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On the nature of plasma microturbulence

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Trieste, Italy

Some introductory remarks

There is a great body of work on fluid turbulence; to which degree is plasma microturbulence similar or different?

I will attempt to present the material in an accessible way

Please feel free to interrupt me if you have a question

Turbulence in fluids and plasmas

What is turbulence?

Leonardo
da Vinci
(1529)

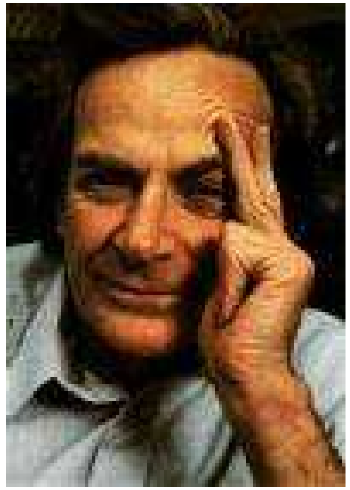
Turbulence…

- is a nonlinear phenomenon
- occurs (only) in open systems
- involves many degrees of freedom
- is highly irregular (chaotic) in space and time
- often leads to a (statistically) quasi-stationary state far from thermodynamic equilibrium

These properties make it a very complicated problem – neither Dynamical Systems Theory nor Statistics applies!

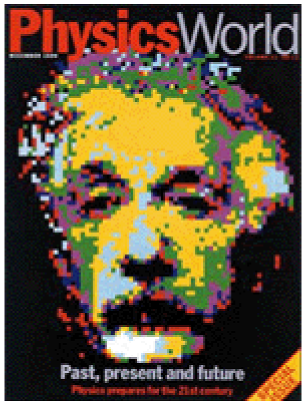
Turbulence – one of the most important unsolved problems in physics

According to a famous statement by Richard Feynman…



…and a survey by the British “Institute of Physics” among many of the leading physicists world-wide…

“Millennium Issue”
(December 1999)



TURBULENCE:
A challenging topic for both basic and applied research

How to approach turbulence?

Many physicists – including Heisenberg, von Weizsäcker, Onsager, Feynman, and many others – have attempted to tackle turbulence **purely analytically** but with only **very limited success**.

Today, **supercomputers** help to unravel the “mysteries” of turbulence in the spirit of **John von Neumann**:

„There might be some hope to 'break the deadlock' by extensive, but well-planned, computational efforts...“

The Navier-Stokes equation

The NSE in its ‘classical’ form:

$$(\partial_t + \vec{v} \cdot \nabla)\,\vec{v} = -\nabla p + \nu \nabla^2 \vec{v} \qquad \nabla \cdot \vec{v} = 0$$

Expressed in terms of vorticity $\vec{\Omega} = \nabla \times \vec{v}$:

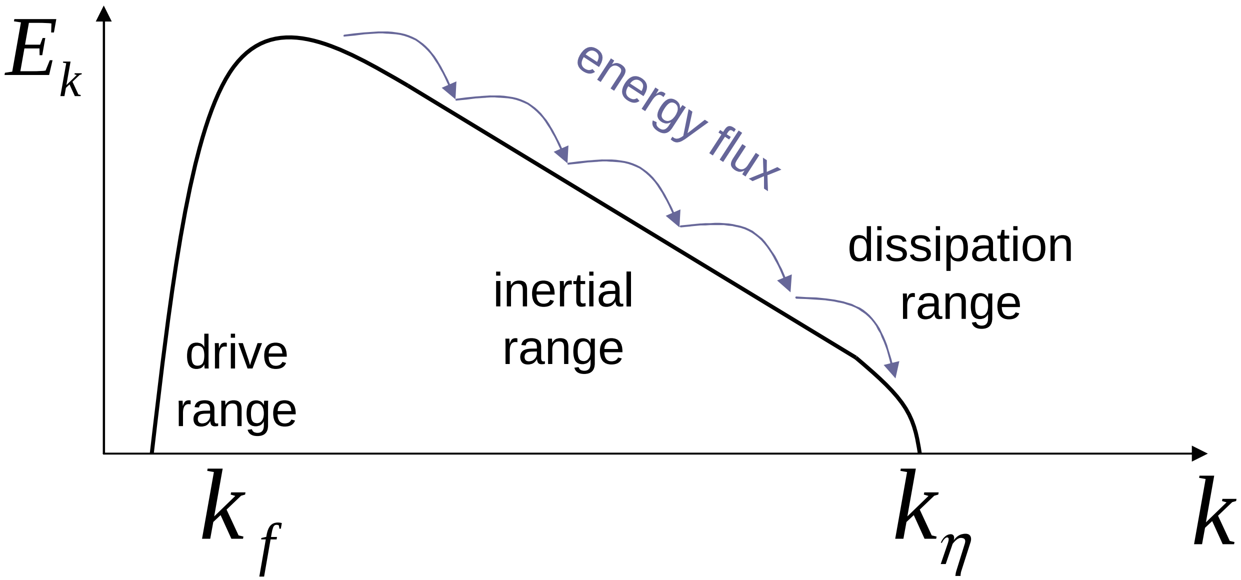
$$(\partial_t + \vec{v} \cdot \nabla)\,\vec{\Omega} = (\vec{\Omega} \cdot \nabla)\,\vec{v} + \mathrm{Re}^{-1} \nabla^2 \vec{\Omega}$$

Reynolds number as single dimensionless parameter:

$$\mathrm{Re} = \frac{LU}{\nu}$$

The Richardson cascade

Turbulence as a **local cascade in wave number space**…



$$\frac{k_\eta}{k_f} \sim \mathrm{Re}^{3/4}$$

Computational effort $\sim \mathrm{Re}^3$

„Big whorls have little whorls, little whorls have smaller whorls that feed on their velocity, and so on to viscosity"

Much turbulence research addresses the **cascade** problem

Kolmogorov's theory from 1941

K41 is based merely on intuition and dimensional analysis – it is *not* derived rigorously from the Navier-Stokes equation

Key assumptions:

- Scale invariance – like, e.g., in critical phenomena
- Central quantity: energy flux ε

$$E = \frac{1}{2V}\int v^2\, d^3x = \int_0^\infty E(k)\, dk$$

$$E(k) = C\, \varepsilon^{2/3}\, k^{-5/3}$$

Quantity	Dimension
Wave number	1/length
Energy per unit mass	$\text{length}^2/\text{time}^2$
Energy spectrum $\mathcal{E}(k)$	$\text{length}^3/\text{time}^2$
Energy flux ε	$\text{energy}/\text{time} \sim \text{length}^2/\text{time}^3$

This is the most famous turbulence result: the "-5/3" law.

However, K41 is fundamentally wrong: scale invariance is broken!

Global Gyrokinetic Simulation of Turbulence in

ASDEX Upgrade

GENE

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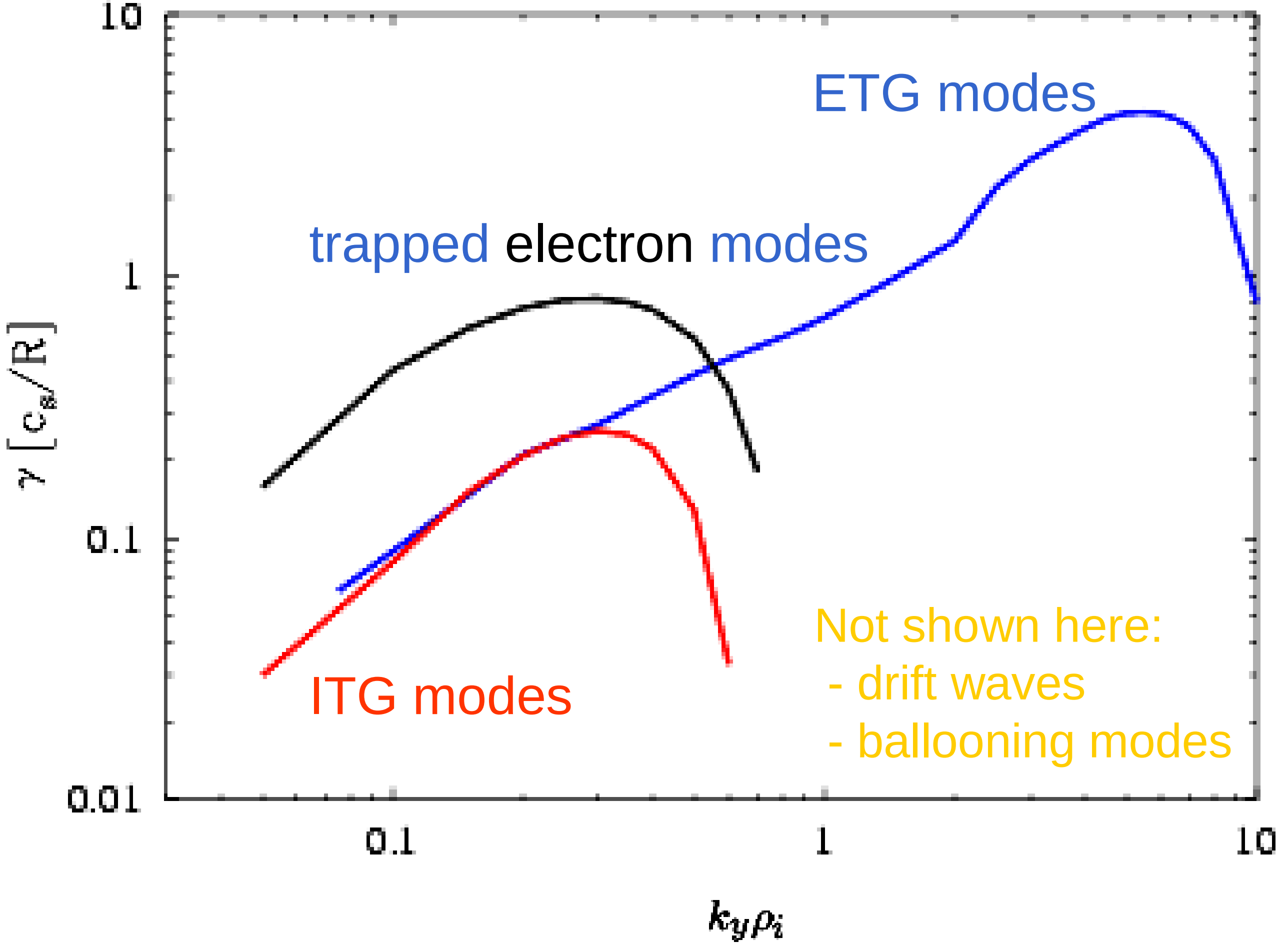
Plasma microturbulence: Linear drive

Some important microinstabilities

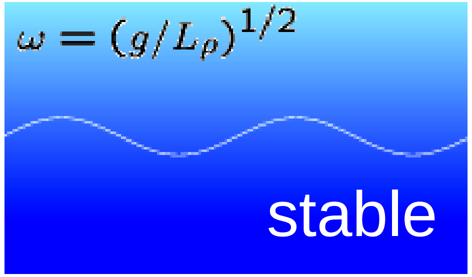


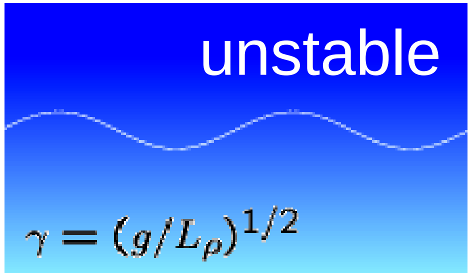
Wide drive range

Universality?

Gradient-driven microinstabilities

Perpendicular dynamics: de-/stabilization in out-/inboard regions



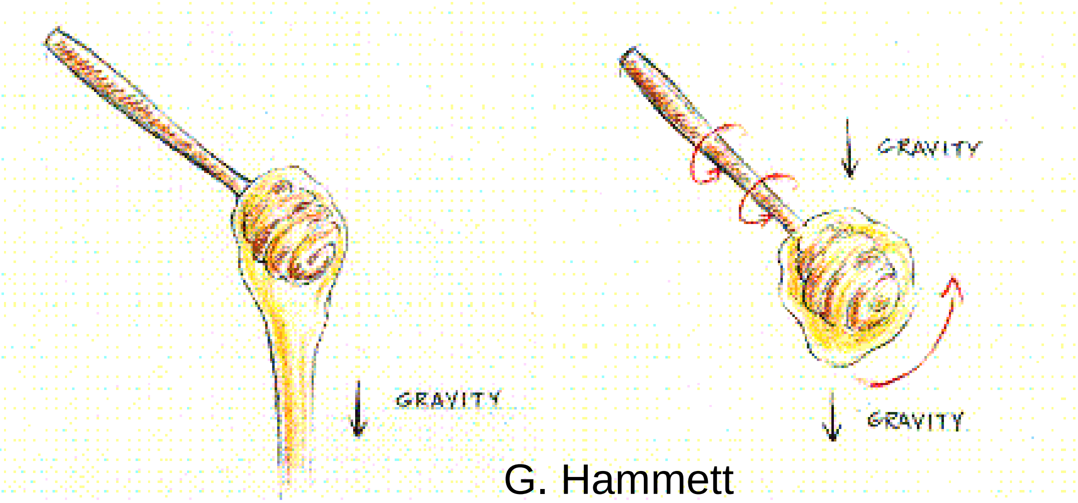


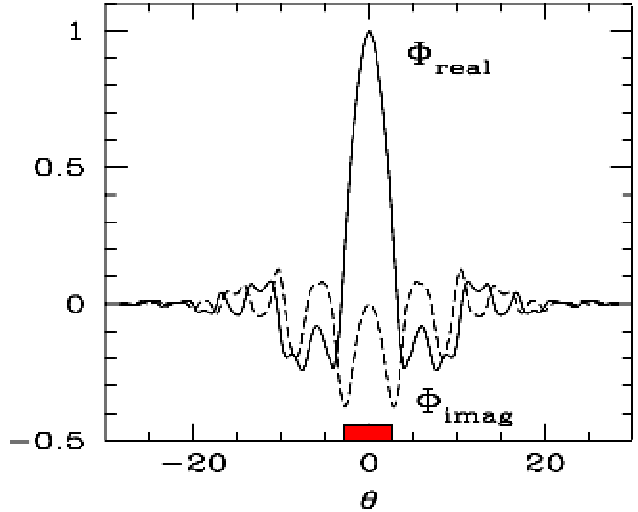
Rayleigh-Taylor instability

Analogy in a plasma:

$$g_{\text{eff}} = v_t^2/R$$

Parallel dynamics: localization in outboard regions





Basic properties of microturbulence

$$\gamma_{\text{eff}} \approx (k_\perp \rho_i)\,\frac{v_t}{L_T} - \mathcal{C}\,\frac{v_t}{R}$$

Existence of critical temperature gradients

$$k_\perp \rho_i \approx 1 \; : \quad \gamma_{\text{eff}} > 0 \quad \leftrightarrow \quad \frac{R}{L_T} > \left(\frac{R}{L_T}\right)_{\text{crit}}$$

Temperature profiles tend to be 'stiff' (cp. solar convection zone).

Typical space scales: several ion gyroradii (not system size)

$$\frac{R}{L_T} \sim \left(\frac{R}{L_T}\right)_{\text{crit}} \; : \quad \gamma_{\text{eff}} > 0 \quad \leftrightarrow \quad (k_\perp \rho_i) > (k_\perp \rho_i)_{\text{crit}}$$

ETG / ITG modes: Critical gradients

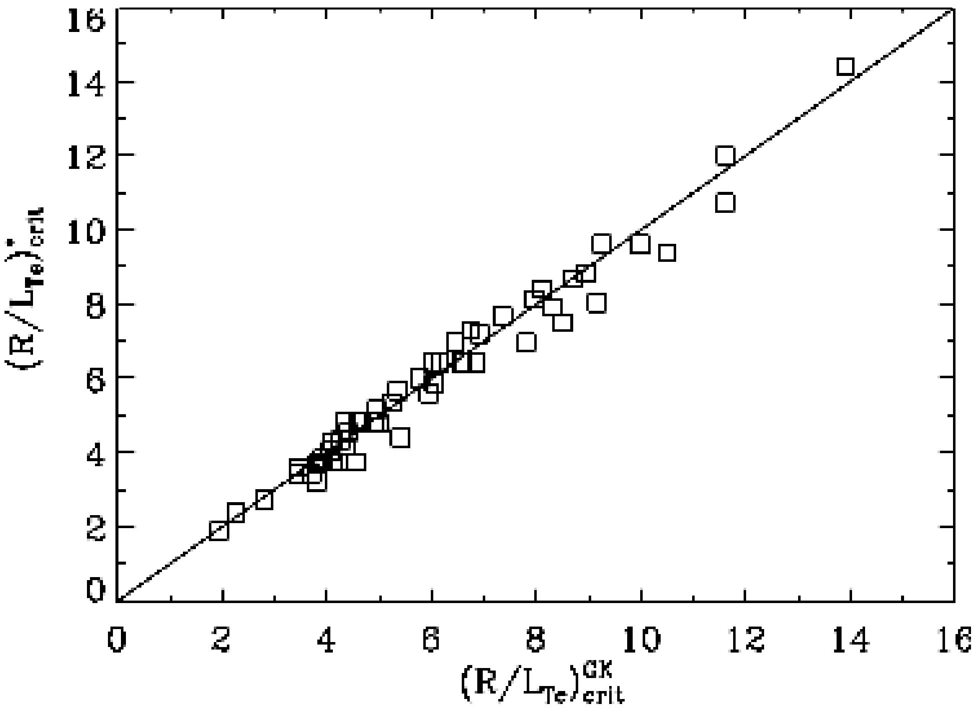
Linear stability of ETG / ITG modes [Jenko *et al.* 2001]

Linear gyrokinetic simulations:

$$(R/L_{T_j})_{\mathrm{crit}} \approx (1+\tau_j)\,(1.33 + 1.91\,\hat{s}/q)$$

$$\tau_e \equiv T_e/T_i \equiv 1/\tau_i$$

Limiting cases (analytical results):
- Hahm & Tang 1989 (for high s/q)
- Romanelli 1989 (for low s/q)



Thousands of linear GK simulations condense into one simple formula…

Plasma microturbulence: Nonlinear saturation?

F. Jenko, Physics Letters A **351**, 417 (2006)

2D Hasegawa-Mima equations: ZFs

Hasegawa & Mima, PRL 1977

Standard HME (ETG)

$$\frac{d}{dt}(\phi - \nabla^2 \phi - x) = 0$$

Modified HME (ITG)

$$\frac{d}{dt}(\phi - \langle \phi \rangle - \nabla^2 \phi - x) = 0$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} - \frac{\partial \phi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial \phi}{\partial x}\frac{\partial}{\partial y}$$

4-mode analysis

ITG-type HME in Fourier space

$$(1+k^2)\dot{\Phi}_{\mathbf{k}} + ik_y\Phi_{\mathbf{k}} = \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} [\hat{\mathbf{z}}\cdot(\mathbf{k}_1\times\mathbf{k}_2)(1+k_2^2)]\Phi_{\mathbf{k}_1}\Phi_{\mathbf{k}_2}$$

Reduction to just 4 modes (and their CC's)

streamer $(k_x, k_y) = (0, q)$

zonal flow $(k_x, k_y) = (p, 0)$

sidebands $(k_x, k_y) = (p, q)$

$(k_x, k_y) = (p, -q)$

Resulting amplitude equations

$$\dot{\phi}_q + i\Omega_q \phi_q = 0,$$

$$\dot{\phi}_0 = -qp(\phi_q \phi_- - \phi_q^* \phi_+),$$

$$\dot{\phi}_+ + i\Omega_+ \phi_+ = \frac{qp(1+q^2-p^2)}{1+q^2+p^2} \phi_q \phi_0,$$

$$\dot{\phi}_- + i\Omega_- \phi_- = -\frac{qp(1+q^2-p^2)}{1+q^2+p^2} \phi_q^* \phi_0.$$

$$\Omega_+ = -\Omega_- = \frac{q}{1+q^2+p^2}, \quad \Omega_q = \frac{q}{1+q^2}.$$

Zonal flow growth rate

If the streamer amplitude exceeds a certain threshold, the zonal flow becomes unstable.

Its growth rate is given by:

$$\gamma_0 = \sqrt{\frac{2q^2p^2(1+q^2-p^2)}{(1+q^2+p^2)}|\phi_q|^2 - \Delta\Omega^2}$$ ITG case

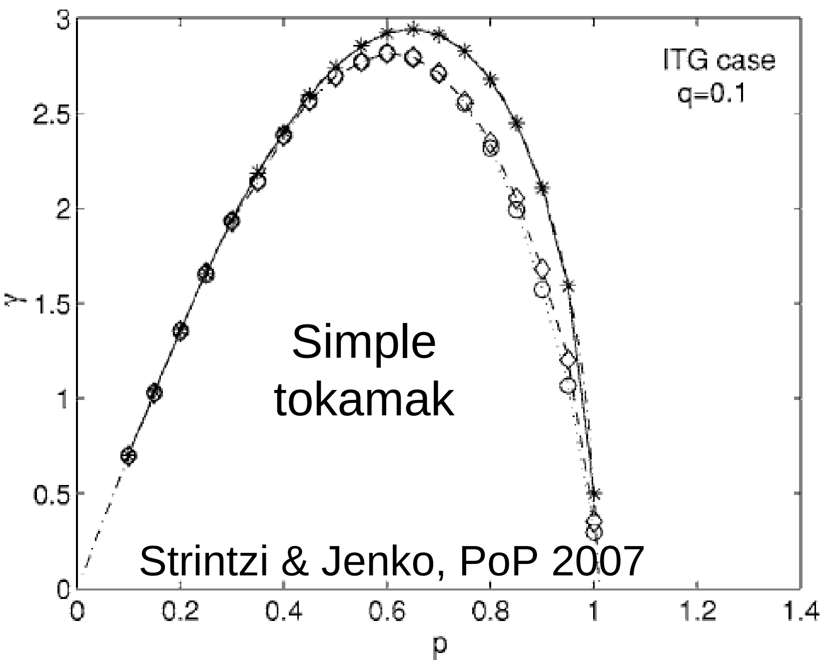
$$\gamma_0 = \sqrt{\frac{2q^2p^4(q^2-p^2)}{(1+p^2)(1+q^2+p^2)}|\phi_q|^2 - \Delta\Omega^2}$$ ETG case

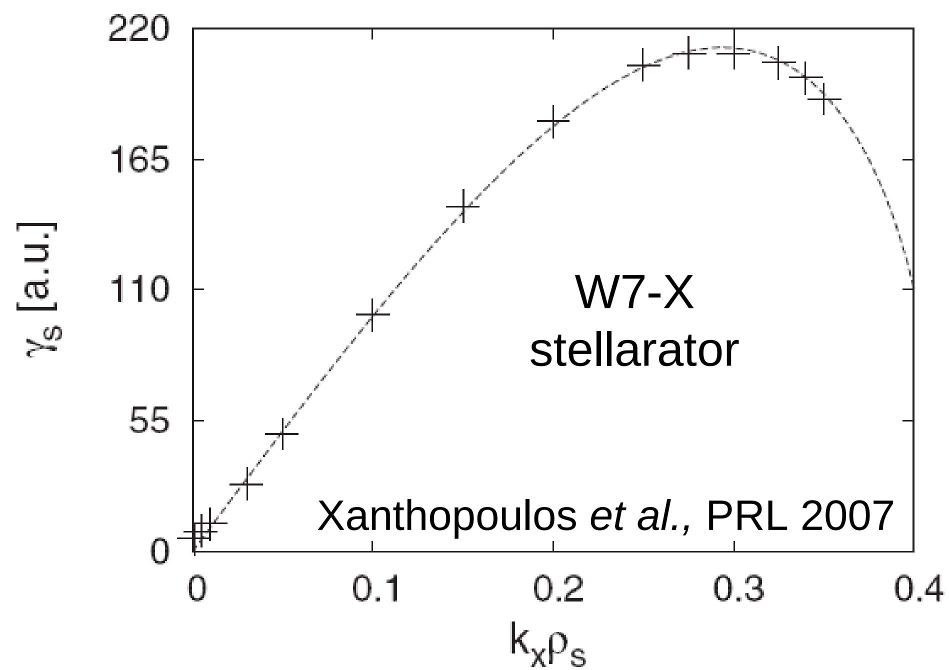
Strintzi & Jenko, PoP 2007

Secondary instabilities & ZF generation

- Large-amplitude streamers are Kelvin-Helmholtz unstable

  [Cowley at al. 1991; Dorland & Jenko PRL 2000]
- This secondary instability contains a zonal-flow component
- Near-equivalence to 4-mode and wave-kinetic approaches

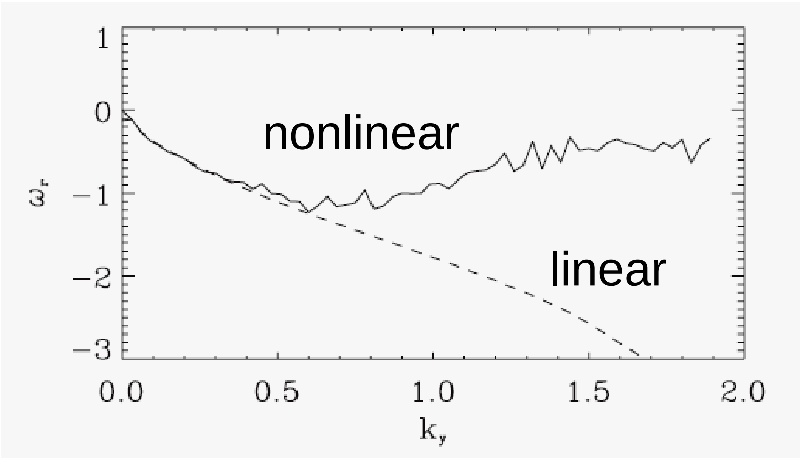




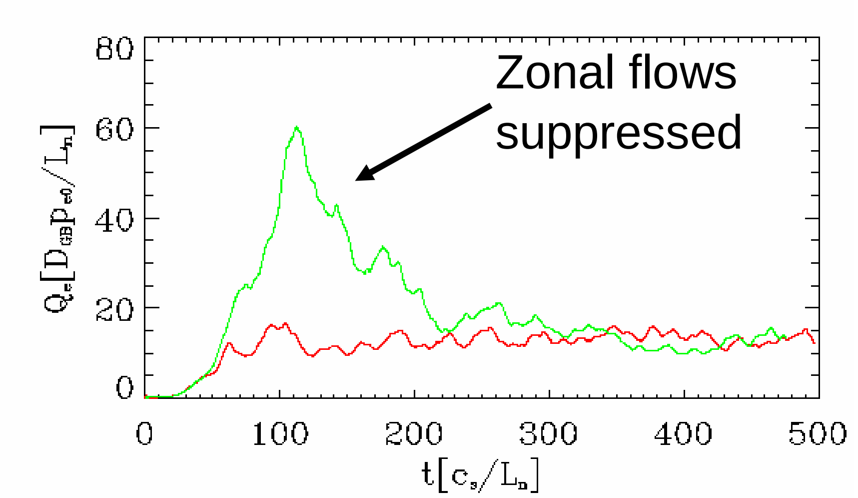
A different story: TEM turbulence

Saturated phase of TEM turbulence simulations:

- In the drive range, nonlinear and linear frequencies are identical



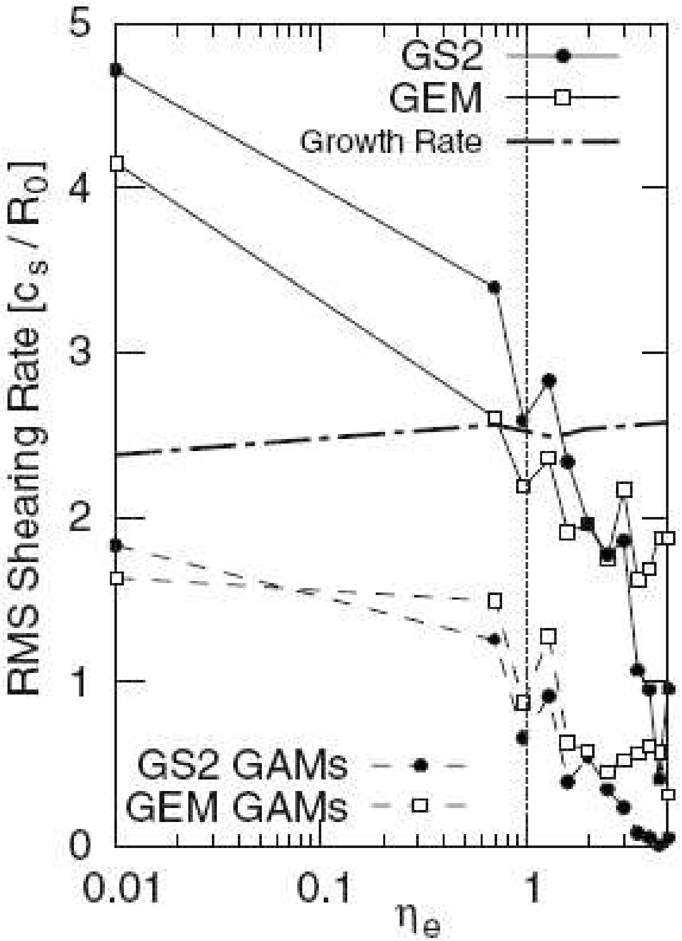
- In the drive range, there is no significant shift of cross phases w.r.t. linear ones



– No dependence of transport level on zonal flows [Dannert & Jenko 2005]

ZF / Non-ZF regimes

Ernst et al., PoP 2009



ExB shearing rates exceed the growth rate *only* for $\eta_e < 1$

For mainly temperature gradient driven TEM turbulence, ZFs (and GAMs) are “unimportant”

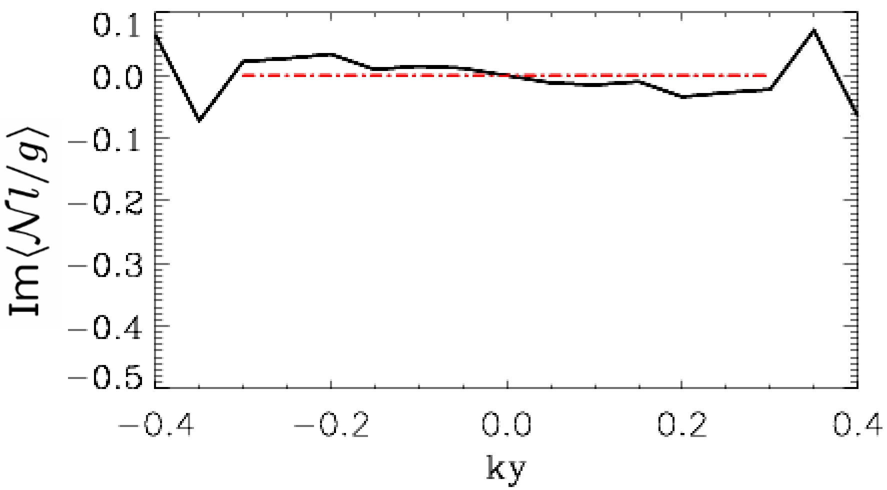
Thus, in a wide region of parameter space, the standard drift-wave / ZF paradigm does not hold

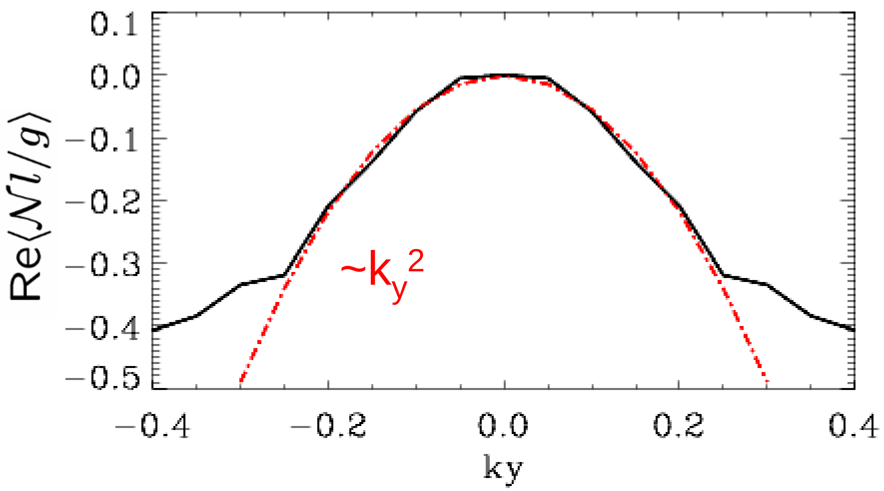
Saturation of TEMs: “eddy damping”

Merz & Jenko, PRL 2008

Low-k_y drive range: large transport contributions, but small random noise; here, one finds:

$$\mathcal{N}l[g] \simeq D(-k_\perp^2)g = D\nabla_\perp^2 g$$





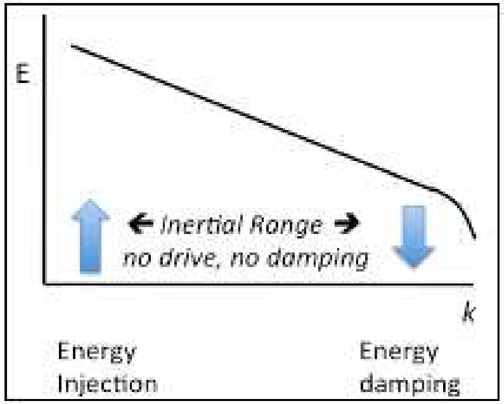
This is in line with various theories, including Resonance Broadening Theory (Dupree), MSR formalism (Krommes), Dressed Test Mode Approach (Itoh).

Dissipation & cascades in plasma microturbulence

Hatch, Terry, Jenko, Merz & Nevins, PRL 2011

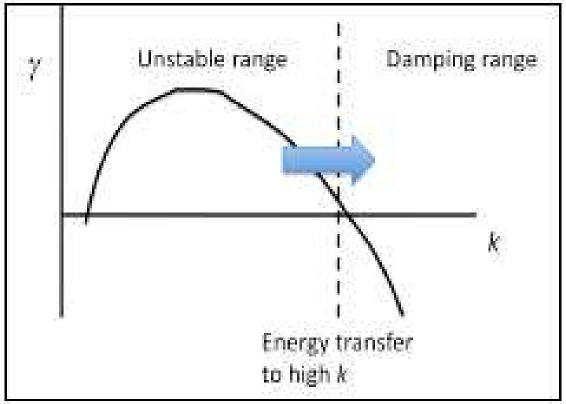
Turbulence in fluids and plasmas – Three basic scenarios

1. Hydrodynamic cascade



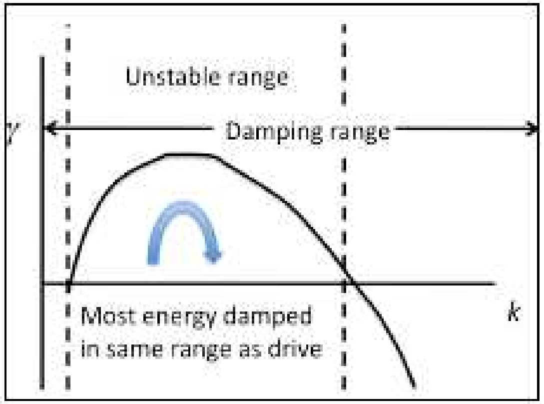
Inertial range
→ no dissipation
→scale invariant dynamics
→power law spectrum

2. Conventional μ-turbulence



Energy transfer to high *k*
like hydro – no inertial range
adjacent unstable,
damping ranges

3. Saturation by damped eigenmode



Energy can go to high *k*
but most of it is lost at
low *k* in driving range

Saturation via damped eigenmodes

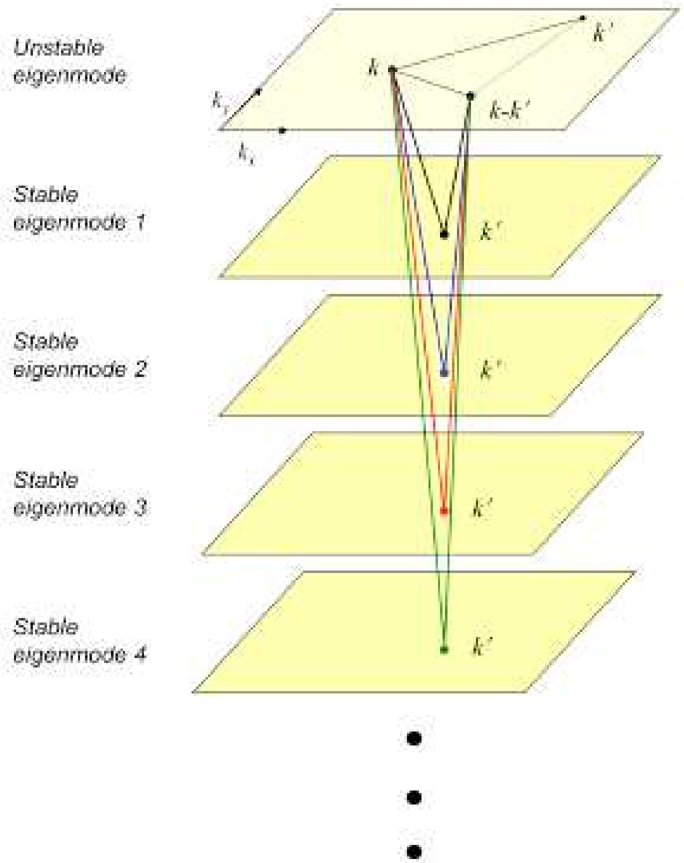
Plasma dispersion relation has multiple roots

- One root unstable → drives turbulence (TEM, ITG, ETG…)
- Other roots can be damped for all *k*
- Fluid models: one root per equation
- Gyrokinetics: infinite in principle;
  discretization yields large but finite number

3-wave interactions drive damped eigenmodes

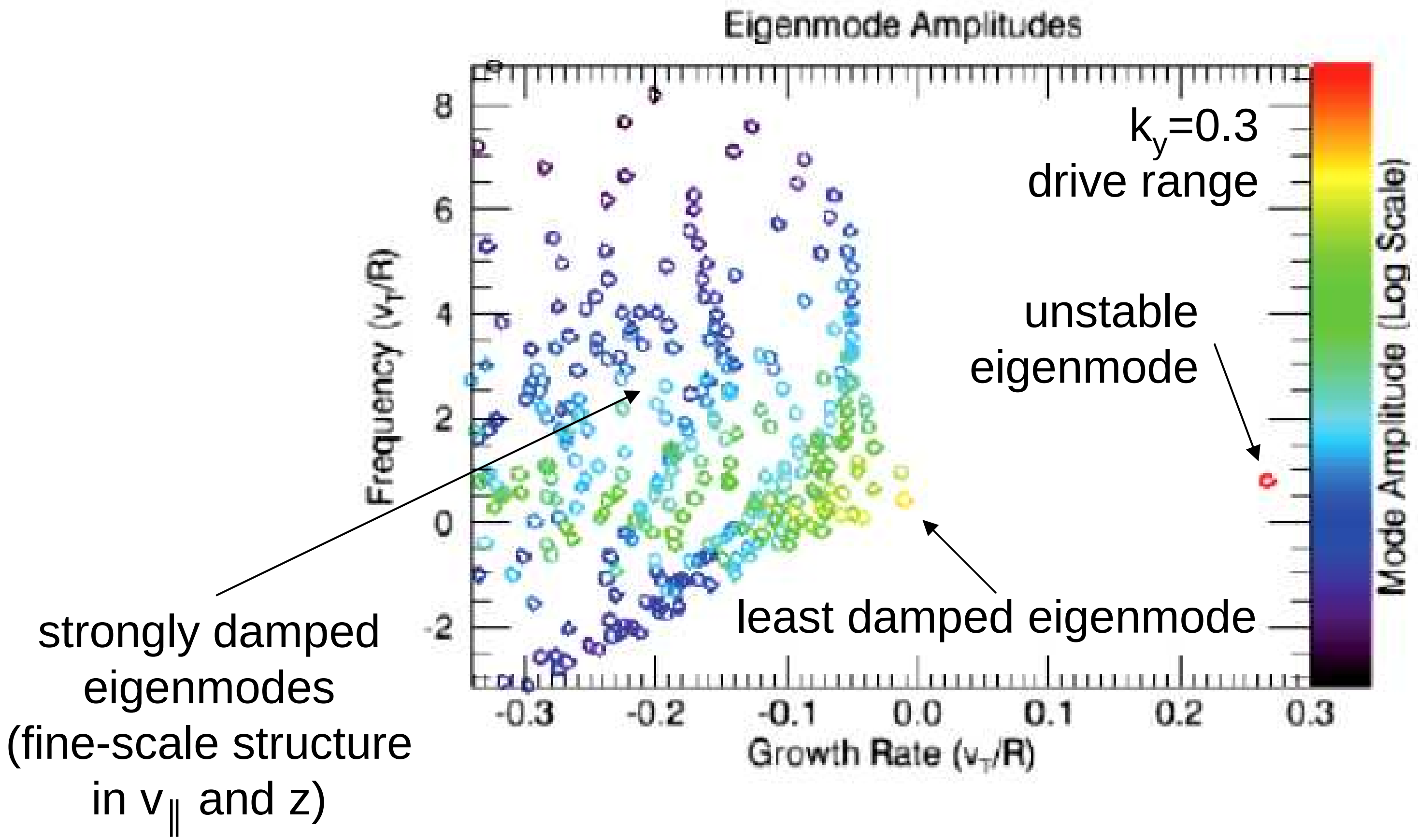
- Pumped by unstable mode through parametric instability
  Only condition: $Amp_{damp} << Amp_{ustable}$ initially
- Each eigenmode driven by combo of all nonlinearities
  => Large *multiplicity* of coupling channels
  => Many eigenmodes are excited

Consistent phenomenology across many models



Excitation of damped eigenmodes

Using GENE as a linear eigenvalue solver to analyze nonlinear ITG runs via projection methods, one finds…



Energetics

Turbulent free energy consists of two parts:

$$\mathcal{E}_f = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2}, \qquad \mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}$$

Drive and damping terms:

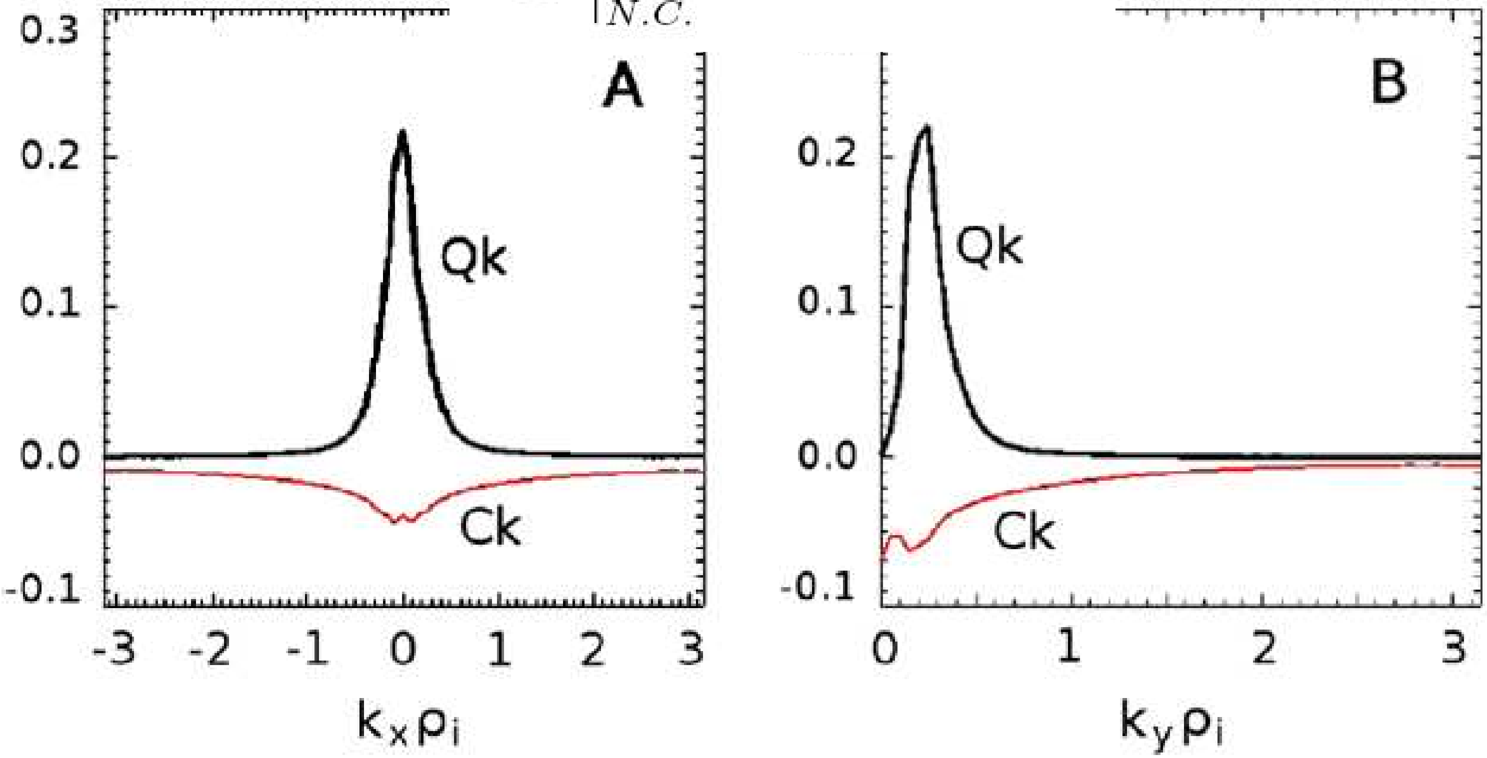
$$\frac{\partial \mathcal{E}}{\partial t} = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j \frac{\partial f_j}{\partial t} = \mathcal{G} - \mathcal{D} \qquad h_j = f_j + (q_j \bar{\phi}_1 / T_{0j}) F_{0j}$$

$$\mathcal{G} = -\sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j \cdot \left[\omega_n + \left(v_\parallel^2 + \mu B_0 - \frac{3}{2} \right) \omega_{Tj} \right] \times F_{0j} \frac{\partial \bar{\phi}_1}{\partial y}$$

$$\mathcal{D} = -\sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j (\mathcal{D}_z f_j + \mathcal{D}_{v_\parallel} f_j)$$

Energetics in wavenumber space

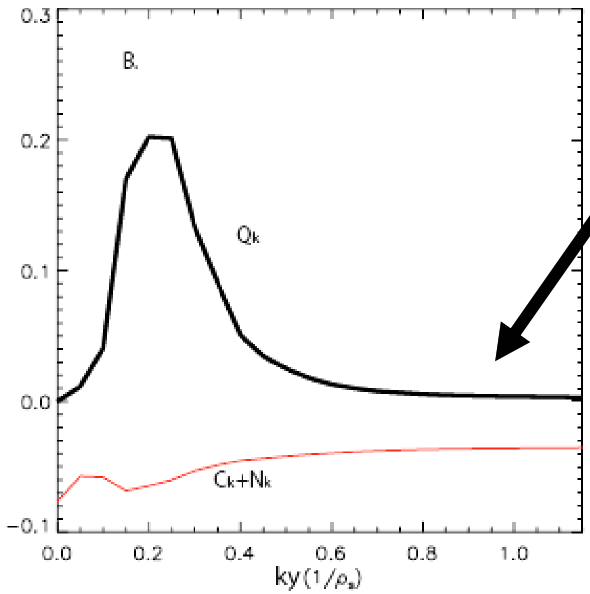
$$\left.\frac{\partial E_k}{\partial t}\right|_{N.C.} = Q_k + C_k + N_k$$



Damped eigenmodes are responsible for significant dissipation in the drive range (!)

Some energy escapes to high k

From finite amplitude dissipation rate diagnostic, high k dissipation is constant in *k*



Calculate spectrum of residual of energy that is transferred to high *k*

Use attenuation condition:

d/dk (transfer rate) = Energy dissipation rate

Do simple calculation for flow field

Dissipation rate = const. $E(k) = \alpha\, E(k)$

$$E(k) = \int dx\, v^2 e^{ikx}$$

Transfer rate = $T(k) = v_k^3 k$

Use closure of Terry and Tangri, PoP '09

Resulting spectrum decays exponentially @lo k, asymptotes to power law @hi k

Spectrum from k space attenuation of $T(k)$ by dissipation $\alpha E(k)$:

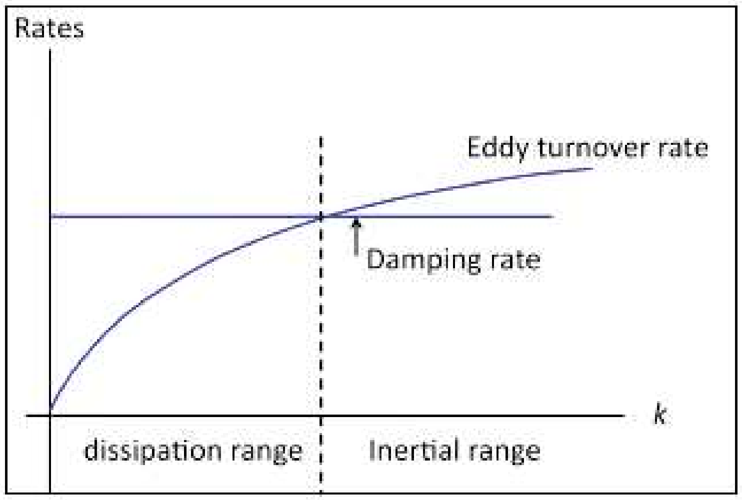
$$\frac{dT(k)}{dk} = \frac{d(v_k^3 k)}{dk} = aE(k)$$

Corrsin closure procedure: $v_k^3 k = v_k^2 \cdot v_k k = E(k)k \cdot \varepsilon^{1/3} k^{-1/3} k$

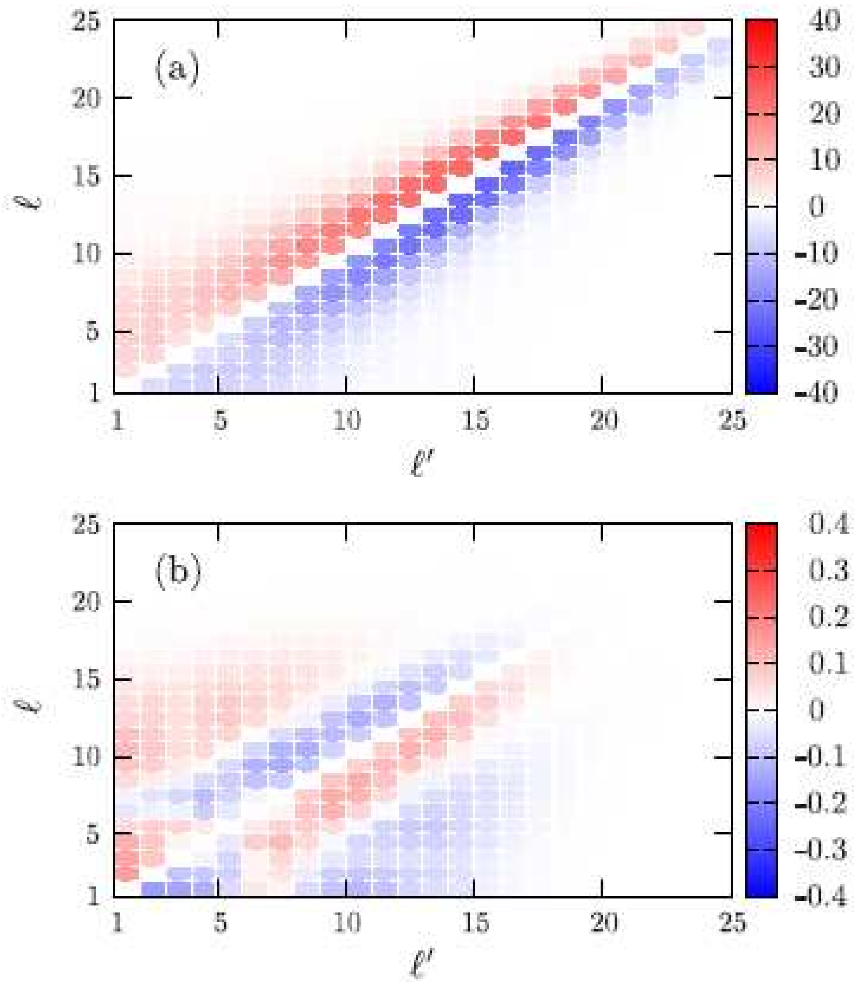
Solving attenuation ODE:

$$E(k) = \beta \varepsilon^{2/3} k^{-5/3} \exp\left[\frac{3}{2} \alpha \varepsilon^{-1/3} k^{-2/3}\right]$$

Spectrum becomes power law in range where eddy turnover rate exceeds constant dissipation rate



Shell-to-shell transfer of free energy



$$\mathcal{E}_f = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2},$$

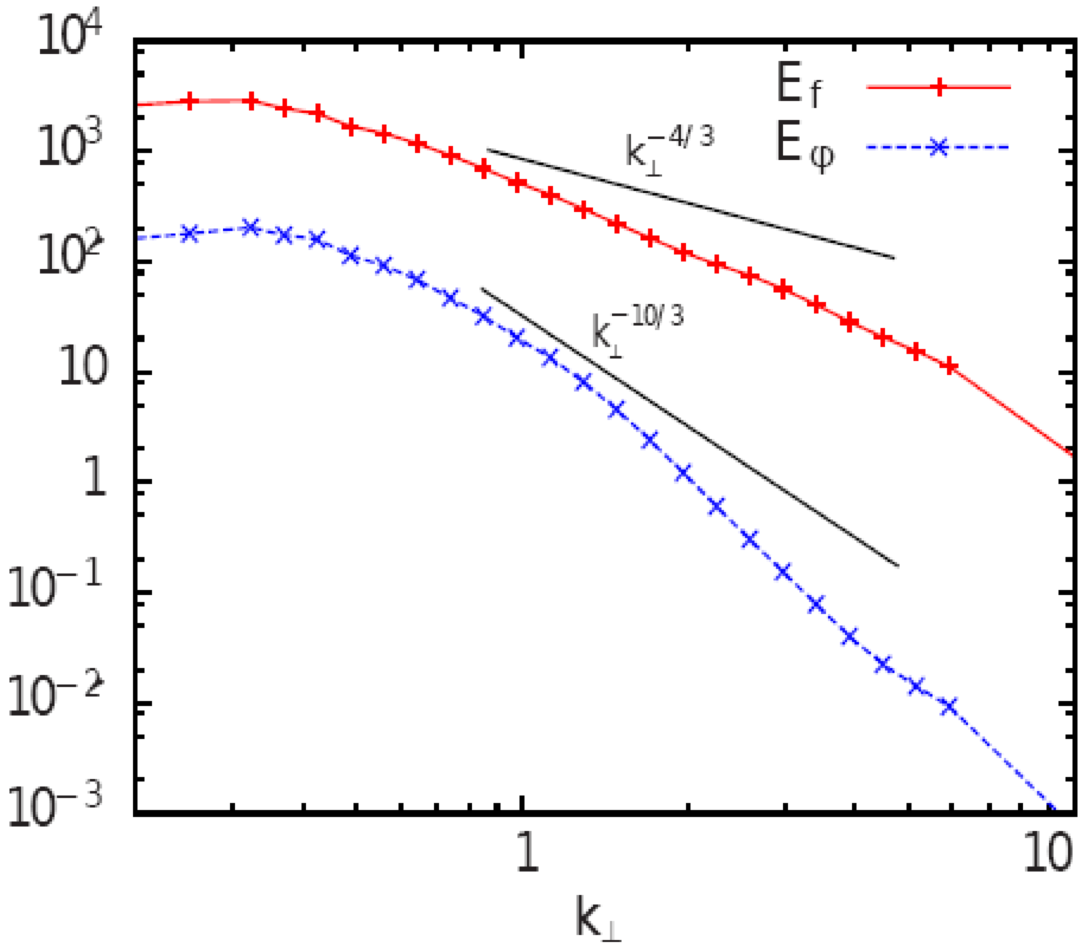
ITG turbulence (adiabatic electrons);
logarithmically spaced shells

Entropy contribution dominates;
exhibits very local, forward cascade

$$\mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}.$$

Banon Navarro *et al*., PRL 2011

Free energy wavenumber spectra



Asymptotic self-similarity coincides with power law spectra

Measured exponents are relatively close to those of a 2D GK scaling theory [Schekochihin *et al.*, 2009]

Banon Navarro *et al.*, PRL 2011

Application: Gyrokinetic LES models

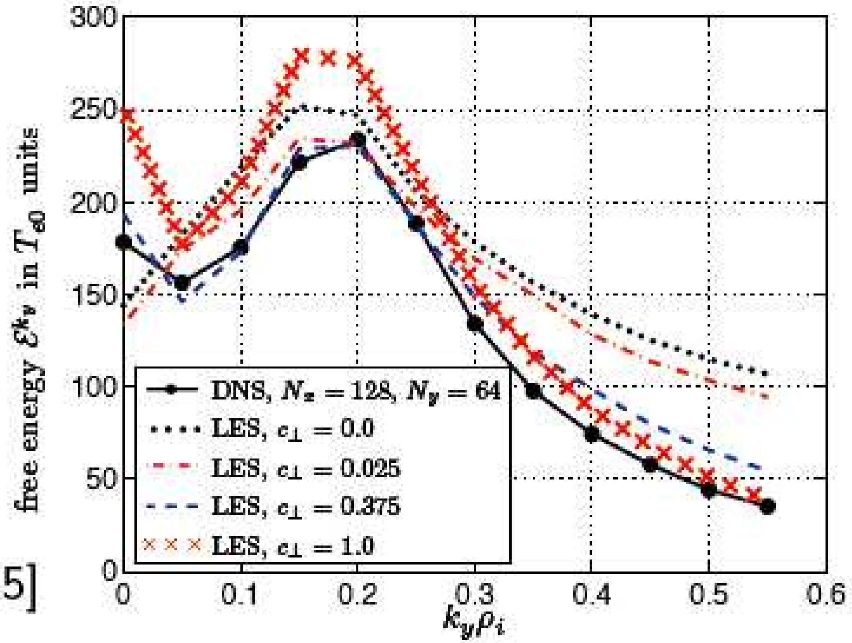
Model: $M[c_\perp, \overline{f}] = -c_\perp k_\perp^4 \overline{f}$

Unknown free parameter: $c_\perp$

Free energy spectra vs $c_\perp$:

Cyclone Base Case (ITG)

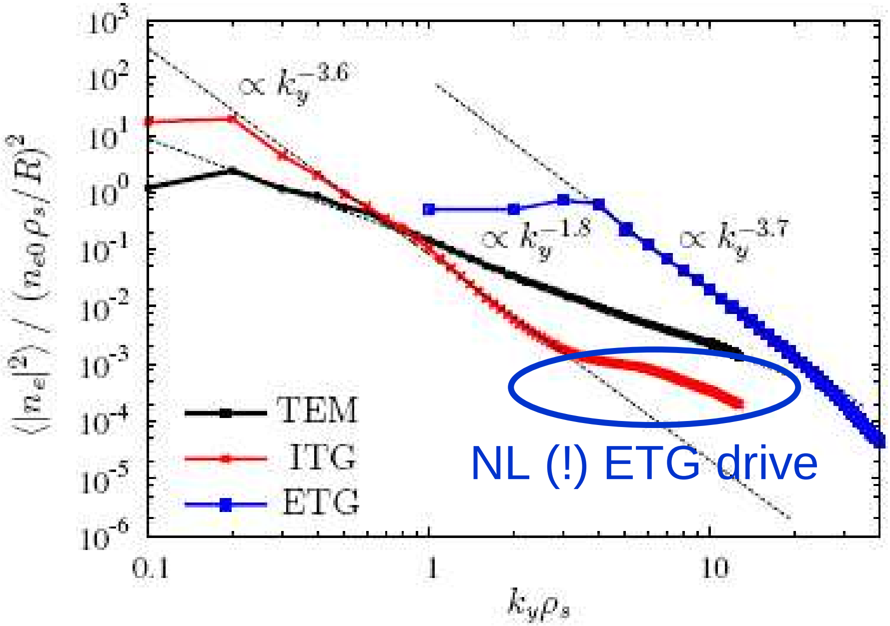
- ⋆ $c_\perp$ too small
  - $\Rightarrow$ not enough dissipation
- ⋆ $c_\perp$ too strong
  - $\Rightarrow$ overestimates injection
- ⋆ $c_\perp = 0.375$ good agreement
  - $\rightarrow$ "plateau" for $c_\perp \in [0.25, 0.625]$
  - $\rightarrow$ holds for k_x



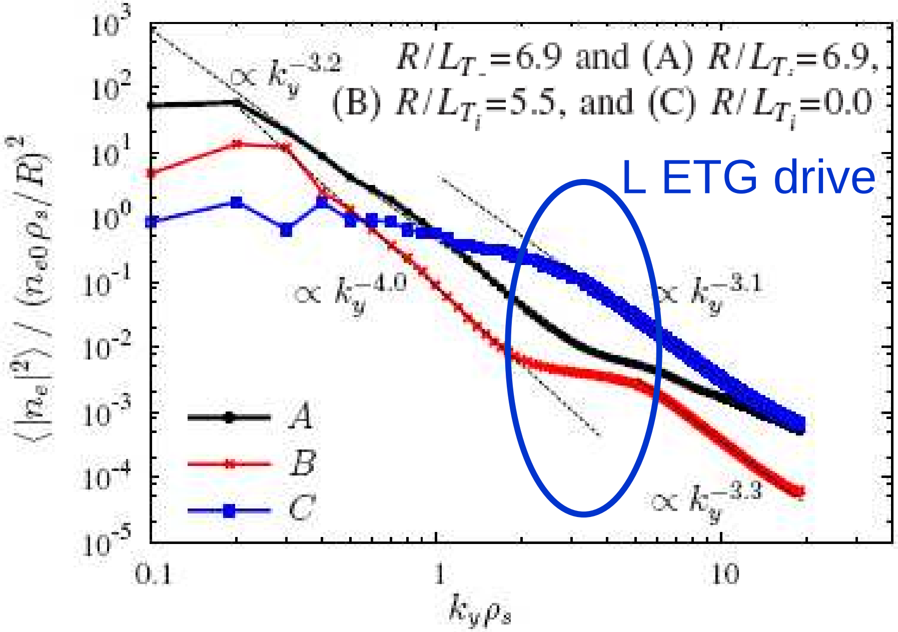
Morel *et al.*, submitted

Multiscale wavenumber spectra

Poloidal wavenumber spectra of density fluctuations for pure TEM / ITG / ETG turbulence



Poloidal wavenumber spectra of density fluctuations for mixed TEM / ITG – ETG turbulence



Universality?

Görler & Jenko, PoP **15**, 102508 (2008)

Summary and outlook

Some outroductory remarks

More info: http://gene.rzg.mpg.de

Goal of this second lecture:

Introduction to the physics of plasma microturbulence

Key insights:

Nonlinear saturation may have different faces

Drive and dissipation ranges overlap (damped modes!)

Question of universality in plasma microturbulence

Topic of next lecture:

On multi-scale aspects of plasma microturbulence