauge invariant, of the form:

$$\mathcal{O} = \text{tr}(A\psi_B)$$

is gauge invariant, but would naively have $\Delta(\mathcal{O}) = \frac{3}{2}$. This is of **too high a dimension** to produce a non-Fermi liquid by coupling to the semi-holographic “c” fermion.

* So, it is more promising to look for composite gauge-invariant fermionic operators combining defect and itinerant fields. In fact, operators of the form:

$$\mathcal{O}_{\text{defect}} = \chi_1 \psi_A \tilde{\chi}_2$$

(which form a four-plet after considering all such combinations of defect and bulk fermions) would naively have $\Delta = 1$ and be perfect candidates for producing a marginal Fermi liquid, after coupling to “c” fermions.

At strong coupling, one can see that this logic is precisely borne out. The masses of defect KK modes are related to scaling dimensions in the field theory via:

$$m_{\text{localized}}^2 = \Delta(\Delta - 1) .$$

So to compute the spectrum of scalar operators arising from one relevant tower of M2' brane excitations, one can check brane fluctuations in the transverse compact dimensions of the geometry. E.g. consider $k=1$:

The M2' brane wraps an $AdS_2 \times S^1$ and can fluctuate in six transverse directions related by an $SO(6)$ symmetry.

If we let r denote the AdS2 radial direction we can perform a KK reduction on the circle to find the scalar modes which live in AdS2:

$$\delta x^I(r, \phi) = \sum_l \delta x^{I,l}(r) e^{il\phi}$$

The resulting Laplace equation shows a mass spectrum

$$m_l^2 = -\frac{1}{4} + \frac{l^2}{4}$$

resulting in a tower of dual scalar operators with

$$\Delta_l = \frac{1}{2} + \frac{l}{2}.$$

The lowest bosonic operator in this tower has dimension $1/2$; its fermionic superpartners have dimension 1 and the right properties to be the operators we identified using weakly coupled intuition.

So we've succeeded in the goal of giving examples of microscopic marginal Fermi liquids, **in the leading large N approximation.**

(I have glossed over the fact that because of the bosonic defect modes, we actually have to **use global symmetries** to guarantee that “ c ” couples to these fermionic operators; there is a lower dimension fermionic operator with different symmetry properties in another KK tower. This would not happen in the D2-D6 lattice system, presumably).

V. Comments on finite N effects

We used large N in two crucial ways in the semi-holographic story; to see that there is a locally critical fixed point (i.e. AdS₂ geometries of the defects), and to compute the “ c ” correlation functions after dressing them with interactions with the large N sector.

What would change at finite values of N ?

* In all of these kinds of lattice systems, if the lattice spacing is “ L ”, one expects the free energy per unit area to take a schematic form:

$$\mathcal{F} = N^a T^3 + N^b \frac{T}{L^2}$$

(with $a=2$, $b=1$ in a standard gauge theory with the kind of field content we wrote down).

So one should expect backreaction of the lattice to become important at an energy scale that goes like some inverse power of N ($N^{-1/2}$ in this case).

Equivalently, as one scales to the IR, one includes more and more lattice points, till backreaction becomes relevant.

Here I just make some elementary remarks about what this backreaction does.

The most basic question is: does local quantum criticality survive? In the gravity regime, this becomes the question: is there an exact solution including the lattice of M2' branes and an AdS₂ factor in the infrared?

Let's think about this loosely, using an energetics argument. We are looking for a stable solution of the form:

$$AdS_2 \times T^2 \times X$$

(where we compactified the field theory spatial dimensions for convenience). Call the radii of the three factors in the geometry A, T and S. The effective action for these radions reduced to 1+1 dimensions is of rough form:

$$\mathcal{S} = \int d^2x \left(-T^2 S^7 + A^2 T^2 S^5 - N_2' A^2 S - \frac{N_2^2 A^2 T^2}{S^7} \right). \quad (17)$$

The four terms come from the AdS and internal curvatures; the M2' brane tensions; and the 7-form flux from the M2 branes. We have smeared the M2' branes, averaging their energy over the internal directions.

There is an extremum of this schematic action, with:

$$A \sim S \sim N_2^{1/6}, \quad T \sim N_2'^{1/2} / N_2^{1/3}.$$

Physically, what's happening is that the M2' branes provide a force opposing the contraction of the "T2" directions (which would contract in the AdS4 solution), helping to drive the system to a fixed point with local criticality.

Now, this is correct logic, but in our supersymmetric microscopic system, we did **NOT** smear the M2' branes over the internal dimensions, but rather wrapped them on a preferred circle. To more accurately think about energetics, we should include a radion for the circle, thinking of the sphere as a Hopf fibration.

Including this one additional mode, one finds no extremum; whatever backreaction does, there is no AdS2 solution with radii in the regime where gravity is reliable.

Brane models better represented by the theory with an extremum can be constructed but are non-supersymmetric. One would have to study them in much more detail to be sure there aren't other instabilities.

One can also study the backreaction directly in the field theory. The basic issue is that defects will “talk to each other” via bulk exchange; integrating out bulk modes would be expected to induce gradients in the lattice action, destroying local criticality.

For the special case of fermionic defects coupled via Chern-Simons gauge fields, things aren't as grim as they could be. E.g. consider the theory with action

$$\int dt d^2z [A_0(\partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z) - A_z(\partial_0 A_{\bar{z}} - \partial_{\bar{z}} A_0) + A_{\bar{z}}(\partial_0 A_z - \partial_z A_0)] + \sum_n \delta^{(2)}(z - z_n) \chi_n^\dagger A_0 \chi_n$$

Integrating out the gauge fields at tree level does not induce any interactions between the defect fermions. Similarly the bulk A,B fields of the ABJM model couple to the defects quadratically, and do not induce couplings at tree-level. Especially in the $k=1$ case where there is a lot of SUSY, corrections may be very highly constrained.

Speaking more generally, there **IS** a good reason to think that local criticality will **never be absolutely stable down to zero temperature**. The density of states in a locally critical theory takes the form:

$$\rho(E) = A\delta(E) + B/E$$

* The delta function term would be present even in a trivial defect theory, and reflects the $T=0$ ground state degeneracy.

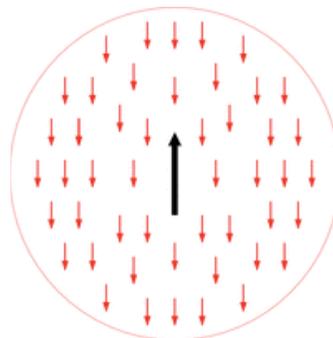
* The second term would contribute a divergence in the number of states as one approaches $E=0$; this must always be cut off in an exact treatment.

Happily, the competition between the constant entropy and the log divergence doesn't kick in until **exponentially low energy scales**; this in principle allows for the existence of natural models which work down to very low energies.

Thus, although backreaction becomes important in our lattice models at some scale which is power law in $1/N$, in principle one should be able to construct models which remain critical after including backreaction down to much lower scales.

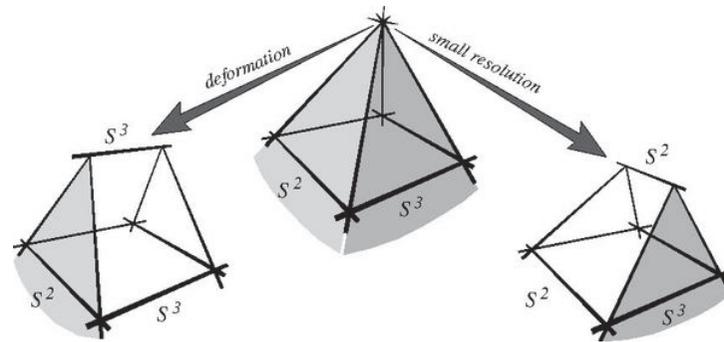
* Final note on backreaction in class B (D3/D5 case) :

In the case of M stacks of $k(i)$ D5s wrapping different polar angles in the 5-sphere all at one point in the field theory spatial dimensions, one can solve geometrically for the backreacted space-time.



Harrison, SK, Torroba to appear;
D'Hoker, Estes, Gutperle

The theory undergoes a geometric transition: each D5 stack “squishes” a four-sphere in the five-sphere, resulting in a space-time with $M+1$ five-spheres and M three-spheres.



The final solution is an $AdS_2 \times S^2 \times S^4$ fibration over a Riemann surface of genus g . It mirrors the expected Kondo flow in the dual field theory, and exhibits a geometrization of the Affleck-Ludwig “ g -function.”

Take away lesson: backreaction becomes important in the lattice models at some scale which is power law in $1/N$, but in principle one should be able to construct models that remain critical down to exponentially lower scales.

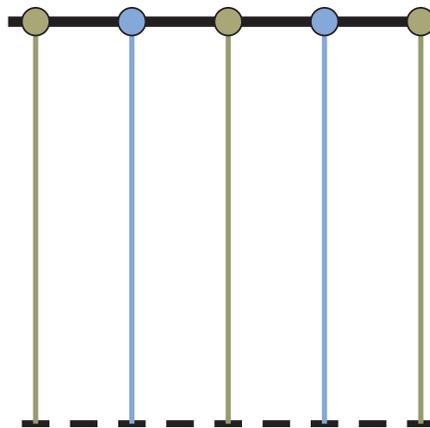
VI. Models with purely fermionic defects and FL/NFL transitions

Now, let's move on to illustrate something new in the class of models "B," coming from type IIB string theory, with purely fermionic defects. Recall in the simplest microscopic setup there:

$$S_{\text{field theory}} = S_{\mathcal{N}=4} + \int dt \left[i\chi_b^\dagger \partial_t \chi^b + \chi_b^\dagger \left\{ (A_0(t, \vec{0}))_c^b + v^I (\phi_I(t, \vec{0}))_c^b \right\} \chi^c \right],$$

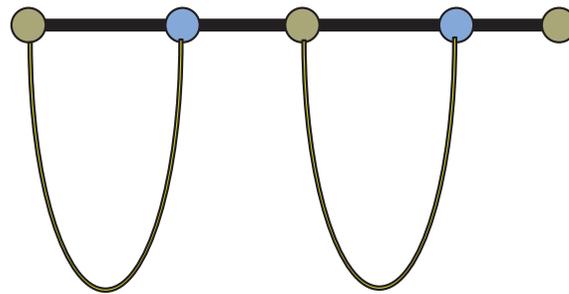
In the microscopic string theory, the defect fermions χ (in the fundamental of $SU(N)$) come from D5-brane defects stretching down AdS_2 slices of AdS_5 , and wrapping a four-sphere in the “extra” five-sphere.

We could instead consider a bipartite lattice of D5s and anti-D5s, giving fundamentals and anti-fundamentals of $SU(N)$ at neighboring lattice sites:



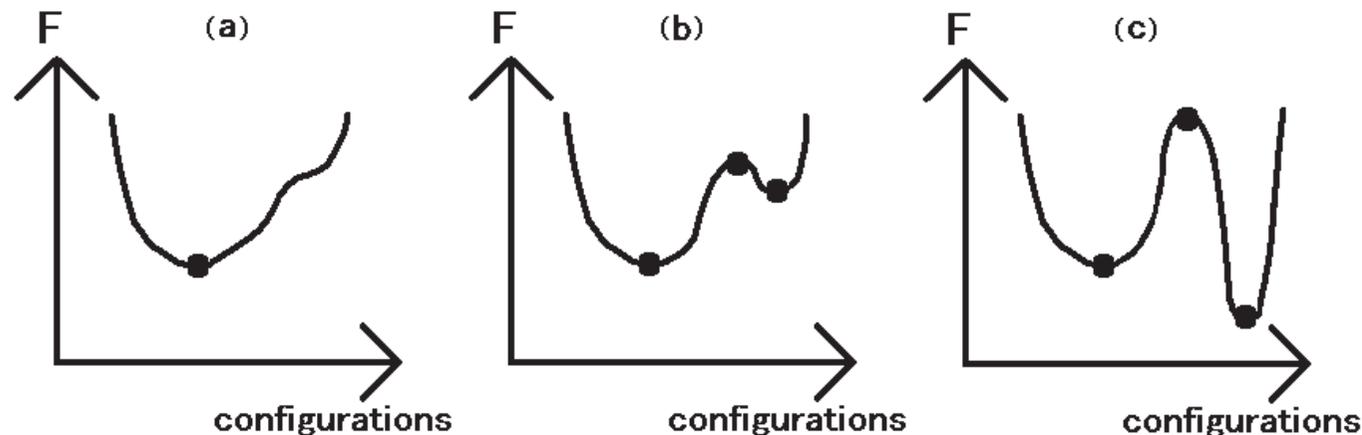
Now, the free-energy of a neighboring D5/anti-D5 pair both stretching straight down to the horizon, is clearly just twice the free-energy of a single D5 (at leading order in large N).

However, supersymmetry is broken in this background; and we expect integrating out the bulk SYM fields to introduce interactions between the neighboring spins. A new kinematically possible configuration:



may then become thermodynamically favorable, depending on the temperature.

Let us call the separation between the brane/anti-brane pair Δx . Then a simple computation reveals that for large Δx (relative to the scale set by T , i.e. the location of the horizon) the “straight down” configuration is preferred. For smaller Δx , two reconnected configurations appear - one a local maximum of F , and one the true minimum:



The phase transition occurs (numerically) at:

$$\frac{r_+ \Delta x}{L^2} \approx 0.7, \quad \text{where}$$

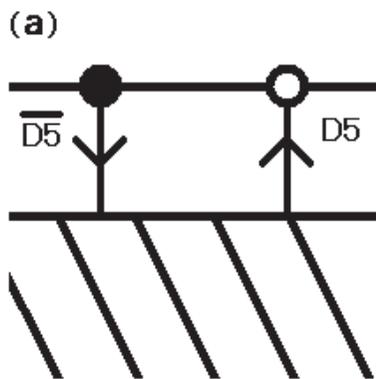
$$\frac{r_+}{L^2} = \pi T \quad \rightarrow \quad T_c \sim \frac{2}{\Delta x}.$$

The dimerized phase dominates at low temperature, when intuitively, the branes can save tension energy by reconnecting rather than stretching all the way to the distant horizon.

(We note that we could modify this to a $T=0$ phase transition at finite charge density, by instead varying the chemical potential in the D3 field theory. At large charge, the horizon also moves out towards the boundary.)

Now, let us consider the behaviour of two-point functions for a semi-holographic fermion, coupled to the large N CFT (with defects) that we've just been discussing; i.e., we are replacing the M2/M2' system from the first 2/3 of this talk, with the D3/D5 system. Let us call the lowest-dimension fermionic operator arising on the defects, \mathcal{O}_J^F . Consider its behaviour in the two phases:

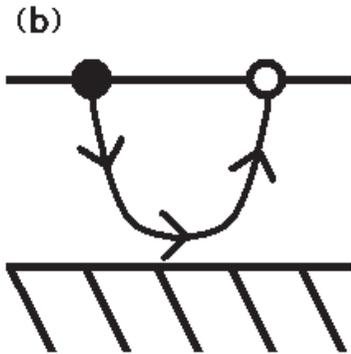
- a) In the phase: The operator lives on an AdS_2 slice of the bulk geometry. The two-point functions are constrained to behave as:



$$\int dt e^{i\omega t} \langle \mathcal{O}_J^F(t) \mathcal{O}_{J'}^{F\dagger}(0) \rangle = i\delta_{J,J'} \mathcal{G}(\omega),$$

$$\mathcal{G}(\omega) \sim \omega^{2\Delta-1}$$

b) In the phase: The submanifold wrapped by the probes deviates from AdS_2 in the IR. The resulting two-point functions are governed by a **gap**:



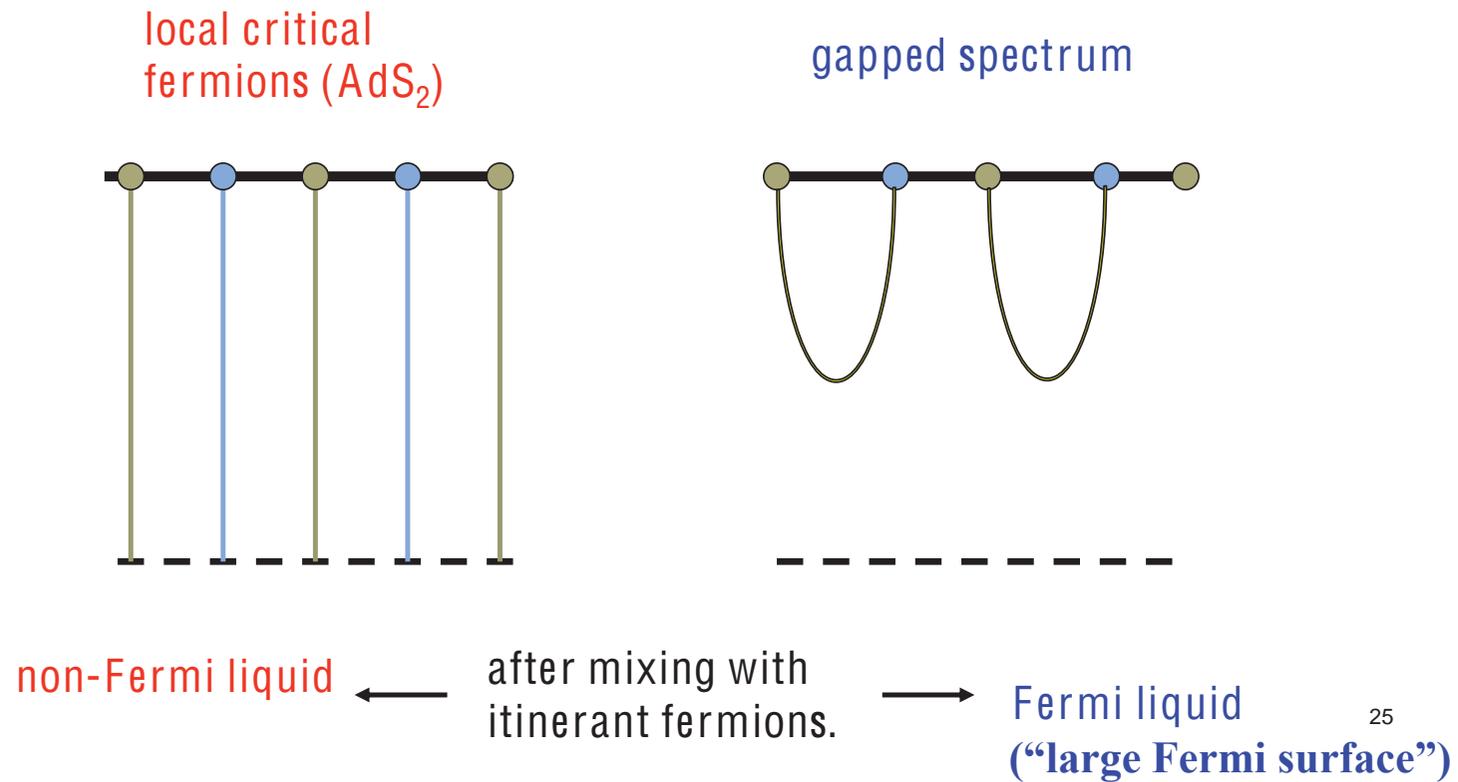
$$\lim_{\omega \rightarrow 0} \int dt e^{i\omega t} \langle \mathcal{O}_J^F(t) \mathcal{O}_{J'}^{F\dagger}(0) \rangle = iA_{J,J'}.$$

Consequently, the behaviour of the dressed c-fermion is dramatically different in the a) and b) phases of our holographic Kondo lattice model:

a) phase can generically yield NFL behaviour.

b) phase generically yields Fermi liquid with expanded Fermi surface (relative to free c fermion).

Or in pictures, once again: Fermi surface reorganization:



Several important notes and caveats:

i. In the model I described in detail, the transition occurs as a function of T . The NFL behaviour only occurs for frequency large compared to T , which would mean we would need to tune parameters so the transition temperature is low compared to the Fermi momentum. We can do this in the model, but it is annoying.

ii. In the literal model coming out of the D3/D5 system, as opposed to D5 probes of other known AdS/CFT dual pairs, the lowest dimension attainable for a fermionic operator on the defect is

$$\Delta(\mathcal{O}) = \Delta(\chi_a^\dagger \lambda^{ab} \chi_b) = \frac{3}{2}$$

This would give non-Fermi liquid behaviour of the decay width, but not a vanishing quasiparticle residue. To get vanishing quasiparticle residue from the “top down,” one would have to study D5 embeddings in more general AdS solutions.

iii. A **basic limitation** of this entire approach is the need for a large N bath which strongly renormalizes the properties of the UV Fermi liquid to make an IR non-Fermi liquid. I am not sure how to circumvent this limitation with **ANY** of the approaches currently under discussion using string/gravity techniques.

i and ii are not basic problems (indeed ii is circumvented in the M2/M2' model I described); iii seems to me the point which requires serious thought.

Summary (my view of holographic approach to NFLs):

- * One of very few controlled approaches to construct novel non-Fermi liquid phases which are seen increasingly cleanly in experiment; can we beat them to seeing some striking new phenomena, or at least give theoretically controlled models of conjectured novel phases?
- * Large N “hidden sector” that renormalises the electrons is the rather embarrassing crutch in use. Questions of finite N effects need to come to the fore if this subject is to advance further.
- * More parochially: can we find a soluble Kondo lattice model? This would be of striking interest to a good fraction of the field-theoretic CM community.