Introduction to Digital Design

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Outline

- Digital CMOS design
- Arithmetic operators
- Sequential functions

Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers

Boolean Algebra

- English mathematician 1815 - 1864

  1854 : *Introduction to the Laws of Thought*

Boolean Algebra

- Let **B = {0 , 1}** — **B** is called the Boolean set
  0, 1 are the Boolean constants

- Let $\mathbf{x} \in \mathbf{B}$ — **x** is a Boolean variable

Boolean Algebra

- Unary functions : $\mathbf{B} \longrightarrow \mathbf{B}$

Unary function *0* : $\forall\, \mathbf{x} \in \mathbf{B},\ \mathbf{x} \longmapsto \mathbf{0}$

Unary function *1* : $\forall\, \mathbf{x} \in \mathbf{B},\ \mathbf{x} \longmapsto \mathbf{1}$

Unary function *Identity* : $\forall\, \mathbf{x} \in \mathbf{B},\ \mathbf{x} \longmapsto \mathbf{x}$

Unary function *Not* :

$$\mathbf{0} \longmapsto \mathbf{1}$$
$$\mathbf{1} \longmapsto \mathbf{0}$$

Not (**x**) is denoted $\overline{\mathbf{x}}$

Boolean Algebra

- Binary functions : $\mathbf{B}^2 \longrightarrow \mathbf{B}$

  function *And* :

  $\forall$ **x, y** $\in$ **B**, *And* (**x, y**) **= 1** if and only if **x = 1** and **y = 1**

  And (**x, y**) is also called *Min* is denoted **x.y**

  function *Or* :

  $\forall$ **x, y** $\in$ **B**, *Or* (**x, y**) **= 0** if and only if **x = 0** and **y = 0**

  Or (**x, y**) is also called *Max* is denoted **x+y**

Boolean Algebra

- Other binary functions can be defined using *And*, *Or* and *Not*

function *Nand* : $\quad Nand\,(\mathbf{x}, \mathbf{y}) = Not\,(And\,(\mathbf{x}, \mathbf{y}))$

function *Nor* : $\quad Nor\,(\mathbf{x}, \mathbf{y}) = Not\,(Or\,(\mathbf{x}, \mathbf{y}))$

function *Xor* : $\quad Xor\,(\mathbf{x}, \mathbf{y}) = \bar{\mathbf{x}}.\mathbf{y} + \mathbf{x}.\bar{\mathbf{y}}$

$\boxed{Xor\,(\mathbf{x}, \mathbf{y}) \text{ is denoted } \mathbf{x} \oplus \mathbf{y}}$

Boolean Algebra

- Let $\mathbf{B} = \{0 , 1\}$ — **B** is called the Boolean set; **0, 1** are the Boolean constants
- Let $\mathbf{x} \in \mathbf{B}$ — **x** is a Boolean variable
- Let $\mathbf{v} \in \mathbf{B}^n$ — **v** is a Boolean vector

Boolean Algebra

- $\mathbf{v} \in \mathbf{B}^{\mathbf{n}}$, $\mathbf{v} = (\mathbf{x_1}, \ldots, \mathbf{x_i}, \ldots, \mathbf{x_n})$
  $\mathbf{u} \in \mathbf{B}^{\mathbf{n}}$, $\mathbf{u} = (\mathbf{y_1}, \ldots, \mathbf{y_i}, \ldots, \mathbf{y_n})$

The number of Boolean variables that are different between **v** and **u** is called the Hamming distance (**v**, **u**)

Hd ((**0,0,0,1**) , (**1,0,1,0**)) = 3

Boolean Algebra

To vectors are said adjacent when their Hamming distance = 1

Hd ((**0,0,0,1**) , (**1,0,0,1**)) = 1

Boolean Algebra

- Let $\mathbf{B} = \{0 , 1\}$ — $\mathbf{B}$ is called the Boolean set
  $\mathbf{0, 1}$ are the Boolean constants
- Let $\mathbf{x} \in \mathbf{B}$ — $\mathbf{x}$ is a Boolean variable
- Let $\mathbf{v} \in \mathbf{B}^{\mathbf{n}}$ — $\mathbf{v}$ is a Boolean vector
- Let $\mathbf{f} : \mathbf{B}^{\mathbf{n}} \longrightarrow \mathbf{B}$ — $\mathbf{f}$ is a Boolean function
- $\mathbf{B}_{\mathbf{n}}$ is the set of Boolean Functions

$$\boxed{\mathit{card}\,(\mathbf{B}_{\mathbf{n}}) = 2^{(2^{n})}}$$

Boolean Algebra

- *Card* ($\mathbf{B}^n$) is finite

A Boolean function **f** may be defined by giving the value **f** (**v**) of each Boolean vector **v** (Truth table)

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean Algebra

- A Boolean function **f** may be defined by giving a Boolean expression

$$f = \bar{x}.y.z + \bar{x}.\bar{y}.z + x.z$$

$$f = x.\bar{y} + y.z$$

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

There is not a unique expression

Boolean Algebra

- Let $\mathbf{f} \in \mathbf{B_n}$

$$\mathbf{f} = \sum (\alpha_j \,.\, \Pi \tilde{\mathbf{x}}_\mathbf{i})$$

$$\mathbf{f} = \bar{x}.y.z + \bar{x}.\bar{y}.z + \bar{x}.y.z + x.y.z$$

min-term

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean Algebra

- Let $\mathbf{f} \in \mathbf{B_n}$

$$\mathbf{f} = \prod (\beta_j + \sum \tilde{\mathbf{x}}_\mathbf{i})$$

$$f = (x+y+z) \,.\, (x+\bar{y}+z) \,.$$

$$(x+\bar{y}+z) \,.\, (\bar{x}+\bar{y}+z)$$

max-term

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean Algebra

- Let $\mathbf{f} \in \mathbf{B_n}$ **f** is said independent from the variable $\mathbf{x_i}$

$$\forall\, v \in B^n,\ v = (x_1, \ldots, x_i, \ldots, x_n)$$

$$f\,(x_1, \ldots, x_i, \ldots, x_n) = f\,(x_1, \overline{\ldots}, x_i, \ldots, x_n)$$

Boolean Algebra

- Let $f \in \mathbf{B}_n$

  $\exists!\ f_{i0}\ ,\ f_{i1}$ independent from the variable x_i

  $$f = x_i \,.\, f_{i1} + \overline{x_i} \,.\, f_{i0}$$

  Shannon decomposition

Boolean Algebra

- Let $f \in \mathbf{B_n}$

$$f = x_i \,.\, f_{i1} + \overline{x_i} \,.\, f_{i0}$$

$$f = x.(\overline{y}+z) + \overline{x}.(y.z)$$

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean Algebra

- Let $\mathbf{f = x_i \,.\, f_{i1} + \overline{x_i} \,.\, f_{i0}}$

  if $\mathbf{f}$ is independent from the variable $\mathbf{x_i}$ $\mathbf{f = f_{i0} = f_{i1}}$

  $$\mathbf{f_{i0} \oplus f_{i1} = 0}$$

  if $\mathbf{f_{i0} \oplus f_{i1} = 0}$ then $\mathbf{f}$ is insensitive to $\mathbf{x_i}$

  notion of derivative

Boolean Algebra

- Let $f = x_i \cdot f_{i1} + \overline{x_i} \cdot f_{i0}$

$$\boxed{\frac{\partial f}{\partial x_i} = f_{i0} \oplus f_{i1}}$$

Boolean Algebra

- Let $f = x_i \cdot f_{i1} + \overline{x_i} \cdot f_{i0}$

  f may be sensitive to $\mathbf{x_i}$ in two ways

$$\frac{\partial f}{\partial x_i} = f_{i1} \cdot \overline{f_{i0}} + \overline{f_{i1}} \cdot f_{i0}$$

$f_{i1} \cdot \overline{f_{i0}}$ and $\overline{f_{i1}} \cdot f_{i0}$ cannot be **1** for the same vector

Boolean Algebra

- $f = x_i \cdot f_{i1} + \overline{x_i} \cdot f_{i0}$ $\qquad \dfrac{\partial f}{\partial x_i} = f_{i1} \cdot \overline{f_{i0}} + \overline{f_{i1}} \cdot f_{i0}$

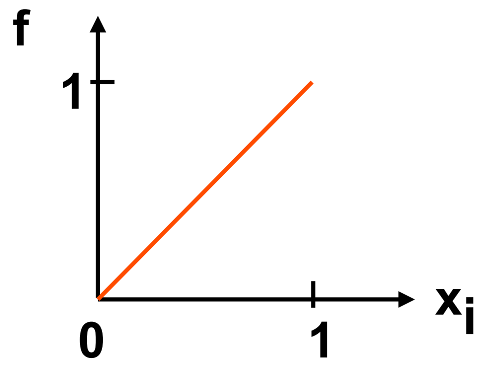
if $f_{i1} \cdot \overline{f_{i0}}\,(v) = 1$, f varies in a direct way with x_i
f is a positive function of x_i

if $\overline{f_{i1}} \cdot f_{i0}\,(v) = 1$, f varies in an opposite way with x_i
f is a negative function of x_i

$$\boxed{\frac{\partial f^{+}}{\partial x_i} = f_{i1} \cdot \overline{f_{i0}}} \qquad \boxed{\frac{\partial f^{-}}{\partial x_i} = \overline{f_{i1}} \cdot f_{i0}}$$

Boolean Algebra

- $$\frac{\partial f^{+}}{\partial x_i} = f_{i1} \cdot \overline{f_{i0}}$$



- $$\frac{\partial f^{-}}{\partial x_i} = \overline{f_{i1}} \cdot f_{i0}$$