The Abdus Salam International Centre for Theoretical Physics

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2265-11

Advanced School on Understanding and Prediction of Earthquakes and other Extreme Events in Complex Systems

26 September - 8 October, 2011

Extreme Events: Dynamics, Statistics and Prediction

Michael Ghil

ENS & UCLA
U.S.A.

Adv. School: Understand & Predict EQs & Other Extreme Events in Complex Systems

ICTP, Trieste, 27 Sept. 2010

Extreme Events: Dynamics, Statistics and Prediction

Michael Ghil (ENS & UCLA),

with special thanks to V. I. Keilis-Borok and his MITPAN colleagues

+ P. Yiou (LSCE) and the full E2C2 cast

LETTRES SCIENCES ÉCOLE NORMALE SUPÉRIEURE

THE UNIVERSITY OF CALIFORNIA UCLA

Pls. see these sites for further info.

http://www.atmos.ucla.edu/tcd/ (TCD);

http://www.environnement.ens.fr/(CERES); http://e2c2.ipsl.jussieu.fr/ (E2C2)

Outline

A. Extreme events: Causes and consequences (E2C2)

- overview
- selected results

B. Dynamic coupling of the climate

and socio-economic systems

- endogenous business cycles (EnBC)
- extreme events impacting EnBCs
  - vulnerability paradox and nonlinear FDT

C. Some questions for the Discussion sessions

Extreme events: Causes and consequences (E2C2)
An overview and selected results

Motivation

1. The rarity of extreme events: an obstacle and an incentive.
2. Extreme events as a manifestation of complexity.
3. Means, variances, and extrema: statistical analysis and modeling – deterministic and stochastic.
4. Integrated analysis and modeling: Earth System Modeling (ESM) and beyond – coupling socio-economic and natural phenomena
5. A new integrated modeling tool: Boolean delay equations (BDEs): simpler, more flexible.
6. Pattern recognition and complex system modeling: a pathway to prediction?

Pls. visit the E2C2 web site: http://e2c2.ipsl.jussieu.fr

Outline

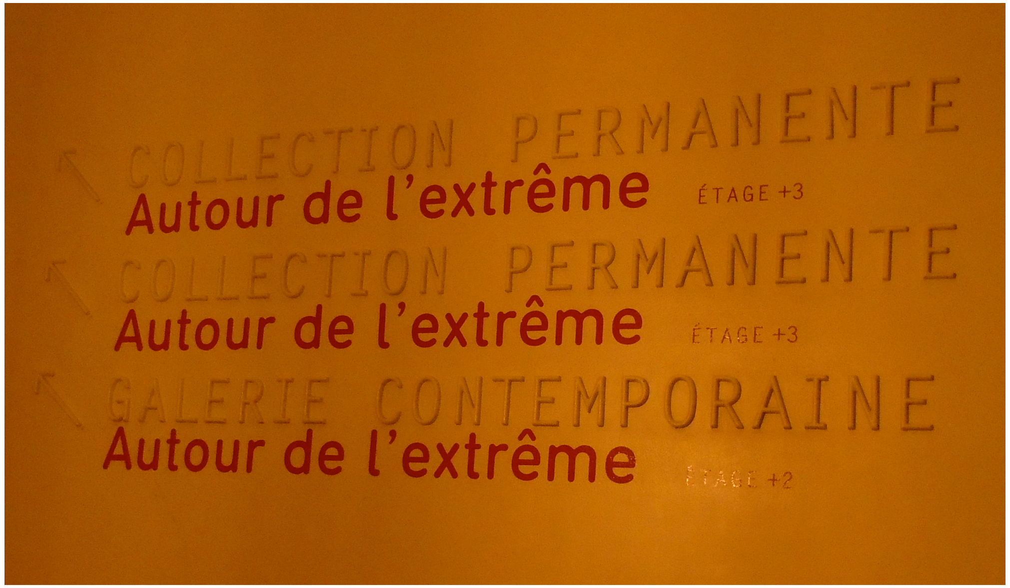
- What we started with.
- What we did.
- What we found out.
- What we'd like to know.

Outline

- Who we were and what we started with ➔

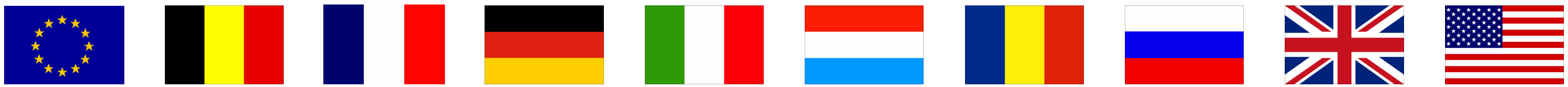
A few fashionable extremes

EXTREME PIZZA ®



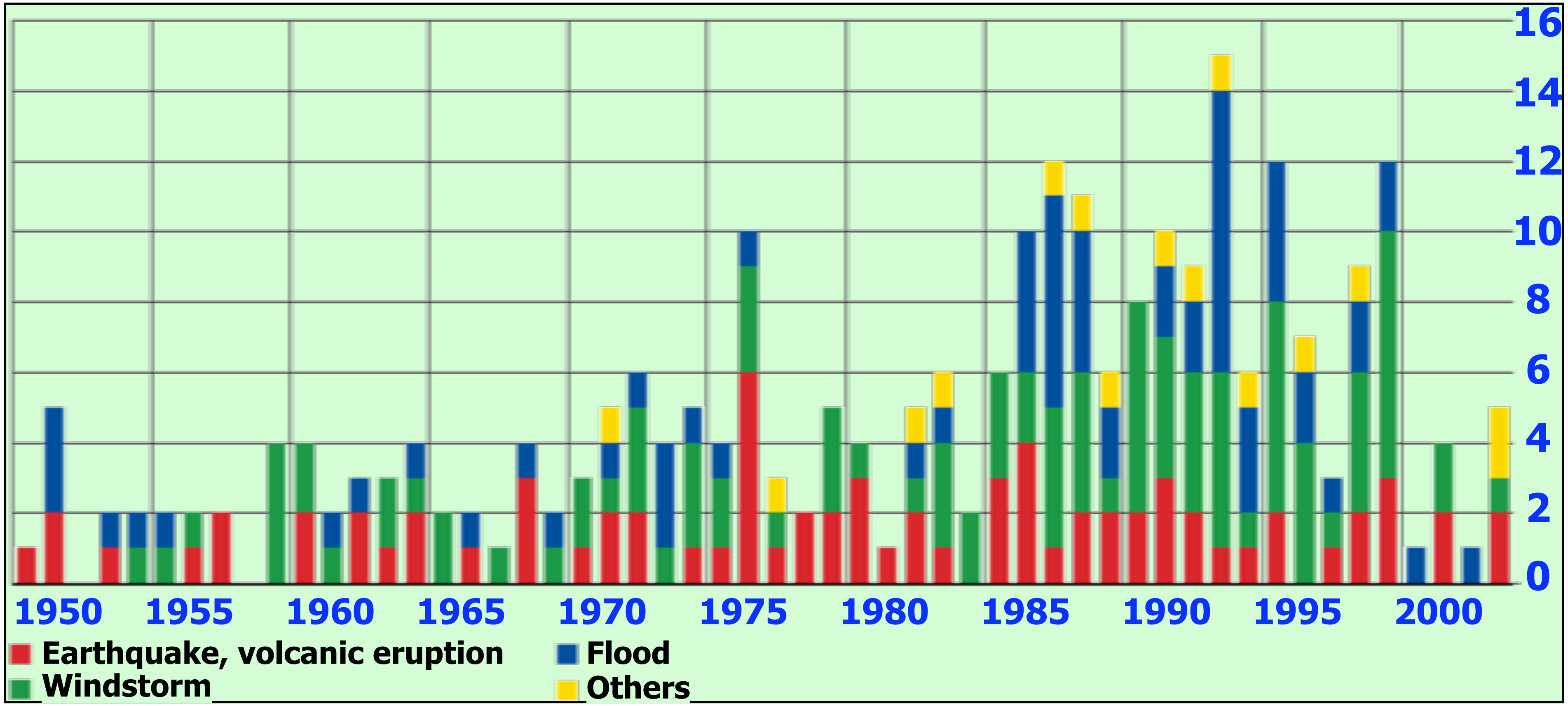
Extreme Events:
Causes and Consequences (E2-C2)

- EC-funded project bringing together researchers in mathematics, physics, environmental and socio-economic sciences.
- €1.5M over 3.5 years (March 2005–August 2008).
- Coordinating institute: Ecole Normale Supérieure.
- 17 ‘partners’ in 9 countries.
- 72 scientists + 17 postdocs/postgrads.
- PEB: M. Ghil (ENS, Paris, P.I.), S. Hallegatte (CIRED), B. Malamud (KCL, London), A. Soloviev (MITPAN, Moscow), P. Yiou (LSCE, Gif s/Yvette, Co-P.I.)



Belgium *France* *Germany* *Italy* *Luxembourg* *Romania* *Russia* *UK* *USA*

Great Natural Catastrophes 1950–2003



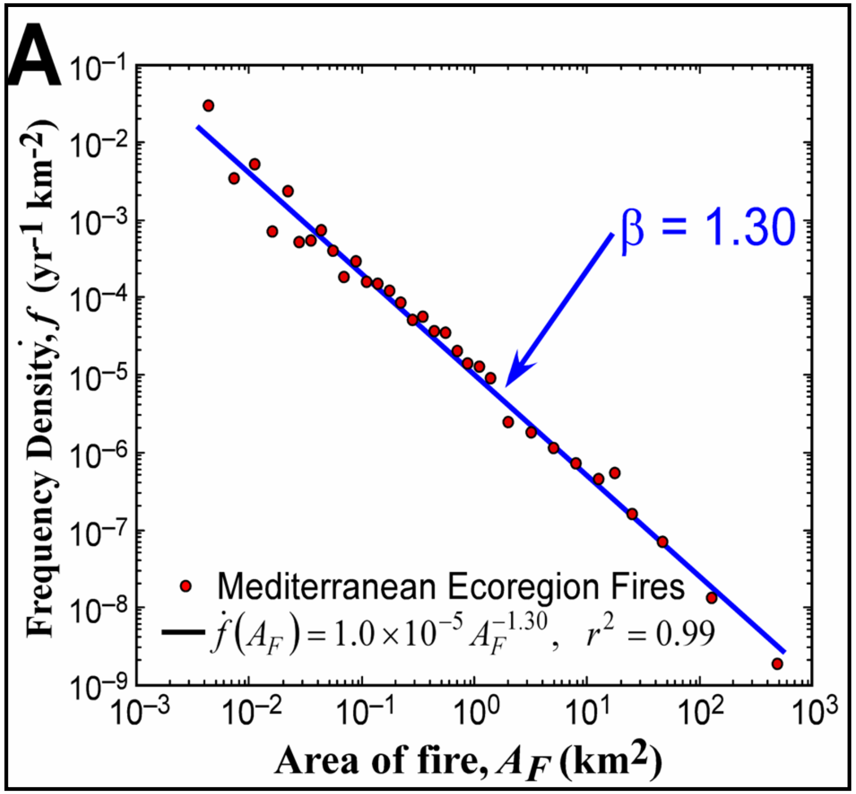
Number of major natural catastrophes, by year and type of event (from *Munich Re, Topics Geo 2003*)

E2-C2 Summary & Key Ideas

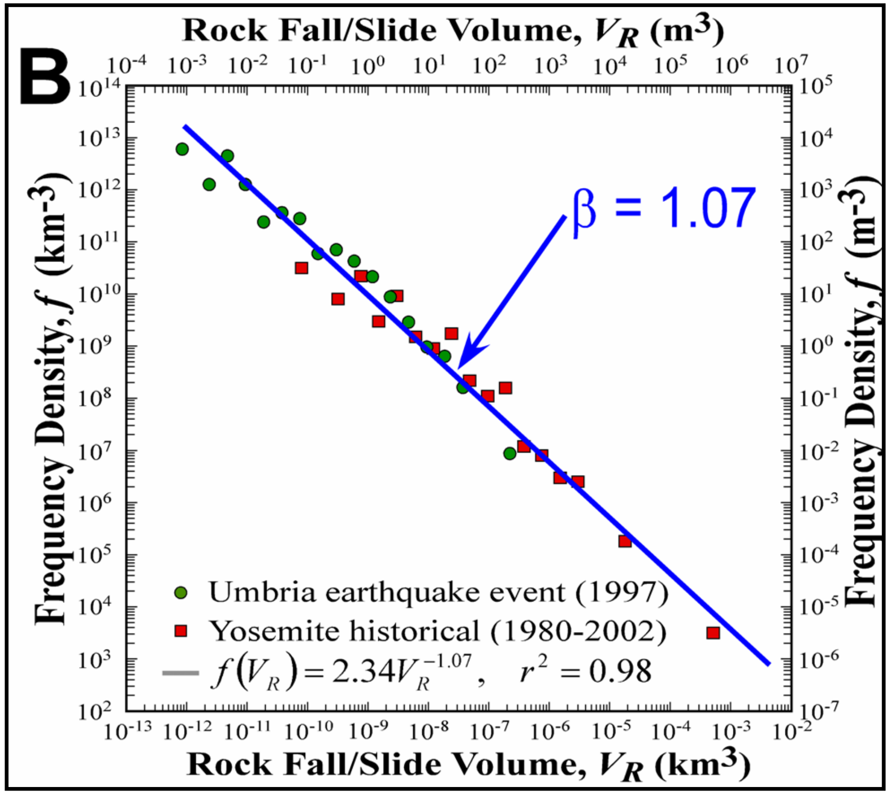
- Extreme events a key manifestation of complex systems.
- Describe, understand & predict extreme events.
- Combine expertise in complex systems with broad knowledge in the natural and social sciences.
- Main study areas included:
  - Natural disasters (earthquakes, wildfires, landslides, climatic extremes, etc.)
  - Socio-economic crises
  - Interaction between economic & climatic changes
- Six scientific work-packages bridging the natural and social sciences.
- Outcomes included:
  - Validated data sets
  - Novel insights
  - Forecast algorithms

U.S. Wildfires

U.S. & Italian Rockfalls



Malamud, Morein & Turcotte (1998, *Science*)



Malamud, Turcotte, Guzzetti & Reichenbach (2004, *ESPL*)

Frequency-size distributions for natural hazards

➔ probabilistic hazard forecasting

Outline

- What we started with.
- What we did →

Special Issue: ***Extreme Events: Nonlinear Dynamics and Time Series Analysis***
Journal: **Nonlinear Processes in Geophysics (*NPG*)**
Editors: Henning Rust, Pascal Yiou and Bruce D. Malamud

(0) *Overall Review Paper* : **Extreme Events: Dynamics, Statistics and Prediction**
M. Ghil, P. Yiou, + WG leaders and all other contributors in alphabetical order, in preparation.

(1) **Recurrence and interoccurrence behavior of self-organized complex phenomena.**
S. G. Abaimov, D. L. Turcotte, R. Shcherbakov, and J. B. Rundle. *NPG*, **14**, 455-464, 2007.

(2) **Spatial dependences among precipitation maxima over Belgium.**
S. Vannitsem and P. Naveau. *NPG*, **14**, 621-630, 2007.

(3) **Analysis of global geomagnetic variability.**
V. Anh, Z.-G. Yu, and J. A. Wanliss. *NPG*, **14**, 701-708, 2007.

(4) **Modeling pairwise dependencies in precipitation intensities.**
M. Vrac, P. Naveau, and P. Drobinski. *NPG*, **14**, 789-797, 2007.

(5) **Sequence of eruptive events in the Vesuvio area recorded in shallow-water Ionian Sea sediments.**
C. Taricco, S. Alessio, and G. Vivaldo. *NPG*, **15**, 25-32, 2008.

(6) **Detecting spatial patterns with the cumulant function – Part 1: The theory.**
A. Bernacchia and P. Naveau. *NPG*, **15**, 159-167, 2008.

(7) **Detecting spatial patterns with the cumulant function – Part 2: An application to El Niño.**
A. Bernacchia, P. Naveau, M. Vrac, and P. Yiou. *NPG*, **15**, 169-177, 2008.

(8) **Transformation of frequency-magnitude relation prior to large events in the model of block structure dynamics.**
A. Soloviev. *NPG*, **15**, 209-220, 2008.

(9) **Loading rates in California inferred from aftershocks.**
C. Narteau, P. Shebalin, and M. Holschneider. *NPG*, **15**, 245-263, 2008.

(10) **Weather regime dependence of extreme value statistics for summer temperature and precipitation.**
P. Yiou, K. Goubanova, Z. X. Li, and M. Nogaj. *NPG*, **15**, 365-378, 2008.

(11) **A delay differential model of ENSO variability: parametric instability and the distribution of extremes.**
M. Ghil, I. Zaliapin, and S. Thompson. *NPG*, **15**, 417-433, 2008.

(12) **Extreme event return times in long-term memory processes near 1/f.**
R. Blender, K. Fraedrich, and F. Sienz. *NPG*, **15**, 557-565, 2008.

(13) **Multivariate non-normally distributed random variables in climate research – introduction to the copula approach.**
C. Schölzel and P. Friederichs. *NPG*, **15**, 761-772, 2008.

Nonlin. Processes Geophys., 18, 295–350, 2011
www.nonlin-processes-geophys.net/18/295/2011/
doi:10.5194/npg-18-295-2011

Nonlinear Processes in Geophysics

Extreme events: dynamics, statistics and prediction

M. Ghil[1,2], P. Yiou[3], S. Hallegatte[4,5], B. D. Malamud[6], P. Naveau[3], A. Soloviev[7], P. Friederichs[8], V. Keilis-Borok[9], D. Kondrashov[2], V. Kossobokov[7], O. Mestre[5], C. Nicolis[10], H. W. Rust[3], P. Shebalin[7], M. Vrac[3], A. Witt[6,11], and I. Zaliapin[12]

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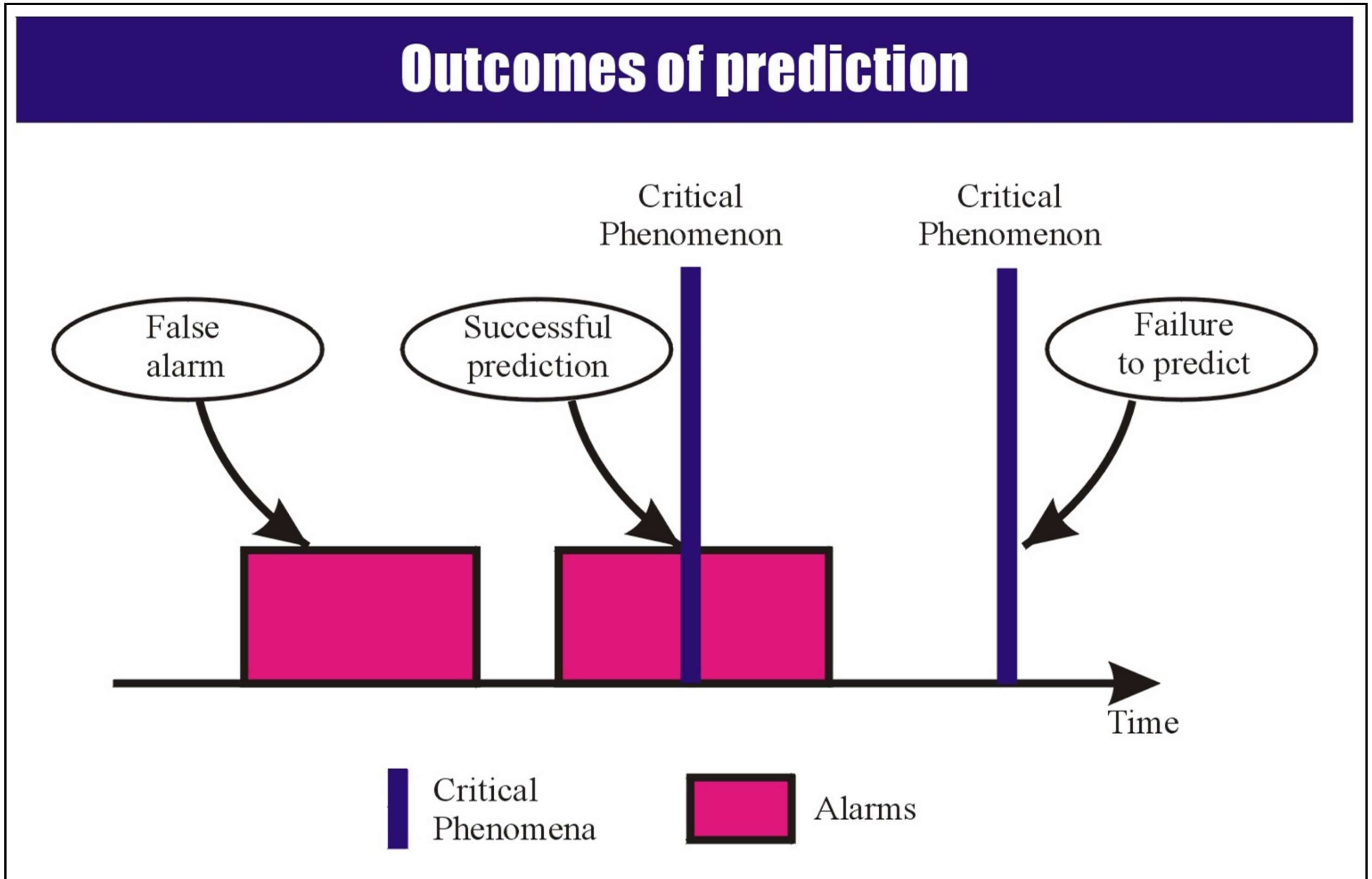
Outline

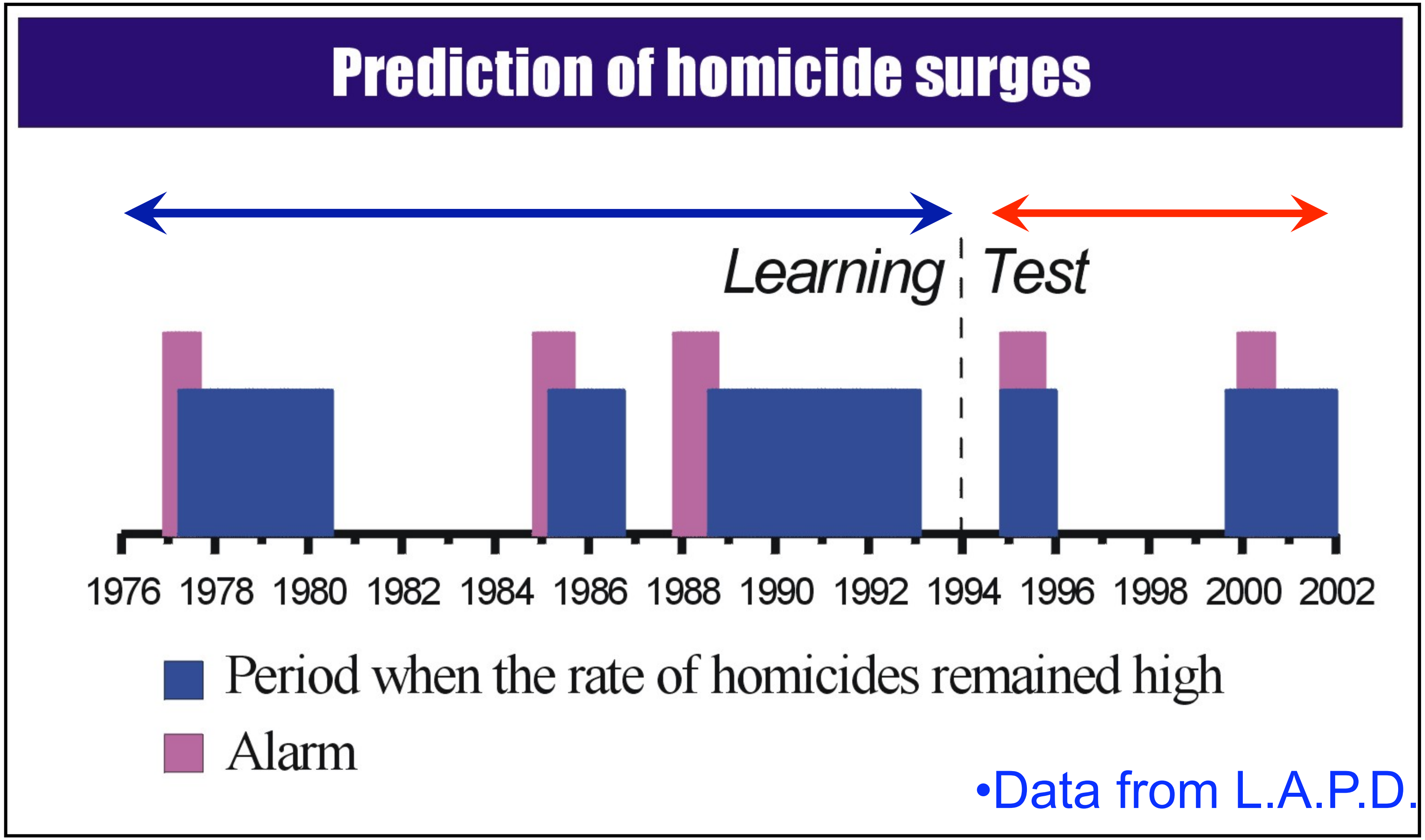
- What we started with.
- What we did.
- What we found out →



Possible outcome of "yes" or "no" prediction. Its probabilistic nature is reflected in estimated probabilities of false alarms and failures to predict.

Forecasting algorithm for natural & social systems: can we beat statistics-based approaches?

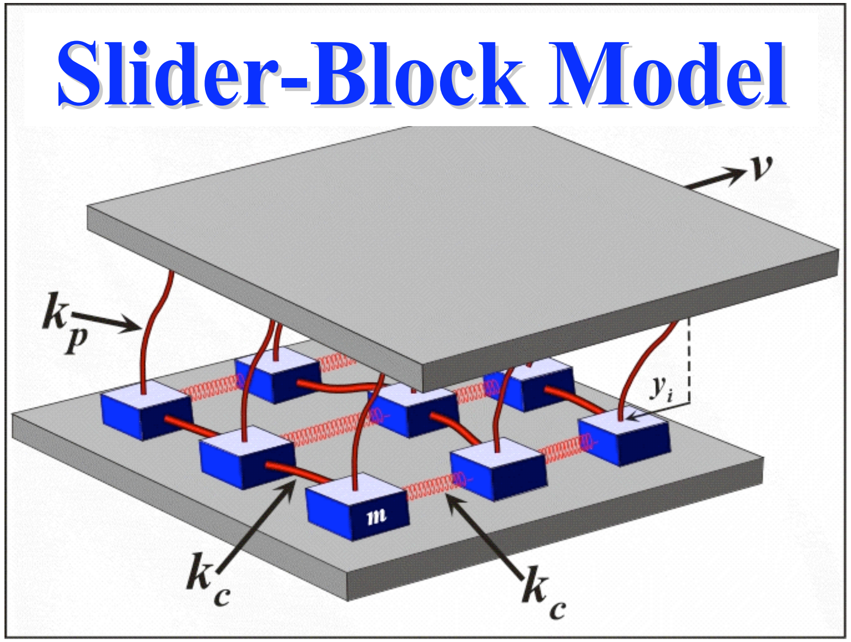
Ghil & Robertson, 2002, *PNAS*; Keilis-Borok. 2002, *Annu. Rev. EPS*.



Forecasting algorithm example for social systems

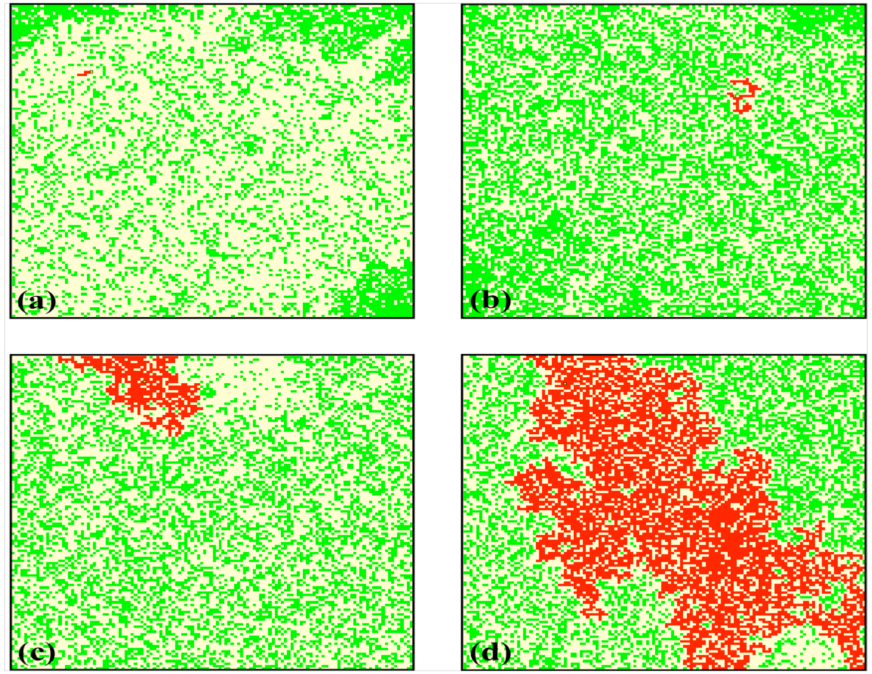
Keilis-Borok, Gascon, Soloviev + 3 (2003,
in *T. Beer & A. Ismail-Zadeh, Eds.*, Kluwer)

Minimal model for seismic events



Burridge & Knopoff (1967, *BSSA*)

CA models of forest fires



Malamud & Turcotte (2000, *IEEE Trans. CSE*);
Spyratos, Bourgeron & Ghil (2007, *PNAS*)

Simple models (ODEs, cellular automata, and BDEs) can help us understand and predict complex interactions in "real" systems

Boolean Delay Equations (BDEs)
as models for increasing complexity

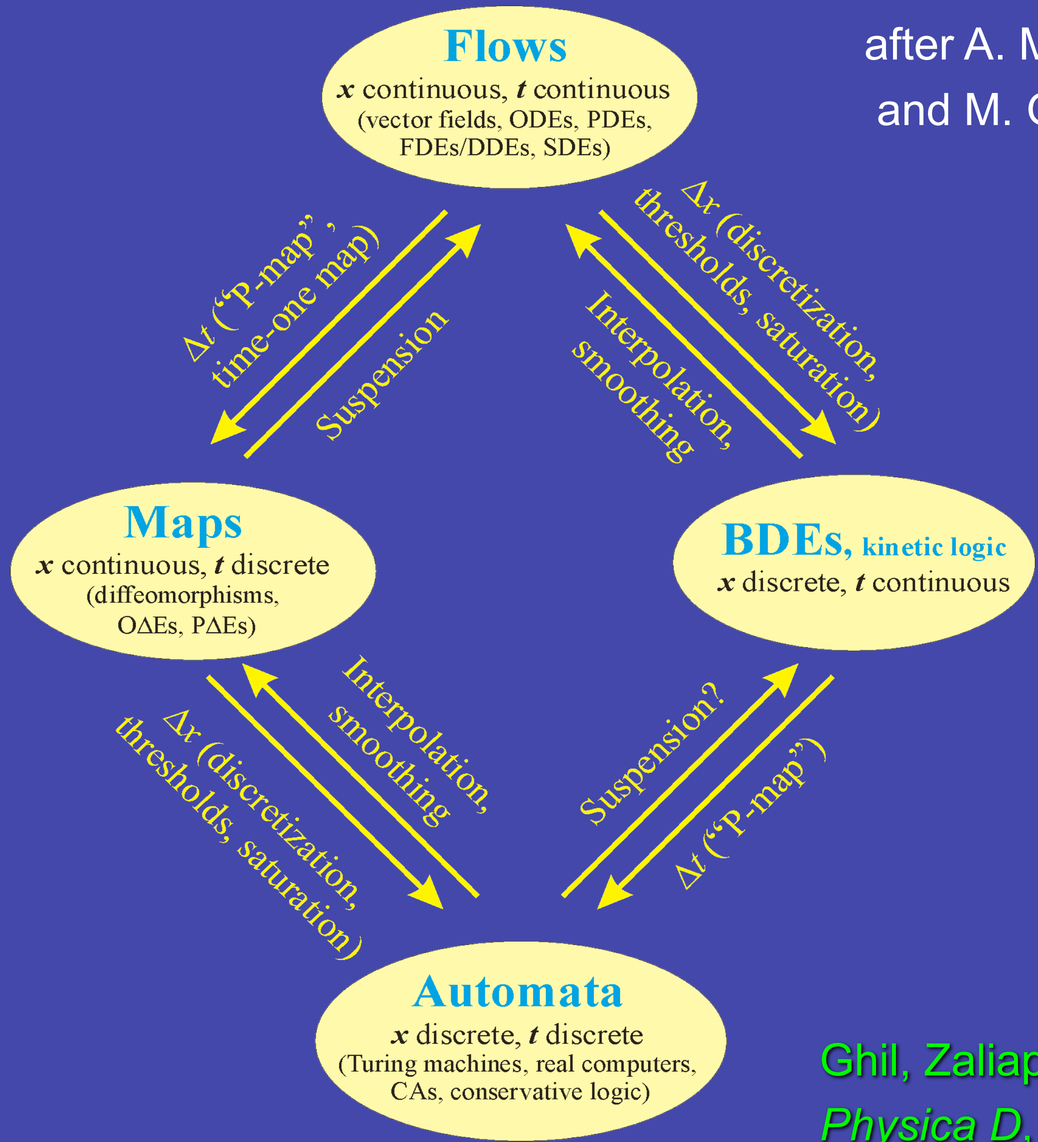
Motivation

1. Complexity of the phenomena and feedback networks in the geosciences, life sciences and socio-economic problems.
2. Difficulty in formulating "classical" models (ODEs, PDEs, SDEs), ascertaining parameter values, and analyzing even qualitative behavior for such models.
3. Availability of new modeling tool: **Boolean Delay Equations (BDEs)** – simpler, more flexible
   – easier to formulate and analyze

Work with *B. Coluzzi* (ENS, Paris), *D. Dee* (ECMWF, U.K.), *F.-f. Jin* (U. Hawaii), *V. Keilis-Borok* (IGPP, UCLA, & MITPAN, Moscow), *A.P. Mullhaupt* (Wall Street), *J.D. Neelin* (UCLA), *P. Pestiaux* (Total, France), *A.W. Robertson* (IRI, Columbia), *A. Saunders* (UCLA & L.A. School District), & *I. Zaliapin* (U. Nevada, Reno).

The place of BDEs in dynamical systems theory

after A. Mullhaupt (1984) and M. Ghil *et al.* (2008)



Ghil, Zaliapin & Coluzzi, 2008: *Physica D*, **237**, 2967–2986.

OECD Global Science Forum

Wkshop on Complexity Science & Public Policy Erice, 5–7 Oct. 2008

Dynamic Coupling of the Climate and Socio-Economic Systems

Michael Ghil (ENS & UCLA)

with B. Coluzzi, A. Groth & G. Weisbuch (ENS),
P. Dumas, S. Hallegatte & J.-Ch. Hourcade (CIRED),
L. Sella, P. Terna & G. Vivaldo (U. of Torino)

Pls. see these sites for further info.

http://www.atmos.ucla.edu/tcd/ (TCD and IPCC)

http://www.environnement.ens.fr/

The need for models with endogenous dynamics

“The currently prevailing paradigm, namely that financial markets tend towards equilibrium, is both false and misleading; our current troubles can be largely attributed to the fact that the international financial system has been developed on the basis of that paradigm.”

George Soros,
The New Paradigm for Financial Markets:
The Credit Crisis of 2008 and What It Means,
BBS, PublicAffairs, New York, 2008

Extreme Events:
Causes and Consequences (E2C2)
WP4: Economic impacts of extremes

A tale of two theories: the "real" cycle and the endogenous cycle theories

- ***In the real cycle theory, business cycles and economic fluctuations arise from exogenous "real" (i.e. not monetary) shocks, like changes in productivity or in energy prices, or from fiscal shocks.***

  Aside from these exogenous shocks, the economic system is stable:
  all markets are at equilibrium, and there is no involuntary unemployment.
  Deviations from equilibrium are damped more or less rapidly.
  Acting on the economy, therefore (e.g., recovery policies), is not useful.

- ***In endogenous business cycle (EBC) models, cyclical behavior originates from endogenous instabilities in the economic system.***

  Several instabilities have been proposed:
  - profitability-investment instability
  - delays in investment
  - income distribution

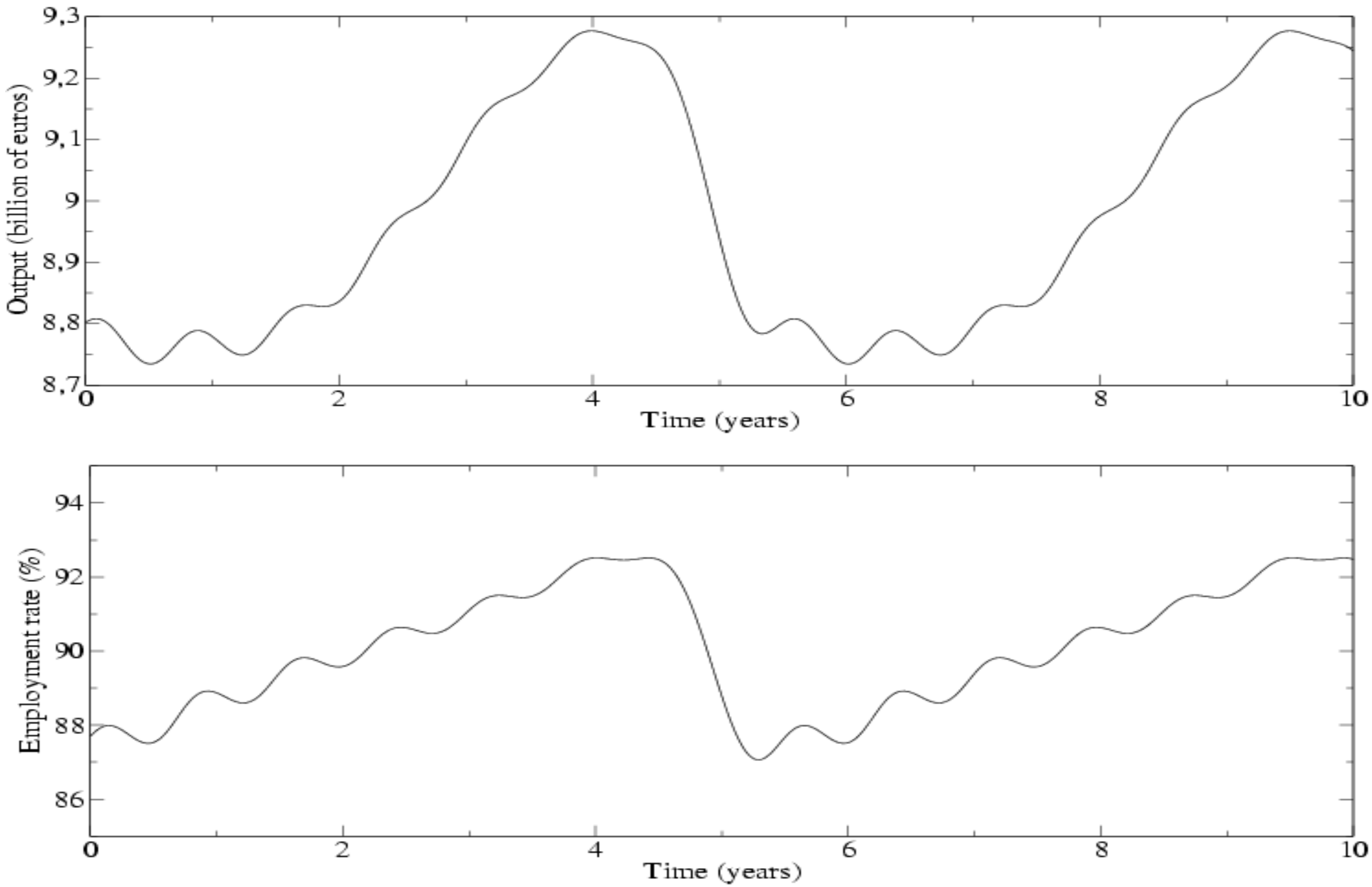
  Acting on the economy can, therefore, have positive effects,
  by stabilizing it or by shifting its mean state.

NEDyM (Non-equilibrium Dynamic Model)

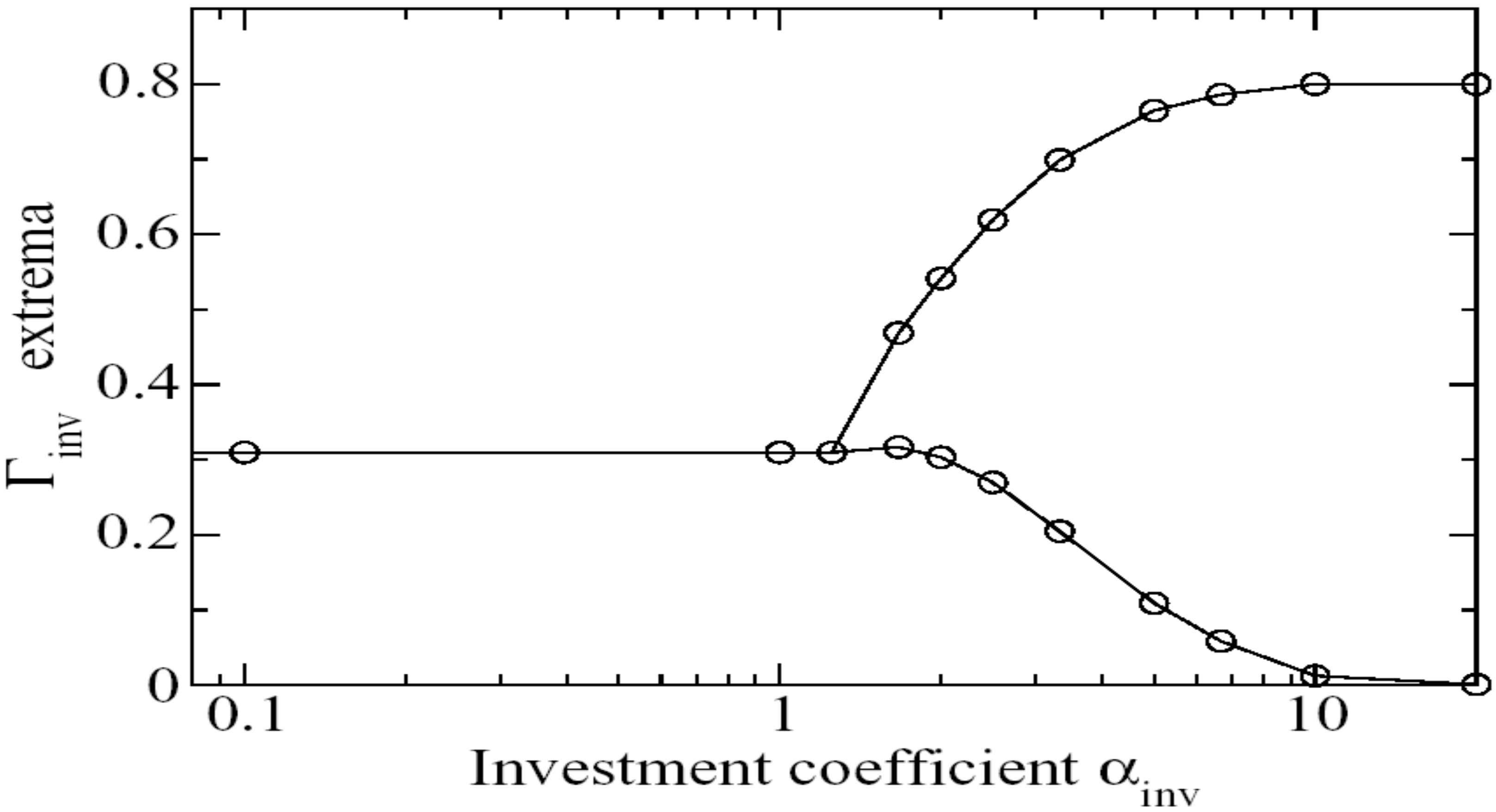
- Represents an economy with one producer, one consumer, one goods that is used both to consume and invest.
- Based on the Solow (1956) model, in which all equilibrium constraints are replaced by dynamic relationships that involve adjustment delays.
- The NEDyM equilibrium is neo-classical and identical to that in the original Solow model. If the parameters are changing slowly, NEDyM has the same trajectories as the Solow model.
- Because of market adjustment delays, NEDyM model dynamics exhibits Keynesian features, with transient trajectory segments, in response to shocks.
- NEDyM possesses endogenous business cycles!

Hallegatte, Ghil, Dumas & Hourcade (*J. Econ. Behavior & Org.*, 2008)

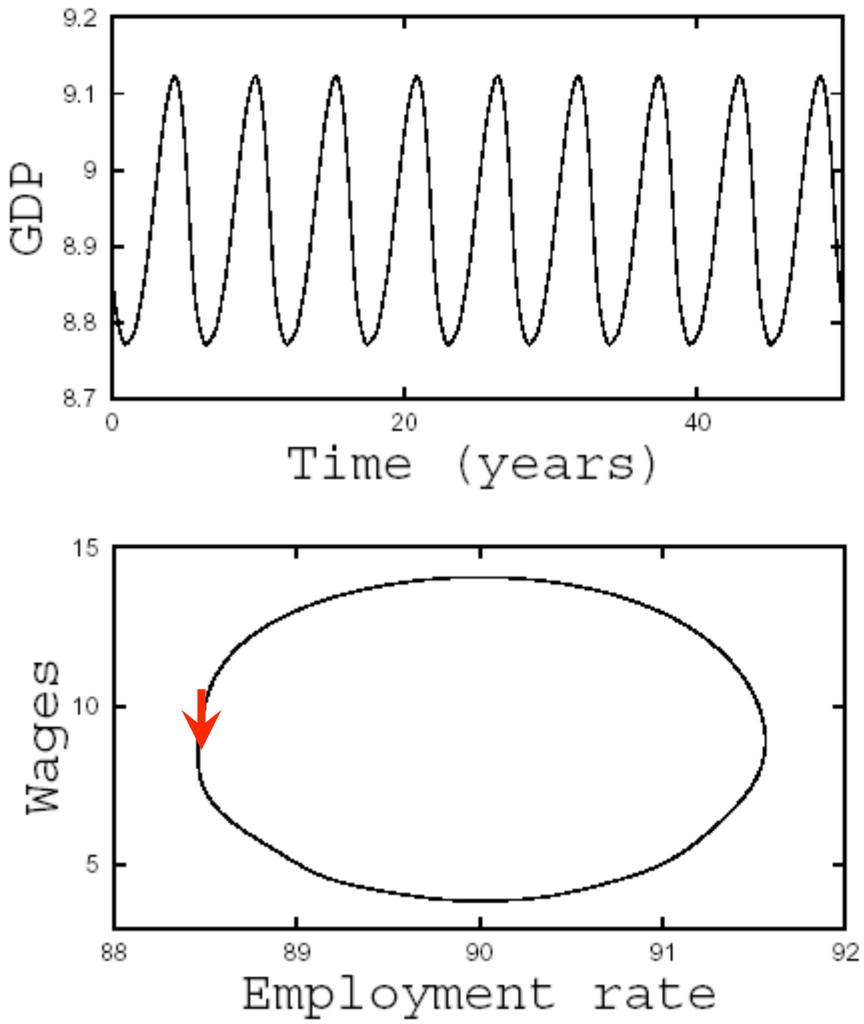
Endogenous dynamics: an alternative explanation for business cycles



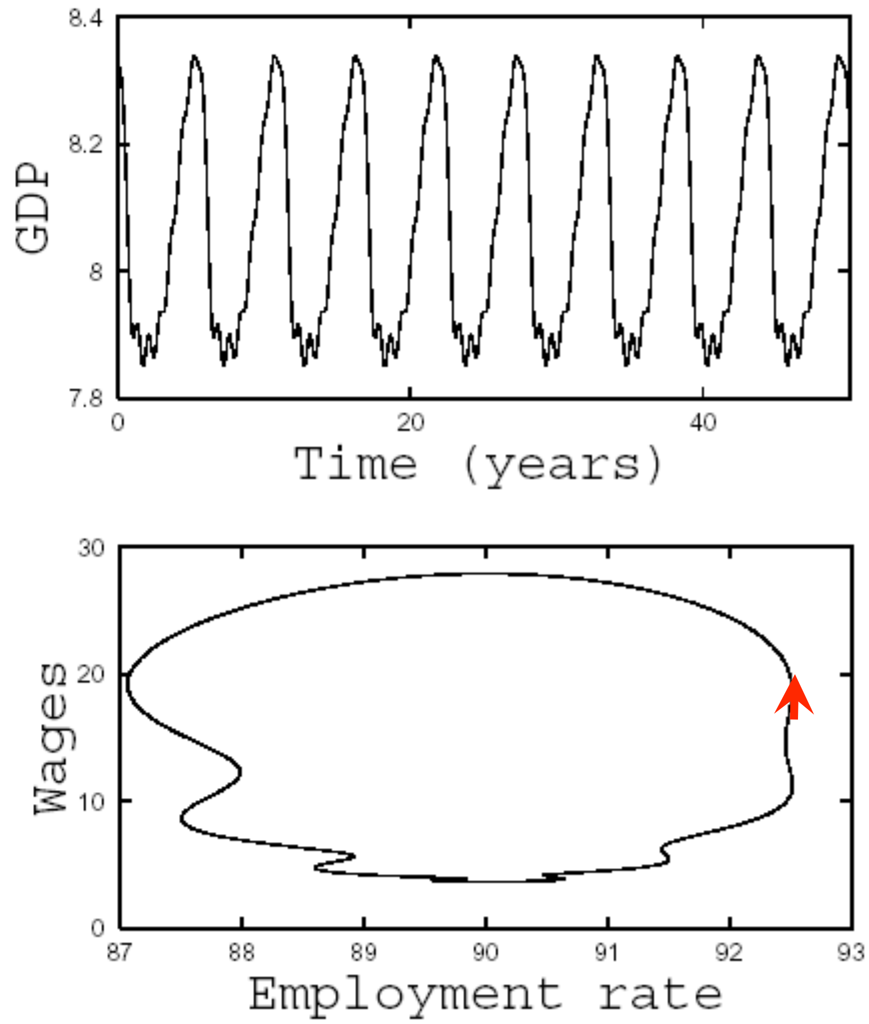
Hopf bifurcation ("tipping point") from stable equilibrium to a limit cycle ("business cycle")



α_{inv} = 1.7: purely periodic behavior (limit cycle)

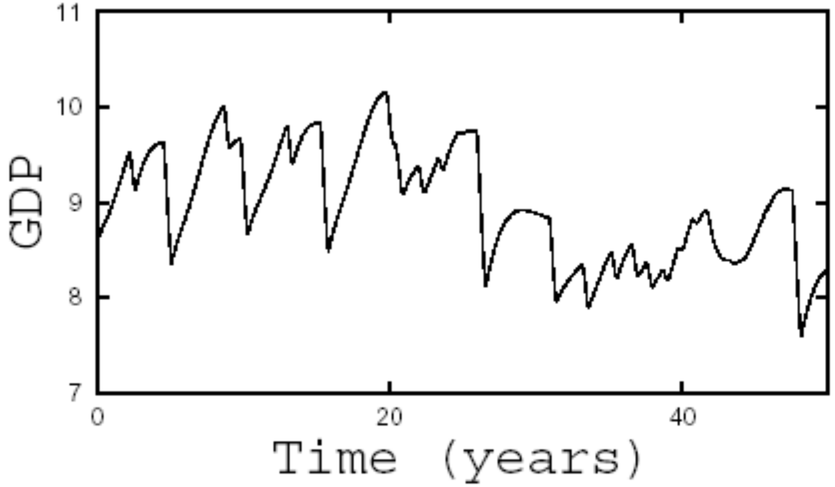


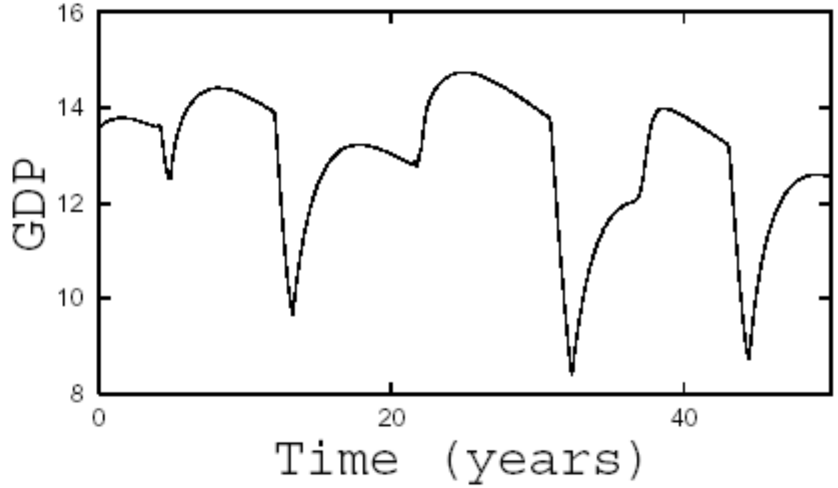
α_{inv} = 2.5: transition to chaos (irregular behavior)



α_{inv} = 10: irregular orbit (kinky torus)

α_{inv} = 20: very asymmetric business cycle (relaxation oscillation)





Endogenous business cycles (EnBCs) in NEDyM

- Business cycles originate from the profit–investment relationship
  (oscillations with a 5–6-year period) – Fukuyama (1989–92)?!

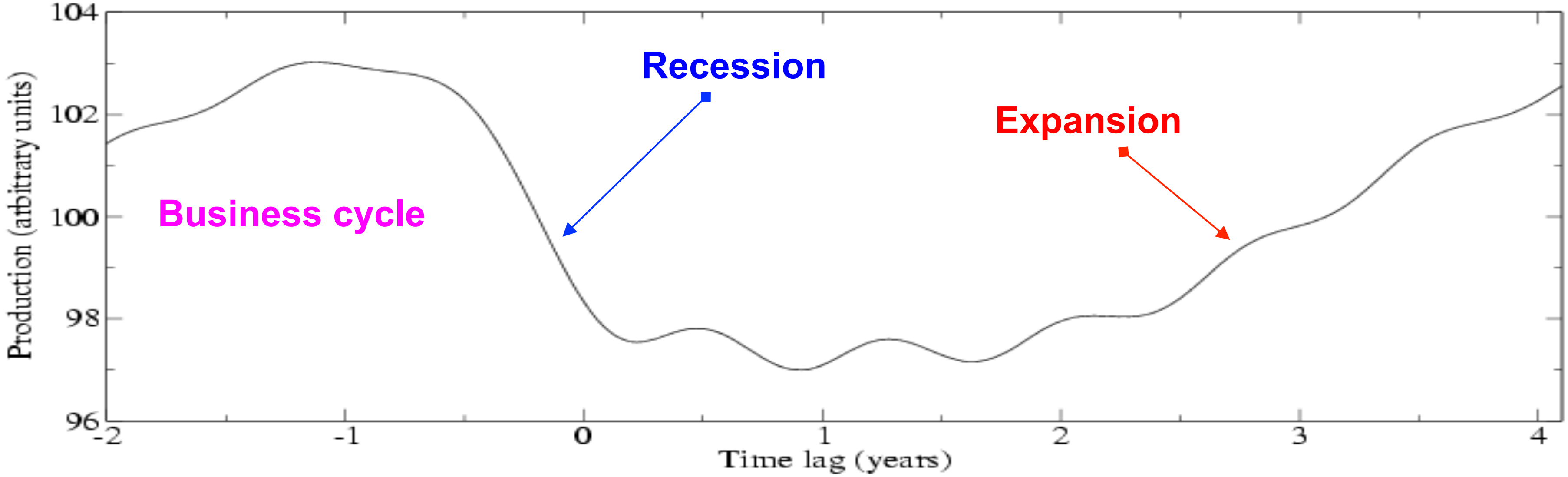
  higher profits => more investments => larger demand => higher profits

- Business cycles are limited in amplitude by three processes:
  - increase in labor costs when employment is high;
  - constraints in production and the consequent inflation in goods prices when demand increases too rapidly;
  - financial constraints on investment.

- EnBC models need to be calibrated and validated
  - harder than for real business cycle models (RBCs):
    fast and slow processes =>
    need a better definition of the business cycles =>
    study of BEA & NBER data!

Catastrophes and the state of the economy – I

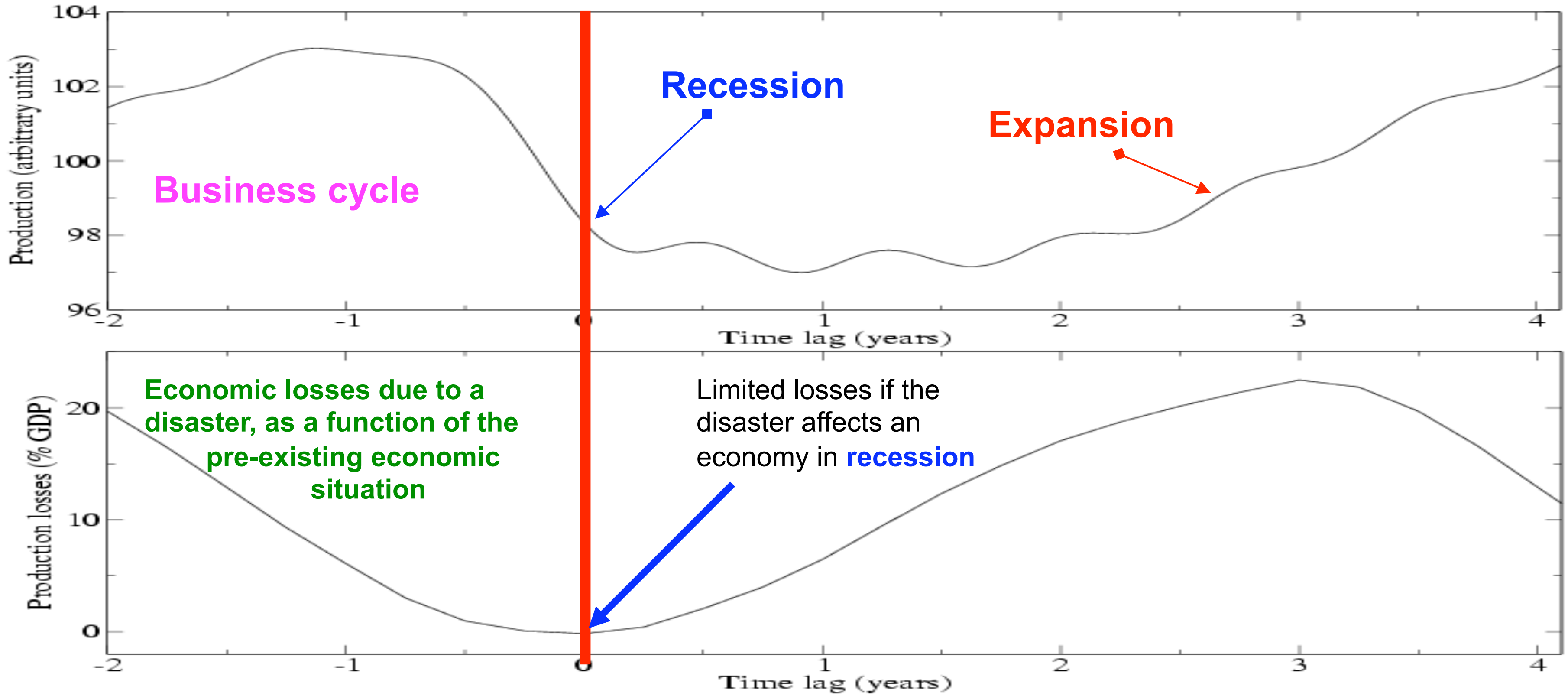
A vulnerability paradox: When does a disaster cause greater **long-term damage** to an economy, during its **expansion phase** or during a **recession**?



Hallegatte & Ghil, 2008, *Ecol. Econ.*, **68**, 582–592, doi:10.1016/j.ecolecon.2008.05.022

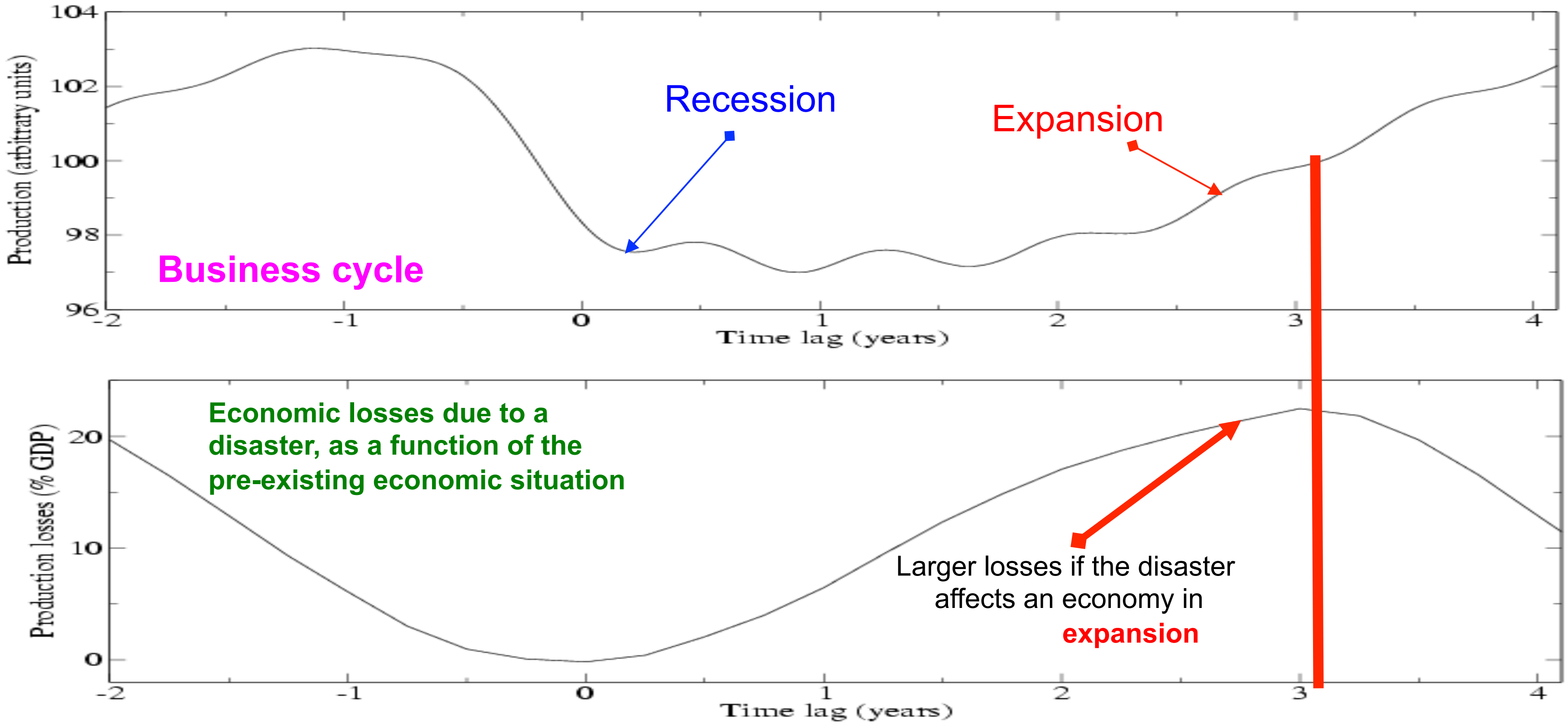
Catastrophes and the state of the economy – II

A vulnerability paradox:
A disaster that affects an economy during its
recession phase…



Catastrophes and the state of the economy – III

… causes **fewer** long-term damages
than if it occurs during an **expansion!**



Hallegatte & Ghil, 2008, *Ecol. Econ.*, **68**, 582–592, doi:10.1016/j.ecolecon.2008.05.022

Stylized Facts of a Business Cycle – I

Need a more objective, quantitative description of the "typical business cycle." To do so we use two complementary approaches:

1. synchronization methods from dynamical systems ("chaos"); and
2. Advanced methods of time series analysis (SSA and M-SSA)

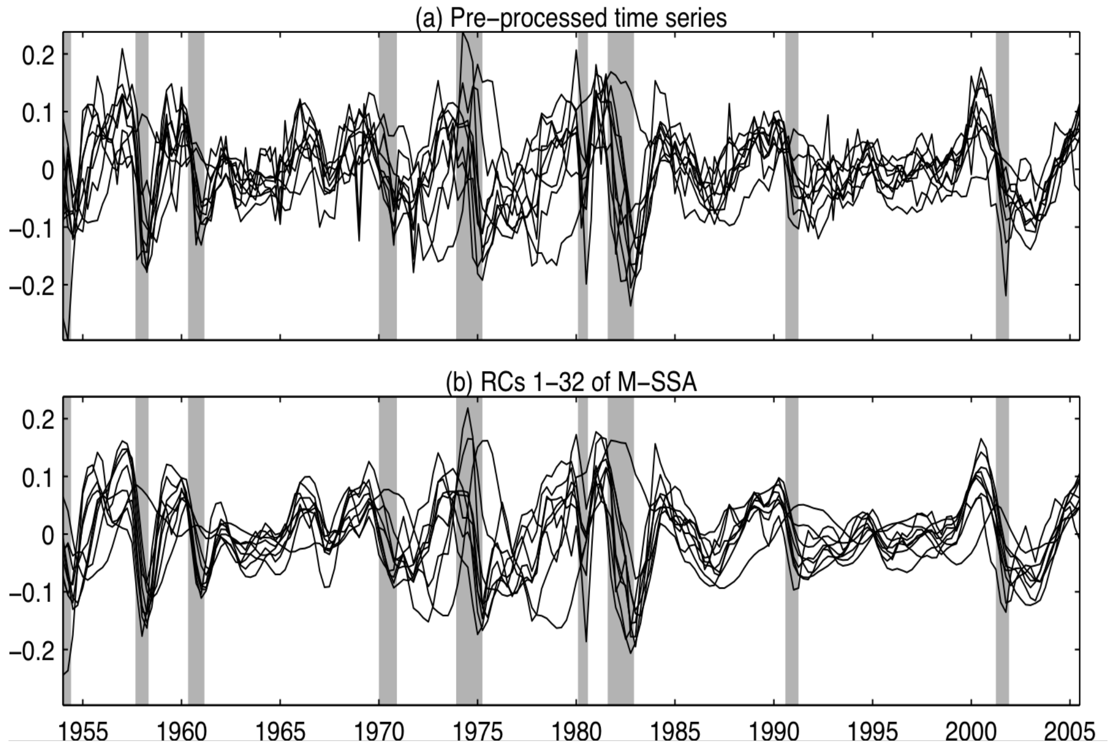
Bureau of Economic Analysis, www.bea.gov; 1947–2005.

9 variables:

gross domestic product (GDP), investment, consumption, employment rate (in %), price, total wage, imports, exports, and change in private inventories.

Groth, Ghil, Hallegatte and Dumas, submitted

Raw data, detrended and standardized

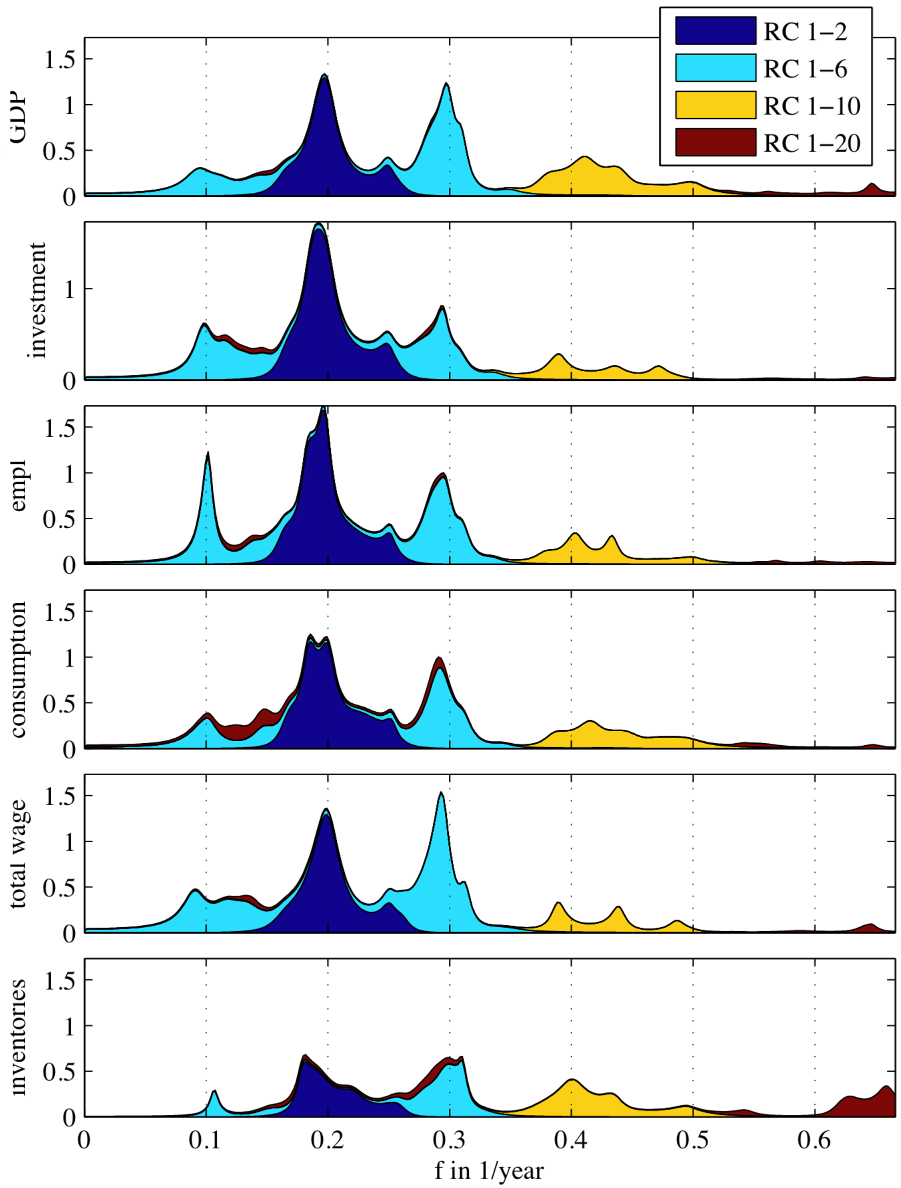


9-channel SSA ($D = 9$, $M = 24$ quarters)

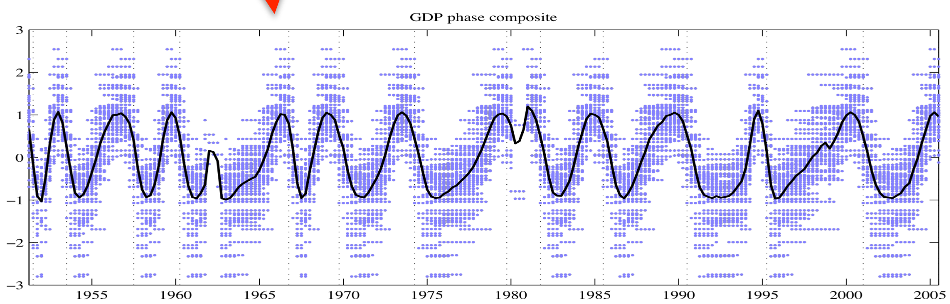
Adaptive filtering, via multichannel singular-spectrum analysis (M-SSA);
vertical shaded bars are NBER-defined recessions

Stylized Facts of a Business Cycle – II

Stack spectrum of reconstructed components (RCs)
2 distinct periodicities:
a) the dominant business cycle (5 years)
b) a 3-year cycle.



Spread of points around the "mean cycle":
nonlinear fluctuation-dissipation theorem?



Dumas, Ghil, Groth & Hallegatte,
Math. Social Sci., to appear.

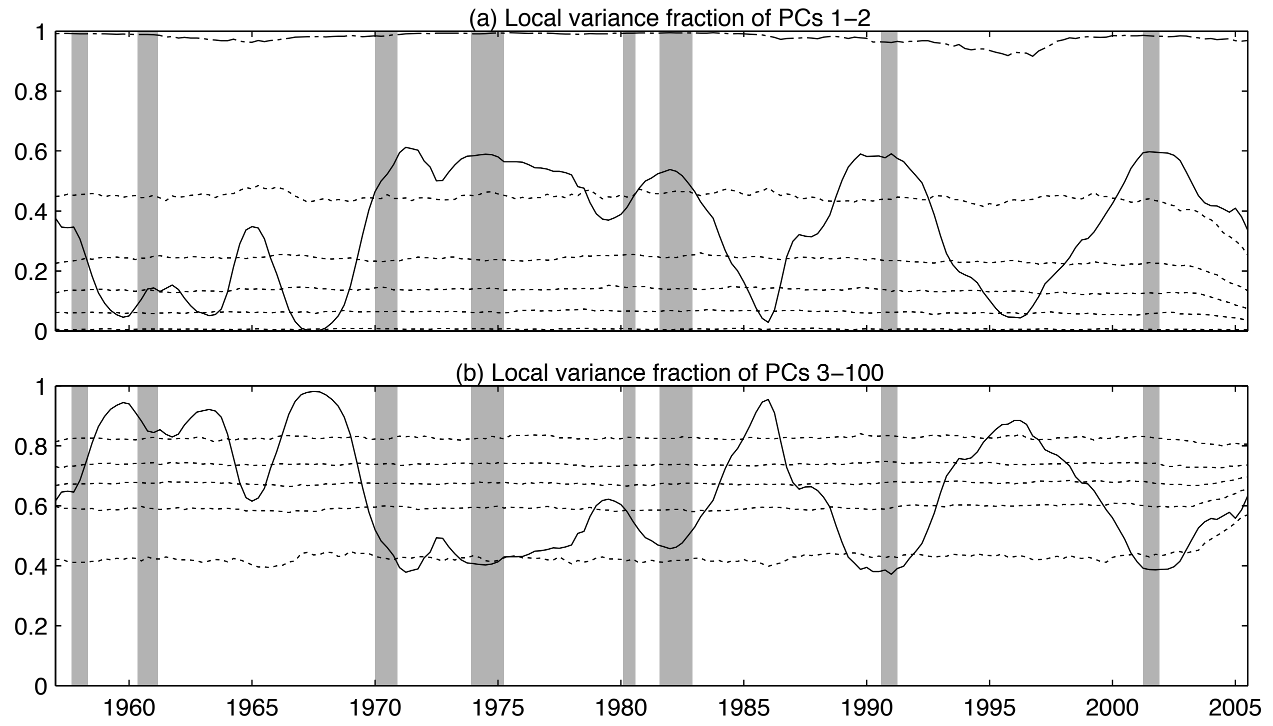
Stylized Facts of a Business Cycle – III

Consider the local variance fraction $V_{\mathcal{K}}(t)$
with $D = 9$, $M = 100$, and $A_k(t)$ the PCs:

$$V_{\mathcal{K}}(t) = \frac{\frac{\sum_{k\in\mathcal{K}} A_k(t)^2}{DM}}{\sum_{k=1} A_k(t)^2}$$

The "signal" fraction is largest during the recessions

The "noise" fraction is largest during the expansions



Vertical shaded bars are NBER-defined recessions

Groth, Ghil, Hallegatte and Dumas, submitted

Conclusions and outlook:
a hierarchy of economic models and data analysis methods

1. The highly idealized **NEDyM model** exhibits fairly realistic, **endogenous business cycles (EBCs):** period = 5–6 years, seasaw shape, good phasing of indices.
2. NEDyM displays a **vulnerability paradox**:
   - extreme-event consequences depend on the state of the economy;
   - they are more severe during an expansion than a recession.
3. This paradox is supported by
   - consequences of **Izmit (Marmara) earthquake, 1999**;
   - reconstruction process after the **2004 and 2005 hurricane seasons in Florida**.
4. **U.S. economic data** (BEA, 1947–2005) tentatively support **a nonlinear fluctuation-dissipation theorem (FDT)** à la Ruelle.
5. **EBC model calibration** is an issue => sequential **data-assimilation methods** are being developed by P. Dumas and A. Groth.
6. Need **a better, quantitative characterization of business cycles**: U.S. + Euro-data, synchronization and spectral methods (A. Groth, L. Sella, G. Vivaldo)
7. Need more detailed, regional and sectorial models: B. Coluzzi, M. G., S.H., and G. Weisbuch are using simplified, **Boolean models to study the economy as a network of businesses** (suppliers and clients, etc.).

The deeper motivations of economic modeling

"Really, Karl! Can't I mention the high price of kohlrabi without getting a manifesto?"

Outline

- What we started with.
- What we did.
- What we found out.
- What we'd like to know →

Some key questions on extreme events

- Are extreme events similar in nature to all other events, only larger?
  (cf. Scott Fitzgerald's "The Great Gatsby")

- Do standard statistical theories of extreme-value distribution do justice to all types of phenomena, or are there differences?
  - "deterministic" vs. "stochastic" processes

- Can long-tailed distributions of events and periodic features co-exist in a time series?

- Can we gain confidence in predicting extreme events from deterministic and stochastic models of the underlying mechanisms?

- Topics for Panel Discussion?

Some real mathematics of large deviations

Varadhan's Lemma

There is a simple lemma due to Laplace that is useful in evaluating limits of integrals: For every continuous function h on $[0,1]$

$$\lim_{n\to\infty} \frac{1}{n}\log\int_0^1 e^{-nh(x)}\,dx = -\inf h(x).$$

(The common fact $\lim_{p\to\infty}\|f\|_p = \|f\|_\infty$ can be used to get a one-line proof of this lemma:

$$\lim_{n\to\infty}\log\|e^{-h}\|_n = \log\|e^{-h}\|_\infty = \log\sup e^{-h(x)} = -\inf h(x).)$$

Now suppose we are given a family of probability measures and are asked to evaluate the limit

$$\lim_{n\to\infty} \frac{1}{n}\log\int_0^1 e^{-nh(x)}\,d\mu_n(x).$$

In his 1966 paper Varadhan argues that if we have

$$d\mu_n(x) \sim e^{-nI(x)}dx,$$

then by Laplace's lemma this limit would be

$$-\inf\,[h(x)+I(x)].$$

The function $I(x)$ is now called the *rate function*. It is defined for spaces much more general than the unit interval $[0,1]$.

Let X be any complete separable metric space (Polish space). A rate function I is a lower semicontinuous function from X into $[0,\infty]$ such that for every $\ell<\infty$ the level-set $\{x : I(x)\le \ell\}$ is compact. A family $\{\mu_n\}$ of probability measures on X is said to satisfy the *large-deviation principle* (LDP) with the rate function I if

(i) for every open set U

$$\underline{\lim}\ \frac{1}{n}\log\mu_n(U) \ge -\inf_U I(x),$$

(ii) for every closed set F

$$\overline{\lim}\ \frac{1}{n}\log\mu_n(F) \le -\inf_F I(x).$$

Varadhan's Lemma says that if $\{\mu_n\}$ satisfy the LDP, then for every bounded continuous function h on X

$$\lim_{n\to\infty}\frac{1}{n}\log\int_X e^{-nh(x)}\,d\mu_n(x) = -\inf\,[h(x)+I(x)].$$

There is an amazing variety of situations where the LDP holds. Finding the rate function is a complex art that Varadhan has developed over the years.

The Abel Prize 2007

Srinivasa S. R. Varadhan has won the 2007 Abel prize. Image © Cheryl Sylivant/The Abel Prize/The Norwegian Academy of Science and Letters.

Some prizes you win for your achievements and others you win through sheer luck. The Abel prize belongs to the former category and it has just been won by the mathematician Srinivasa S. R. Varadhan for his work on the maths describing those rare chance events that produce lucky lottery winners but can also spell death and disaster. The Norwegian Academy of Science and Letters awarded the prize "for Varadhan's fundamental contributions to probability theory and in particular for creating a unified theory of large deviation".

If you are a wildly optimistic person or someone crippled by pessimism, then you have probably often been told by rational and reasonable people to consider the laws of probability. For every lottery winner there are millions of people who don't win anything at all and for every potential time you walk down the street and a brick falls on your head from out of nowhere there are millions of times you've walked down the street safely. Everything evens out in the end and therefore your hopes or fears are completely unjustified.

The rational and reasonable person will have been right. There are rigorous results in probability theory that prove this. The *law of large numbers* and the *central limit theorem* state, loosely speaking, that if you repeat the same experiment — playing the lottery or walking down the street — a large number of times, then the observed outcomes — the number of times you win or the

The mathematical theory of large deviations is connected to stochastic processes, martingales, parabolic PDEs, and maximum principles. Could it help us with our applications?

Singular Spectrum Analysis (SSA)

Spatial EOFs

x -- space

$$\phi(x,t) = \sum a_k(t)e_k(x)$$

$$C_\phi(x,y) = E\phi(x,\omega)\phi(y,\omega) = \frac{1}{T}\int_o^T \phi(x,t)\phi(y,t)dt$$

$$C_\phi e_k(x) = \lambda_k e_k(x)$$

Colebrook (1978); *Weare & Nasstrom* (1982);
Broomhead & King (1986: BK); *Fraedrich* (1986)

BK+VG: Analogy between Mañe-Takens embedding and the Wiener-Khinchin theorem

SSA

s -- lag

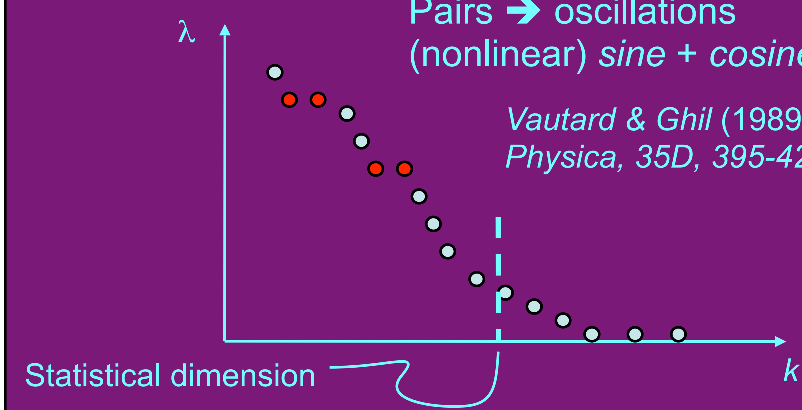
$$X(x+s) = \sum a_k(t)e_k(s)$$

$$C_X(s) = EX(t+s,\omega)\phi(s,\omega) = \frac{1}{T}\int_o^T X(t)X(t+s)dt$$

$$C_X e_k(s) = \lambda_k e_k(s)$$

Pairs ➔ oscillations
(nonlinear) *sine* + *cosine* pair

Vautard & Ghil (1989: VG)
Physica, 35D, 395-424

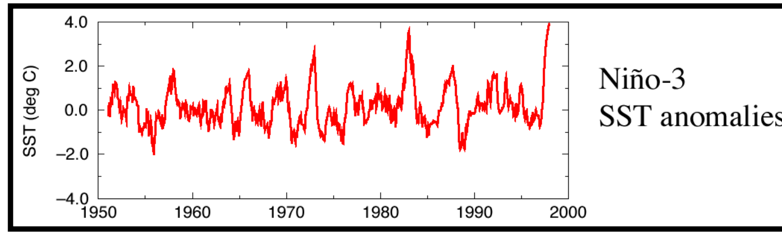


Singular Spectrum Analysis (SSA)

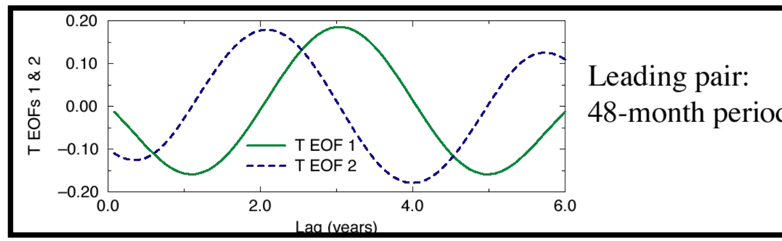
SSA decomposes (geophysical & other) time series into *Temporal EOFs* (T-EOFs) and *Temporal Principal Components* (T-PCs), based on the series' lag-covariance matrix

Selected parts of the series can be reconstructed, via *Reconstructed Components* (RCs)

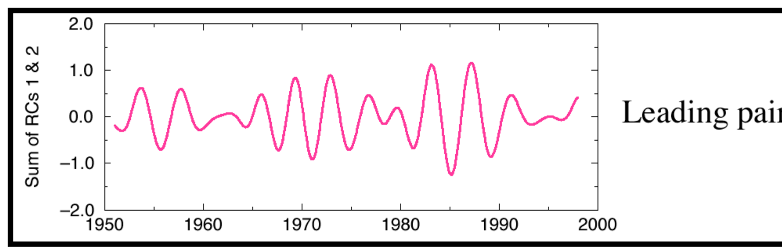
Time series



T-EOFs



RCs



- SSA is good at isolating oscillatory behavior via paired eigenelements.
- SSA tends to lump signals that are longer-term than the window into
  - one or two trend components.

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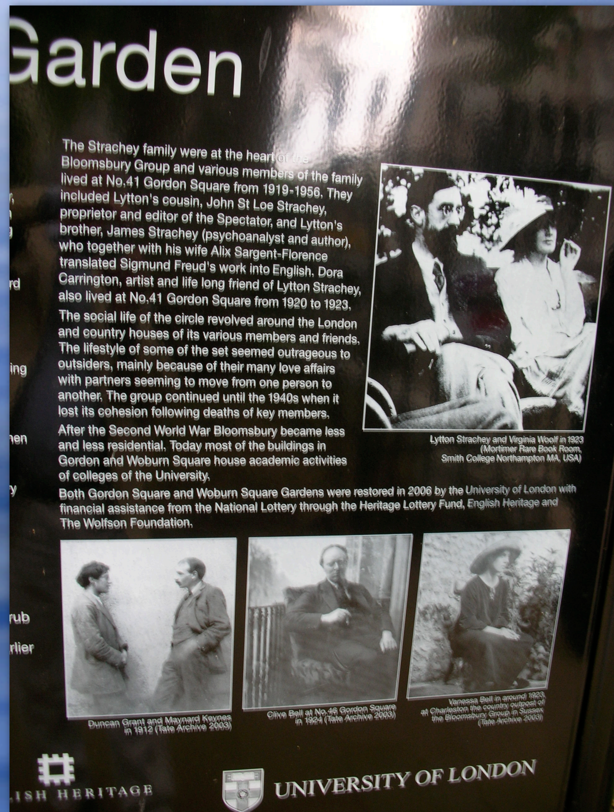
Reserve slides

The blessings of interdisciplinarity

♥ *Photo with lover*
Duncan Grant



♥ *John M. Keynes's*
home in Bloomsbury

photos M.G., May 2008