Relation between the adhesion and elastic forces for fractal contact surfaces with different Hurst exponent

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Outline

- Reduced dimensionality and contact problem for random surface;
- Numerically generated random potentials and numerical procedure;
- Illustrative movies;
- Mutual relation between adhesion and elastic forces;
- Conclusion

Reduced dimesionality
(previous publications)

1. Geike T. and V.L. Popov, **Mapping of three-dimensional contact problems into one dimension.** - *Phys. Rev. E., 2007, v.* ***76****, 036710 (5 pp.).*

2. Geike T. and V.L. Popov. **Reduction of three-dimensional contact problems to one-dimensional ones.** - *Tribology International, 2007, v. 40, 924-929.*

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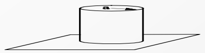
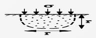
1. Popov V.L. and S.G. Psakhie. **Numerical simulation methods in tribology -** *Tribology International, 2007, v. 40,* 916–923

2. Popov V.L. , Filippov A.E., **Applicability of a reduced model to description of real contacts between rough surfaces with different Hurst exponents.** - *Tech. Phys. Lett., 2008, v. 34, No. 8, pp.722-724* .

3. Popov V.L. , Filippov A.E., **Statistics of contacts and the dependence of their total length on the normal force for fractal surfaces with different Hurst exponents**.- *Tech. Phys. Lett., 2008, v. 34, No. 9, pp.792-794.*

The basic idea: reduction of dimensionality

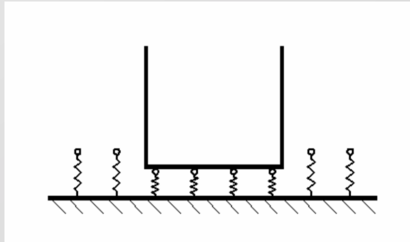




$$F = 2aE^* u_z = {\color{red} d} \cdot E^* u_z$$

The force is proportional to the **diameter** of the contact, not to its area.

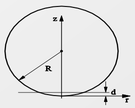
$$E^* = \frac{E}{1-\nu^2}$$



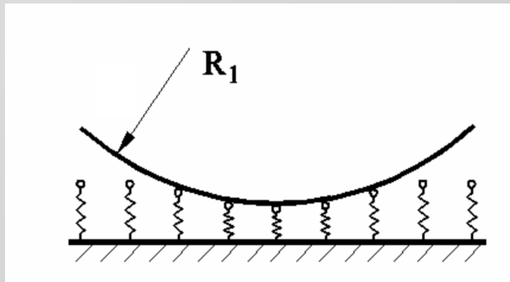
This property allows simulate this 3D system by a 1D system. In a 1D system **the force is „automatically“ proportional to the diameter of the contact.**

Effective stiffness per unit length should be

$$\boxed{\Delta c = E^* \Delta x}$$



Contact of a rigid sphere with an elastic half space – Hertz' problem



$$F_N = \frac{4}{3} E^* R^{1/2} d^{3/2}$$

$$a = \sqrt{Rd}$$

$$F_N(d) = \frac{4\sqrt{2}E^*}{3} R_1^{1/2} d^{3/2}$$

$$\boxed{R_1 = R/2}$$

$$a = \sqrt{Rd}$$

Adhesion in the approach of reduction of dimensionality

(by Markus Hess)

M. Heß, Über die Abbildung ausgewählter dreidimensionaler Kontakte auf Systeme mit niedrigerer räumlicher Dimension, Dissertation an der Technischen Universität Berlin, 2010.

rough elastic bodies show a finite adhesion force if the condition $l\nabla z \leq \gamma_{12}/E^*$ is fulfilled, where l is the root mean square (RMS) height of roughness, ∇z is the RMS value of the surface slope, γ_{12} is the relative surface energy of contacting bodies, and $E^* = E/(1-\nu^2)$ is the effective modulus of elasticity with E being Young's modulus and ν the Poisson ratio.

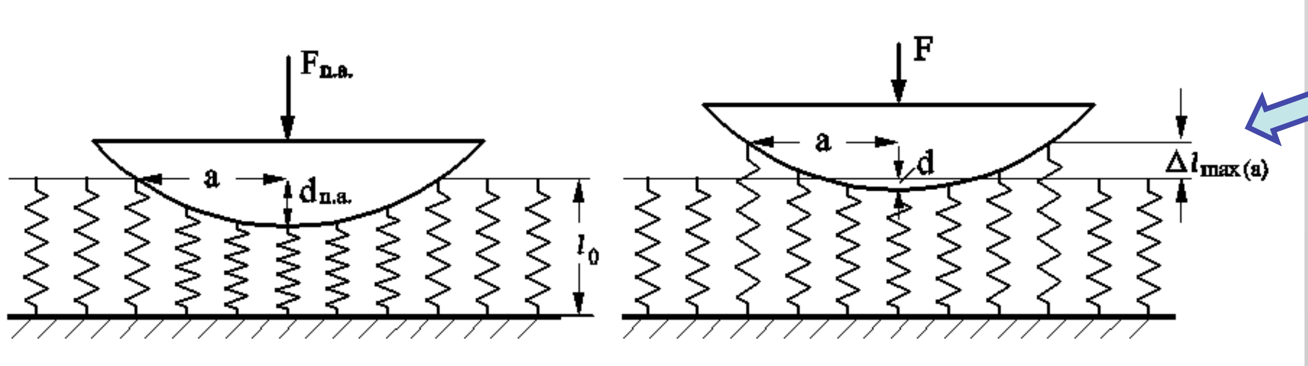
Popov, V.L.: Contact Mechanics and Friction, Physical Principles and Applications. Springer-Verlag, Berlin (2010)

$$F = F_{n.a.} - \sqrt{8\pi E^* a^3 \gamma_{12}} \ .$$

$$d = d_{n.a.} - \sqrt{\frac{2a\pi\gamma_{12}}{E^*}} \ ,$$

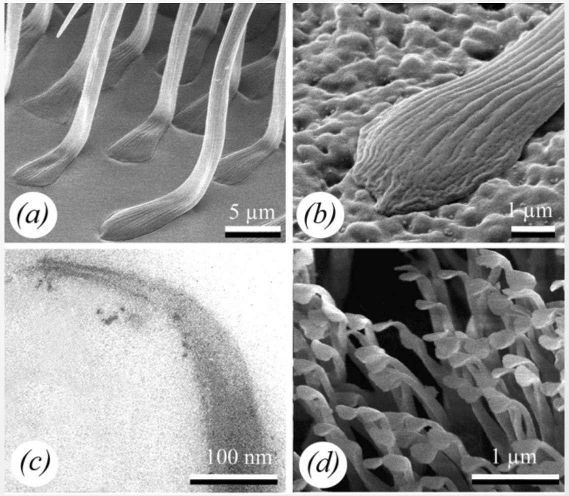
$$F_{n.a.} - F = 2E^* a \left(d_{n.a.} - d \right) .$$

Contact with adhesion is determined by the contact without adhesion by these relations.



$$\Delta l_{max}(a) = \sqrt{\frac{2a\pi\gamma_{12}}{E^*}} \ .$$

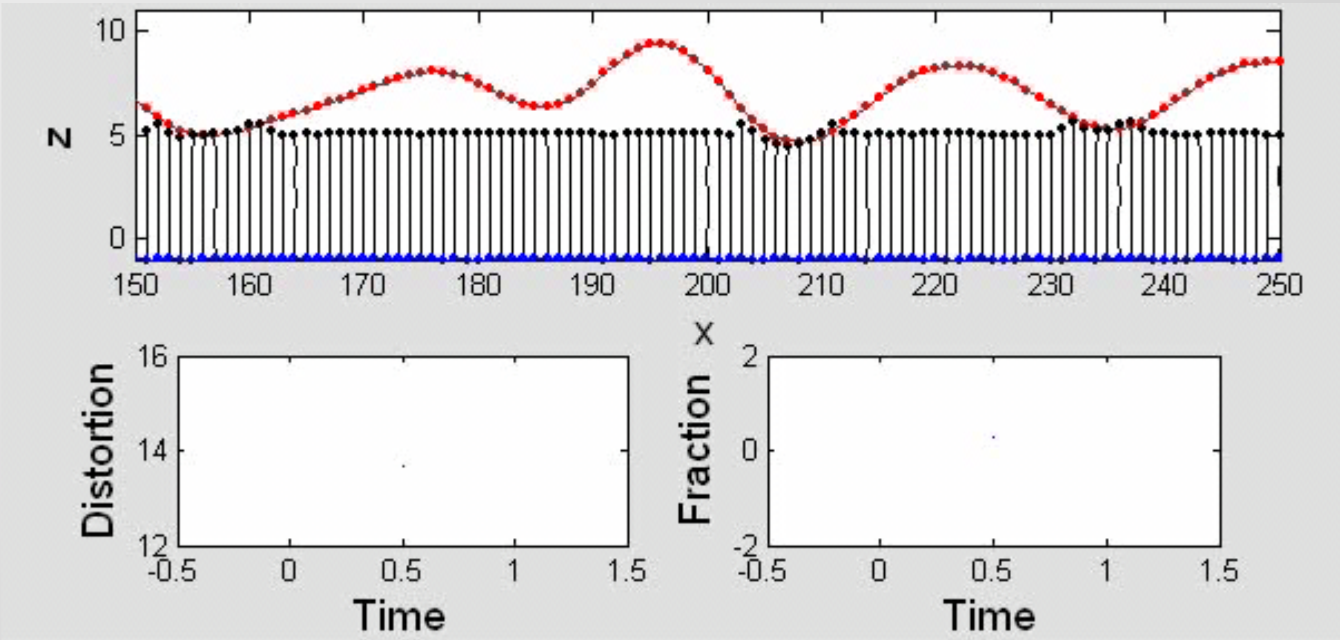
Practical demands on simpler approach



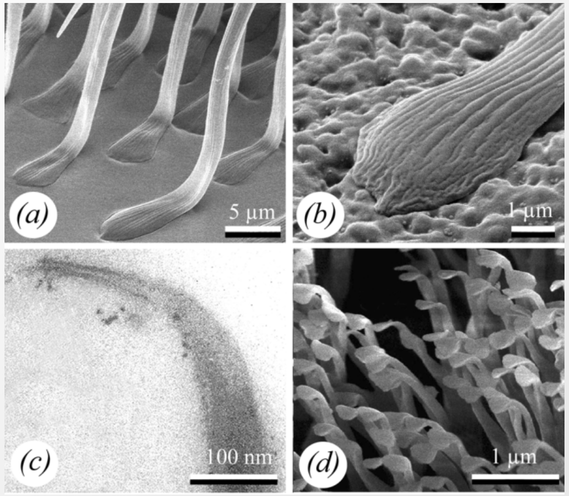
Artificial tissue mimicking Gecko foot hair

Additional degree of freedom = additional dimensionality

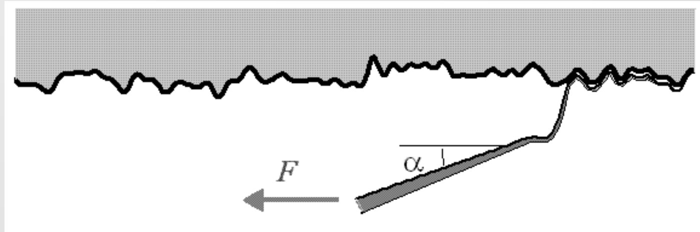
A.Filippov. V.Popov, J.Phys. Cond. Matter (2007) CM/ 235571/PAP/157764

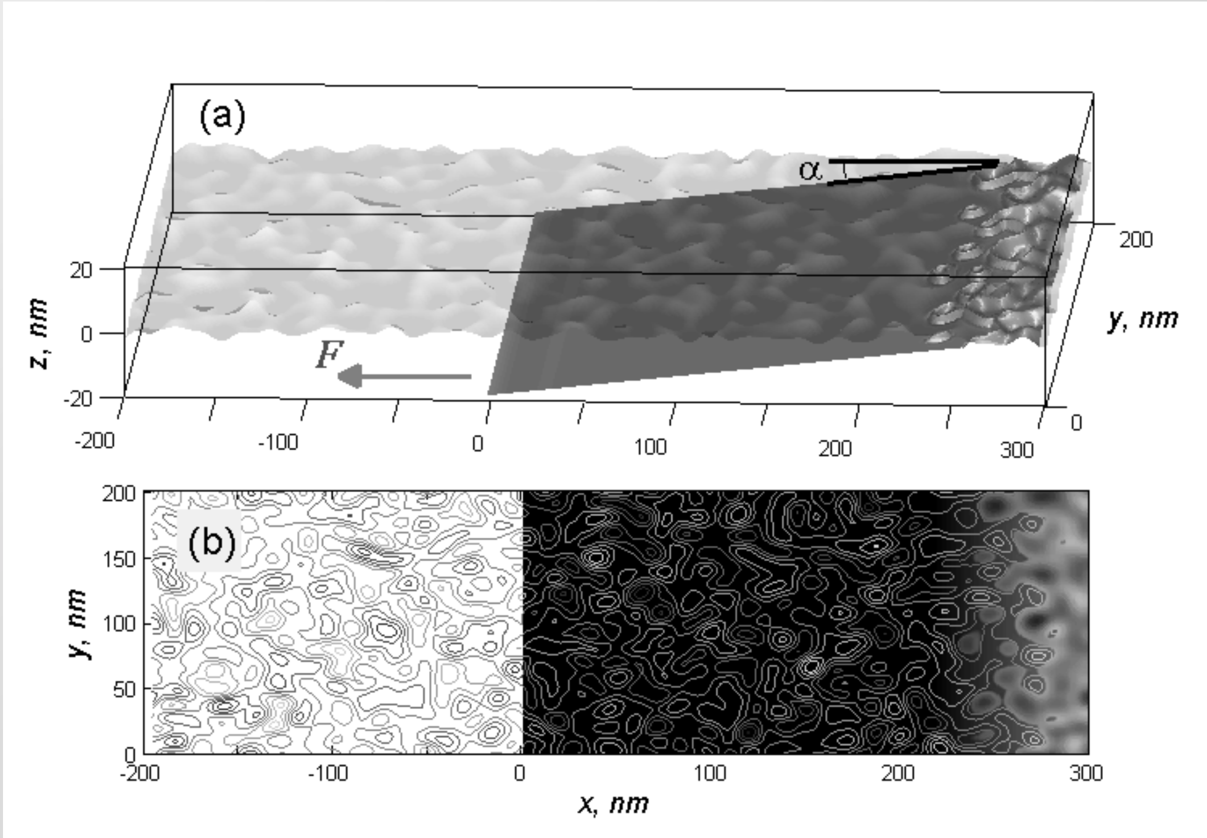


Practical demands on simpler approach



Adhesion of the spatula





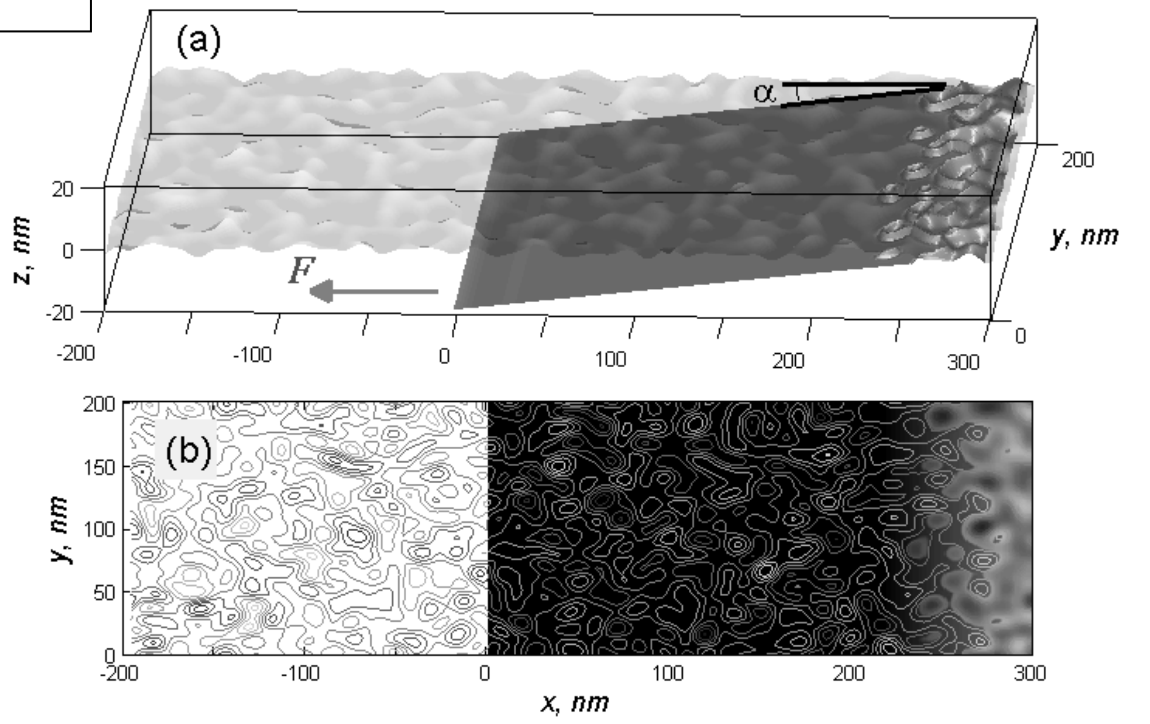
A.Filippov. V.Popov, S.Gorb. Journal of Theor. Biology Vol. 276 (2011) p. 126–131

Practical demands on simpler approach

$$W_{elastic} = \frac{E}{24(1-\nu^2)} \iint dxdy h^3(x,y) \left\{ \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)^2 + 2(1-\nu) \left[\left(\frac{\partial^2 z}{\partial x \partial y} \right)^2 - \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} \right] \right\}$$

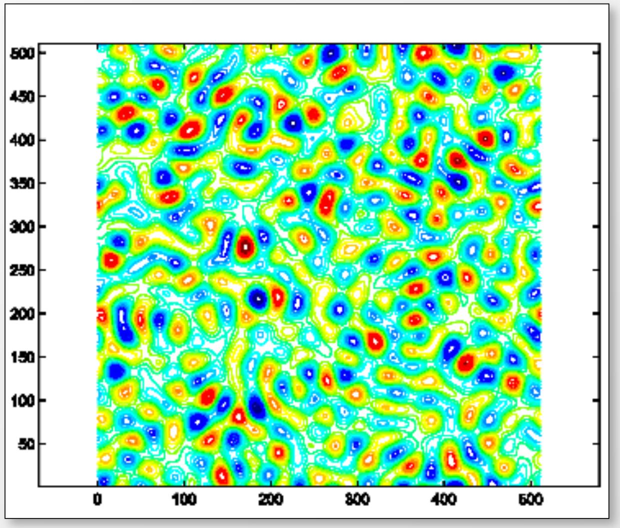
$$\gamma \frac{\partial z(x,y)}{\partial t} = -\frac{\delta W_{elastic}[z]}{\delta z} - \frac{\delta U_{VdW}[z]}{\delta z}$$

$$F_{VdW}^{x} = -\delta U_{VdW}[z(x)] / \delta x$$



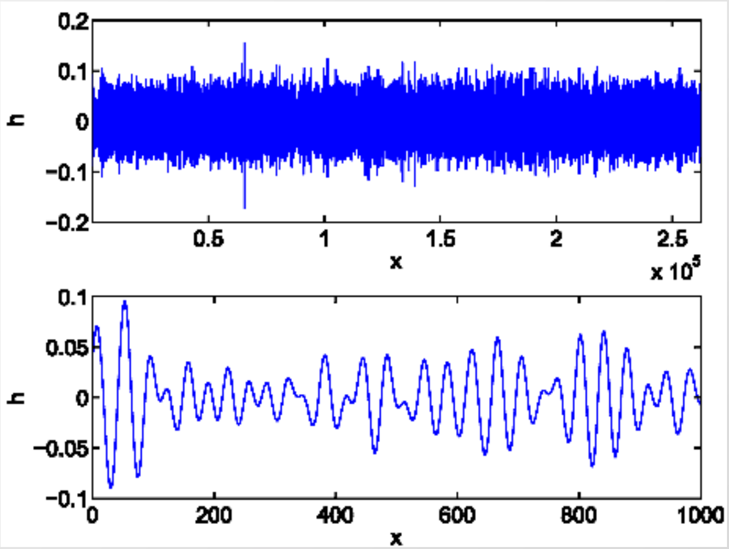
A.Filippov. V.Popov, S.Gorb. Journal of Theor. Biology Vol. 276 (2011) p. 126–131

Reduction of dimensionality for rough surfaces



$$C_{2D}(q) = \frac{1}{(2\pi)^2} \int \langle h(\vec{\mathbf{x}}) h(0) \rangle e^{-i\vec{\mathbf{q}}\cdot\vec{\mathbf{x}}} d^2x$$

$$C_{1D}(q) = \frac{1}{2\pi} \int \langle h(x) h(0) \rangle e^{-iqx} dx$$



Height dispersion 2D $\langle h^2 \rangle_{1D} = 2\int_0^\infty C_{1D}(q)\,dq$

Height dispersion 1D $\langle h^2 \rangle_{2D} = 2\pi\int_0^\infty q C_{2D}(q)\,dq$

They are equal if $C_{1D}(q) = \pi q C_{2D}(q)$

Numerically generated fractal *1D* “surface” and Hurst exponent

$$h(\vec{x}) = \sum_{\vec{q}} B_{2D}(\vec{q}) \exp\left(i\left(\vec{q}\cdot\vec{x} + \zeta(\vec{q})\right)\right),$$

$$h(x) = \sum_{q} B_{1D}(q) \exp\left(i\left(qx + \zeta(q)\right)\right),$$

$$B_{2D}(\vec{q}) = \frac{2\pi}{L}\sqrt{C_{2D}(\vec{q})} = \bar{B}_{2D}(-\vec{q}),$$

$$B_{1D}(q) = \sqrt{\frac{2\pi}{L} C_{1D}(q)} = \bar{B}_{1D}(-q).$$

$$\langle \zeta(q)\zeta(q')\rangle = 2\pi\delta(q-q').$$

⇨ Random phase

*Gamma/E=1 / (pi*10^4)*
*q1=2pi*0.02; q2=2pi*0.25;*

$$B_{1D}(q) \sim q^{\beta}$$

⇨ Spectral density

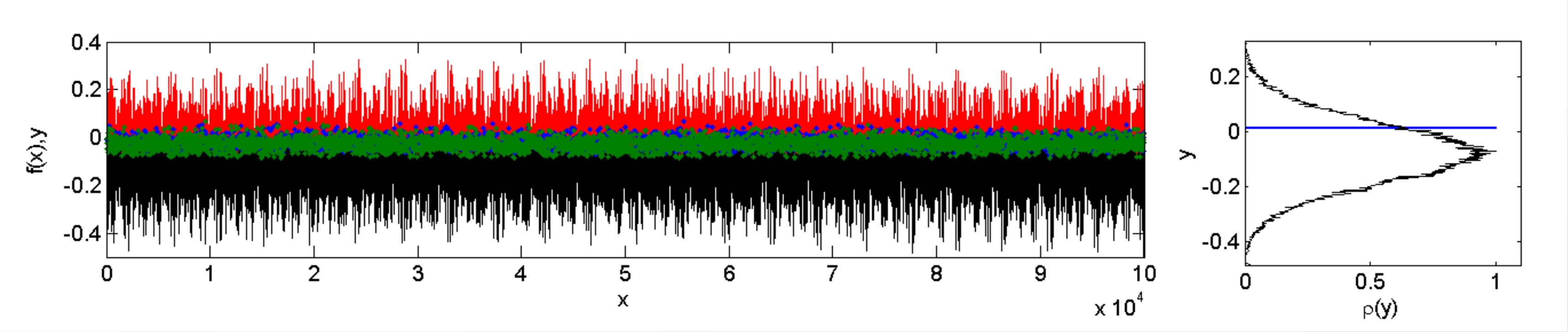
$$\beta = H + \tfrac{1}{2}$$

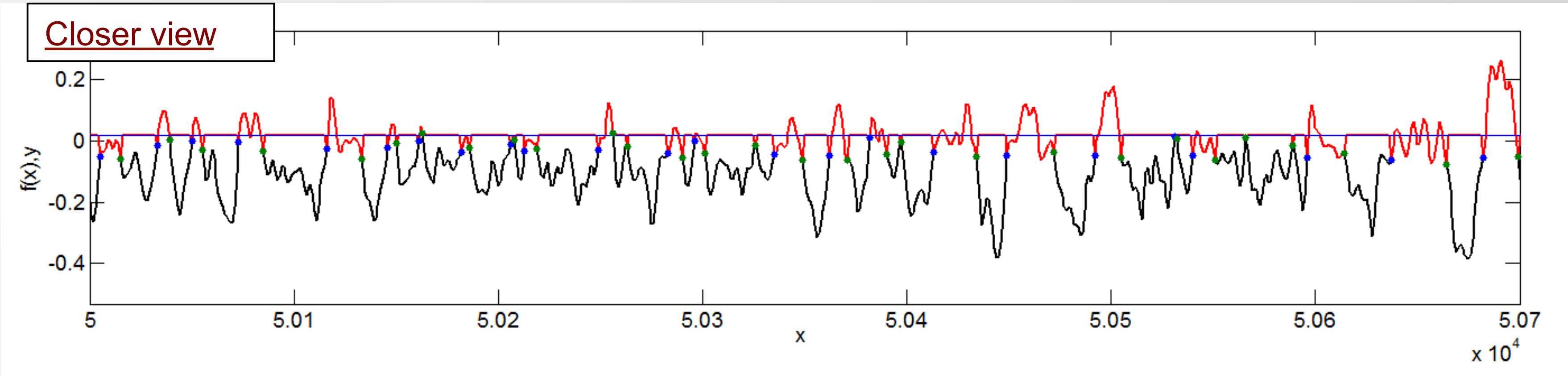
H is the Hurst exponent of the 2D fractal surface $f(\mathbf{x})$

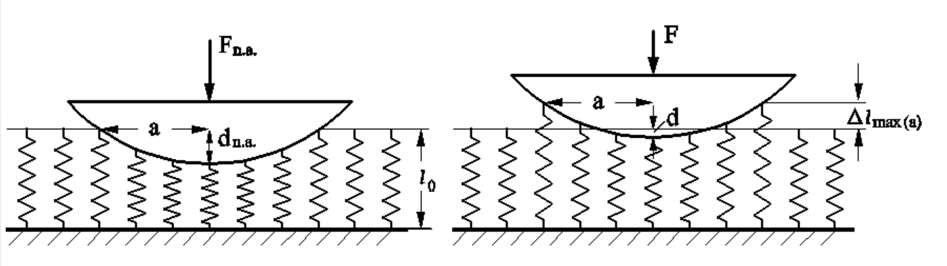
Physical values of Hurst exponent varies in limited interval ⇨ $0 \le H \le 1$, so: $\frac{1}{2} \le \beta \le \frac{3}{2}$.

$D_f = 3 - H$

Typical intermediate space configuration of the soft surface and height distribution of the hard surface.

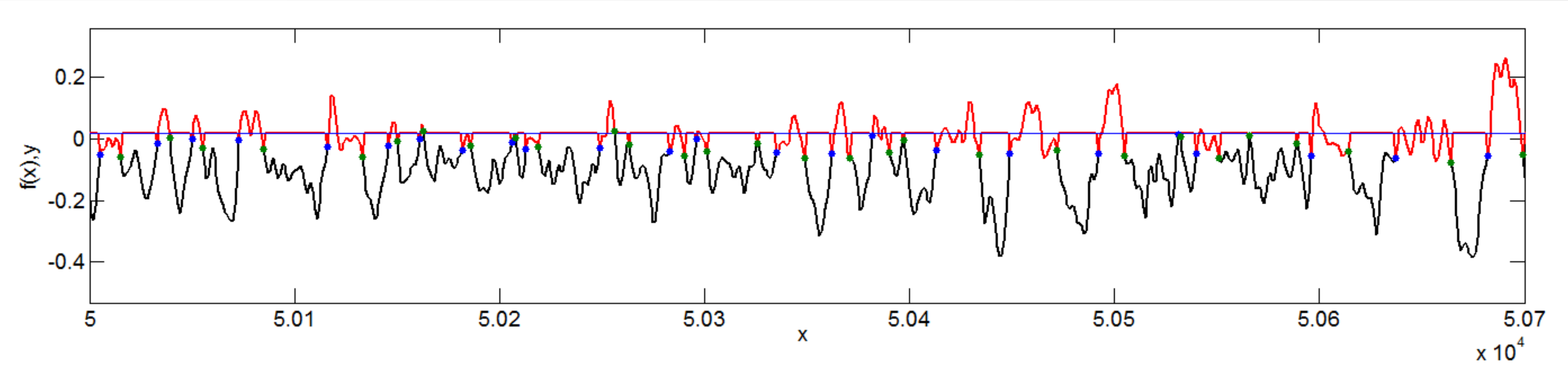






Typical intermediate space configuration of the soft surface and height distribution of the hard surface.

Closer view

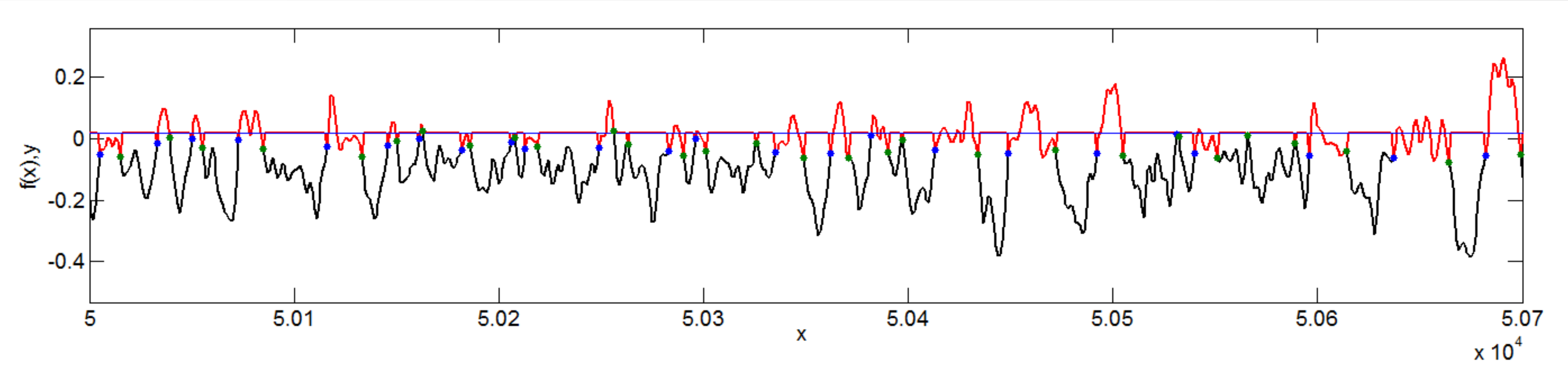


The adhesion intervals are calculated in following scenario:

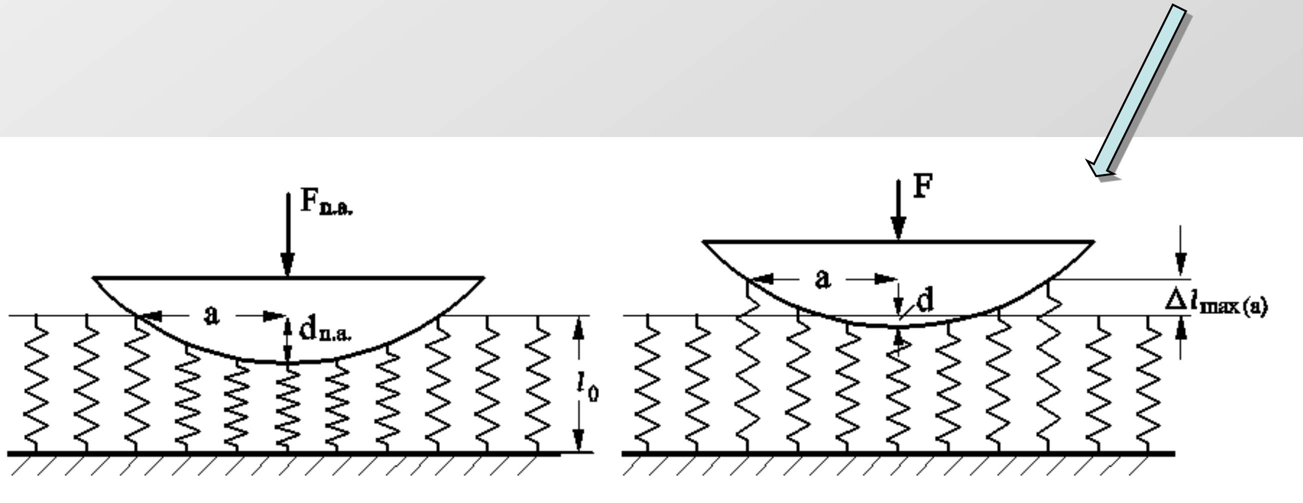
1. At every instant ***y***-position we find all the segments of soft surface which are formally below the hard one;
2. After, we take next segments on the left and right sides (marked by green and blue points respectively) and check for them adhesion condition at given contact length between each pair of such points;
3. If the condition is satisfied, we take next pair of the segments and check the condition again, so on;
4. For every ***y*** value we continue this procedure while at least one of the contact regions is still expanding.

Typical intermediate space configuration of the soft surface and height distribution of the hard surface.

Closer view

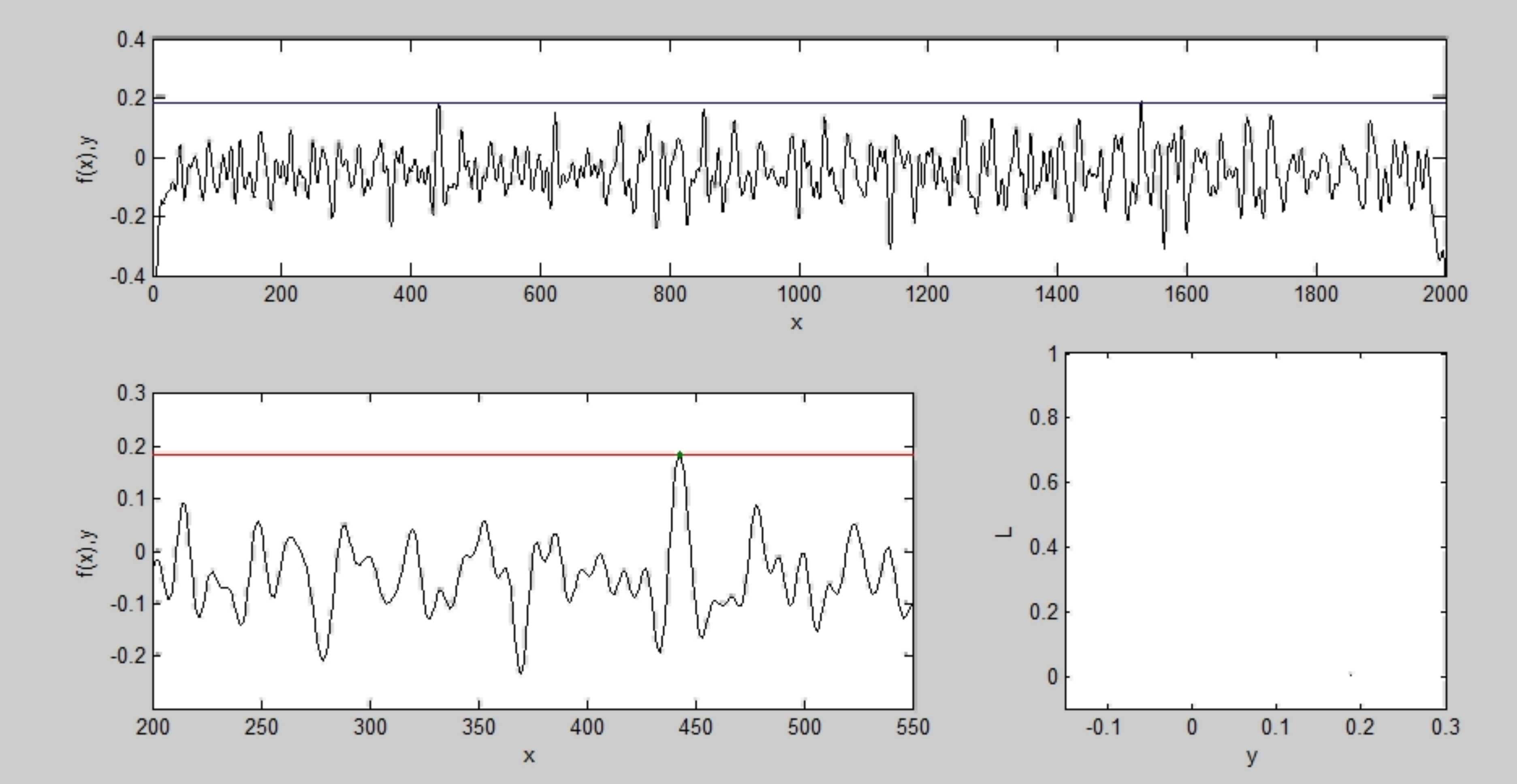


…For every ***y*** value we continue this procedure while at least one of the contact regions is still expanding.



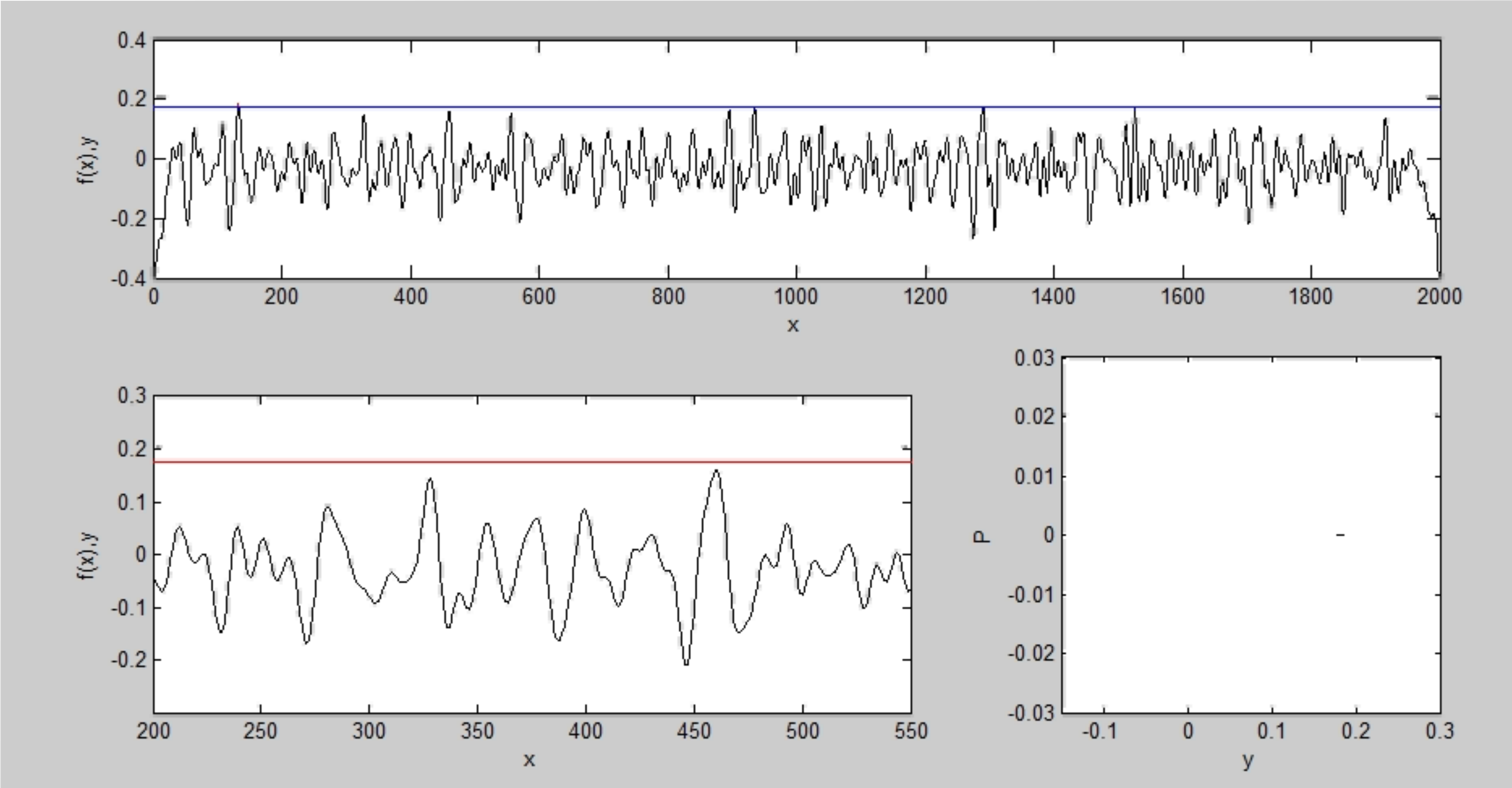
$$\Delta l_{max}(a) = \sqrt{\frac{2a\pi\gamma_{12}}{E^*}}\ .$$

Movies illustrating the numerical procedure.



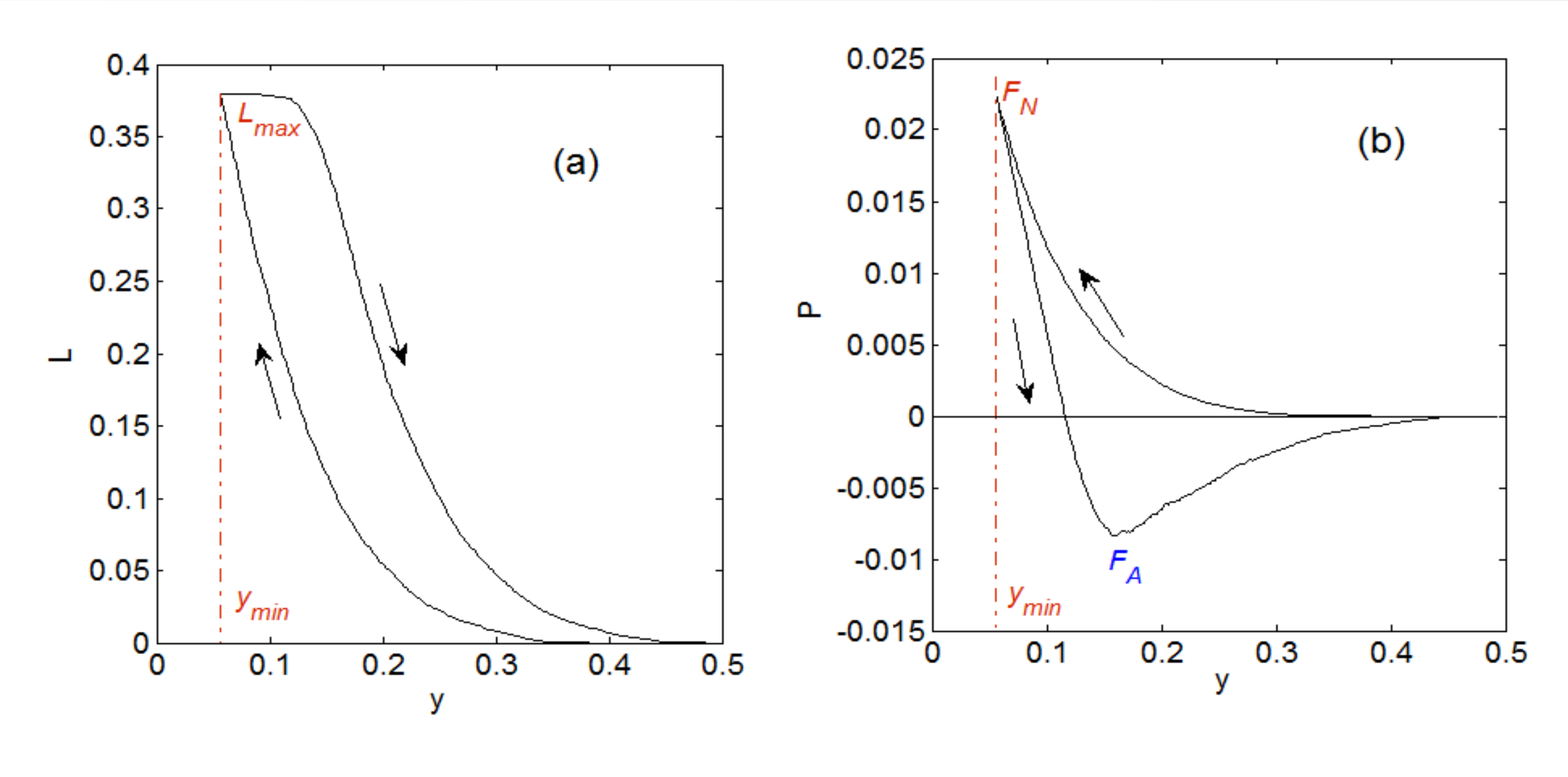
Contact length loop in the system with adhesion

Movies illustrating the numerical procedure.



Total contact force (elastic and adhesion)
in the system under consideration

Typical loops of the force and contact length

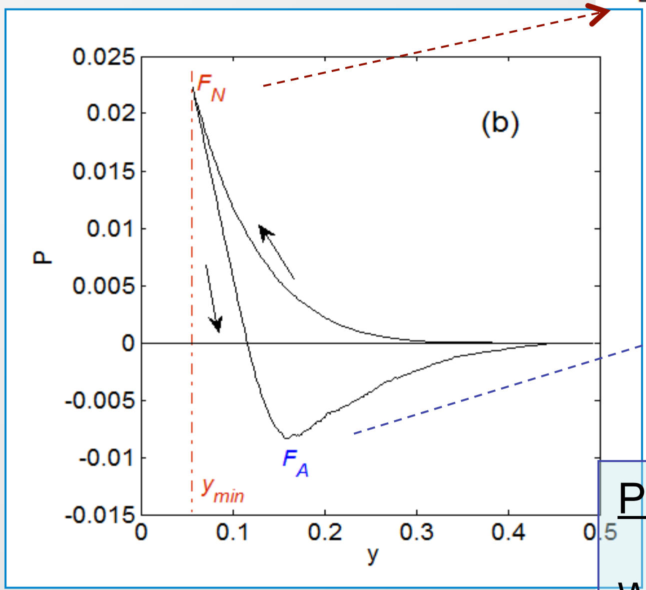


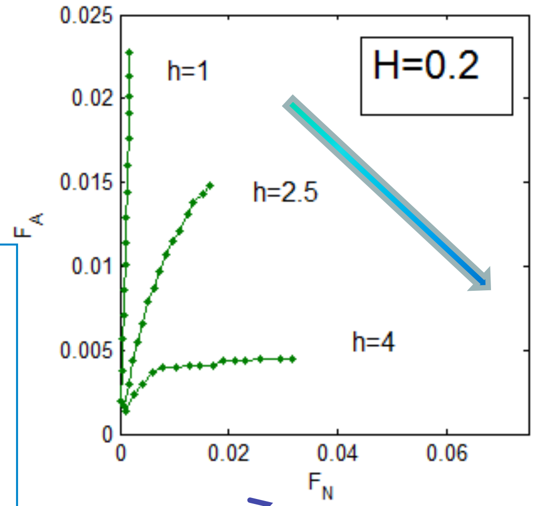
<u>The scenario looks as follows:</u>

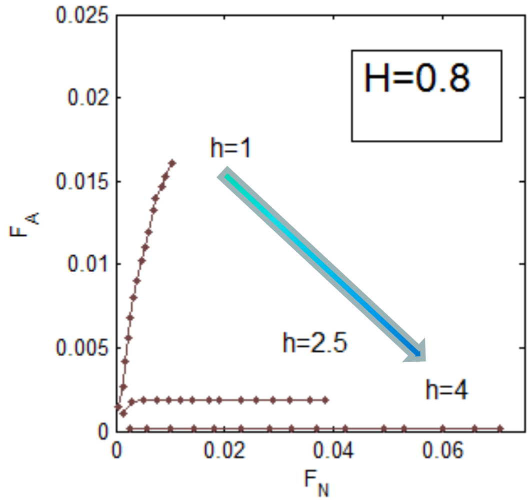
Trial position *y* of the elastic surface goes down to some minimal value ymin and returns back to a position at which last segment of the elastic surface finally detaches from the hard surface.

Calculation of the forces at different y_{min} and multiplicative constant h

General tendencies with variation of the multiplicative constant *h* and Hurst exponent *H*



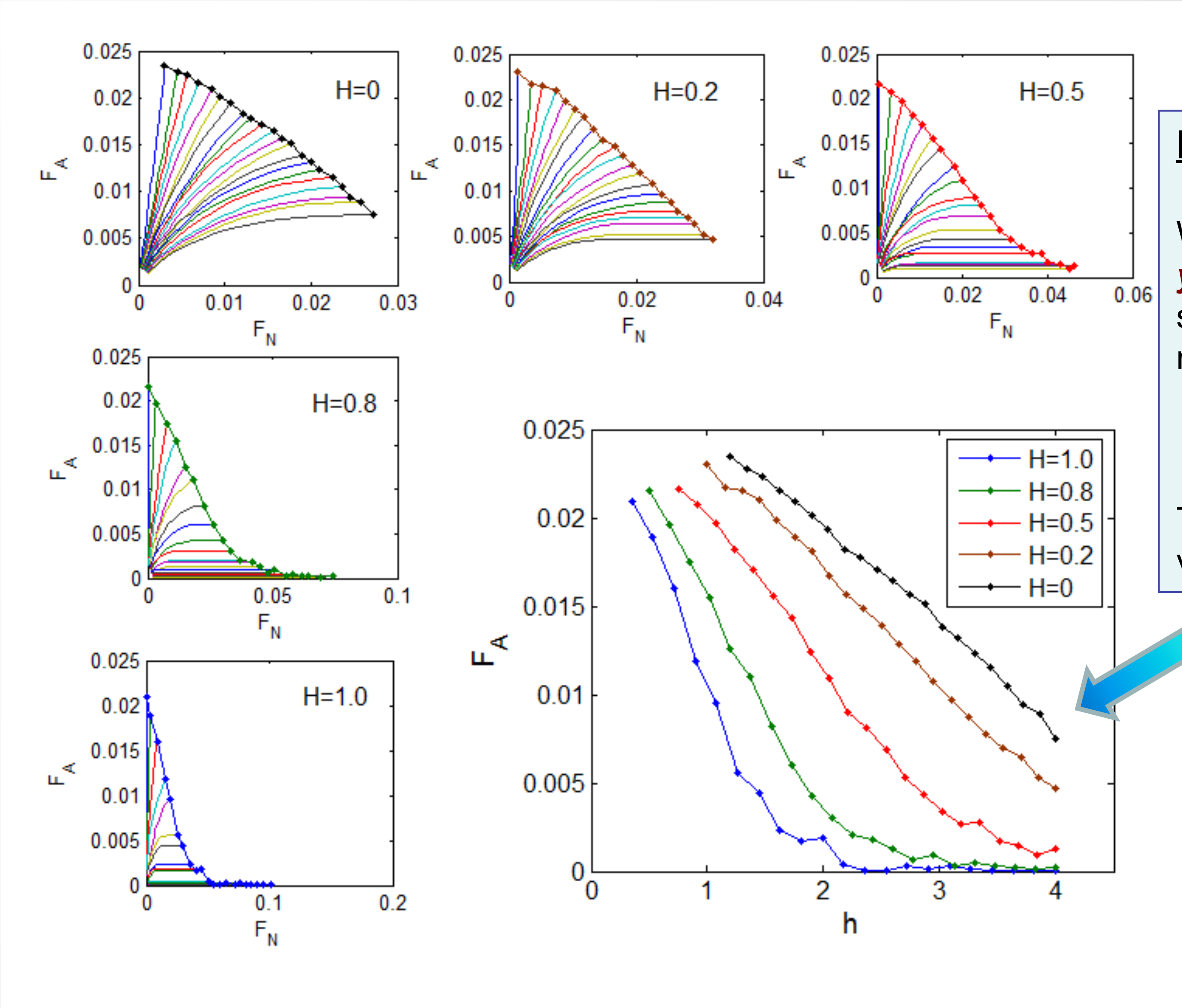




Procedure:

We fix exponent H and vary y_{min} step by step down to the same value at every multiplicative constant h.

Adhesion force curves at varied amplitudes *h* and different Hurst exponents

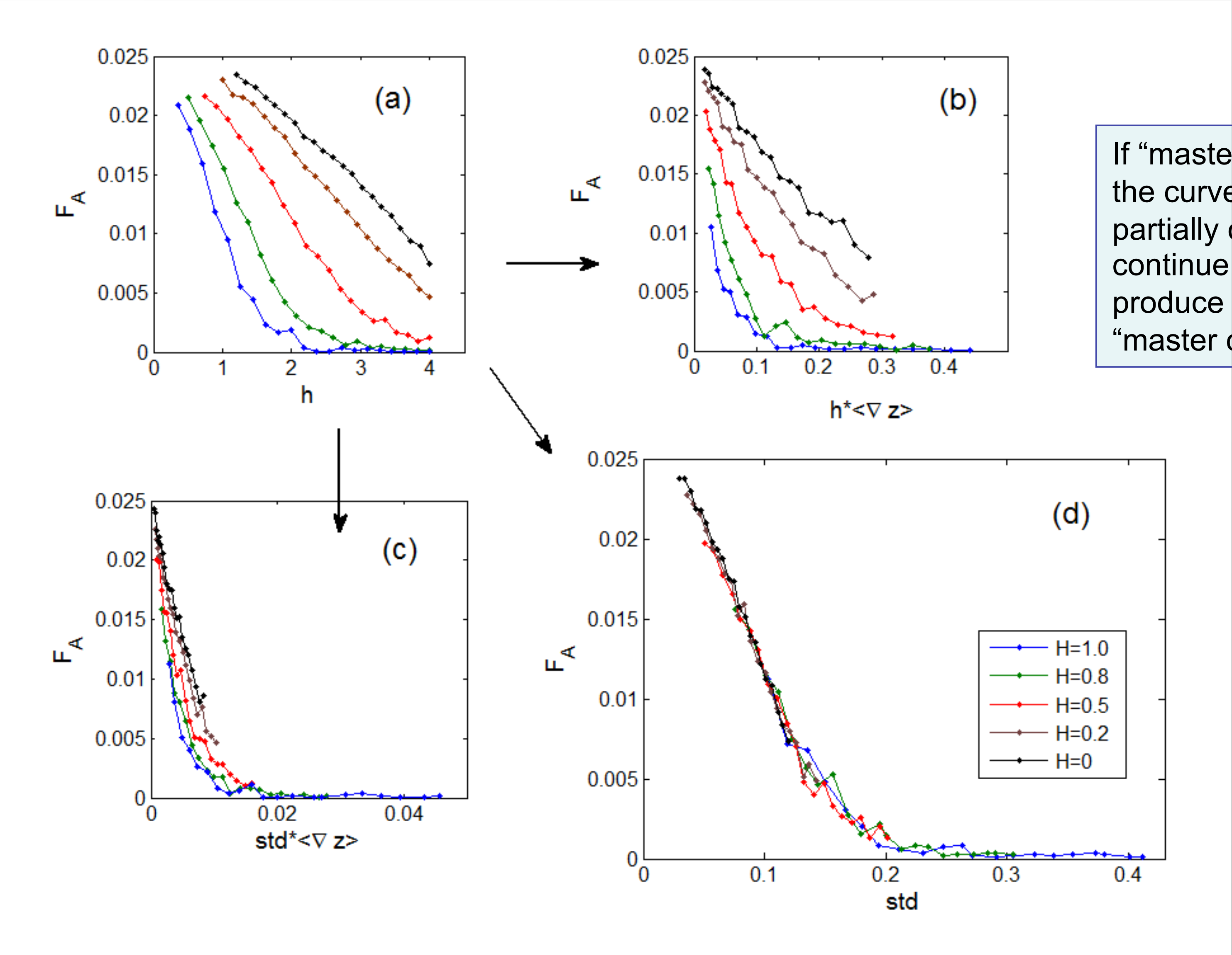


<u>Procedure:</u>

We fix exponent ***H*** and vary ***y_{min}*** step by step down to the same value at every multiplicative constant ***h***.

Then we plot a family of final values ***F_A*** vs ***h.***

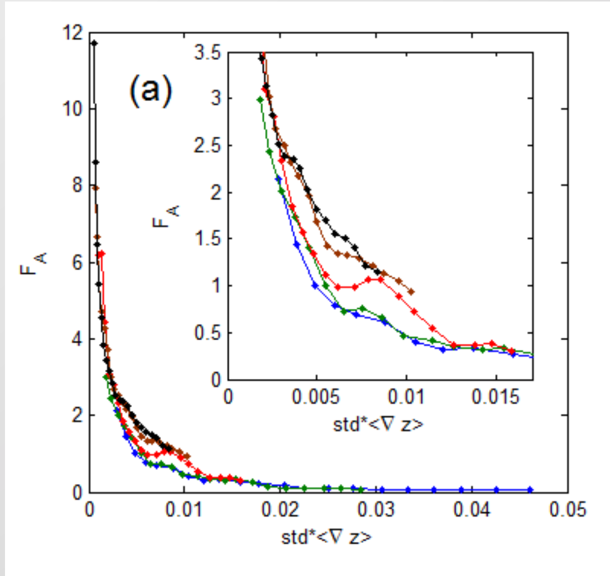
Search for a “**master curve**”
(and respective “master parameter”)

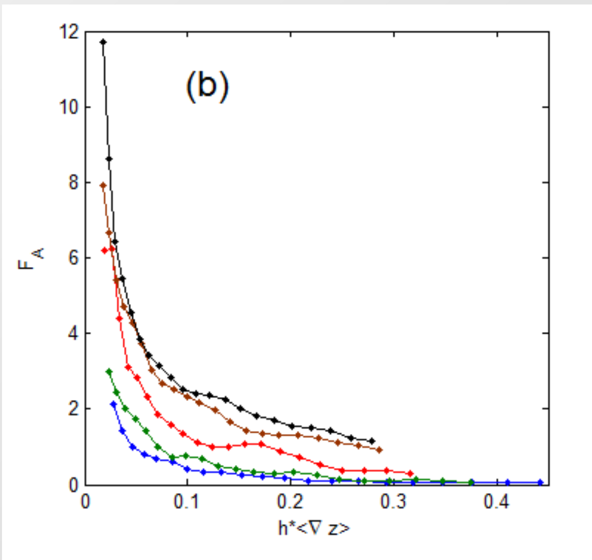


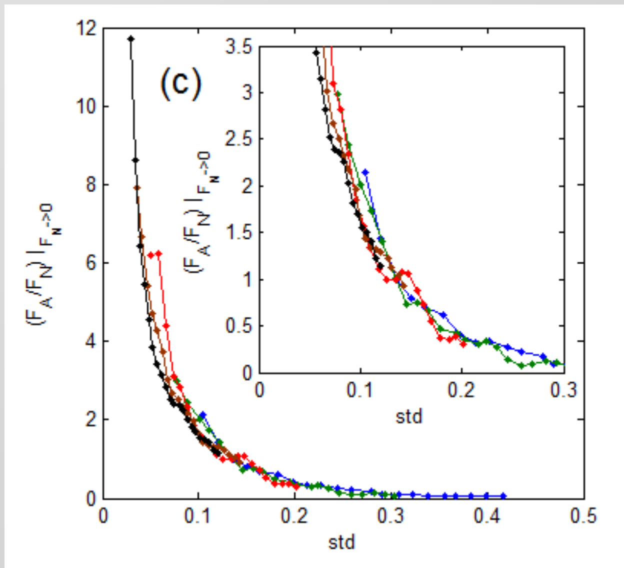
If “master variable” exists the curves of ***F_A*** should partially overlap, partially continue one another to produce corresponding “master curve”*.*

Adhesion coefficient

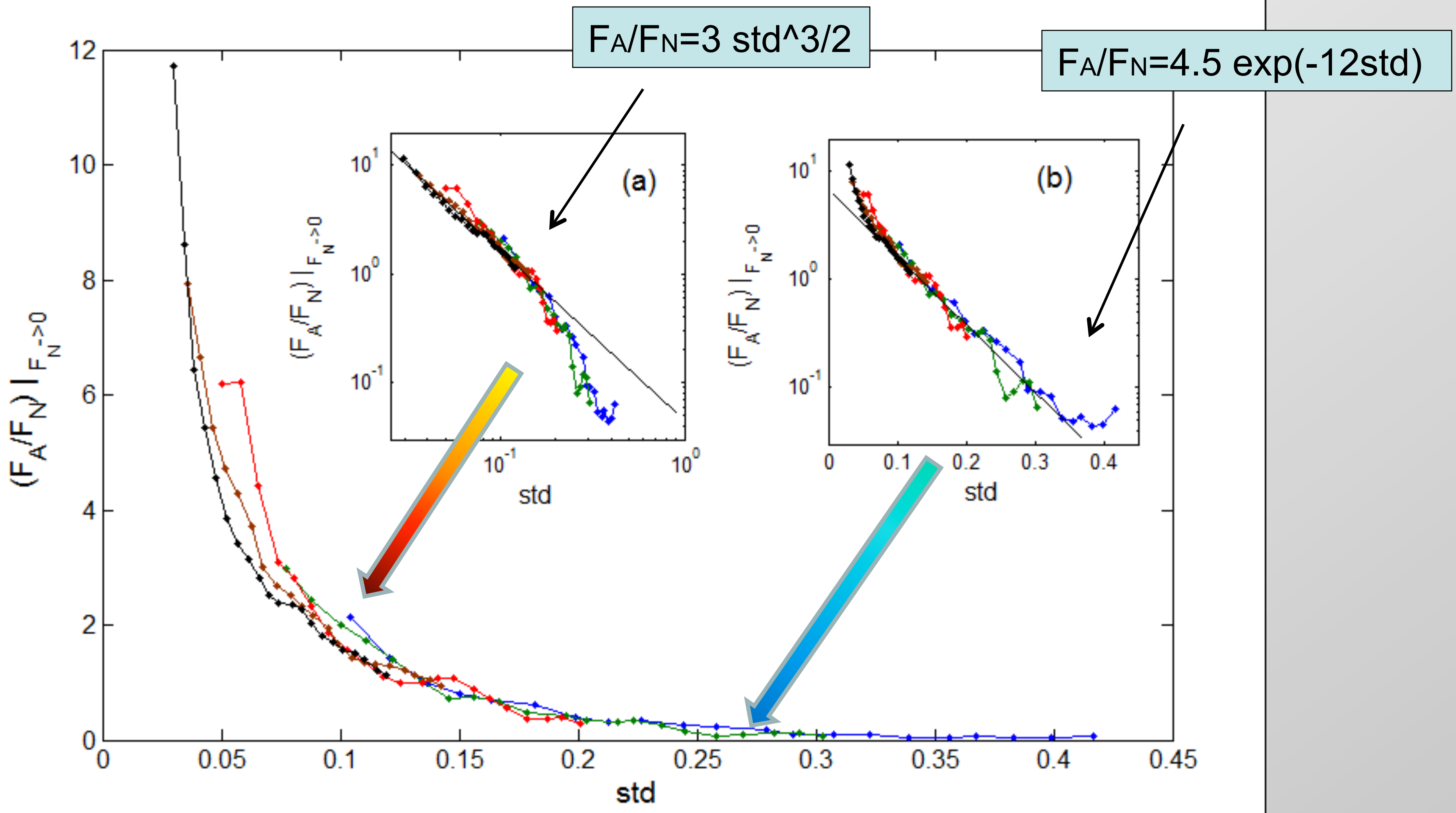
To the choice of a best variable for “master-curve” (best overlap of the F_A curves).





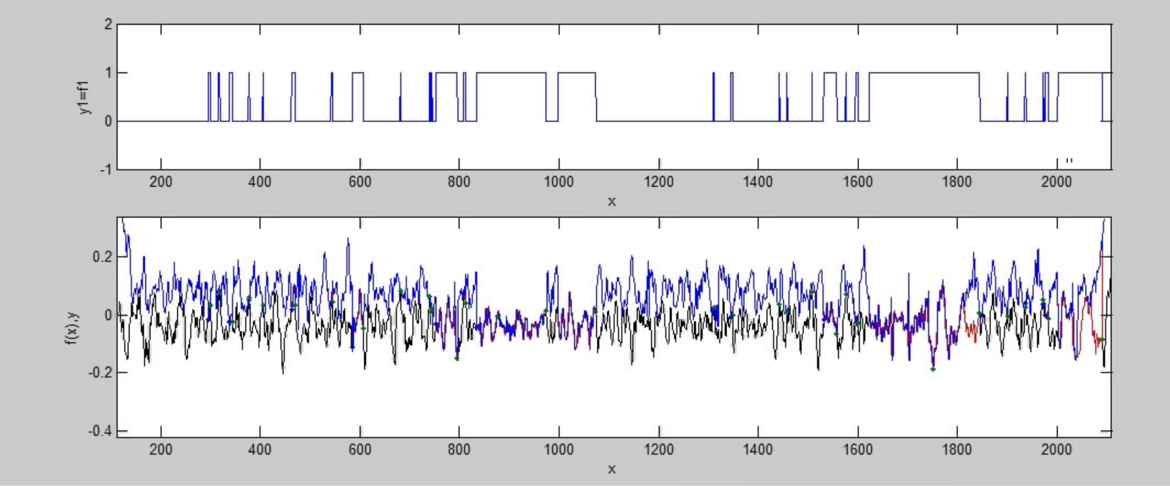


On a relation between pressing and adhesion forces.

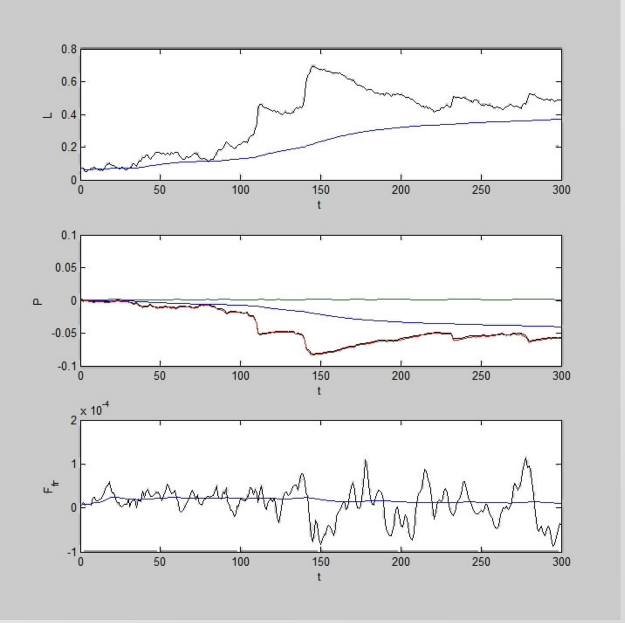


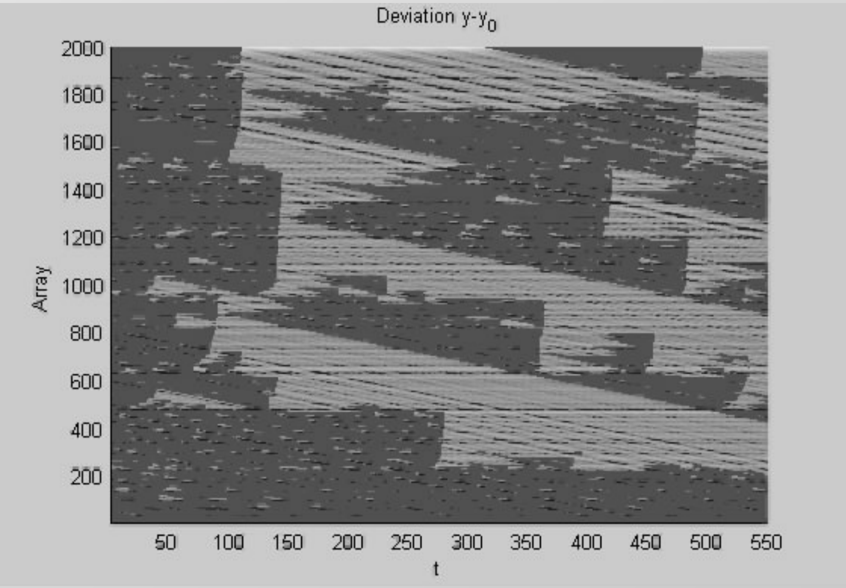
It is not universal and shows **different asymptotic behavior** of the adhesion at different standard deviation.

Application to friction



Now the model is ready to use…





Conclusions

- In some cases **3D** contact problem can be mapped by **1D** calculation

- Numerically generated potentials can be used to check an applicability of the 1D approach to study **adhesion** problem;

- Some general properties of fractal adhesive surfaces are studied at **different values of Hurst exponent** and nontrivial relation between **adhesion coefficient** and **surface roughness** is found;