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Self-accelerating universe in massive gravity

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Self-accelerating universe in massive gravity

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Non-linear massive gravity theory

$$\mathcal{L} = \frac{M_{Pl}^2}{2}\sqrt{-g}\left(R - 2\Lambda - m^2\mathcal{U}\right) \qquad \mathcal{U} = m^2\left[\mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4\right]$$

$$\begin{aligned}
\mathcal{U}_2 &= \mathrm{tr}\,(\mathcal{K}^2) - (\mathrm{tr}\,\mathcal{K})^2, \\
\mathcal{U}_3 &= (\mathrm{tr}\,\mathcal{K})^3 - 3(\mathrm{tr}\,\mathcal{K})(\mathrm{tr}\,\mathcal{K}^2) + 2\mathrm{tr}\,\mathcal{K}^3, \\
\mathcal{U}_4 &= (\mathrm{tr}\,\mathcal{K})^4 - 6(\mathrm{tr}\,\mathcal{K})^2(\mathrm{tr}\,\mathcal{K}^2) + 8(\mathrm{tr}\,\mathcal{K})(\mathrm{tr}\,\mathcal{K}^3) + 3(\mathrm{tr}\,\mathcal{K}^2)^2 - 6\mathrm{tr}\,\mathcal{K}^4
\end{aligned}$$

$$\mathcal{K}_\mu^{\ \nu} \equiv \delta_\mu^{\ \nu} - \left(\sqrt{g^{-1}\left[g - H\right]}\right)_\mu^{\ \nu}$$

$$H_{\mu\nu} = h_{\mu\nu} + \eta_{\beta\nu}\partial_\mu\pi^\beta + \eta_{\alpha\mu}\partial_\nu\pi^\alpha - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta,$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$u_n \propto \det_{(n)}(K_\nu^\mu), \quad K_\nu^\mu = H_\nu^\mu + O(H^2) + \ldots$$

Parameters: m, α_3, α_4

(de Rham, Gadadadze & Tolley '10)

Spherically symmetric solutions

- Metric ansatz in unitary gauge $\pi_\mu = 0$

$$ds^2 = -C(r)\,dt^2 + A(r)\,dr^2 + 2D(r)\,dtdr + B(r)d\Omega^2$$

- Two branches

$$0 = D(r)\,T^{\mathcal{U}}_{tt} + C(r)\,T^{\mathcal{U}}_{tr}$$

$$= m^2 \frac{D(r)\left(2r - 3\sqrt{B(r)}\right)\sqrt{\Delta(r)}}{\sqrt{B(r)}\left(A(r) + C(r) + 2\sqrt{\Delta(r)}\right)^{1/2}} \qquad (\alpha_3 = \alpha_4 = 0)$$

$$D(r) = 0$$

$$\Delta(r) = A(r)C(r) + D^2(r) \equiv \Delta_0 = \text{const.}$$

$$B(r) = 4\,r^2/9$$

- (KK, Niz, Tasinato '10)

Diagonal metric branch

- Metric in the unitary gauge

$$ds^2 = -N(r)^2 dt^2 + F(r)^{-1} dr^2 + r^2 H(r)^{-2} d\Omega^2$$

- Perturbations

$$N = 1 + n, \quad F = 1 + f, \quad H = 1 + h \qquad \rho = \frac{r}{H(r)}$$

$$ds^2 = -(1+2n)dt^2 + (1-\tilde{f})d\rho^2 + \rho^2 d\Omega^2$$

- Linear solutions - vDVZ discontinuity

$$2n = -\frac{8GM}{3\rho}e^{-m\rho}, \qquad \tilde{f} = -\frac{4GM}{3\rho}(1+m\rho)e^{-m\rho}$$

cf. GR solutions

$$2n = -\frac{2GM}{\rho}, \qquad \tilde{f} = -\frac{2GM}{\rho}$$

Strong interactions

- Keep all non-linear terms in h in the limit $m\rho \ll 1$ and at the leading order in GM

$$\tilde{f} = -2\frac{GM}{\rho} - (m\rho)^2 \left[h - (1+3\alpha_3)h^2 + (\alpha_3+4\alpha_4)h^3\right]$$

$$2\rho\, n' = \frac{2GM}{\rho} - (m\rho)^2 \left[h - (\alpha_3+4\alpha_4)h^3\right]$$

$$\frac{\rho_V}{\rho}\left[1-3(\alpha_3+4\alpha_4)h^2\right] = -\left\{\frac{3}{2}h - 3(1+3\alpha_3)h^2 + \left[(1+3\alpha_3)^2 + 2(\alpha_3+4\alpha_4)\right]h^3 - \frac{3}{2}(\alpha_3+4\alpha_4)^2 h^5\right\}.$$

- Vainshtein radius $\rho_V = \left(\frac{GM}{m^2}\right)^{1/3}$

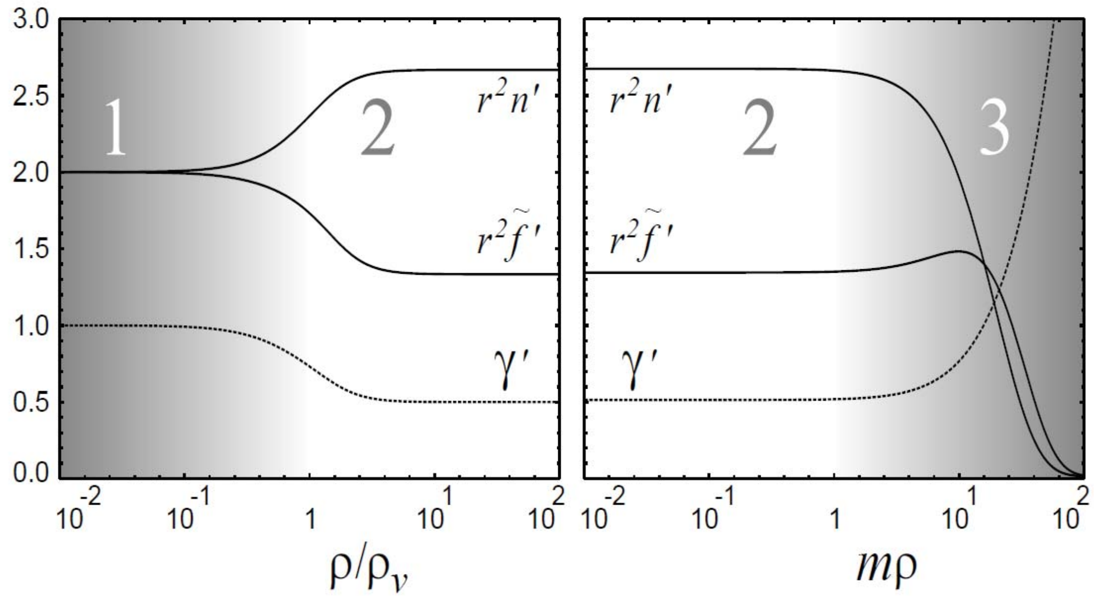
Solutions

$$\alpha_3 = -4\alpha_4$$

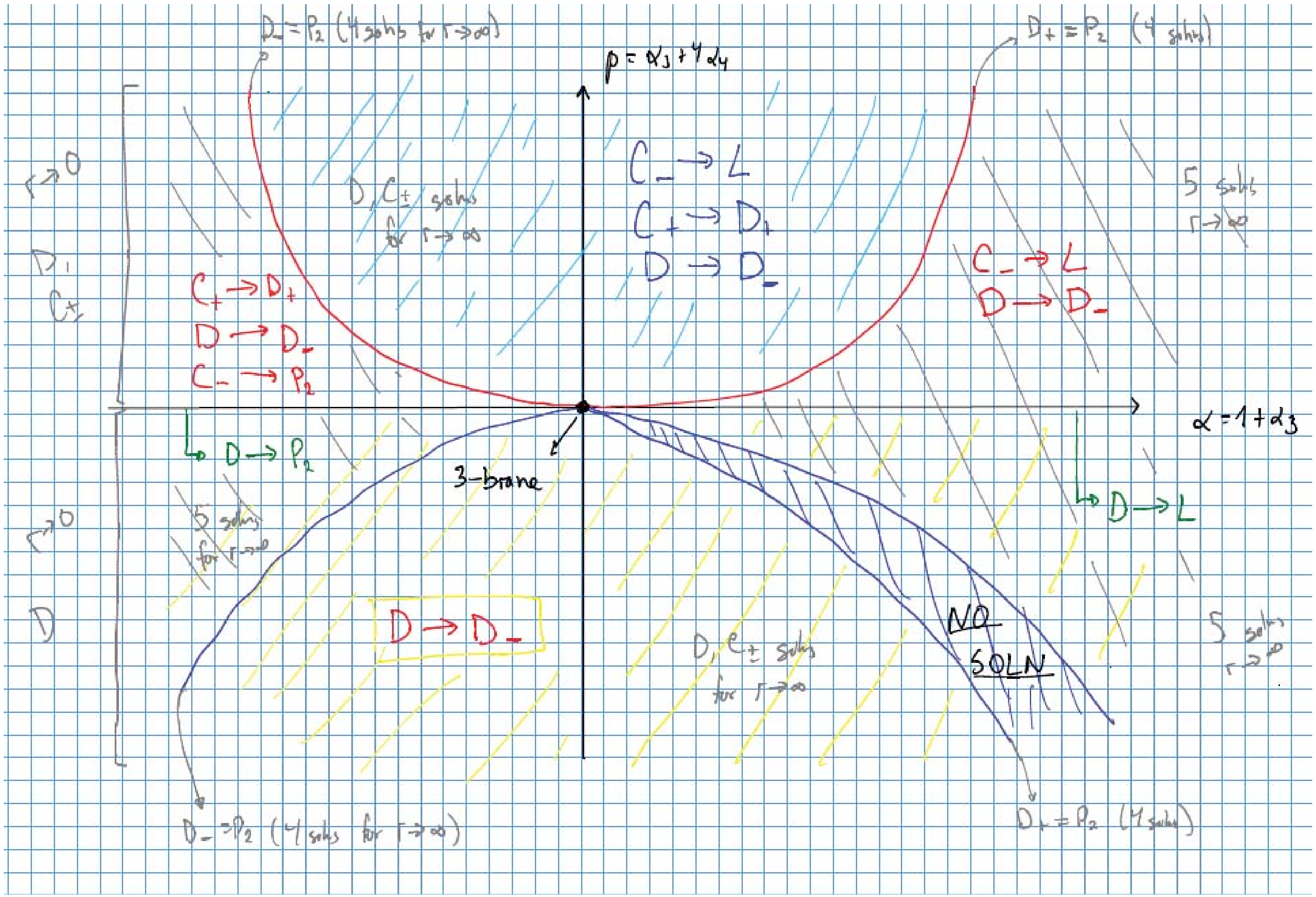
- Inside Vainshtein radius $\rho << \rho_V$ $\quad h = -\frac{1}{(1-12\alpha_4)^{2/3}}\frac{\rho_V}{\rho}$

$$2n = -\frac{2GM}{\rho}\left[1 - \frac{1}{2(1-12\alpha_4)^{2/3}}\left(\frac{\rho}{\rho_V}\right)^2\right] \qquad \rho_V = \left(\frac{GM}{m^2}\right)^{1/3}$$

$$\tilde{f} = -\frac{2GM}{\rho}\left[1 - \frac{1}{2(1-12\alpha_4)^{1/3}}\left(\frac{\rho}{\rho_V}\right)\right]$$



Classification of solutions



(Sbisa, Niz, KK, Tasinato ‘11) Ask Fulvio Sbisa in the audience for details

Non-diagonal metric branch

- Metric in the unitary gauge (cf. . Salam & Strathdee '77)

$$ds^2 = -C(r)\,dt^2 + 2D(r)\,dtdr + A(r)\,dr^2 + B(r)\,d\Omega^2$$

for simplicity we take $\alpha_3 = -4\alpha_4$

$$A(r) = \frac{9}{4}\Delta_0\left(\frac{1-8\alpha_4}{1-12\alpha_4}\right)^2[p(r)+\gamma+1] \quad , \quad B(r) = \frac{4}{9}\left(\frac{1-12\alpha_4}{1-8\alpha_4}\right)^2 r^2$$

$$C(r) = \frac{9}{4}\Delta_0\left(\frac{1-8\alpha_4}{1-12\alpha_4}\right)^2[1-p(r)] \quad ,$$

$$D(r) = \frac{9\Delta_0}{4}\left(\frac{1-8\alpha_4}{1-12\alpha_4}\right)^2\sqrt{p(r)\,(p(r)+\gamma)}$$

$$p(r) \equiv \frac{\mu}{r} + \frac{(1-12\alpha_4)\;m^2 r^2}{9\,(1-8\alpha_4)^2} \quad , \quad \gamma \equiv \frac{16}{81\Delta_0}\left(\frac{1-12\alpha_4}{1-8\alpha_4}\right)^4 - 1.$$

Δ_0 : integration constant

Diagonal metric in non-unitary gauge

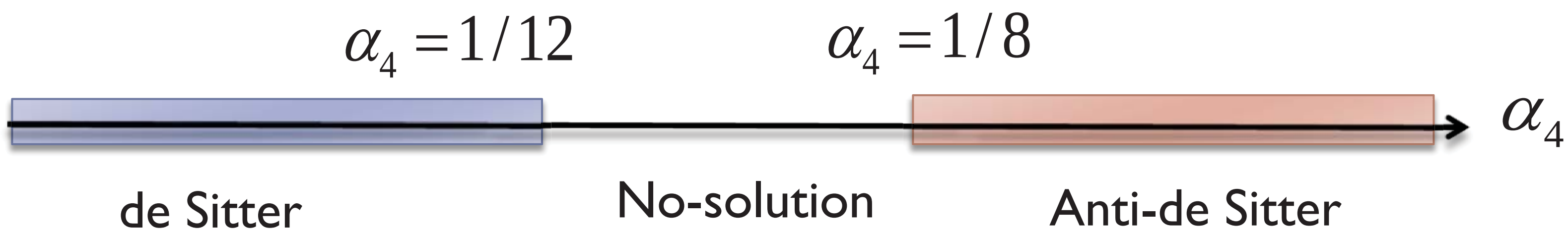
- Coordinate transformation (non-unitary gauge)

$$ds^2 = -C(r)dt^2 + \tilde{A}(r)dr^2 + B(r)\,d\Omega^2 \qquad \pi^\mu = (\pi_0(r), 0, 0, 0)$$

$$\tilde{A}(r) = \frac{4}{9}\left(\frac{1-12\alpha_4}{1-8\alpha_4}\right)^2 \frac{1}{1-p(r)} \quad , \quad \pi_0'(r) = -\frac{\sqrt{p(r)(p(r)+\gamma)}}{1-p(r)}$$

$$C(r) = \frac{9}{4}\Delta_0\left(\frac{1-8\alpha_4}{1-12\alpha_4}\right)^2 [1-p(r)] \qquad p(r) \equiv \frac{\mu}{r} + \frac{(1-12\alpha_4)\, m^2 r^2}{9\,(1-8\alpha_4)^2}$$

Solutions are exactly Schwartzchild-(anti)de Sitter solutions



- (KK, Niz, Tasinato '10)

Time-dependent metric

- We can add a bare cosmological constant Λ

$$p(r) = \frac{\mu}{r} + \frac{(1-12\alpha_4)}{9\,(1-8\alpha_4)^2}\left[m^2 + \frac{4}{3}\,(1-12\alpha_4)\,\Lambda\right] r^2$$

- Coordinate transformation for $\mu = 0$,

$$t = F_t(\tau,\rho) \text{ and } r = F_t(\tau,\rho)$$

$$F_t(\tau,\rho) = \frac{4}{3\,\Delta_0^{1/2}\,\tilde{m}}\left(\frac{1-12\alpha_4}{1-8\alpha_4}\right)\operatorname{arctanh}\left(\frac{\sinh\left(\frac{\tilde{m}\tau}{2}\right) + \frac{\tilde{m}^2\rho^2}{8}e^{\tilde{m}\tau/2}}{\cosh\left(\frac{\tilde{m}\tau}{2}\right) - \frac{\tilde{m}^2\rho^2}{8}e^{\tilde{m}\tau/2}}\right)$$

$$F_r(\tau,\rho) = \frac{3}{2}\left(\frac{1-8\alpha_4}{1-12\alpha_4}\right)\rho\, e^{\tilde{m}\tau/2}.$$

de Sitter metric

$$ds^2 = -d\tau^2 + e^{\tilde{m}\tau}\,\left(d\rho^2 + \rho^2 d\Omega^2\right) \quad H = \frac{\tilde{m}}{2} = \frac{1}{2(1-12\alpha_4)^{\frac{1}{2}}}\left[m^2 + \frac{4}{3}\,(1-12\alpha_4)\,\Lambda\right]^{\frac{1}{2}}$$

General solutions

- Two branches

$$
\begin{aligned}
B(r) &= b_0\, r^2 \,, \\
C(r) &= c_0 + \frac{c_1}{r} + c_2\, r^2 \\
A(r) + C(r) &= Q_0 \,, \\
D^2(r) + A(r)C(r) &= \Delta_0 .
\end{aligned}
$$

$$
b_0 = \left(\frac{1 + 6\alpha_3 + 12\alpha_4 + \Gamma_\pm}{3(1 + 3\alpha_3 + 4\alpha_4)} \right)^2 , \qquad c_0 = \frac{\Delta_0}{b_0}
$$

$$
\Gamma_\pm \equiv \pm\sqrt{1 + 3\alpha_3 + 9\alpha_3^2 - 12\alpha_4} .
$$

$$
c_2 = \frac{\Delta_0 m^2}{9 b_0 (1 + 3\alpha_3 + 4\alpha_4)^2} \left[1 - 2\Gamma_\pm + 4\alpha_4(2\Gamma_\pm - 7) + 2\alpha_3(1 - 18\alpha_4 - 2\Gamma_\pm) + \alpha_3^2(15 - 6\Gamma_\pm) + 18\alpha_3^3 \right]
$$

$\Rightarrow$ $\Lambda_{eff} = m\, F_\pm(\alpha_3, \alpha_4)$ $\qquad$ Δ_0 : integration constant

Cosmological solutions

- Self-accelerating de Sitter solution $\Lambda_{eff} = m\, F_{\pm}(\alpha_3, \alpha_4)$
- Some issues with our solution in the FRW slicing
  - The solution for π^t is singular at horizon and it does not respect the FRW symmetry
- No go theorem (D'Amico et.al.'11)
  - In fact, it is not possible to find flat/closed FRW solutions that respect the FRW symmetry for π^μ
  - However, this does not apply to the open FRW solutions

    (Gumrukcuoglu, Lin and Mukohyama '11)

For a given solution of physical metric $g_{\mu\nu}$, there are many possible solutions for π^t

Differences show up for perturbations (?)

Decoupling limit

(Arkani-Hamed, Georgi & Schwartz '03)

(de Rham, Gadadadze '10)

- Canonically normalised fields $\pi^{\mu} = \eta^{\mu\nu}\left(\partial_{\nu}\hat{\pi} + \hat{A}_{\nu}\right)$

$$h_{\mu\nu} \to M_{Pl}\,\hat{h}_{\mu\nu} \quad , \quad A_{\mu} \to m M_{Pl}\,\hat{A}_{\mu} \quad , \quad \pi \to m^2 M_{Pl}\,\hat{\pi}$$

take a limit

$$m \to 0 \quad , \quad M_{Pl} \to \infty \quad , \quad \Lambda_3 \equiv m^2 M_{Pl} = \text{fixed}$$

Perform coordinate transformation so that metric is finite

$$ds^2 = \left[1 - \frac{m^2}{8}(r^2 - t^2)\right](-dt^2 + dr^2 + r^2 d\Omega^2) + \mathcal{O}(m^3) \qquad (\alpha_3 = \alpha_4 = 0)$$

$$\hat{A}_{\mu} = \left(-\frac{\Lambda_3 r^2}{24}\sqrt{\frac{16}{\Delta_0} - 81} + \mathcal{O}(m^2), 0, 0, 0\right) ,$$

$$\hat{\pi} = -\frac{\Lambda_3}{4}(r^2 - t^2) + \frac{3\,\Lambda_3}{4}\left(\frac{4}{9\sqrt{\Delta_0}} - 1\right)t^2 + \mathcal{O}(m^2) .$$

Full solutions with different π^t converge to these solutions

(de Rham, Gadadadze, Heisenberg, Pirtskhalava '10)

Decoupling limit with vector (KK, Niz, Tasinato '11)

- There are solutions with a vector charge if $\Delta_0 \neq \frac{16}{81}$
- Decoupling theory without vector

$$K^{\nu}_{\mu} = \delta^{\nu}_{\mu} - \sqrt{(\delta^{\nu}_{\mu} - \partial^{\nu}\partial_{\mu}\pi)^2} = \partial^{\nu}\partial_{\mu}\pi$$

Include vectors

$$\delta^{\nu}_{\mu} - K^{\nu}_{\mu} = \sqrt{P^2(1 + mQ_1(A) + m^2 Q_2(A))^2} \qquad P^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \partial^{\mu}\partial_{\nu}\pi$$

$$= P(1 + mQ_1(A) + m^2 Q_2(A)) + mD + m^2 E$$

$$2\,D \;=\; -P^{-1}\,\eta^{-1}dA\,P + 2\sum_{n=2}^{\infty}(-1)^n\;P^{-n}\,\eta^{-1}dA\,P^n \qquad \mathrm{tr}\,E \;=\; -\frac{1}{2}\,\mathrm{tr}\,\left(P^{-1}\,D^2\right)$$

$$+\,\eta^{-1}dA^T + 2\sum_{n=1}^{\infty}(-1)^n\;P^n\,\eta^{-1}dA^T\,P^{-n}$$

$$g \equiv g_{\mu\nu},\; \eta = \eta_{\mu\nu},\; \eta^{-1} = \eta^{\mu\nu} \qquad dA = \partial_{\mu}A_{\nu}$$

Vector-scalar coupling

- Infinite series of coupling

$$L = \left(-\frac{1}{4} + O(\pi) + O(\pi^2) + \dots. \right) F_{\mu\nu} F^{\mu\nu}$$

- In some special cases, we can sum up all terms non-perturbatively

$$\pi = \pi(t,r), \quad A_\mu = (A_t(t,r), A_r(t,r), 0, 0) \qquad (\Lambda_3 = 1)$$

$$L = -\frac{1}{2} \frac{(1 + \pi'/r)}{2 + \ddot{\pi} - \pi''} F_{\mu\nu} F^{\mu\nu}$$

- Tensor-scalar coupling terms are finite

$$L = h_{\mu\nu} \left(\overset{(1)}{X}{}^{\mu\nu}(\pi) + \overset{(2)}{X}{}^{\mu\nu}(\pi) + \overset{(3)}{X}{}^{\mu\nu}(\pi) \right)$$

Decoupling theory with vector

- Special ansatz

$$g_{\mu\nu} = \left(1 + \frac{\hat{f}(t,r)}{M_{Pl}}\right)\eta_{\mu\nu}$$

$$\hat{A}_\mu = (\hat{A}_t(r), 0, 0, 0)$$
$$\hat{\pi} = \hat{\pi}_t(t) + \hat{\pi}_r(r).$$

$$\mathcal{L}_{dec} = \frac{r^2}{2}\left[\frac{F_1(\pi)}{m^2} + \mathcal{L}_R^{(2)}(\hat{f}) + \Lambda_3 F_2(\pi)\hat{f}(t,r) + F_3(\pi)(\partial_r \hat{A}_t)^2\right]$$

Total derivative

Coupling between scalar and tensor

Coupling between scalar and vector

$$F_1(\pi) = (2\pi'' + \frac{\pi'}{r})\frac{\pi'}{r} - (\pi'' + \frac{2\pi'}{r})\ddot{\pi}$$
$$F_2(\pi) = 2(3 + \frac{\pi'}{r})\frac{\pi'}{r} + (3 + \frac{4\pi'}{r})\pi'' - (3 + 2\pi'' + 4\frac{\pi'}{r})\ddot{\pi}$$
$$F_3(\pi) = \frac{r + 2\pi'}{r(2 + \ddot{\pi} - \pi'')}.$$

$$\hat{f} = -\frac{\Lambda_3}{8}(r^2 - t^2),$$
$$\hat{A}_t = \frac{\Lambda_3 Q_0}{2} r^2,$$
$$\hat{\pi} = -\frac{\Lambda_3}{4}(r^2 - t^2) + \frac{3\Lambda_3}{4}\left(\sqrt{1 + \frac{16 Q_0^2}{9}} - 1\right) t^2$$

Stability analysis

- Spherically symmetric "scalar" perturbations

$$\pi = \pi_0 + \pi(t,r), \quad A_\mu = (A_{t0}(r) + A_t(t,r), A_r(t,r), 0, 0)$$

$$L = -\frac{1}{2}\frac{(1+\pi_0'/r)}{2+\ddot{\pi}_0 - \pi_0''} F_{\mu\nu}F^{\mu\nu} = 0$$

(cf. de Rham, Gadadadze, Heisenberg, Pirtskhalava '10)

- Without the background vector charge

  vector field perturbations do not propagate

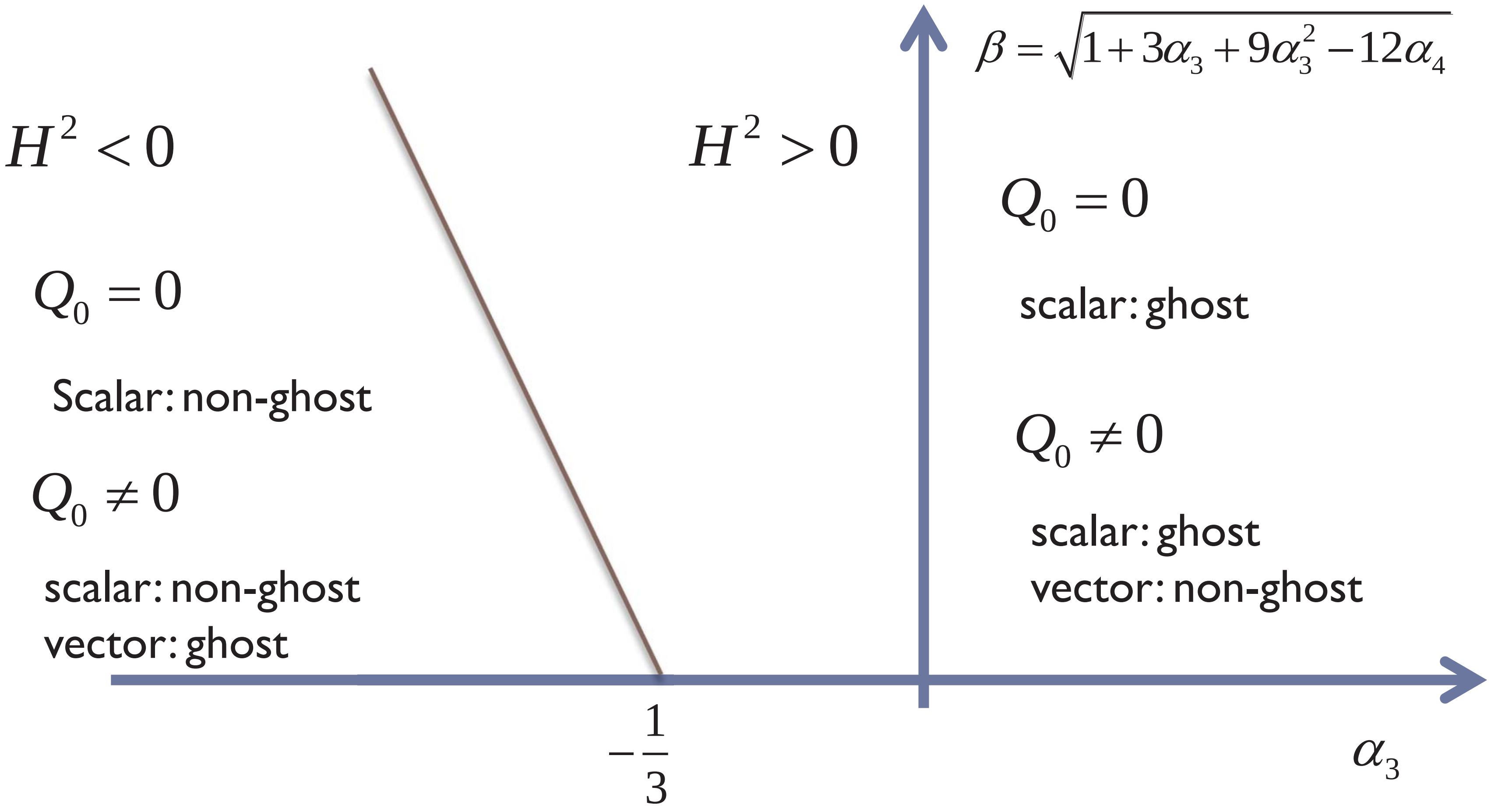
- With the background vector charge

  The vector-scalar coupling generates vector field perturbations

$$L = Q_0 f(Q_0, \alpha_3, \alpha_4)\, \dot{\pi} \dot{A}_t$$

Preliminary result

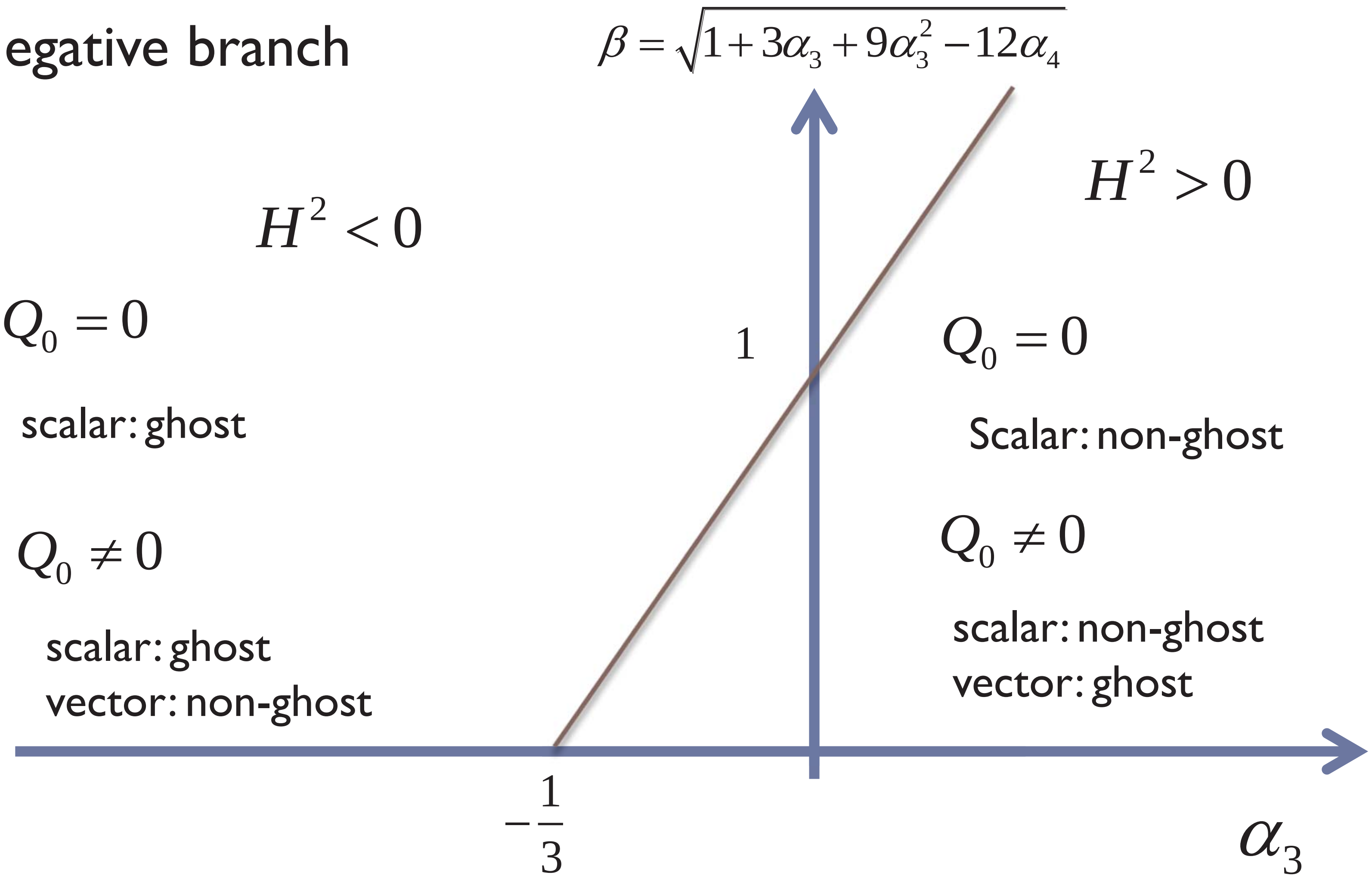
- Positive branch (include de Sitter solution for $\alpha_3 = \alpha_4 = 0$)



(cf. de Rham, Gadadadze, Heisenberg, Pirtskhalava '10)

Preliminary result

- Negative branch



(cf. de Rham, Gadadadze, Heisenberg, Pirtskhalava '10)

Conclusion

- Self-accelerating solution in non-linear massive gravity

$$\Lambda_{eff} = m\, F_{\pm}(\alpha_3, \alpha_4)$$

Solutions for $\pi^t(t, r)$ are not unique

- Decoupling limit solutions are unique but there can be a vector charge

- Preliminary results indicate that the ghost appears if the background vector charge is present

- If there is no vector charge, there is a parameter region without ghost

Open questions

- Is there a symmetry to forbid the background vector charge?
  (e.g. de Sitter invariance for π^{μ})

$$\pi_0 = -\frac{1}{4}(r^2 - t^2) + \frac{3}{4}\left(\sqrt{1+\frac{16Q_0^2}{9}} - 1\right)t^2$$

- Are the vector field perturbations strongly coupled?

$$L = -\frac{1}{2}\frac{(1+\pi_0'/r)}{2+\ddot{\pi}_0 - \pi_0''} F_{\mu\nu}F^{\mu\nu} = 0$$

- Vector-scalar coupling is non-trivial even in decoupling limit
  Stability around a non-trivial background supported by matter?

- Cosmological consequences of the self-accelerating universe
  Can we distinguish it from LCDM perturbations?
  work in progress
  cf. in decoupling limit, helicity-0 mode does not couple to matter
  (cf. de Rham, Gadadadze, Heisenberg, Pirtskhalava '10)