The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2264-3

Workshop on Infrared Modifications of Gravity

26 - 30 September 2011

Nonlocal metric formulations of MOND with sufficient lensing

R. Woodard

University of Florida
U.S.A.

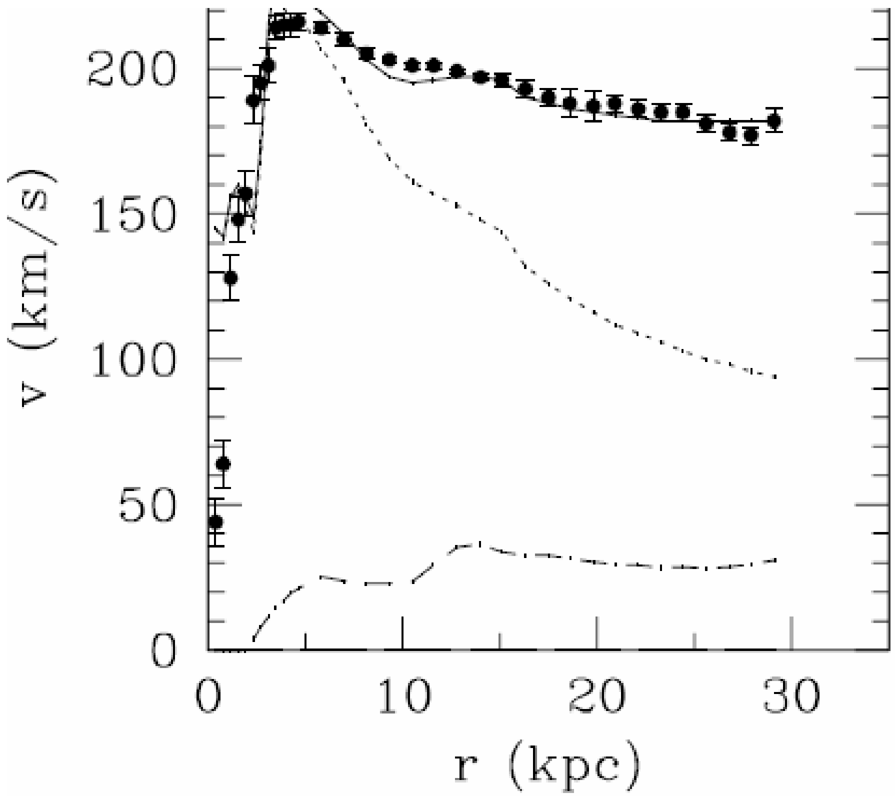
Nonlocal Metric Formulation of MOND with Sufficient Lensing

arXiv:1106.4989

Cedric Deffayet (APC, Paris)
Gilles Esposito-Farese (IAP, Paris)

Dark Matter vs Mod. Gravity

- $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ works for solar system
- But not for galaxies
- Theory: $v^2 = GM/r$
- Obser: $v^2 \sim (a_0 GM)^{1/2}$
- Maybe missing Mass
- Or modified gravity



MOND (Milgrom 1983)

- $\rho(x,y,z) \equiv$ mass in stars and gas

  ➔ $g_N^i \equiv$ Newtonian acceleration
- $g^i \equiv$ actual acceleration

  ➔ $g^i \mu(|g|/a_0) = g_N^i$
- $a_0 \sim 10^{-10}$ m/s^2
- GR regime: $\mu(x) = 1$ for $x >> 1$
- MOND regime: $\mu(x) = x$ for $x << 1$

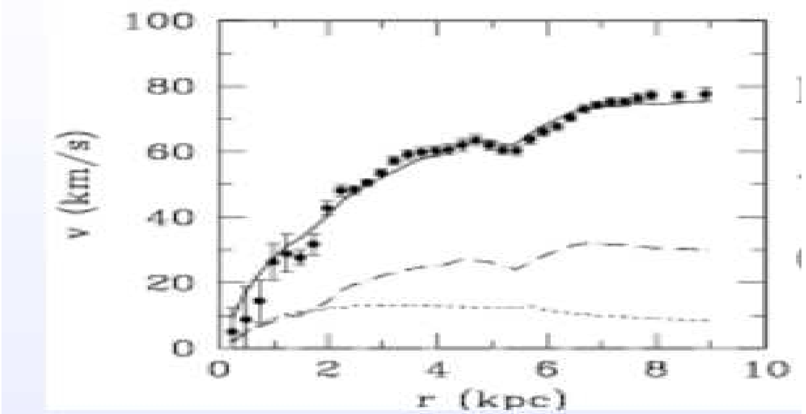
  ➔ Eg. $\mu(x) = x/(1+x)$, or $\tanh(x)$, . . .

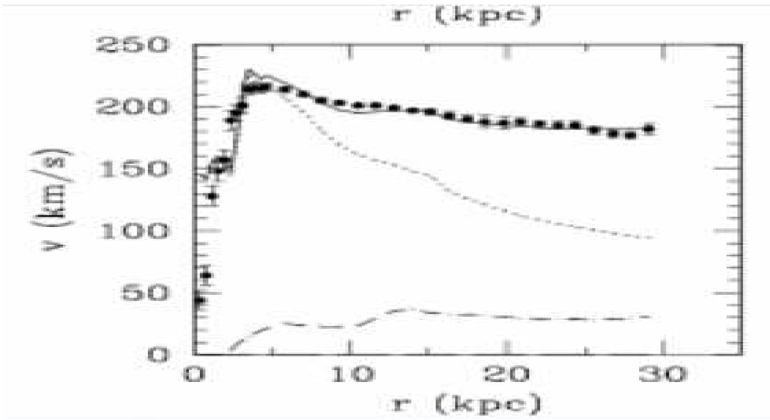
MOND Successes for Rotationally Supported Systems

- Asymp. flat curves
- Milgrom's Law: need dark mat. for $g \sim a_0$
- Freeman's: $\Sigma < a_0/G$
- Sancisi's: bumps trace baryons

  cf. NCG 1560 (LSB)

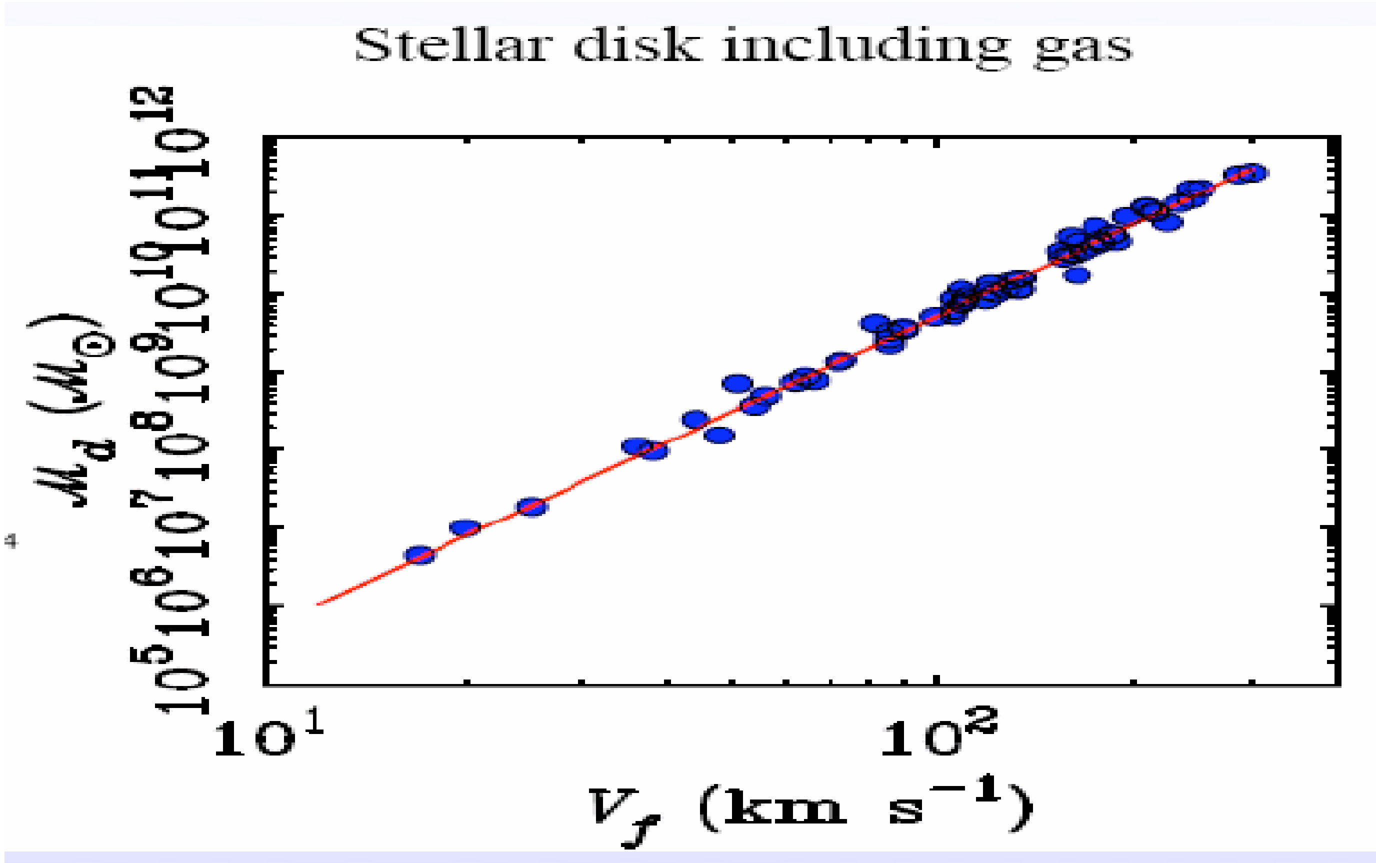
  & NGC 2903 (HSB)





Baryonic Tully-Fisher Relation

$GMa_0 = (v_\infty)^4$ (arXiv:1102.3913)



Breathe-taking . . . but need relativistic model for

- Gravitational Lensing
- Recently disturbed systems
  - The Bullet Cluster!
- Cosmology

Previous models have new fields

  - TeVeS (Bekenstein 2004)
  - Another form of dark matter?

Our Goal: A purely metric version

Metric potentials for static, spherically symmetric

- $ds^2 = -B(r)c^2dt^2 + A(r)dr^2 + r^2d\Omega^2$
- $b(r) = B(r) - 1$ ➔ Rotation curves
  - $rb'(r) = 2v^2/c^2 \rightarrow [4GMa_0/c^4]^{1/2}$
- $a(r) = A(r) - 1$ ➔ Lensing
  - Data ➔ $a(r) \sim + rb'(r)$

GR vs MOND for a MONDian ρ(r)

- $M(r) = 4\pi/c^2 \int^r dr' \, r'^2 \rho(r')$
  - MONDian ➔ $GM(r)/r^2 \ll a_0$
- GR ➔ $a(r) = rb'(r) = 2GM(r)/c^2 r$
  - $\delta S_{GR}/\delta b = (c^4/16\pi G)[(ra)' + O(h^2)] - \frac{1}{2} r^2 \rho$
  - $\delta S_{GR}/\delta a = (c^4/16\pi G)[-rb' + a + O(h^2)]$
- MOND ➔ $a(r) = rb'(r) = [4GM(r)a_0/c^4]^{1/2}$
  - $\partial_r(a^2) = \partial_r(rb')^2 = (16\pi G a_0/c^4)\, r^2 \rho$

$\mathcal{L}_{MOND}$ to cancel h^2 from GR & add h^3 for MOND

- $\mathcal{L}_{GR} = -\frac{1}{2} r^2 \rho b + (c^4/16\pi G)[-ab' + \frac{1}{2} a^2]$
- $\mathcal{L}_{MOND} = r^2 (c^4/16\pi G)[ab'/r - \frac{1}{2}(a/r)^2$
  $+ c^2/a_0 \, [-1/6 \, (b')^3 + k(b' - a/r)^3 + \ldots]$
  - h^3/r^2 of GR « $c^2/a_0 \, (h/r)^3$ of MOND for $r \ll r_H$
- $S = \int dr \, [\mathcal{L}_{GR} + \mathcal{L}_{MOND}]$
  - $\partial_r (rb')^2 - 6k \partial_r (rb'-a)^2 = (16\pi G a_0/c^4) \, r^2 \rho$
  - $-6k/r \, (rb'-a)^2 = 0$

No Local Model

- Abstractions: $a \sim b \sim h$ & $h/r \sim h'$
- $\mathcal{L}_{\mathrm{MOND}} \sim r^2(c^4/16\pi G)[(h')^2 + c^2/a_0\,(h')^3]$
  - Problem is the cubic term
  - An odd # of ∂_r's
- All curvatures $\sim h'' + O(h'^2)$
  - Only even # of ∂_r's
  - No way to act odd # of extra ∂_r 's

Nonlocality removes ∂'s

- $\Box = (-g)^{-1/2}\, \partial_\mu(\sqrt{-g}\, g^{\mu\nu}\, \partial_\nu)$
- $F = \Box^{-1} f \;\rightarrow\; F = \Box f$ (retarded BC)
  - $F(r) = -\int_r dr'/r'^2 (A/B)^{1/2} \int^{r'} dr'' r''^2 (AB)^{1/2} f$
- Eg $R = (r)^{-2}\, \partial_r[-r^2 b' + 2ra] + O(h'^2)$
  - $\Box^{-1} R = \int_r dr'[b' - 2a/r'] + O(h^2)$
  - $g^{\mu\nu} \partial_\mu(\Box^{-1} R) \partial_\nu(\Box^{-1} R) = (b' - 2a/r)^2 + O(hh'^2)$

Was Newton wrong about action-at-a-distance?

- We don't think so
  - Fundamental theory is local
  - But quantum effective field eqns are not
  - M=0 loops could give big IR corrections
- Primordial inflation → IR gravitons
  - $N(t,k) \sim [Ha(t)/2kc]^2$ for every k
  - Perhaps their attraction stops inflation
  - Perhaps MOND is from vacuum polarization

A Nonlocal 4-Velocity Field

- $\mathcal{V}[g](x)$ = inv. volume past l-cone of x^μ
  - To finite time initial value surface
  - Guaranteed to grow in timelike direction
- $u^\mu[g](x) = -g^{\mu\nu}\partial_\nu\mathcal{V}/[-g^{\alpha\beta}\,\partial_\alpha\mathcal{V}\,\partial_\beta\mathcal{V}]^{½}$
  - Static, spherical ➔ $u^\mu = \delta^\mu{}_0/B$
  - The TeVeS vector without a real vector
- $R_{\mu\nu}u^\mu u^\nu$ ➔ $½b'' + b'/r + O(h'^2)$

A class of models based on two scalars

- $X[g] = g^{\mu\nu}[\partial_\mu \Box^{-1}(R_{\alpha\beta}u^\alpha u^\beta - \frac{1}{2}R)]$
  $\times\ [\partial_\nu \Box^{-1}(R_{\rho\sigma}u^\rho u^\sigma - \frac{1}{2}R)]$ → $(b' - a/r)^2$
- $Y[g] = g^{\mu\nu}[\partial_\mu \Box^{-1}(2R_{\alpha\beta}u^\alpha u^\beta)]$
  $\times\ [\partial_\nu \Box^{-1}(2R_{\rho\sigma}u^\rho u^\sigma)]$ → b'^2
- $\mathcal{L}_{MOND} = (c^4/16\pi G)\sqrt{-g}\,[\frac{1}{2}(-X + Y)$
  $+ c^2/6a_0\,(X^{3/2} - Y^{3/2}) + \ldots]$

Dimensionless variables

- $x[g] = c^2/3a_0 \sqrt{|X[g]|} \rightarrow c^2/3a_0 \, |b' - a/r|$
  - $x = 0$ for GR in empty space
- $y[g] = c^2/3a_0 \sqrt{|Y[g]|} \rightarrow c^2/3a_0 \, |b'|$
  - y is large for GR & small for MOND
- $\mathcal{L}_{MOND} = 9a_0^2/32\pi G \, [-x^2 + y^2 + x^3 - y^3 + \ldots]\sqrt{-g}$
- Not necessary to cancel the x^2 term
  - $\mathcal{L}_{MOND} = 9a_0^2/32\pi G \, y^2 e^{-y} \sqrt{-g}$ seems ok
- $3a_0/c^2 \rightarrow D_\mu u^\mu$ for cosmology

Conclusions

- MOND superb for static structures
- But needs a relativistic generalization
  - Could add new fields
  - We use only $g_{\mu\nu}$
- Force law & lensing fix MONDian eqns
- No local model but many nonlocal ones
  - Some are even pretty!

Open Questions

- Stability
  - New DOF
  - $\mathcal{L}_{\mathrm{MOND}} \sim h^3$
- Cosmology (Costas Skordis)
- MOND problems
  - Bullet cluster
  - Cluster cores
- Deriving $\mathcal{L}_{\mathrm{MOND}}$ from fundamental theory