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Emergent low-energy Lorentz invariance in theories with dynamical preferred frame

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Emergent low-energy Lorentz invariance with theories with dynamical preferred frame

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Workshop of Infrared modifications of Gravity, Trieste, 2011

SUPER-AETHER

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Theoretical framework:

Lorentz invariance (LI) is not a fundamental symmetry of nature but emerges only as a low-energy property

Motivations

- In models of modified gravity LI is often broken

  explicitly: ghost condensate, LV massive gravity, Einstein-aether, ...

  implicitly: theories with superluminal propagation, e.g. Galileons, 'Lorentz invariant' massive gravity, ...

- Hints from quantum gravity, e.g. *Horava, 2009*

- Probably, LV has been observed (*OPERA Collaboration, 2011*) ...

But how then LI emerges ???

$$|c_e - c_\gamma| < 10^{-15}\,, \qquad |c_p - c_\gamma| < 10^{-20}$$

With the help of SUPERSYMMETRY !

Groot Nibbelink & Pospelov, 2005
Bolokhov, Groot Nibbelink & Pospelov, 2005

Non-relativistic SUSY

Spatial momenta P_a , $a = 1, 2, 3$, energy P_0 and $SO(3)$ rotations J_a . No boosts

Add a supercharge Q_α - the $SO(3)$ spinor, and its complex conjugate $\bar{Q}^\alpha$, $\alpha = 1, 2$

The most general SUSY algebra:

$$\{Q_\alpha, Q_\beta\} = 2A\sigma^a_{\alpha\beta} P_a \ ,$$

$$\{\bar{Q}^\alpha, \bar{Q}^\beta\} = -2A^*(\sigma^a)^{\alpha\beta} P_a \ ,$$

$$\{Q_\alpha, \bar{Q}^\beta\} = 2B(\sigma^a)_\alpha^\beta P_a - 2C\delta_\alpha^\beta P_0 \ ,$$

$$[P_a, Q_\alpha] = [P_a, \bar{Q}^\alpha] = [P_0, Q_\alpha] = [P_0, \bar{Q}^\alpha] = 0$$

NB. C has dimension of velocity

Non-relativistic SUSY (cntd)

- Set $A = 0$ by

$$Q_\alpha \mapsto \tilde{Q}_\alpha = a_1 Q_\alpha + a_2 \varepsilon_{\alpha\beta} \bar{Q}^\beta$$

- Set $B = C = 1$ (if $B,\ C \neq 0$)
- Redefine $\bar{Q}^\alpha \mapsto \bar{Q}_{\dot\alpha}$

⇒ recover the standard 4d SUSY !

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^m_{\alpha\dot\beta} P_m \ ,$$
$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = [P_m, Q_\alpha] = [P_m, \bar{Q}_{\dot\alpha}] = 0 \ ,$$

BUT without boosts

It is enough to construct the superspace

Superfield Lagrangians

MSSM contains chiral superfields $\Phi_{(I)}$ and real superfields $V_{(J)}$

NB. They don't carry Lorentz indices

Dimensions of the objects in the Lagrangian:

matter field $\Phi_{(I)}$	1
gauge field $V_{(J)}$	0
gauge field strength $W_{(J)\,\alpha}$	3/2
space-time derivative ∂_m	1
supercovariant derivatives $D_\alpha,\ \bar{D}_{\dot{\alpha}}$	1/2
chiral measure $\int d^2\theta$	1
super-space measure $\int d^2\theta d^2\bar{\theta}$	2

Superfield Lagrangians (cntd)

- Cannot write any Kahler term with external Lorentz indices of dimension < 4
- Can write a unique superpotential term of dimension 4

$$\kappa_{(IJ)} \int d^2\theta \, \Phi_{(I)} \partial_0 \Phi_{(J)} + \text{h.c.} \, , \quad I \neq J$$

but it's not gauge invariant for charged $\Phi_{(I)}$

NB. This term appears for sterile neutrinos. Can be used to assess the OPERA results (*Giudice, S.S., Strumia, 2011*)

Within MSSM LI emerges as an accidental symmetry

SUSY breaking ⟹ violation of LI at the level $(m_{soft}/M_{LV})^2$

Dynamical aether

The theory can be written as formally LI with the preferred frame set by the global vector $u^m = (1,0,0,0)$ - aether

When coupled to gravity aether must become dynamical

Dynamical aether (cntd)

Jacobson, Mattingly, 2000

$$S_{æ} = -\frac{M_{æ}^2}{2}\int \sqrt{-g}\Big(c_1 \nabla_n u_m \nabla^n u^m + c_2 (\nabla_m u^m)^2 + c_3 \nabla_n u_m \nabla^m u^n$$
$$-c_4\, u^r u^s \nabla_r u_m \nabla_s u^m + \lambda(u_m u^m + 1)\Big) d^4x$$

Lagrange multiplier:
enforces* $u_m u^m = -1$

- c_2 - and c_3 - terms are equivalent in flat space
- $M_{æ}$ sets the scale of Lorentz violation

  ⇔ corrections to GR (e.g. PPN parameters) are proportional to $(M_{æ}/M_{pl})^2$
- All PPN parameters except α_1 , α_2 are as in GR

* The metric signature is **(-,+,+,+)**

Super - aether: the choice of multiplet

Constant aether must break LI, but not SUSY

⇒ u^m is the lowest component of the multiplet

Consider chiral vector superfield $\bar{D}_{\dot{\alpha}} U^m = 0$

$$U^m = u^m(y) + \sqrt{2}\theta^\alpha \eta^m_\alpha(y) + \theta^2 G^m(y)$$

$$y^m = x^m + i\theta\sigma^m\bar{\theta}$$

NB. The aether vector u^m is now complex

We want to impose the constraint

$$U^m U_m = -1$$

Super - Aether: the Lagragian

$$\mathcal{L} = M_{æ}^2 \left[\int d^2\theta d^2\bar{\theta}\; f(U^m \bar{U}_m) + \left(\int d^2\theta\; \Lambda(U^m U_m + 1) + \text{h.c.} \right) \right]$$

Lagrange multiplier
(chiral field)

Symmetry: Lorentz invariance $\times$ internal $SO(3,1)$

➡ again accidental LI at low energy

Will be broken by higher order operators, or coupling to other fields, or gravity

Super - Aether: bosonic part and vacua

$$\mathcal{L}_{bos} = M_{æ}^2 \Big[- f^{mn}\, \partial_r \bar{u}_m \partial^r u_n + \big[H (u_m u^m + 1) + \text{h.c.} \big] \Big]$$

$$f^{mn} = f'(|u|^2)\, \eta^{mn} + f''(|u|^2)\, u^m \bar{u}^n$$

Restricting to real aether:

$$c_1 = 2 f'(-1) \ , \quad c_2 + c_3 = c_4 = 0$$

Super - Aether: bosonic part and vacua

$$\mathcal{L}_{bos} = M_{æ}^2 \Big[- f^{mn}\, \partial_r \bar{u}_m \partial^r u_n + \big[H(u_m u^m + 1) + \text{h.c.} \big] \Big]$$

$$f^{mn} = f'(|u|^2)\, \eta^{mn} + f''(|u|^2)\, u^m \bar{u}^n$$

General complex aether:

$$u_m u^m = -1 \quad \Longrightarrow \quad \begin{cases} u_{\rm R}^m u_{{\rm R}\,m} - u_{\rm I}^m u_{{\rm I}\,m} = -1 \\ u_{\rm R}^m u_{{\rm I}\,m} = 0 \end{cases}$$

Two families of inequivalent vacua:

$u^m_{vac} = (\cos\alpha,\ 0,\ 0,\ i\sin\alpha)$ OK ✔

$u^m_{vac} = (0, 0, \sinh\beta, i\cosh\beta)$ ghosty ✘

NB. A general vacuum breaks spatial isotropy

Effects of SUSY breaking

$$\mathcal{L}_{SB} = -M_{æ}^2 \int d^2\theta d^2\bar{\theta} \left[S_{(1)}\, g_{(1)}(U^m \bar{U}_m) + S_{(2)}\, g_{(2)}(U^m \bar{U}_m) \right]$$

spurions

$$S_{(1)} = m_{(1)}^2\, \theta^2 \bar{\theta}^2$$

$$S_{(2)} = m_{(2)}(\theta^2 + \bar{\theta}^2)$$

Imaginary part of the aether and fermions acquire masses of order $m_{(i)}$

CONCLUSIONS

- Non-relativistic SUSY ensures emergence of LI at low energy

- It is possible to realize this mechanism in theories with dynamical preferred frame → super-aether model

- Super-aether exhibits rich dynamics. General vacua break spatial isotropy (interesting applications to inflation ?)

- SUSY breaking gives masses to aether partners

OPEN ISSUES

- Coupling to (super)gravity. Does SUSY ensure $\alpha_2 = 0$?
- Extending notion of SUSY to theories with anisotropic scaling
- Supersymmetrizing other Lorentz violating models (ghost condensate, khrono-metric model, ...) compatibly with emergent LI