



The Abdus Salam  
International Centre for Theoretical Physics



2269-25

**Workshop on New Materials for Renewable Energy**

*17 - 21 October 2011*

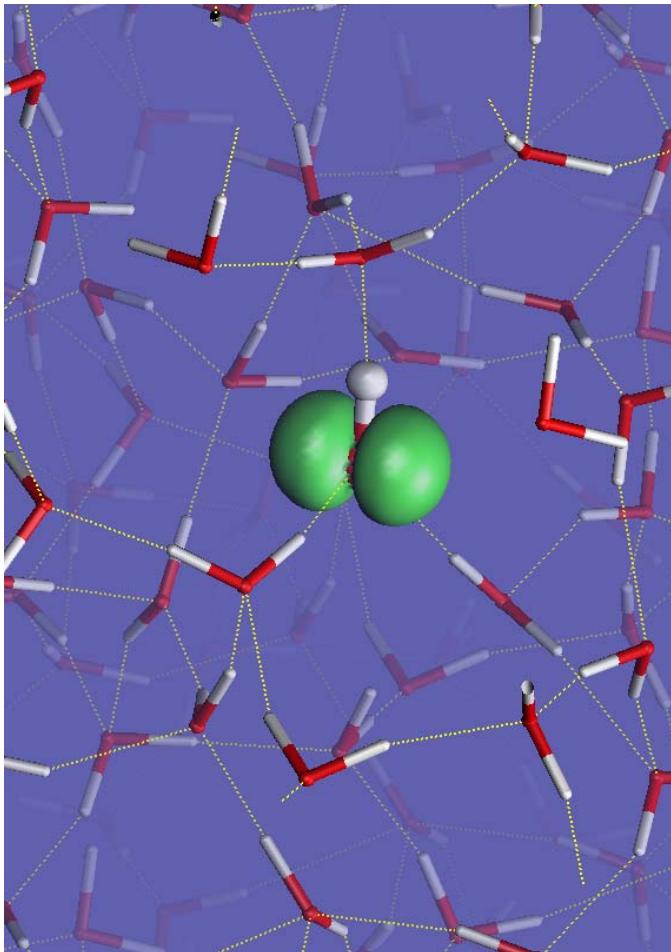
**All-atom density functional thoery of the O<sub>2</sub> Latimer series of dioxygen**

Michiel SPRIK

*Dept. of Chemistry, University of Cambridge  
U.K.*

# All-atom DFT of the O<sub>2</sub> Latimer series

*using a molecular dynamics normal hydrogen electrode*



**Collaboration:**

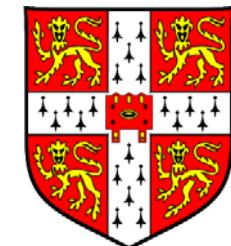
**Jun Cheng**

**Marialore Sulpizi**

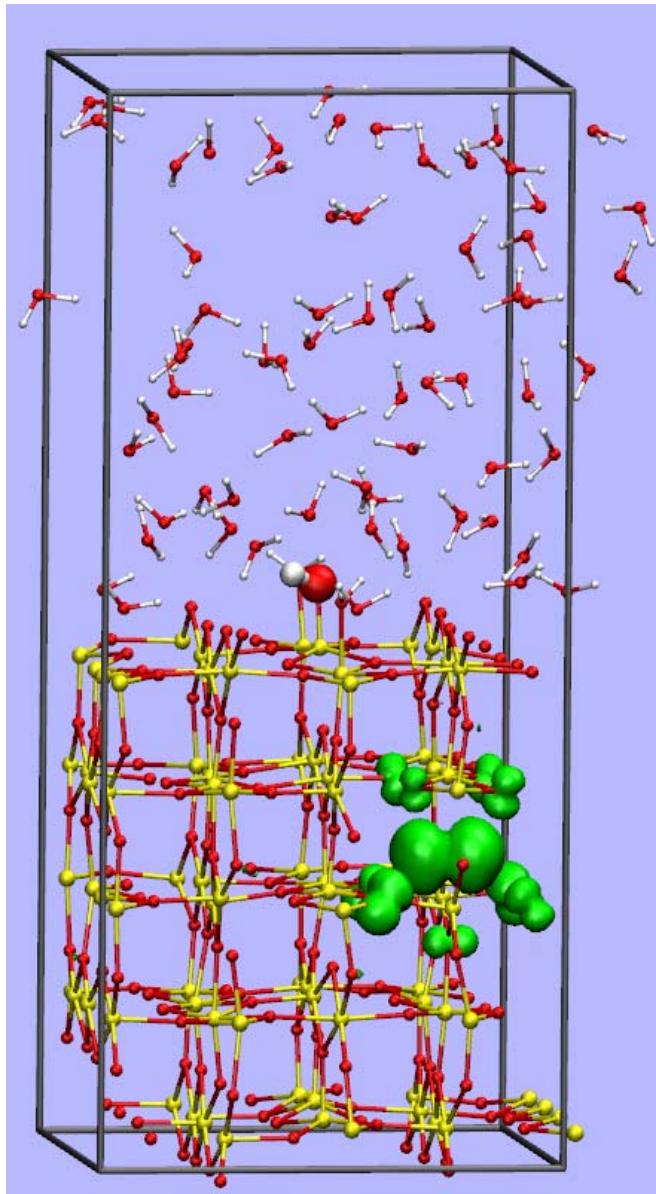
**Joost VandeVondele**

**Funding:** EPSRC, UKCP

Department of Chemistry  
University of Cambridge



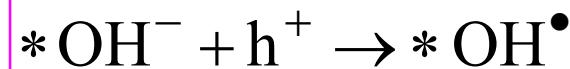
# Motivation: formation of a surface hydroxyl group



Oxidative dehydrogenation of water



*Hole trapping by surface hydroxide*

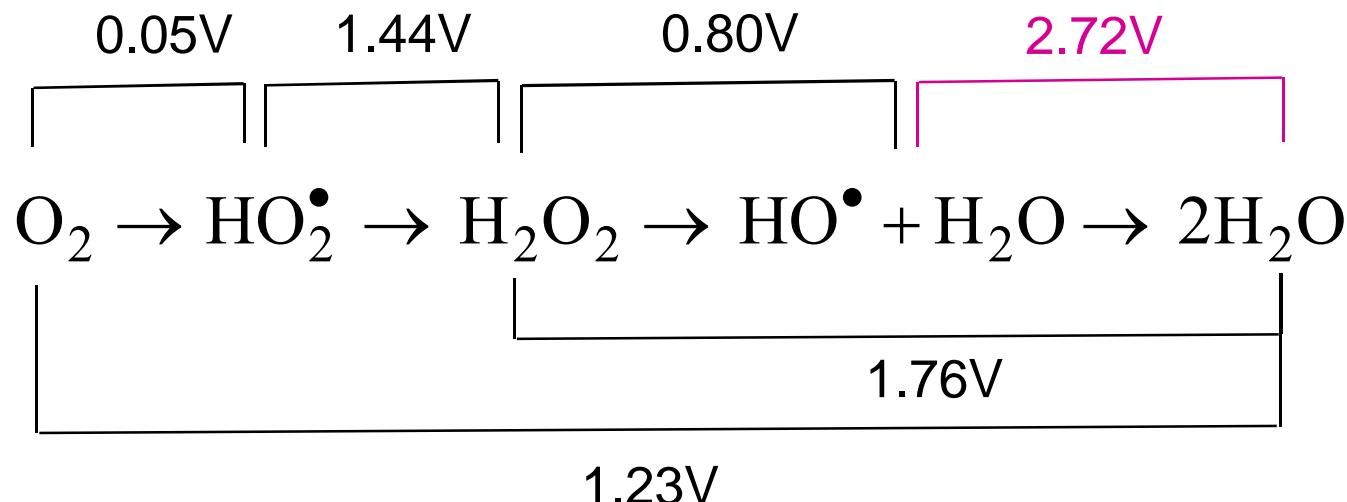


*Spin density in TiO<sub>2</sub>*

*(Jun Cheng)*

# Reduction $O_2 \leftrightarrow$ oxidation $H_2O$ in solution

Successive addition/removal of  $e^-$  and  $H^+$ .

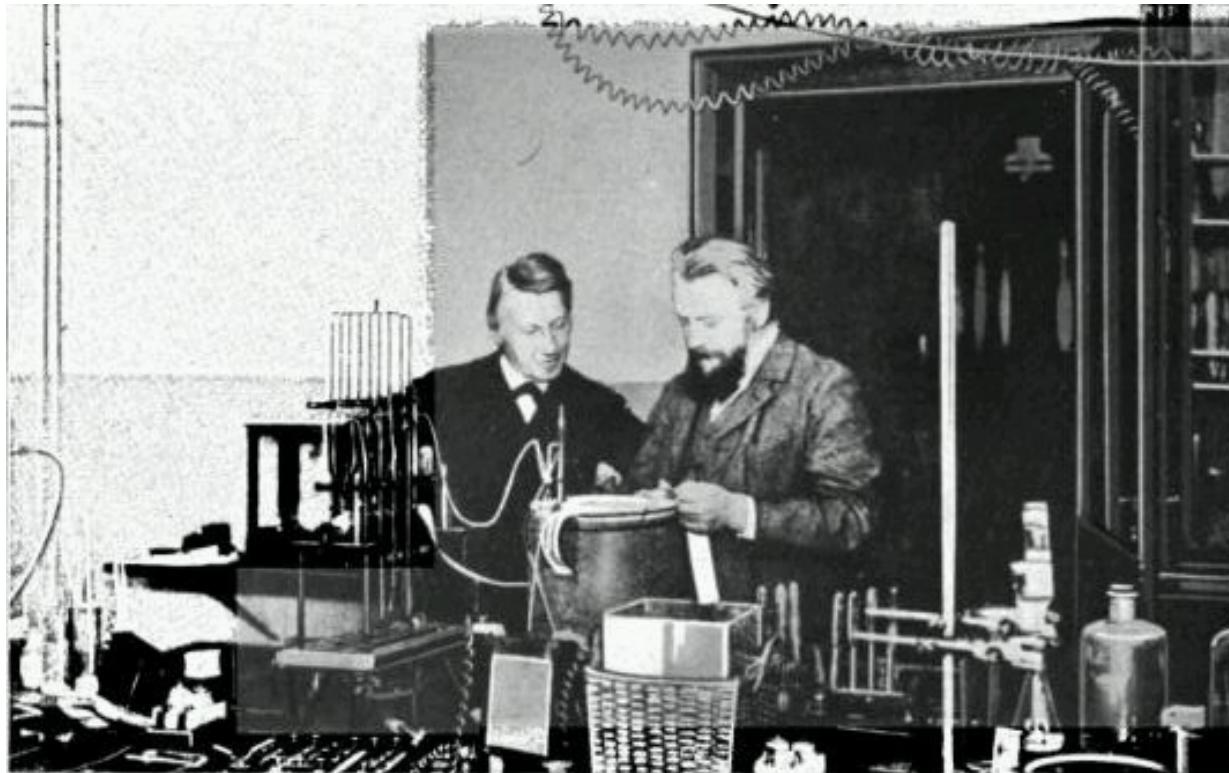


$$E^\circ = -\frac{\Delta G^\circ}{Fn}$$

*First H abstraction the most costly*

# Interfacial Electrochemistry:

*Is molecular simulation finally ready for  
19<sup>th</sup> century science?*



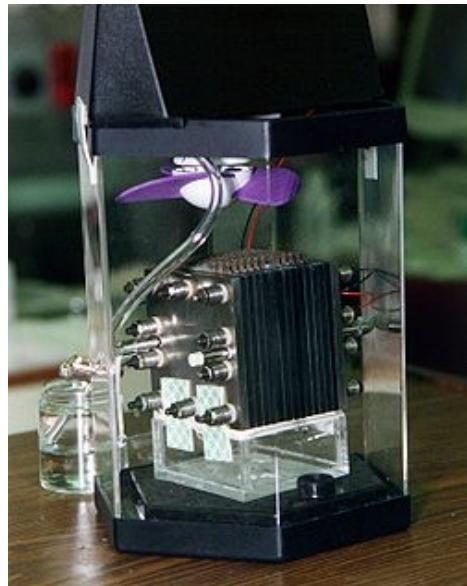
Ostwald and van't Hoff  
Founding fathers of  
Physical Chemistry

# Electrochemistry in the 19<sup>th</sup> century: highlights

Volta	1800	Volta pile, first electrochemical device
Nicholson and Carlisle	1800	Electrolysis of water
Von Grotthuss	1805	Mechanism of conductance
Faraday	1834	Laws of electrolysis
Daniell	1836	Chemical cells
Grove	1839	Fuel Cells
Hittorf	1853	Migration of ions
Clausius	1857	Existence of free ions
Kohlrausch	1869	Conductance-concentration relationships
Helmholtz	1871	Fixed double layer
Lippmann	1873	Electrocapillary effect
Planté	1879	Storage battery
Ostwald	1883	Definition of electrode potential
Van 't Hoff	1887	Osmotic pressure of electrolyte solutions
Arrhenius	1888	Electrolytic dissociation
Tafel	1905	Overvoltage

# What is Electrochemistry?

*Chemical reactions generating electricity  
and electricity driving chemical reactions*

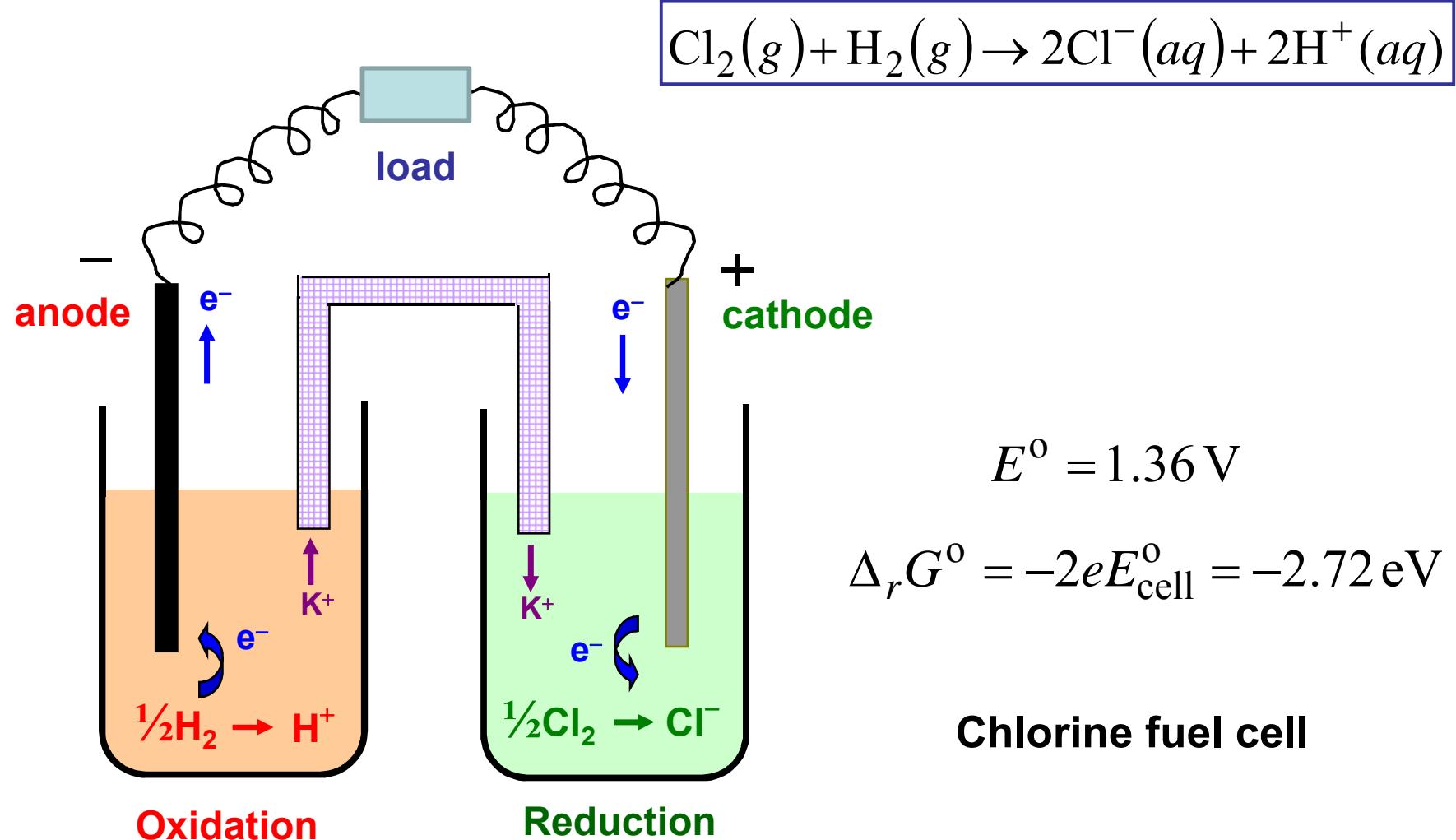


Direct methanol fuel cell

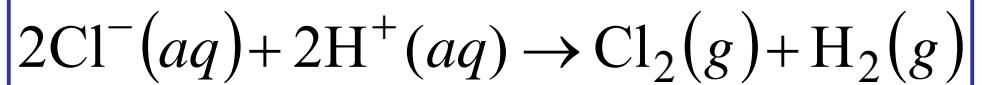
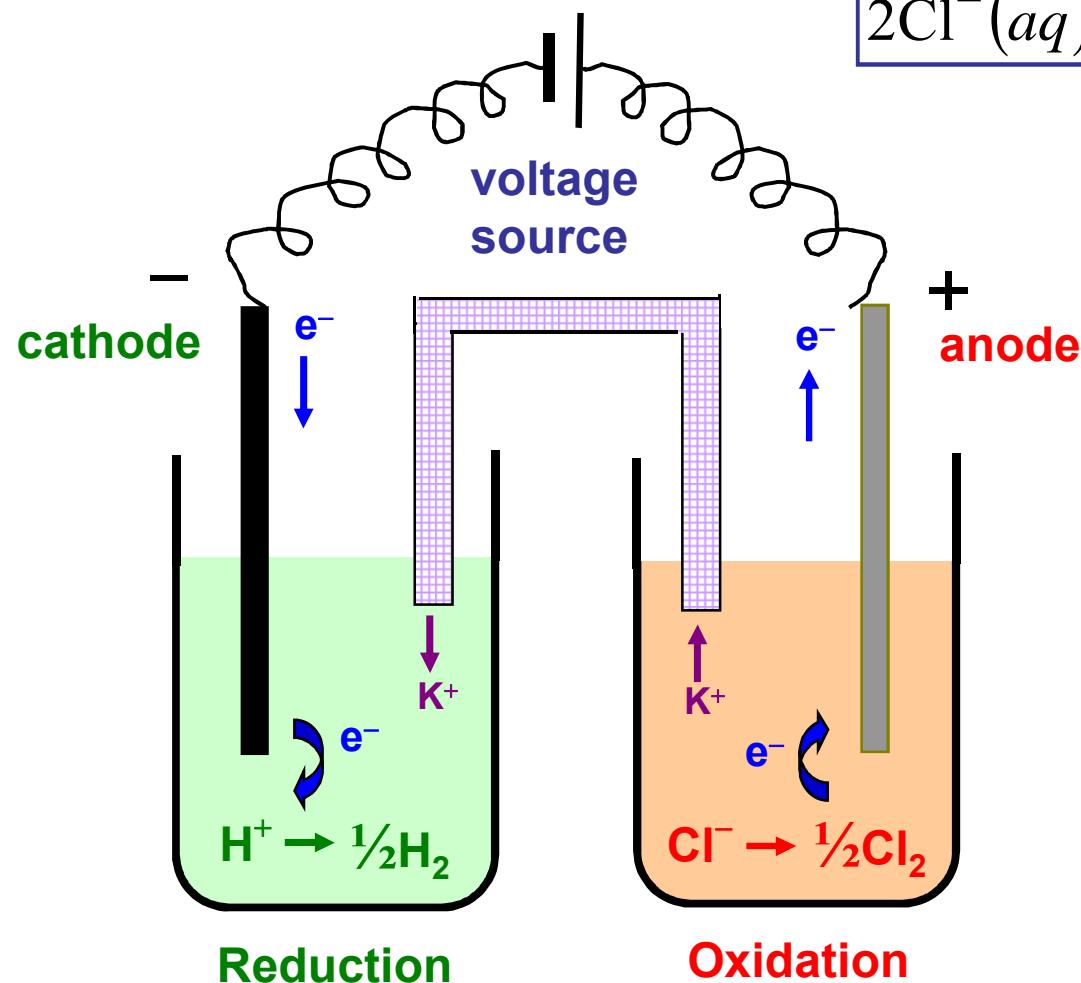


Industrial electrolysis of aluminium oxide

# Chemical reactions generating electricity



# Electricity driving chemical reactions



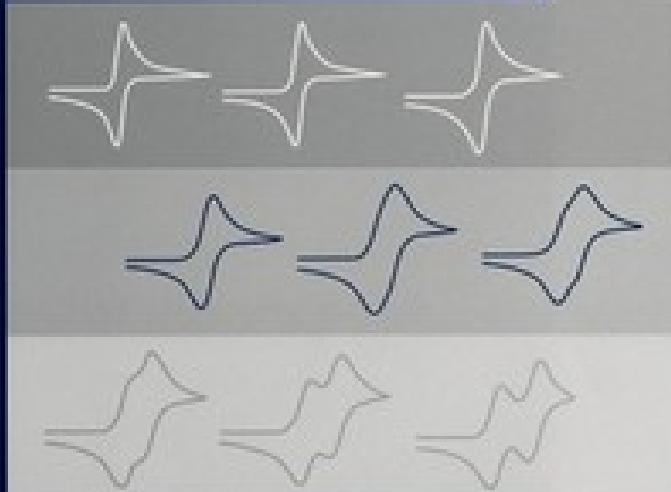
$$E^0 = 1.36 \text{ V}$$

$$\Delta_r G^0 = 2eE_{\text{cell}}^0 = 2.72 \text{ eV}$$

**Chloride oxidation**

ELECTROCHEMICAL  
METHODS  
Fundamentals  
and  
Applications

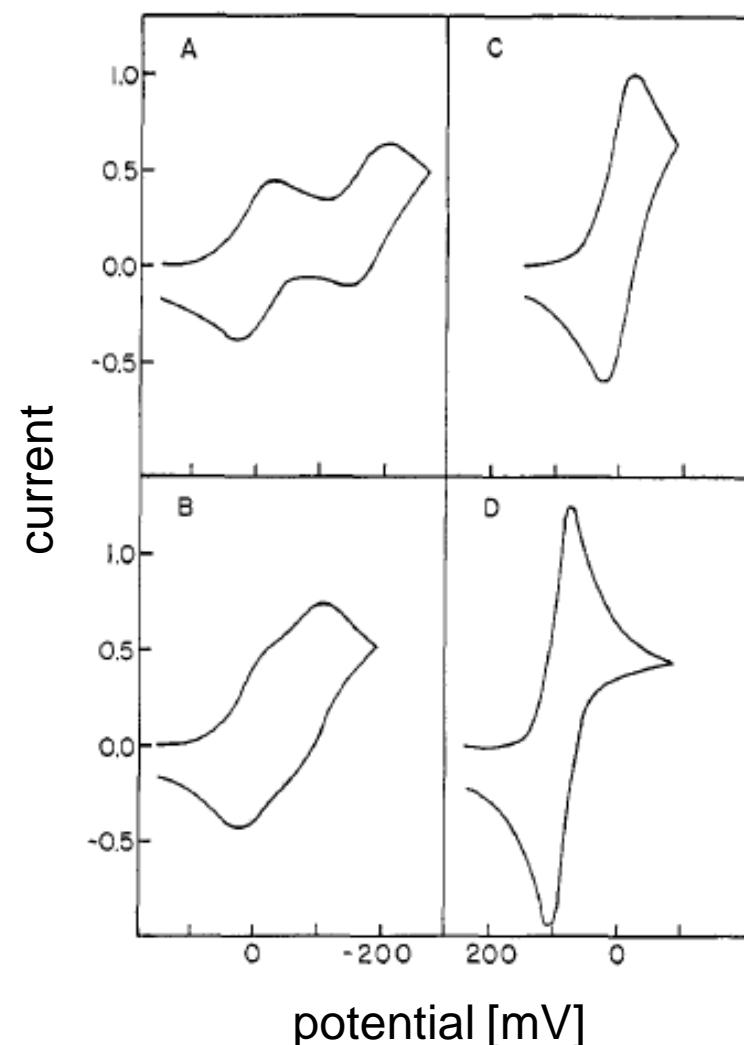
SECOND EDITION



ALLEN J. BARD

LARRY R. FAULKNER

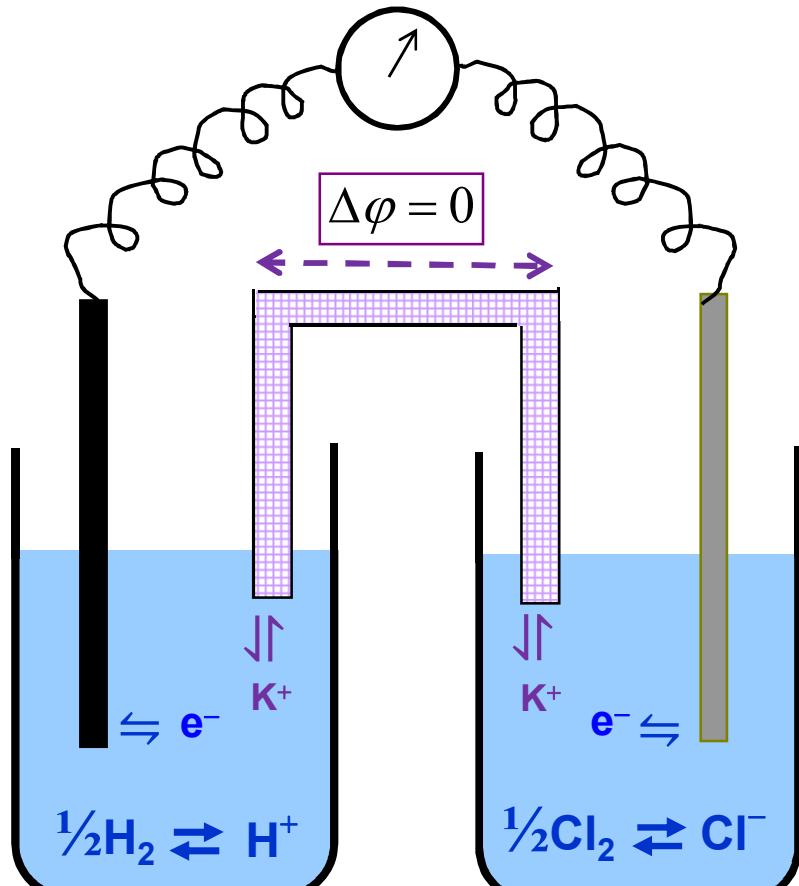
# Cyclic Voltammetry



***Not ready for this yet***

# Open circuit voltage of redox electrodes

Electrode reactions in equilibrium



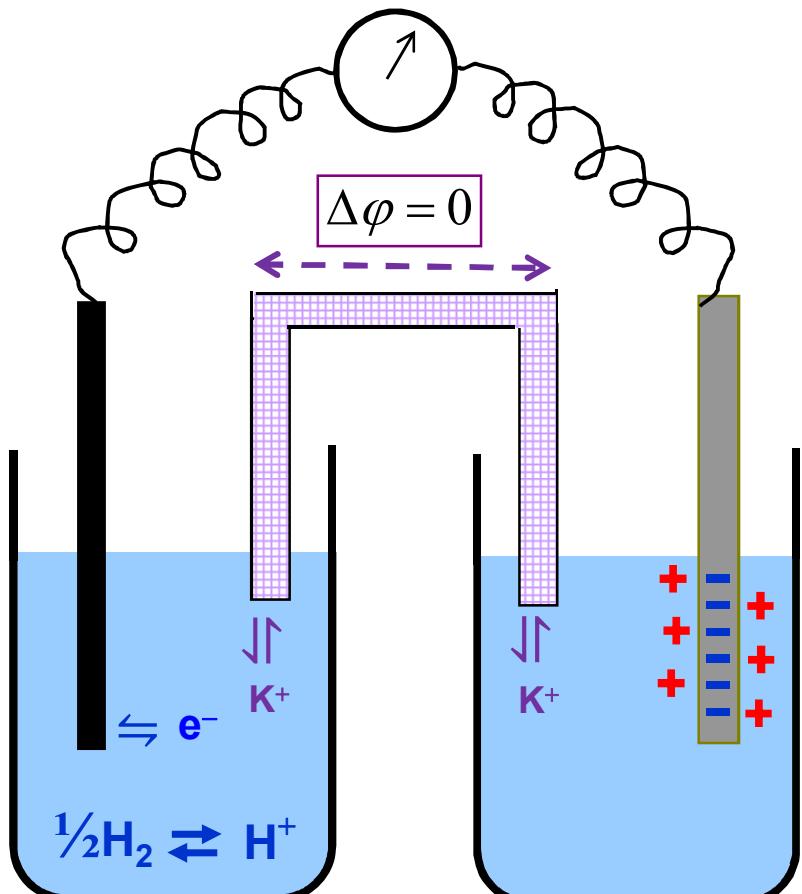
No current

$$E = -\frac{\Delta_r G}{2e}$$

$$E = E^0 + \frac{k_B T}{2e} \ln \frac{p_{\text{H}_2} p_{\text{Cl}_2}}{[\text{H}^+]^2 [\text{Cl}^-]^2}$$

Nature of electrodes irrelevant

# No control by a redox process



Redox reference

Double layer

Electrode acts as a capacitor

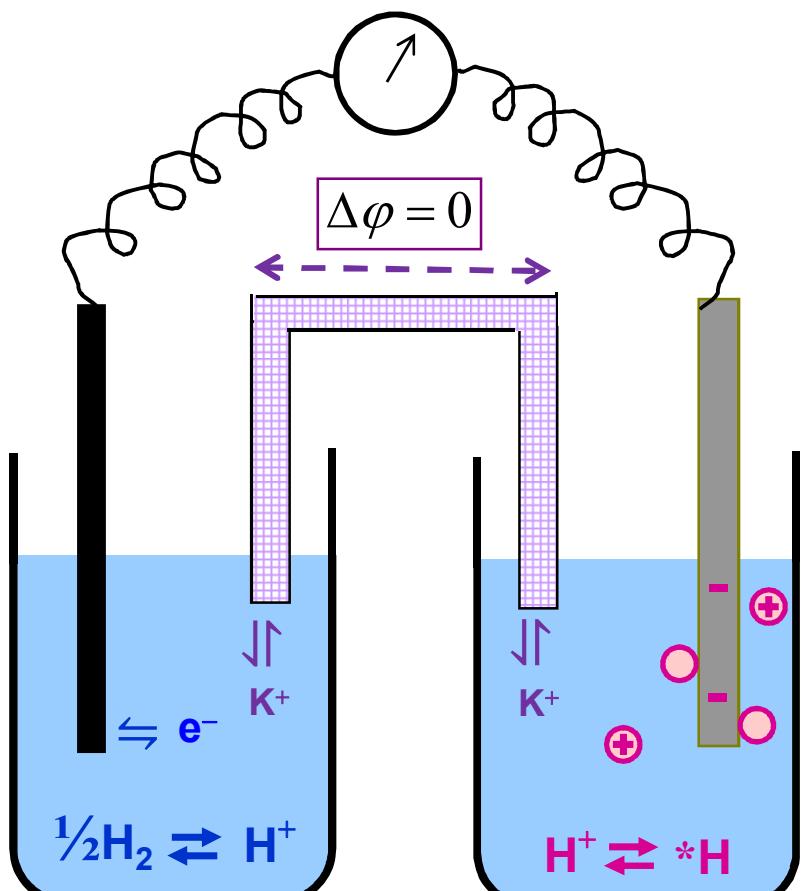
$$E = \varphi_{\text{pzc}} + \frac{Q}{C(Q)}$$

Nature of electrode matters.

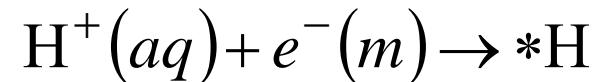
$$\varphi_{\text{pzc}} \propto \epsilon_{\text{fermi}}$$

Point of zero charge for metals

# Redox active species adsorbed on surface



Example: Volmer reaction



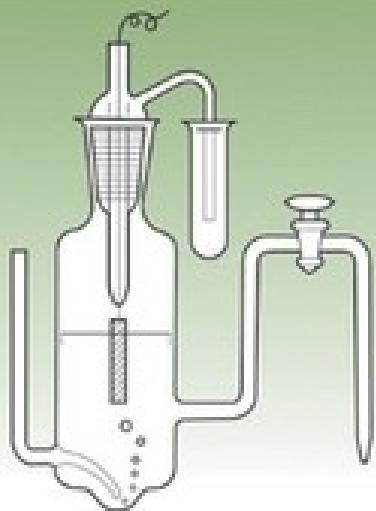
Step in Hydrogen Evolution Reaction



The Electrochemical Society Series

# FUNDAMENTALS OF ELECTROCHEMISTRY

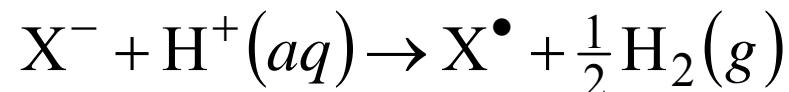
SECOND EDITION



V. S. BAGOTSKY

## We need a common energy reference

Normal hydrogen electrode (NHE)



Oxidation free energy of  $X^-$  by  $H^+$

$$eE_X^0 = \Delta_{ox}G_{X^-}^0$$

**How to construct a molecular dynamics normal hydrogen electrode?**

**NHE half cell**

# Priorities in computational electrochemistry

## *Development of*

- A computational normal hydrogen electrode.
- Methods for constant potential simulation.

## *Some of the leading groups*

Norskov, Rossmeisl (Stanford, Lyngby)

Neurock (Virginia)

Sugino, Morikawa (Tokyo, Osaka)

Anderson (Cleveland)

Gross (Ulm)

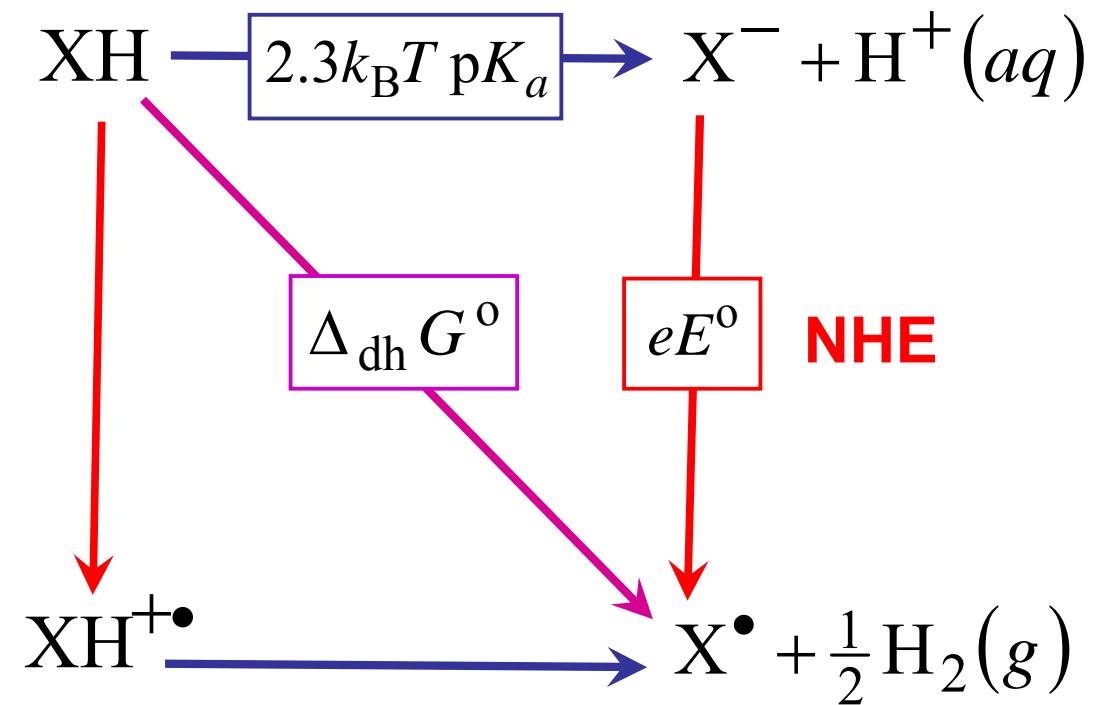
Marzari (EPFL)

Liu (Fudan)

**All of them starting from a computational surface science perspective**

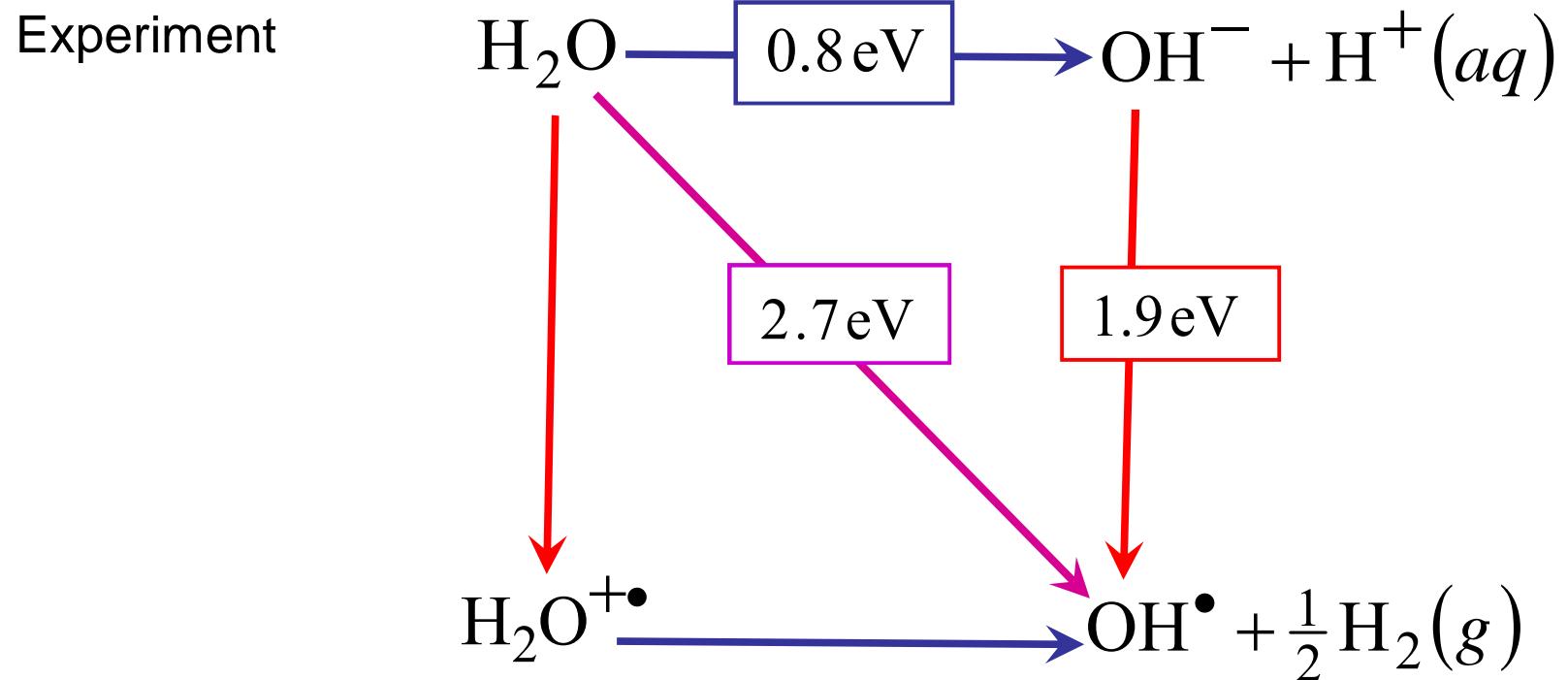
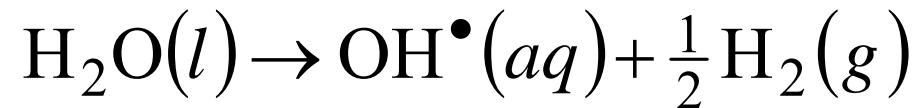
# Our starting point: Solution chemistry

Proton coupled oxidation (PCET)



$$\Delta_{\text{dh}} G^{\circ} = 2.3k_{\text{B}}T \text{ p}K_a + eE^{\circ}$$

# Abstracting the first hydrogen from water



# Density functional molecular dynamics method

## *Credo*

***Treat solute and solvent at the same level of density functional theory***

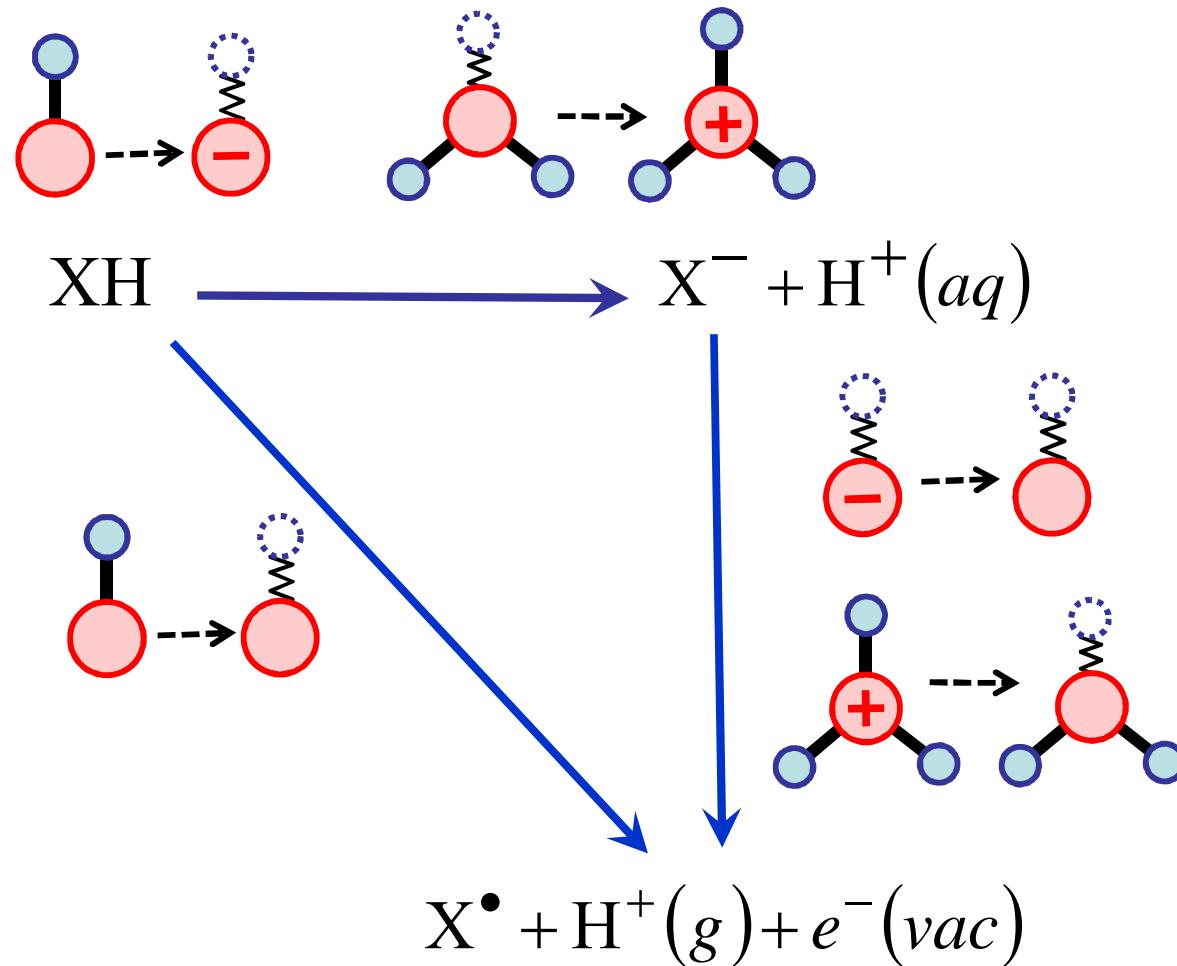
## **Code**

**CP2K** ([cp2k.berlios.de](http://cp2k.berlios.de))

- Gaussian basis set (+ plane waves for density)
- GGA and now also Hybrid functionals
- Born-Oppenheimer dynamics
- Free energy calculation

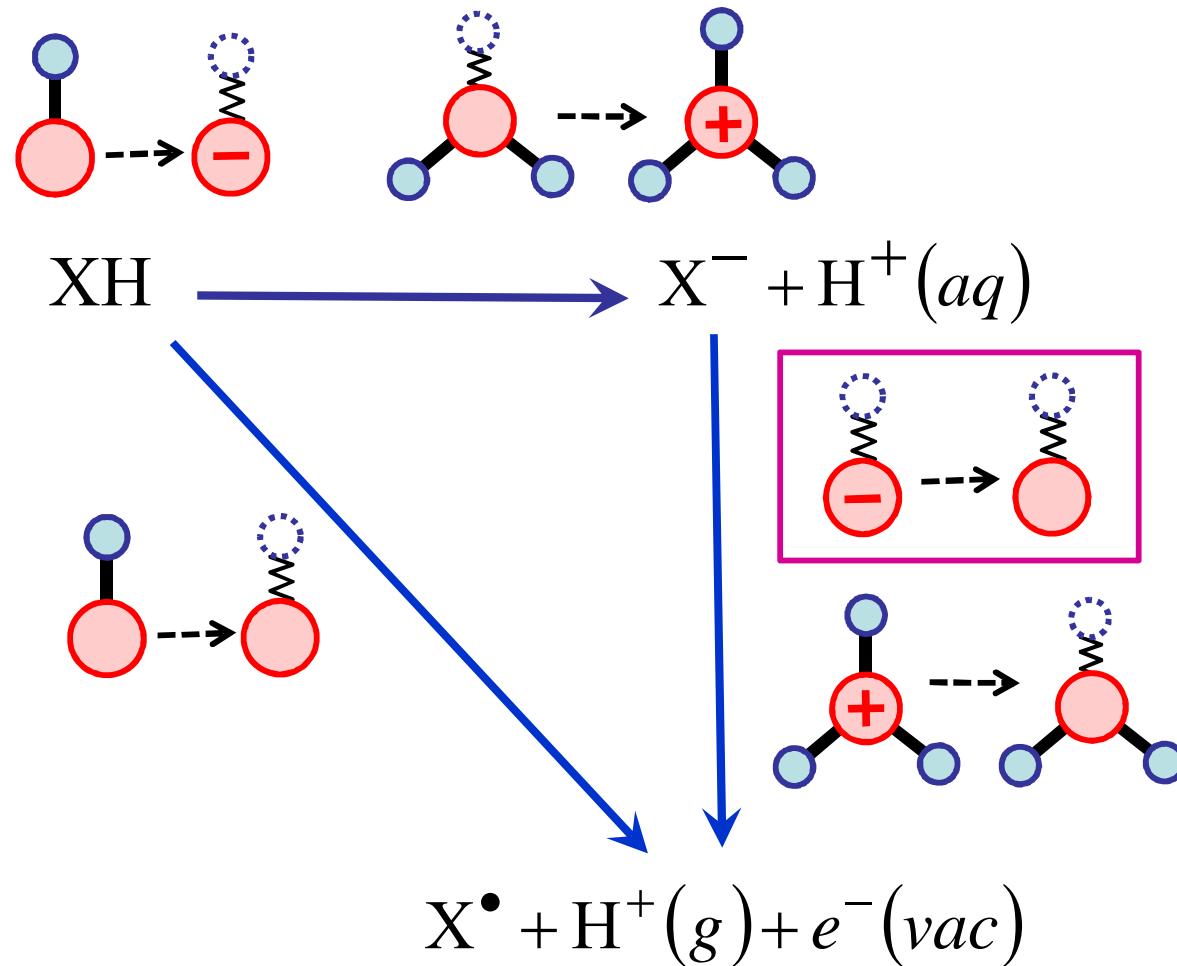
*J. VandeVondele, M. Krack, F. Mohamed, M. Parrinello, T. Chassaing, J. Hutter,  
Comp. Phys. Comm. (2005), 167, 103.*

# Reversible discharge of proton + electron



Free energy perturbation plus analytical thermochemical corrections

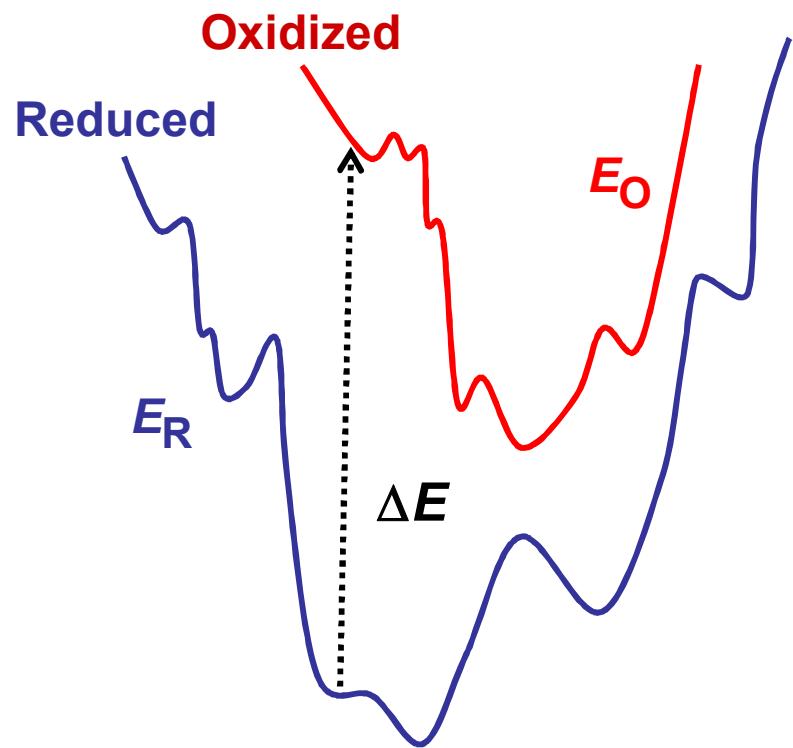
# Dehydrogenation by double reversible discharge



Free energy perturbation plus analytical thermochemical corrections

# Vertical ionization energy

Atomic position fixed



$$\Delta E(\mathbf{R}^N) = E_O(\mathbf{R}^N) - E_R(\mathbf{R}^N)$$

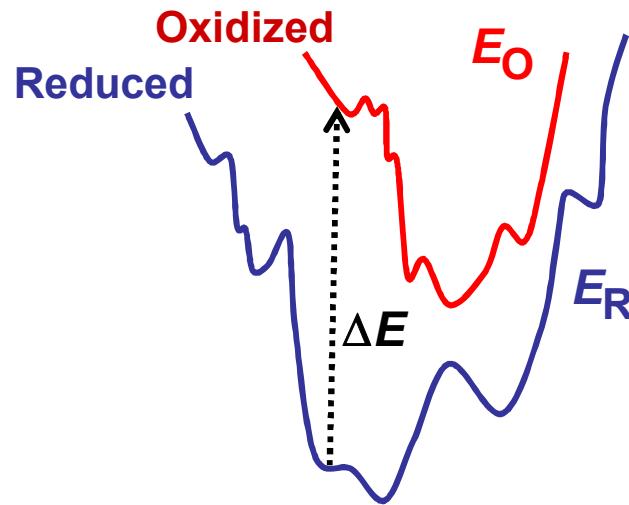
$E_R(\mathbf{R}^N)$  : PES **reduced**

$E_O(\mathbf{R}^N)$  : PES **oxidized**

However we also need the  
adiabatic ionization potential

*N* atom configurational coordinate  $\mathbf{R}^N$

# Free energy perturbation method



Two potential energy surfaces

$E_R(\mathbf{R}^N)$  : PES **reduced**

$E_O(\mathbf{R}^N)$  : PES **oxidized**

**Vertical energy gap**

$$\Delta E(\mathbf{R}^N) = E_O(\mathbf{R}^N) - E_R(\mathbf{R}^N)$$

**Mapping potential**

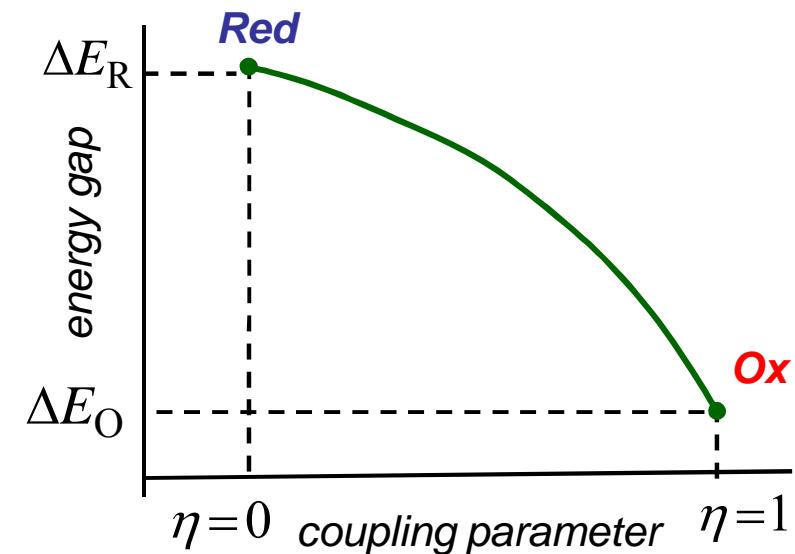
$$E_\eta(\mathbf{R}^N) = \eta E_O(\mathbf{R}^N) + (1-\eta)E_R(\mathbf{R}^N)$$

Coupling parameter derivative

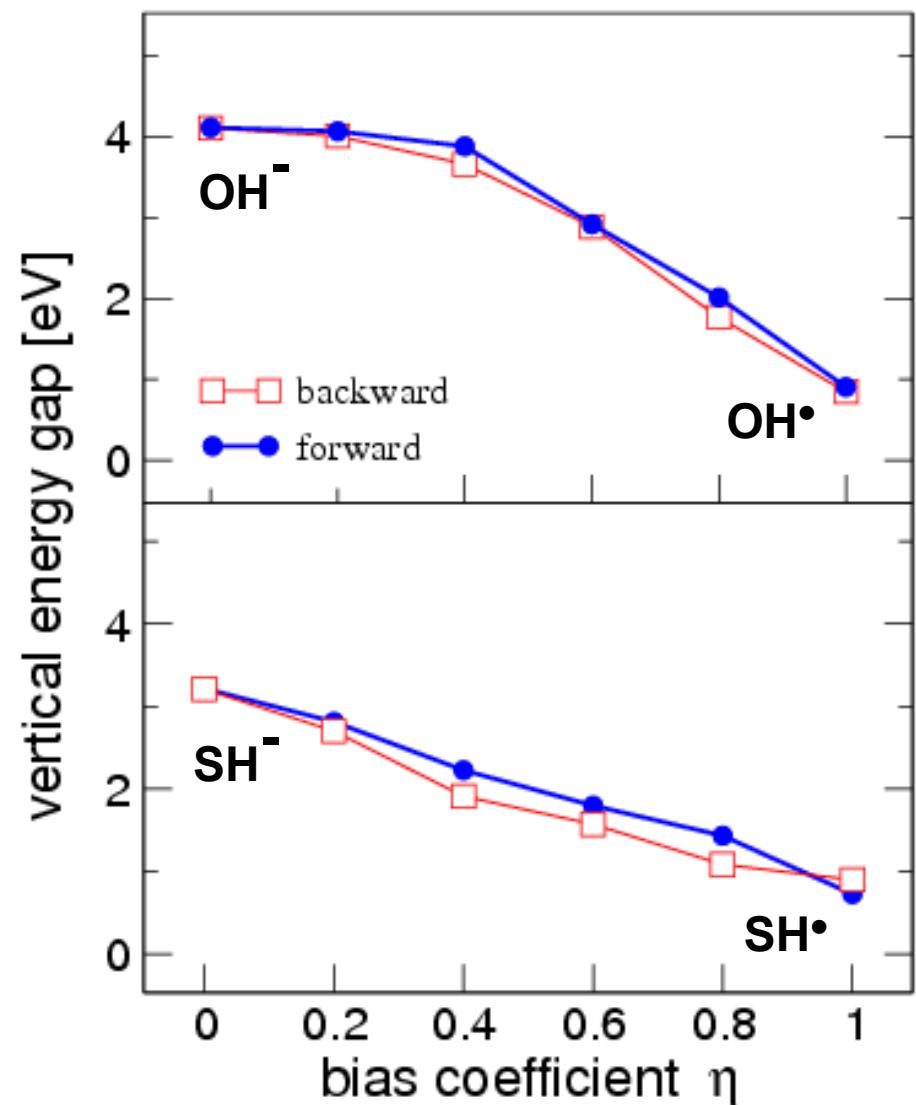
$$\frac{\partial}{\partial \eta} E_\eta(\mathbf{R}^N) = \Delta E(\mathbf{R}^N)$$

Oxidation free energy

$$\Delta A = A_O - A_R = \int_0^1 d\eta \langle \Delta E(\mathbf{R}^N) \rangle_\eta$$



# Example: two aqueous radicals



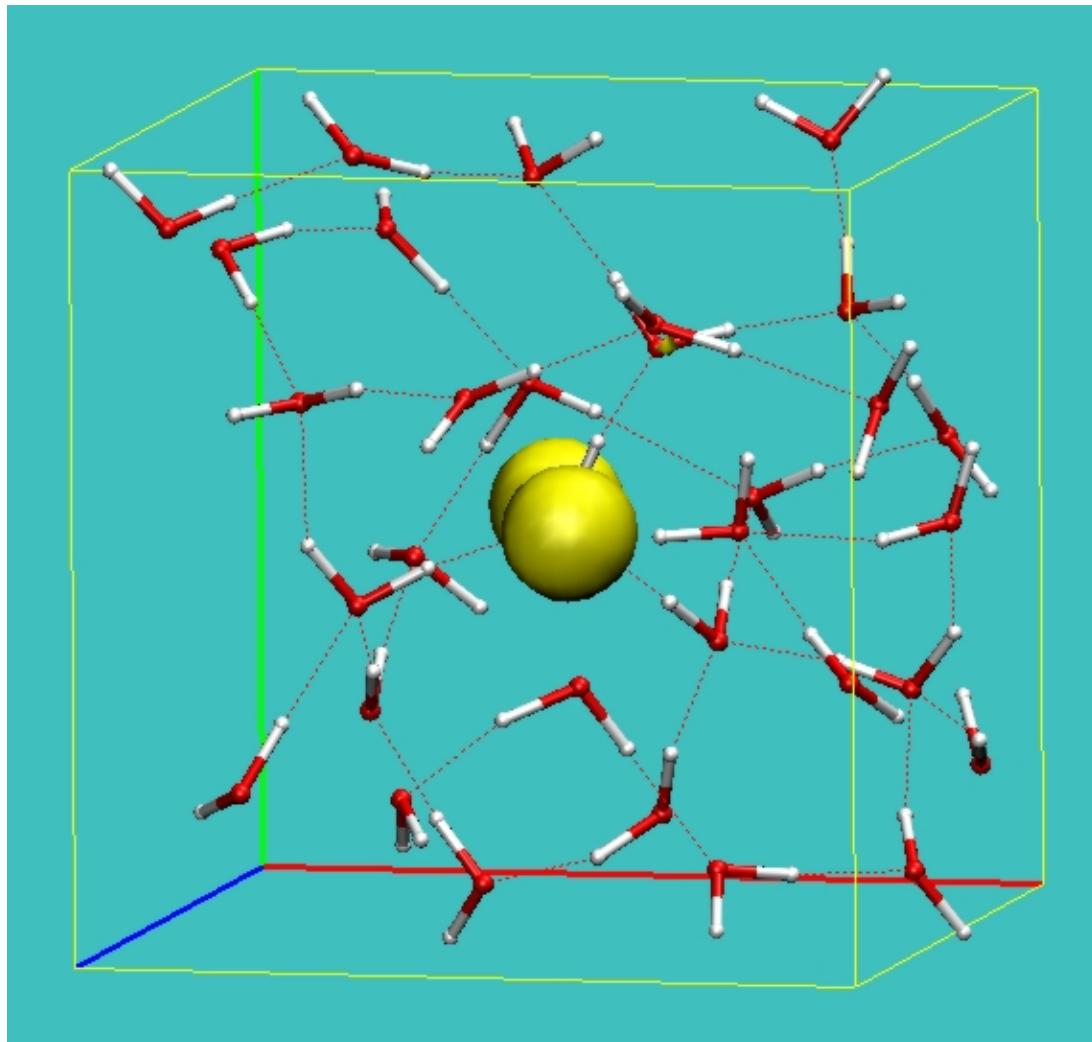
Non-linear solvent response:



Linear solvent response

Run length  $\approx 10$  ps per data point

# DFTMD model system for homogeneous OH<sup>•</sup>/H<sub>2</sub>O



## DFTMD Model

1 OH<sup>•</sup> radical

31 H<sub>2</sub>O

In cubic box of

L = 9.86 Å

(periodic boundaries)

10-50 ps MD runs

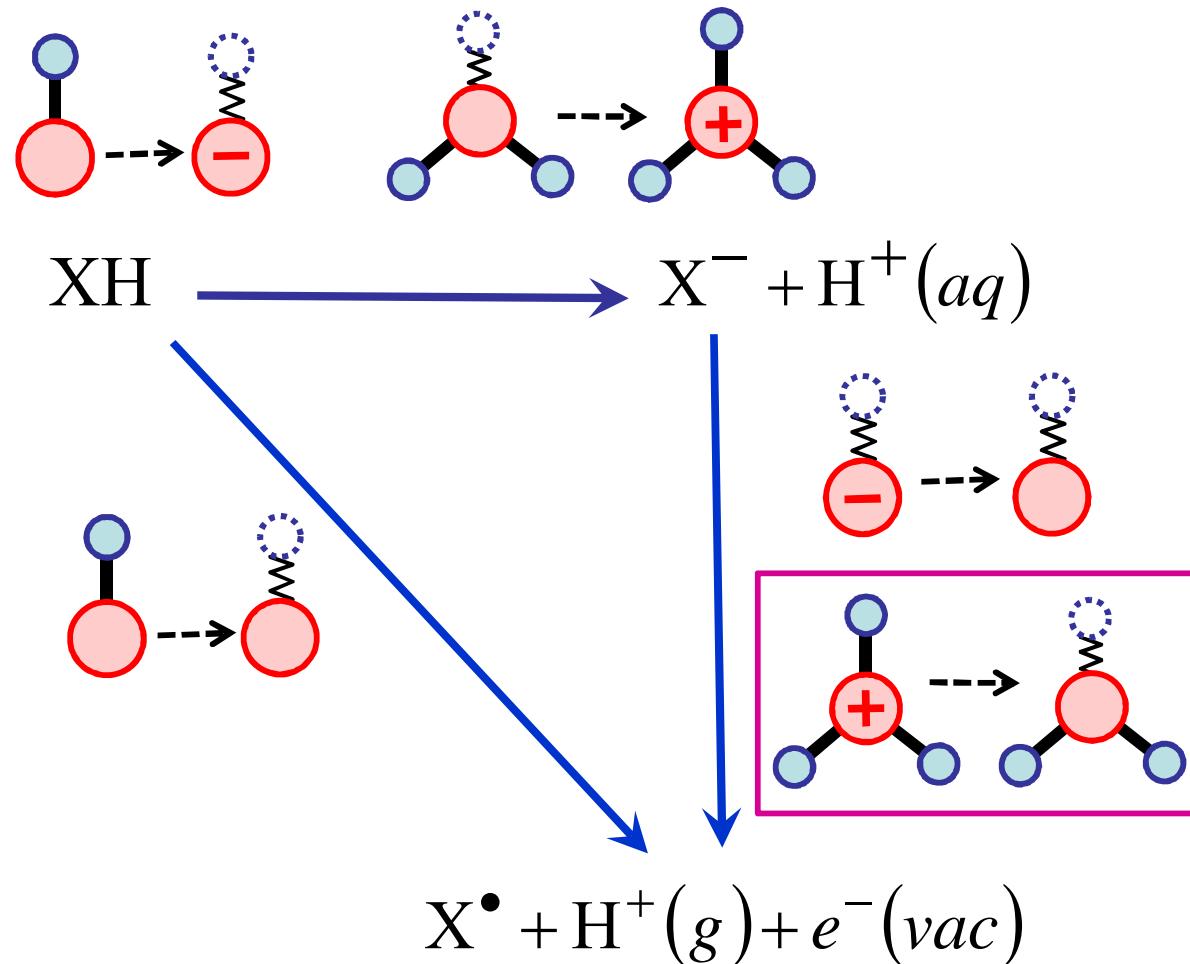
## Electronic structure

TZV2P basis,

280 Ry (density cutoff)

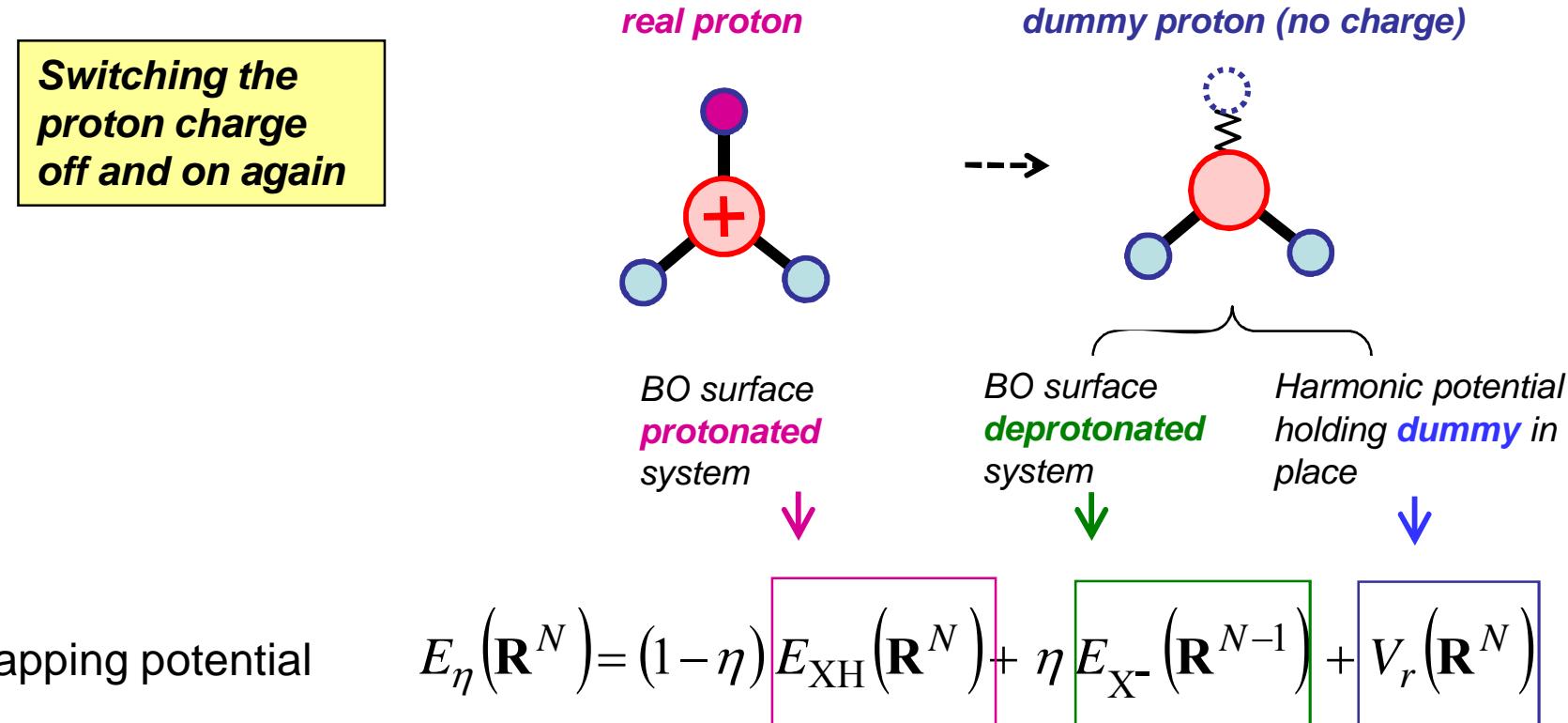
**No explicit counterion**

# Dehydrogenation by double reversible discharge



Free energy perturbation plus analytical thermochemical corrections

# Reversible deprotonation



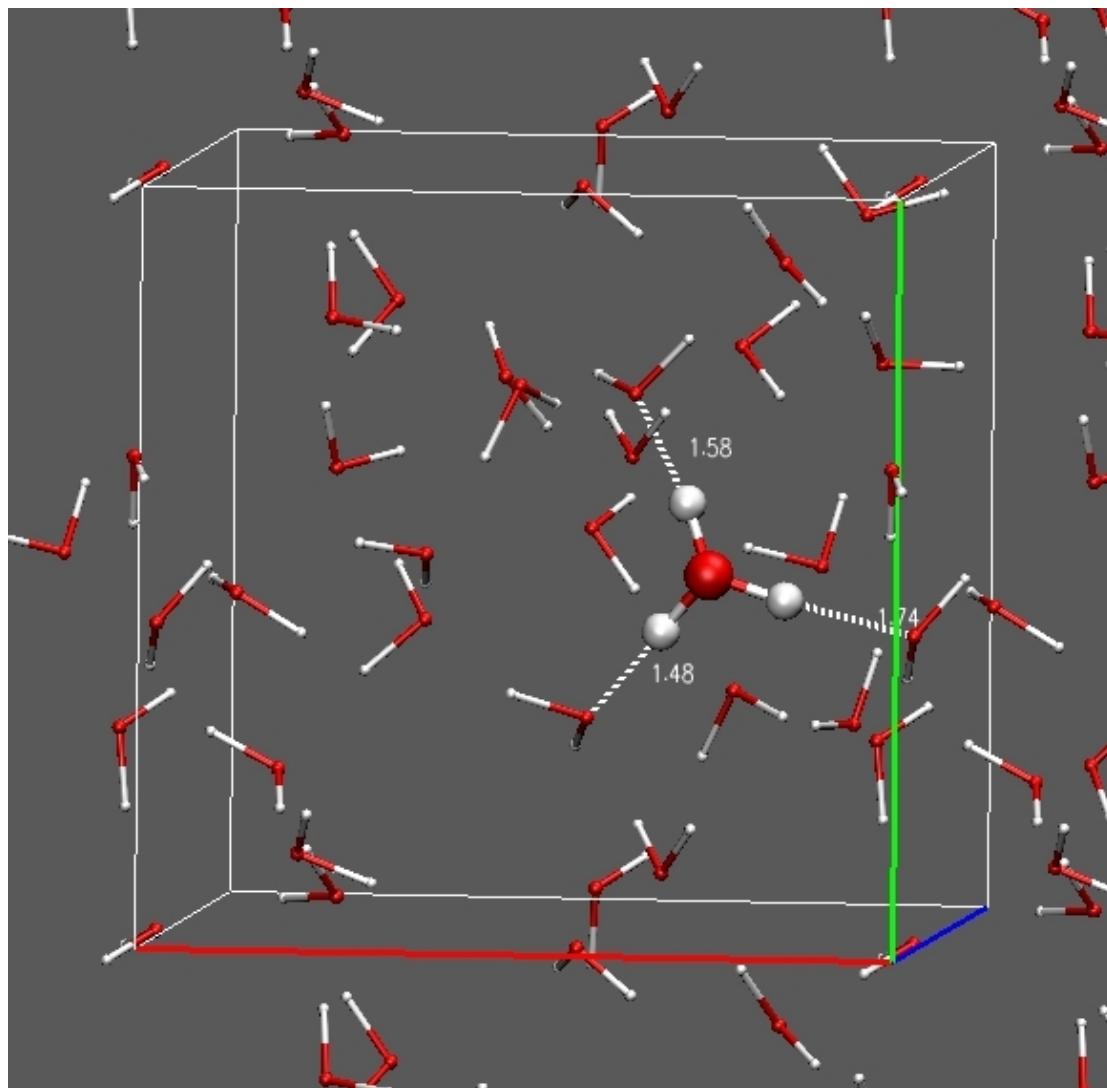
Vertical proton gap:

$$\frac{\partial}{\partial \eta} E_\eta(\mathbf{R}^N) = E_{\text{X}^-}(\mathbf{R}^{N-1}) - E_{\text{XH}}(\mathbf{R}^N) = \Delta E(\mathbf{R}^N)$$

**Restrained**  
deprotonation  
free energy

$$\Delta A = A_{\text{X}^-} - A_{\text{XH}} = \int_0^1 d\eta \langle \Delta E \rangle_{r\eta}$$

# $\text{H}_3\text{O}^+/\text{H}_2\text{O}$ model system



DFTMD Model

$32 \text{ H}_2\text{O} \pm \text{H}^+$

In cubic box of

$L = 9.86 \text{ \AA}$

(periodic boundaries)

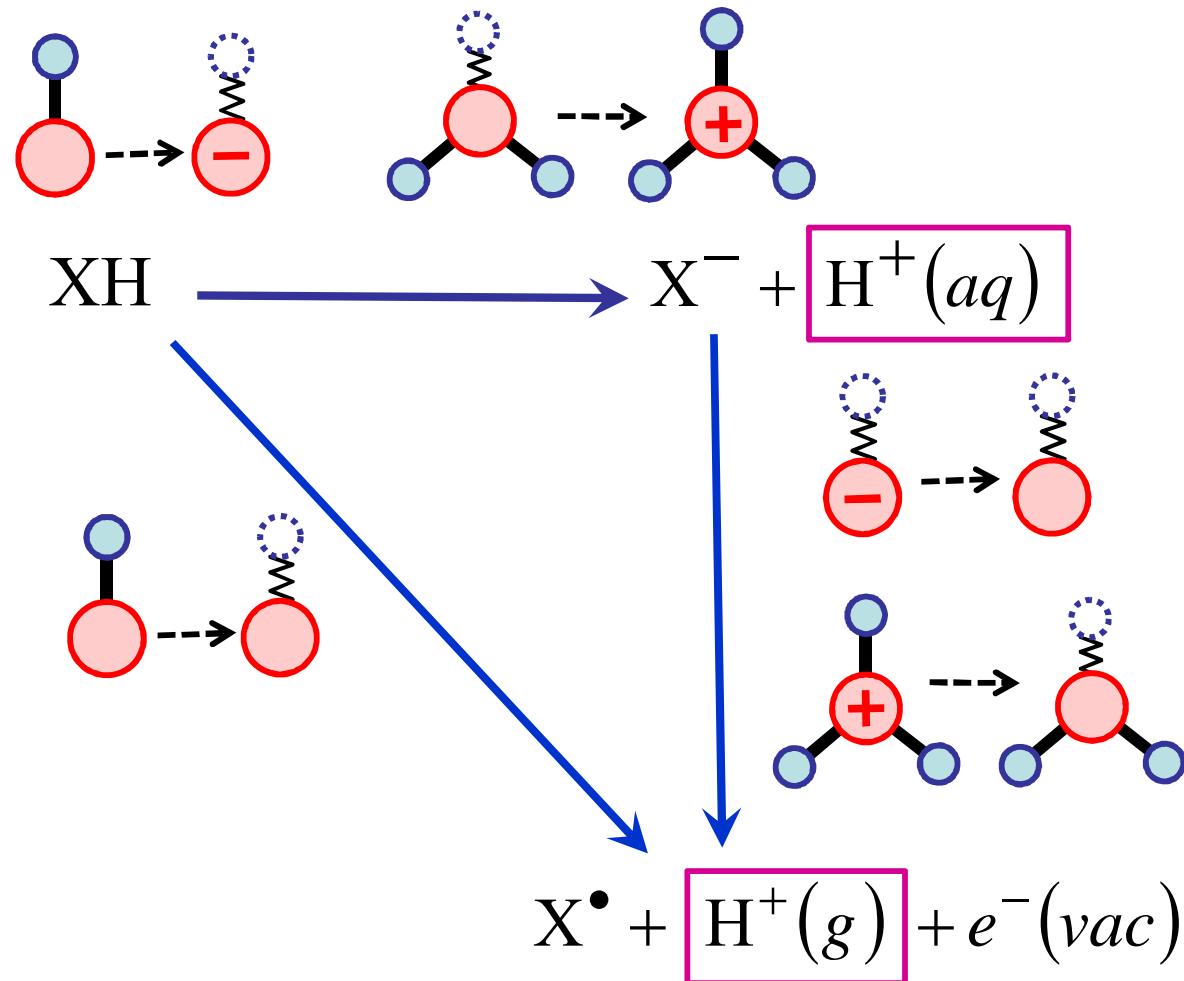
20-50 ps MD runs

Electronic structure

TZV2P basis,  
280 Ry (density cutoff

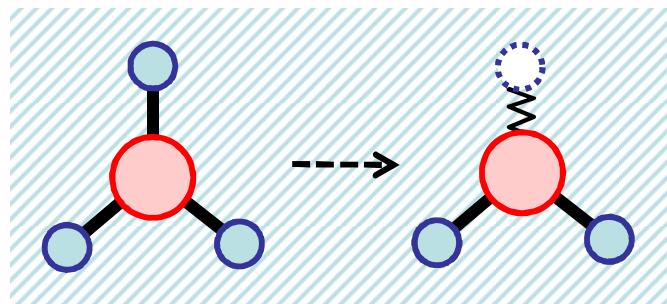
No explicit counter ion

# Dehydrogenation by double reversible discharge



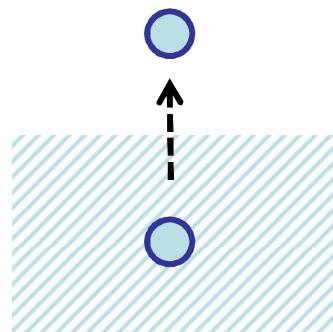
Free energy perturbation plus **analytical thermochemical corrections**

# Desolvating H<sup>+</sup> by deprotonating H<sub>3</sub>O<sup>+</sup>



What we calculate: discharge energy

$$\int_0^1 d\eta \left\langle \Delta E_{\text{H}_3\text{O}^+} \right\rangle_{r\eta}$$



What we want: desolvation energy

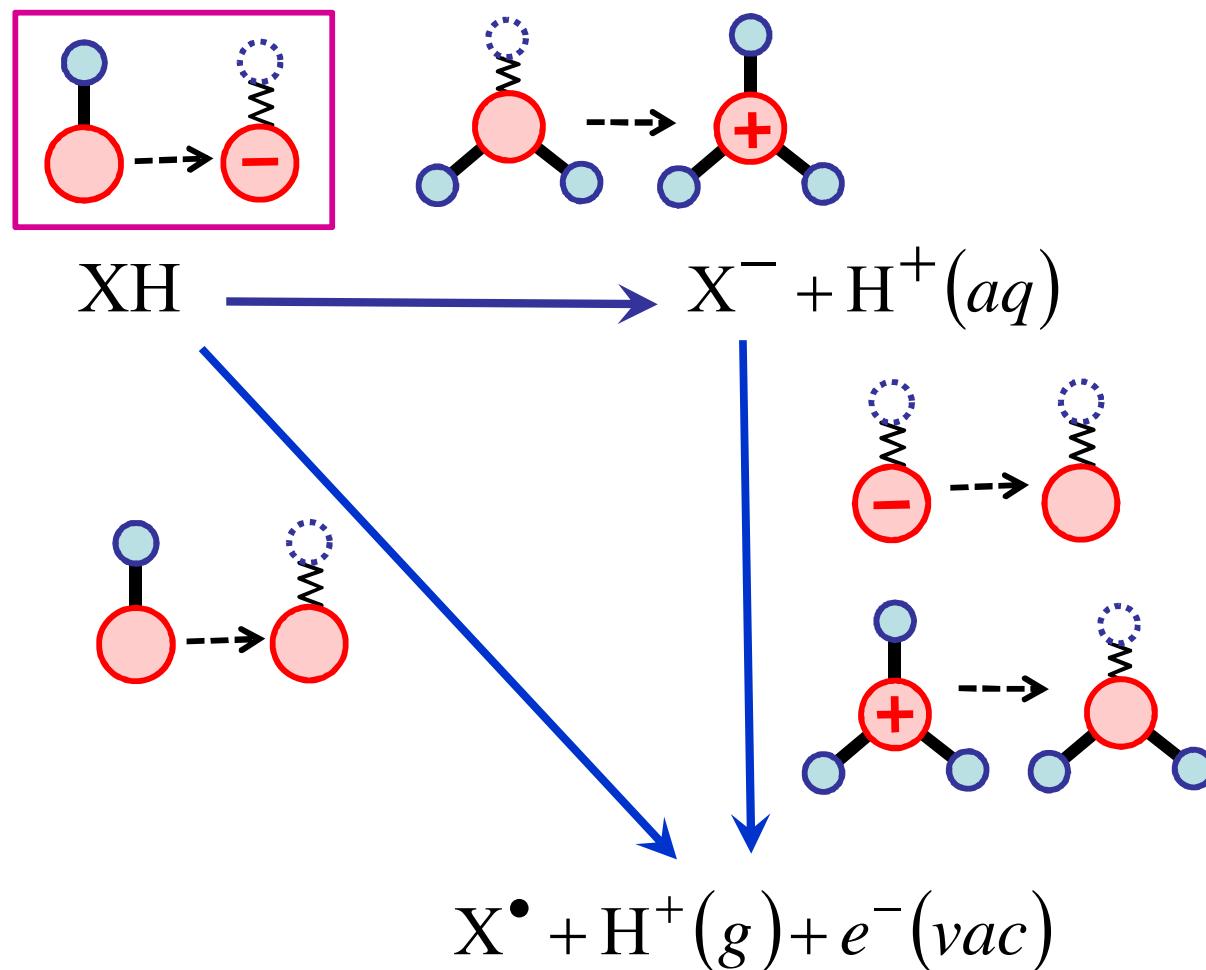
$$-\Delta_s G_{\text{H}^+} > 0$$

Desolvation free energy (in 0 order approximation)):

$$-\Delta_s G_{\text{H}^+} = \int_0^1 d\eta \left\langle \Delta E_{\text{H}_3\text{O}^+} \right\rangle_{r\eta} - \boxed{\Delta_{\text{zp}} E_{\text{H}^+}}$$

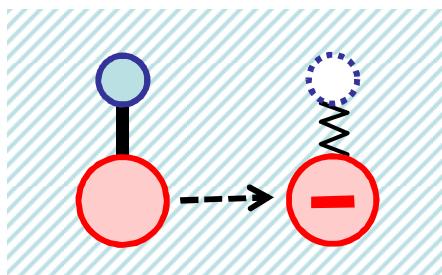
*Zero point motion correction ≈ 350 meV*

# Dehydrogenation by double reversible discharge



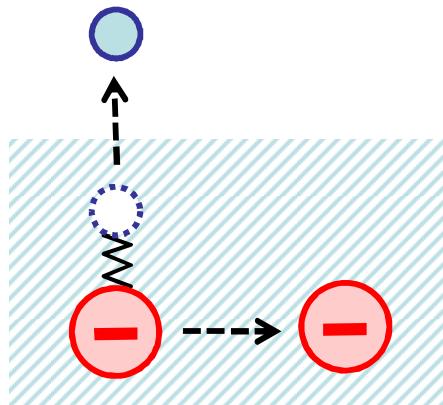
Free energy perturbation plus analytical thermochemical corrections

# Proton transfer to virtual gas-phase



Discharging integral

$$\int_0^1 d\eta \langle \Delta E_{\text{XH}} \rangle_{r\eta}$$



Thermochemical correction

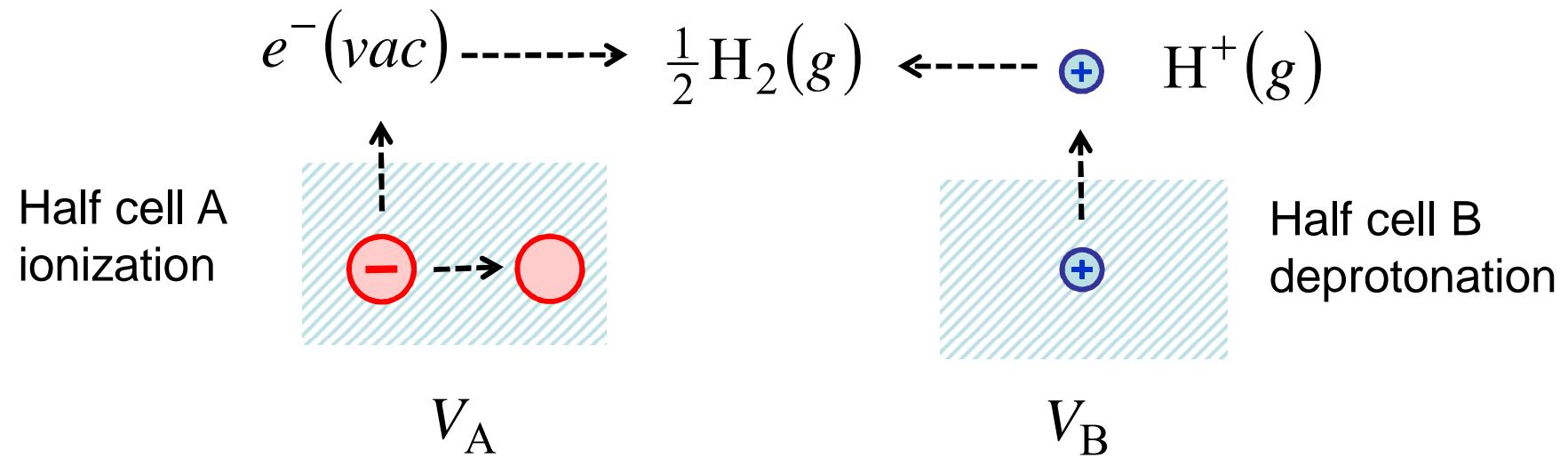
$$k_B T \ln \left[ c^\circ \Lambda_{\text{H}^+}^3 \right] - \Delta_{\text{zp}} E_{\text{H}^+}$$

*ideal chemical potential H<sup>+</sup> ≈ -190 meV*

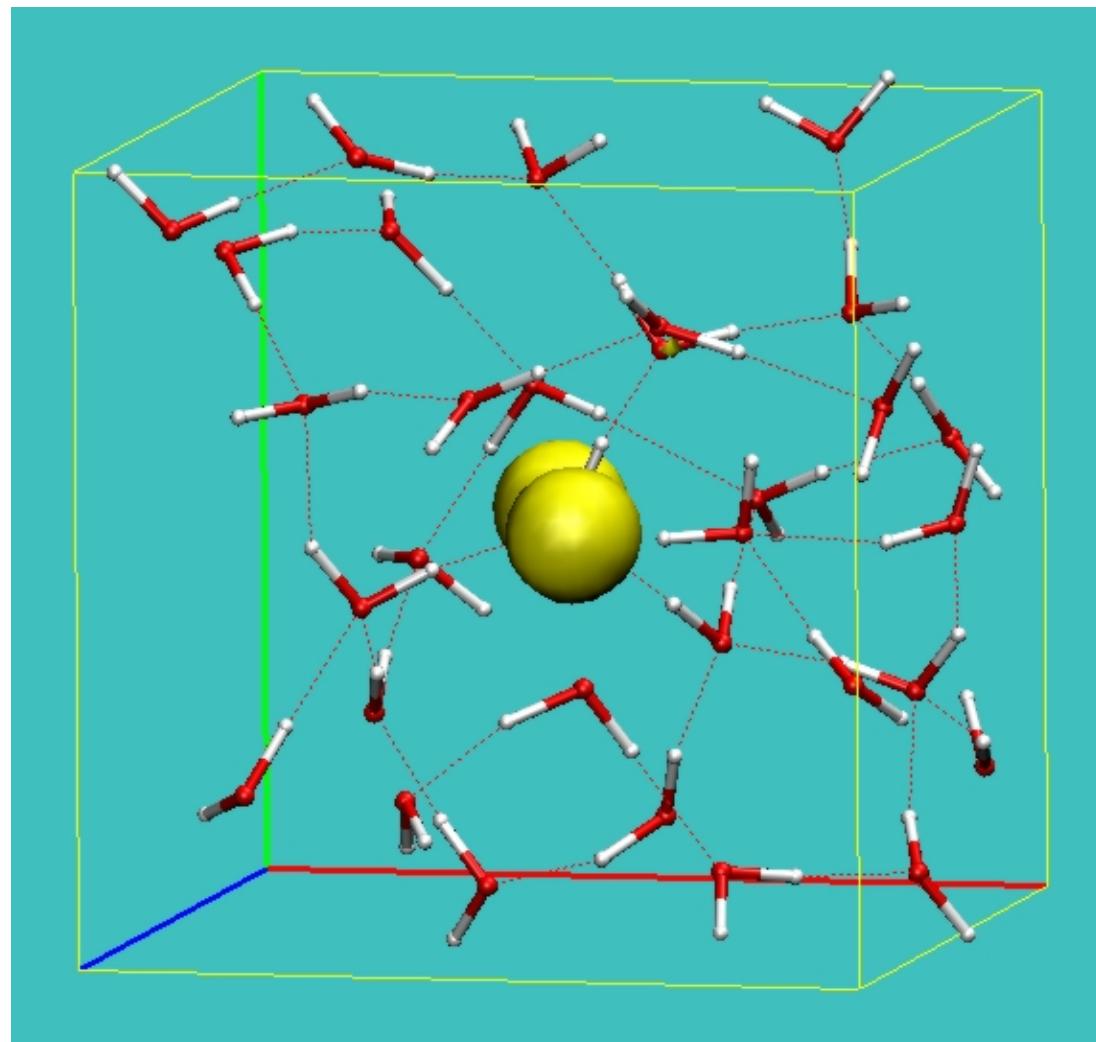
Deprotonation free energy (in 0 order approximation)

$$\Delta_{\text{dp}} G_{\text{XH}} = \int_0^1 d\eta \langle \Delta E_{\text{XH}} \rangle_{r\eta} + k_B T \ln \left[ c^\circ \Lambda_{\text{H}^+}^3 \right] - \Delta_{\text{zp}} E_{\text{H}^+}$$

# Half cells with virtual gas-phase interface



# Half cell for OH<sup>-</sup>/OH<sup>•</sup> oxidation



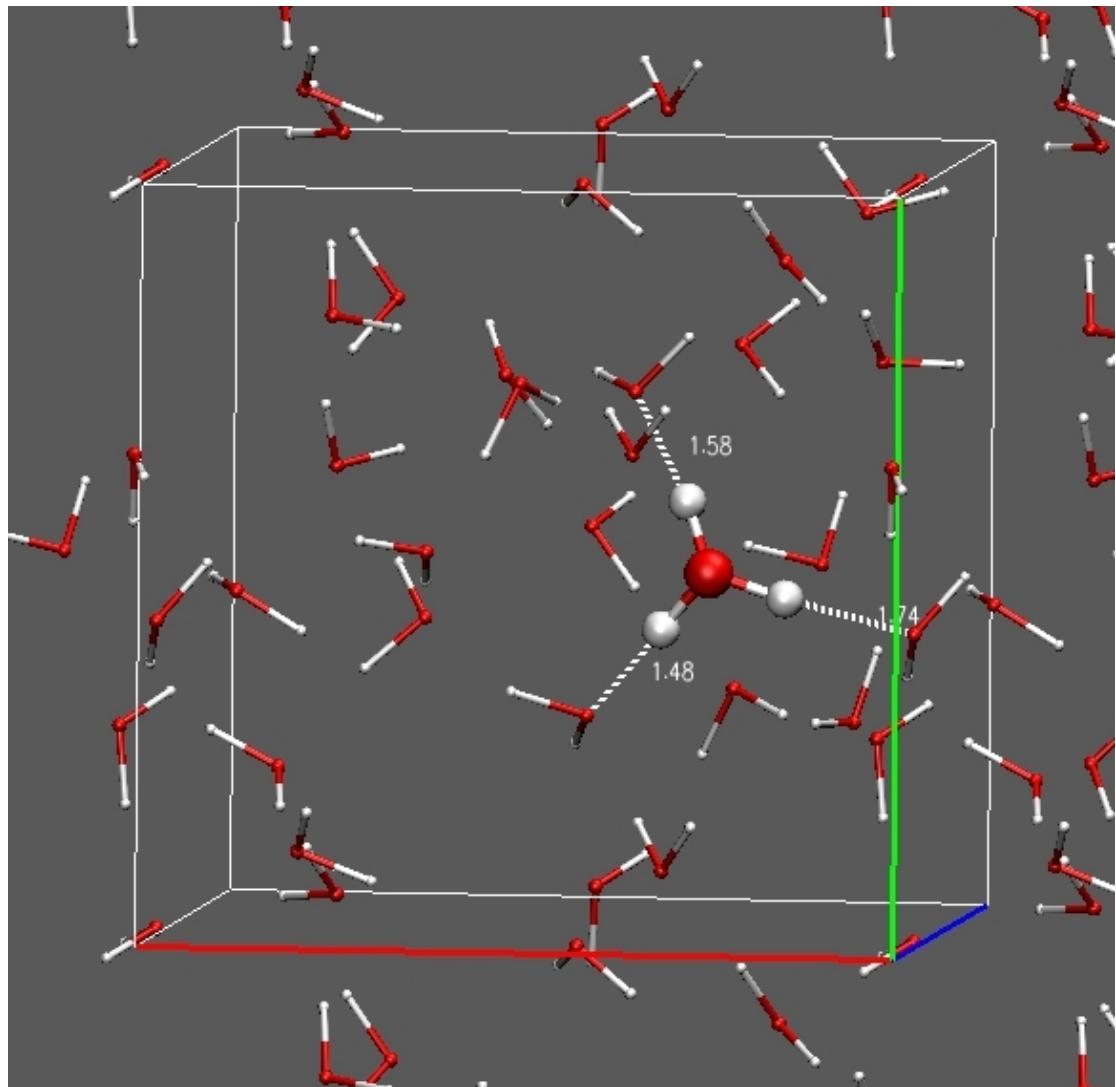
DFTMD Model

1 OH<sup>•</sup> radical  
31 H<sub>2</sub>O

In periodic cubic box of  
 $L = 9.86 \text{ \AA}$

No explicit counter ion

# Half cell for $\text{H}_3\text{O}^+/\text{H}_2\text{O}$ deprotonation



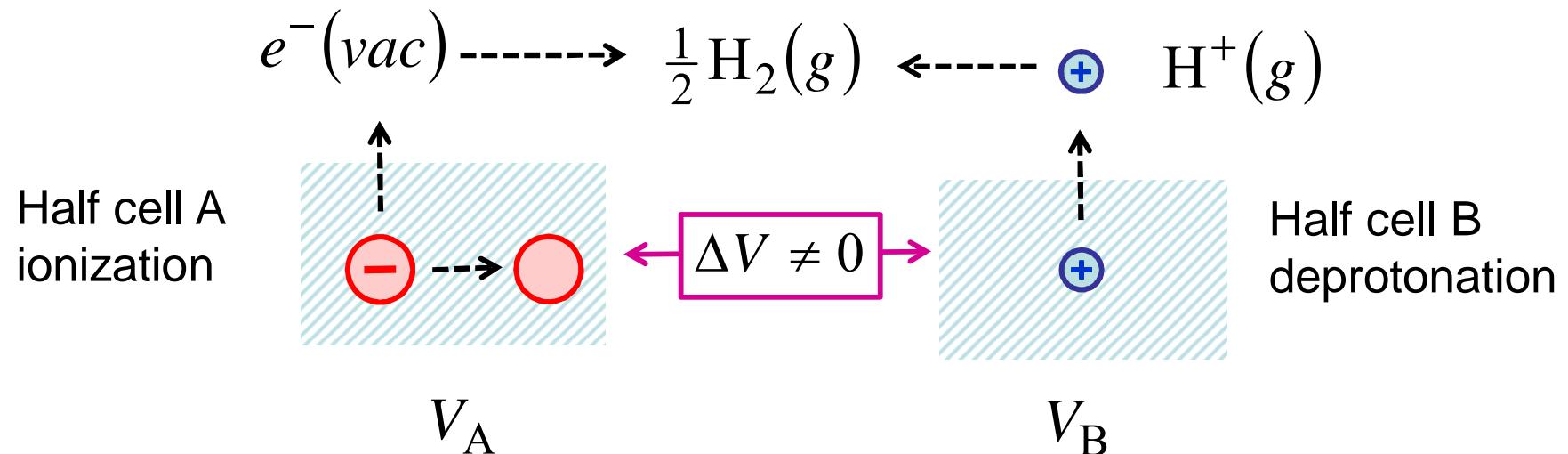
DFTMD Model

$32 \text{ H}_2\text{O} \pm \text{H}^+$

In periodic cubic box of  
 $L = 9.86 \text{ \AA}$

*No explicit counter ion*

# Half cells with virtual gas-phase interface

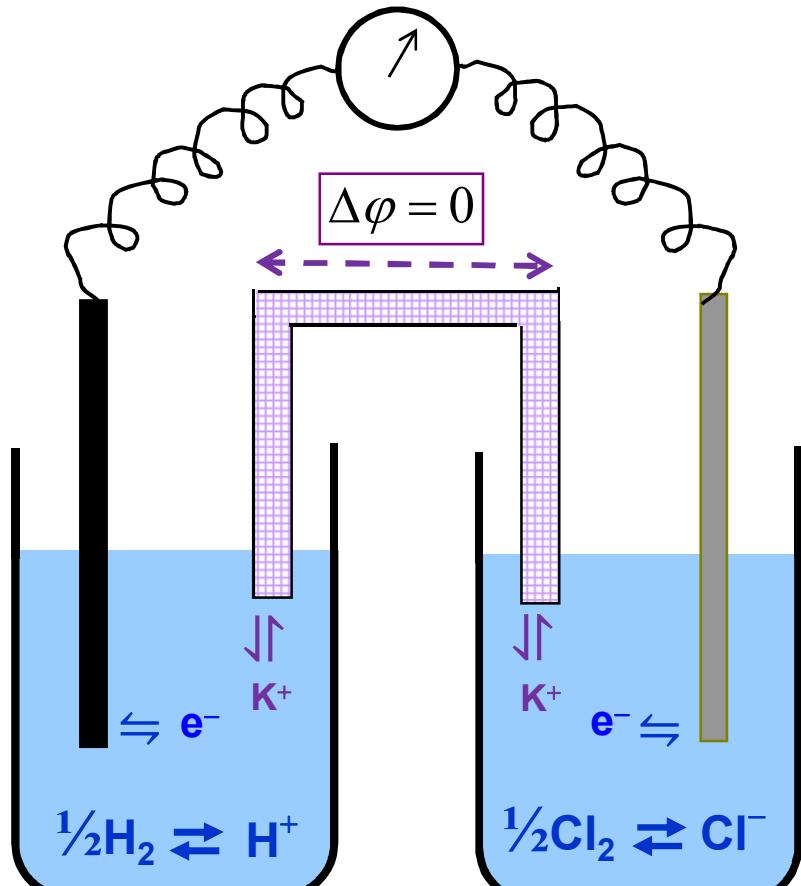


Reference for electrostatic potential in cell A and B may not be the same

***Level alignment (band offset) problem  
under periodic boundary conditions***

# Open circuit voltage of redox electrodes

Electrode reactions in equilibrium



No current

$$E = -\frac{\Delta_r G}{2e}$$

$$E = E^0 + \frac{k_B T}{2e} \ln \frac{p_{\text{H}_2} p_{\text{Cl}_2}}{[\text{H}^+]^2 [\text{Cl}^-]^2}$$

Nature of electrodes irrelevant

# Electrochemical potential

For a **finite charged conducting phase**

$$\frac{\partial G}{\partial n_i} = \bar{\mu}_i$$

- $\bar{\mu}_i$  the **electrochemical potential** of the phase

$$\bar{\mu}_i = \mu_i + z_i e \varphi$$

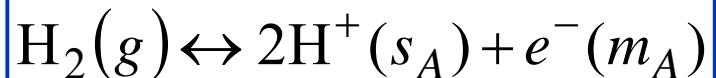
- $\mu_i$  the **chemical potential** of component  $i$

$$\mu_i = \mu_i^\circ + k_B T \ln a_i$$

- $\varphi$  the **inner or Galvani potential** of the phase

*Separation in chemical and electrostatic terms experimentally not possible*

# Chemical equilibrium at anode and cathode



$$\mu_{\text{H}_2}^g = 2\bar{\mu}_{\text{H}^+}^{s_A} + 2\bar{\mu}_{e^-}^{m_A}$$

$$\mu_{\text{H}_2}^g = 2\left(\mu_{\text{H}^+}^{s_A} + e\varphi^{s_A}\right) + 2\bar{\mu}_{e^-}^{m_A}$$

$$-2\bar{\mu}_{e^-}^{m_A} = 2\mu_{\text{H}^+}^{s_A} - \mu_{\text{H}_2}^g + 2e\varphi^{s_A}$$



$$\mu_{\text{Cl}_2}^g + 2\bar{\mu}_{e^-}^{m_C} = 2\bar{\mu}_{\text{Cl}^-}^{s_C}$$

$$\mu_{\text{Cl}_2}^g + 2\bar{\mu}_{e^-}^{m_C} = 2\left(\mu_{\text{Cl}^-}^{s_C} - e\varphi^{s_C}\right)$$

$$2\bar{\mu}_{e^-}^{m_C} = 2\mu_{\text{Cl}^-}^{s_C} - \mu_{\text{Cl}_2}^g - 2e\varphi^{s_C}$$

Adding

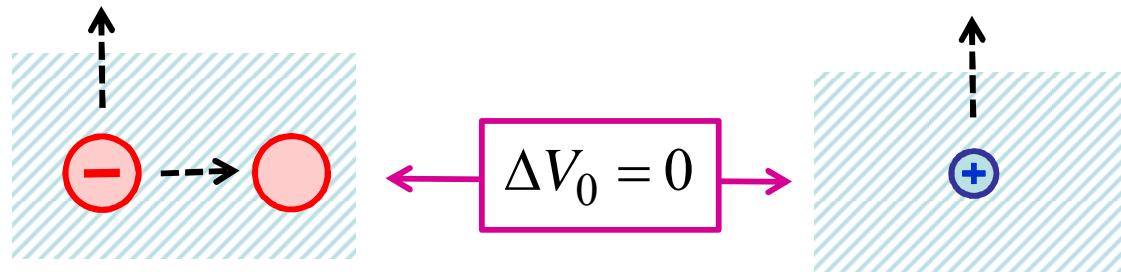
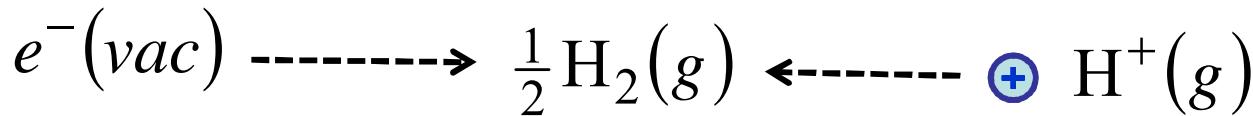
$$2\left(\bar{\mu}_{e^-}^{m_C} - \bar{\mu}_{e^-}^{m_A}\right) = 2\mu_{\text{Cl}^-}^{s_C} + 2\mu_{\text{H}^+}^{s_A} - \mu_{\text{Cl}_2}^g - \mu_{\text{H}_2}^g + 2e\left(\varphi^{s_A} - \varphi^{s_C}\right)$$

$$-2eE = \Delta_r G + 2e\left(\varphi^{s_A} - \varphi^{s_C}\right)$$

= 0 electrical equilibrium  
between anode and cathode

$$E = -\frac{\Delta_r G}{2e}$$

# Average potential half cells determined by solvent



$$\Delta_{ip} A - \boxed{V_0}$$

Adiabatic ionization potential

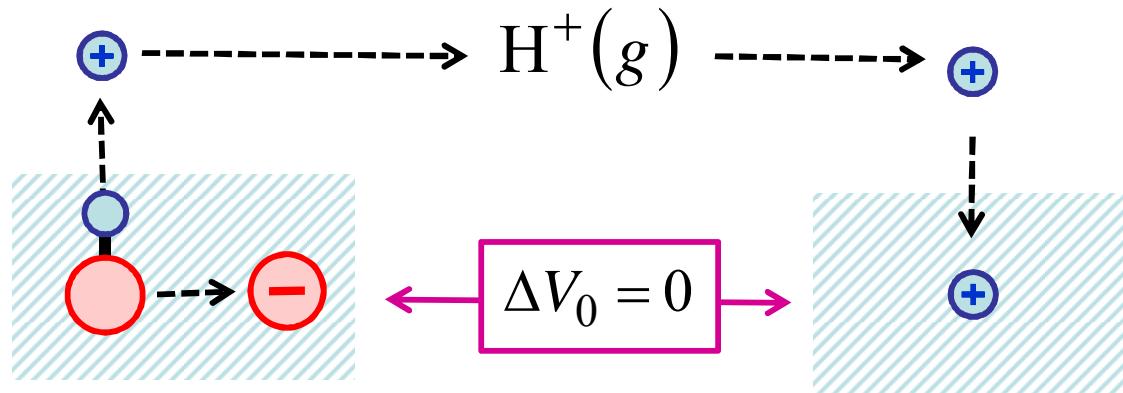
$$- \Delta_s G_{H^+} + \boxed{V_0}$$

Desolvation free energy  $H^+$

$$eE^\circ = \int_0^1 d\eta \left\langle \Delta_{ip} E_{X^-} \right\rangle_\eta + \int_0^1 d\eta \left\langle \Delta_{dp} E_{H_3O^+} \right\rangle_{r\eta} - \Delta_{zp} E_{H^+} - \Delta_f G_{H^+(g)}$$

**Bias  $V_0$  in the electrostatic reference should cancel**

# Half cell method for determination pKa



$$\Delta_{dp} A + \boxed{V_0}$$

Adiabatic deprotonation energy

$$\Delta_s G_{H^+} - \boxed{V_0}$$

Solvation free energy  $H^+$

$$2.3 k_B T \text{p}K_a = \int_0^1 d\eta \left\langle \Delta_{dp} E_{XH} \right\rangle_{r\eta} - \int_0^1 d\eta \left\langle \Delta_{dp} E_{H_3O^+} \right\rangle_{r\eta} + k_B T \ln [c^o \Lambda_{H^+}^3]$$

**Bias  $V_0$  in the electrostatic reference should cancel**

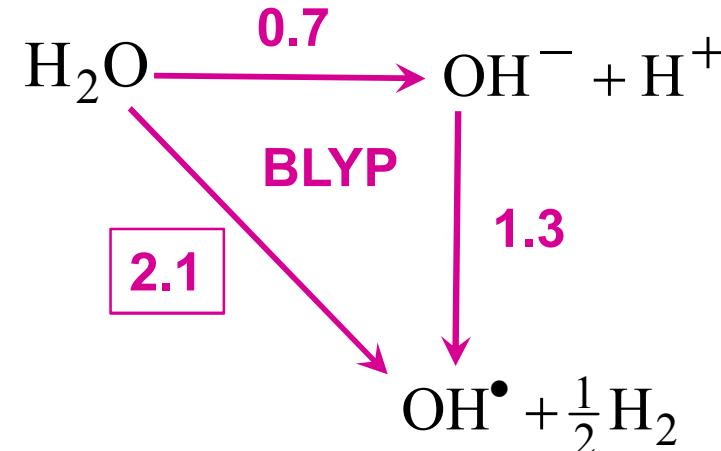
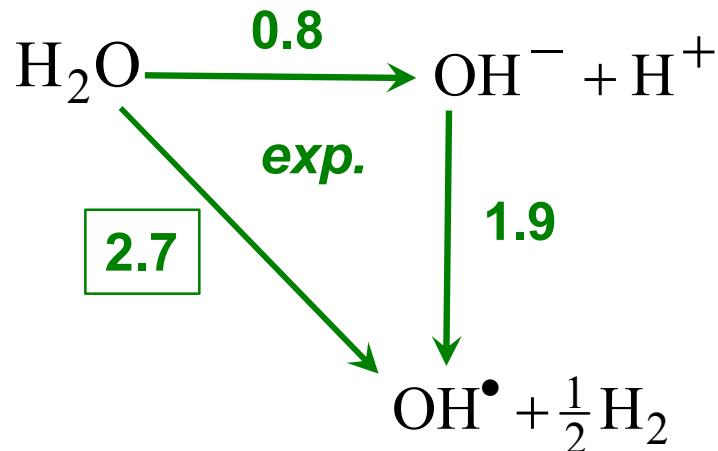
# Small acids: Comparison to experiment

Energies eV	$\Delta_{\text{dp}} A_{\text{XH}}$	$\text{p}K_a$	exp.	$\Delta \text{p}K_a$
HCl	15.1	-7	-7	0
$\text{H}_3\text{O}^+$	15.35	-3.2	-1.74	-1.5
HCOOH	15.8	5	3.75	1
$\text{H}_2\text{S}$	16.0	8	7.0	1
$\text{CH}_3\text{SH}$	16.1	11	10.3	1
$\text{NH}_4^+$	16.1	11	10.2	1
Errors	statistical uncertainty < 0.1 eV			<b>MUE = 1</b>

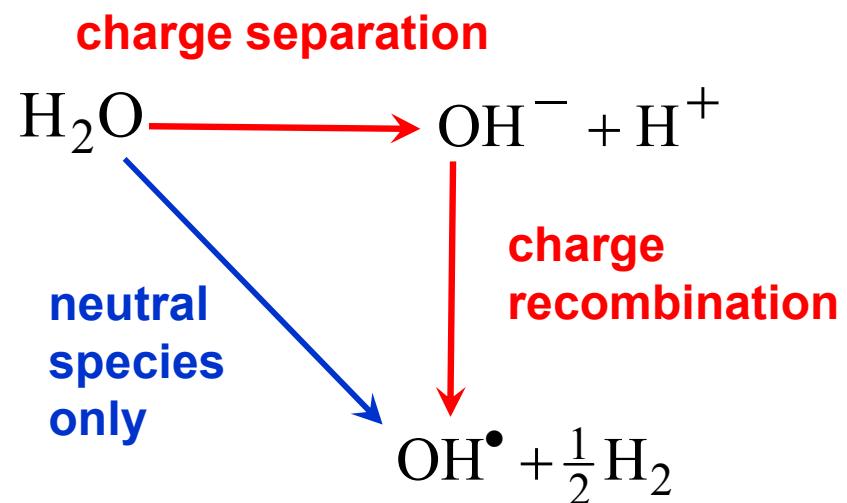
$$\Delta_{\text{dp}} A_{\text{XH}} = \int_0^1 d\eta \left\langle \Delta_{\text{dp}} E_{\text{XH}} \right\rangle_{r\eta}$$

(*Sulpizi*)

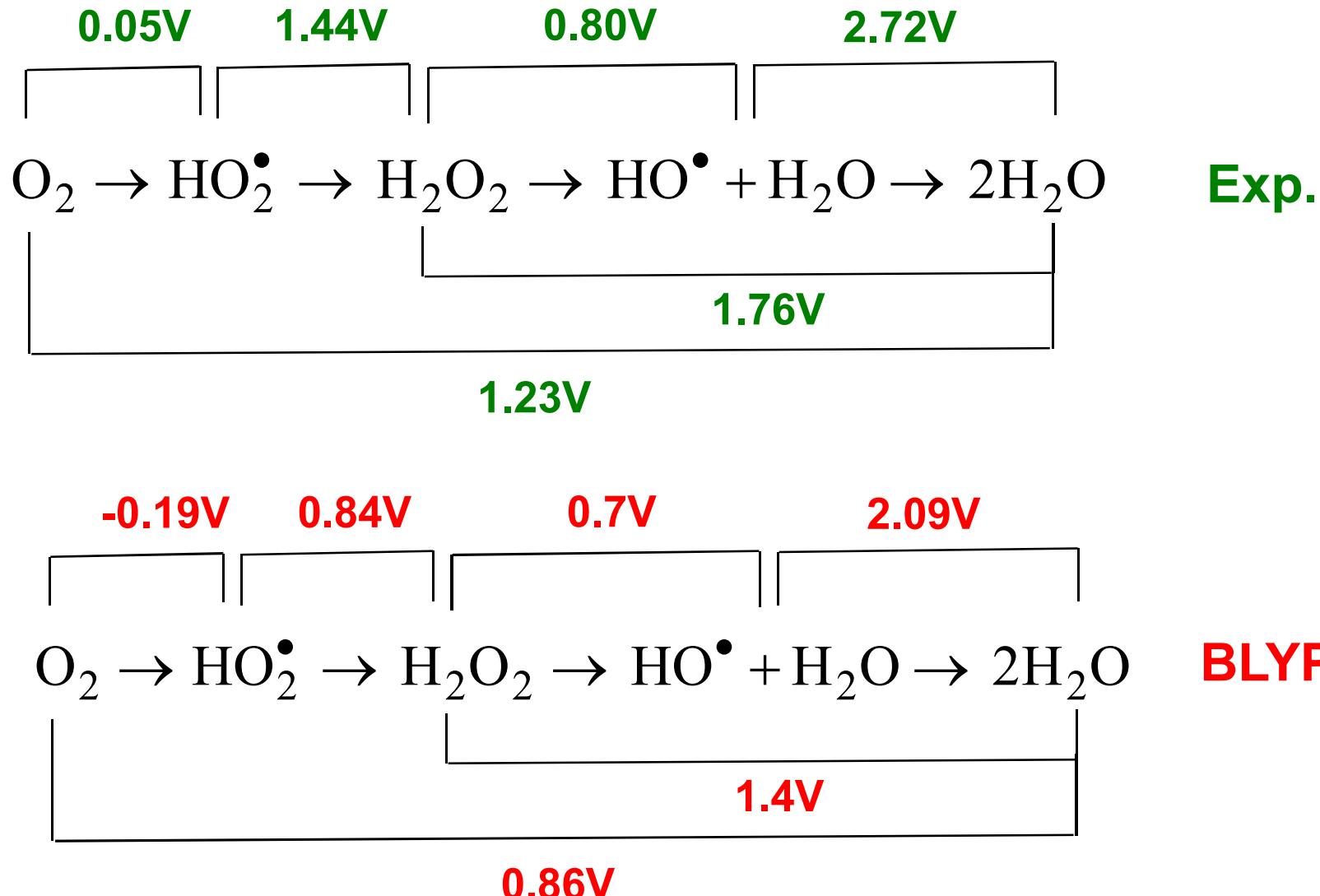
# Homogeneous $\text{H}_2\text{O} \rightarrow \text{OH}^- + \text{H}^+$ : our benchmark



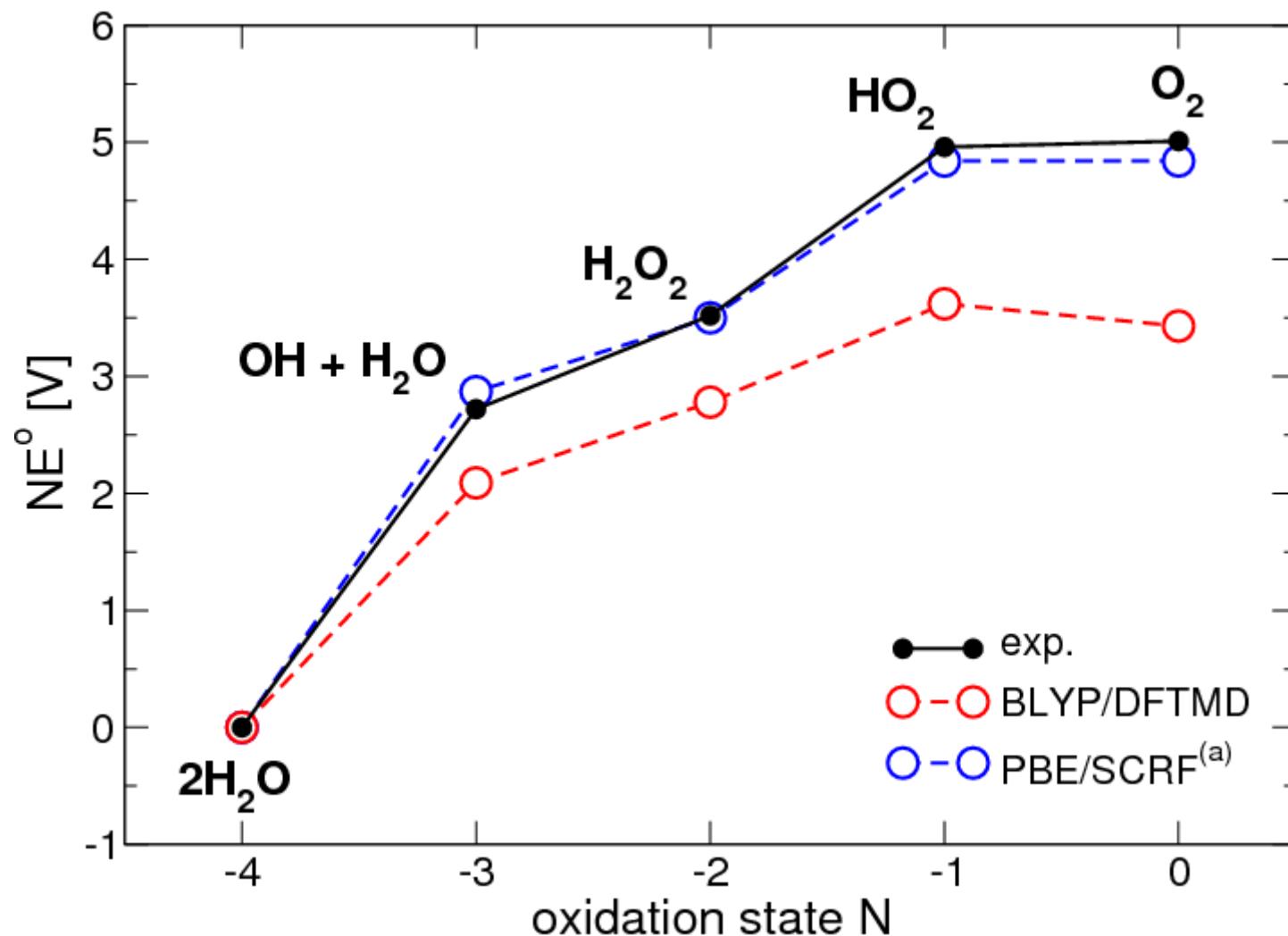
*Finite size and time  
errors “under control”*



# Reduction O<sub>2</sub> : BLYP/DFTMD performance

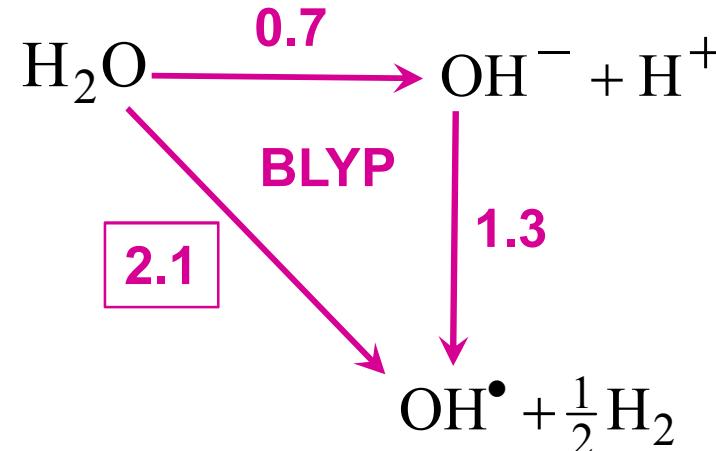
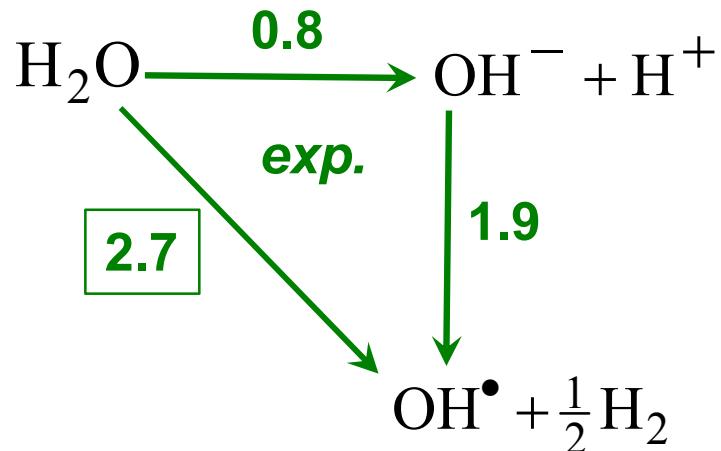


# Frost diagram reduction O<sub>2</sub>

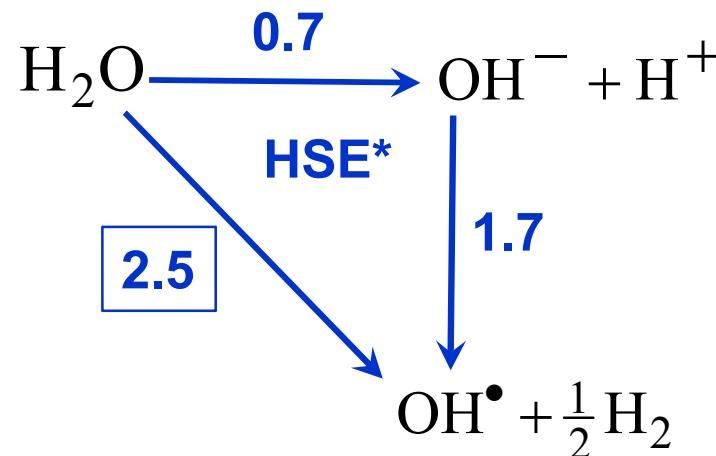


(a) R. Jinouchi and A. B. Anderson, *Phys. Chem. C* **112** (2008), 8747

# $\text{H}_2\text{O} \rightarrow \text{OH}^- + \text{H}^+$ in solution: Effect of HF exchange

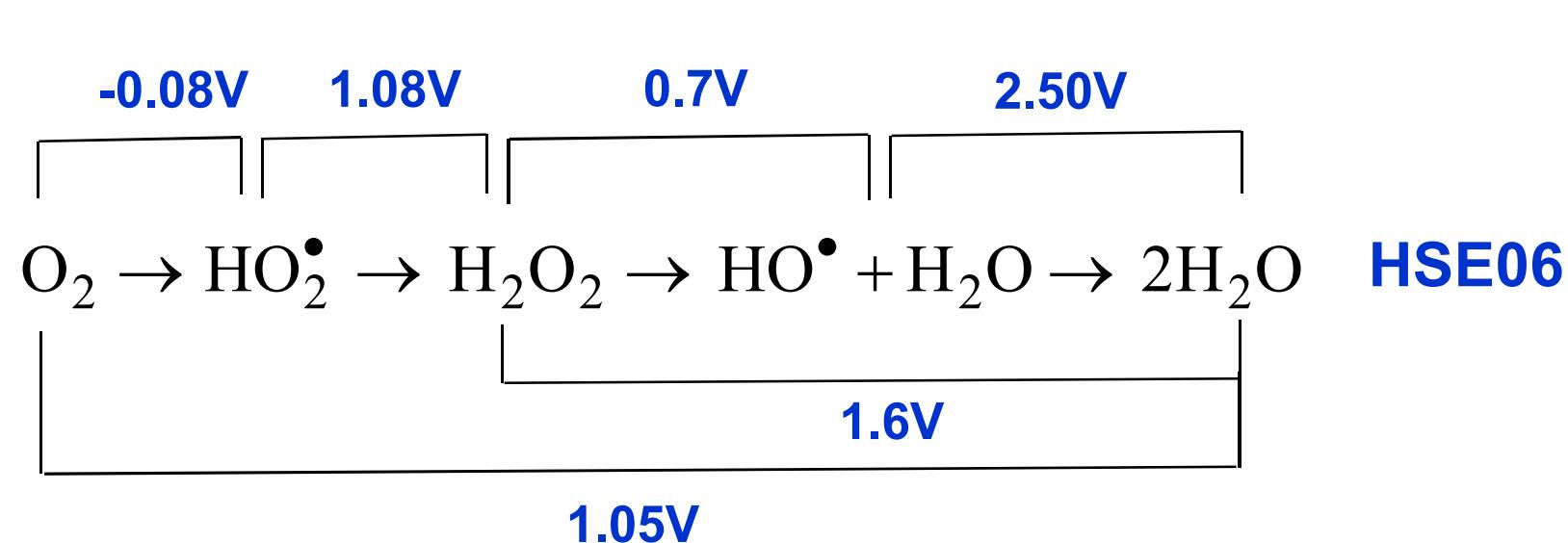
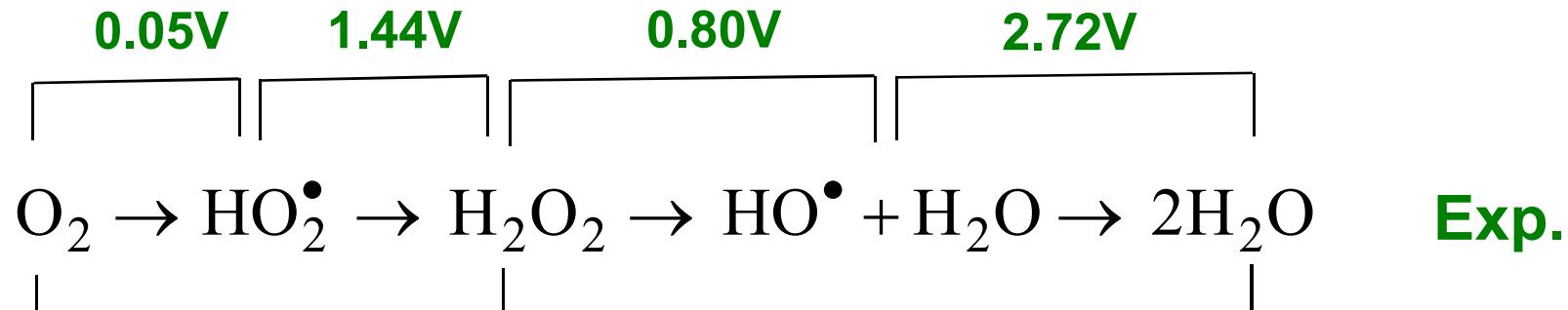


Energies in eV



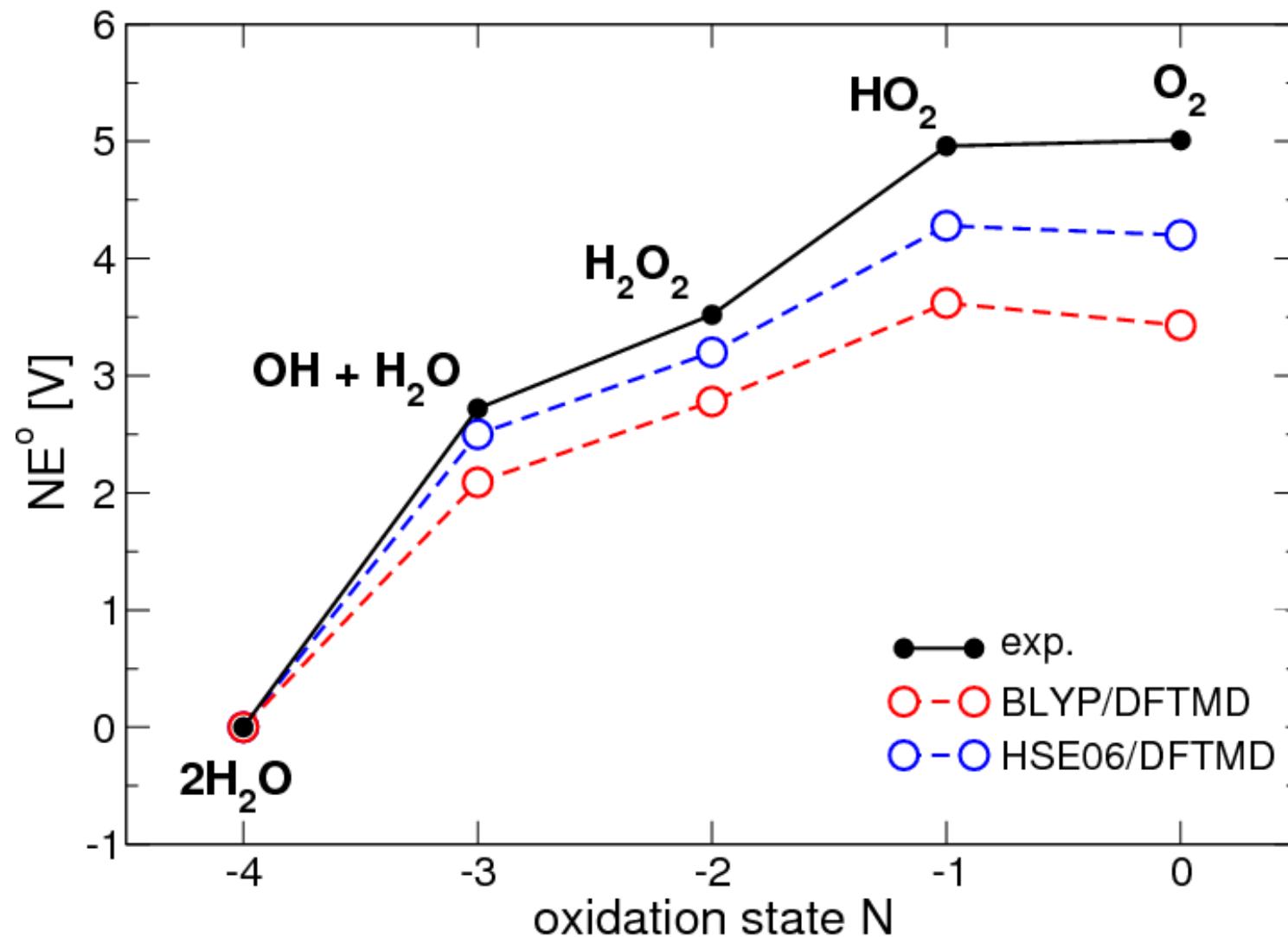
\*Guidon, M.; Hutter, J.; VandeVondele, J.  
“Auxiliary Density Matrix Methods for  
Hartree-Fock Exchange Calculations.”  
*J. Chem. Theory Comput.* (2010), 6, 2348.

# Reduction O<sub>2</sub> :HSE06/DFTMD performance



(Cheng)

# Frost diagram reduction O<sub>2</sub>



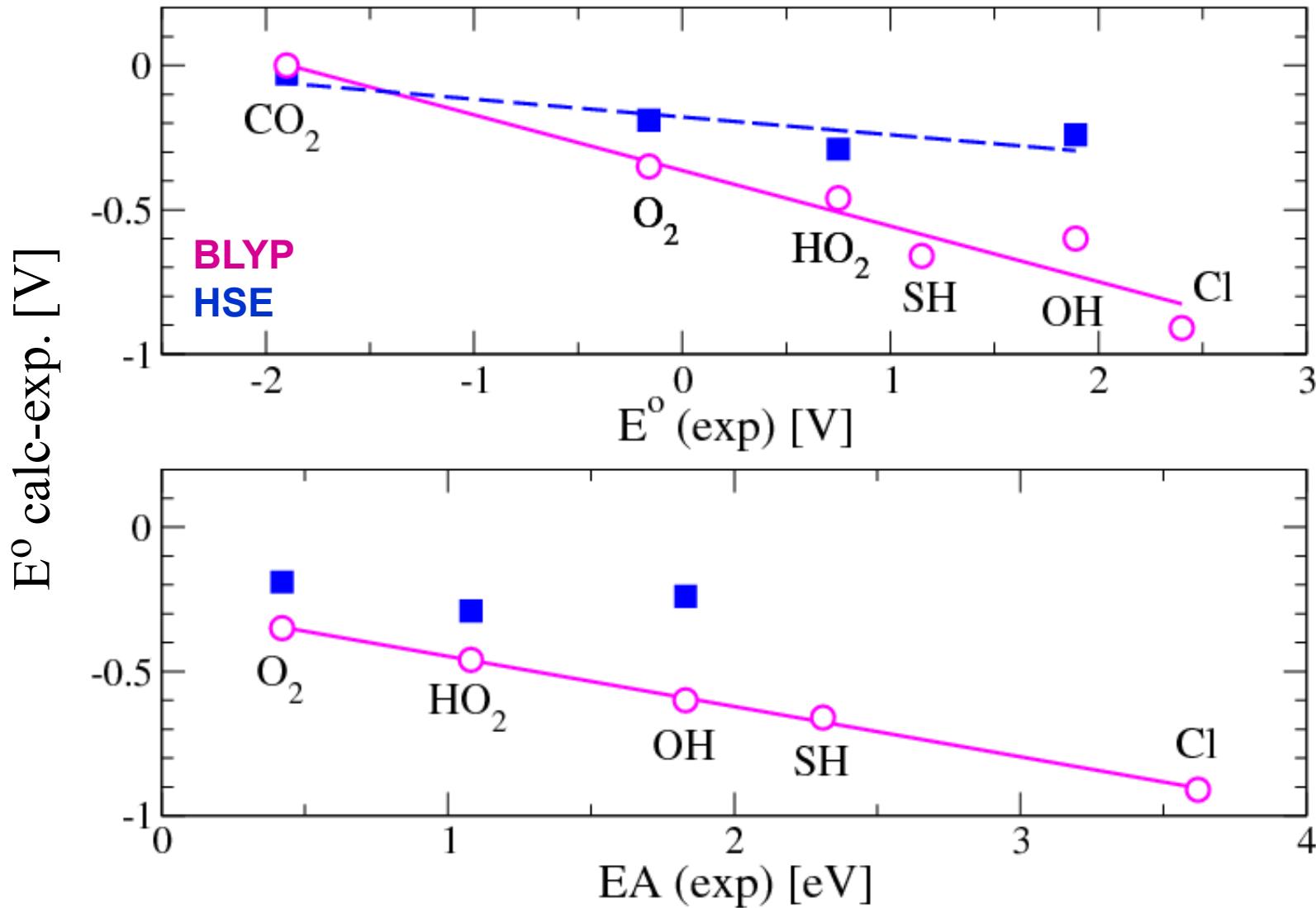
# BLYP electron affinities in vacuum and solution

Units V	EA vacuum (eV)		Reduction potential vs NHE (V)		
	BLYP	exp.	BLYP	exp.	diff.
O <sub>2</sub> /O <sub>2</sub> <sup>-</sup>	0.52	0.42	-0.5	-0.16	-0.4
OOH <sup>•</sup> /OOH <sup>-</sup>	0.96	1.08	0.3	0.75	-0.5
OH <sup>•</sup> /OH <sup>-</sup>	1.80	1.83	1.3	1.89	-0.6
SH <sup>•</sup> /SH <sup>-</sup>	2.19	2.31	0.5	1.15	-0.6
Cl <sup>•</sup> /Cl <sup>-</sup>	3.52	3.62	1.5	2.4	-0.9
MUE	0.09				0.5

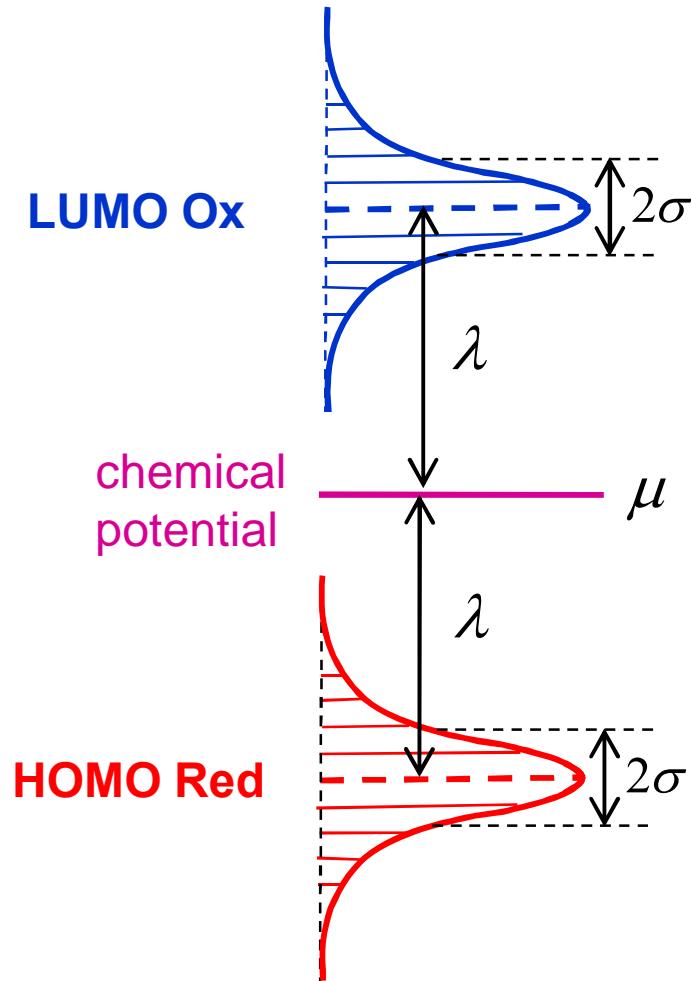
*Why the much larger error in solution?*

(Adriaanse)

# Correlating the DFT error



# One-electron orbital picture (Marcus-Gerischer)



**Electronic chemical potential**

$$\mu = \frac{1}{2}(\varepsilon_{\text{HOMO}} + \varepsilon_{\text{LUMO}})$$

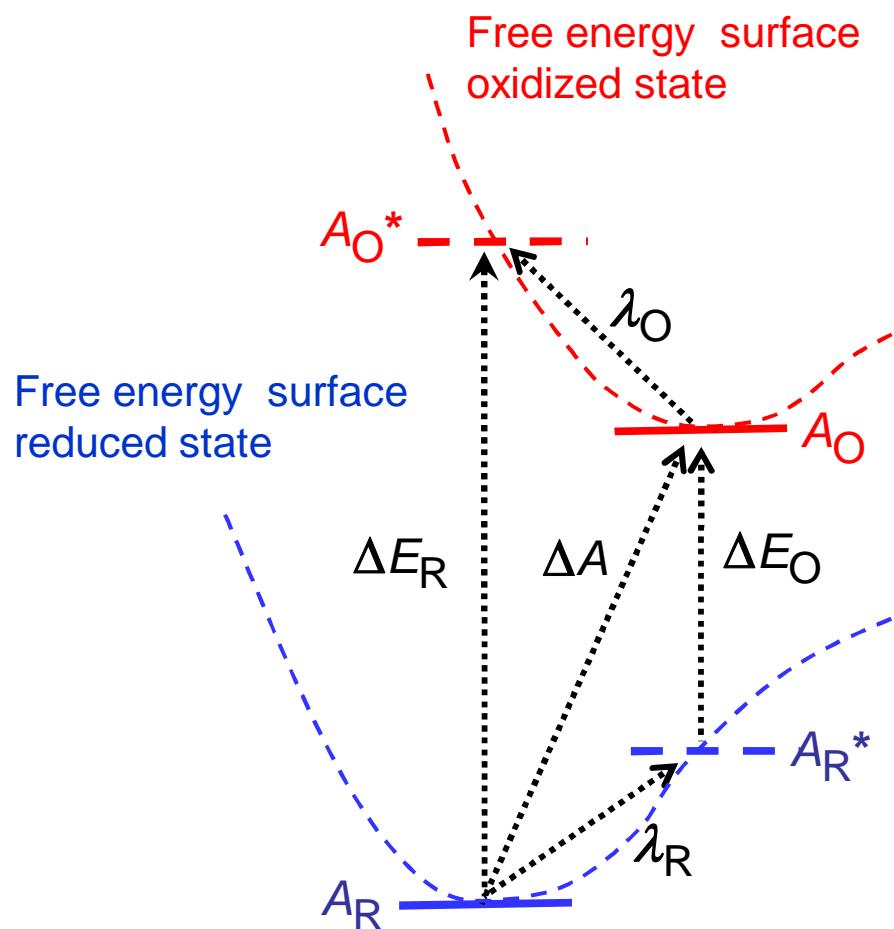
**Reorganization energy**

$$\lambda = \frac{1}{2}(\varepsilon_{\text{LUMO}} - \varepsilon_{\text{HOMO}})$$

**Solvent fluctuations**

$$\sigma^2 = 2k_B T \lambda$$

# Total energy (potential of mean force) picture



Non-linear solvent response

$$\Delta E_R = \Delta A + \lambda_O$$

$$\Delta A = \lambda_R + \Delta E_O$$

Linear response approximation

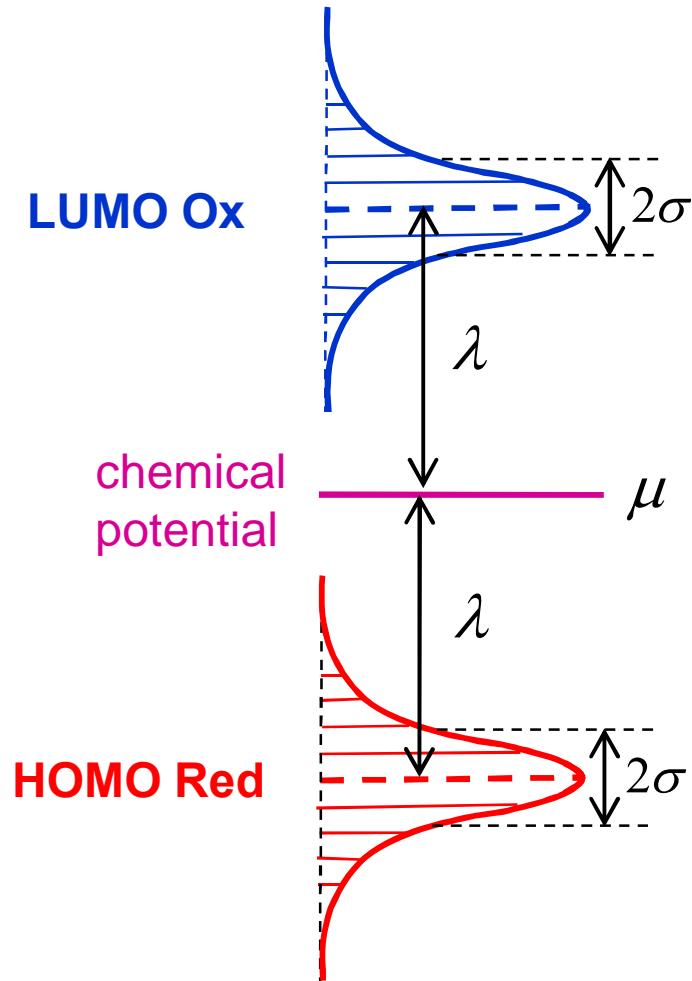
$$\lambda_R = \lambda_O = \lambda$$

$$\lambda = \frac{1}{2}(\Delta E_R - \Delta E_O)$$

$$\Delta A = \frac{1}{2}(\Delta E_R + \Delta E_O)$$

(Zhou and Szabo, JCP 1995)

# One-electron orbital picture (Marcus-Gerischer)



**Electronic chemical potential**

$$\mu = \frac{1}{2}(\varepsilon_{\text{HOMO}} + \varepsilon_{\text{LUMO}})$$

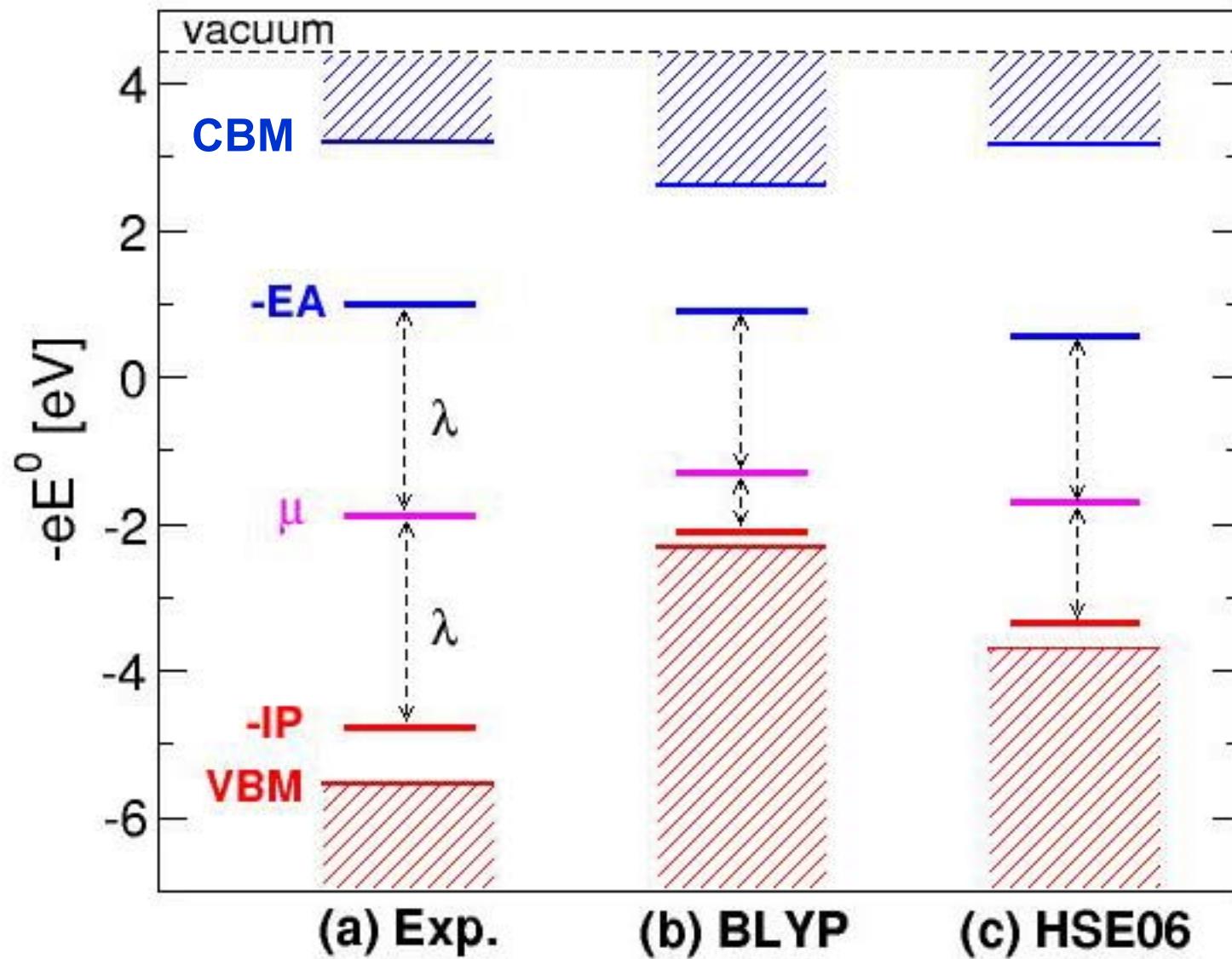
**Reorganization energy**

$$\lambda = \frac{1}{2}(\varepsilon_{\text{LUMO}} - \varepsilon_{\text{HOMO}})$$

**Solvent fluctuations**

$$\sigma^2 = 2k_B T \lambda$$

# Vertical and adiabatic levels OH/water



# Interfacial Electrochemistry:

*Is molecular simulation finally ready for  
19<sup>th</sup> century science?*



**We have made a start**

**Jun Cheng**

**Marialore Sulpizi**

**Joost VandeVondele**

**Regla Ayala**

**Francesca Costanzo**

**John Kattirzi**