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Workshop on Infrared Modifications of Gravity

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A Paradise Island for Deformed Gravity

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A Paradise Island for Deformed Gravity

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F.B., D.D. Dietrich, S. Hofmann: JCAP 1011 (2010) 018

F.B., D.D. Dietrich, S. Hofmann: Phys. Rev. Lett. 106 (2011) 191102

F.B., D.D. Dietrich, S. Hofmann: JCAP 09 (2011) 024

F.B., D.D. Dietrich, S. Hofmann, F. Kühnel, P. Moyassari: arXiv:1106.3566 [hep-th]

Motivation

Point Of View

- ★ The vacuum energy density can only be inferred by employing gravity.
- ★ The technical naturalness challenge is communicated exclusively via gravity.
- ★ The resolution of this challenge might originate from the gravitational sector.
- ★ If so, relevant deformations are required to consistently break gravity's democratic foundation, i.e. the principle of equivalence.
- ★ Massive Gravity is an example of such a theory.

New dofs in the IR?

General „Massive“ Deformations

★ We are attacking cosmological questions.

→ Consider linear theory in a cosmological background.

$$S = \frac{1}{2} \int_M d^4x \sqrt{|g_0|} \left(h_{\mu\nu} \left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0, \nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu} \right] h_{\alpha\beta} + T^{\mu\nu} h_{\mu\nu} \right)$$

$g_0^{\mu\nu}$ standard FRW metric.

★ Question: Is there a unique choice for $\mathcal{M}$ like for a Minkowski background?

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Linearized Einstein Hilbert.

$g_0^{\mu\nu}$ standard FRW metric.

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like for a Minkowski background?

General „Massive“ Deformations

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Deformation term.

$$S = \frac{1}{2}\int_M d^4x\,\sqrt{|g_0|}\,\left(h_{\mu\nu}\left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0,\nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu}\right]h_{\alpha\beta} + T^{\mu\nu}h_{\mu\nu}\right)$$

Linearized Einstein Hilbert.

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★ Question: Is there a unique choice for $\mathcal{M}$ like for a Minkowski background?

Stability Analysis

Naive FP

Higuchi Bound

★ The most simple ansatz would be the naive FP

$$\mathcal{M}^{\mu\nu\alpha\beta} = m^2 \left(g_0^{\mu\nu} g_0^{\alpha\beta} - g_0^{\mu\beta} g_0^{\nu\alpha} \right)$$

★ On a deSitter background, Higuchi has shown that

$$m^2 > H^2 = const.$$

to guarantee the absence of negative norm states.

★ On FRW: $H \to H(t)$ Implications?

Stability Analysis: Technique

★ Using Bianchi: $\nabla_\mu \mathcal{M}^{\mu\nu\alpha\beta} h_{\alpha\beta} = 0$

→ This constraint influences the kinetic structure of the theory.

★ Reveal by introducing Stückelberg fields:

$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_{(\mu} A_{\nu)} + \nabla_\mu \nabla_\nu \phi$$

★ Gives the gauge symmetries:

$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_{(\mu} \zeta_{\nu)} \quad , \quad A_\mu \to A_\mu - \zeta_\mu$$

$$A_\mu \to A_\mu + \nabla_\mu \xi \quad , \quad \phi \to \phi - \xi$$

→ Constraint becomes the equation of motion of A_μ .

Results of Naive FP in FRW I

★ At high energies the action can be diagonalized:

$$\mathcal{L} \supset A(t)\dot{\phi}^2 + B(t)(\vec{\nabla}\phi/a)^2$$

1. Sign of $A(t)$ determines the norm in Fock-space:

→ $$\left[a(\mathbf{k}), a^\dagger(\mathbf{k'})\right] = \mathrm{sign}(\mathrm{A})\delta^{(3)}(\mathbf{k} - \mathbf{k'})$$

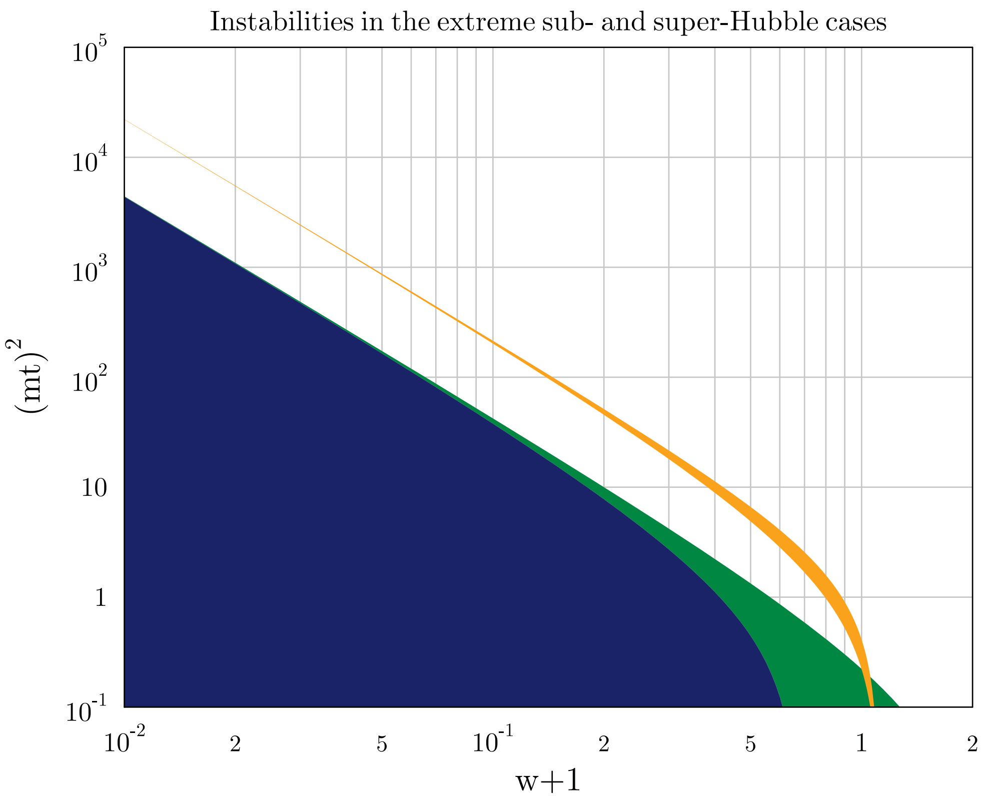
→ Unitarity bound: $m^2 > H^2 + \dot{H}$

2. Sign of $B(t)$ implies classical (in)stability.

→ Stability bound: $m^2 > H^2 + \frac{1}{3}\dot{H}$

Results of Naive FP in FRW II

- Additionally, we performed a complete cosmological perturbation analysis.
  - Valid at all energies.
  - Incorporates all degrees of freedom.



Orange: Classically unstable for zero momentum.

Green: Classically unstable for high momenta.

Blue: Unitarity violating.

Self-Protection

★ The stability bound is stronger than the unitarity bound for non-phantom matter $\dot{H} < 0$.

→ System self-protects from direct unitarity violation.

★ Violation of stability bound

→ Large fluctuations.

→ Formation of a new background.

★ How to avoid the classical instability?

Try $m \to m(t)$! Or even more general ....

General

case

The "Deformation Matrix"

★ Covariance and symmetry constrain the IR leading terms of the deformation matrix as:

$$\mathcal{M}^{\mu\nu\alpha\beta} = (m^2 + \alpha R_0)\left(g_0^{\mu\nu} g_0^{\alpha\beta} - g_0^{\mu\beta} g_0^{\nu\alpha}\right)$$

$$+\beta\left(R_0^{\mu[\nu} g_0^{\beta]\alpha} + R_0^{\alpha[\beta} g_0^{\nu]\mu}\right)$$

$$+\gamma R_0^{\mu\alpha\nu\beta}$$

$$S = \frac{1}{2}\int_M d^4x\,\sqrt{|g_0|}\,\left(h_{\mu\nu}\left[\mathcal{E}^{\alpha\beta\mu\nu}(g_0,\nabla) + \mathcal{M}(g_0)^{\alpha\beta\mu\nu}\right]h_{\alpha\beta} + T^{\mu\nu}h_{\mu\nu}\right)$$

★ On FRW: $\gamma = 0$ (vanishing Weyl tensor)

Stability Analysis: Technique

★ Use the gauge:

$$h_{0\mu} = 0 \quad , \quad A_0 = 0$$

★ Singular points of the resulting kinetic matrix

$$K_{ij}^{\mu\nu}(\nabla_\mu \Psi_i)(\nabla_\nu \Psi_j) \quad , \quad (\Psi_i) = (\phi, A_1, A_2, A_3, h_{11}, h_{12}, ...)$$

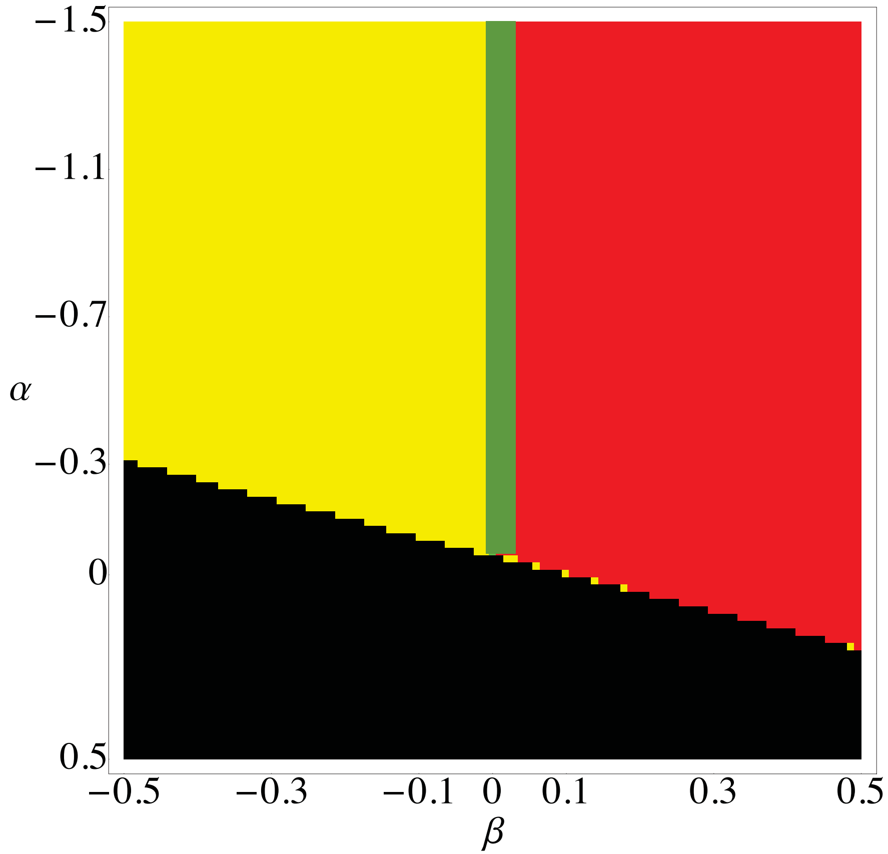
signal the following instabilities:

1. Spatial directions: Classical instability
   - ★ System evolves into a new background.
2. Time direction: Unitarity violation
   - ★ Quantum theory loses probabilistic interpretation.
   - ★ Equation of motion becomes singular.

Stability Analysis: General Case

Bounds in the general case $\alpha \neq 0$, $\beta \neq 0$ are much more complicated.

$m = 0$



Green: Classically stable and unitary.

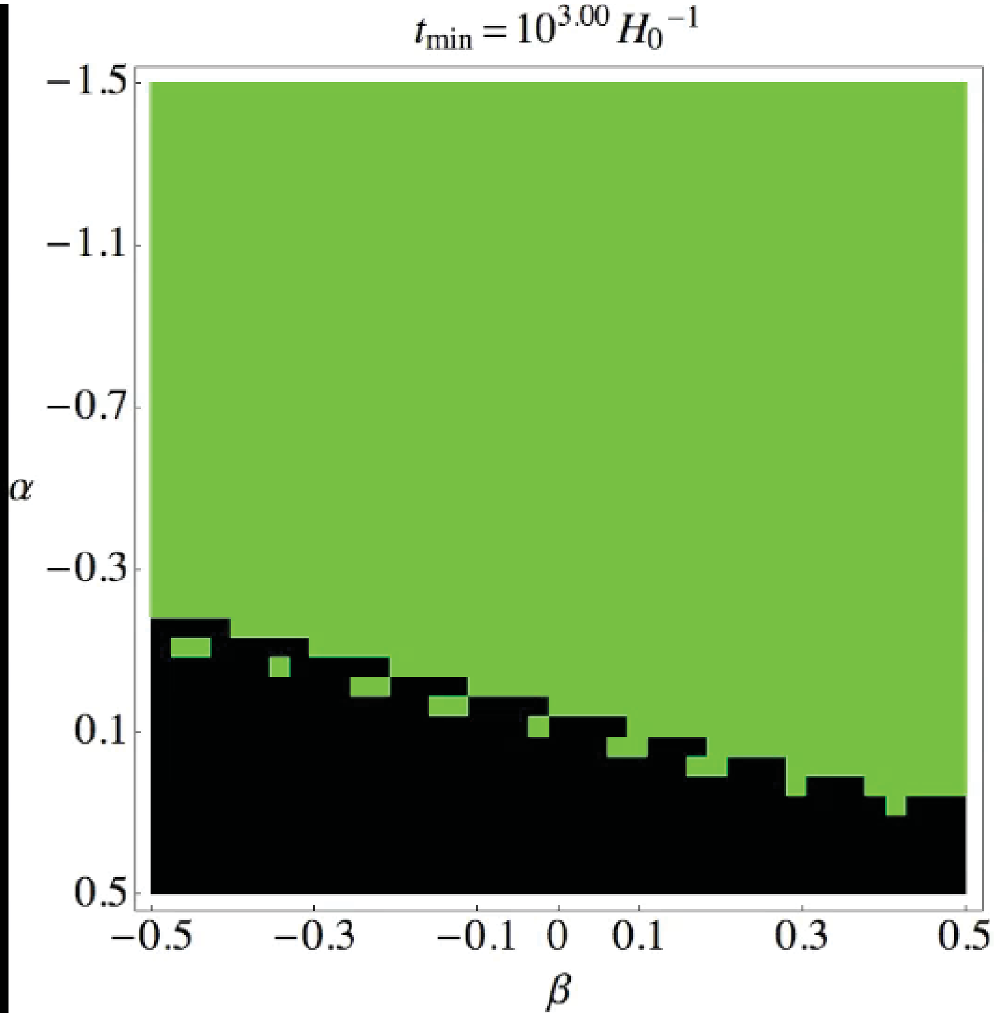
Yellow: Self-Protection.

Red: Unitarity violation.

Black: No stability or unitarity today.

Stability Analysis: Time Dependence

$$m = 0$$



Stability Analysis: Conclusion

★ ONLY the “running mass“ deformation

$$\mathcal{M}^{\mu\nu\alpha\beta} = (m_0^2 + \alpha R_0)\left({g_0}^{\mu\nu}{g_0}^{\alpha\beta} - {g_0}^{\mu\beta}{g_0}^{\nu\alpha}\right)$$

will yield a stable theory.

★ Absolute stability requires proper covariantization of the deformation matrix!

★ α must be sufficiently negative.

★ The form and parameters of the theory are constrained UNIQUELY like in Minkowski!