Friction and Every Day Life

- ☞ **Allows us to walk and drive**
- ☞ **Holds thread, nails, screws, bolts, bricks, …**
- ☞ **Determines how things feel, texture of food**

- ☞ **Wastes energy → ~20% in car engine**
- ☞ **Produces wear → abrades material**
  → destroys lubricants

Economic cost of poor friction control
more than 6% of GNP > $400 billion/year

Answering Questions About Friction Complicated

Friction determined by processes on a wide range of scales

- Surfaces rough on nm to mm scales

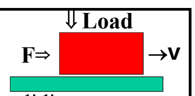
Area and geometry of contacting regions determined by roughness and long-range elastic and plastic deformation.

- Friction comes from interactions between atoms in contacting regions → sensitive to exact chemical makeup, impurities, surface coatings, … that are often unknown

Computer simulations allow controlled "experiments"
Explore trends, discover unanticipated mechanisms

No general theory for behavior far from equilibrium
Equilibrium ⇒ stable state minimizes free energy
Far from equilibrium ⇒ must solve dynamical equations

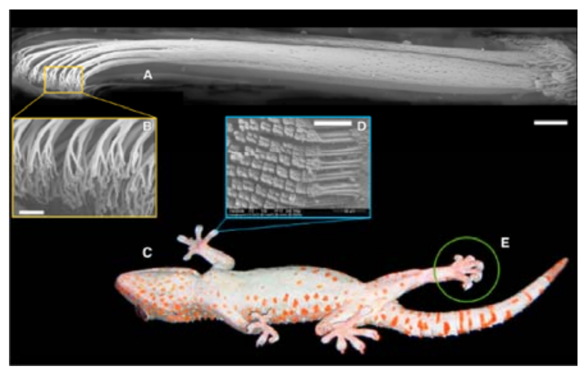
Typical measurement of friction ➔



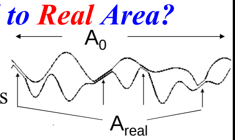
Static friction F_s
→ minimum force needed to initiate sliding.
Kinetic friction $F_k(v)$
→ force to keep sliding at velocity v.
Typically, $F_k(v)$ varies only as log(v) and $F_s > F_k(v)$ at low v
Amontons' Laws (1699):
- Friction ∝ load → constant $\mu = F/\text{Load}$.
- Friction force independent of apparent contact area A_{app}.

But: Amontons coated all surfaces with pork fat
$F \propto A_{app}$ for soft, flat solids, polymers
μ often changes with load ⇒ friction for load ≤ 0
Friction depends on history (rate-state models)
Laws violated in nanoscale experiments & simulations
⇒ solids slide like fluids, fluids stick like solids

Many Systems Have Friction with Load ≤ 0
Geckos, tape, putty, … stick on walls or ceilings



Is Friction Proportional to Real Area?

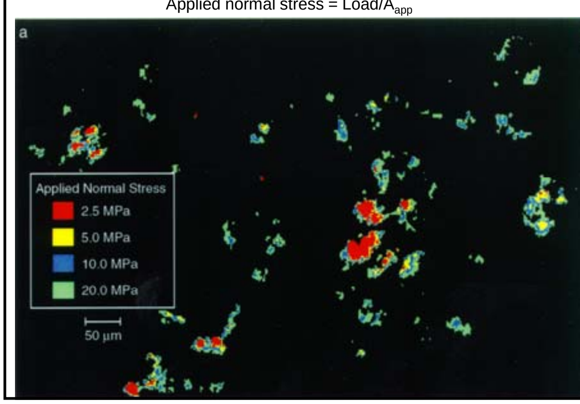


Common view since mid 1900's
Surfaces rough on many length scales
and usually find $A_{real} << A_0$
Measurements and theory ⇒ $A_{real} \propto$ Load in many cases
⇒ get Amontons' laws if constant shear stress τ_{shear}
friction = $A_{real}\, \tau_{shear} \propto$ Load
Also explains many exceptions to Amontons' laws
Adhesion ⇒ A_{real} nonzero at zero load, still have friction
Friction ∝ A_0 for soft materials because $A_{real} \approx A_0$

First describe recent progress in continuum theory for A_{real} then difficulty in defining A_{real} and explaining τ_{shear} at atomic scale

Contact area ∝ Load (Dieterich & Kilgore)

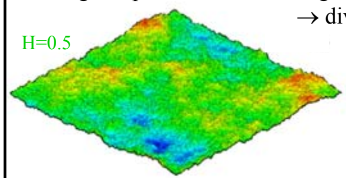
Applied normal stress = Load/A_{app}

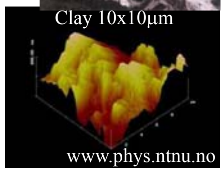


Surfaces Often Rough on Many Scales ⇒ Self-Affine

Height variation δh over length $\ell \rightarrow \delta h \propto \ell^{H}$ $0<H<1$

Average slope $\delta h/\ell \propto \ell^{-(1-H)}$ → goes to zero as ℓ increases
→ diverges as ℓ decreases



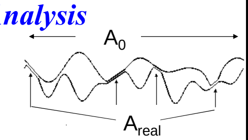


Fractured and polished surfaces self-affine over large range of scales H~0.6-0.8

Find contact area using finite-element continuum code for wide range of H, etc.

Hyun, Pei, Molinari, & Robbins, *Phys. Rev. E* **70**, 026117, '04; J. Mech. Phys. Sol. **53**, 2385, '05; Trib Int. **40**, 1413, '07

Dimensional Analysis



Assume area ∝ load (N=load)

Only material property is contact modulus $E' = E/(1-\nu^2)$

$N/(A_{real} E')$ is dimensionless

Roughness only enters through dimensionless measure
→ rms slope $\Delta = |\nabla h|^{1/2}$ → sensitive to small scales
→ independent of h_{rms}= rms roughness, system size

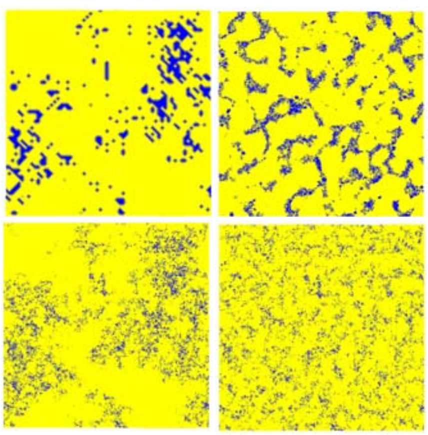
$N/(A_{real} E') = \Delta/\kappa$, κ a constant

Numerical solution: $1.8 < \kappa < 2.2$ for all H, Δ, ν, …

Very different models, similar analytic predictions for κ

Bearing area – Greenwood-Williamson $\kappa = (2\pi)^{1/2} \approx 2.5$

Persson's scaling theory $\kappa = (8/\pi)^{1/2} \approx 1.6$



Very different surface roughness profiles give same κ=2.0

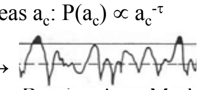
Results here are for different synthetic and experimental surfaces at A/A_0~0.1

Models Predict Very Different Contact Geometry For Same Rough Surface and A_{real}

Power law distribution of connected areas a_c: $P(a_c) \propto a_c^{-\tau}$

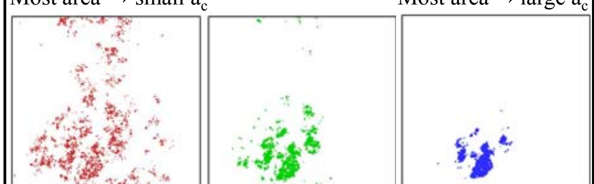
Connected regions are fractal $a_c \propto r^{Df}$

Inconsistent with bearing area model →



Ideal Elastic	Perfectly Plastic	Bearing Area Model
$\tau > 2$, $D_f \approx 1.6$	$\tau \approx 2$, $D_f \approx 1.8$	$\tau = (2-H/2)$, $D_f = 2$

Most area → small a_c … Most area → large a_c



Contact and Pressure Correlation Functions

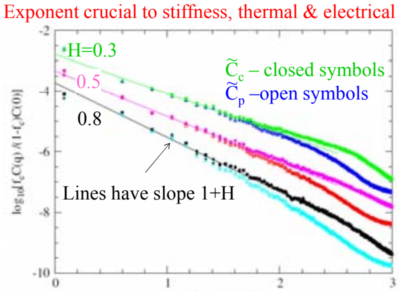
$C_c(r-r') = \langle c(r)c(r')\rangle_{r'}$ where c(r)=1 in contacts, 0 otherwise

$C_p(r-r') = \langle p(r)p(r')\rangle_{r'}$ where p(r) is normal pressure

Bearing area & Greenwood-Williamson $\tilde{C}(q) \sim q^{-(2+H)}$

Full numerical and Persson $\tilde{C}(q) \sim q^{-(1+H)}$ q = wavevector

Exponent crucial to stiffness, thermal & electrical conductance



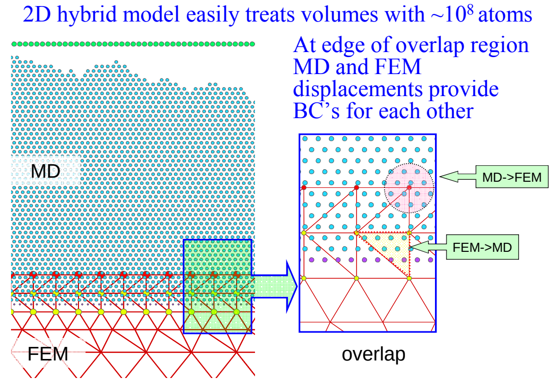
Campañá, Müser & Robbins, J. Phys. Cond. Matt. **20**, 354013 (2008)

Ramisetti, Campañá, Anciaux, Molinari, Müser, Robbins, J. Phys. Cond. Matt. 23, 215004 (2011)

What Happens With Atoms?

2D hybrid model easily treats volumes with ~10^8 atoms

At edge of overlap region MD and FEM displacements provide BC's for each other



Luan & MOR, Tribol. Lett., **36**, 1 ('09), Luan, Hyun, Molinari, Bernstein, MOR PRE74, 046710 ('06)

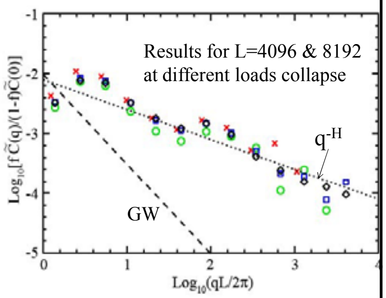
Atomic Simulations, Same Large Scale Correlations

Pressure correlation function C(q) scales as power of q

GW: $C(q) \sim q^{-(1+H)}$

Persson: $C(q) \sim q^{-H}$

Also consistent with Persson in 3D simulations



BUT Contact area sensitive to structure at atomic scale, discrete surface slopes, ratio of lattice constants, etc..

Find larger A_{real}, $\kappa \rightarrow \kappa \sim 15$ in 2D, up to 4 in 3D

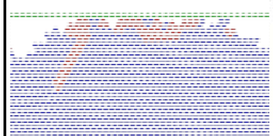
Plasticity Irrelevant as System Grows

Continuum ⇒ plasticity for $\Delta > \sigma_y/E' < 0.1$-$0.3$

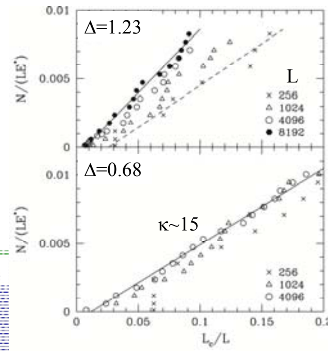
No plasticity at Δ=0.68 for LJ

For Δ=1.23 find $L_c \propto N$, but L_c/N converges as L rises

Scale dependent surface flattening dominates, dislocations inhibited by lowered slope



Red atoms have new neighbors



Scale of flattening << L as L increases

Response approaches elastic behavior

Dimensional Analysis of Contact Stiffness

Contacts often dominate macroscopic stiffness → jet engine mounts

Electrical and thermal conductance scale like stiffness

Normal stiffness $k_N \equiv -dN/du$ with u=mean surface separation

Tangential stiffness $k_T = -dF/dx$ with x=lateral disp.

Dimensional analysis $k_N = N/\gamma h_{rms}$ with γ a constant and h_{rms}=rms height → sensitive to system size

Integrate $-dN/du = N/\gamma h_0 \Rightarrow N = cA_0E' \exp(-u/\gamma h_{rms})$

Continuum predicts lateral stiffness $k_T = k_N 2(1-\nu)/(2-\nu)$ if isotropic

Experiment:

Lateral - Berthoud, Baumberger, Proc. R. Soc. Lond. A454, 1615 '98.

Normal - Benz, Rosenberg, Kramer, Israelachvili, J. Chem. Phys. '06.

Lorenz and Persson. J. Phys. Condens. Matter 21, 015003 '09.

Theory: Pei, Hyun, Molinari, Robbins, JMPS 53, 2385 (2005).

Persson, Phys. Rev. Lett. 99, 125502 (2007), …

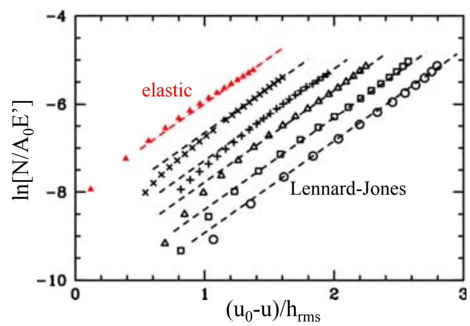
3D simulations of Lennard-Jones atoms (up to $5 \cdot 10^7$)

Akarapu, Sharp, Robbins PRL 106, 001504301 (2011)

Load Varies Exponentially With Separation u

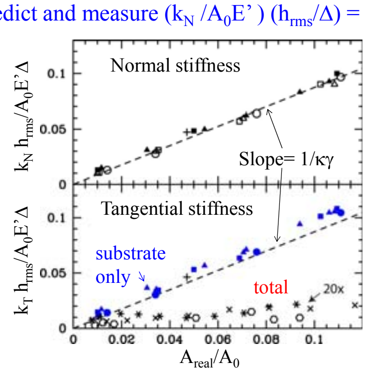
Find $F_N = cA_0E' \exp(-u/\gamma h_{rms})$ with γ=0.48

For all H, ν, system size, elastic continuum &Lennard-Jones



Normal Stiffness ∝ Load ∝ Area

Predict and measure $(k_N/A_0E')(h_{rms}/\Delta) = (\kappa\gamma)^{-1} A_{real}/A_0$



Results for k_N with different H, L, xtal structure collapse

Tangential stiffness ~100 times smaller

Contribution from substrate ~k_N

Lateral motion between atoms on opposing interfaces dominates total k_T!

k_T varies with xtal surface, atomic spacing

Pressure distribution for sphere on flat

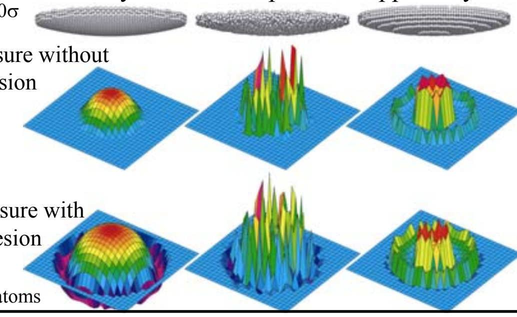
Atomic scale roughness qualitatively changes pressure, yield

Bent crystal agrees with Hertz/JKR, more realistic tips do not

R~30nm ~100σ

Bent crystal Amorphous Stepped Crystal

Pressure without adhesion



Pressure with adhesion

~10^7 atoms

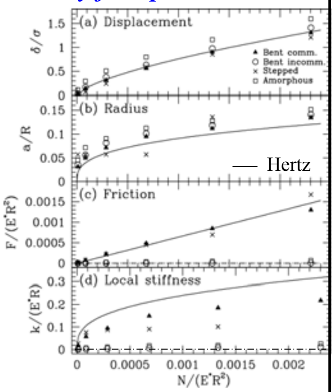
Comparison to Hertz Theory for Sphere on Flat

Fits of δ to Hertz give good E^* and R (10%)

Contact radius a is shifted several σ away from Hertz, leading to 100% error for small contact radius.

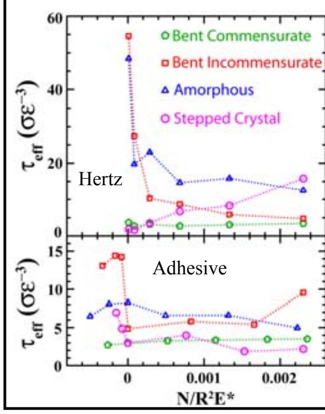
Friction varies by factor of 200
Commens.: $F \propto N$, μ=0.7
Incomm.: F~0
Amorphous: $F \propto A$, and small

Lateral stiffness varies 10 fold due to atomic scale lateral motion at interface.
Agrees with Hertz if don't allow sliding at interface.



Luan & Robbins, Nature 435, 929 (2005)

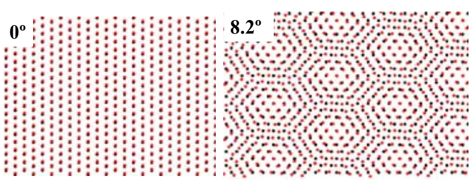
Does Ratio F/k² Give Shear Stress?



Continuum: $F=\tau_{eff}\pi r^2$, k=8Gr
$\rightarrow \tau_{eff} = (F/k^2)\ (64G^2/\pi)$
Find τ_{eff} relatively constant, but 10-100 times true τ !
and τ not constant for stepped
Similar curves in experiments, including rise at small loads
$\rightarrow$ Not reliable way of estimating shear stress under general conditions.
Main difficulty is that k is dominated by interface which is not included in continuum expression.

Nanoscale Studies of Flat Surfaces

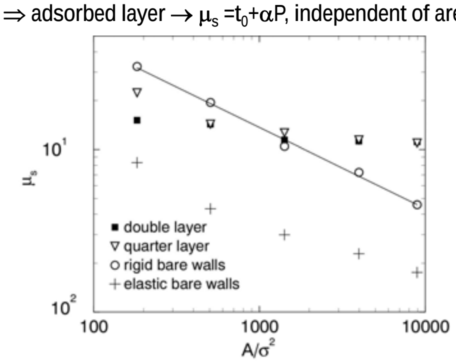
Clean, flat surfaces expected to have zero static friction
Hirano & Shinjo – Contacting crystals typically incommensurate,
$\Rightarrow$No common period $\rightarrow$ lateral force averages to zero, F_s=0
Consistent with many experiments & simulations
F_s=0 for incommensurate monolayers on substrate (Krim et al.)
Solids more slippery than fluid of same element
F_s~0 for misaligned mica, graphite, MoS_2, amorphous antimony
(Hirano et al. '91, Dienwiebel et al. '04, Martin et al., Dietzel et al. '08)



commensurate incommensurate

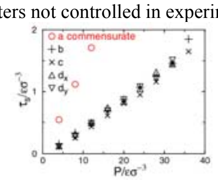
Amorphous walls

$\Rightarrow$ bare surfaces $\rightarrow \mu_s$ vanishes as 1/area$^{1/2}$
$\Rightarrow$ adsorbed layer $\rightarrow \mu_s = t_0 + \alpha P$, independent of area



"Dirt" Leads to Static Friction

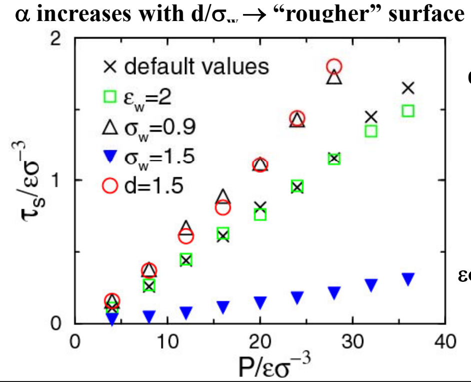
Molecules adsorbed from air, wear debris, elastomer segments, and other mobile "third bodies" lock surfaces together, $F_s \neq 0$
Find $\tau_s = \tau_0 + \alpha\ p \Rightarrow F_s = \tau_0\ A_{real} + \alpha$ N (He, Müser, Robbins, Science '99)
$\Rightarrow$ can explain Amontons's laws, exceptions with adhesion, rate-state BUT not a constant τ_{shear}
$\Rightarrow \alpha$ ~ independent of many parameters not controlled in experiment



Mica, graphite, MoS_2, antimony show higher friction, less variability when exposed to air and adsorbed molecules
Nanometer fluid films act like solids in surface force apparatus
(Israelachvili, Granick, Klein)

Effect of Potential

α indep. of coverage, chain length ($n \le 6$), ε_w or r_c
α increases with $d/\sigma_w \rightarrow$ "rougher" surface



default:
ε_w=1
σ_w=1
d=1.2

$\varepsilon\sigma^{-3} \approx$ 40MPa

Geometric Explanation

If pressure high enough → hard sphere limit

Repulsive force balances pressure

$F \sim P/c \sim 48\,(\varepsilon_w/\sigma_w)(\sigma_w/r)^{13}$ where c=coverage

$\Rightarrow r \sim \sigma_w\,(c\,\varepsilon_w\,/P\,\sigma_w)^{1/13}$

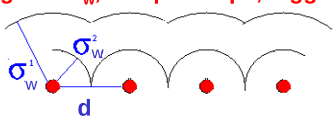
Effective hard-sphere radius: insensitive to c, ε_w , P

almost linear in σ_w

Surface of closest approach depends on d/σ_w

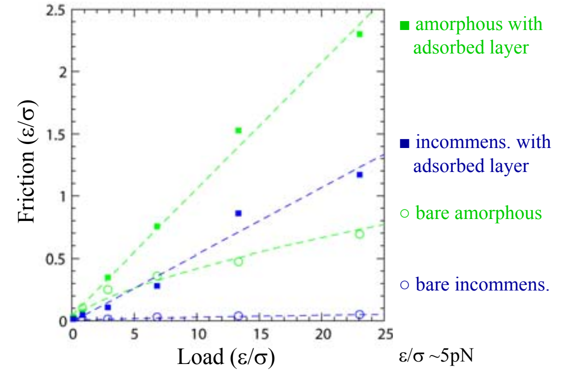
$\alpha \propto$ maximum slope as in geometric model

→ larger d/σ_w, steeper slope, bigger α



Analytic theory: Müser, Wenning, Robbins PRL 86, 1295, '01

Adsorbed layers give F∝load for AFM tips and decrease variability of friction with tip geometry



Monolayer ⇒ Huge Increase in Contact Area

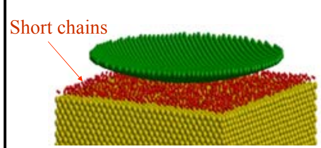
Monolayer : much bigger contact

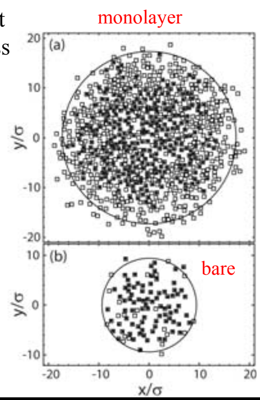
much smaller normal stiffness

⇒ like very compliant layer

(Cheng and Robbins, PRE 2010)

90% of load and friction come from 10% of contacting atoms





What Is Contact at Atomic Scales?

Attraction at large separations

⇒ Entire surface interacts: $A_c = A_0$

Common alternative

⇒ Contact = Repulsive interaction

Separation ~ atomic diameter

First idea – $A_c = A_a * N_c$

A_a = Area per atom

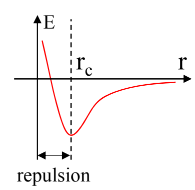
N_c = mean # contacting atoms at any instant

Mo et al. (Nature 2009): found A_c~Load~friction

Concluded could explain with continuum theory for rough solid

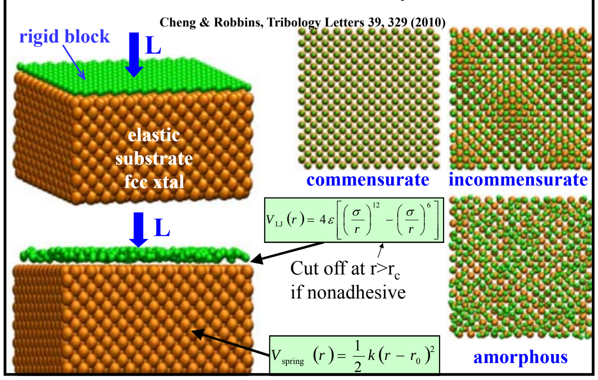
Continuum theory assumes T=0

→ How important are thermal fluctuations?



Simplest Geometry: Parallel Planar Surfaces

Continuum ⇒ full contact at any normal load

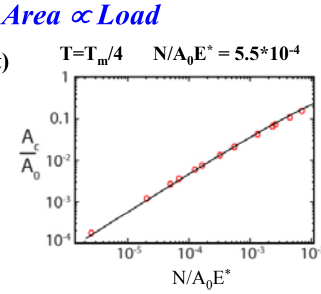


$V_{LJ}(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$

$V_{spring}(r) = \frac{1}{2}k(r-r_0)^2$

Contact Area ∝ Load

pressure field (snapshot)



Although surfaces flat, small fraction contact at any instant

N/A_0E^* ~ normal strain, bulk strain at yield ~10^{-4} to 10^{-2}

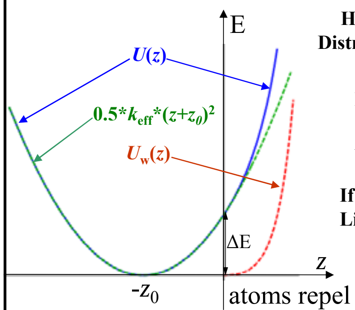
$A_c/A_0 = c_A N/A_0E^*$ with $c_A \sim 20\,[T_m/T]^{1/2}$, T_m=melting T

while rough surface $c_A = \kappa/\Delta$

Theoretical Model

Spring k_{eff} binding atom to substrate + wall potential U_w

$$U(z) = \frac{1}{2} k_{eff} (z+z_0)^2 + U_w(z)$$



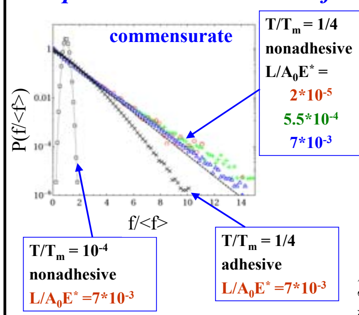
Height Distribution $$p(z) = \frac{e^{-\beta U(z)}}{\int_{-\infty}^{\infty} e^{-\beta U(z)} dz}$$

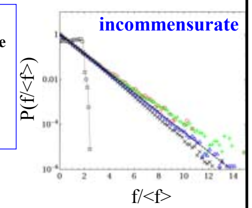
Load: $$N = M_0 \int_0^{\infty} p(z) f(z) dz$$

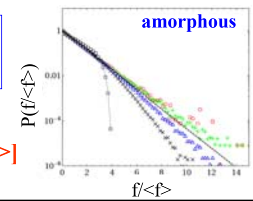
Area: $$A_c(0) = A_0 \int_0^{\infty} p(z) dz$$

If $A_c/A_0 << 1 \Rightarrow \Delta E/k_B T << 1$
Linearize U $\Rightarrow$ analytic sol.
Find $A_c(0) \propto N$
Exponential distribution of instantaneous forces

Exponential Distribution of Instantaneous Force

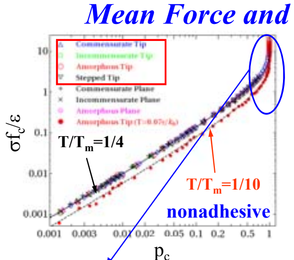


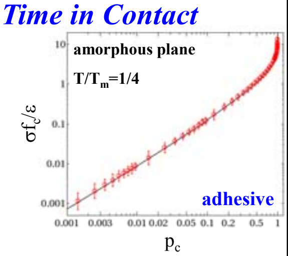


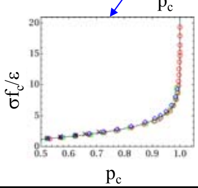


At $T/T_m > 0.05$, $P(f/\langle f \rangle) = \exp[-f/\langle f \rangle]$

Mean Force and Time in Contact







probability in contact: $p_c = \int_0^{\infty} p(z) dz$

time-averaged force: $f_c = \int_0^{\infty} p(z) f(z) dz$

Universal dependence of f_c on p_c

$f_c \propto p_c$ leads to $L \propto A_c(0)$

$p_c \sim 1 \rightarrow f_c \sim 5\text{-}10 \rightarrow p/E^* \sim 0.1\text{-}0.2$

>> macroscopic hardness of 10^{-4}- 10^{-2}

Conclusions for Thermal Effects

- Even for simplest geometry, continuum concept of contact hard to extend to nanometer scale
- Thermal fluctuations lead to strong fluctuations in instantaneous contact force $\Rightarrow$ pure exponential
- Universal relation between contact time and mean force
- Force is not repulsive more than 50% of time until mean pressure is comparable to ideal hardness
- Nonadhesive: $A_c(0)/A_0 = c_A N/E^*A_0$ with $c_A \sim 20 [T_m/T]^{0.5}$ From Lindemann criteria and $k_{eff}/\sigma^3 \sim E^*$
- Only unambiguous definition is time averaged force for adhesive case, but requires $\Delta p \sim$hardness for full contact
- Similar results for spherical tip on flat $A_c(0) <<$ Hertz, linear at small load, …
- Must also coarse-grain in space to approach continuum

Conclusions

- Measured macroscopic friction is a single number, but reflects processes on a wide range of scales.
- Continuum calculations show area proportional to load but do not explain shear stress τ_{shear} that controls friction
- Normal stiffness k_N consistent with continuum theory, but atom scale effects greatly reduce tangential stiffness k_T
- Clean surfaces generally have no friction, small k_T
  Adding adsorbed molecules gives $\tau_s = \tau_0 + \alpha p \neq$ const.
- Rough surfaces: A_{real} difficult to define at atomic scale
  Sensitive to atomic scale geometry, interactions, …
- Thermal fluctuations $\rightarrow$ pressures ~ ideal yield stress
  $\rightarrow$atoms usually out of contact $A_c(0)/A_0 = c_A N/E^*A_0$ with $c_A \sim 20 [T_m/T]^{0.5}$ from Lindemann criteria, $k_{eff}/\sigma^3 \sim E^*$
- Much left to do to understand how roughness, plasticity, temperature and atomic interactions determine τ_{shear}