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Black holes is Einstein-aether and Horava-Lifshitz gravity

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Black holes in Einstein-aether and Hořava-Lifshitz gravity

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Why Lorentz-violating gravity?

Lorentz-violating effects are severely constrained in the matter sector. However,

- Observational constraints are far weaker in the more weakly coupled gravitational sector
- A low energy effective theory of Lorentz-violating gravity is needed for such tests (e.g. Einstein-aether theory)

The might be an additional pay-off:

- Recently it has been claimed that some models of Lorentz-violating gravity are power-counting renormalizable (Hořava-Lifshitz gravity)

Why black holes?

- Simpler to find than other solutions (e.g. stars)
- Potentially a probe for observable deviations from GR
- Intrinsically interesting in theories with Lorentz violations

What characterizes a black hole in this case?

- There can be different modes with different speeds, so there can be multiple horizons
- There can be modified dispersion relations, in which case short wavelength perturbations can travel with arbitrarily high speed

Einstein-aether theory

The action of the theory is

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int d^4x \sqrt{-g}(-R - M^{\alpha\beta\mu\nu}\nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives
- Extensively tested and still viable

Imposing hypersurface orthogonality...

Now assume

$$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T}(g^{TT})^{-1/2} = N\delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{h.o.\,æ} = \frac{1}{16\pi G_H}\int dT d^3x\, N\sqrt{h}\,(K_{ij}K^{ij} - \lambda K^2 + \xi\,{}^{(3)}R + \eta a_i a^i)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_æ} = \xi = \frac{1}{1-c_{13}} \qquad \lambda = \frac{1+c_2}{1-c_{13}} \qquad \eta = \frac{c_{14}}{1-c_{13}}$$

Hořava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H}\int dT d^3x\, N\sqrt{h}\left(L_2 + \frac{1}{M_\star^2}L_4 + \frac{1}{M_\star^4}L_6\right)$$

where

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi\, {}^{(3)}R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

$M_\star : 10^{10}\mathrm{GeV} - 10^{16}\mathrm{GeV}$

Our goal

We are interested in vacuum black hole solutions which are

- spherically symmetric
- static
- asymptotically flat
- and can form from gravitational collapse

Finding such solutions analytically seems unfeasible, so we find them numerically

- Spherically symmetric vectors are hypersurface orthogonal!
- Consequence: Sphericallly symmetric solutions of ae-theory will be also solutions of low-energy HL-gravity

The metric and aether ansaetze

We use Eddington-Finkelstein(-like) coordinates

$$ds^2 = F(r)dv^2 - 2B(r)dvdr - r^2 d\Omega^2$$

$$u^\alpha \partial_\alpha = A(r)\partial_v - \frac{1 - F(r)A^2(r)}{2B(r)A(r)}\partial_r$$

In general there is a 3-parameter family of solutions. However

- asymptotic flatness
- regularity of the spin-0 horizon

will lead to a 1-parameter family.

- Fixing the horizon radius one has a unique solution

Equations and Constraints

The field equations are

$$E^{\mu\nu} \equiv G^{\mu\nu} - T_{æ}^{\mu\nu} = 0 \qquad\qquad Æ^{\mu} = 0$$

but the non-redundant or non-trivial components are

$$E^{vv} = E^{vr} = E^{rr} = E^{\theta\theta} = Æ^{v} = 0$$

However, the combinations

$$C^{\mu} \equiv E^{r\mu} - u^{r} Æ^{\mu} = 0$$

are actually constraint equations, so we will use instead the system

$$E^{vv} = E^{\theta\theta} = Æ^{v} = 0, \quad C^{v} = C^{r} = 0$$

Equations and Constraints

The first three equations can be recast in the form

$$
\begin{aligned}
F'' &= F''(A, A', B, F, F') \\
A'' &= A''(A, A', F, F') \\
B' &= B'(A, A', B, F, F')
\end{aligned}
$$

so we need 5 pieces of initial data. Additional, the constraints

- are automatically preserved if they are imposed initially
- relate initial date

As mentioned, one then generically has 3-parameter solutions.

Horizons and field redefinitions

In general there will be spin-2, spin-1 and spin-0 modes. For the spin-0

$$s_0^2 = \frac{c_{123}(2-c_{14})}{c_{14}(1-c_{13})(2+c_{13}+3c_2)}$$

The horizon for a mode is a null surface of the effective metric

$$g^{(i)}_{\alpha\beta} = g_{\alpha\beta} + (s_i^2 - 1)u_\alpha u_\beta$$

Together with the aether redefinition

$$u^\alpha_{(i)} = \frac{1}{s_i} u^\alpha$$

the metric redefinition leave the action formally invariant

Numerical implementation

- Perform the redefinition just described
- Define the common spin-0/metric horizon by $F(r_H)=0$
- Require regularity of the horizon

$$B' = \frac{b_0}{F} + b_1 + b_2 F \quad \xrightarrow{\text{regularity}} \quad b_0(A, A', B, F')|_H = 0$$

- Impose the constraints: one is trivial on the horizon, the other is

$$C^v(A, A', B, F')|_H = 0$$

- Choose the time coordinate such that $B_H = 1$
- Parametrize the space of initial data by A_H
- Determine the value that leads to asymptotic flatness by a shooting method

Parameter space

We impose the following viability constraints

- Classical and quantum-mechanical stability
- Avoidance of vacuum Cherenkov radiation by matter.
- Exact agreement with Solar system experiments (vanishing preferred frame parameters)

The last constraint is more restrictive than actually required. However,

- It reduces the dimensions of the parameters space to 2
- It provided an important simplification with little given away

Parameter space for ae-theory

Exact agreement with Solar system test requires

$$c_2 = \frac{-2c_1^2 - c_1 c_3 + c_3^2}{3c_1} \qquad\qquad c_4 = -\frac{c_3^2}{c_1}$$

Then, defining

$$c_\pm = c_1 \pm c_3$$

the remaining constraint require

$$0 \leq c_+ \leq 1 \qquad\qquad 0 \leq c_- \leq \frac{c_+}{3(1-c_+)}$$

We have explored the region that additionally satisfies

$$c_- \leq 1$$

Parameter space for HL-gravity

It is more convenient to work in the parametrization

$$\xi = \frac{1}{1-\beta} \qquad \lambda = \frac{1+\mu}{1-\beta} \qquad \eta = \frac{\alpha}{1-\beta}$$

Then, exact agreement with Solar system test requires

$$\alpha = 2\beta$$

and the remaining constraint require

$$0 < \beta < 1/3 \qquad \mu > \frac{\beta(\beta+1)}{1-3\beta}$$

$$0 < \beta < 1/3 \qquad \mu < -\frac{2+\beta}{3}$$

$$1/3 < \beta < 1 \qquad \frac{\beta(\beta+1)}{1-3\beta} < \mu < -\frac{2+\beta}{3}$$

Characteristic quantities

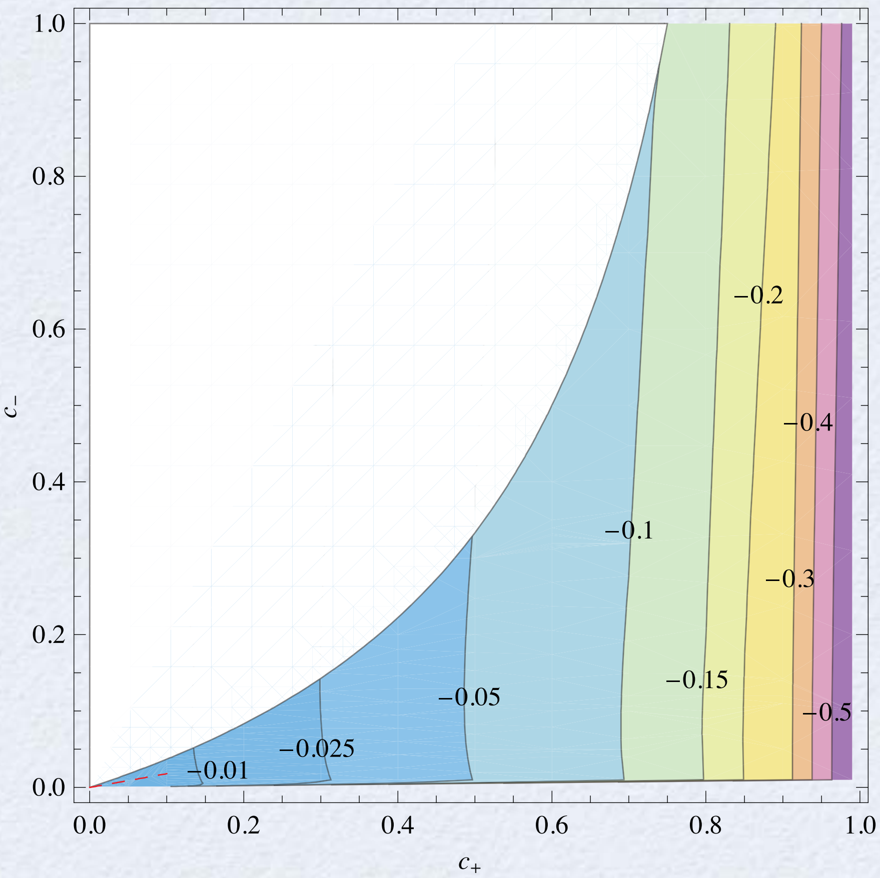
$\omega_{\rm ISCO}\, r_g$: orbital frequency at the ISCO times gravitational radius.
Can be measured using X-ray spectra from accretion or gravitational waves from EMRIs

$z_{\rm max}$: $(= \nu_{\rm emitted}/\nu_{\rm measured} - 1)$ the maximum redshift for a photon emitted at the ISCO. Can be measured using iron-Kα line

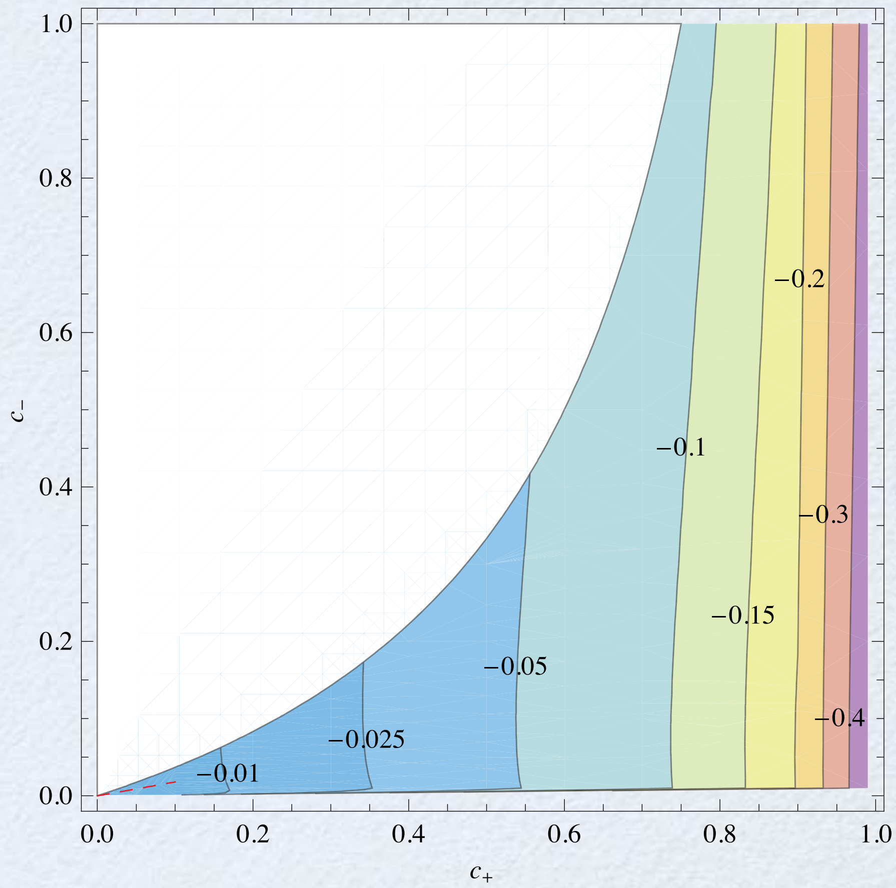
$b_{\rm ph}/r_g$: impact parameter for circular photon orbit/grav. radius
Can be measured by gravitational lensing or in the future via black hole quasinormal modes

r_g/r_H : grav. radius/horizon radius
Not measurable but gives info about near horizon region

Results: ae-theory

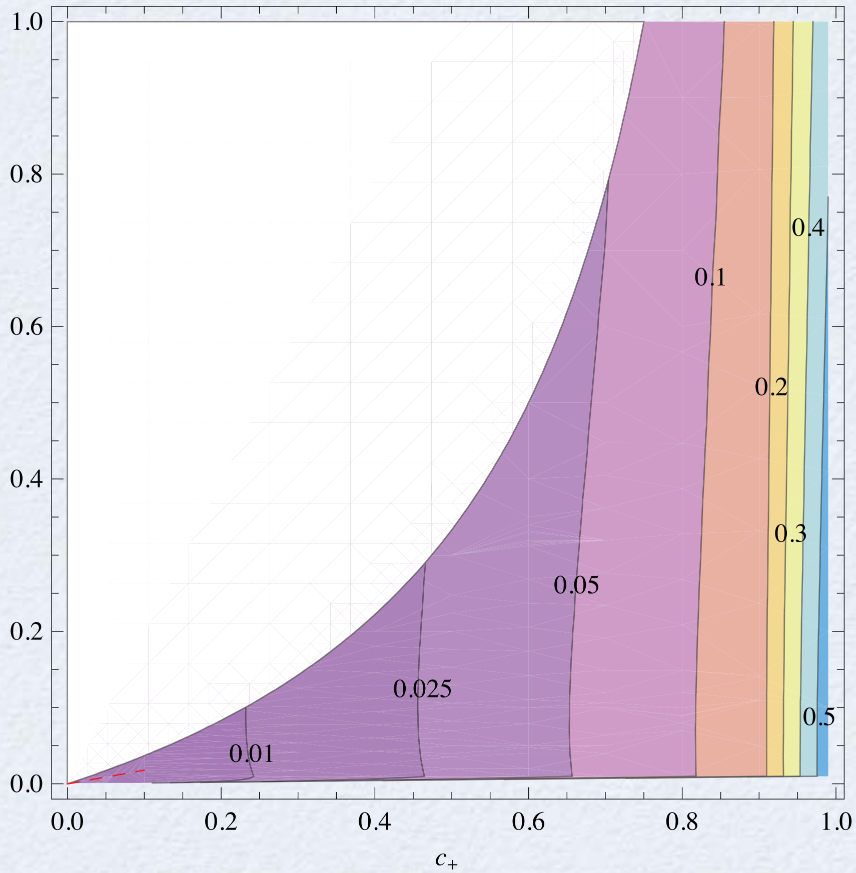


$\omega_{\text{ISCO}}\, r_g$

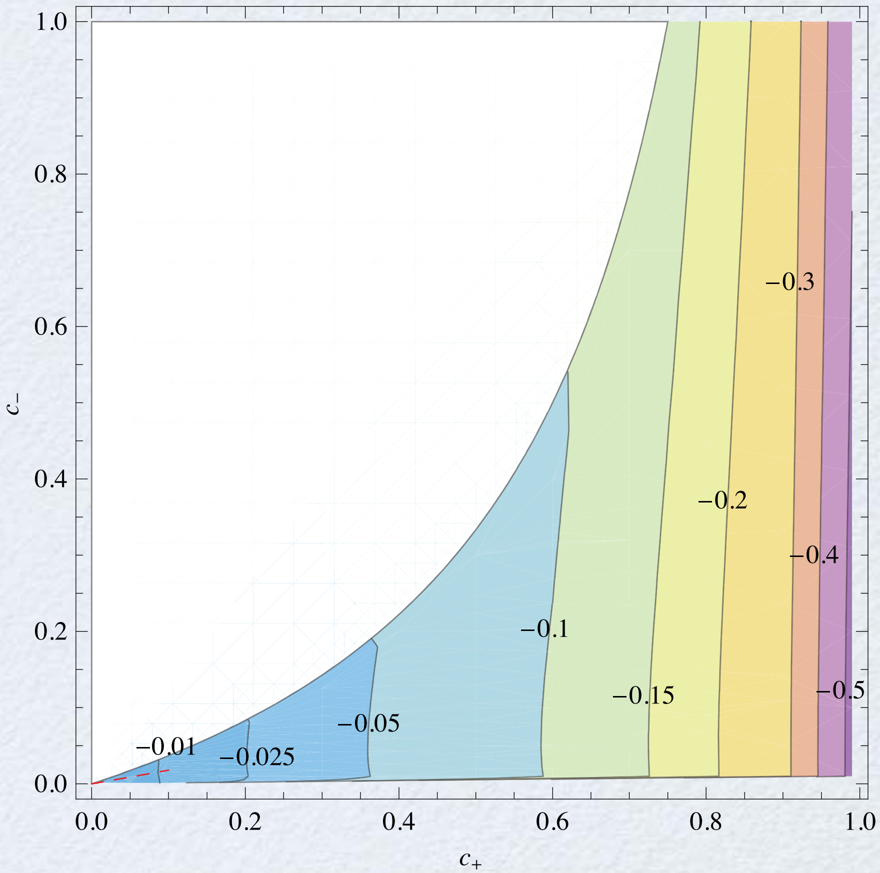


z_{max}

Results: ae-theory

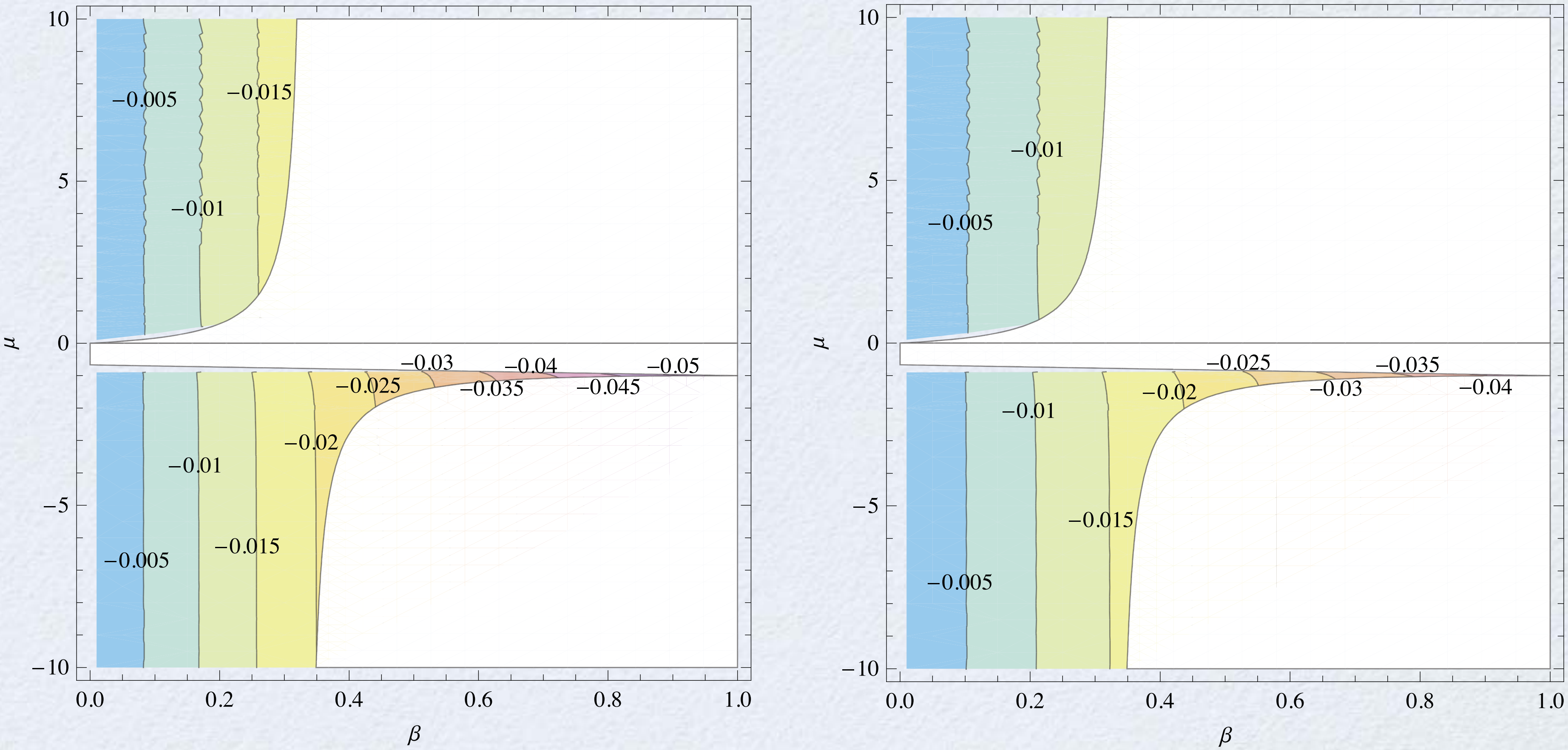


$b_{\rm ph}/r_g$



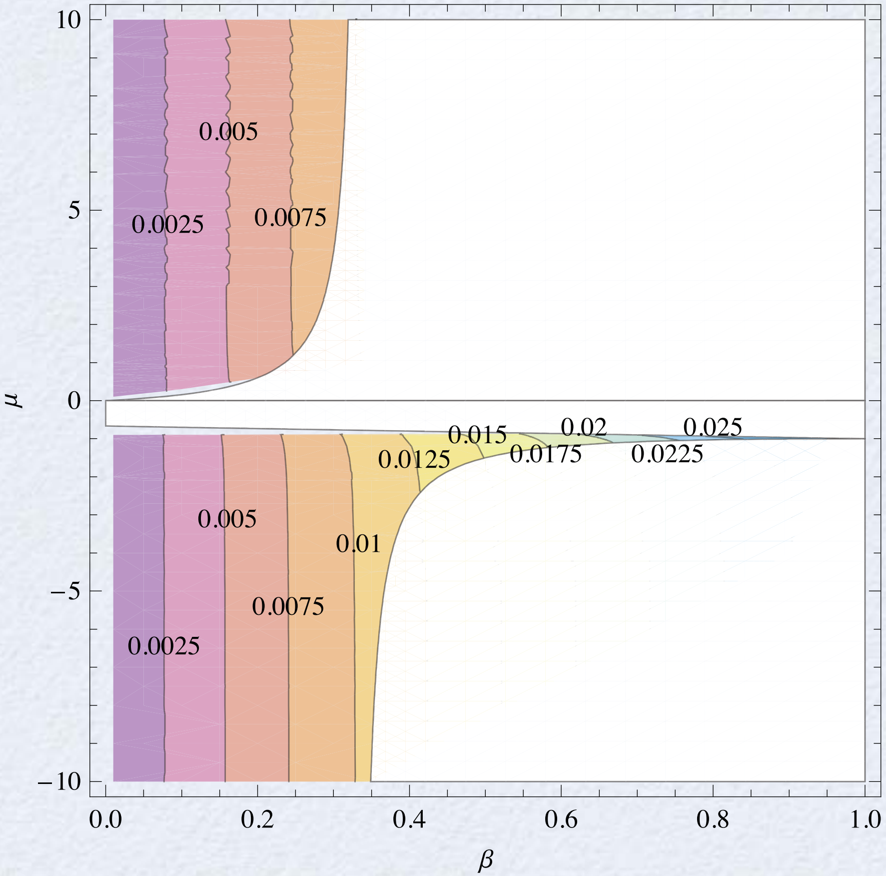
r_g/r_H

Results: HL-gravity

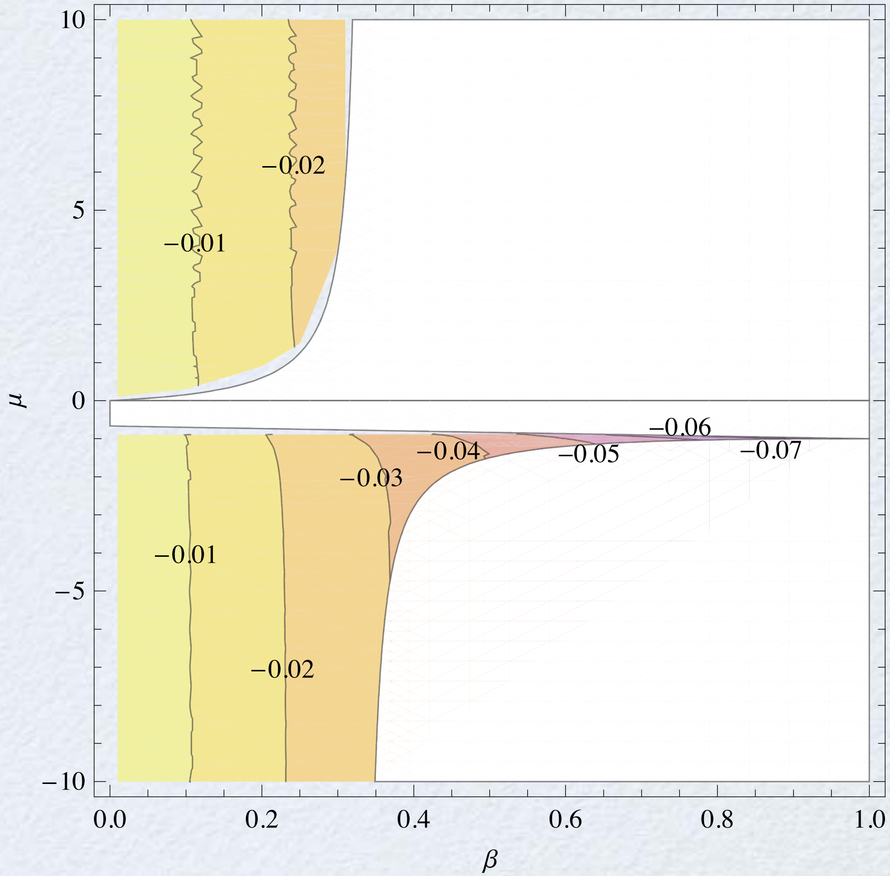


$\omega_{\rm ISCO}\, r_g$ $z_{\rm max}$

Results: HL-gravity



$b_{\rm ph}/r_g$



r_g/r_H

Interior solution

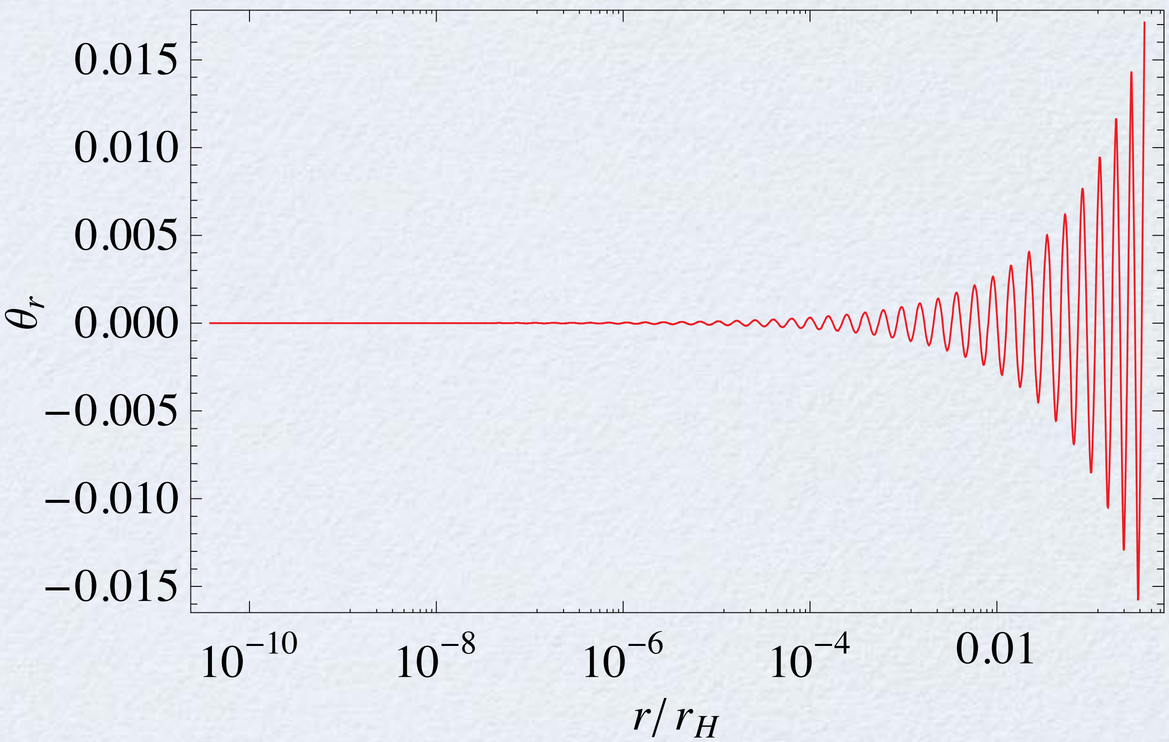
- Curvature singularity at the centre

We have also calculated the Lorentz factor of the aether as measured by the future directed observer orthogonal to $r=$ const. hypersurfaces

$$\gamma_r \equiv u^\alpha_{\rm obs} u_\alpha$$

and the corresponding boost angle

$$\theta_r = {\rm arccosh}\gamma_r$$



Ultimate horizon

- Different modes have different horizons
- When higher spatial derivatives are added short wavelength perturbations can travel at arbitrarily high speed

However,

- Signals cannot travel backwards in time
- Future and past direction are locally defined by the aether
- The aether is orthogonal to constant time hypersurfaces in the preferred foliation
- When the boost angle vanishes the aether is orthogonal to constant r hypersurfaces
- Ultimate horizon!

Conclusions

- We have found a one parameter family of static, spherically symmetric, asymptotically flat black hole solutions with regular spin-0 horizons
- We have covered a significant portion of the allowed parameter space for both theories
- Deviations from general relativity are rather small. However, potentially observable with future observations
- A generic feature of these black holes seems to be the existence of an ultimate horizon (or a succession of them)