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Holographic matter: deconfined string at criticality

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Holographic Matter : Deconfined String at Criticality

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Strongly coupled QFT is hard, but

- There are theories that have weak coupling descriptions in terms of dual variables
  - Original `particles' remain strongly coupled and have short life time, yet they are organized into long-lived (weakly coupled) collective excitations
  - Duality provides new windows into strong coupling physics
  - Dual variable may carry new (sometimes fractional) quantum numbers : fractionalization
  - Dual variable may live in different space : holography

Plan

- Fractionalization
  - Interacting boson model
  - Gauge theory with compact one-form gauge field
  - Quantum order in fractionalized phase
- Holography
  - Gauged matrix model in D-dimensions
  - Closed string field theory with compact two-form gauge field in (D+1)-dimensions
  - Quantum order in holographic phase

A model for fractionalization

[Anderson]

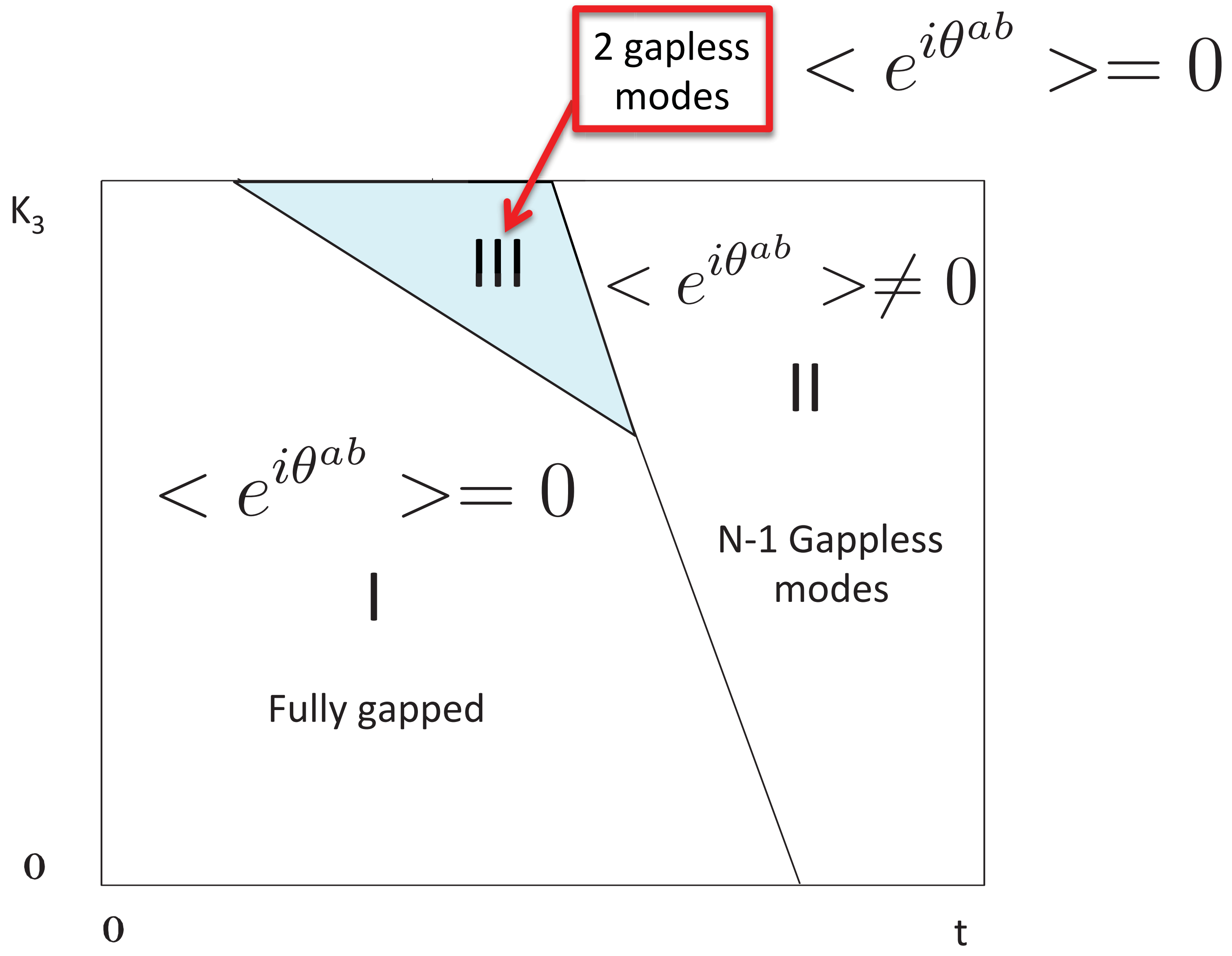
$$S = -t \sum_{<i,j>} \sum_{a,b} \cos\left(\theta_i^{ab} - \theta_j^{ab}\right)$$

$$- K_3 \sum_{i} \sum_{a,b,c} \cos\left(\theta_i^{ab} + \theta_i^{bc} + \theta_i^{ca}\right)$$

[SL and Patrick Lee (05)]

- i, j : lattice sites in 4D lattice (3 space + discrete time)
- θ_i^{ab} : boson with flavor a and anti-flavor b (a,b=1,2,…,N)
  with constraints, $\theta_i^{ab} + \theta_i^{ba} = 0$
- $U(1)^{(N-1)}$ global symmetry : $\theta_i^{ab} \rightarrow \theta_i^{ab} + \phi^a - \phi^b$

Phase diagram (for large N)



Quantum order [Wen]

- `Order' in the pattern of long range entanglement
- Provide `explanation' for why there exist gapless modes whose robustness is not from any microscopic symmetry
- Can be used to classify phases of matter beyond the symmetry breaking scheme
  - In particular, phases with different quantum order form different universality classes
- Associated with the suppression of topological defects

$$Z = \sum e^{-S}$$

$$\rightarrow \quad Z = \sum e^{-S}$$

Gauge theory

$$S = -t \sum_{<i,j>} \sum_{a,b} \cos\left(\theta_i^{ab} - \theta_j^{ab}\right)$$

$$- K_3 \sum_{i} \sum_{a,b,c} \cos\left(\theta_i^{ab} + \theta_i^{bc} + \theta_i^{ca}\right)$$

Strong coupling (K3 >> 1) : Dynamical constraint

$$\theta^{ab} \qquad (S^1)^{N(N-1)/2}$$

$$\theta^{ab} + \theta^{bc} + \theta^{ca} = 0 \quad \Downarrow$$

$$\theta^{ab} = \phi^a - \phi^b \qquad \frac{(S^1)^N}{S^1}$$

Parton fields

U(1) redundancy

Gauge theory (cont'd)

$$S = -t \sum_{<i,j>} \left[\sum_a e^{i(\phi_i^a - \phi_j^a)} \right] \left[\sum_b e^{-i(\phi_i^b - \phi_j^b)} \right] + c.c$$

No bare hopping : slave-particles can hop only through mutual hoppings

⇩

$$S = t \sum_{a<b} \sum_{<i,j>} \left[|\eta|^2 - \eta e^{-i(\phi_i^b - \phi_j^b)} - \eta^* e^{i(\phi_i^a - \phi_j^a)} \right],$$

$\eta = |\eta| e^{i a_{ij}}$: complex auxiliary field

- Compact U(1) gauge theory coupled with N bosons
- a_{ij} : gauge field associated with local symmetry
- Bare gauge coupling is infinite (auxiliary field)

Fractionalization

$$S = \int dx \left[|(\partial_\mu - a_\mu)\Phi_a|^2 + V(\Phi_a) + \frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} \right]$$

$$\Phi_a = e^{i\phi^a}$$

- Quantum fluctuations generates the Maxwell's term
- For a large N, $g^2 \sim 1/N$
- Compactness of gauge field ➔topological defect (monopole)
- Monopole mass ~ g^2 ~ N : Deconfinement phase
- Non-trivial quantum order : dF=0
- Low energy modes in the Coulomb phase :
  fractionalized particles, gapless gauge boson
- Although fractionalized particles are not gauge invariant objects, they become `classical' in the large N limit
- Emergence of internal space

Gauge-string duality

[Maldacena; Gubser, Klebanov, Polyakov; Witten]

$$
\begin{aligned}
Z[J(x)] &= \int D\phi(x) e^{-S_{field\ theory}[\phi]} && \text{D-dimensional gauge theory} \\
&= \int D \text{ ``}J(x,z)\text{''}\, e^{-S'[J(x,z)]}\Big|_{J(x,0)=J(x)} && \text{(D+1)-dimension string theory}
\end{aligned}
$$

- Best understood in the maximally supersymmetric gauge theory in 4D
  - Weak coupling description for strongly coupled QFT
  - Non-perturbative definition of string theory (quantum gravity)
- Believed to be a general framework for a large class of QFT's

[Das, Jevicki; Gopakumar; Heemskerk, Penedones, Polchinski; Lee; Faulkner, Liu, Rangamani; Douglas, Mazzucato, Razamat]

Q. Do those states that admit holographic description possess non-trivial quantum orders ?

A. **Yes.** A non-trivial quantum order in holographic phases of matter is responsible for

1) emergent external space with an extra dimension,
2) deconfined string in the bulk and
3) an operator with a protected scaling dimension

Holographic states form distinct universality classes !

Gauged Matrix Model

Gauged matrix model

$$S[U] = NM^2 \sum_{<i,j>} \mathrm{tr}(U_{ij}^\dagger U_{ij}) + N^2 V[W_C/N]$$

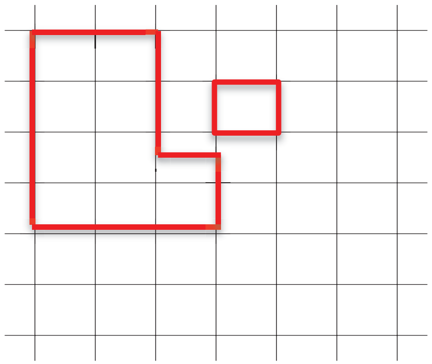
$$V = -\sum_{n=1}^{\infty} N^{-n} \sum_{\{C_1,..,C_n\}} J_{\{C_1,..,C_n\}} \prod_{k=1}^{n} W_{C_k}$$

U_{ij} : N × N complex matrices

U(N) gauge symmetry : $U_{ij} \to V_i^\dagger U_{ij} V_j$

$$W_C = \mathrm{tr} \prod_{<i,j>\in C} U_{ij}$$

: Wilson loop



D-dimensional Euclidean lattice

$\mathcal{J}_C$: Sources for single-trace operators

$\mathcal{J}_{C_1,C_2}$: Sources for double-trace operators

⋮

$\mathcal{J}_{\{C_1,..,C_n\}}$: Sources for general multi-trace operators

Gauged matrix model

$$S[U] = NM^2 \sum_{<i,j>} \mathrm{tr}(U_{ij}^{\dagger} U_{ij}) + N^2 V[W_C/N]$$

$$V = -\sum_{n=1}^{\infty} N^{-n} \sum_{\{C_1,..,C_n\}} J_{\{C_1,..,C_n\}} \prod_{k=1}^{n} W_{C_k}$$

- A ``linear sigma model" for gauge theory :

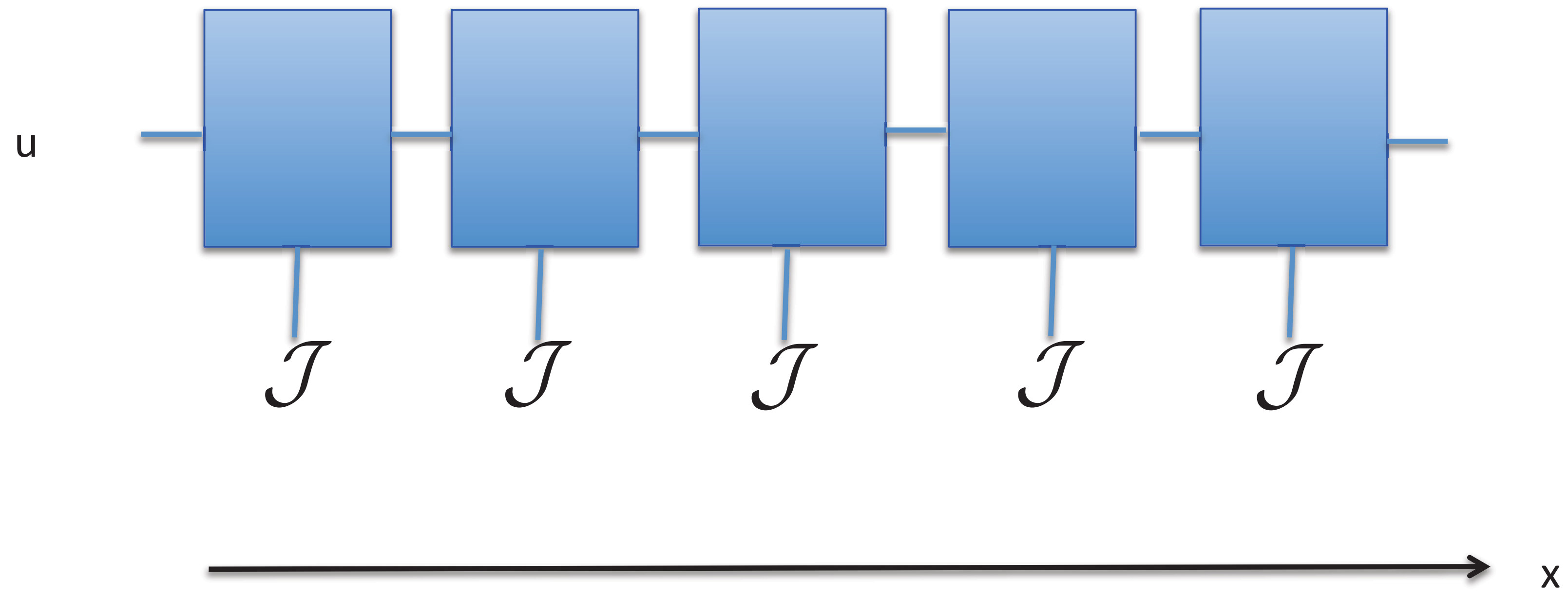
$$N^2 V = \sum_{<i,j>} \left[-NM_0^2 \ \mathrm{tr}(U_{ij}^{\dagger} U_{ij}) + Nv \ \mathrm{tr}(U_{ij}^{\dagger} U_{ij} U_{ij}^{\dagger} U_{ij}) + v' \ \left\{ \mathrm{tr}(U_{ij}^{\dagger} U_{ij}) \right\}^2 \right] - NJ \sum_{\Box} W_{\Box}$$

  - $M_0 < M$: gapped phase
  - $M_0 > M$: low energy manifold is spanned by unitary matrices
    ➔ U(N) gauge theory

General Construction of Holographic Dual

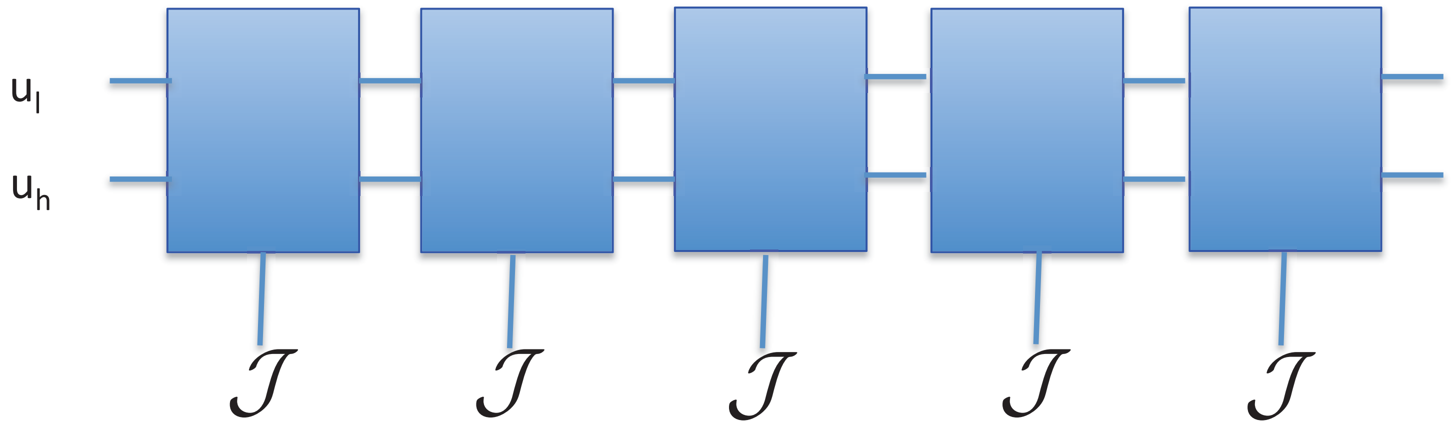
Construction of holographic theory

Partition function can be viewed as contractions of an D-dimensional array of tensors which depend on external sources



Construction of holographic theory

Integration on each bond can be done in infinitesimal steps, rather than doing it once



$$I = \int du \quad e^{-(M^2u^2+Ju^4)}$$

$$= \int du d\tilde{u} \quad e^{-(M^2u^2+Ju^4)-\mu^2\tilde{u}^2}$$

$$= \int du_l du_h \quad e^{-\left(M_l^2u_l^2+M_h^2u_h^2+J(u_l+u_h)^4\right)}$$

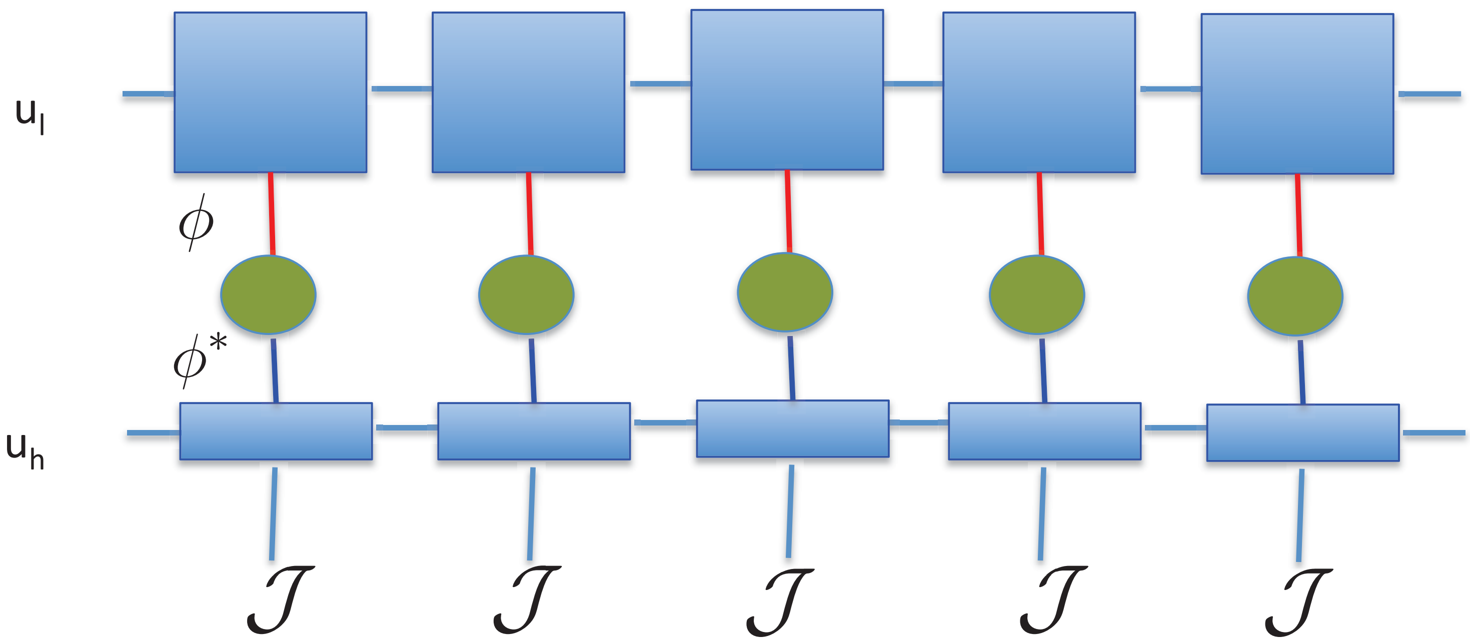
$$u = u_l + u_h$$

$$\tilde{u} = Au_l + Bu_h$$

$$M_l^2 = M^2 e^{2\alpha dz}, \qquad M_h = \frac{M^2}{2\alpha dz}$$

Construction of holographic theory

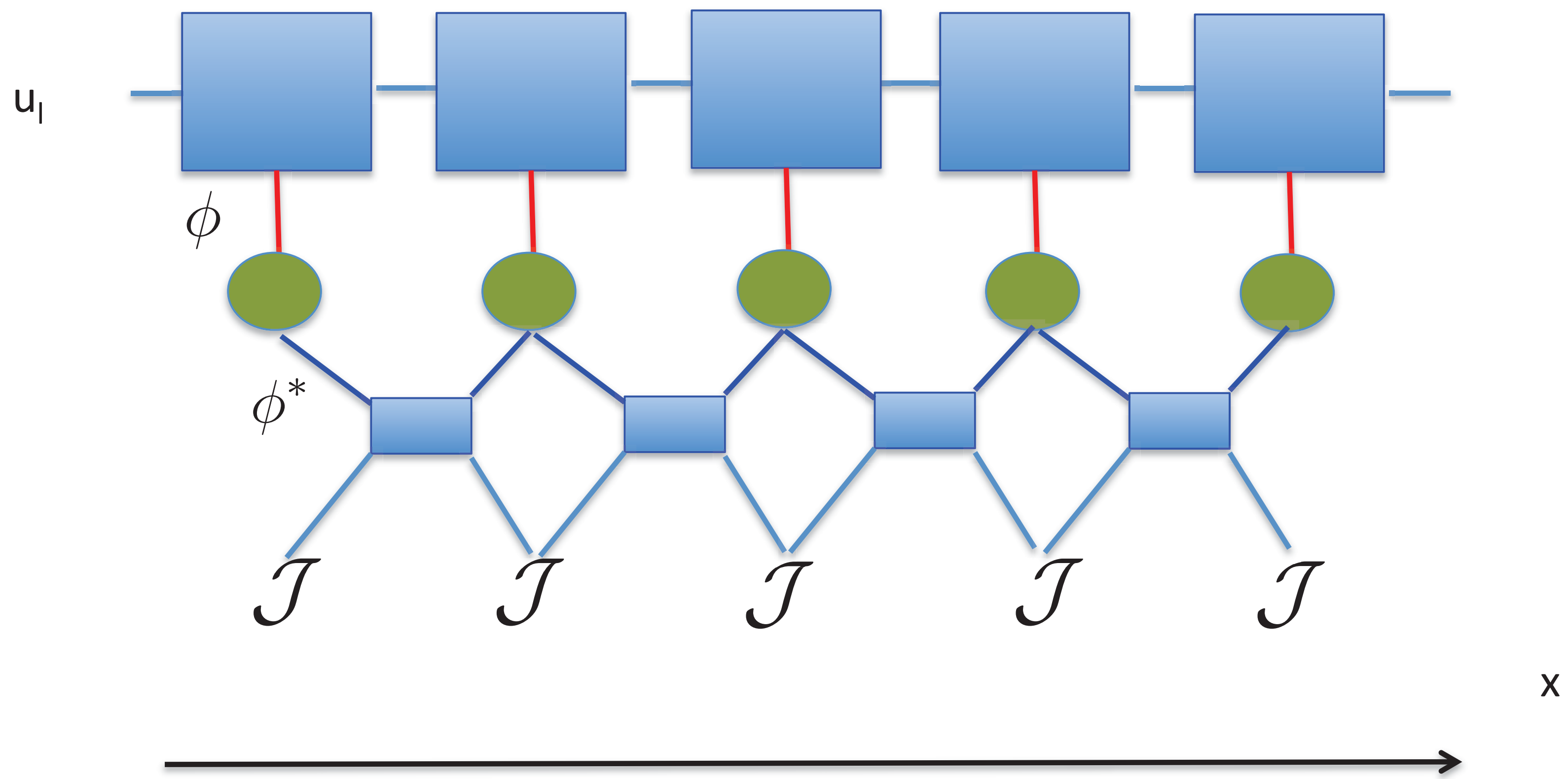
- High energy fields can be viewed as fluctuating sources for the low energy fields



ϕ : an auxiliary field that plays the role of fluctuating source for low energy field
ϕ^* : a Lagrangian multiplier than impose the constraint between ϕ and J

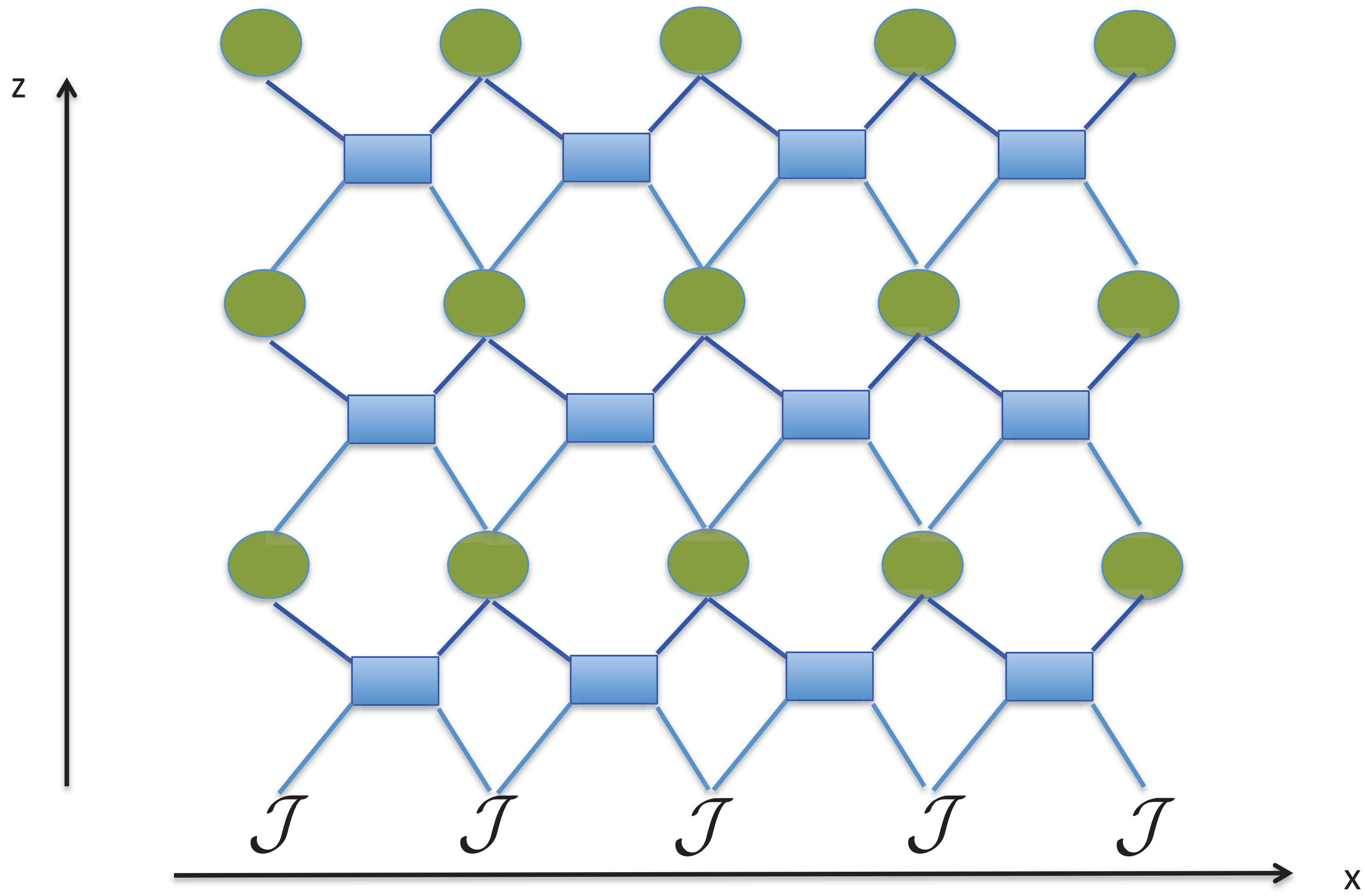
Construction of holographic theory

Integrating out high energy modes generate dynamical action for the source and its conjugate field

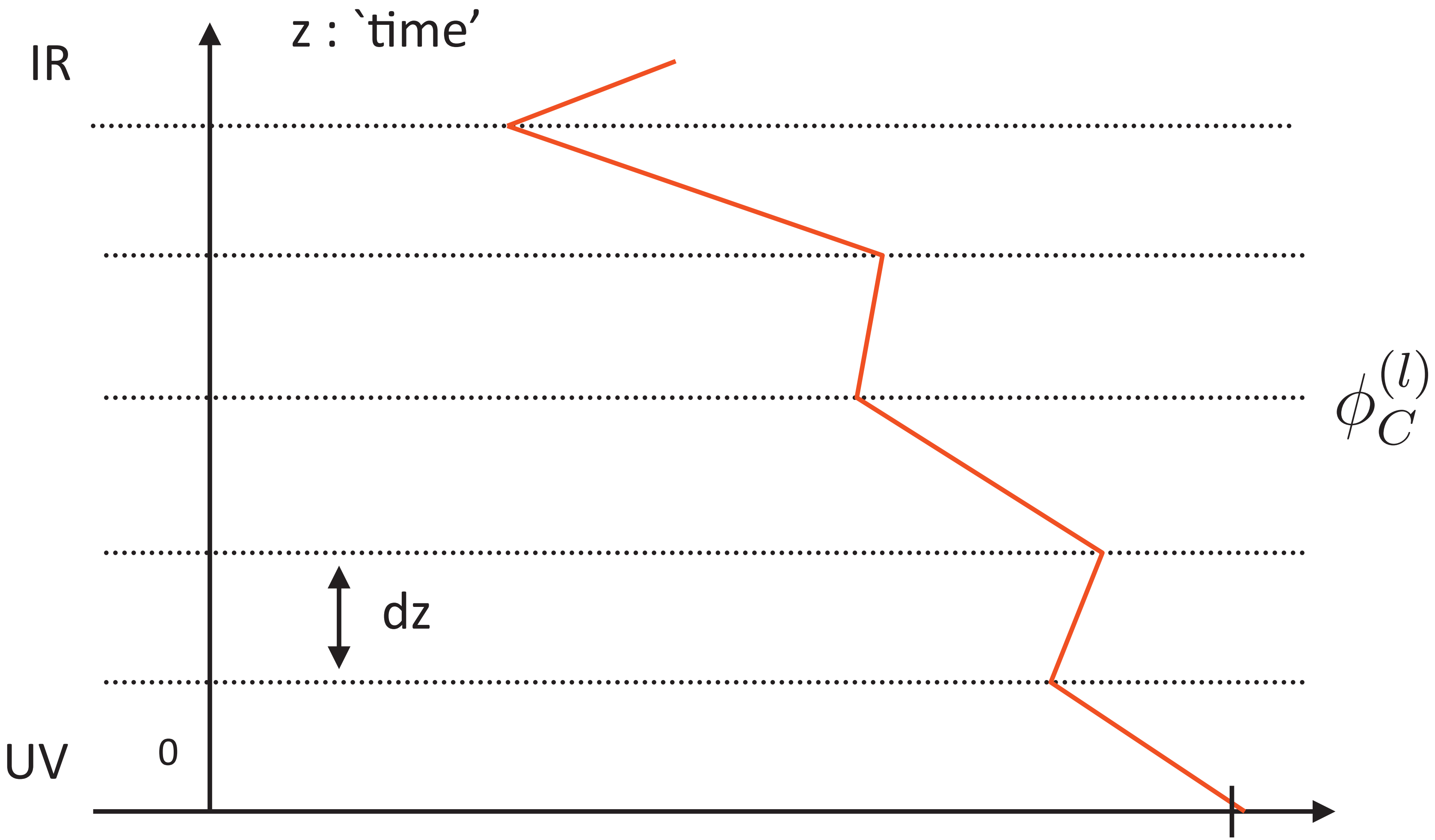


Construction of holographic theory

Repetition of these step leads to contractions of (D+1)-dimensional array of matrices for the partition function



Extra dimension as a length scale



Key features

- An exact change of variable
- D-dimensional partition function can be written as (D+1)-dimensional partition for dynamical source fields and their conjugate fields (vev's)
- For matrix model, the sources are defined in the space of loops : field theory of loops

(D+1)-dimensional field theory of closed loops

$$Z = \int D\phi_C D\phi_C^* \;\; e^{-\left(S_{bulk}[\phi_C^*(z),\phi_C(z)] + N^2\phi_C^*(0)\phi_C(0) + N^2 V[\phi_C^*(0)] + V'[\phi_C(\infty)]\right)}$$

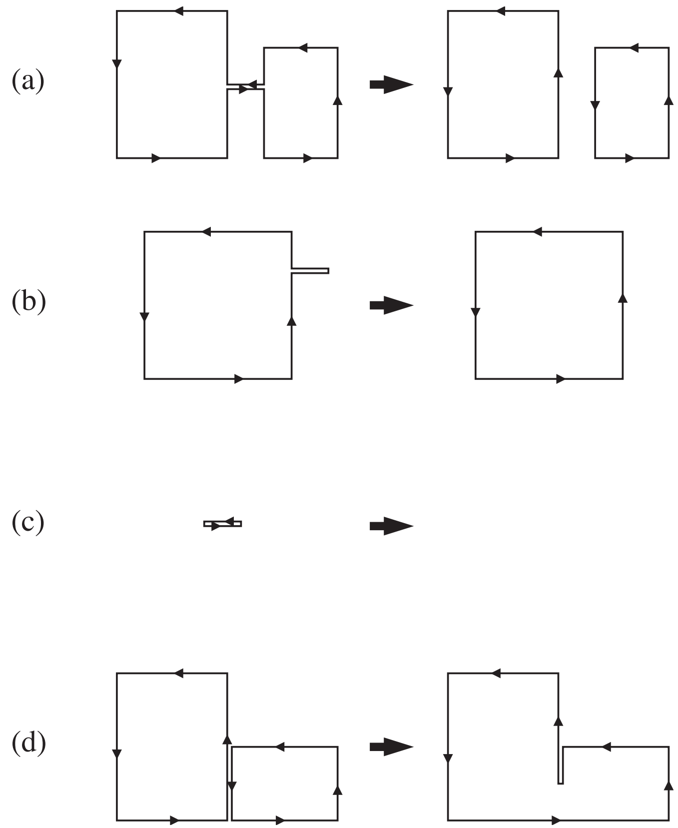
$$S_{bulk} = N^2 \int_0^\infty dz \Big[\phi_C^* \partial_z \phi_C + \alpha L_C \phi_C^* \phi_C$$
$$- \frac{\alpha}{M^2}\Big(F_{ij}[C_1, C_2]\phi_{C_1}^* \phi_{C_2}^* \phi_{[C_1+C_2]_{ij}} + G_{ij}[C_1, C_2]\phi_{(C_1+C_2)_{ij}}^* \phi_{C_1}\phi_{C_2}\Big)\Big]$$

- V : J_C dependent action for the UV(z=0) boundary fields
- V' : universal action for the IR(z=∞) boundary fields
- S_{bulk} : action for closed loop fields in (D+1)-dimensions
- $\phi_C(z), \phi_C^*(z)$: coherent fields for annihilation/ creation operators of loop

Loop Hamiltonian in the bulk

$$Z = \lim_{\beta \to \infty} < \Psi_f | e^{-\beta H} | \Psi_i >$$

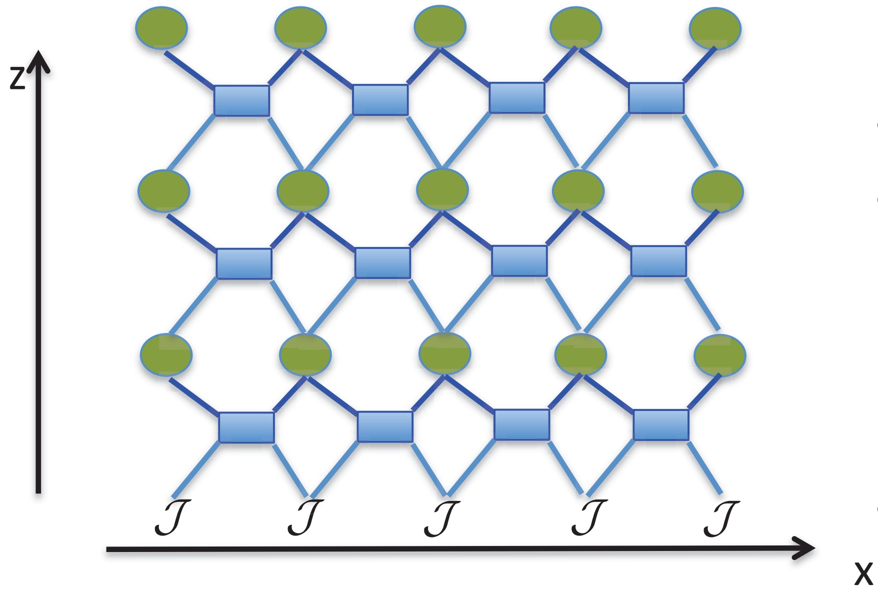
$$H = \alpha L_C a_C^\dagger a_C - \frac{\alpha}{NM^2}\left(F_{ij}[C_1, C_2] a_{C_1}^\dagger a_{C_2}^\dagger a_{[C_1+C_2]_{ij}} + G_{ij}[C_1, C_2] a_{(C_1+C_2)_{ij}}^\dagger a_{C_1} a_{C_2} \right)$$

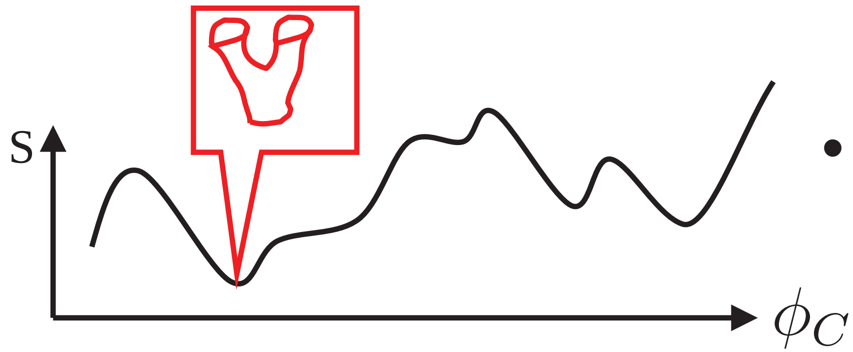


– Tension

– Joining/splitting

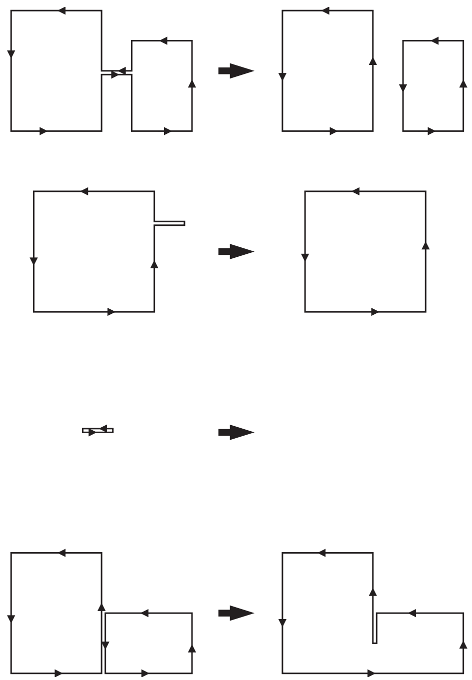
Saddle point and beyond





- S ~ N² (…)
- Fluctuations of loop fields around a saddle point describe weakly interacting closed strings in (D+1)-dimensional space for a large N
- The background (metric and the two-form gauge field) for closed strings are determined by the saddle point solution
- Key question : When is the saddle point solution stable ?

Gauge symmetry

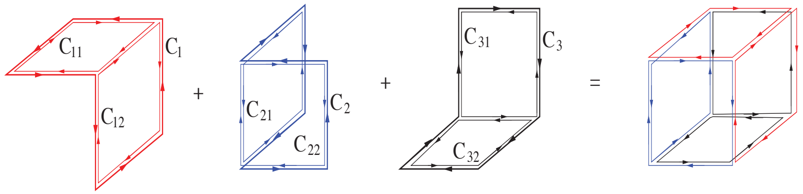


- No-quadratic hopping : flux conservation
- The cubic interactions between loops generate the kinetic term for strings

$$-\frac{\alpha < \phi_C >}{M^2} a^{\dagger}_{C+C'} a_{C'}$$

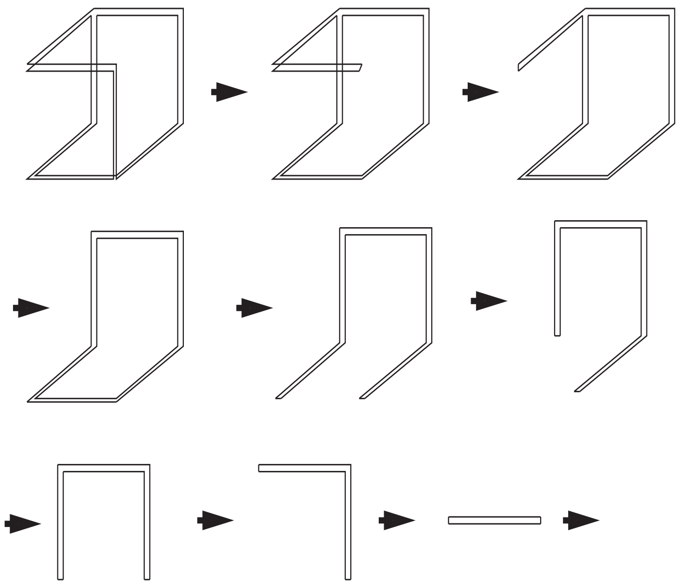
- The phase of the background loop field provides a Berry phase for strings that moves in space $\phi_C = |\phi_C| e^{ib_C}$
- $b_C = \int_{A_C} B$ ← compact two-form gauge field $b_C \sim b_C + 2\pi$

Quantum fluctuations generate kinetic energy for the two-form gauge field



(a)

- Integrating out heavy (long) loops generate the kinetic energy for the two-form gauge field

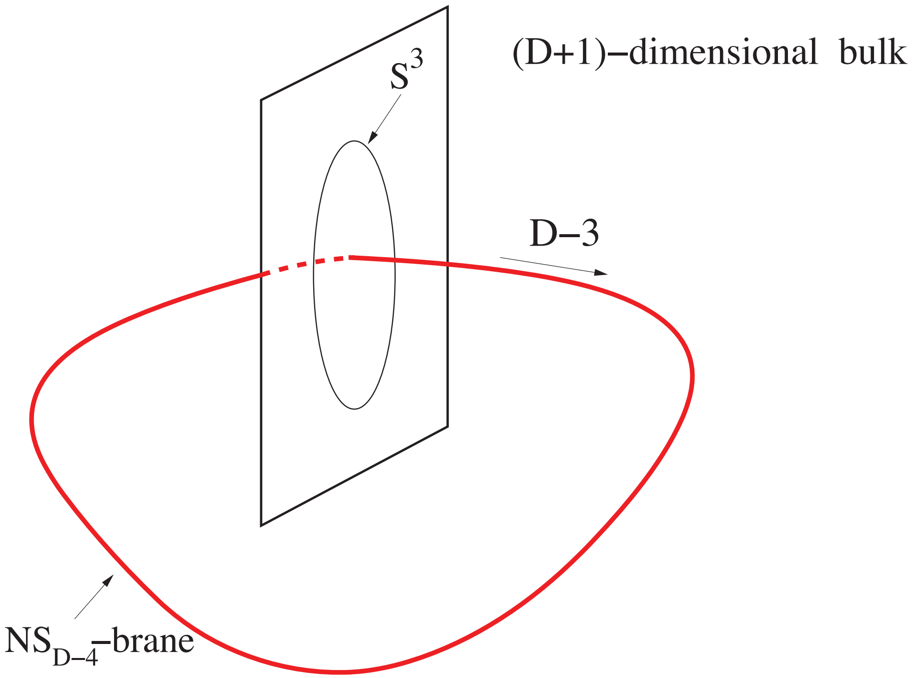

(b)

$$S_{eff} = \frac{1}{g_{KR}^2} \int dz \Big(\sum_{\square} (\partial_z B_{\mu\nu})^2$$

$$- \sum_{\text{cubes}} \cos\left[a^3 (\Delta_\mu B_{\nu\lambda} + \Delta_\nu B_{\lambda\mu} + \Delta_\lambda B_{\mu\nu}) \right] \Big)$$

$$g_{KR}^2 \sim 1/(|\phi_\square|^6 N^2)$$

Topological defect for the compact two-form gauge field



$$H = dB$$

$$\int_{S^3} H = 2\pi$$

- Tension of the brane ~ N^2

NS-brane determines the fate of string

Gapped NS-brane

- Emergent Bianchi identity dH=0 at long distances
- Strings are deconfined
- Emergent space
- Two-form gauge field remains light even at strong coupling
- Non-trivial quantum order!

Condensed NS-brane

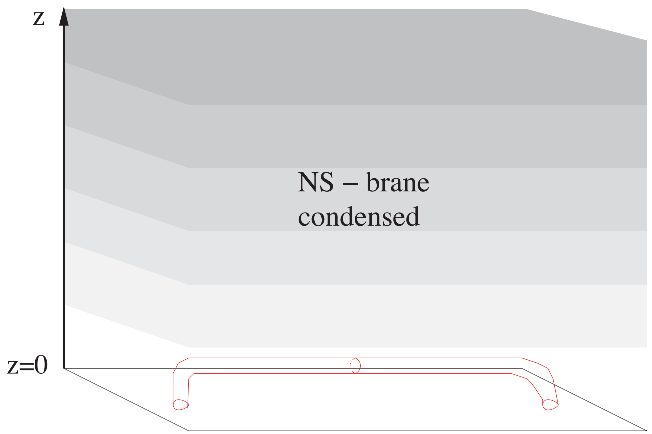
- Bianchi identity is violated at all distance scales
- Strings are confined
- No emergent space
- No light propagating mode deep inside the bulk
- No quantum order

Possible phases

Two parameters

- $1/N^2$ controls quantum fluctuations
  - N^2 : tension of NS-brane
- J_C (inverse of 't Hooft coupling) controls the magnitude of loop fields
  - Magnitude of loop fields controls the size (1/mass) of string
  - NS-brane is always suppressed in the UV region because of external sources (Higgs fields)

Confinement phase

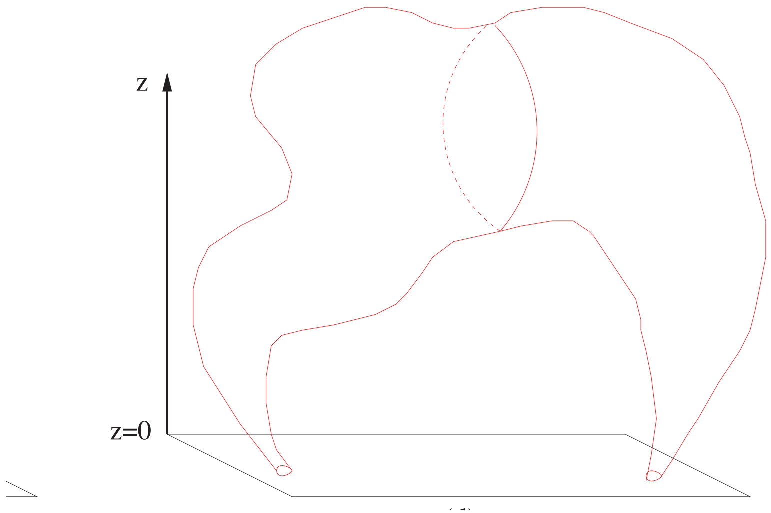


Small J_C

- NS-branes are condensed inside the bulk
- Loop amplitudes are small : kinetic term for string fields is small
- World sheet of string inserted at the UV boundary form a straight line : exponentially decaying correlation function for Wilson-loop operators

Confinement of 2-form gauge field [Polyakov; S.-J. Rey, …]

Deconfinement (IR free) phase

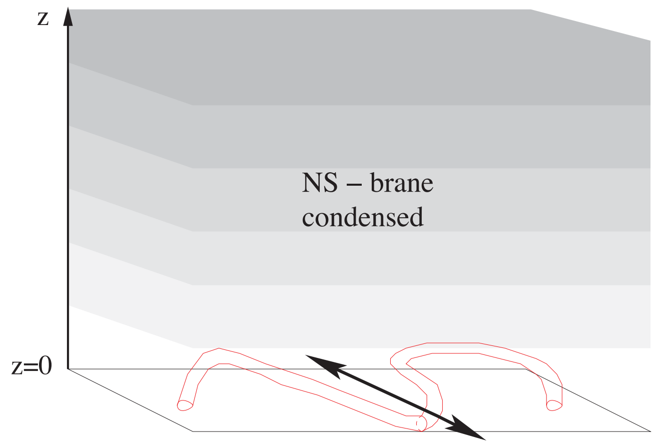


Large J_C

- Loop fields with all sizes acquire non-zero expectation values in the bulk
- Strings in the bulk become non-local because of non-local hoppings medicated by large loop fields
- Locality in the bulk is lost

Higgs phase of 2-form gauge field [S.-J. Rey; P. Yi]

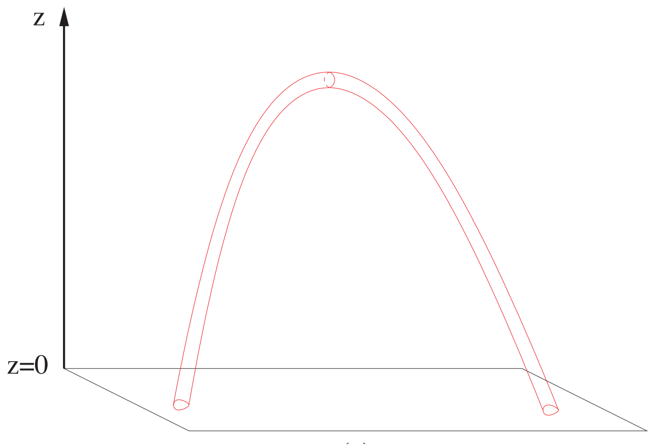
Non-holographic critical phase



Small N
critical J_C

- NS-branes remain condensed in the bulk
- Loop amplitudes are large near the UV boundary : kinetic term for string fields is large
- Strings are delocalized along the D-directions near in the UV region : algebraically decaying correlation function for Wilson-loop operators

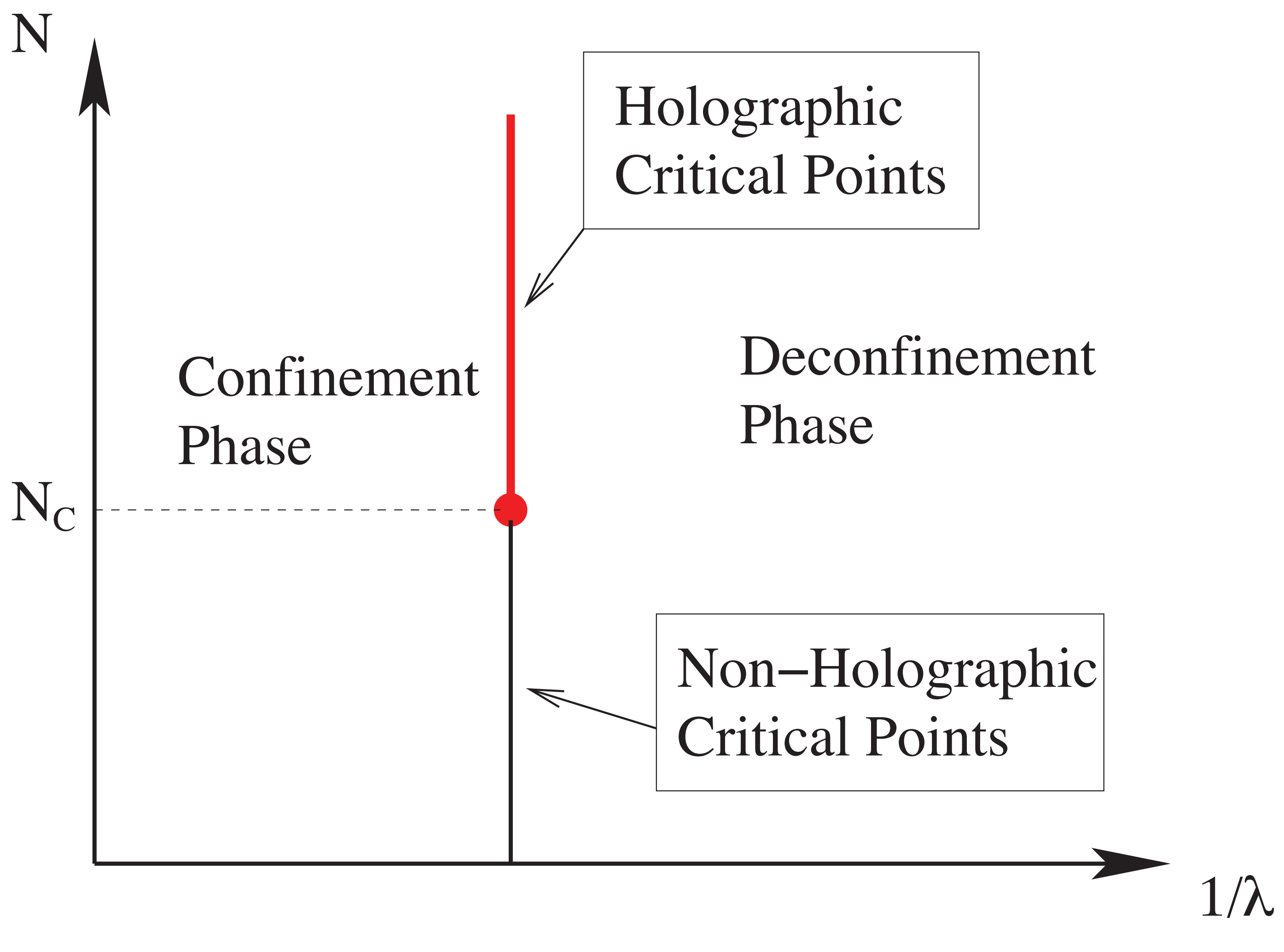
Holographic critical phase



Large N
critical J_C

- NS-branes are gapped out in the bulk
- Only loop fields with finite size are condensed
- Strings can propagate deep inside the bulk, mediating critical correlation between Wilson loop operators
- The scaling dimension of the phase fluctuations of Wilson loop operators is determined by the mass of the two-form gauge field
- The two-form gauge field remains light in the large N limit (even at strong coupling limit) : the scaling dimension is protected

A proposed phase diagram for a pure bosonic gauged matrix model in D>4



Summary

- General D-dimensional gauged matrix model can be mapped into (D+1)-dimensional string field theory which include compact two-form gauge field
- Those phases that admit holographic description have a distinct quantum order
  - Emergent space
  - Deconfined string
  - Protected scaling dimension
- (not discussed here) Open string and 1-form gauge field may emerge as fractionalized excitation of closed string