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Workshop on Infrared Modifications of Gravity

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The MOND paradigm

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The MOND paradigm

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IR modifications of gravity, Trieste, September 2011

MOND introduced

A theory of dynamics (gravity/inertia) involving a new constant a_0 (beside $G, ...$)

$\mathrm{Standard\ limit}\ (a_0 \to 0)$: The Newtonian limit

$\mathrm{MOND\ limit:}\ a_0 \to \infty, \ \ G \to 0, \ \ Ga_0\ fixed$:

Scale invariance: $(t, \mathbf{r}) \to \lambda(t, \mathbf{r})$

a_0 is analog to c in relativity or $\hbar$ in QM

Example:

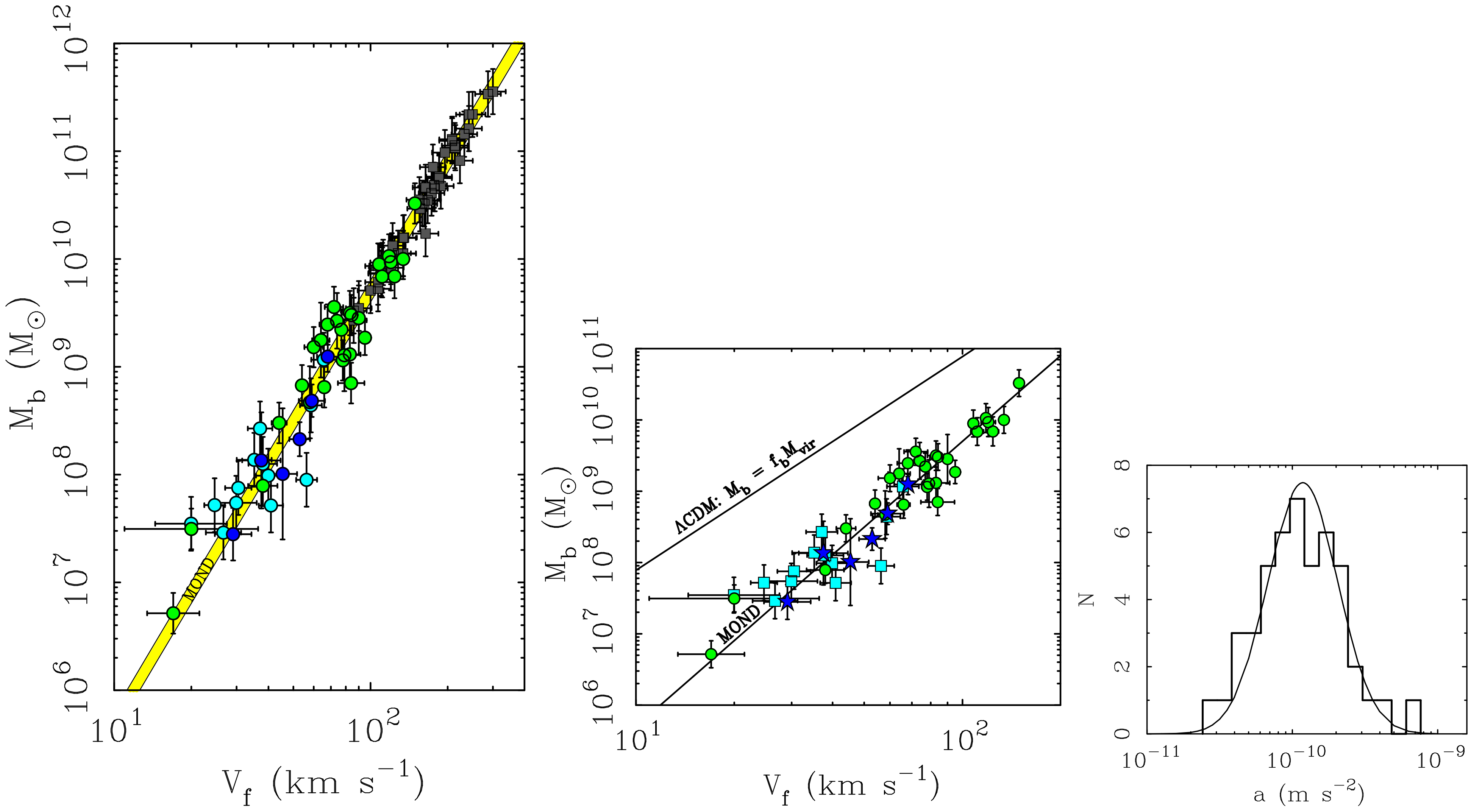
Point-like central mass: $a = \frac{MG}{R^2} f\left(\frac{MG}{R^2 a_0}\right)$

$$a \approx \begin{cases} MG/R^2 & : \quad a \gg a_0 \\ (MGa_0)^{1/2}/R & : \quad a \ll a_0 \end{cases}$$

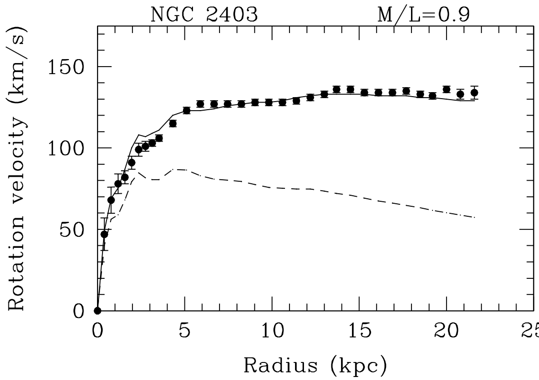
Independent Kepler-like laws in galaxies

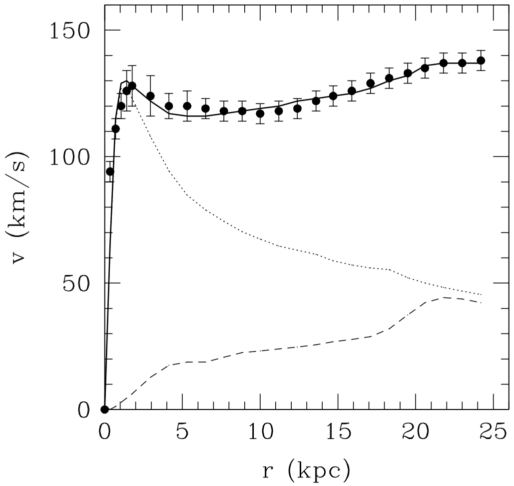
- Asymptotic constancy of orbital velocity: $V(r) \rightarrow V_\infty$
- The velocity mass relation: $V_\infty^4 = MGa_0$
- $\sigma^4 \sim MGa_0$ relation ("isothermal" spheres, deep MOND virial relation)
- Discrepancy appears always at $V^2/R = a_0$
- Isothermal spheres have surface densities $\bar{\Sigma} \lesssim a_0/G$
- The central surface density of "dark halos" is $\approx a_0/2\pi G$
- Disc galaxies have a disc AND a spherical "DM" components
- Scale invariance $(+a_0)$ $\rightarrow$ Nonlinearity $\rightarrow$ External-field effect (EFE)
- Full rotation curves from baryon distribution alone

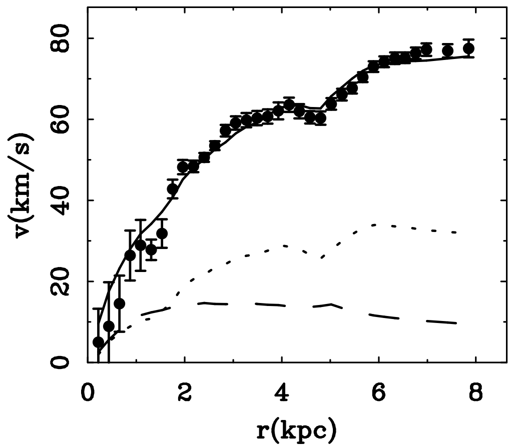
Test of the mass-asymptotic-speed prediction–McGaugh 2011

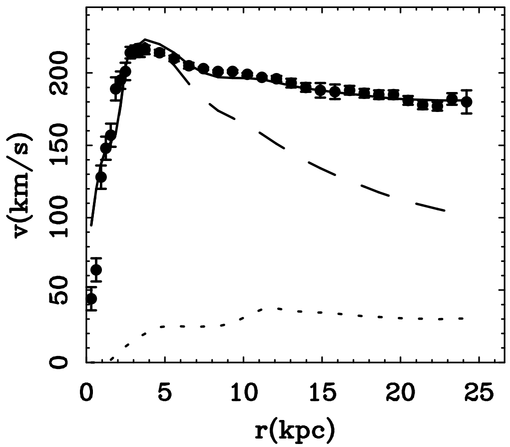


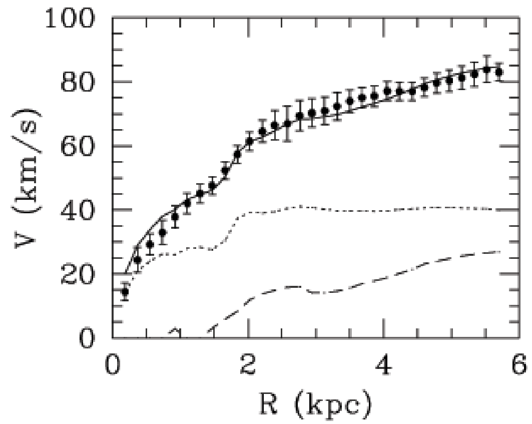
Rotation Curves of Disc Galaxies

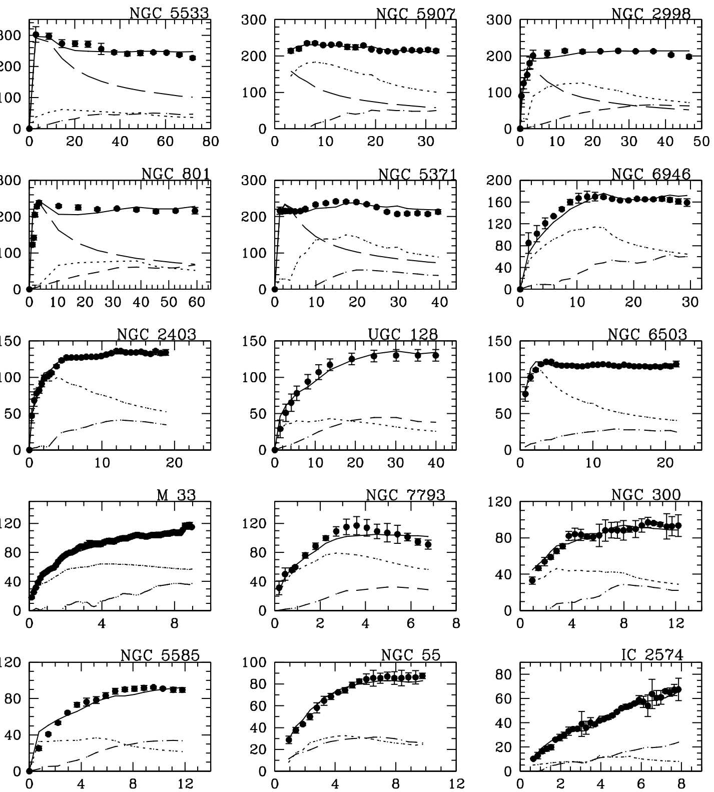


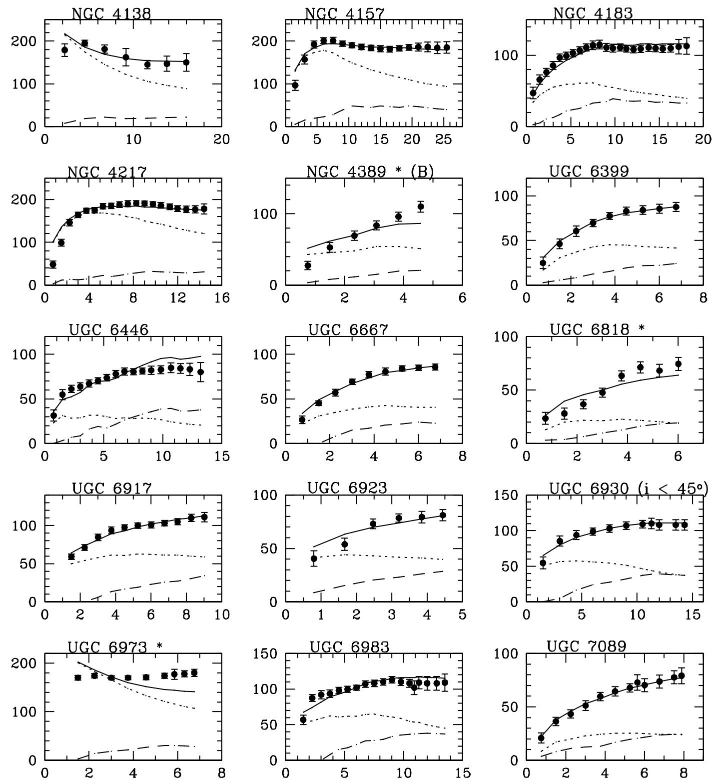




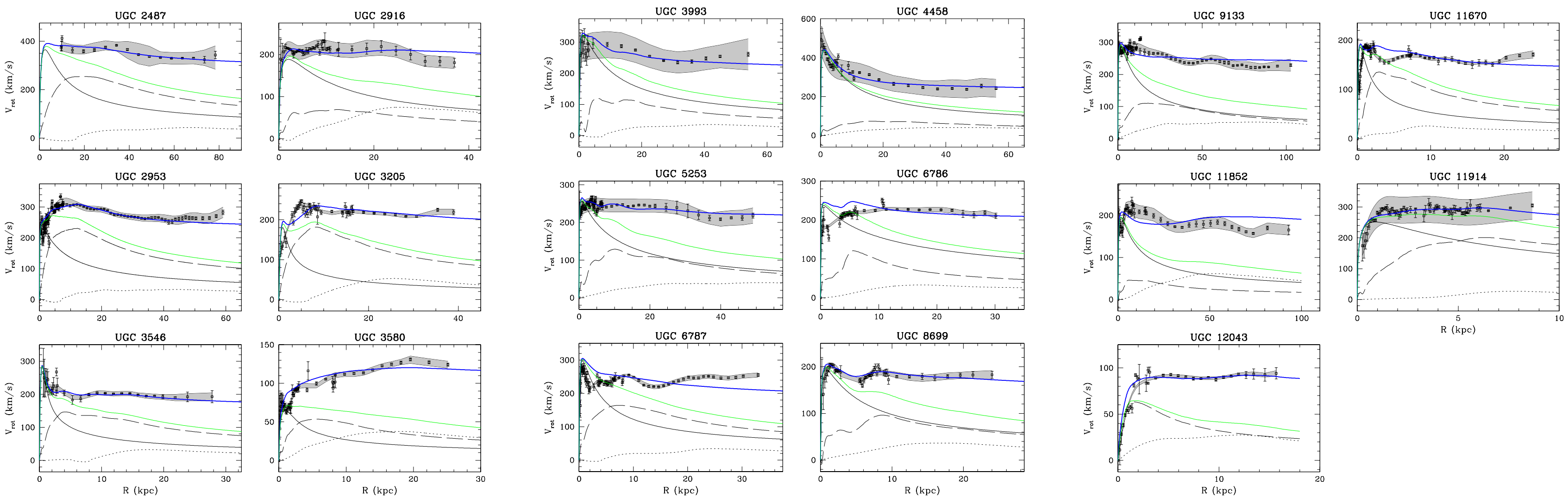








from Sanders and McGaugh 2002



From Sanders and Noordermeer (2007). The grey shaded bands give the allowed range due to inclination uncertainties. The thin green (grey) line gives the Newtonian sum of the individual components. The bold blue (grey) lines gives the total MOND rotation curve

$a_0 =?$

a_0 can be derived in several independent ways:

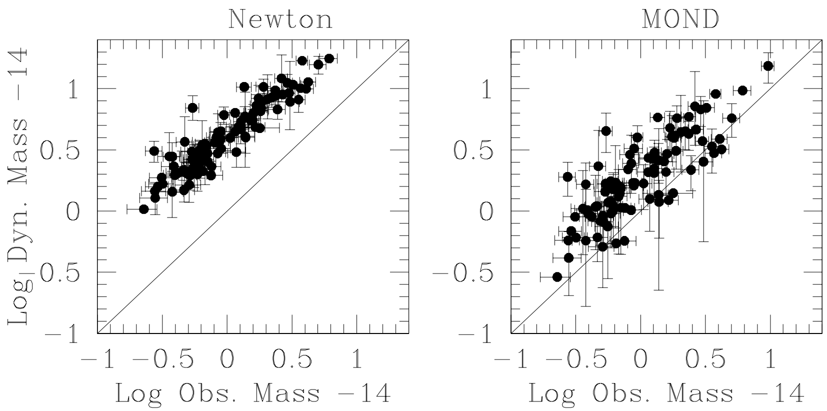
$$a_0 \approx 1.2 \times 10^{-8} \text{ cm s}^{-2}$$

- $\bar{a}_0 \equiv 2\pi a_0 \approx c\bar{H}_0$
- $\bar{a}_0 \approx c(\Lambda/3)^{1/2}$

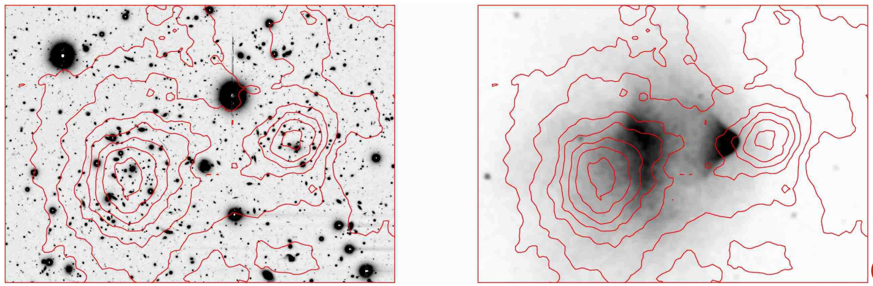
A low acceleration black hole $R_S \gg R_{Hubble}$

All is not roses

- Galaxy clusters



Sanders 1999



Clowe et al. 2006

- Cosmological DM

Nonrelativistic theories

Newtonian dynamics: $\mathbf{a} = -\vec{\nabla}\phi$ $\qquad$ $\vec{\nabla} \cdot [\vec{\nabla}\phi] = 4\pi G\rho$

Modified gravity: $\mathbf{a} = -\vec{\nabla}\phi$ $\qquad$ $\vec{\nabla} \cdot [\mu(\frac{|\vec{\nabla}\phi|}{a_0})\vec{\nabla}\phi] = 4\pi G\rho$

Modified inertia: $\mathsf{A}[\{\mathbf{r}(t)\}, a_0] = -\vec{\nabla}\phi$ $\qquad$ $\vec{\nabla} \cdot [\vec{\nabla}\phi] = 4\pi G\rho$

Conformal invariance

Limits of relativistic theories

QUMOND

$$\mathcal{L} = -\frac{1}{8\pi G}\{2\vec{\nabla}\phi \cdot \vec{\nabla}\phi^* - a_0^2\mathcal{Q}[(\vec{\nabla}\phi^*/a_0)^2]\} + \rho(\frac{1}{2}\mathbf{v}^2 - \phi)$$

$$\Delta\phi^* = 4\pi G\rho, \qquad \Delta\phi = \vec{\nabla} \cdot [\nu(|\vec{\nabla}\phi^*|/a_0)\vec{\nabla}\phi^*]$$

Relativistic theories

- Tensor-Vector-Scalar Gravity (TeVeS–Bekenstein 2004, after Sanders 1997) Gravity is described by $g_{\alpha\beta}$, $\mathcal{U}_\alpha$, ϕ: $\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta}+\mathcal{U}_\alpha\mathcal{U}_\beta)-e^{2\phi}\mathcal{U}_\alpha\mathcal{U}_\beta$

  Reproduces NR modified gravity on galactic scales ($a_0 \propto k\hat{k}^{-1/2}$). Lensing: Similar to the GR result with modified potential Cosmology and structure formation: preliminary work (Dodelson and Liguori, Skordis et al.) CMB: preliminary work: has potential to mimic aspects of cosmological DM (Skordis et al.).

- MOND adaptations of Aether theories (Zlosnik, Ferreira, & Starkman 2007)

$$\mathcal{L}(A,g) = \frac{a_0^2}{16\pi G}\mathcal{F}(\mathcal{K}) + \lambda(A^\mu A_\mu + 1), \tag{1}$$

where

$$\mathcal{K} = a_0^{-2}\mathcal{K}^{\alpha\beta}_{\gamma\sigma}A^\gamma{}_{;\alpha}A^\sigma{}_{;\beta}. \tag{2}$$

$$\mathcal{K}^{\alpha\beta}_{\gamma\sigma} = c_1 g^{\alpha\beta}g_{\gamma\sigma} + c_2\delta^\alpha_\gamma\delta^\beta_\sigma + c_3\delta^\alpha_\sigma\delta^\beta_\gamma + c_4 A^\alpha A^\beta g_{\gamma\sigma},$$

- Galileon k-mouflage MOND adaptation (Babichev, Deffayet, & Esposito-Farese 2011)

  Also a tensor-vector-scalar theory. Said to improve on TeVeS in various regards (e.g., small enough departures from GR in high-acceleration environments)

- Nonlocal metric MOND theories (Soussa & Woodard 2003; Deffayet, Esposito-Farese, & Woodard 2011) Pure metric, but highly nonlocal in that they involve $F(\Box)$.

BIMOND

$$I = I_{EH} + I_M + \hat{I}_{EH} + \hat{I}_M + I_{Int}$$

$$I = -\frac{1}{16\pi G}\int [\beta g^{1/2}R + \alpha \hat{g}^{1/2}\hat{R} - 2(g\hat{g})^{1/4}a_0^2\mathcal{M}]d^4x + I_M(g_{\mu\nu},\psi_i) + \hat{I}_M(\hat{g}_{\mu\nu},\chi_i)$$

$\mathcal{M}$ a dimensionless scalar a function of (quadratic) scalars of

$$a_0^{-1}C^{\alpha}_{\beta\gamma}, \qquad C^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma} - \hat{\Gamma}^{\alpha}_{\beta\gamma}$$

$$\Upsilon_{\mu\nu} = C^{\gamma}_{\mu\lambda}C^{\lambda}_{\nu\gamma} - C^{\gamma}_{\mu\nu}C^{\lambda}_{\lambda\gamma}$$

$$\Upsilon = g^{\mu\nu}\Upsilon_{\mu\nu}, \quad \hat{\Upsilon} = g^{\hat{\mu}\nu}\Upsilon_{\mu\nu}$$

Limits (for $\beta = 1$)

- The high-acceleration limit

  $\mathcal{M} \to \mathcal{M}(\infty) = const$: we get GR with a CC $\propto a_0^2 \mathcal{M}(\infty)$

  Implications for the solar system, binary pulsar, etc.

- Metric equality: GR with a CC: $\Lambda \sim \mathcal{M}(0) a_0^2$

  For example, a double Schwarzschild solution

Nonrelativistic limit

Assume a symmetric cosmology; then locally

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\phi\delta_{\mu\nu} + h_{\mu\nu}, \qquad \hat{g}_{\mu\nu} = \eta_{\mu\nu} - 2\hat{\phi}\delta_{\mu\nu} + \hat{h}_{\mu\nu}$$

With $\Upsilon,\ \bar{\Upsilon}$ field equations give $h_{\mu\nu} = \hat{h}_{\mu\nu} = 0$

$$\phi = \frac{1}{2}\tilde{\phi} + \bar{\phi}, \qquad \hat{\phi} = \frac{1}{2}\tilde{\phi} - \bar{\phi}$$

$$\Delta\tilde{\phi} = 4\pi G(\rho + \hat{\rho}), \qquad \vec{\nabla} \cdot \{\tilde{\mu}(|\vec{\nabla}\bar{\phi}|/a_0)\vec{\nabla}\bar{\phi}\} = 4\pi G(\rho - \hat{\rho})$$

Space conformal invariance in the deep-MOND limit

Matter-Twin matter interactions

- No MOND for $\rho = \hat{\rho}$
- Full MOND when $\hat{\rho} = 0$
- No interaction in Newtonian regime for $\beta = 1$ $[\tilde{\mu}(\infty) = 2]$: $\phi = \phi_N$, $\hat{\phi} = \hat{\phi}_N$
- Repulsion in the MOND regime
- Light bending as in GR, but with the MOND potential
- No strong lensing by TM; acts as diverging lens in the MOND regime

“Microscopic” approaches

- DM with novel, unexpected properties, that may behave as dictated by MOND:
  - ▷ Polarized dark medium (Blanchet 2007, Blanchet & Le Tiec 2009)
  - ▷ Novel baryon-DM interactions (Bruneton & al. 2008)
  - ▷ Dark Fluid (Zhao 2008)

- Entropic effect (Verlinde): (Klinkhamer & Kopp 2011, Pikhitsa Ho & al. 2010, Li & Chang 2010), others

- Vacuum effects (Milgrom 1999)

- Membranes with gravitational DoF extra coordinates (Milgrom 2002)

- Horava gravity (Romero & al. 2010), Sanders (2011), Blanchet & Marsat (2011)

Modified inertia

- No complete theory of MI yet, but an important viable option for MOND
- Has to be nonlocal (in time); plus nonlinearity inherent in MOND
- beyond the basic "Kepler" laws can give very different MOND predictions (e.g., on the EFE, solar system, etc.)
- General result for rotation curves: $a\mu(a/a_0) = -\partial\phi^N/\partial r$

Easy to construct nonlocal, linear, causal theories that hinge on a frequency:

$$\mathbf{a}(t) = \int_{-\infty}^{\infty} L(t-t')\mathbf{f}(t')dt', \quad \hat{\mathbf{a}}(\omega) = \hat{L}(\omega)\hat{\mathbf{f}}(\omega), \quad L(t<0) = 0$$

A toy theory

$$\mathcal{A}(\omega) \equiv 2^{-1/2} \int_{-|\omega|}^{|\omega|} |\hat{\mathsf{a}}(\omega')| d\omega' = 2^{1/2} \int_0^{|\omega|} |\hat{\mathsf{a}}(\omega')| d\omega'$$

$$\mathcal{A}(\omega)\mu[\mathcal{A}(\omega)/a_0] = \mathcal{A}_N(\omega)$$

- Acausal. Conservation laws?
- Solutions for linear and harmonic problems
- Good initial value problem
- Center of mass motion
- EFE, Oort problem
- Pioneer anomaly

possible connection with the asymptotic de Sitter geometry of our universe

If cosmic acceleration is due to cosmological constant, Λ, our universe is asymptotically de Sitter of radius $R_{ds} = c(\Lambda/3)^{-1/2}$

The symmetry group of dS space time, $SO(4,1)$, is isomorphic to the conformal group in 3-D Euclidean space, or on the boundary of the dS (two S^3s)

Connection with deep MOND:

$$\bar{a}_0 \approx c^2/R_{ds}$$

$SO(4,1)$ is the symmetry of the MOND limit nonlinear Poisson equation

The symmetry is broken in the actual cosmos: does deep MOND prevail in local physics in an exact dS cosmos??

What is behind the phenomenological success of MOND?

- DM?
  - ▷ DM distribution is determined from that of the baryons.
  - ▷ But DM to baryon ratio varies greatly and also differs from cosmological value.
  - ▷ It is inconceivable that CDM will ever explain MOND: for individual galaxies the outcome depends on the unknown history of formation, interactions/mergers, ejection of most baryons, etc..

Summary

- MOND is a paradigm still under construction that replaces DM with new physics (or novel DM) at accelerations below $a_0 \sim cH_0 \sim c\Lambda^{1/2}$.

- Strongly anchored in symmetry (NR space-time scaling, de Sitter symmetry)

- Several theoretical directions; can differ greatly on second-rank predictions (e.g., EFE, solar system)

- There are some important things that it was not yet shown with certainty to do (e.g. replacing cosmological DM–some preliminary work).

- Still, it does a lot, and it does it extremely well.

- Rather inconceivable that MOND phenomenology can be explained as some organizing principle for CDM.