The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

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Workshop on Infrared Modifications of Gravity

26 - 30 September 2011

Solar System Tests of Relativistic Gravity: Recent Progress and Possible Future Directions

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Solar System Tests of Relativistic Gravity:

Recent Progress and Possible Future Directions

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"Workshop on Infrared Modifications of Gravity",
The Abdus Salam International Centre for Theoretical Physics, 26-30 September 2011, Trieste, Italy

Outline: Solar System Tests of Gravity

The talk will cover:

- Theoretical Landscape in the 20th Century:
  - (brief…) History of the tests of general relativity
  - Frameworks used: the PPN formalism and Robertson-Mansouri-Sexl
  - Recent progress in the tests of general relativity
- Beginning of the 21st Century…:
  - Motivations for high-precision tests of gravity
  - What to expect in the near future? and some proposed experiments
- Main objective:
  - Remind where we came from and what lessons we learned
- Themes for discussion:
  - Are the solar system tests still useful?
  - Is there a discovery potential? Or what is the importance of new improved limits?
  - What tests are most valuable?

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

not a complete list…

Newton 1686	Poincaré 1890				
Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915	
Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943	
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956	
Brans & Dicke 1961	Yilmaz 1962	Whitrow & Morduch 1965	Kustaanheimo & Nuotio 1967		
Page & Tupper 1968	Bergmann 1968	Deser & Laurent 1968	Nordtvedt 1970	Wagoner 1970	
Bollini et al. 1970	Rosen 1971	Will & Nordtvedt 1972	Ni 1972	Hellings & Nordtvedt 1972	
Ni 1973	Yilmaz 1973	Lightman & Lee 1973	Lee, Lightman & Ni 1974	Rosen 1975	
Belinfante & Swihart 1975	Lee et al. 1976	Bekenstein 1977	Barker 1978	Rastall 1979	
Coleman 1983	Hehl 1997	Overlooked (20th century)			

- Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
- Some theories are just variations of others
- Some theories were proposed in the 1910s/20s; many theories in the 1960s/70s
- Overlooked: this is not a complete list!

Theory must be:

- **Complete**: not a law, but a theory. Derive experimental results from first principles
- **Self-consistent**: get same results no matter which mathematics or models are used
- **Relativistic**: Non-gravitational laws are those of Special Relativity
- **Newtonian**: Reduces to Newton's equation in the limit of low gravity and low velocities

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that fail already

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | Will & Nordtvedt 1972 | Ni 1972 | Hellings & Nordtvedt 1972

Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | Hehl 1997 | Overlooked (20th century)

- Newton (1686) - non-relativistic: implicit action at a distance - incompatible with special relativity
- Poincare (1890) and conformally flat theory of Whithrow-Morduch (1965) - incomplete: do not mesh with non-gravitational physics (Maxwell)
- Fierz & Pauli (1939) ["spin-2 field theory"] - inconsistent: field equations -> all gravitating bodies move along straight lines, equation of motion -> gravity deflects bodies
- Birkhoff (1943) - not Newtonian: demands *speed of sound = speed of light*.
- Milne (1948) – incomplete - no gravitational red-shift prediction
- Kustaanheimo-Nuotio (1967) – inconsistent – grav. redshift for photons, but not for light waves.

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate the Einstein's Equivalence Principle

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | Will & Nordtvedt 1972 | Ni 1972 | Hellings & Nordtvedt 1972

Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | Hehl 1997 | Overlooked (20th century)

Einstein's Equivalence Principle (EEP):

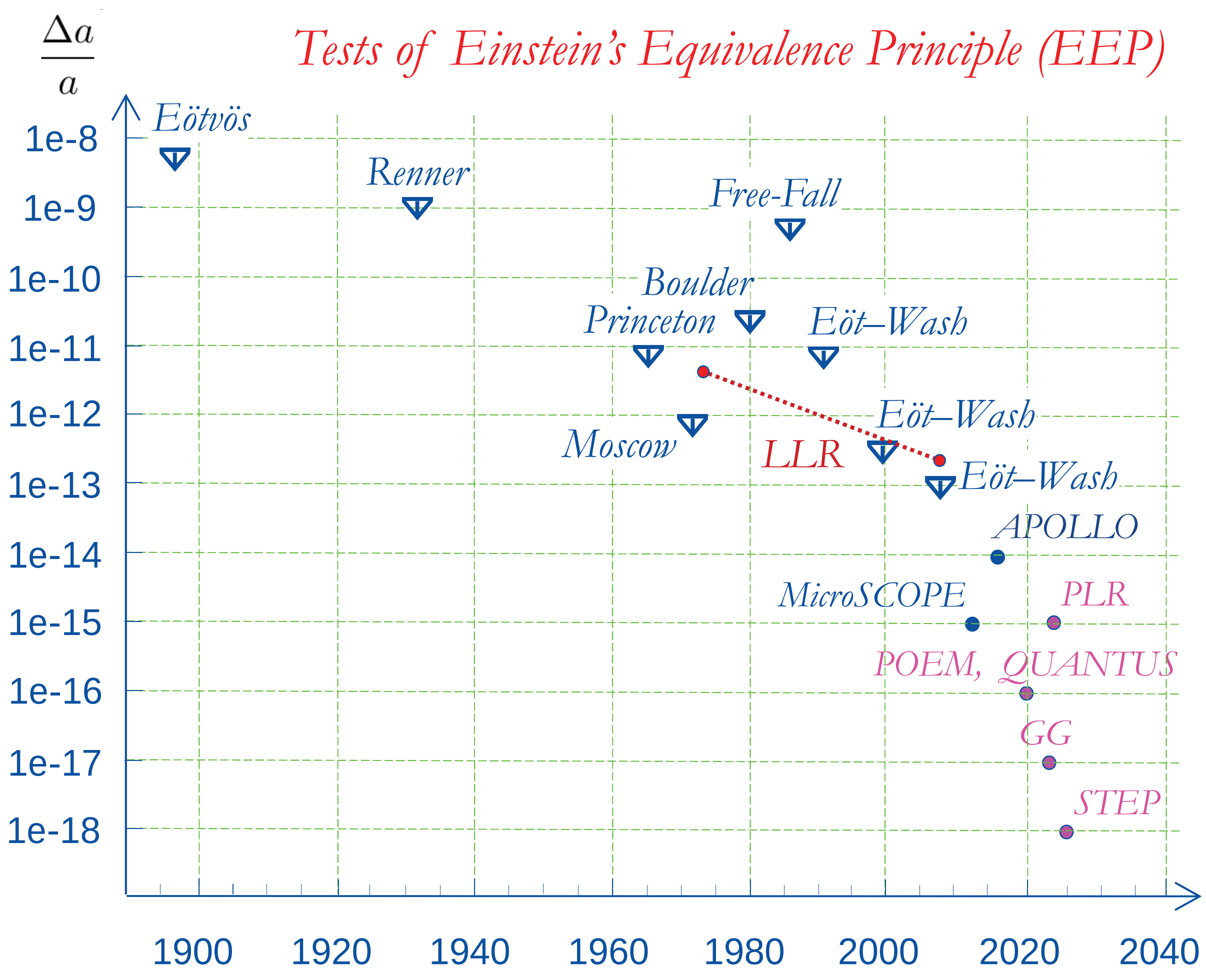
- Uniqueness of Free Fall
- Local Lorentz Invariance
- Local Position Invariance

Only metric theories are viable:

- Belinfante & Swihart (1975): not a metric theory
- Kaluza-Klein (1932): violates EEP
- Still too many theories around…

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment



- • funded projects
- • proposed projects
- *LLR*, *APOLLO*, and *PLR* testing the Strong Equivalence Principle (SEP)

$$\left[\frac{m_G}{m_I}\right]_{\rm SEP} = 1 + \eta\left(\frac{\Omega}{mc^2}\right)$$

$$\frac{\Delta a}{a} = (4\beta - \gamma - 3)\left\{\left[\frac{\Omega}{mc^2}\right]_1 - \left[\frac{\Omega}{mc^2}\right]_2\right\}$$

Uniqueness of Free Fall

(≡ Weak Equivalence Principle):

$$\vec{F} = m_I\vec{a} = m_G\vec{g}$$

$$\Rightarrow m_I = m_G$$

All bodies fall with the same acceleration

Define the test parameter that signifies a violation of the **WEP**

$$\frac{\Delta a}{a} = \frac{(a_1 - a_2)}{\frac{1}{2}(a_1 + a_2)} = \left[\frac{m_G}{m_I}\right]_1 - \left[\frac{m_G}{m_I}\right]_2$$

Let Ω is the gravitational binding energy of a test body, then the test parameter that signifies a violation of the **SEP** is

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate Local Lorentz Invariance (LLI)

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

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Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

Coleman 1983 | Hehl 1997 | Overlooked (20th century)

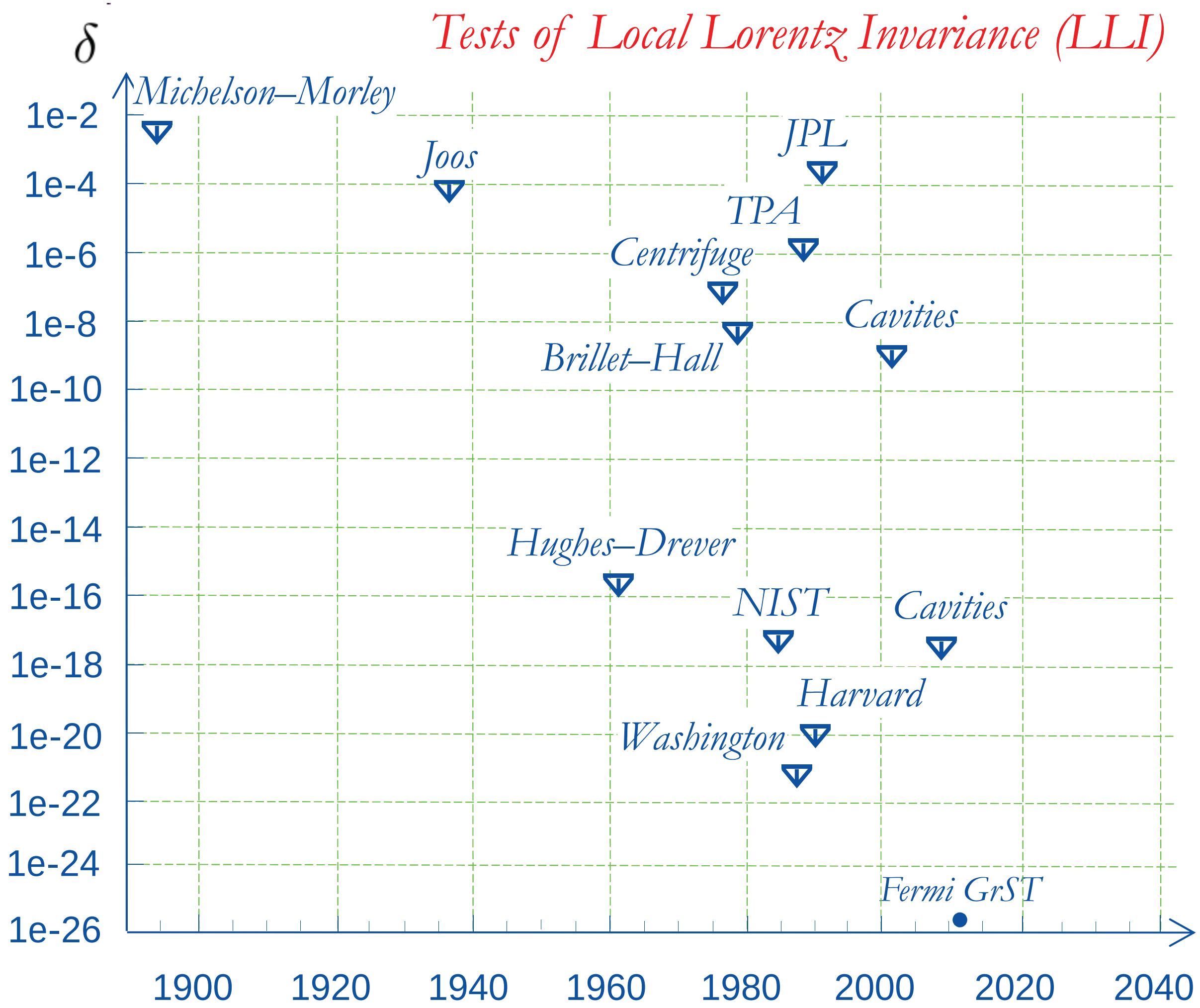
Quasi-linear theories:

- Deser & Laurent (1968), Bollini, Giambiagi & Tiomno (1970) both predict existence of a preferred reference frame (i.e., $\xi=1$)
- Whitehead (1922) predicts time-dependence for ocean tides in violation of everyday experience

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment

Tests of Local Lorentz Invariance (LLI)



- Michelson-Morley, Joos, Brillet-Hall: Round-trip propagation
- Centrifuge, TPA, JPL: One-way propagation
- Rest: Hughes-Drever experiments

$$\delta \equiv \frac{c}{c_0} - 1$$

Local Lorentz Invariance:

- Extended frameworks by Kostelecky et al., Jacobson et al.

Future experiments:

- Clock comparisons
- Clocks vs microwave cavities
- Time of flight of high energy photons
- Birefringence in vacuum
- Neutrino oscillations
- Threshold effects in particle physics

Test of one-way speed of light:

- Important to fundamental physics, cosmology, astronomy and astrophysics

Laboratory tests of **Lorentz Invariance**:
search for preferred-frame effects

frame1 : $S(T, X)$ e.g. CMB $v_{sol} \approx 377\,\mathrm{km/s}$

frame2 : $s(t, x)$ laboratory $RA, dec = (11.2, -6.4^\circ)$

Mansouri & Sexl, 1977

$$dT = \frac{1}{a}(dt + \frac{v}{c^2}dx) \qquad a = 1 + \alpha\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dX = \frac{1}{b}dx + \frac{v}{a}(dt + \frac{v}{c^2}dx) \qquad b = 1 + \beta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dY = \frac{1}{d}dy,\ dZ = \frac{1}{d}dz \qquad d = 1 + \delta\frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

Special Theory of Relativity: $\alpha = -1/2,\ \beta = 1/2,\ \delta = 0$

Clock comparison experiments:

$$P_{MM} = (\tfrac{1}{2} - \beta + \delta)$$ Michelson-Morley: orientation dependence

$$P_{KT} = (\beta - \alpha - 1)$$ Kennedy-Thorndike: velocity dependence

$$P_{IS} = |\alpha + \tfrac{1}{2}|$$ Ives-Stillwell: contraction, dilation

Precision tests of Lorentz Invariance:

$$P_{MM} = -1.6(\pm 6.1) \times 10^{-12}$$ Eisele et al, PRL 103 (2009) 090401

$$P_{KT} = 3.1(\pm 6.9) \times 10^{-7}$$ Wolf et al, PRL 90 (2003) 060402

$$P_{IS} < 2.2 \times 10^{-7}$$ Saathoff et al, PRL 91 (2003) 190403

Tests of isotropy of the speed of light:

$$\Delta c_\theta / c \lesssim 1 \times 10^{-17}$$ Herrmann et al, PRD 80 (2009) 105011

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Theories that violate Local Position Invariance (LPI)

Newton 1686 | Poincaré 1890

Einstein 1912 | Nordstrøm 1912 | Nordstrøm 1913 | Einstein & Fokker 1914 | Einstein 1915

Whitehead 1922 | Cartan 1923 | Kaluza & Klein 1932 | Fierz & Pauli 1939 | Birkhoff 1943

Milne 1948 | Thiry 1948 | Papapetrou 1954 | Jordan 1955 | Littlewood & Bergmann 1956

Brans & Dicke 1961 | Yilmaz 1962 | Whitrow & Morduch 1965 | Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | Bergmann 1968 | Deser & Laurent 1968 | Nordtvedt 1970 | Wagoner 1970

Bollini et al. 1970 | Rosen 1971 | Will & Nordtvedt 1972 | Ni 1972 | Hellings & Nordtvedt 1972

Ni 1973 | Yilmaz 1973 | Lightman & Lee 1973 | Lee, Lightman & Ni 1974 | Rosen 1975

Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977 | Barker 1978 | Rastall 1979

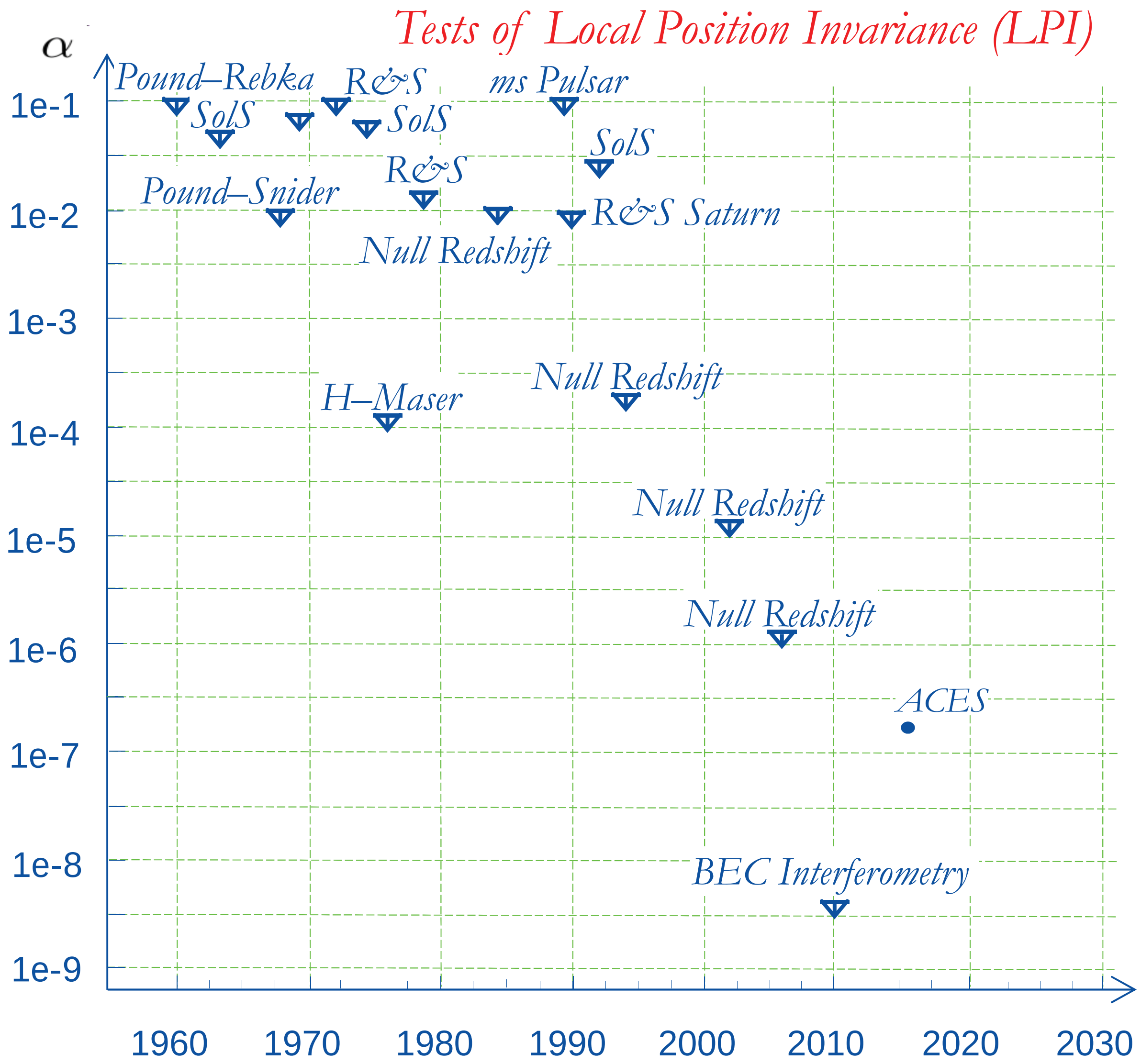
Coleman 1983 | Hehl 1997 | Overlooked (20th century)

Stratified theories with time-orthogonal time slices all predict $\xi \neq 0$:

- Einstein (1912), Papapetrou (1954) (actually two theories)
- Yilmaz (1962), Whithrow & Morduch (1965)
- Page & Tupper (1968), Rosen (1971)
- Ni (1972), Coleman (1983)

Empirical Foundations of General Relativity:

Confrontation Between the Theory and Experiment



– SolS: Solar Spectra; – R&S: Rockets and Spacecraft

– Null Redshift: comparison of different atomic clocks

Gravitational redshift:

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2}$$

Local Position Invariance:

– The outcome of any local non-gravitational experiment is independent of where & when in the universe it is performed

Splits into:

– spatial invariance

– temporal invariance

– The best clock result is by Ashby et al., Phys. Rev. Lett. 98, 070802 (2007)

$$|\alpha| < 1.4 \times 10^{-6}$$

– Current best result is by Müller, Peters, and Chu, Nature 463, 926 (2010):

$$|\alpha| < 7 \times 10^{-9}$$

Parameterized Post-Newtonian (PPN) formalism

PPN Formalism: Eddington, Fock, Chandrasekhar, Dicke, Nordtvedt, Thorne, Will,…

$$
\begin{aligned}
g_{00} &= 1 - \frac{2}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \frac{2\beta}{c^4}\Big[\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\Big]^2 - \frac{1+2\gamma}{c^4}\sum_{j\neq i}\frac{\mu_j \dot{r}_j^2}{r_{ij}} + \\
&\quad + \frac{2(2\beta-1)}{c^4}\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} - \frac{1}{c^4}\sum_{j\neq i}\mu_j\frac{\partial^2 r_{ij}}{\partial t^2} + \mathcal{O}(c^{-5}) \\
g_{0\alpha} &= \frac{2(1+\gamma)}{c^3}\sum_{j\neq i}\frac{\mu_j \dot{\mathbf{r}}_j^{\alpha}}{r_{ij}} + \mathcal{O}(c^{-5}) \\
g_{\alpha\beta} &= -\delta_{\alpha\beta}\Big(1 + \frac{2\gamma}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}} + \frac{3\delta}{2c^4}\Big[\sum_{j\neq i}\frac{\mu_j}{r_{ij}}\Big]^2\Big) + \mathcal{O}(c^{-5})
\end{aligned}
$$

- Assumption: Local Lorentz Invariance (LLI) and local position invariance (LPI) hold, thus, preferred frame parameters α_1, α_2, α_3 are not included…
- General case, there are 10 PPN parameters: $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- γ are β the Eddington's parameterized post-Newtonian (PPN) parameters:

General relativity: $\gamma = \beta = 1$ Brans-Dicke theory: $\gamma = \frac{1+\omega}{2+\omega}, \ \beta = 1$

- δ is the post-PPN parameter – important for next generation of light propagation tests.

C. M. Will, Living Rev. Relat. **9** (2006), gr-qc/0510072

PPN Equations of Motion (a part of the model)

$$\ddot{\mathbf{r}}_i = \sum_{j\neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3}\Big\{ \Big[\frac{m_G}{m_I}\Big]_i - \frac{2(\beta+\gamma)}{c^2}\sum_{l\neq i}\frac{Gm_l}{r_{il}} - \frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{Gm_k}{r_{jk}} +$$

$$+\gamma\Big(\frac{\dot{r}_i}{c}\Big)^2 + (1+\gamma)\Big(\frac{\dot{r}_j}{c}\Big)^2 - \frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\dot{\mathbf{r}}_j + \frac{\dot{G}\cdot t}{G} -$$

$$-\frac{3}{2c^2}\Big[\frac{(\mathbf{r}_i-\mathbf{r}_j)\dot{\mathbf{r}}_j}{r_{ij}}\Big]^2 + \frac{1}{2c^2}(\mathbf{r}_j - \mathbf{r}_i)\ddot{\mathbf{r}}_j\Big\} +$$

$$+\frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{r_{ij}^3}\Big\{\Big[\mathbf{r}_i - \mathbf{r}_j\Big]\cdot\Big[(2+2\gamma)\dot{\mathbf{r}}_i - (1+2\gamma)\dot{\mathbf{r}}_j\Big]\Big\}(\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j) +$$

$$+\frac{3+4\gamma}{2c^2}\sum_{j\neq i}\frac{Gm_j\ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m=1}^{3}\frac{Gm_m(\mathbf{r}_m - \mathbf{r}_i)}{r_{im}^3} + \sum_{c,s,m}\mathbf{F}_{\text{asteroids}}$$

Possible EP violation

Possible temporal dependence of *G*

$$\Big[\frac{m_G}{m_I}\Big]_{\text{SEP}} = 1 + \eta\Big(\frac{\Omega}{mc^2}\Big)$$

$$\eta = 4\beta - \gamma - 3$$

$$\Omega_i = -\frac{G}{2}\int_i d^3x\rho_i U_i = -\frac{G}{2}\int_i d^3x d^3x' \frac{\rho_i(\mathbf{r})\rho_i(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

- In general theory of relativity $\gamma=\beta=1$, thus $\eta=0$ (this is not the case for scalar-tensor theories of gravity, for instance, where these parameters can have different values).

$$t_2 - t_1 = \frac{r_{12}}{c} + (1+\gamma)\sum_i \frac{\mu_i}{c^3}\ln\Bigg[\frac{r_1^i + r_2^i + r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}}{r_1^i + r_2^i - r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}}\Bigg] + \mathcal{O}(c^{-5})$$

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

Cassini (2003): $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$

Theories that predict $\gamma = 0$ or -1 fail

Newton 1686 Poincaré 1890

Einstein 1912 | **Nordstrøm 1912** | **Nordstrøm 1913** | **Einstein & Fokker 1914** | **Einstein 1915**

Whitehead 1922 | **Cartan 1923** | Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 | **Thiry 1948** | Papapetrou 1954 | **Jordan 1955** | **Littlewood & Bergmann 1956**

Brans & Dicke 1961 | Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 | **Bergmann 1968** | Deser & Laurent 1968 | **Nordtvedt 1970** | **Wagoner 1970**

Bollini et al. 1970 Rosen 1971 | **Will & Nordtvedt 1972** | **Ni 1972** | **Hellings & Nordtvedt 1972**

Ni 1973 | **Yilmaz 1973** | **Lightman & Lee 1973** | **Lee, Lightman & Ni 1974** | **Rosen 1975**

Belinfante & Swihart 1975 | **Lee et al. 1976** | **Bekenstein 1977** | **Barker 1978** | **Rastall 1979**

Coleman 1983 | **Hehl 1997** | **Overlooked (20th century)**

Parameterized Post-Newtonian Formalism (PPN):

- Solar system is the main arena to test weak gravity:
- Expand the metrics; identify various potentials
- They have 10 PPN parameters in front

$$\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$$

- Calculate those parameters & Compare with experiments

[**2011**: A need for Cosmological "PPN formalism"?]

Conformally-flat theories fail test of time delay and deflection of light:

- Nordstrom (1912)
- Nordstrom (1913)
- Einstein & Fokker (1914)
- Littlewood & Bergmann (1956)
- Ni (1972)

List of PPN Parameters for Competing Theories

Competing theories of Gravity	γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
Scalar Field theories – Note: in Page-Tupper (1968): parameter *d* is defined as $\Delta = 1-\gamma$										
Einstein (1912) [not GR]	0	0	–	–4	0	–2	0	–1	0	0*
Whitrow-Morduch (1965)	0	–1	–	–4	0	0	0	–3	0	0*
Rosen (1971)	λ	$\frac{3}{4}+\frac{\lambda}{4}$	–	$-4(1-\lambda)$	0	–4	0	–1	0	0
Papapetrou (1954a, 1954b)	1	1	–	–8	-4	0	0	2	0	0
Ni (1972) (stratified)	1	1	–	–8	0	0	0	2	0	0
Yilmaz (1958, 1962)	1	1	–	–8	0	–4	0	–2	0	–1*
Page-Tupper (1968)	γ	β	–	-4Δ	0	-2Δ	0	ζ_2	0	ζ_4
Nordström (1912, 1913)	–1	½	–	0	0	0	0	0	0	0*
Einstein-Fokker (1914)	–1	½	–	0	0	0	0	0	0	0
Ni (1972) (flat)	–1	$1-q$	–	0	0	0	0	ζ_2	0	0*
Whitrow-Morduch (1960)	–1	$1-q$	–	0	0	0	0	q	0	0*
Littlewood (1953), Bergman (1956)	–1	½	–	0	0	0	0	–1	0	0*

– Note: * The theory is incomplete, and ζ_4 can take one of two values. The value closest to zero is listed.

List of PPN Parameters for Competing Theories

Competing theories of Gravity	γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
Scalar-Tensor theories										
Bergmann (1968), Wagoner (1970)	$\frac{1+\omega}{2+\omega}$	β	0	0	0	0	0	0	0	0
Nordtvedt (1970), Bekenstein (1977)	$\frac{1+\omega}{2+\omega}$	β	0	0	0	0	0	0	0	0
Brans-Dicke (1961)	$\frac{1+\omega}{2+\omega}$	1	0	0	0	0	0	0	0	0
Vector-Tensor theories										
Hellings-Nordtvedt (1973)	γ	β	0	α_1	α_2	0	0	0	0	0
Will-Nordtvedt (1972)	1	1	0	0	α_2	0	0	0	0	0
Bimetric theories – Note: in Rosen (1975): parameter k_2 is defined as $k_2 = (c_0/c_1) - 1$										
Rosen (1975)	1	1	0	0	k_2	0	0	0	0	0
Rastall (1979)	1	1	0	0	α_2	0	0	0	0	0
Lightman-Lee (1973)	γ	β	0	α_1	α_2	0	0	0	0	0
Stratified theories										
Lee-Lightman-Ni (1974)	ac_0/c_1	β	ξ	α_1	α_2	0	0	0	0	0
Ni (1973)	ac_0/c_1	bc_0	0	α_1	α_2	0	0	0	0	0

Unlikely Scalar-Tensor Theories

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 **Thiry 1948** Papapetrou 1954 **Jordan 1955** Littlewood & Bergmann 1956

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Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**

Ni 1973 **Yilmaz 1973** **Lightman & Lee 1973** **Lee, Lightman & Ni 1974** **Rosen 1975**

Belinfante & Swihart 1975 **Lee et al. 1976** **Bekenstein 1977** **Barker 1978** **Rastall 1979**

Coleman 1983 **Hehl 1997** **Overlooked (20th century)**

Scalar-Tensor theories are extremely constrained by Viking (1976) and Cassini (2003) results on γ:

- Thiry (1948), Jordan 1955
- Brans & Dicke (1961): ω >6500 (Viking, 1976), ω >40,000 (Cassini, 2003)
- Bergmann (1968), Nordtvedt (1970)
- Wagoner (1970), Bekenstein (1977)
- Barker (1978)

Theoretical Landscape of the 20th Century:
Competing Theories of Gravity

Mercury's Perihelion: Theories that fail

Newton 1686 · Poincaré 1890

Einstein 1912 · Nordstrøm 1912 · Nordstrøm 1913 · Einstein & Fokker 1914 · **Einstein 1915**

Whitehead 1922 · **Cartan 1923** · Kaluza & Klein 1932 · Fierz & Pauli 1939 · Birkhoff 1943

Milne 1948 · Thiry 1948 · Papapetrou 1954 · Jordan 1955 · Littlewood & Bergmann 1956

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Page & Tupper 1968 · Bergmann 1968 · Deser & Laurent 1968 · Nordtvedt 1970 · Wagoner 1970

Bollini et al. 1970 · Rosen 1971 · **Will & Nordtvedt 1972** · Ni 1972 · **Hellings & Nordtvedt 1972**

Ni 1973 · **Yilmaz 1973** · **Lightman & Lee 1973** · **Lee, Lightman & Ni 1974** · **Rosen 1975**

Belinfante & Swihart 1975 · **Lee et al. 1976** · Bekenstein 1977 · Barker 1978 · **Rastall 1979**

Coleman 1983 · **Hehl 1997** · **Overlooked (20th century)**

Stratified theories predict preferred frame effects on perihelion shift:

– Ni (1973)
– Lee, Lightman & Ni (1974)

$$\dot{\pi} = (2+2\gamma-\beta)\frac{GM_\odot n_M}{c^2 a_M(1-e_M^2)} + \frac{3}{4}\left(\frac{R_\odot}{a_M}\right)^2 \frac{J_{2\odot} n_M}{(1-e_M^2)^2}(3\cos^2 i_M - 1), \quad ''/\text{cy}$$

$$\dot{\pi} = 42''.98\left[\frac{1}{3}(2+2\gamma-\beta) + 0.296\cdot J_{2\odot}\times 10^4\right], \quad ''/\text{cy}$$

$J_{2\odot} \simeq 2\times 10^{-7}$ from helioseismology; confirmed by Konopliv et al., 2010

GW & Binary Pulsar: Theories that fail

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

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Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**

Ni 1973 **Yilmaz 1973** **Lightman & Lee 1973** Lee, Lightman & Ni 1974 **Rosen 1975**

Belinfante & Swihart 1975 **Lee et al. 1976** Bekenstein 1977 Barker 1978 **Rastall 1979**

Coleman 1983 **Hehl 1997** **Overlooked (20thcentury)**

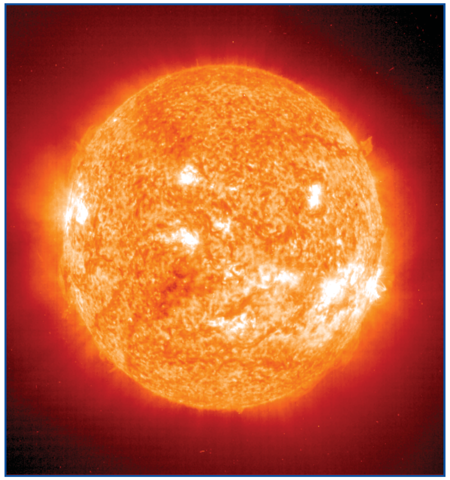
Bi-metric Theories predict a dipole radiation. Can't be…:

- Rosen (1975)
- Lee et al. (1976)
- Rastall (1979)
- Lightman & Lee (1973)

The Current Values of the PPN Parameters

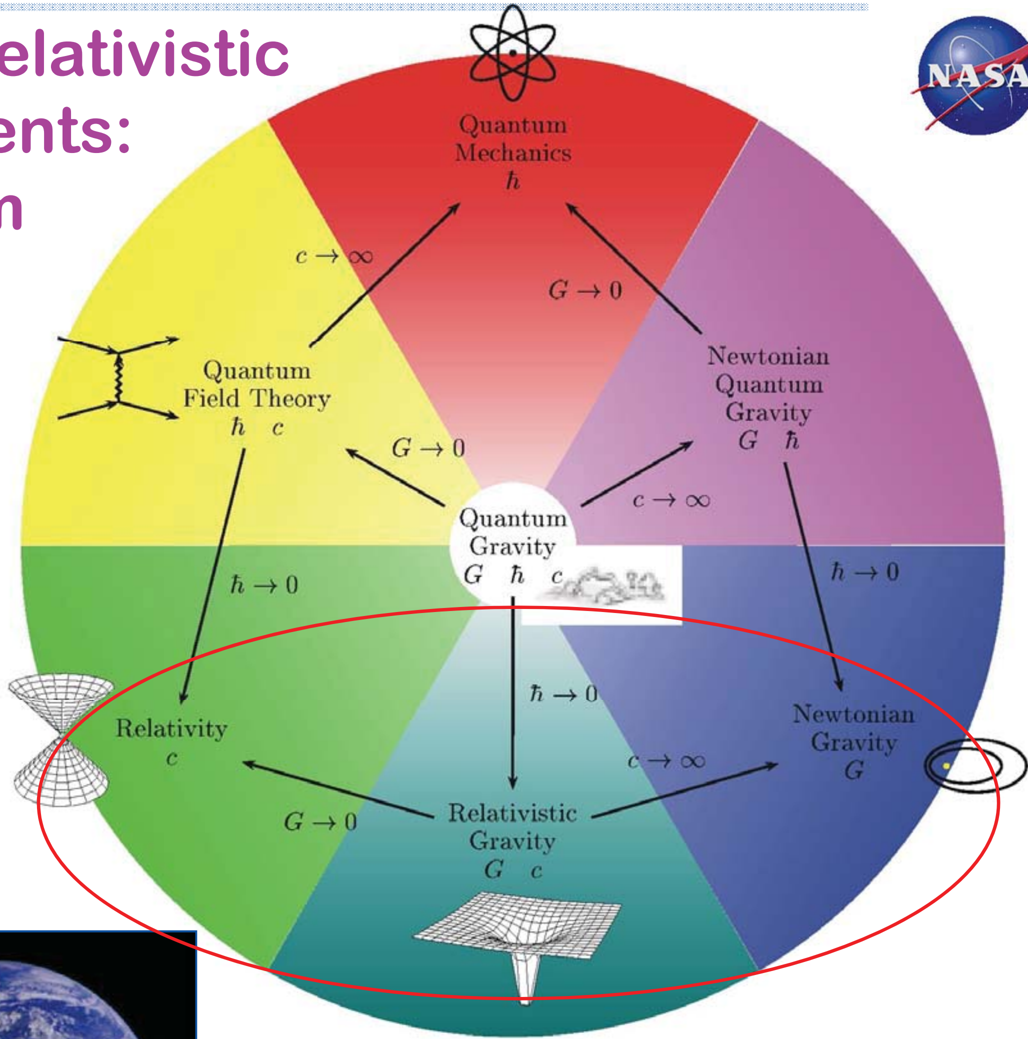
Parameter	What it measured relative to General Relativity?	Current value	Effects	Experiments
γ	Measure of space curvature produced by unit mass	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
β	Measure of non-linearity in gravitational superposition	1.1×10^{-4}	Nordtvedt effect, perihelion shift	Lunar laser ranging
ξ	Measure of existence of preferred location effects	1×10^{-3}	Earth tides	Gravimeter data
α_1	Measure the existence of preferred frame effects	1×10^{-4}	Orbit polarization	Lunar laser ranging
α_2		4×10^{-7}	Spin precession	Sun axis' alignment w/ ecliptic
α_3		4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	Measure (plus α_3) of the failure of conservation laws of energy, momentum and angular momentum	2×10^{-2}	–	Combined PPN bounds
ζ_2		4×10^{-5}	Binary pulsar acceleration	Pulsar: PSR 1913+16
ζ_3		1×10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4		6×10^{-3}	–	Kreuzer experiment

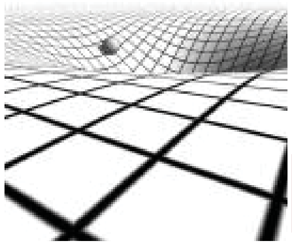
Laboratory for Relativistic Gravity Experiments: Our Solar System



Strongest gravity potential

$$\frac{GM_{Sun}}{c^2 R_{Sun}} \sim 10^{-6}$$





$$\frac{GM_{\oplus}}{c^2 R_{\oplus}} \sim 10^{-9}$$

Most accessible region for gravity tests in space:

- ISS, LLR, SLR, free-fliers

Technology is available to conduct tests in the immediate solar proximity

40 Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:
- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

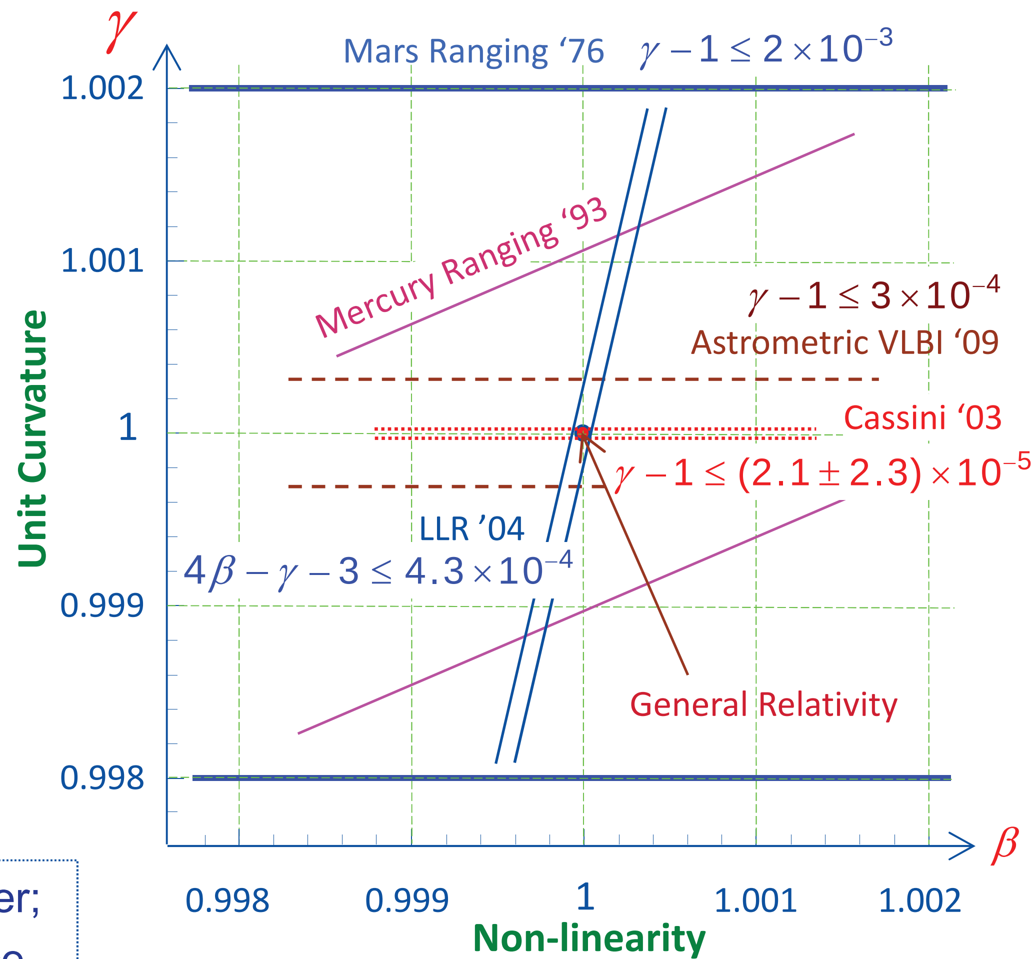
Laser:
- SLR, LLR, interplanetary, etc.

Dedicated Gravity Missions:

- **LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76,'92; GP-B, '04; LARES, '12; LISA, 2025+(?)

New Engineering Discipline – Applied General Relativity:

- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation & astrometry (SIM, Gaia,....).



A factor of 100 in 40 years is impressive, but is not enough for the near future!

Theoretical Landscape of the 20th Century:

Competing Theories of Gravity

"Aesthetics-Based" Conclusion for 20th Century

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

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Coleman 1983 Hehl 1997 Overlooked (20th century)

- "Among all bodies of physical law none has ever been found that is simpler and more beautiful than Einstein's geometric theory of gravity"

  – Misner, Thorne and Wheeler, 1973

- "[...] Unfortunately, any finite number of effects can be fitted by a sufficiently complicated theory. [...] Aesthetic or philosophical motives will therefore continue to play a part in the widespread faith in Einstein's theory, even if all tests verify its predictions."

  – Malcolm MacCallum, 1976

Theoretical Landscape of the 21st Century:

Competing Theories of Gravity

First decade of 21'st century… they are back!

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 Cartan 1923 **Kaluza & Klein 1932** Fierz & Pauli 1939 Birkhoff 1943

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Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20th century) **Scalar-Tensor Theories**

Arkani-Hamed, Dimopoulos & Dvali 2000 **Dvali, Gabadadze & Poratti 2003** **Strings theory?**

Bekenstein 2004 **Moffat 2005** **Multiple f(R) models 2003-10** **Bi-Metric Theories**

Need for new theory of gravity:

- Classical GR description breaks down in regimes with large curvature
- If gravity is to be quantized, GR will have to be modified or extended

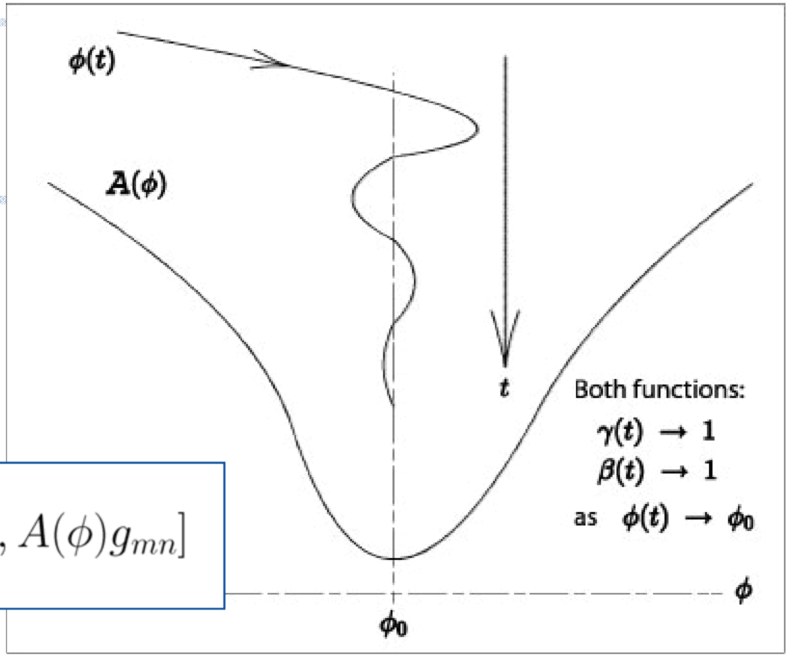
Other challenges:

- Dark Matter
- Dark Energy
- Neutrino velocity

Motivations for new tests of GR:

- GR is a fundamental theory
- Alternative theories & models
- New ideas & techniques require comprehensive investigations

Theoretical Motivation for New Gravity Tests



Long-range massless [or low-mass] scalar:

The low-energy limit of the String Theory in 'Einstein Frame' (Damour-Nordtvedt-Polyakov 1993) suggests:

$$S = -\frac{1}{16\pi G}\int dx^4\sqrt{-g}\Big(R - 2g^{mn}\nabla_m\phi\nabla_n\phi\Big) + S_M[\psi_M, A(\phi)g_{mn}]$$

Expansion $A(\phi)$ around background value ϕ_0 of the scalar leads:

$$\ln A(\varphi) = \ln A(\varphi_0) + \alpha_0(\varphi - \varphi_0) + \frac{1}{2}k_0(\varphi-\varphi_0)^2 + \mathcal{O}(\Delta\varphi^3)$$

Slope α_0 measures the coupling strength of interaction between matter and the scalar.

$$\gamma - 1 \;=\; \frac{-2\alpha_0^2}{1+\alpha_0^2} \simeq -2\alpha_0^2$$

$$\beta - 1 = \frac{1}{2}\frac{\alpha_0^2 k_0}{(1+\alpha_0^2)^2} \simeq \frac{1}{2}\alpha_0^2 k_0 \simeq \frac{1}{4}(1-\gamma)k_0$$

Scenario for cosmological evolution of the scalar (Damour, Piazza & Veneziano 2002):

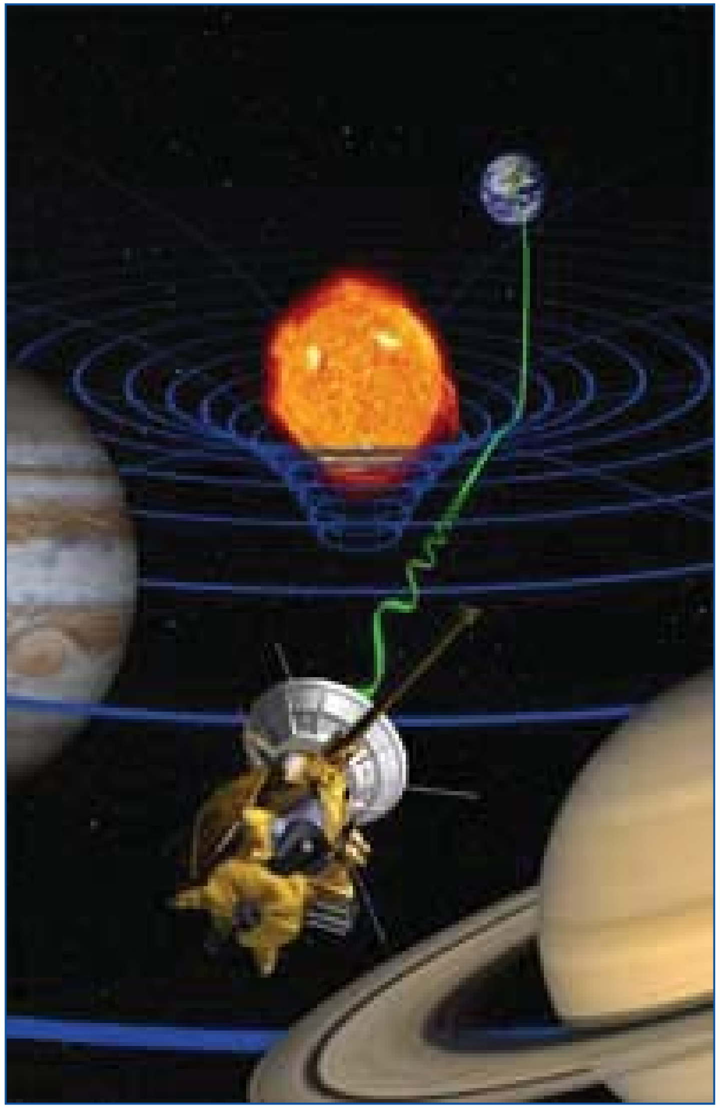
$$\gamma - 1 \sim 7.3\times 10^{-7}\left(\frac{H_0}{\Omega_0^3}\right)^{\frac{1}{2}} \quad\Rightarrow\quad \gamma - 1 \sim 10^{-5} - 10^{-7}$$

The unit curvature PPN parameter γ is the most important quantity to test

Cassini 2003: Where Do We Go From Here?

Cassini Conjunction Experiment:

- Spacecraft—Earth separation > 1 billion km
- Doppler/Range: X~7.14GHz & Ka~34.1GHz
- Result: $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$



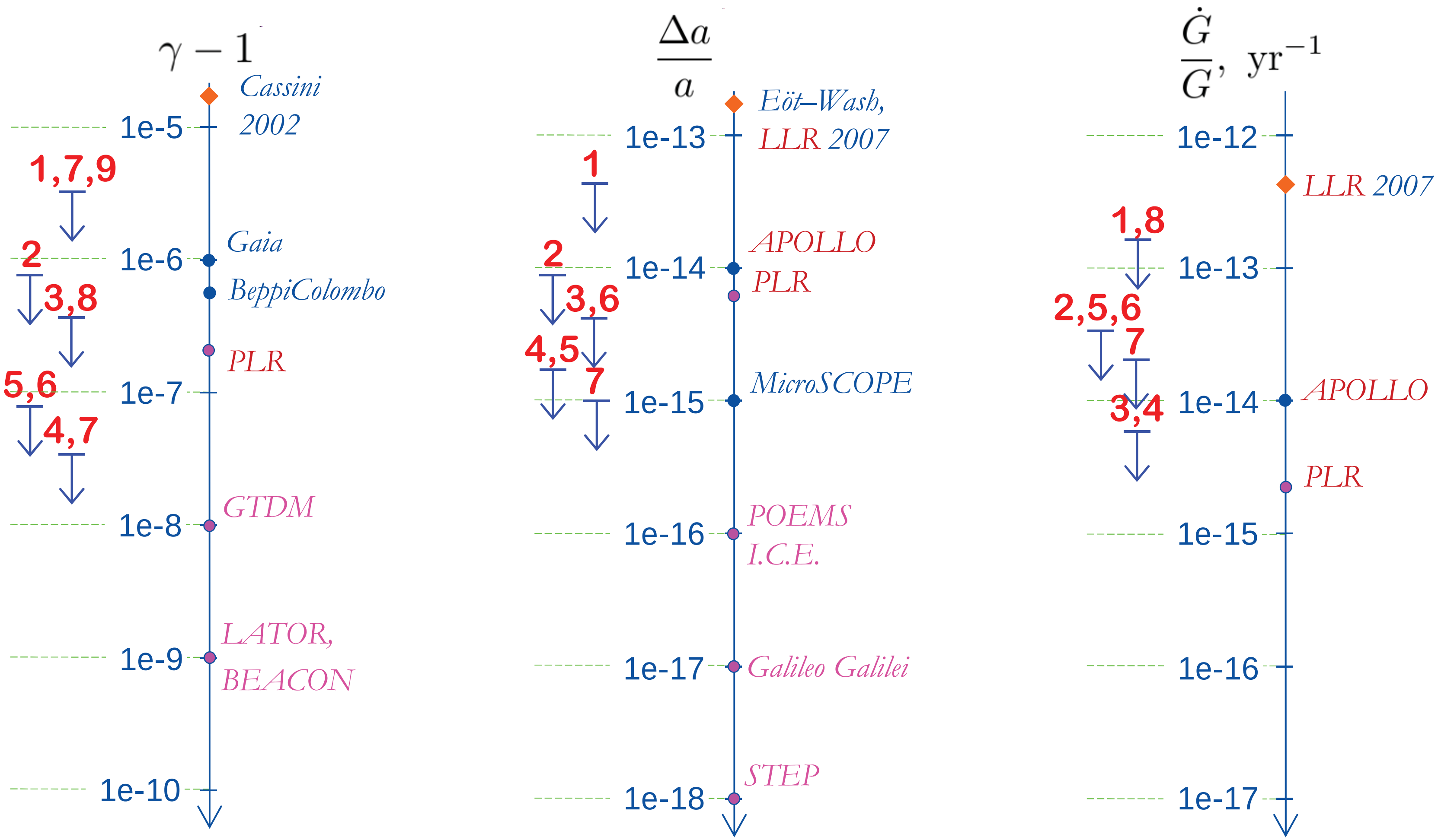
Possible with Existing Technologies?!

- VLBI [current $\gamma = 3 \times 10^{-4}$]: limited to $\sim 1 \times 10^{-4}$:
  - uncertainty in the radio source coordinates
- LLR [current $\eta = 4 \times 10^{-4}$]: in 5 years $\sim 3 \times 10^{-5}$:
  - mm accuracies [APOLLO] & modeling efforts
- μ-wave ranging to a lander on Mars $\sim 6 \times 10^{-6}$
- tracking of BepiColombo s/c at Mercury $\sim 2 \times 10^{-6}$
- Optical astrometry [current $\gamma = 3 \times 10^{-3}$]:

  Gaia mission $\sim 1 \times 10^{-6}$ (2016/18?)

One needs a dedicated mission to explore accuracies better than 10^{-6} for both PPN parameters γ (and β). Interplanetary laser ranging is a possibility.

Theoretical Landscape of the 21th Century:
Confrontation Between Theory and Experiment



New Theories & Future Tests

1 *Damour-Polyakov-Nordtvedt 1993*
2 *Damour-Esposito-Farese 1996*
3 *Damour-Piazza-Veneziano 2002*
4 *Arkani-Dimopoulos-Dvali 2000*
5 *Dvali-Gabadadze-Poratti 2003*
6 *F(R) gravity models 2003-07*
7 *Bekenstein 2004*
8 *Moffat 2005*
9 *Jaekel-Reynaud 2006*

Conclusions

- Recent technological progress: **arXiv:0902.3004 [gr-qc]**
  - Resulted in new instruments with unique performance
  - Could lead to major improvements in the tests of relativistic gravity
  - Already led to a number of recently proposed gravitational experiments
- Challenges for solar system tests of gravity:
  - Dedicated space-based experiments are very expensive – the science must worth the cost… – *EP, G-dot and PPN γ tests are most relevant.*
  - Motivation for the tests in a weak gravity field is a challenge: there is no strong expectation to see deviations from GR in the solar system (we are looking for anomalies…) – *access to strong(er) gravity regime is needed!*
  - GR is very hard to modify, embed, extend or augment (whatever your favorite verb is…) – *thus, perhaps, those anomalies are important…*
  - PPN formalism becomes less relevant for modern gravity research…
  - Looking to Cosmos for help? There is none: Little or no correspondence between cosmological tests and physical principles in the foundation of tests of PPN gravity – *EP, LLI, LPI, energy-momentum conservation, etc…*