United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2264-13

Workshop on Infrared Modifications of Gravity

26 - 30 September 2011

Galileons on curved backgrounds

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Generalizing Galileons

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Workshop on Infrared
Modifications of Gravity

ICTP, Trieste
9/29/2011

Overview

- Some quick motivations
- Galileons - a quick reminder - this talk is tightly tied to the previous one by Kurt Hinterbichler
- Galileons on Curved Spaces - Cosmological Backgrounds
- Multi-Galileons and Higher Co-Dimension Branes
- Future work and comments.

Motivations

- Scalar fields appear useful in particle physics and are ubiquitous in cosmology

- Used to break the electroweak symmetry, solve the strong CP problem, inflate the universe, accelerate it at late times, ...

- In most incarnations, the sweet properties of these scalars are offset by their tendency to be most unruly in the face of quantum mechanics.

- Attempts to do away with scalars for some of these tasks, such as modifying gravity, seem to yield scalars in any case, in certain limits, or as part of the construction (see many of the talks of the last few days).

- As you have seen and will see a little more, Galileons are an intriguing new class of scalars that *may* have a shot of addressing some of these problems, and seem to be tied at some level to attempts to modify gravity such as massive gravity.

- We'll see, but it is turning out to be great fun trying.

Galileons on
Cosmological Spaces

The Decoupling Limit (of, e.g. DGP)

$$S = \frac{M_5^3}{2r_c}\int d^5x\sqrt{-G}\ R^{(5)} + \frac{M_4^2}{2}\int d^4x\sqrt{-g}\ R$$

Much of interesting phenomenology of DGP captured in the *decoupling limit:*

$M_4,\ M_5 \to \infty$ $\qquad \Lambda_{\rm strong} \equiv (M_4 r_c^{-2})^{1/3}$ kept finite

Only a single scalar field - the brane bending mode - remains

Very special symmetry, inherited from combination of:

- 5d Poincare invariance, and
- brane reparameterization invariance

$$\pi(x) \to \pi(x) + c + b_\mu x^\mu$$

The *Galilean* symmetry!

Galileons

Can consider this symmetry as interesting in its own right

- Yields a novel and fascinating 4d effective field theory
- Relevant field referred to as the *Galilean*

(Nicolis, Rattazzi, & Trincherini 2009)

$$\mathcal{L}_1 = \pi \qquad \mathcal{L}_2 = (\partial\pi)^2 \qquad \mathcal{L}_3 = (\partial\pi)^2\Box\pi$$

$$\mathcal{L}_4 = \partial_\mu\pi^I\partial_\nu\pi_I\left(\partial^\mu\partial_\rho\pi^J\partial^\nu\partial^\rho\pi_J - \partial^\mu\partial^\nu\pi^J\Box\pi_J\right) + \cdots \qquad \mathcal{L}_5 = \cdots$$

There is a separation of scales

- Allows for classical field configurations with order one nonlinearities, but quantum effects under control.
- So can study non-linear classical solutions involving galileon terms, and trust solutions

Computing Feynman diagrams - terms of the galilean form cannot receive new contributions! More soon.

Luty, Porrati, Ratazzi (2003); Nicolis, Rattazzi (2004)

Galileons on General Backgrounds

G.Goon, K.Hinterbichler, M.T., *Phys. Rev.Lett.* 106, 231102 (2011).
G.Goon, K.Hinterbichler, M.T., *JCAP* 1107, 017 (2011).

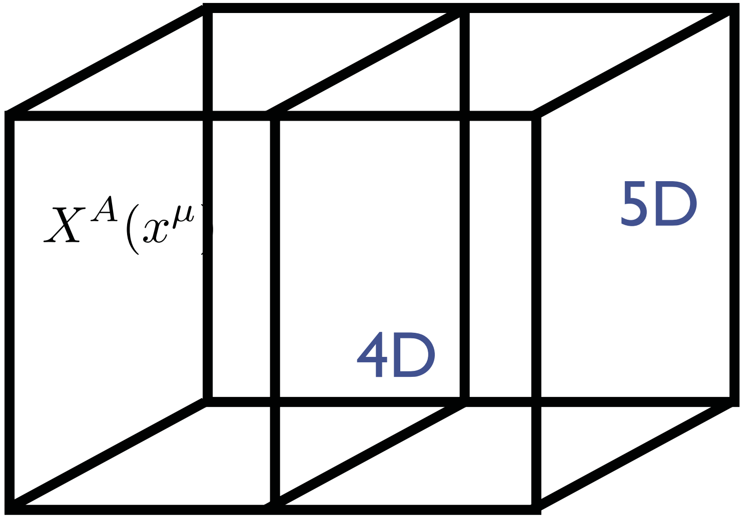
Pick up where Kurt left off - a quick reminder

- Can extend probe brane construction (de Rham & Tolley) to more general geometries. e.g. other maximally-symmetric examples

Bulk $ds^2 = d\rho^2 + f(\rho)^2 g_{\mu\nu}(x)dx^\mu dx^\nu$

Induced on Brane $\bar{g}_{\mu\nu} = f(\pi)^2 g_{\mu\nu} + \nabla_\mu\pi\nabla_\nu\pi$

Bulk Killing Vectors $\delta_K X^A = a^i K_i^A(X) + a^I K_I^A(X)$



Galileons with symmetry

$$(\delta_K + \delta_{g,\mathrm{comp}})\pi = -a^i k_i^\mu(x)\partial_\mu\pi + a^I K_I^5(x,\pi) - a^I K_I^\mu(x,\pi)\partial_\mu\pi$$

The Maximally-Symmetric Taxonomy

Potentially different Galileons corresponding to different ways to foliate a maximally symmetric 5-space by a maximally symmetric 4-d hypersurface

Brane metric

Ambient metric

	AdS_4	M_4	dS_4
AdS_5	AdS DBI galileons $so(4,2) \to so(3,2)$ $f(\pi) = \mathcal{R}\cosh^2(\rho/\mathcal{R})$	Conformal DBI galileons $so(4,2) \to p(3,1)$ $f(\pi) = e^{-\pi/\mathcal{R}}$	type III dS DBI galileons $so(4,2) \to so(4,1)$ $f(\pi) = \mathcal{R}\sinh^2(\rho/\mathcal{R})$
M_5	×	DBI galileons $p(4,1) \to p(3,1)$ $f(\pi) = 1$	type II dS DBI galileons $p(4,1) \to so(4,1)$ $f(\pi) = \pi$
dS_5	×	×	type I dS DBI galileons $so(5,1) \to so(4,1)$ $f(\pi) = \mathcal{R}\sin^2(\rho/\mathcal{R})$
Small field limit	$\downarrow$ AdS galileons	$\downarrow$ normal galileons	$\downarrow$ dS galileons

Galileons on Gaussian Normal Foliations

G.Goon, K. Hinterbichler, M.T., September 2011

Can we foliate a 5-d space in an interesting way such that the resulting theory describes galileons with the appropriate symmetries to propagate on a Friedmann, Robertson-Walker (FRW) background?

- Can actually do a little better - can do a general Gaussian Normal foliation

$$G_{AB}dX^AdX^B = f_{\mu\nu}(x,w)dx^\mu dx^\nu + dw^2$$

$$\bar{g}_{\mu\nu} = f_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$$

Induced
on Brane

$$\mathcal{L}_1 = \int^{\pi(x)} d\pi' \sqrt{-\det f_{\mu\nu}(x,\pi')},$$

$$\mathcal{L}_2 = -\sqrt{-f}\frac{1}{\gamma},$$

$$\mathcal{L}_3 = \sqrt{-f}\Big[-\langle\Pi\rangle + \frac{1}{2}\langle f'\rangle + \gamma^2\left(\langle\pi\Pi\pi\rangle + \frac{1}{2}\langle\pi f'\pi\rangle\right)\Big],$$

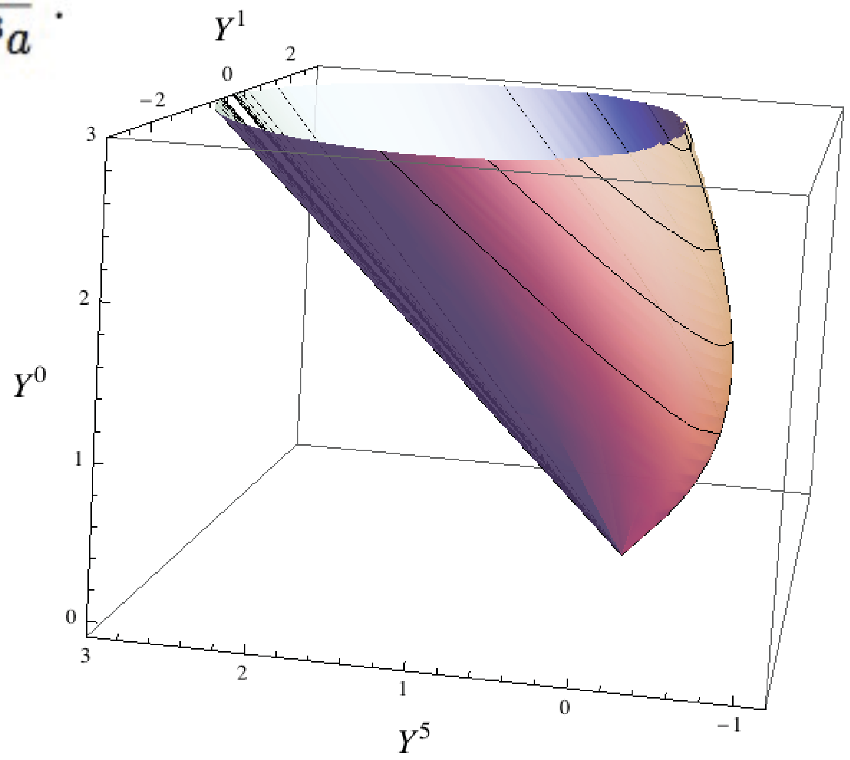
$$\begin{aligned}\mathcal{L}_4 = \sqrt{-f}\Big[&-\frac{1}{2}\langle\pi f'\pi\rangle^2\gamma^3 - \langle f'\rangle\langle\pi\Pi\pi\rangle\gamma^3 - 2\langle\pi\Pi^2\pi\rangle\gamma^3 + 2\langle\pi\Pi\pi\rangle\langle\Pi\rangle\gamma^3\\ &-\frac{1}{2}\langle f'\rangle\langle\pi f'\pi\rangle\gamma^3 + \langle\Pi\rangle\langle\pi f'\pi\rangle\gamma^3 - \frac{\langle f'\rangle^2\gamma}{4} - \langle\Pi\rangle^2\gamma + \frac{\langle f'^2\rangle\gamma}{4}\\ &-\langle\Pi f'\rangle\gamma + \langle f'\rangle\langle\Pi\rangle\gamma + \langle\Pi^2\rangle\gamma + \frac{\langle\pi f'^2\pi\rangle\gamma}{2}\Big],\end{aligned}$$

Embedding 4d FRW in 5d Minkowski

$$ds^2 = -\left(dY^0\right)^2 + \left(dY^1\right)^2 + \left(dY^2\right)^2 + \left(dY^3\right)^2 + \left(dY^5\right)^2$$

$$Y^0 = S(t,w)\left(\frac{x^2}{4} + 1 - \frac{1}{4H^2a^2}\right) - \frac{1}{2}\int dt\,\frac{\dot{H}}{H^3a},$$
$$Y^i = S(t,w)x^i,$$
$$Y^5 = S(t,w)\left(\frac{x^2}{4} - 1 - \frac{1}{4H^2a^2}\right) - \frac{1}{2}\int dt\,\frac{\dot{H}}{H^3a}.$$

$$S(t,w) \equiv a - \dot{a}w$$



Induced Metric on Brane

$$d\tilde{s}^2 = -n^2(t,w)dt^2 + S^2(t,w)\delta_{ij}dx^i dx^j$$

Galileons on Cosmological Backgrounds

G.Goon, K. Hinterbichler, M.T., arXiv:1109.3450 [hep-th] 2011

The form of the first two Lagrangians, for example, is

$$\mathcal{L}_1 = a^3\pi - \frac{a^2\left(3\dot{a}^2 + a\ddot{a}\right)\pi^2}{2\dot{a}} + a\left(\dot{a}^2 + a\ddot{a}\right)\pi^3 - \frac{1}{4}\dot{a}\left(\dot{a}^2 + 3a\ddot{a}\right)\pi^4 + \frac{1}{5}\ddot{a}\dot{a}^2\pi^5,$$

$$\mathcal{L}_2 = -(1 - \frac{\ddot{a}}{\dot{a}}\pi)(a - \dot{a}\pi)^3\sqrt{1 - \left(1 - \frac{\ddot{a}}{\dot{a}}\pi\right)^{-2}\dot{\pi}^2 + (a - \dot{a}\pi)^{-2}(\vec{\nabla}\pi)^2}.$$

and the symmetries are

$$\begin{aligned}
\delta_{v_i}\pi = {} & \frac{1}{2}x^i\dot{a}\int dt\,\frac{\dot{H}}{H^3 a} - \frac{x^i\left(a - \dot{a}\pi + \dot{a}^2\int dt\,\frac{\dot{H}}{H^3 a}\right)}{2\dot{a} - 2\pi\ddot{a}}\dot{\pi} \\
& + \left[\frac{x^i x^i \dot{a}^2 + 1}{4\dot{a}^2} + \frac{\int dt\,\frac{\dot{H}}{H^3 a}}{2a - 2\pi\dot{a}}\right]\partial_i\pi - \sum_{j\neq i}\left[-\frac{x^i x^j}{2}\partial_j\pi + \frac{x^j x^j}{4}\partial_i\pi\right] \\
\delta_{k_i}\pi = {} & x^i\dot{a}\left(\frac{\dot{a}\dot{\pi}}{\dot{a} - \pi\ddot{a}} - 1\right) - \frac{\partial_i\pi}{a - \pi\dot{a}}, \\
\delta_q\pi = {} & \frac{\dot{\pi}\dot{a}^2}{\pi\ddot{a} - \dot{a}} + \dot{a}, \\
\delta_u\pi = {} & \frac{x^2\dot{a}^2 - 1}{4\dot{a}} - \frac{x^2\dot{a}^2 + 1}{4\dot{a} - 4\pi\ddot{a}}\dot{\pi} + \frac{1}{2a - 2\pi\dot{a}}\sum_i x^i\partial_i\pi, \\
\delta_s\pi = {} & -\dot{a}\int dt\,\frac{\dot{H}}{H^3 a} + \frac{\left(a - \dot{a}\pi + \dot{a}^2\int dt\,\frac{\dot{H}}{H^3 a}\right)\dot{\pi}}{\dot{a} - \pi\ddot{a}} - \sum x^i\partial_i\pi,
\end{aligned}$$

These describe covariant versions of Galileons, naturally propagating on FRW backgrounds.

Simple Solutions and Stability

Expand Lagrangians to second order in π, and integrate by parts (a lot)

$$\mathcal{L}_1 = a^3\pi - \frac{1}{2}\left(\frac{\ddot{a}a^3}{\dot{a}} + 3\dot{a}a^2\right)\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_2 = (3a^2\dot{a} + \frac{a^3\ddot{a}}{\dot{a}})\pi + \frac{1}{2}a^3\dot{\pi}^2 - \frac{1}{2}a(\vec{\nabla}\pi)^2 - 3\left(\ddot{a}a^2 + \dot{a}^2 a\right)\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_3 = 6(a\dot{a}^2 + a^2\ddot{a})\pi + 3\dot{a}a^2\dot{\pi}^2 - \left(2\dot{a} + \frac{a\ddot{a}}{\dot{a}}\right)(\vec{\nabla}\pi)^2 - 3\left(3\dot{a}\ddot{a}a + \dot{a}^3\right)\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_4 = 6(\dot{a}^3 + 3a\dot{a}\ddot{a})\pi + 9\dot{a}^2 a\dot{\pi}^2 - 3\left(\frac{\dot{a}^2}{a} + 2\ddot{a}\right)(\vec{\nabla}\pi)^2 - 12\dot{a}^2\ddot{a}\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_5 = 24\dot{a}^2\ddot{a}\,\pi + 12\dot{a}^3\dot{\pi}^2 - 12\frac{\ddot{a}^2\dot{a}}{a}(\vec{\nabla}\pi)^2 + \mathcal{O}(\pi^3)$$

Write

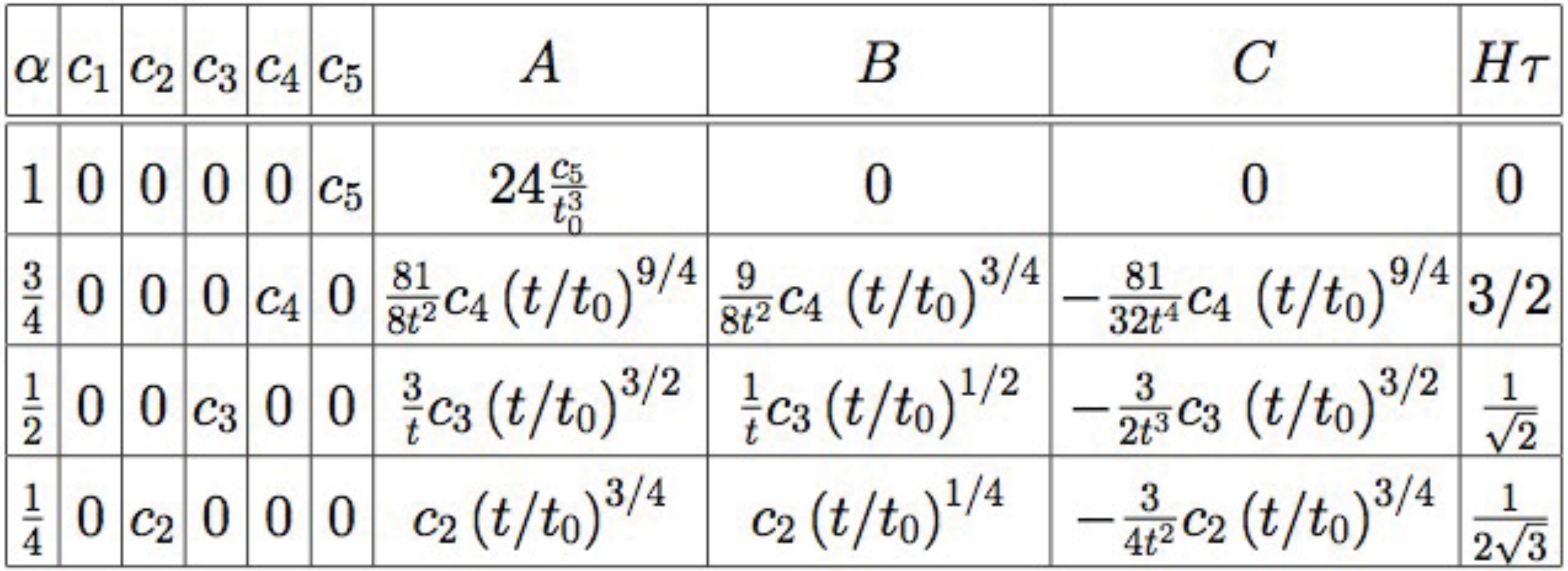
$$\mathcal{L} = \sum_{n=1}^{5} c_n \mathcal{L}_n$$

and just for example, look for combinations for which π=0 is a solution

Fix $a(t) = (t/t_0)^\alpha$ π=0 solutions exist for $\alpha = 1, 3/4, 1/2, 1/4$

Expanding to quadratic order about solution yields
(note - no higher derivatives - one degree of freedom!)

$$\mathcal{L} = \frac{1}{2}A(a(t), c_n)\dot{\pi}^2 - \frac{1}{2}B(a(t), c_n)(\vec{\nabla}\pi)^2 - \frac{1}{2}C(a(t), c_n)\pi^2$$



α	c_1	c_2	c_3	c_4	c_5	A	B	C	$H\tau$
1	0	0	0	0	c_5	$24\frac{c_5}{t_0^3}$	0	0	0
$\frac{3}{4}$	0	0	0	c_4	0	$\frac{81}{8t^2}c_4\left(t/t_0\right)^{9/4}$	$\frac{9}{8t^2}c_4\left(t/t_0\right)^{3/4}$	$-\frac{81}{32t^4}c_4\left(t/t_0\right)^{9/4}$	$3/2$
$\frac{1}{2}$	0	0	c_3	0	0	$\frac{3}{t}c_3\left(t/t_0\right)^{3/2}$	$\frac{1}{t}c_3\left(t/t_0\right)^{1/2}$	$-\frac{3}{2t^3}c_3\left(t/t_0\right)^{3/2}$	$\frac{1}{\sqrt{2}}$
$\frac{1}{4}$	0	c_2	0	0	0	$c_2\left(t/t_0\right)^{3/4}$	$c_2\left(t/t_0\right)^{1/4}$	$-\frac{3}{4t^2}c_2\left(t/t_0\right)^{3/4}$	$\frac{1}{2\sqrt{3}}$

Either marginally stable, or a tachyonic instability, with tachyon timescale ~1/H. Therefore, solutions stable to fluctuations over time scales shorter than the age of the universe.

[Agrees with C. Burrage, C. de Rham, and L. Heisenberg, JCAP 1105 (2011) 025, arXiv:1104.0155.]

Galileon-Like Limit

In maximally symmetric case have small field limits which simplify Lagrangians (To obtain, form linear combinations of original Lagrangians, s.t. perturbative expansion of nth one around constant background order π^n) e.g. flat brane in a flat bulk gives flat space galileons.

Can't do same here - appears to be due to maximal symmetry, but can check our results for dS limit:

Induced Metric on Brane $\bar{g}_{\mu\nu} = (-1 + H\pi)^2 \, g^{(dS)}_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$

Now redefine the field and change coordinates

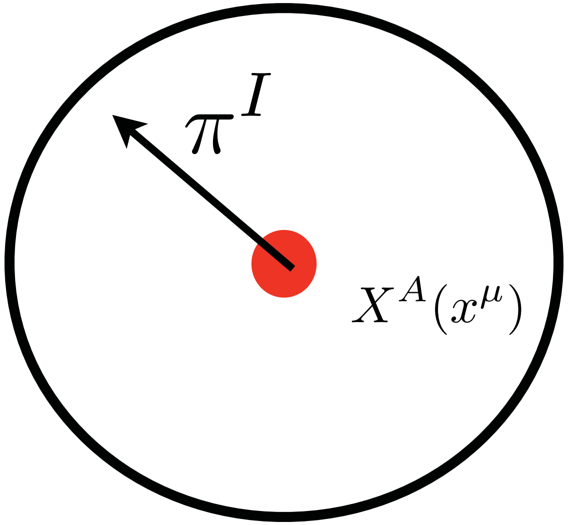
$$\tilde{\pi} = -1 + H\pi \qquad \hat{x}^\mu = H x^\mu$$

Resulting theory is one of the ones Kurt showed you, and the small field limit is the resulting Galileon on a dS background - reassuring!

Multi-field Galileons and Higher co-Dimension Branes

Higher co-Dimension Probe Branes

[K. Hinterbichler, M.T., D. Wesley, *Phys. Rev.* D82 (2010) 124018.]



With some work, can extend probe brane construction to multiple co-dimensions

$$X^\mu(x) = x^\mu, \quad X^I(x) \equiv \pi^I(x)$$

Induced Metric on Brane

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi^I \partial_\nu \pi_I$$

More general version of action de Rham & Tolley wrote (and Kurt explained)

$$S = \int d^4x \, \sqrt{-g} F\left(g_{\mu\nu}, \nabla_\mu, R^i{}_{j\mu\nu}, R^\rho{}_{\sigma\mu\nu}, K^i_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi^I\partial_\nu\pi_I}$$

Technical question. Main differences: extrinsic curvature $K^i_{\mu\nu}$ carries an extra index, associated with orthonormal basis in normal bundle to hypersurface.

Also, covariant derivative has connection, $\beta^j_{\mu i}$ acting on i index. e.g.

$$\nabla_\rho K^i_{\mu\nu} = \partial_\rho K^i_{\mu\nu} - \Gamma^\sigma_{\rho\mu} K^i_{\sigma\nu} - \Gamma^\sigma_{\rho\nu} K^i_{\mu\sigma} + \beta^i_{\rho j} K^j_{\mu\nu}$$

Higher co-Dimension Probe Branes

Covariant Derivative Intrinsic Curvature

$$S = \int d^4x\, \sqrt{-g} F\left(g_{\mu\nu}, \nabla_\mu, R^i{}_{j\mu\nu}, R^\rho{}_{\sigma\mu\nu}, K^i_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi^I\partial_\nu\pi_I}$$

Normal Bundle Curvature

Extrinsic curvature

In co-dimension1, for 2nd order equations, use Lovelock terms and associated boundary terms. Here, for 4d brane, prescription *depends on co-dimension*

1. If N (not = 3) is odd, obtain dimensional continuation of Gibbons-Hawking and Myers terms, with the extrinsic curvature replaced by distinguished normal component of K. (*There is a potential loophole and a project here*)
2. If N = 3, have additional terms involving the extrinsic curvature (and boundary term is not simply dimensional continuation of Myers term.)
3. If N (not = 2) is even, boundary term includes only brane cosmological constant and induced Einstein-Hilbert term.
4. If N = 2, boundary terms include only brane cosmological constant, and

$$\mathcal{L}_{N=2} = \sqrt{-g}\left(R[g] - (K^i)^2 + K^i_{\mu\nu}K_i^{\mu\nu}\right)$$

The Multi-Galileon Limit

[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018;
A.Padilla, P.Saffin, S.Zhou, JHEP 1012, 031 (2010).; C. Deffayet, S. Deser, G. Esposito-Farese, *Phys.Rev.* D82 (2010) 061501]

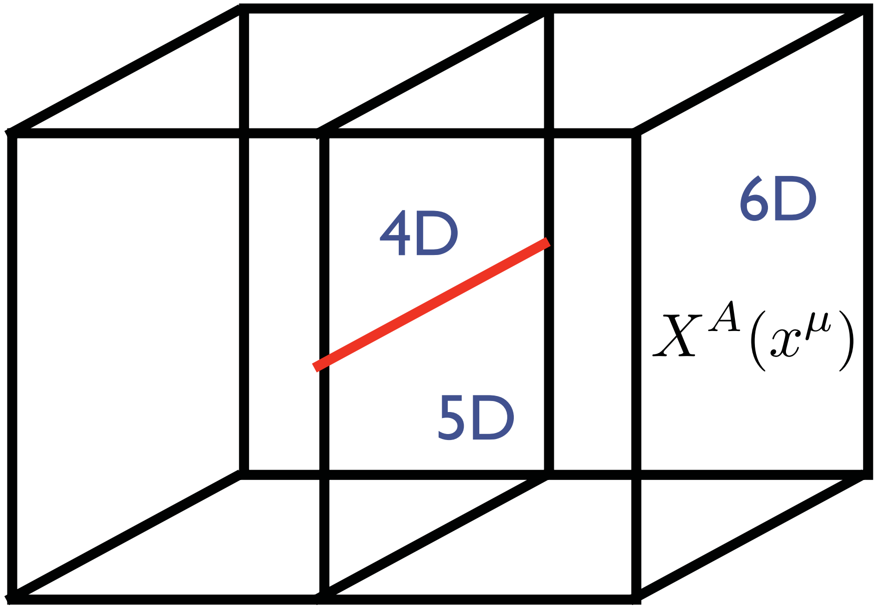
In decoupling limit get a unique multi-Galileon theory, with single coupling, from the brane Einstein-Hilbert action plus a brane cosmological constant:

$$\int d^4x\,\sqrt{-g}\,(-a_2 + a_4 R) \to \int d^4x\,\left[-a_2\,\frac{1}{2}\partial_\mu\pi^I\partial^\mu\pi_I + a_4\,\partial_\mu\pi^I\partial_\nu\pi^J\,\left(\partial_\lambda\partial^\mu\pi_J\partial^\lambda\partial^\nu\pi_I - \partial^\mu\partial^\nu\pi_I\Box\pi_J\right)\right]$$

(In higher dimensions, more terms are possible)

As before, find combined symmetry in small-field limit under which π invariant:

$$\delta\pi^I = \underbrace{\omega^I{}_\mu x^\mu + \epsilon^I}_{\text{Multiple Galileons}} + \underbrace{\omega^I{}_J\pi^J}_{\text{New SO(N) symmetry}}$$



Breaking the SO(N) get a description more appropriate to, for example, cascading gravity.

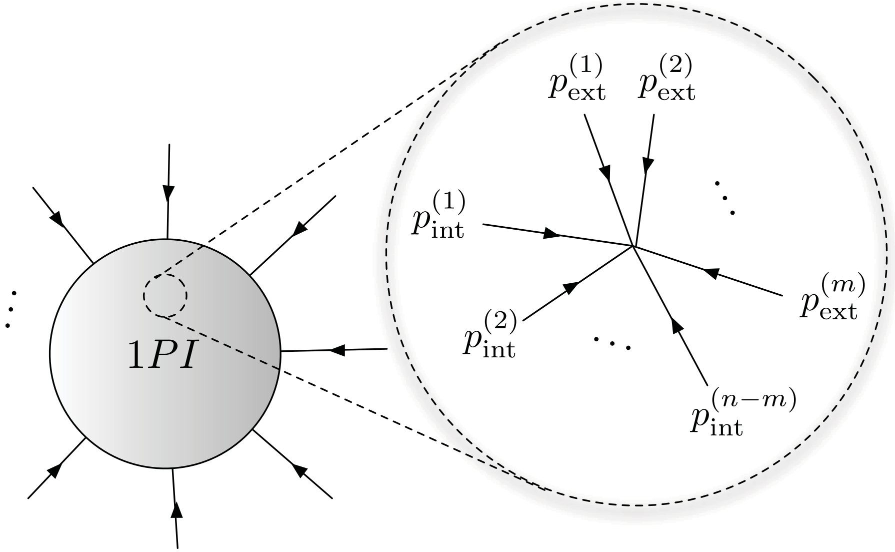
Nonrenormalization!

(as in first talk today)

Remarkable fact about these theories (c.f SUSY theories)

Expand quantum effective action for the classical field about expectation value

$$\Gamma(\pi^c) = \Gamma^{(2)}\pi^c\pi^c + \Gamma^{(3)}\pi^c\pi^c\pi^c + \cdots$$



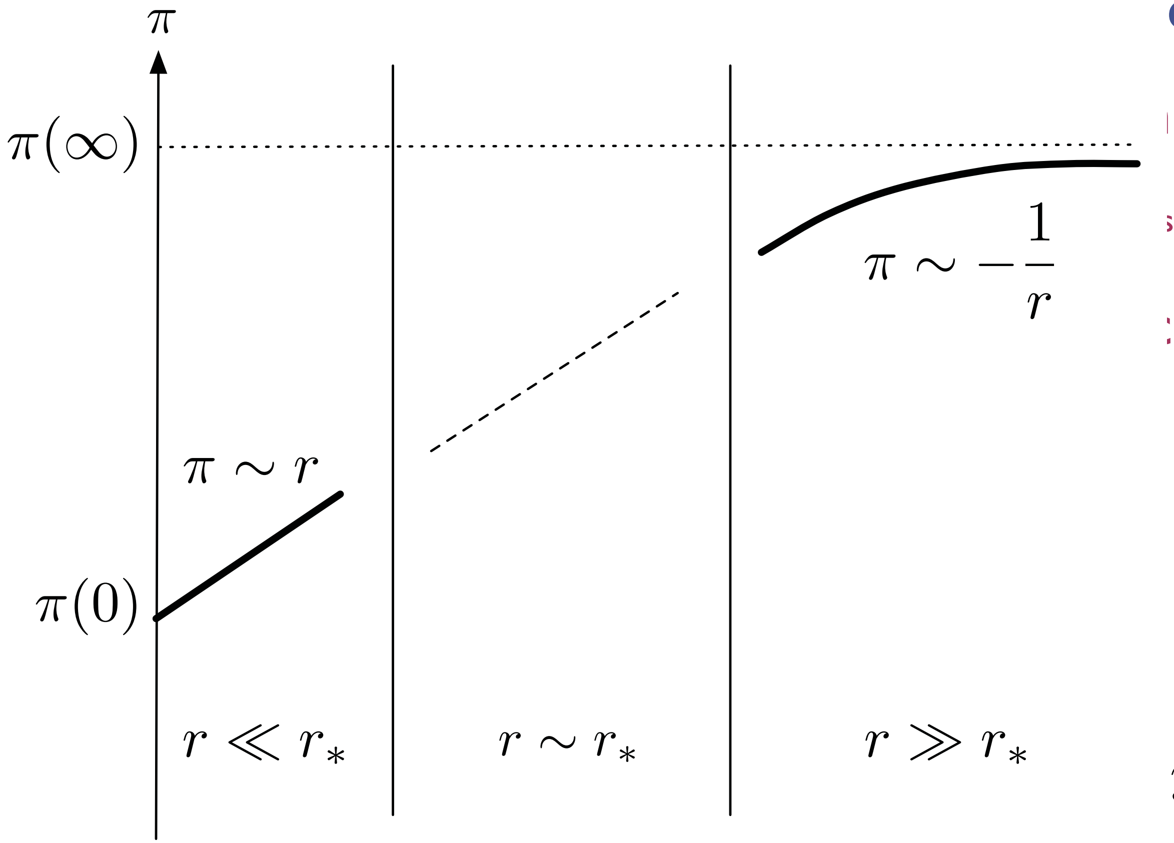
With or without the SO(N), can show, just by computing Feynman diagrams, that at all loops in perturbation theory, for any number of fields, terms of the galilean form cannot receive new contributions.

[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018]

Can even add a mass term and remains technically natural

Coupling to Matter & Stability

[Melinda Andrews, Kurt Hinterbichler, Justin Khoury, & M.T., *Phys.Rev.* D83 (2011) 044042]



For a single G … cts the symmetry

For multi-Gal … $\pi^I \pi^I T$
isn't invariant. … in Lam Hui's talk)
Simplest invar
has no nontri … sources

But for exam … interaction

$\mathcal{L} =$

Solve

$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^3\left(y^I\right.\right.$ … $y^I \equiv \frac{1}{r}\frac{\mathrm{d}\pi^I}{\mathrm{d}r}$

Solution has a Vainshtein radius $\left(\lambda M^2\left[P'\left(\pi^2(0)\right)\pi(0)\right]^2\right)^{1/6}$

BUT: exhibits superluminality and instability. If these are to make sense, better couplings to matter are needed.

[See talk by Andrew Tolley]

Summary

- Higher dimensional models are teaching us about entirely novel 4d effective field theories that may be relevant to cosmology (e.g. talk by Trincherini)
- We have shown how to derive the scalar field theories corresponding to Galileons propagating on fixed curved backgrounds (maximally symmetric and FRW examples).
- Have also shown how to extend the probe brane construction to higher co-dimension branes, yielding multi-Galileon theories.
- Couplings to matter and stabillity still need investigating in generality.

Current Work & the Future

- The cosmological models tell you what Galileons do propagating on cosmological spaces. What about driving cosmology? Need dynamical gravity for that, but would like to retain the nice properties of the Galileons (c.f. covariant galileons). As Kurt said - this may be coming soon!
- What lies behind the nonrenormalized Lagrangians? Provocative thought - may be a topological property. We're investigating that, with interesting preliminary results - stay tuned!

Thank You!