Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers

Representing Numbers

How values can be coded ?

In a digital circuit each signal can take 2 values (0, 1) (Boolean world)

A vector of n bits can represent up to 2^n values

Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

The character 'F'
The character 'Φ'
The number 46
The number 70
The number 123

Any symbol in a set where the *Card* = 256

Representing Numbers

How values can be coded ?

by itself a code has no signification

Representing Numbers

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

Natural numbers
Relative numbers
Rational numbers
Real numbers

Representing Numbers

How can I represent a Natural number ?

I need at least n bits for a Natural ranging from 0 to 2^n-1

Standards

Representing Numbers

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

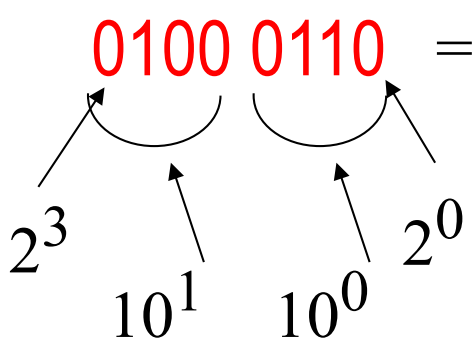
2^7 2^0

Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal :

The bits represent the successive powers of 2
The quartets represent the successive powers of 10



$= (2^1 + 2^2) \times 10^0 + 2^2 \times 10^1 = 46$

packed

Representing Numbers

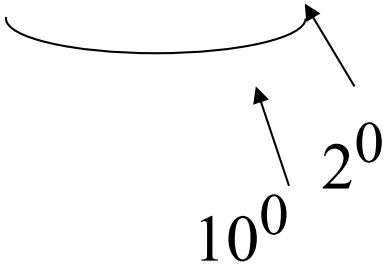
How can I represent a Natural number ?

Binary Coded Decimal - Unpacked :

The bits represent the successive powers of 2
The bytes represent the successive powers of 10
In each byte the 4 Msb are 0

0100 0110 = Illegal

0000 0110 = 6



Representing Numbers

How can I represent a Relative number ?

Sign + Value

The bits represent the successive powers of 2

The Msb represents the sign (1 means negative)

2's complemented

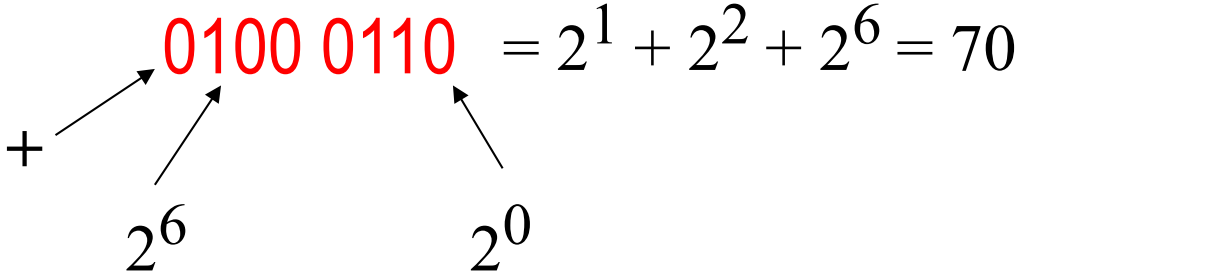
The bits represent the successive powers of 2

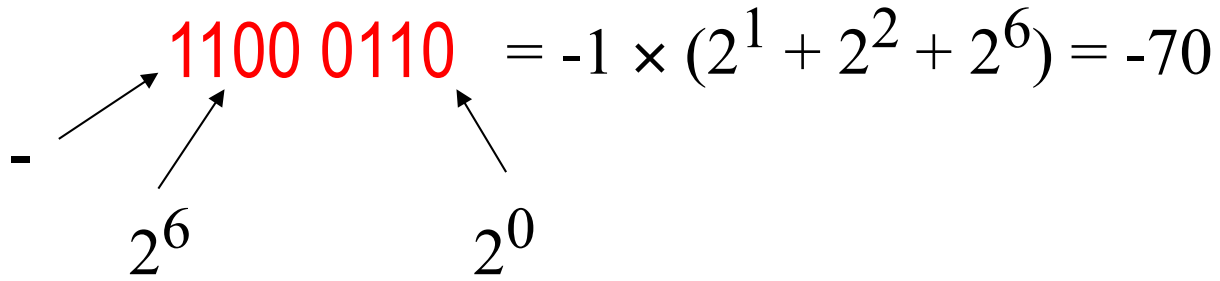
The Msb represents -2^{n-1}

Representing Numbers

How can I represent a Relative number ?

Sign+Value :

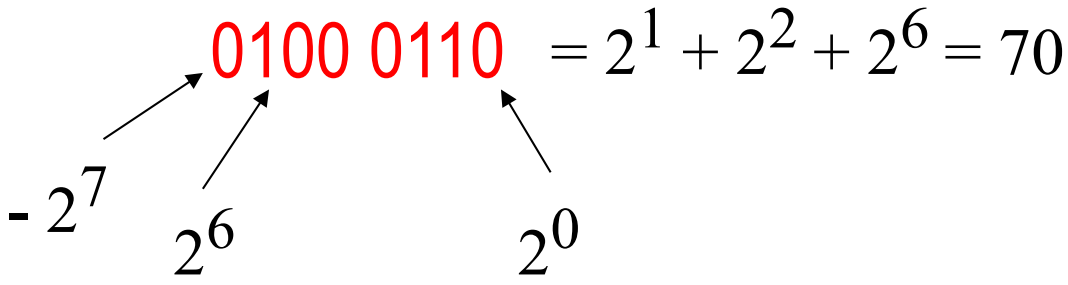




Representing Numbers

How can I represent a Relative number ?

2's complemented :



$1100\ 0110 = 2^1 + 2^2 + 2^6 - 2^7 = -58$

Representing Numbers

How can I represent a Relative number ?

2's complemented :

$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$

70

$2^7 = 2^0+2^1+2^2+2^3+2^4+2^5+2^6+1$

$2^7 = 2^0 \quad +2^3+2^4+2^5 \quad +1 + 70$

$1011\ 1010 \quad -70 = 2^0 \quad +2^3+2^4+2^5 \quad +1 - 2^7$

$-70 = \quad 2^1 \quad +2^3+2^4+2^5 \quad - 2^7$

Representing Numbers

How can I represent a Real number ?

Range

Precision

Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

$$0100.0110 = 2^{-3} + 2^{-2} + 2^{2} = 4.325$$

-2^{3} 2^{0} 2^{-1} 2^{-4}

Representing Numbers

How can I represent a Real number ?

Wide range
High precision

Floating Point :
Logarithmic representation

Representing Numbers

How can I represent a Real number ?

$$R = (-1)^S \times M \times 2^E$$

Normalized scientific representation

S : Sign (1 if negative)

M : Mantissa ($\in [1, 2[$)

E : Exponent (Relative number)

Representing Numbers

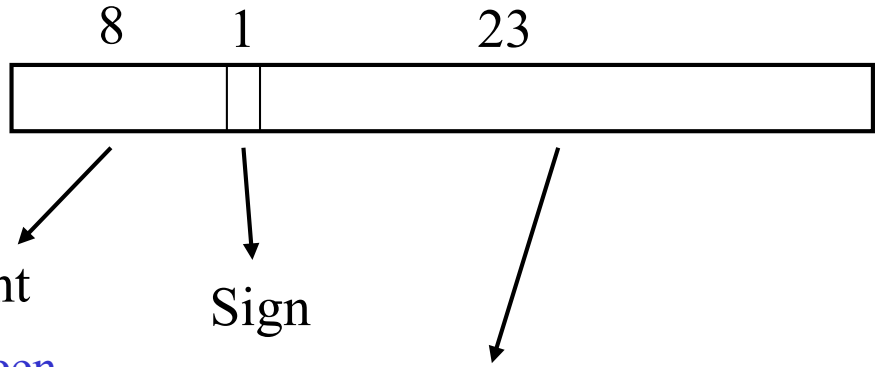
$$R = (-1)^S \times M \times 2^E$$

	Single Precision 32 bits	Double Precision 64 bits
S : Sign (1 if negative)	1 bit	1 bit
M : Mantissa ($\in [1, 2[$)	23 bits	52 bits
E : Exponent	8 bits	11 bits

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision :



Number between
-127 and 127

The code 0000 0000
means -127

Natural Binary Code
by Excess of 127

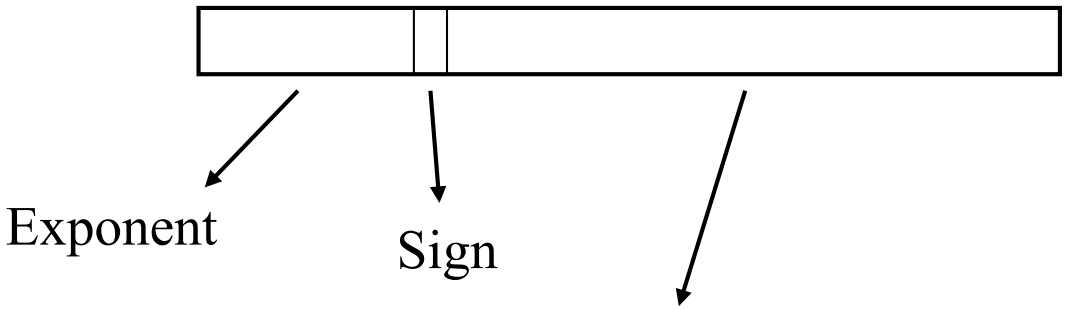
Fixed point positive real number
1. • • •

The 1 is not represented !!

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 1111 1111

(128) means ±∞ or an error

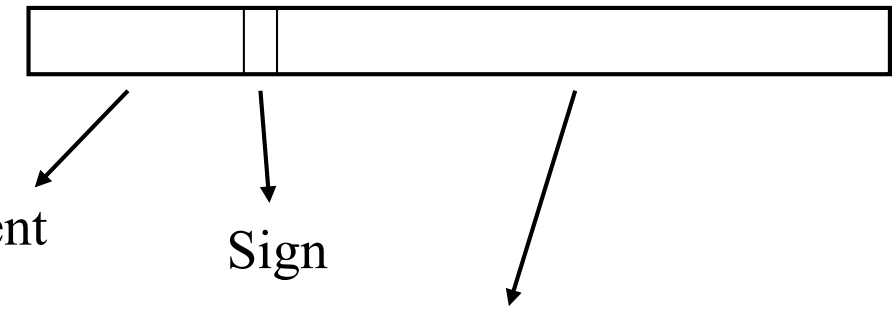
.111 … 111 means ±∞

other values mean error (NaN)

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 0000 0000 (-127) indicates denormalized Mantissa

0.00 … 000 means 0