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Cosmology and GR limit of Horava-Lifshitz gravity

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Cosmology and GR limit of Horava-Lifshitz gravity

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ref. Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199
also arXiv: 1105.0246 with K.Izumi
arXiv: 1109.2609 with E.Gumrukcuoglu & A.Wang

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2 \qquad \int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

  $$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling
  $t \to b^z t$ $(E \to b^{-z} E)$
  $x \to b\, x$
  $\phi \to b^s \phi$
  $z+3-2z+2s = 0$
  $s = -(3-z)/2$
- $s = 0$ if $z = 3$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Cosmological implications

- The z=3 scaling solves the horizon problem and leads to scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (S.Maeda, Mukohyama, Shiromizu 2009).
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- Absence of local Hamiltonian constraint leads to DM as integration “constant” (Mukohyama 2009).

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is shared with “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.

Usual story

- $\omega^2 \gg H^2$: oscillate　　　　H = (da/dt) / a

  $\omega^2 \ll H^2$: freeze　　　　a : scale factor

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \rightarrow b\,t$ ($E \rightarrow b^{-1}E$)

  $x \rightarrow b\,x$　　⇨　　$\delta\phi \propto E \sim H$

  $\phi \rightarrow b^{-1}\,\phi$

  Scale-invariance requires almost const. H, i.e. inflation.

New story with z=3

Mukohyama 2009

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

  OK for $a \sim t^p$ with $p > 1/3$

- Scaling law

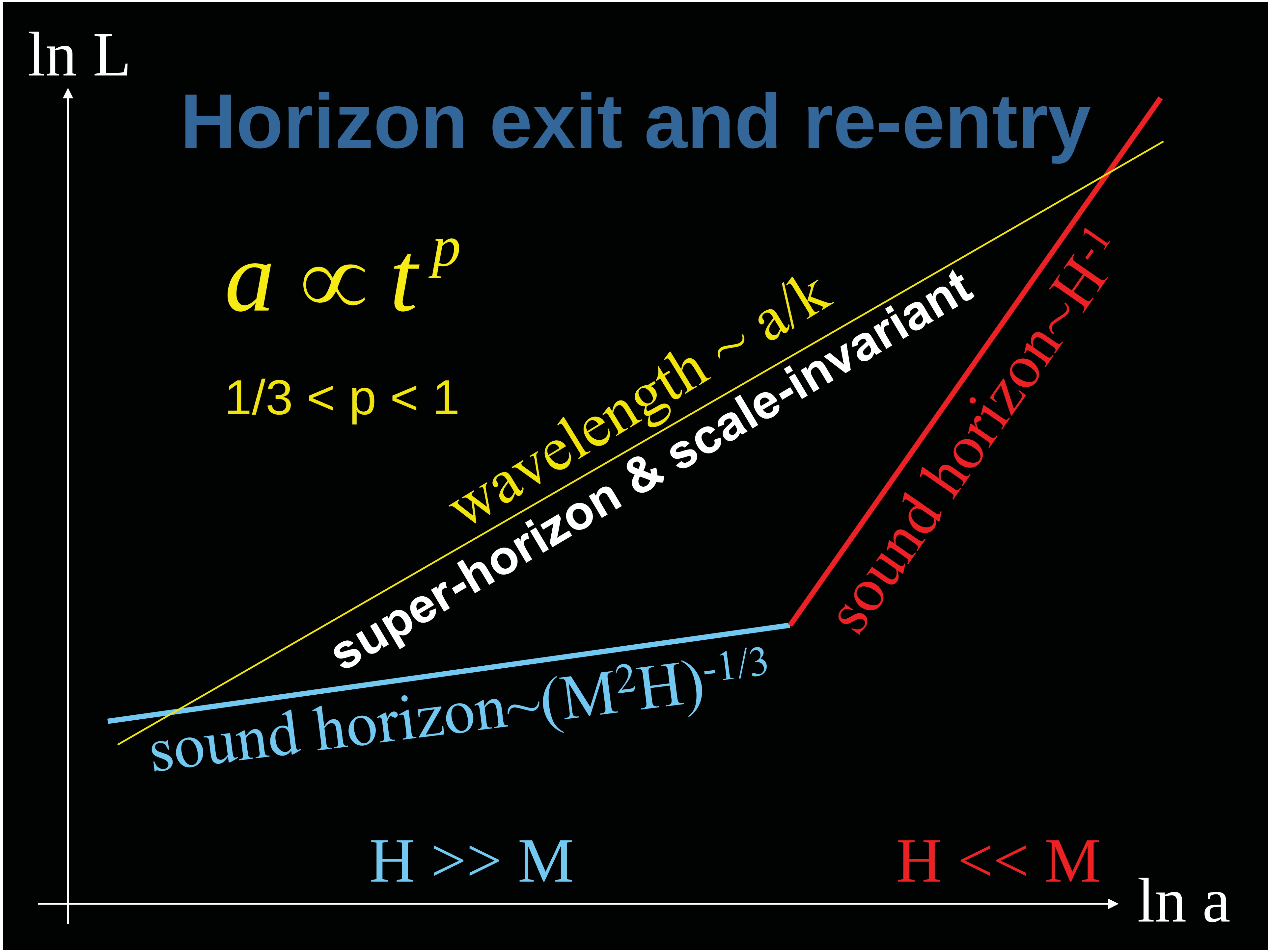
  $t \rightarrow b^3 t$ ($E \rightarrow b^{-3}E$)

  $x \rightarrow b\, x$

  $\phi \rightarrow b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$

  Scale-invariant fluctuations!

Horizon exit and re-entry



Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

See also arXiv:0906.5069 [hep-th]
Caustic avoidance in Horava-Lifshitz gravity

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No "local" Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  can be integrated to give
  Friedmann eq with
  "dark matter as
  integration constant"

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

Summary so far

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+DM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.

IR action

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , seems to recover GR iff λ → 1

kinetic term

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2} + \underbrace{c_g^2 R - 2\Lambda} \right)$$

IR potential

Physical d.o.f.

- (6 + 3) – 3 – 3 = 3
  g_{ij} : 6 components
  N^i : 3 components
  $x^i \rightarrow x'^i(t,x)$: 3 gauge d.o.f.
  $\delta I/\delta N^i=0$: 3 constraints
- 3 = 2 + 1
  tensor graviton: 2 d.o.f.
  scalar graviton: 1 d.o.f.

Perturbative vs non-perturbative regimes

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = a^2 e^{2\zeta_T} \left(e^h\right)_{ij}$$

$$\zeta_T = O(q), \quad h_{ij} = O(q), \quad \boxed{B = O(q^0), \quad n_i = O(q^0)}$$

Momentum constraint

$$\Rightarrow \quad B = \frac{O(1)}{\boxed{O(\lambda - 1) + O(q)}} \partial_t \zeta_T$$

- Perturbative regime: q << (λ-1)
  breakdown in the λ → 1 limit
- Non-perturbative regime: (λ-1) << q << 1
  responsible for recovery of GR

Analogue of Vainshtein effect (mukohyama 2010)

- Spherically symmetric, static ansatz

$$N = 1, \quad N_i dx^i = \beta(x) dx, \quad g_{ij} dx^i dx^j = dx^2 + r(x)^2 d\Omega_2^2$$

$\Downarrow$ $R \equiv \beta^{(\lambda-1)/(2\lambda)} r$ without z>1 terms

$$R'' + \frac{\lambda - 1}{\lambda}\left[\frac{(3\lambda-1)(\beta')^2 R}{4\lambda^2\beta^2} + \frac{(\lambda-1)\beta' R'}{\lambda\beta} - \frac{(R')^2}{R}\right] = 0$$

$$\frac{\beta'}{\beta} - \frac{(\lambda-1)R}{4\lambda R'}\left(\frac{\beta'}{\beta}\right)^2 + \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}} = 0$$

- Two branches

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1 + 4AB}}{2A},$$

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}}$$

- "-" branch recovers GR in the $\lambda \to 1$ limit

Analogue of Vainshtein effect (mukohyama 2010)

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$

⇨ choose the “-” branch

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda}+[(2\lambda-1)\beta^2-1](R')^2}{(3\lambda-1)\beta^2+(\lambda-1)}$$

- $(3\lambda-1)\beta^2 << (\lambda-1)$
  perturbative regime, 1/r expansion
- $(3\lambda-1)\beta^2 >> (\lambda-1)$
  non-perturvative regime, recovery of GR
- $(3\lambda-1)\beta^2 \sim (\lambda-1)$ with $\beta^2 \sim r_g/r$ → $r \sim r_g/(\lambda-1)$
  analogue of Vainshtein radius

Nonlinear cosmological perturbation and λ → 1

arXiv: 1105.0246 [hep-th] with K.Izumi

arXiv: 1109.2609 [hep-th] with E.Gumruhcuglu & A.Wang

- Flat FRW background driven by “DM as an integration constant” + a scalar field
- **Nonlinear cosmological perturbation** in HL gravity with a scalar field (matter)
- Gradient expansion up to any order
- **Regular and continuous in the λ → 1 limit**
- **Recovers GR+DM+scalar field with λ → 1**

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+DM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.
- HL gravity in the $\lambda \rightarrow 1$ limit exhibits analogue of Vainshtein effect: GR+DM is recovered non-perturbatively at least in some simple cases.
  1. spherically-symmetric, static, vacuum configurations
  2. superhorizon cosmological perturbations

Backup slides

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g} d^3x \left(K_{ij}K^{ij} - \lambda K^2 + R - 2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} - 8\pi G_N T_{\mu\nu})n^\mu n^\nu = 0$$

$$n_\mu dx^\mu = -N dt, \quad n^\mu \partial_\mu = \frac{1}{N}(\partial_t - N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu} + \Lambda g^{(4)}_{i\mu} - 8\pi G_N T_{i\mu})n^\mu = 0$$

$$G^{(4)}_{ij} + \Lambda g^{(4)}_{ij} - 8\pi G_N T_{ij} = 0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$