

Autumn College on Non-Equilibrium Quantum Systems
May 2-13, 2011
Buenos Aires, Argentina

Norman Birge, Michigan State University

Measurements out of equilibrium in normal and superconducting metals

- I. Measuring Nonequilibrium $f(E)$ by tunneling spectroscopy
- II. Nonequilibrium experiments in S/N hybrid systems
- III. Spin-triplet supercurrent in S/F/S Josephson junctions

Background – Shot Noise in Mesoscopic Systems

From Marcus Buttiker's lecture:

$$G = \frac{e^2}{2\pi\hbar} \sum_n T_n$$

$$S_2(\omega=0) = \frac{e^2}{\pi\hbar} \left[2k_B T \sum_n T_n^2 + eV \coth\left(\frac{eV}{2k_B T}\right) \sum_n T_n (1 - T_n) \right]$$

$eV \ll k_B T$: $S_2(\omega=0) = 4k_B T G$ Johnson-Nyquist noise

$eV \gg k_B T$: $S_2(\omega=0) = 2FeI$ Shot noise

Fano factor: $F = \frac{\langle T(1-T) \rangle}{\langle T \rangle}$

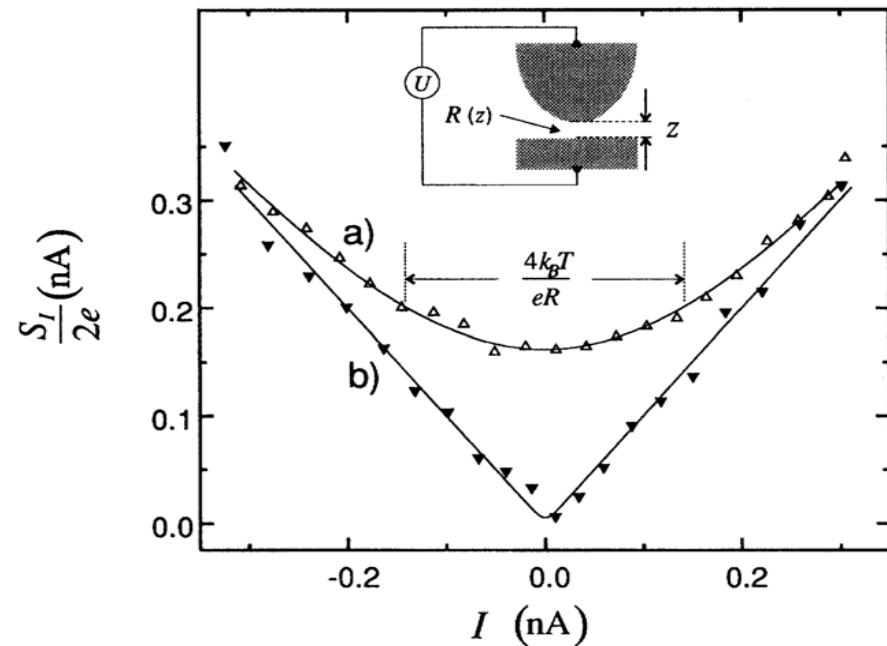
Crossover from Johnson-Nyquist to Shot Noise

Tunnel junction: all T_n 's are small

$$\begin{aligned} S_2(\omega = 0) &= 2eV \coth\left(\frac{eV}{2k_B T}\right) \frac{e^2}{2\pi\hbar} \sum_n T_n \\ &= 2eI \coth\left(\frac{eV}{2k_B T}\right) \end{aligned}$$

$eV \ll k_B T$: Johnson-Nyquist noise

$eV \gg k_B T$: Shot noise

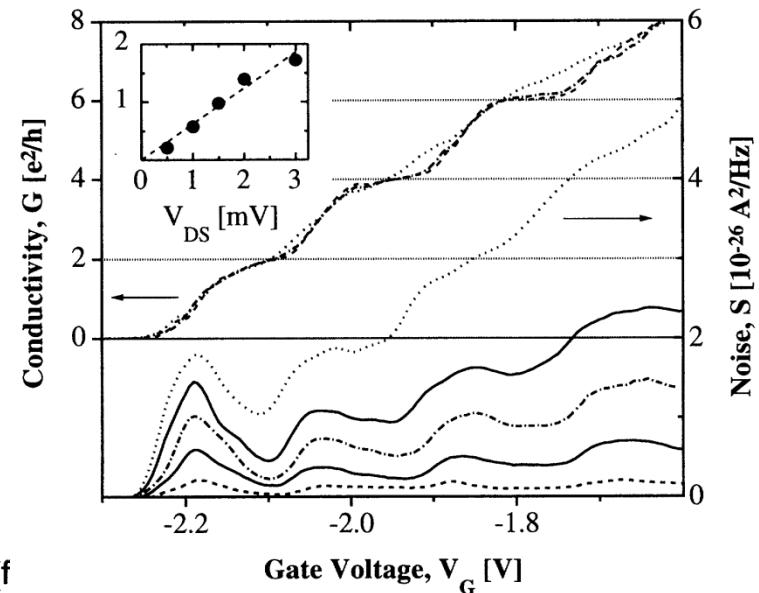
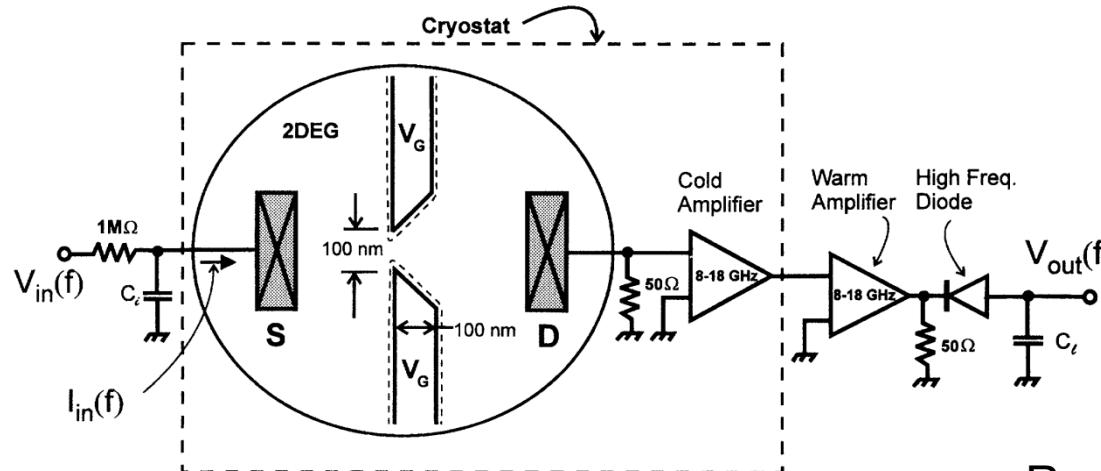


Birk et al., Phys. Rev. Lett. **75**, 1610 (1995).

Shot Noise in a Quantum Point Contact

$$S_2(\omega = 0) = 2FeI$$

Fano factor: $F = 1 - T$
for single channel conductor



Reznikov et al., PRL 75, 3340 (1995)

Shot Noise in a Diffusive Wire

$$S_2(\omega = 0) = 2FeI$$

Fano factor: $F = \frac{\langle T(1-T) \rangle}{\langle T \rangle} = \frac{1}{3}$

Follows from distribution of transmission eigenvalues in diffusive wire:

$$P(T) = \begin{cases} 0 & \text{for } T < T_{\min} \\ \frac{l}{2L} \frac{1}{T\sqrt{1-T}} & \text{for } T_{\min} < T < 1 \end{cases} \quad \text{where} \quad T_{\min} = 4 \exp\left[\frac{-2L}{l}\right]$$

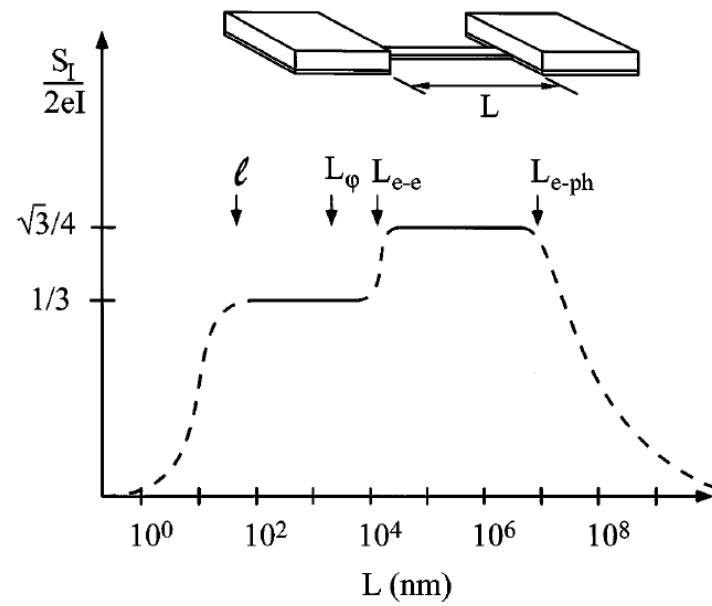
$$\langle T \rangle = \frac{l}{L} \quad \langle T(1-T) \rangle = \frac{l}{3L} \quad l \text{ is mean free path, } L \text{ is length of wire}$$

Dorokhov, Sol. St. Comm. 51, 381 (1984);
Imry, Europhys. Lett. 1, 249 (1986)

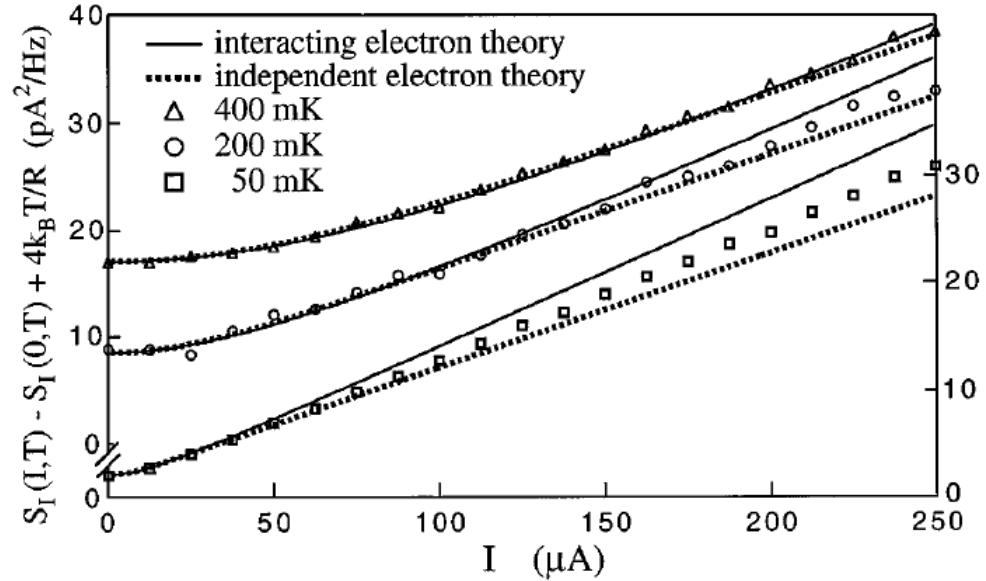
Shot Noise in a Diffusive Wire

$$S_2(\omega = 0) = 2FeI$$

Fano factor: $F = \frac{\langle T(1-T) \rangle}{\langle T \rangle} = \frac{1}{3}$



In presence of strong inelastic e-e scattering, F increases to $\sqrt{3}/4$. With strong e-phonon scattering, F decreases to 0.

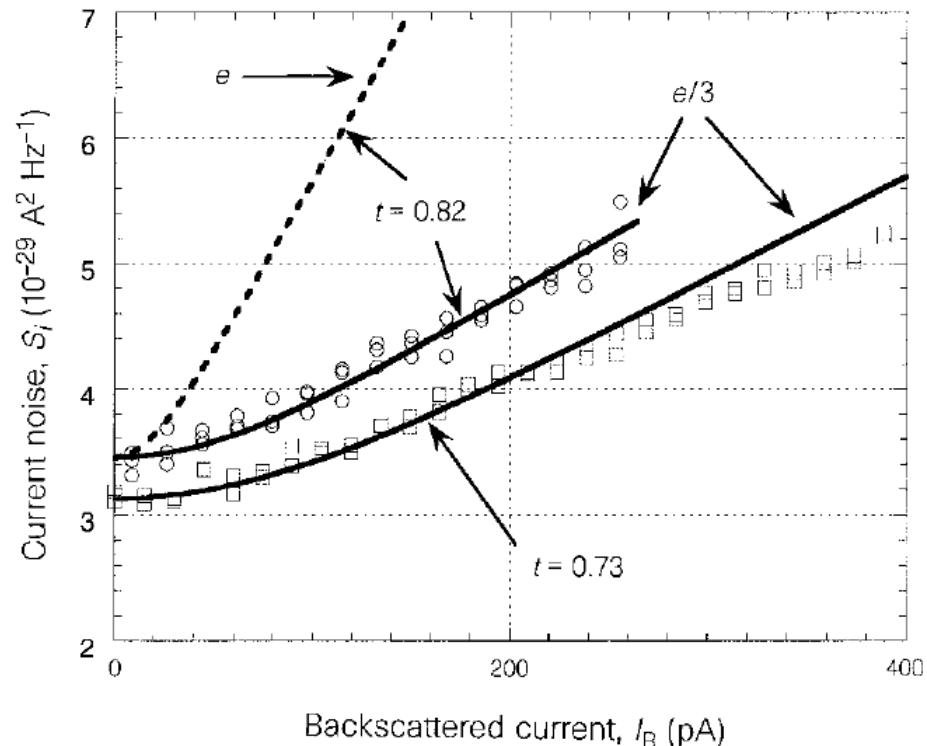
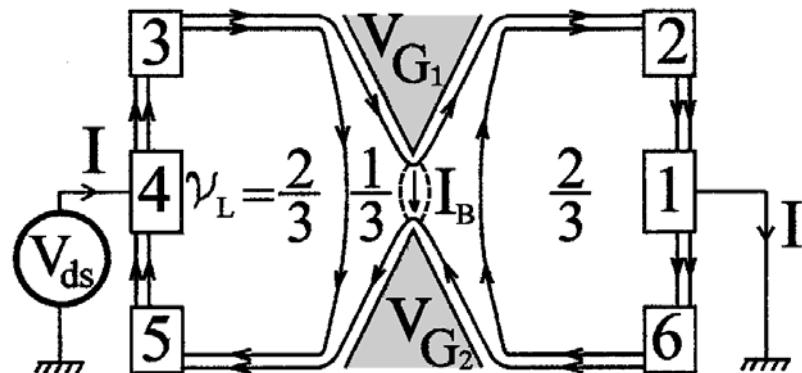


Steinbach et al., PRL 76, 3806 (1996)

Measurement of Q^* in the Fractional Quantum Hall Effect

$$S_2(\omega = 0) = 2FQ^*I \quad Q^* = \frac{e}{3}$$

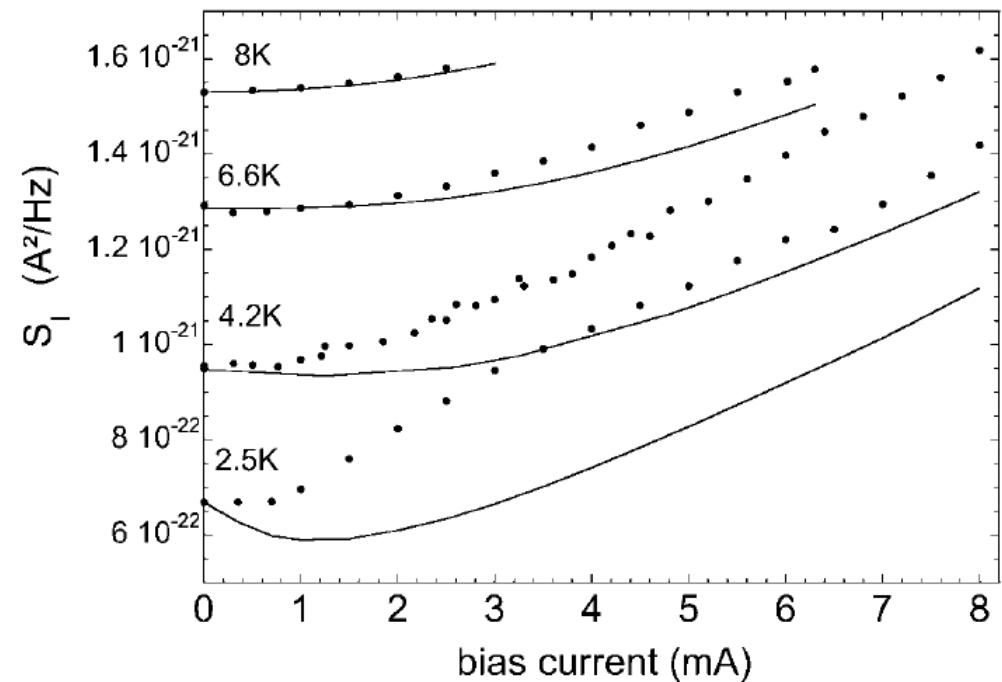
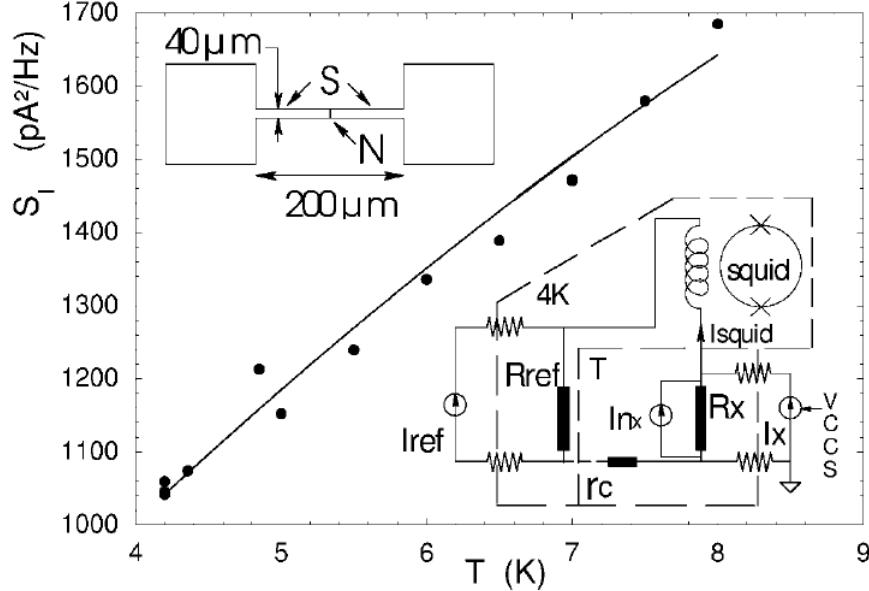
$$S_i = 2g_0t(1-t) \left[QV \coth \left(\frac{QV}{2k_B T} \right) - 2k_B T \right] + 4k_B T g_0 t$$



De-Picciotto et al., Nature 389, 162 (1997)
 Saminadayar et al., PRL 79, 2526 (1997)

Doubling of Shot Noise in a Diffusive S/N junction

$$S_2(\omega = 0) = 2FqI \quad q = 2e$$

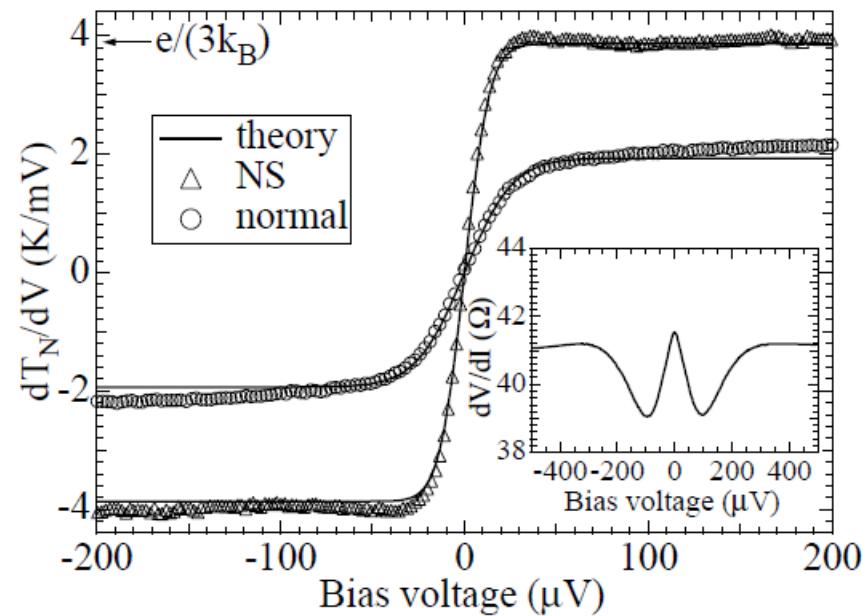
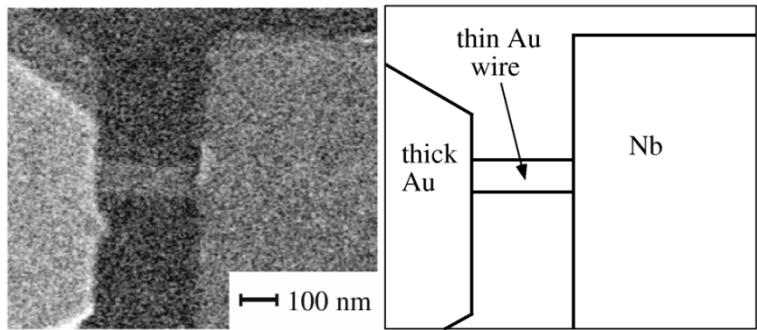


Jehl et al., PRL 83, 3398 (1999)

Doubling of Shot Noise in a Diffusive S/N junction

$$S_2(\omega = 0) = 2FqI \quad \text{Fano factor: } F = \frac{\langle T(1-T) \rangle}{\langle T \rangle} = \frac{1}{3}$$

$$q = 2e$$



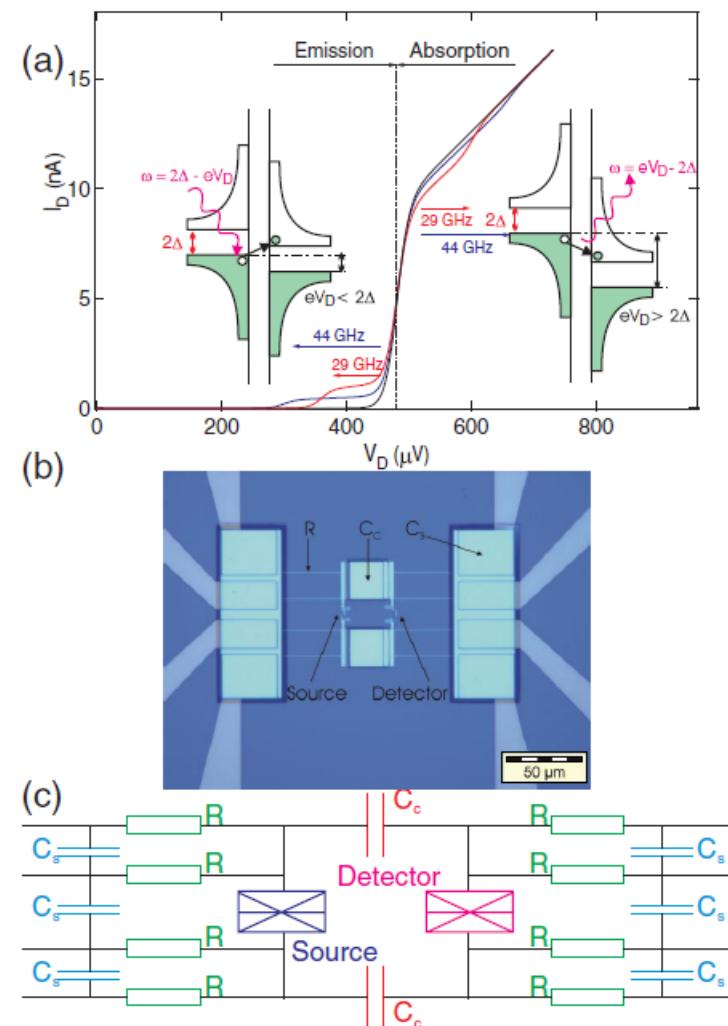
A. A. Kozhevnikov et al., PRL 84, 3398 (2000)

Quantum Noise: Emission vs. Absorption of Photons

$$\hbar\omega > eV, k_B T$$

$$S_2(\hbar\omega) \neq S_2(-\hbar\omega)$$

DeBlock et al., Science 301 (2003)
Billangeon et al., PRL 96, 136804 (2006)



Autumn College on Non-Equilibrium Quantum Systems
May 2-13, 2011
Buenos Aires, Argentina

Norman Birge*, Michigan State University

**Lecture I: Electron Energy Exchange
in Metal Wires and Carbon Nanotubes**

Special thanks to Hugues Pothier and to Nadya Mason, who produced
many of the slides for this lecture!

* Work supported by NSF DMR

Outline

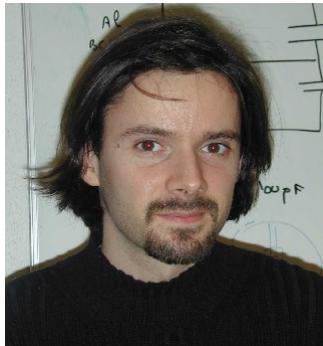
- Part 1: Diffusive Metal Wires
 - The distribution function, $f(E)$, in nonequilibrium situations
 - How to measure $f(E)$ using tunnel spectroscopy
 - First measurements of $f(E)$ in diffusive metal wires
 - Magnetic impurities: theory (Kaminsky & Glazman)
 - Magnetic impurities: experiments
- Part 2: Carbon Nanotubes
 - Introduction to carbon nanotubes
 - $f(E)$ in ballistic vs. diffusive wires
 - Experimental issues with nanotubes
 - First measurements of $f(E)$ in single-wall tubes
 - Future prospects

Collaborators – Part 1

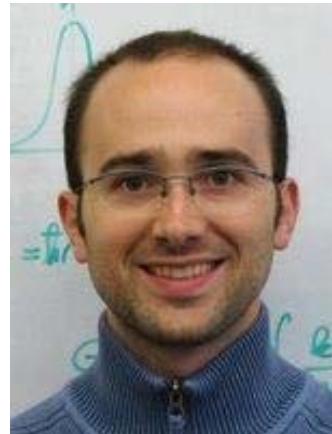
Quantronics Group, CEA-Saclay, France



S. Gueron



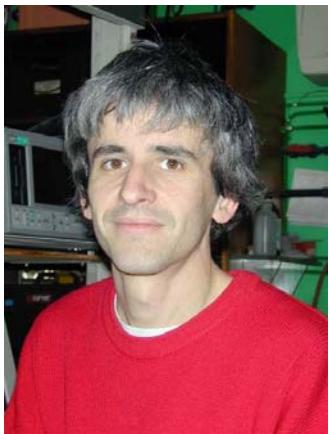
F. Pierre



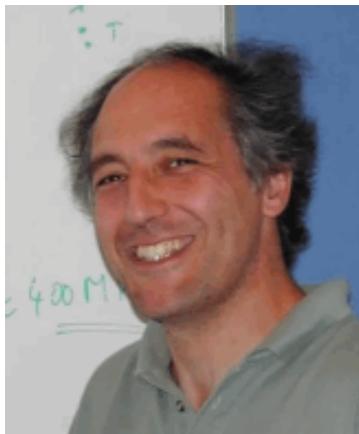
B. Huard



A. Anthore



H. Pothier



D. Esteve



M. Devoret

Collaborators – Part 2

MSU



G. Al-Zoubi
(MSU)

University of Illinois at Urbana-Champaign



T. Dirks

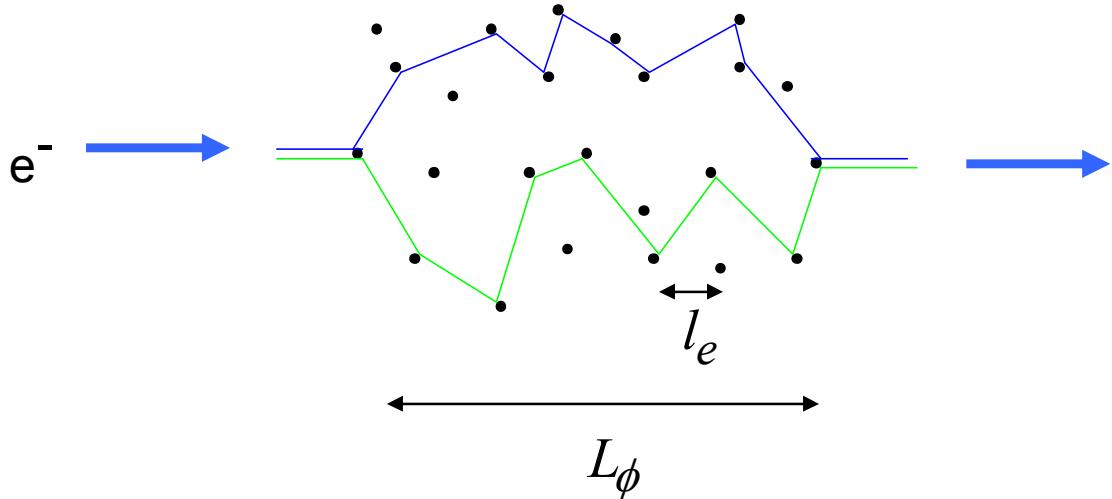


Y.-F. Chen



N. Mason

Electron transport in the diffusive regime



1. Elastic scattering (film boundaries, impurities)

→ diffusive states $D = \frac{1}{3} v_F l_e$ $l_e = v_F \tau_e$

2. Inelastic scattering (phonons, other electrons, spins)

→ loss of phase coherence $L_\phi = \sqrt{D \tau_\phi}$

→ energy exchange between electrons $L_\varepsilon = \sqrt{D \tau_\varepsilon}$

Background: Shot noise in diffusive metal wires

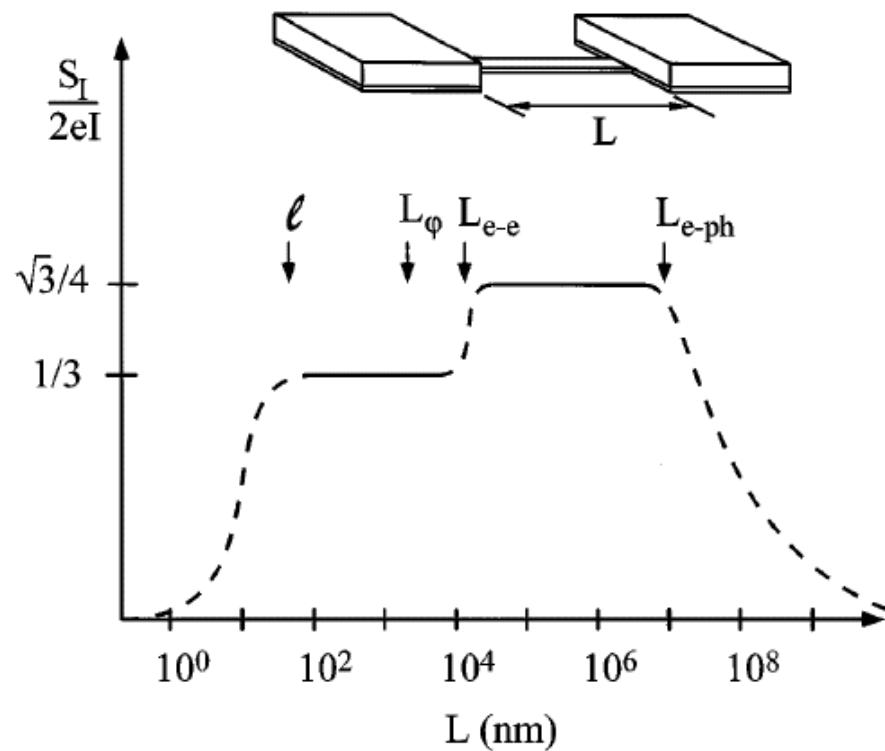
Steinbach, Martinis and Devoret, PRL 76, 3806 (1996)

Classical Noise Theory:

Nagaev 1992, 1995

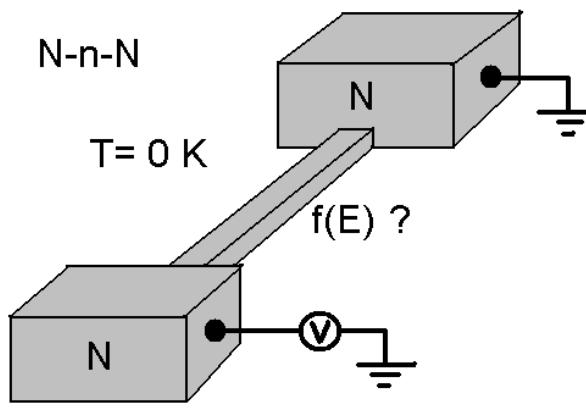
Kozub & Rudin, 1995

$$S_I = \frac{4}{RL} \iint dx dE f(x, E) (1 - f(x, E))$$

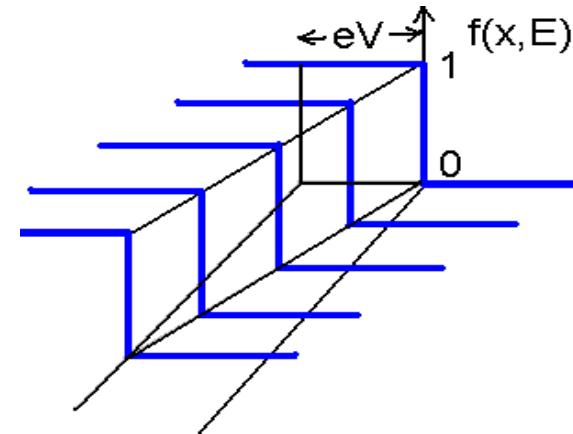


What does $f(x, E)$ look like?

Distribution function -- textbook case (no shot noise)



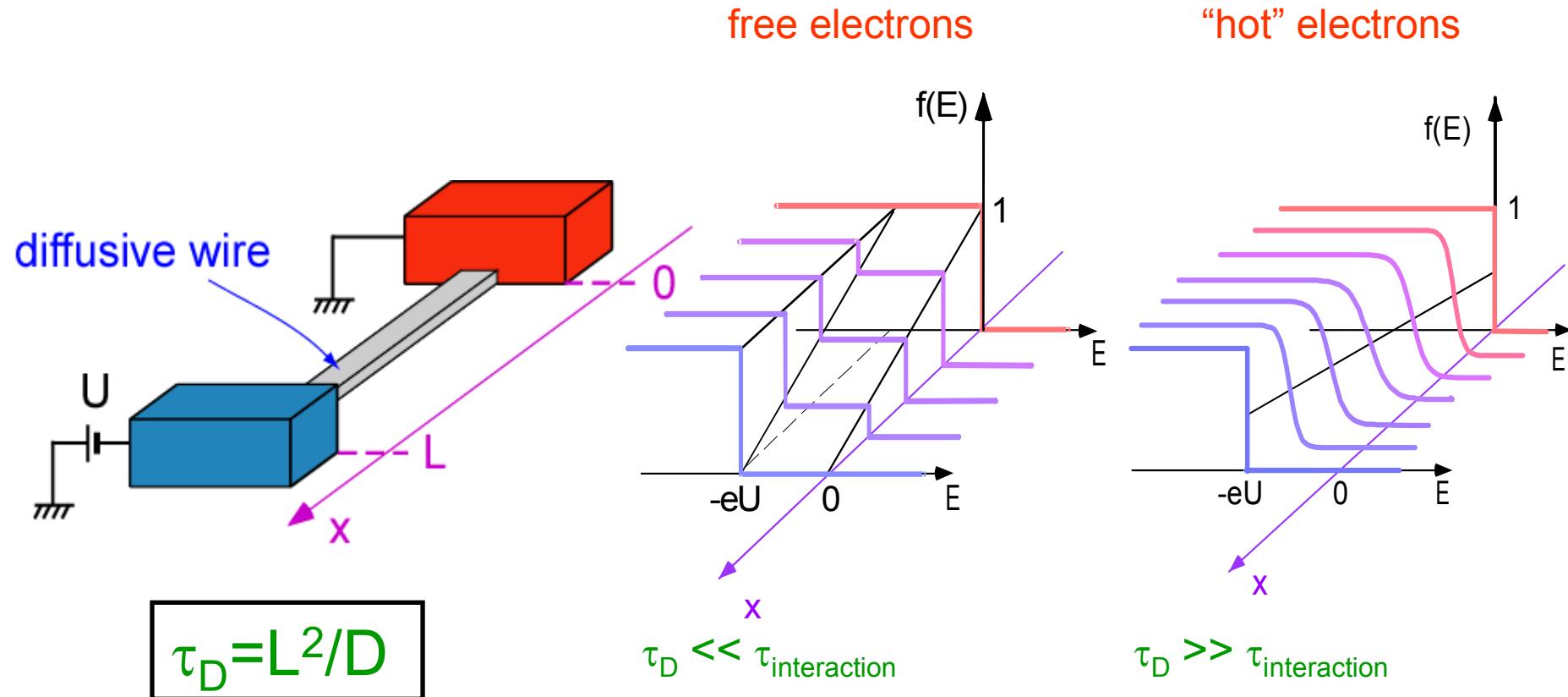
$$\tau_D = L^2/D$$



Assumes complete thermalization -- $\tau_D \gg \tau_{\text{electron-phonon}}$

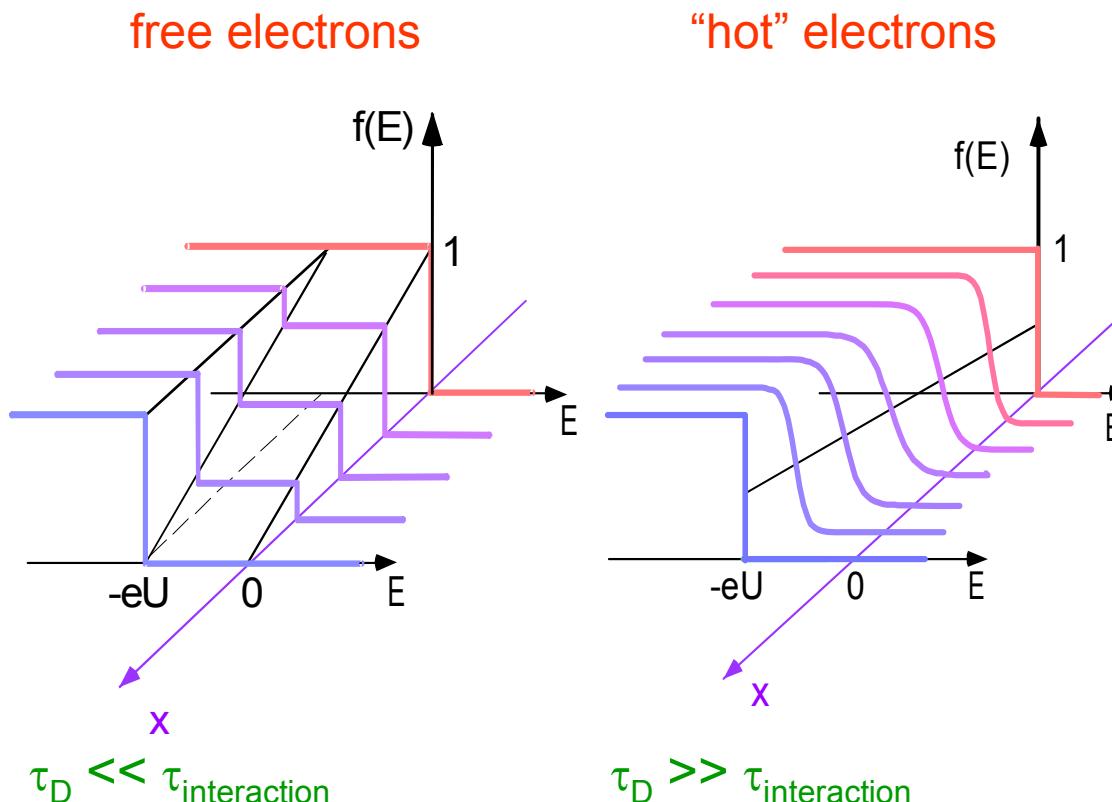
Never true in mesoscopic metal samples at low T!

Distribution function for $\tau_D \ll \tau_{\text{electron-phonon}}$



$f(x,E)$ shaped by energy exchange

The shot noise is almost the same for these very different distribution functions!



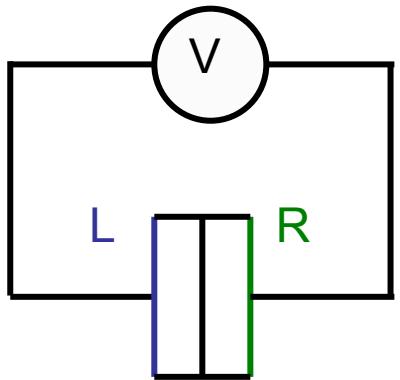
Fano factors: $F = \frac{1}{3} = 0.33\dots$

$$F = \frac{\sqrt{3}}{4} = 0.433$$

How to measure $f(x,E)$ directly?

(local electron energy distribution function)

Aside 1: Current through a tunnel junction

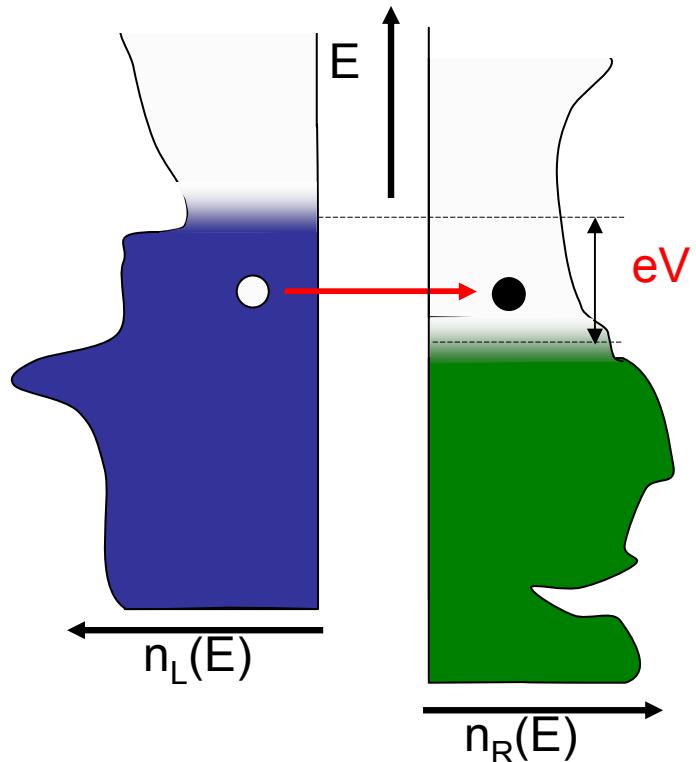


$$I = e(\Gamma_{\rightarrow} - \Gamma_{\leftarrow})$$

normalized densities of states

$$\Gamma_{\rightarrow} = \frac{2\pi v_F^2}{\hbar} \int dE |\langle M \rangle|^2 n_L(E) n_R(E + eV) f_L(E) (1 - f_R(E + eV))$$

$$\Gamma_{\leftarrow} = (1 - f_L(E)) f_R(E + eV)$$



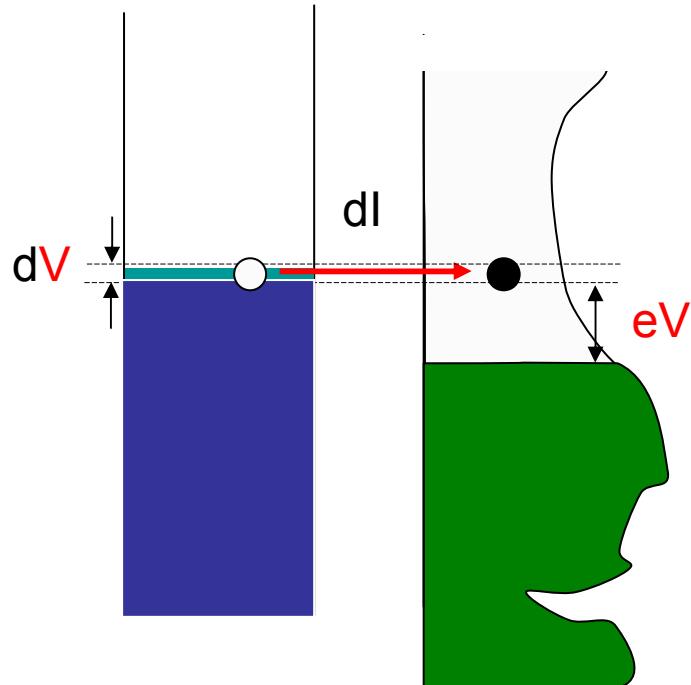
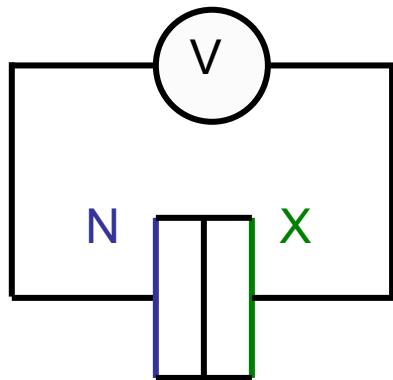
$$I = \frac{1}{eR_T} \int dE n_L(E) n_R(E + eV) \times (f_L(E) - f_R(E + eV))$$

NN junction:

$$n(E) = 1 \quad f(E) = \begin{cases} 0 & E < 0 \\ 1 & E \geq 0 \end{cases}$$

$$\Rightarrow I = \frac{V}{R_T}$$

Conductance of an N-X junction at T=0



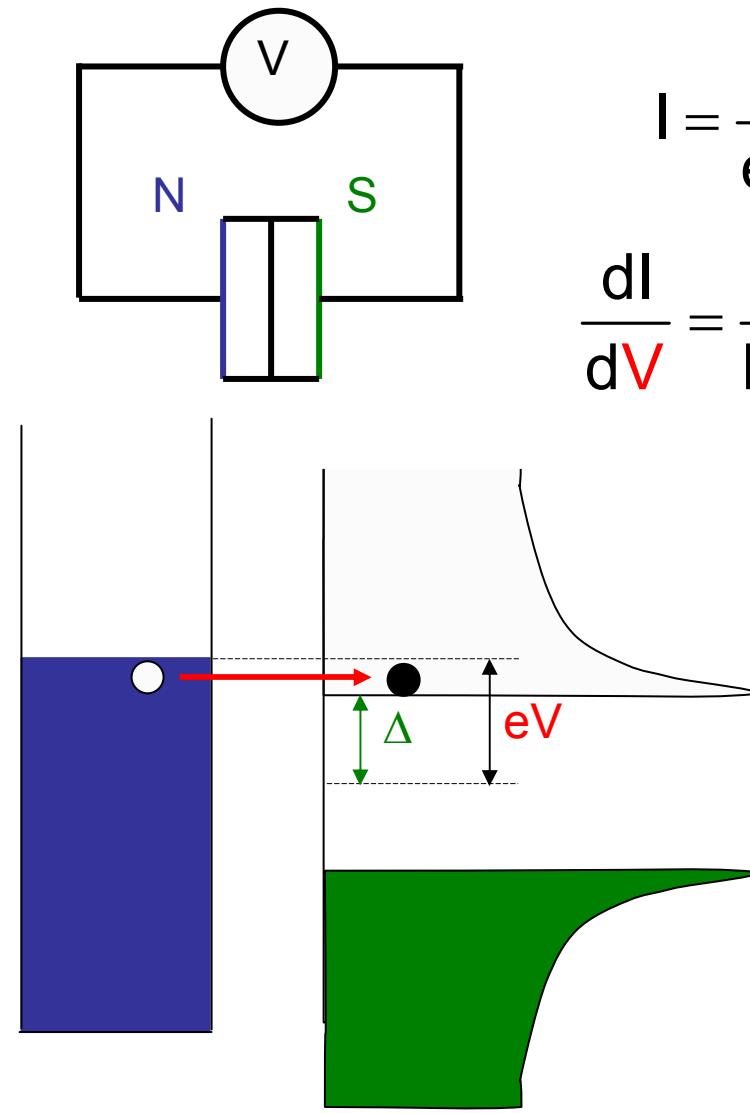
$$\begin{aligned} I &= \frac{1}{eR_T} \int dE n_L(E) n_R(E + eV) \\ &\quad \times (f_L(E) - f_R(E + eV)) \\ &= \frac{1}{eR_T} \int_{-eV}^0 dE n_X(E + eV) \\ &= \frac{1}{eR_T} \int_0^{eV} dE n_X(E) \end{aligned}$$

$$\frac{dI}{dV} = \frac{1}{R_T} n_X(eV)$$

Spectroscopy of n_X

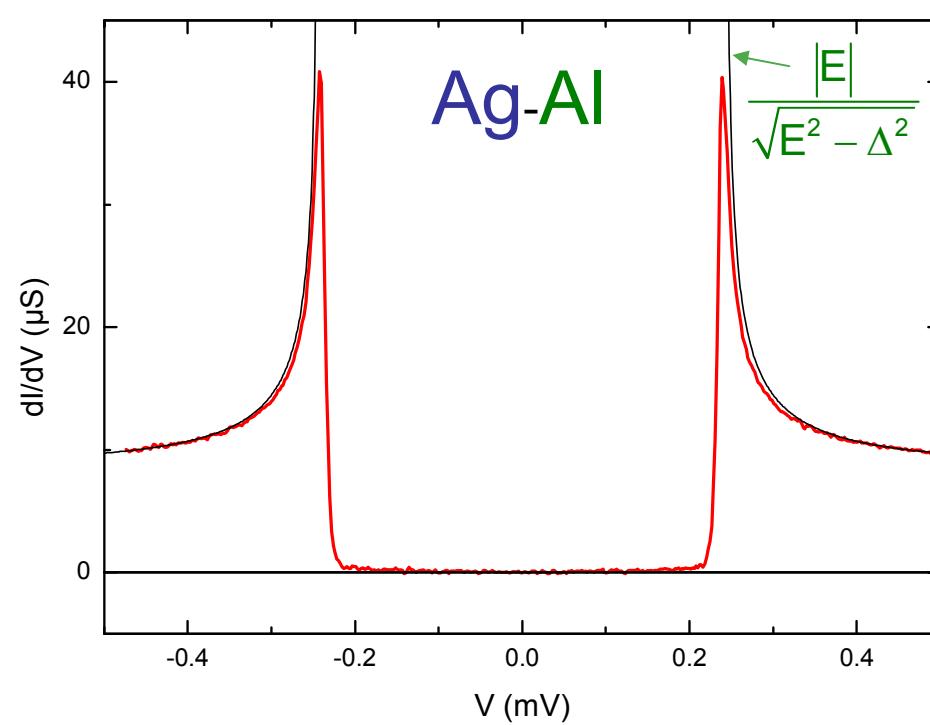
Spectroscopy of n_s of a superconductor:

I. Giaever, PRL 5, 147 (1960)



$$I = \frac{1}{eR_T} \int dE n_s(E) (f_N(E - eV) - f_s(E))$$

$$\frac{dI}{dV} = \frac{-1}{R_T} \int dE n_s(E) f'_N(E - eV)$$



ENERGY GAP IN SUPERCONDUCTORS MEASURED BY ELECTRON TUNNELING

Ivar Giaever

General Electric Research Laboratory, Schenectady, New York

(Received July 5, 1960)

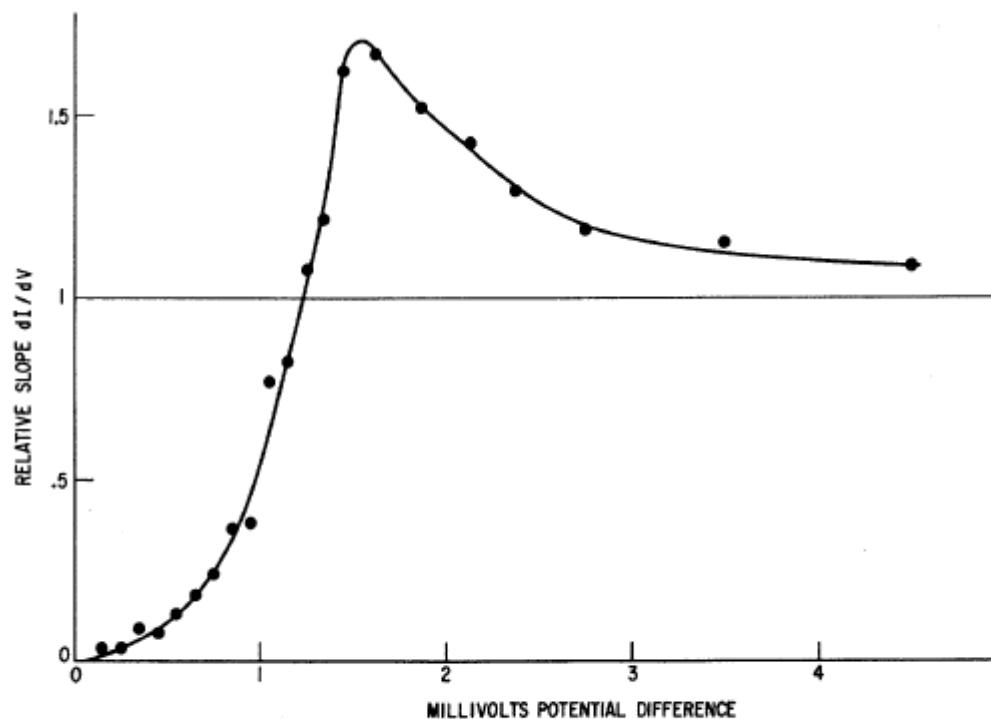
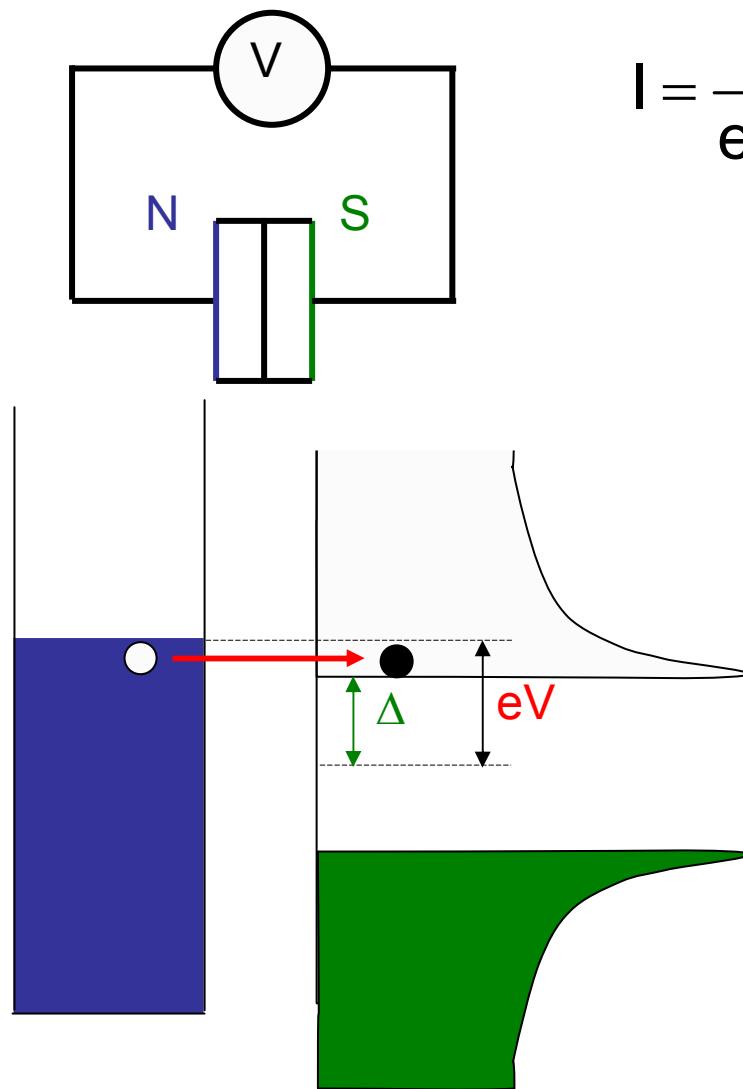
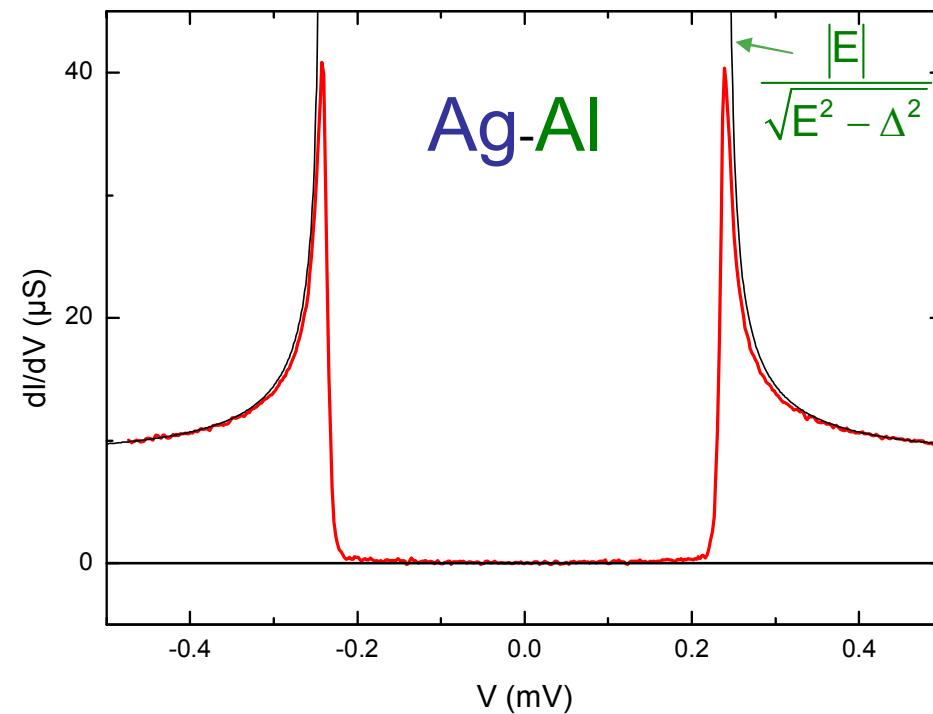


FIG. 2. From Fig. 1, slope dI/dV of curve 5 relative to slope of curve 1.

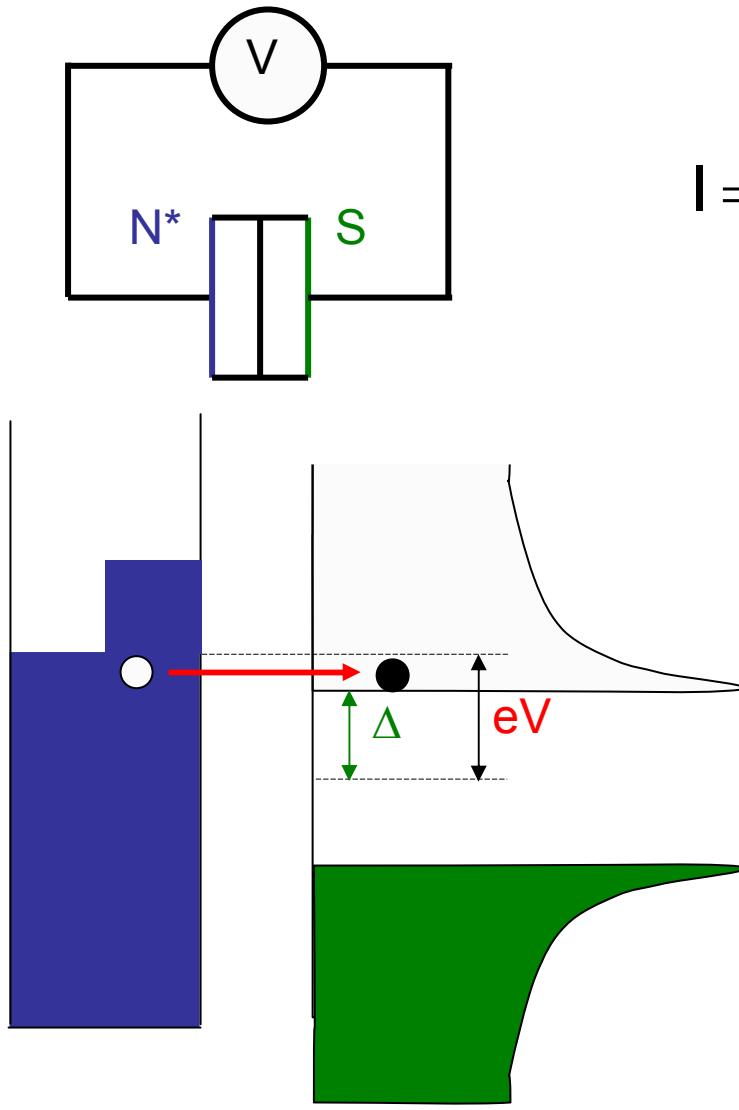
How to measure $f(E)$: tunnel spectroscopy using an N-S junction



$$I = \frac{1}{eR_T} \int dE n_S(E) (f_N(E - eV) - f_S(E))$$

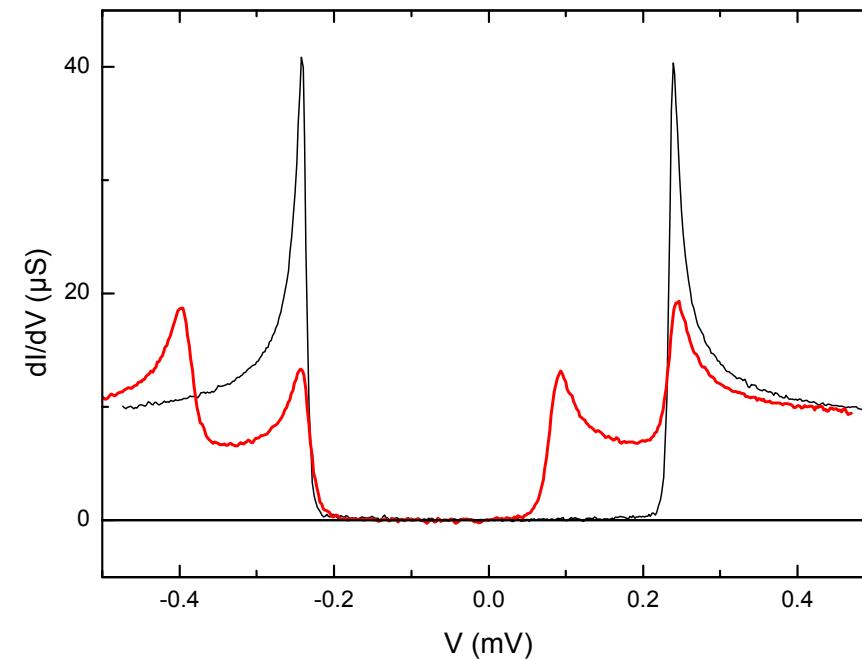


N out of equilibrium: spectroscopy of $f(E)$

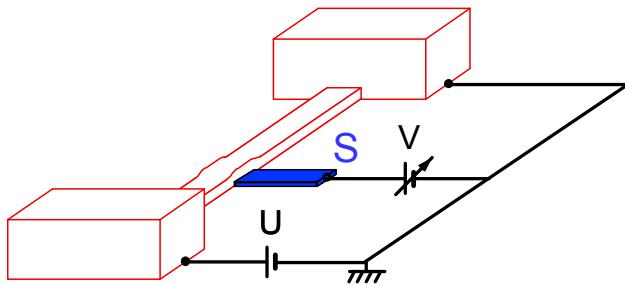


$$I = \frac{1}{eR_T} \int dE n_s(E) (f_N(E - eV) - f_s(E))$$

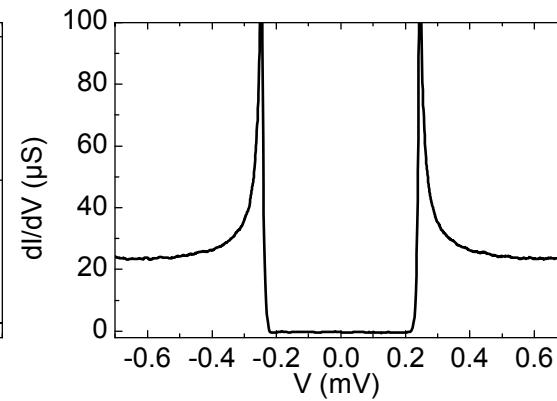
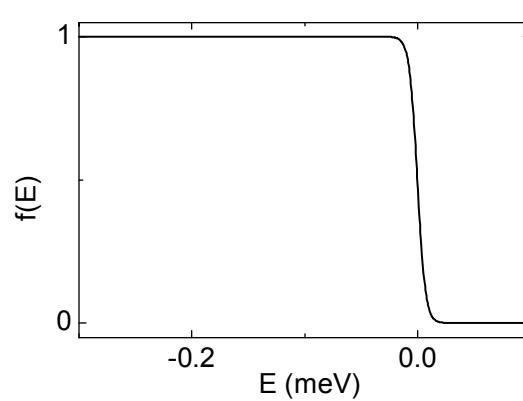
$$\frac{dI}{dV} = \frac{-1}{R_T} \int dE n_s(E) f'_N(E - eV)$$



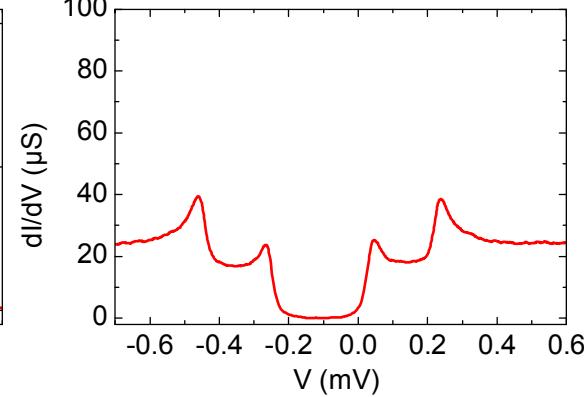
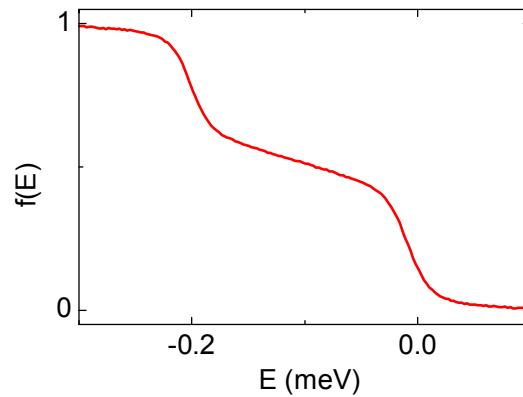
Measurement of $f(E)$ in diffusive wires



$U=0$ mV



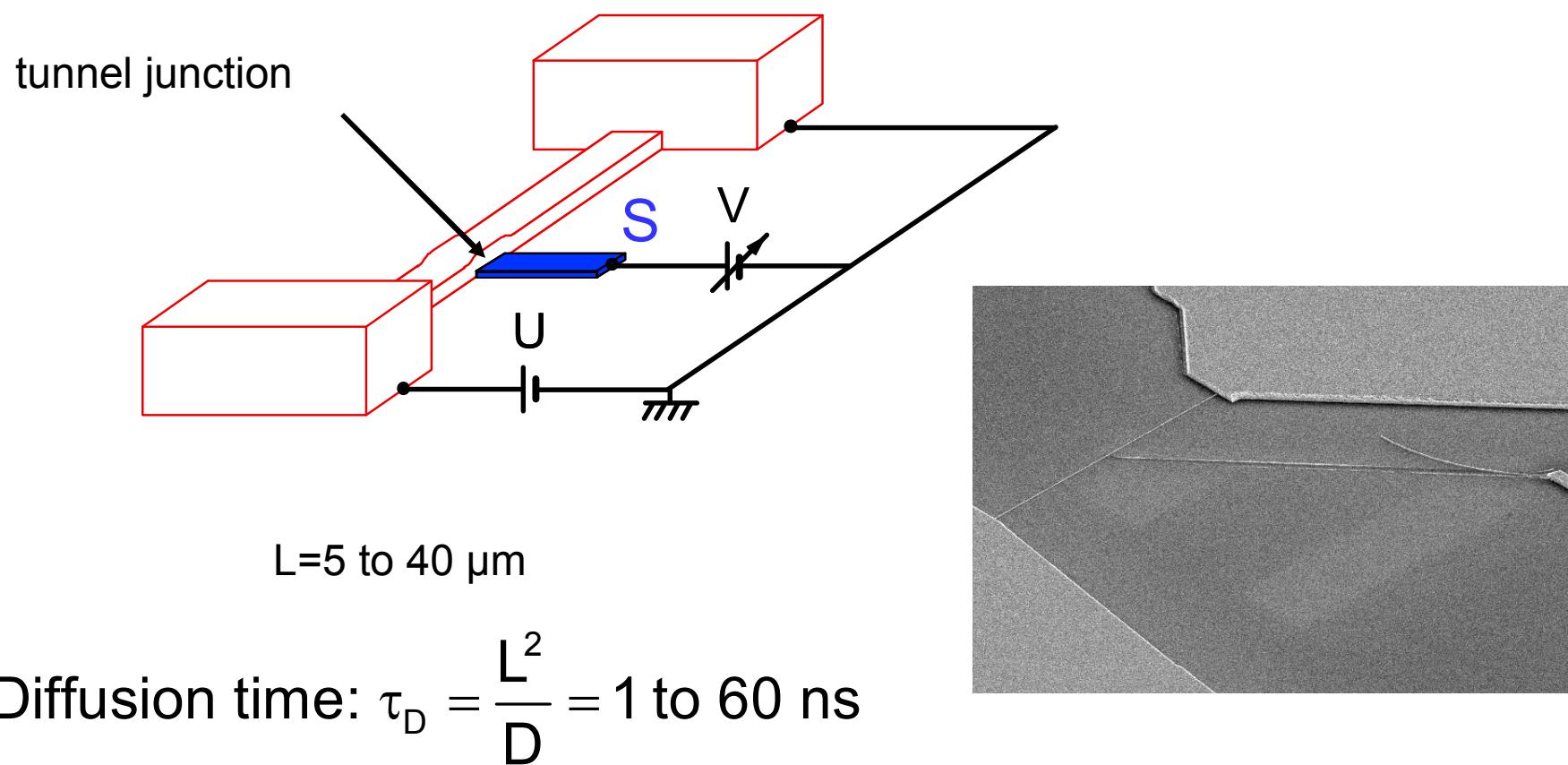
$U=0.2$ mV



$f(E)$

dI/dV

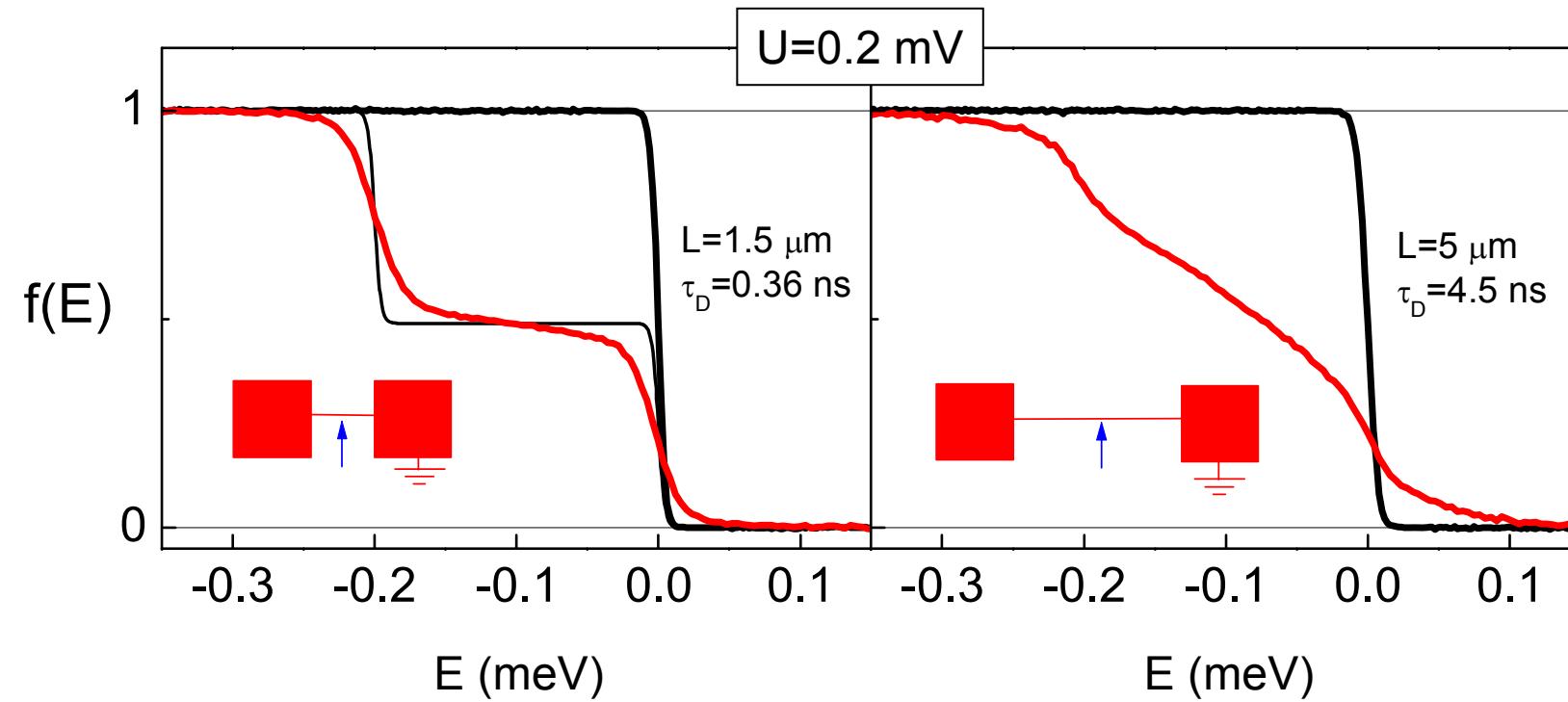
Experimental Setup



$$\text{Diffusion time: } \tau_D = \frac{L^2}{D} = 1 \text{ to } 60 \text{ ns}$$

$$\frac{dI}{dV}(V) \xrightarrow{\substack{\text{numerical} \\ \text{deconvolution}}} f(E)$$

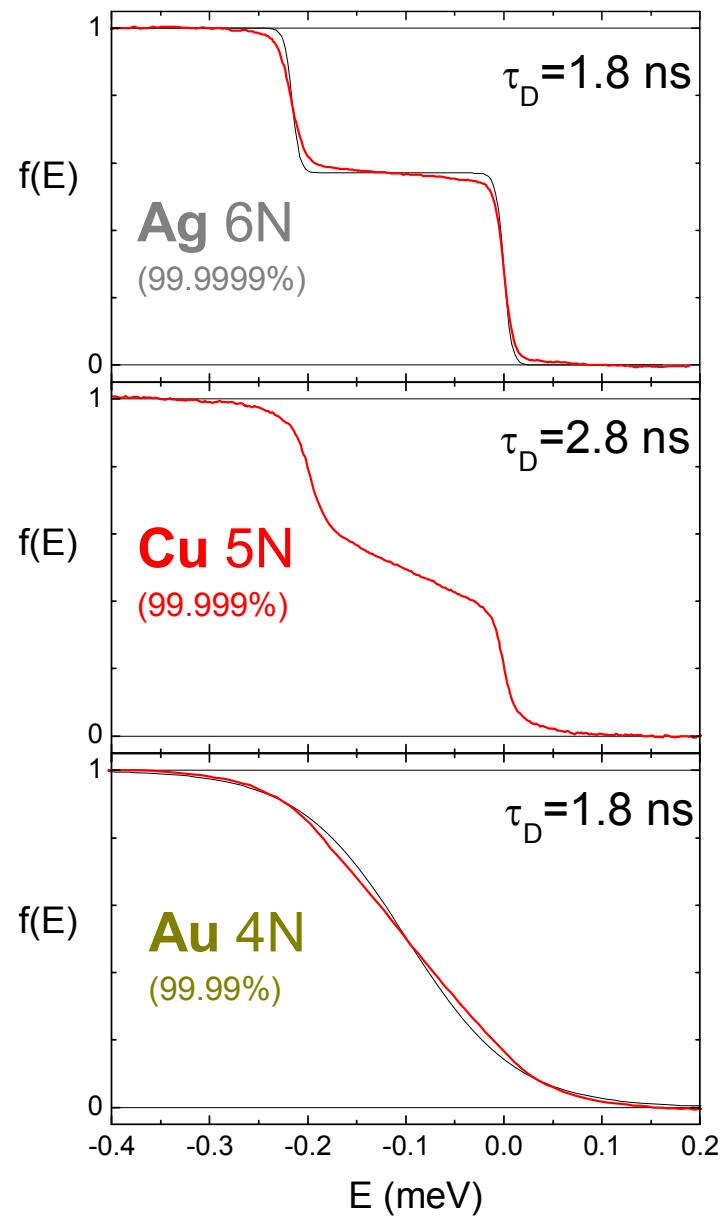
Effect of the diffusion time τ_D on $f(E)$



longer interaction time \Rightarrow more rounding

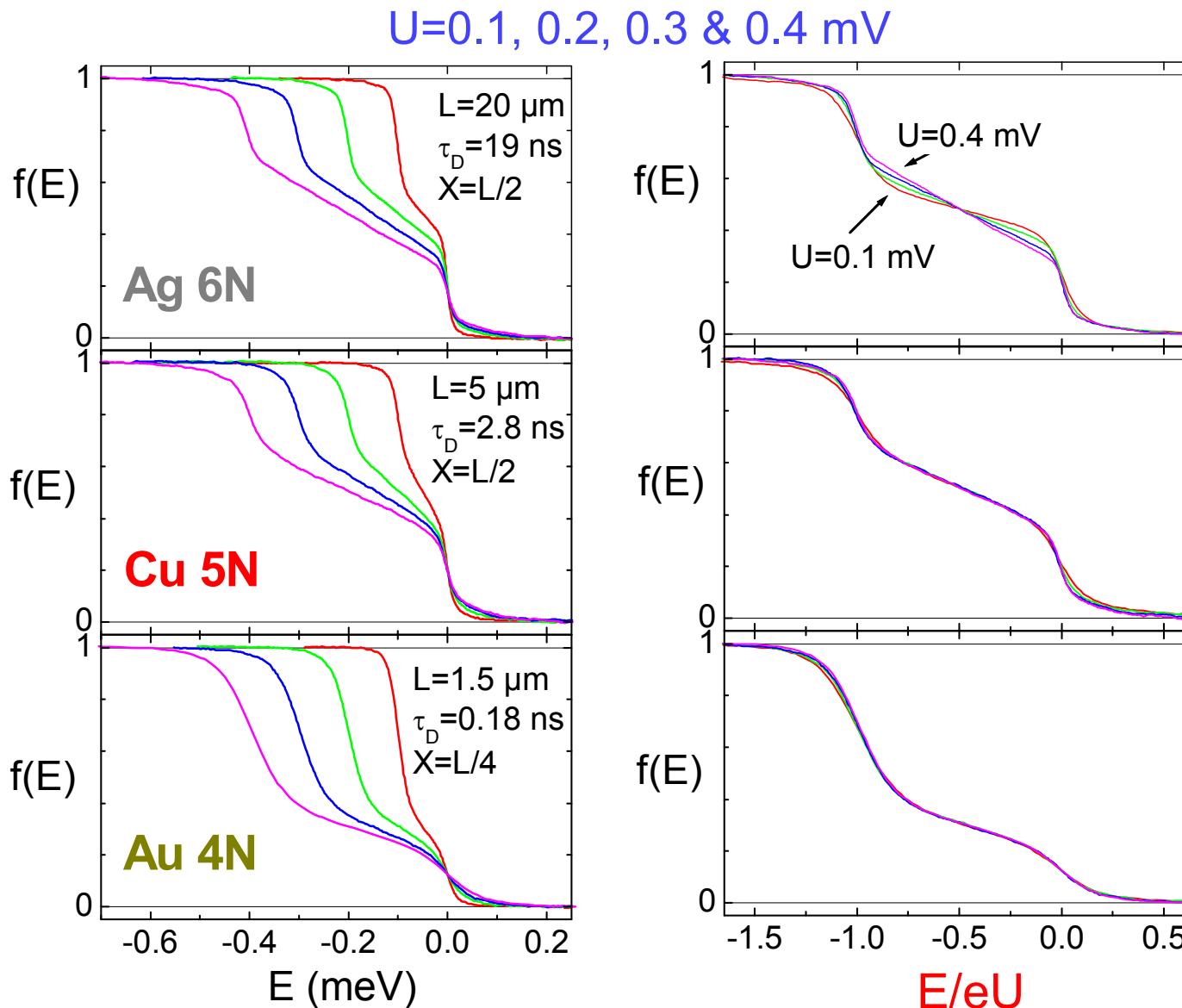
H. Pothier et al., PRL 79, 3490 (1997)

Compare strength of interactions



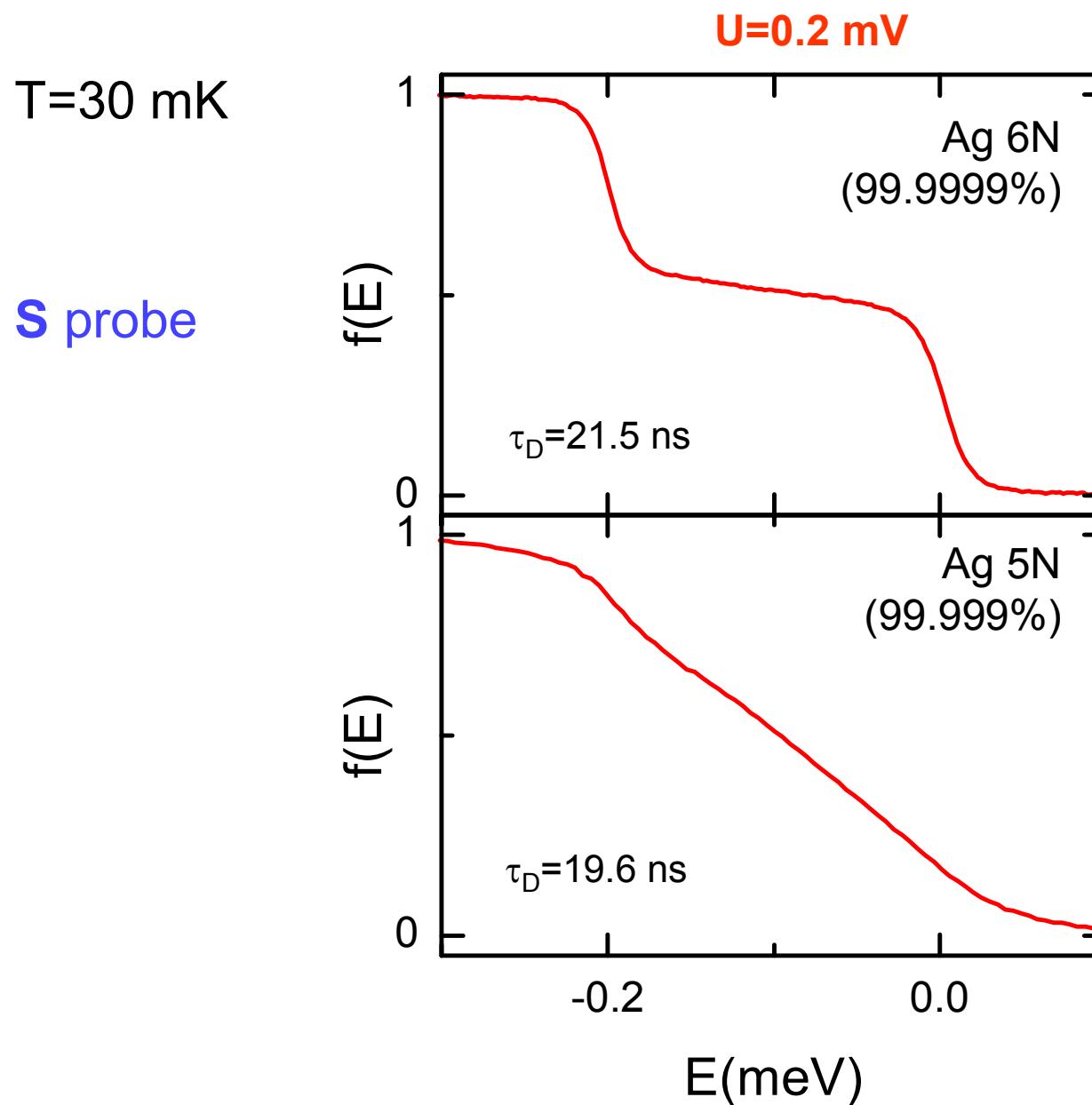
effect of material ?
effect of purity ?

Compare Dependence on U



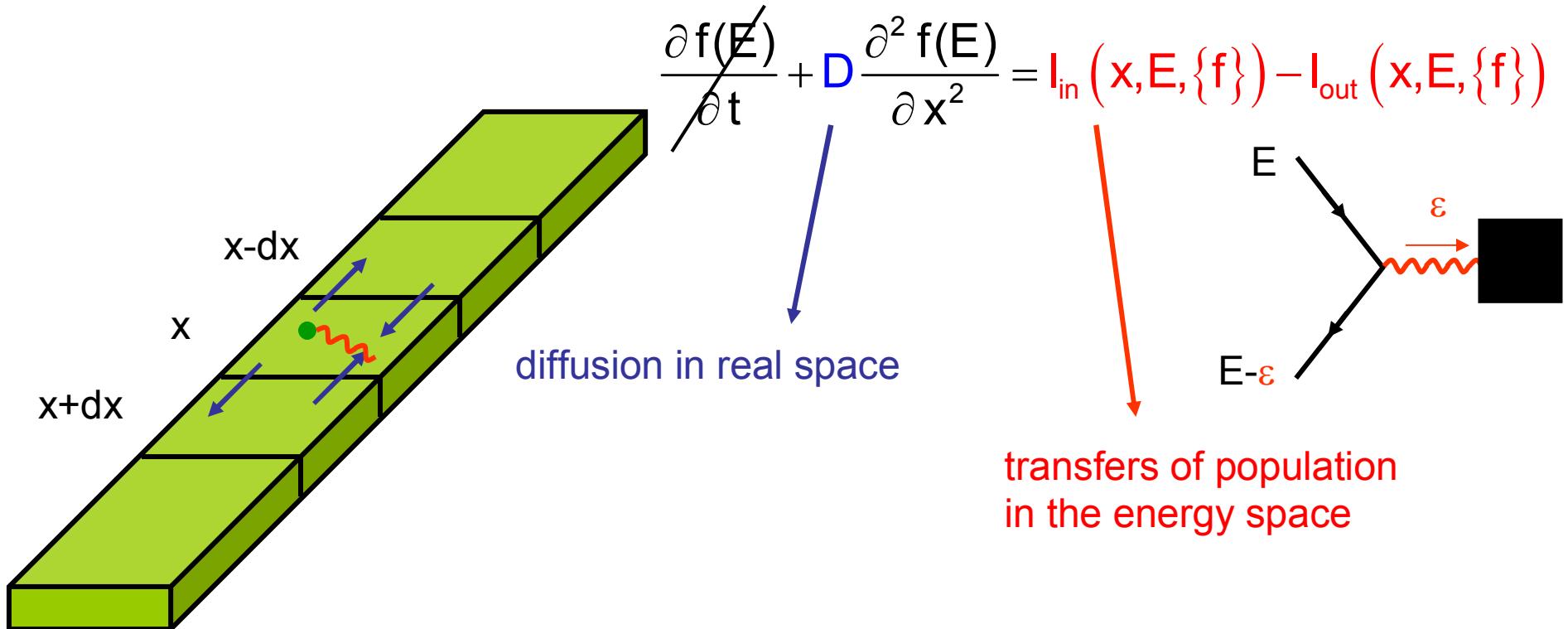
Observe scaling law in **Au 4N** & **Cu 5N** but not in **Ag 6N**

Energy Exchange Rate vs. Sample Purity



Calculation of $f(x, E)$

Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):



transfers of population
in the energy space

Boundary conditions :

$$f_{x=0}(E) = f_{x=L}(E) = \text{Fermi function}$$

Calculation of $f(x, E)$

Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):

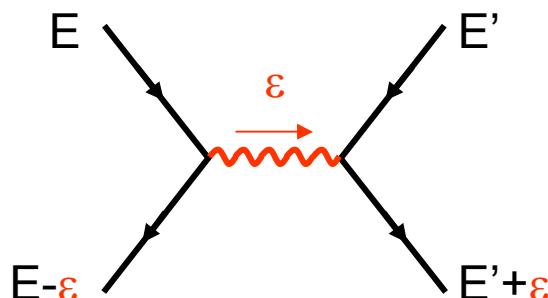
$$D \frac{\partial^2 f(E)}{\partial x^2} = I_{in}(x, E, \{f\}) - I_{out}(x, E, \{f\})$$

e-e interactions :

$$\frac{K}{\varepsilon^{3/2}}$$

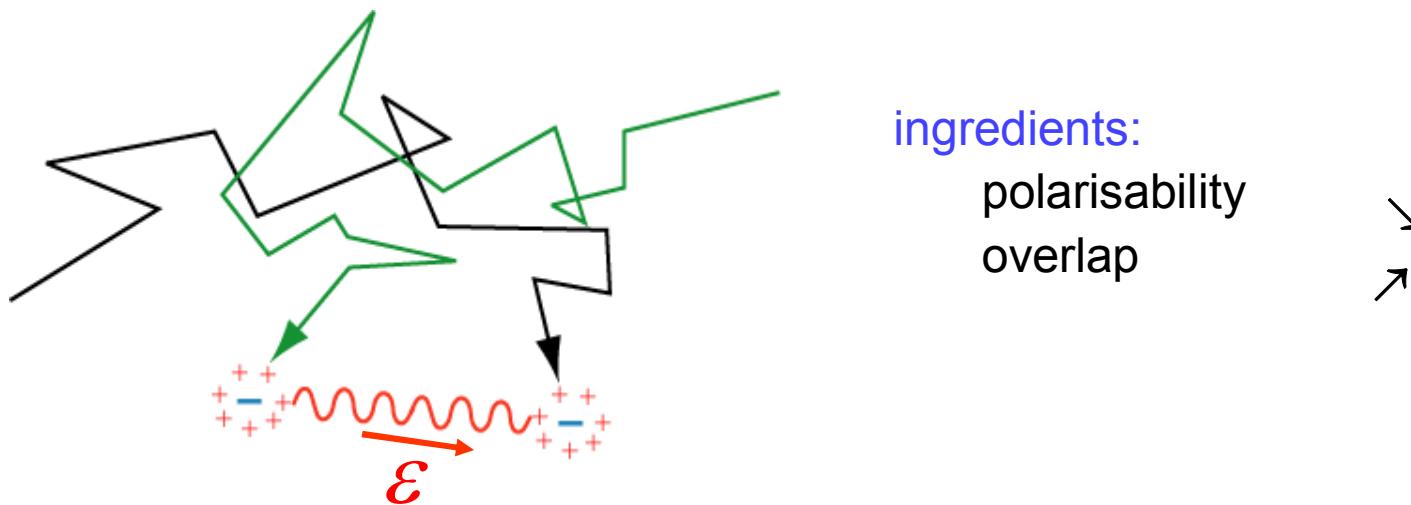
(Altshuler, Aronov,
Khmelnitskii, 1982)

$$I_{out}(x, E, \{f\}) = \int dE' d\varepsilon K(\varepsilon) f(E) [1 - f(E - \varepsilon)] f(E') [1 - f(E' + \varepsilon)]$$



Theory of screened Coulomb interaction in the diffusive regime

Altshuler, Aronov, Khmelnitskii, 1982

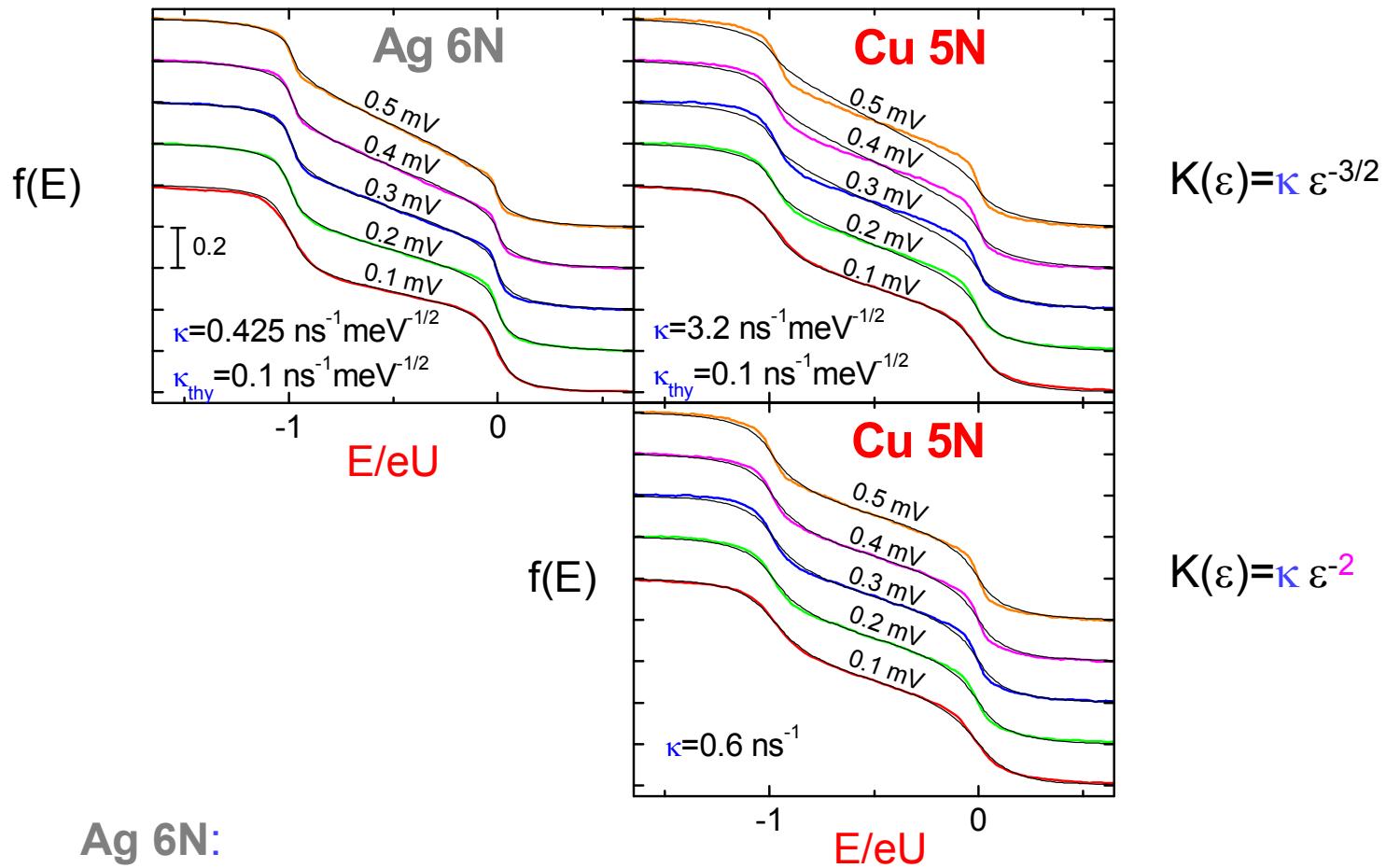


Prediction for 1D wire :

$$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$$

$$\kappa = \left(\sqrt{2D} \pi \hbar^{3/2} \nu_F S_e \right)^{-1}$$

Experiment vs. Theory



Ag 6N:

experiment agrees with theory

Cu 5N, Au 4N, Ag 5N:

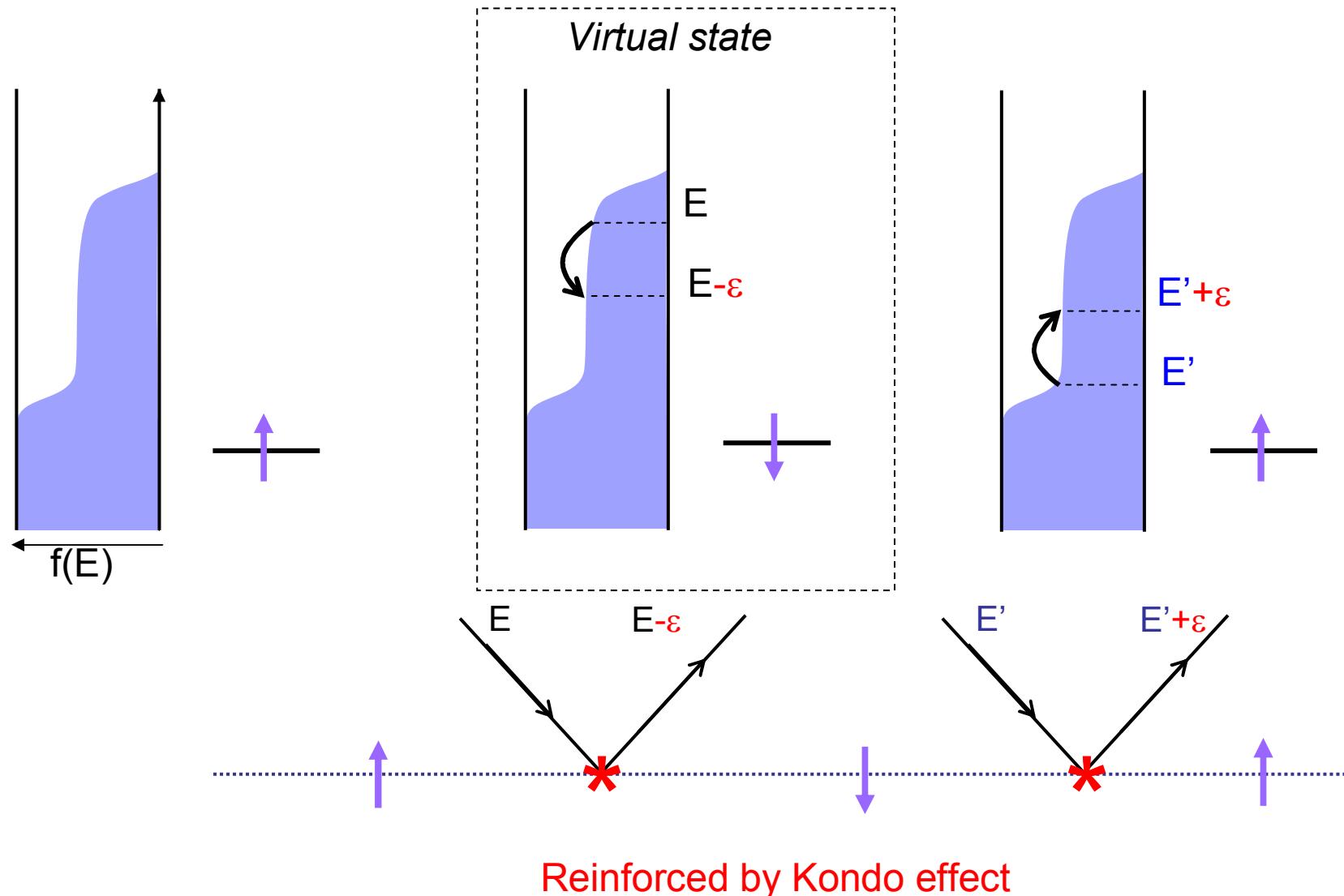
- energy exchange stronger than predicted
- $K(\varepsilon) = \kappa \varepsilon^{-2}$ fits data

$$K(\varepsilon) = \kappa \varepsilon^{-3/2}$$

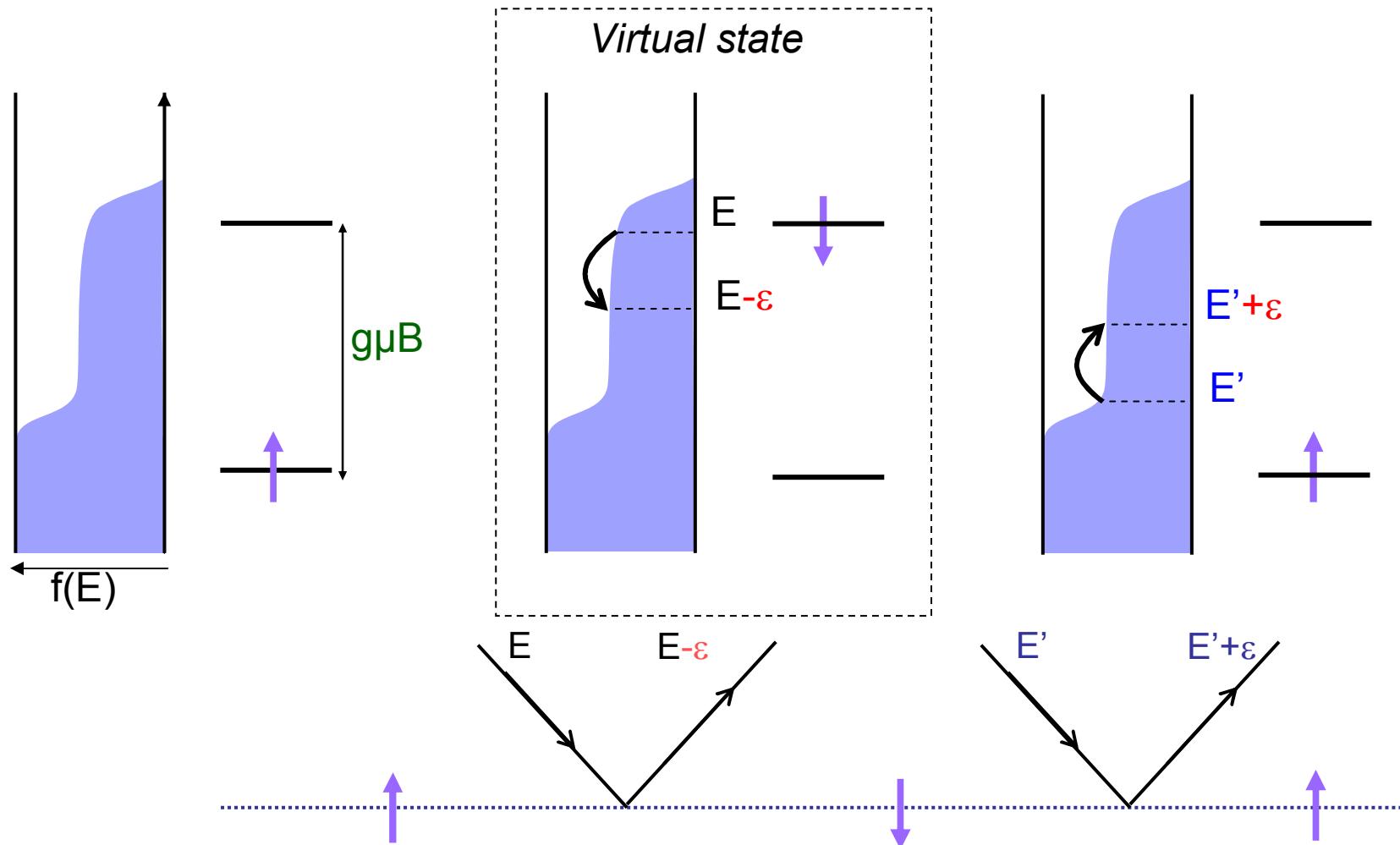
$$K(\varepsilon) = \kappa \varepsilon^{-2}$$

Energy exchange mediated by magnetic impurities

Kaminski and Glazman, PRL 86, 2400 (2001)

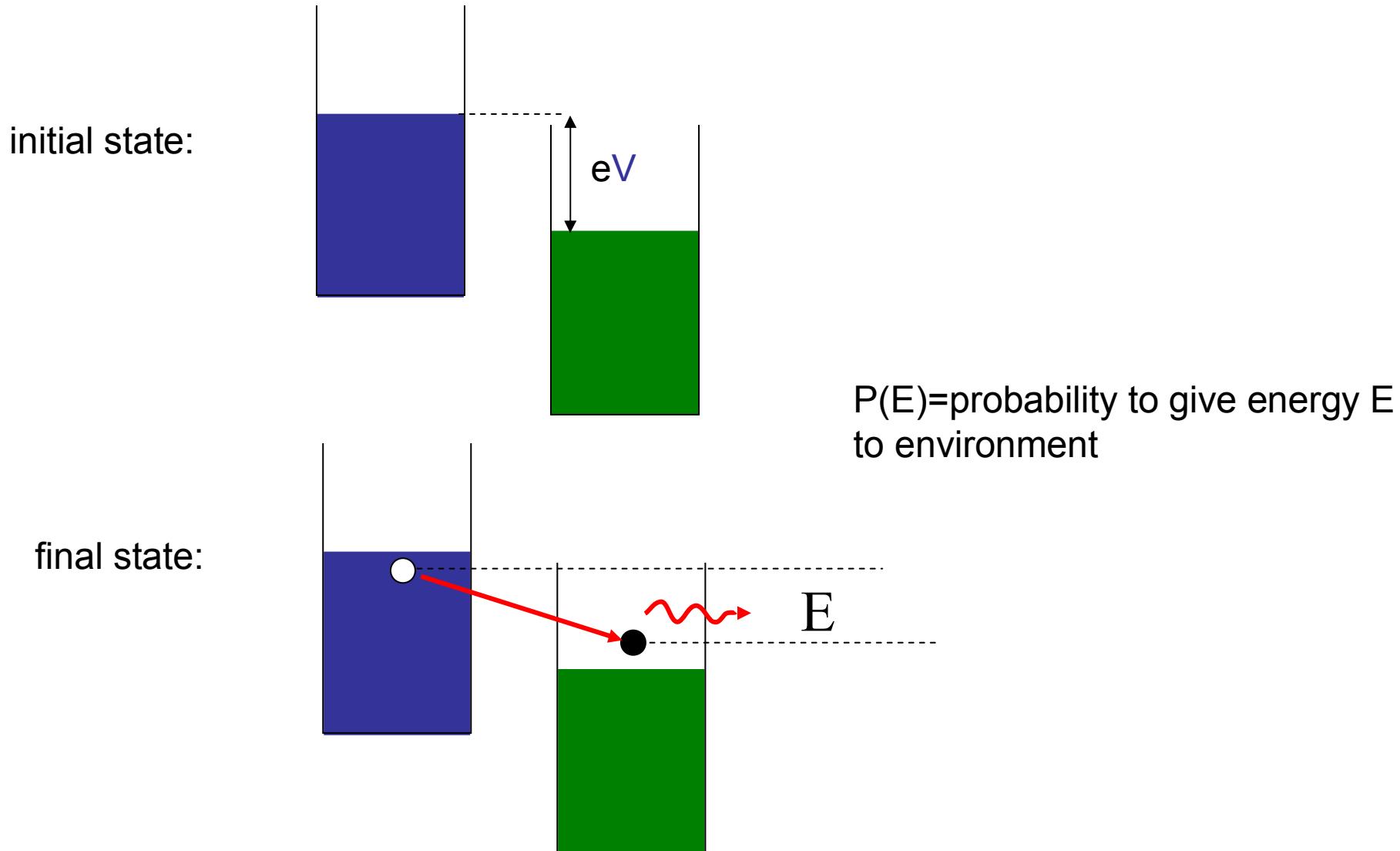


Energy exchange mediated by magnetic impurities vanishes when $g\mu_B B \gg eU$

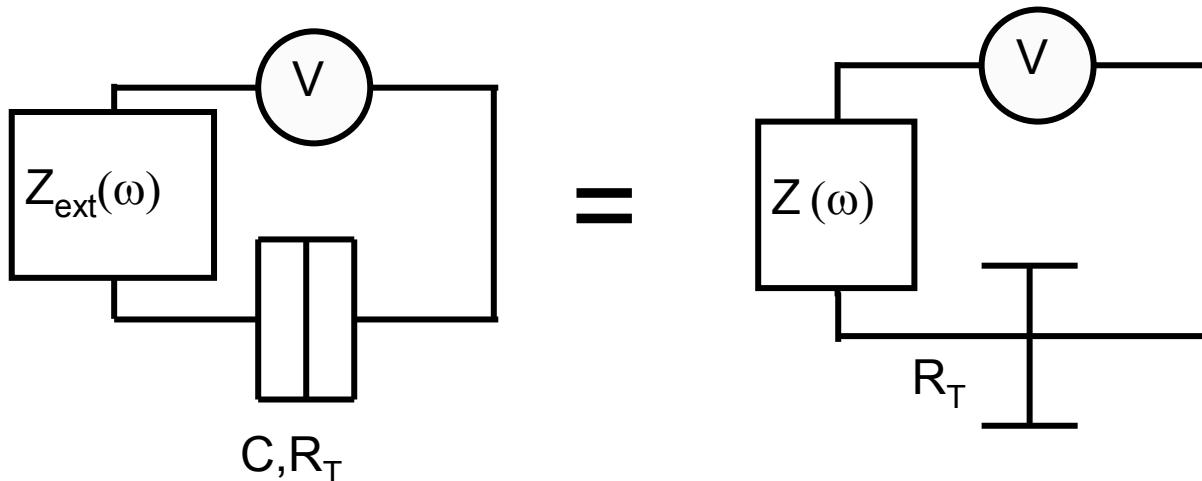


How to measure $f(E)$ in large B , with superconducting probe?

Aside 2: Inelastic tunneling (also called “Dynamical Coulomb Blockade”)



P(E) depends on environmental impedance



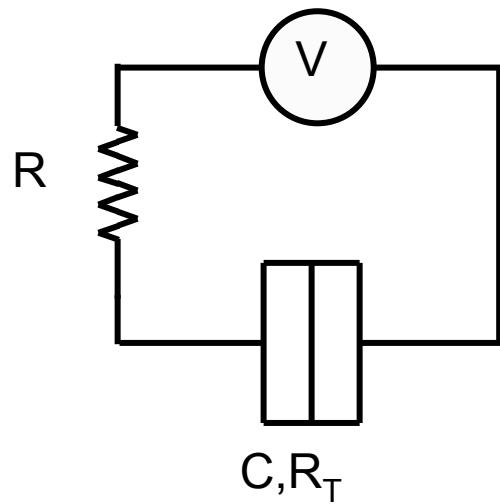
At $T=0$, one obtains :

$$\frac{dI}{dV} = \frac{1}{R_t} \int_0^{eV} P(E) dE$$

$$P(E) = \frac{1}{2\pi\hbar} \int e^{iEt/\hbar + J(t)} dt$$

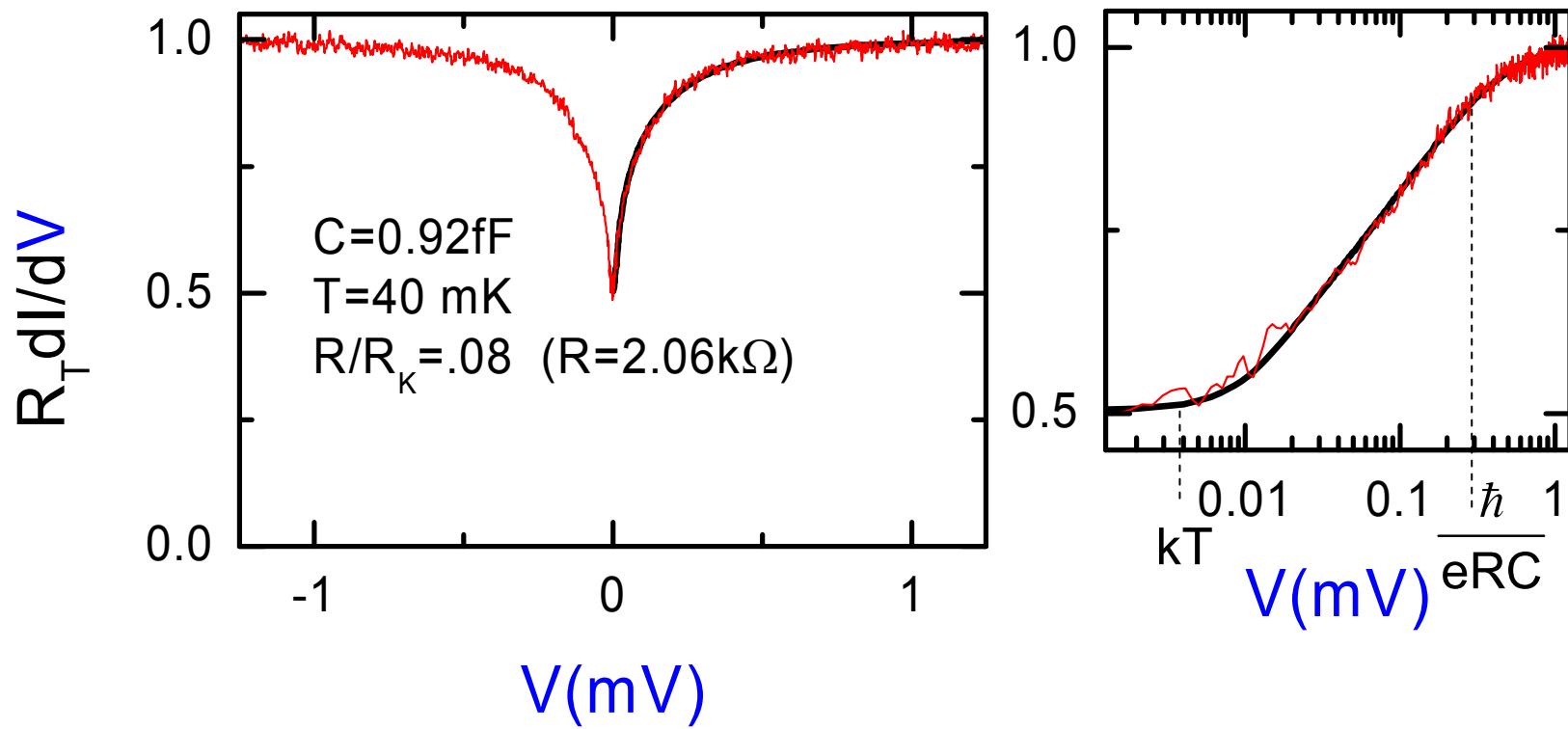
$$J(t) = 2 \int_0^{+\infty} \frac{d\omega}{\omega} \frac{\operatorname{Re}[Z(\omega)]}{R_K} (e^{-i\omega t} - 1)$$

Resistive environment

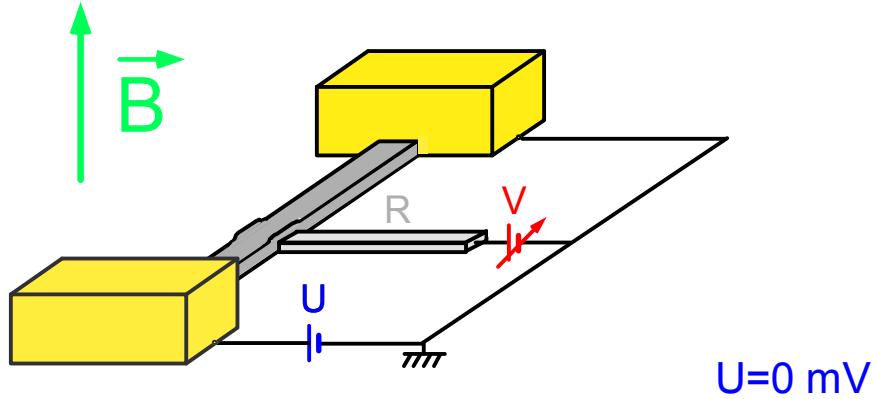


For $eV < \frac{\hbar}{RC}$

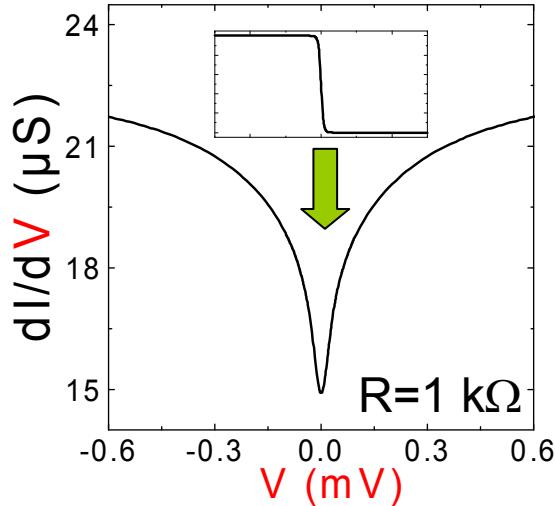
$$\frac{dI}{dV} \propto \left(V^{\frac{2R}{R_K}} + \text{cst.} \right)$$



Measure $f(E)$ at $B \neq 0$ using Zero-Bias Anomaly (Dynamical Coulomb Blockade)

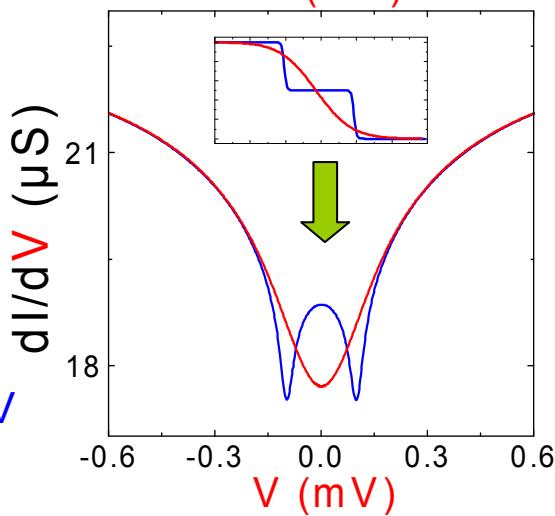


$U=0 \text{ mV}$



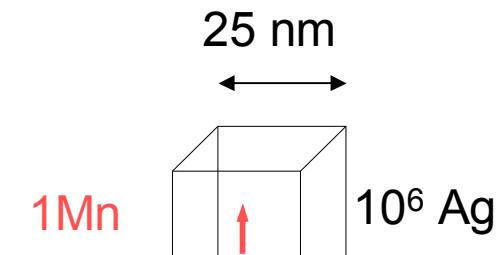
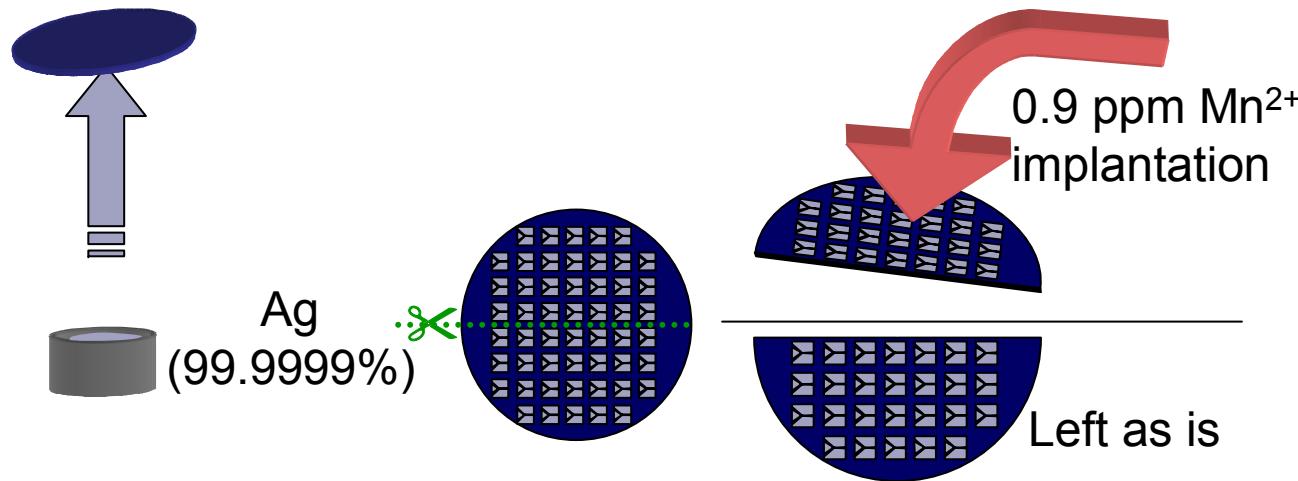
$R=1 \text{ k}\Omega$

$U=0.2 \text{ mV}$



$dI/dV \rightarrow f(E) \rightarrow \text{electron-electron interactions}$

A controlled experiment



implanted

bare

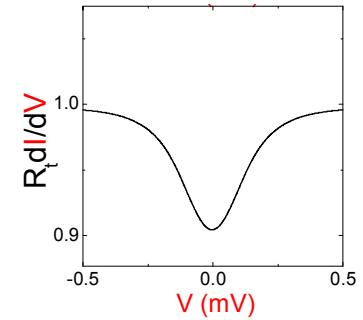
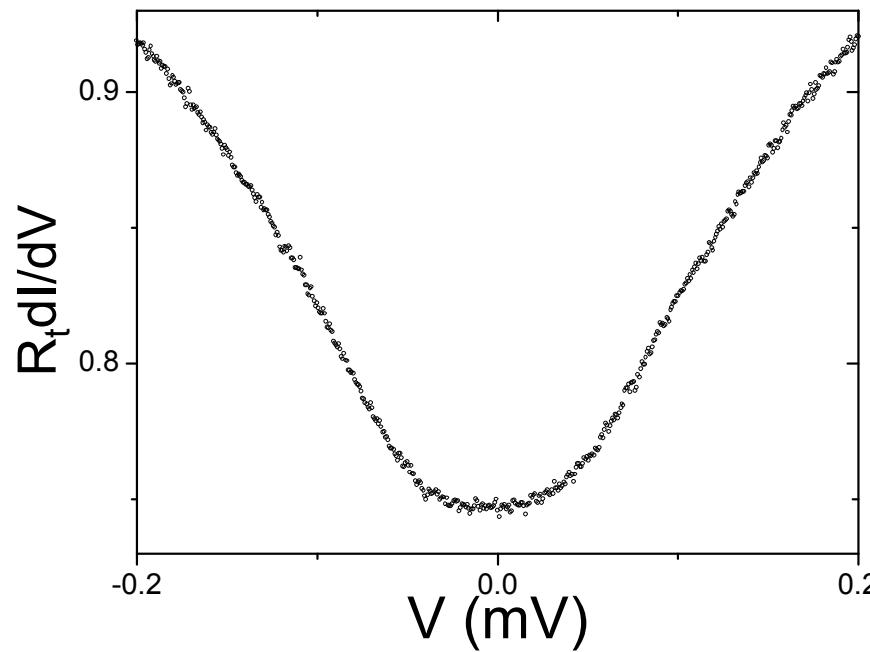
Comparative experiments

Effect of 1 ppm Mn on interactions ?

Experimental data at weak B

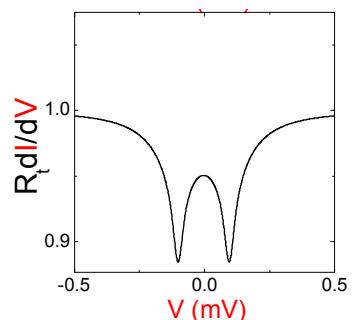
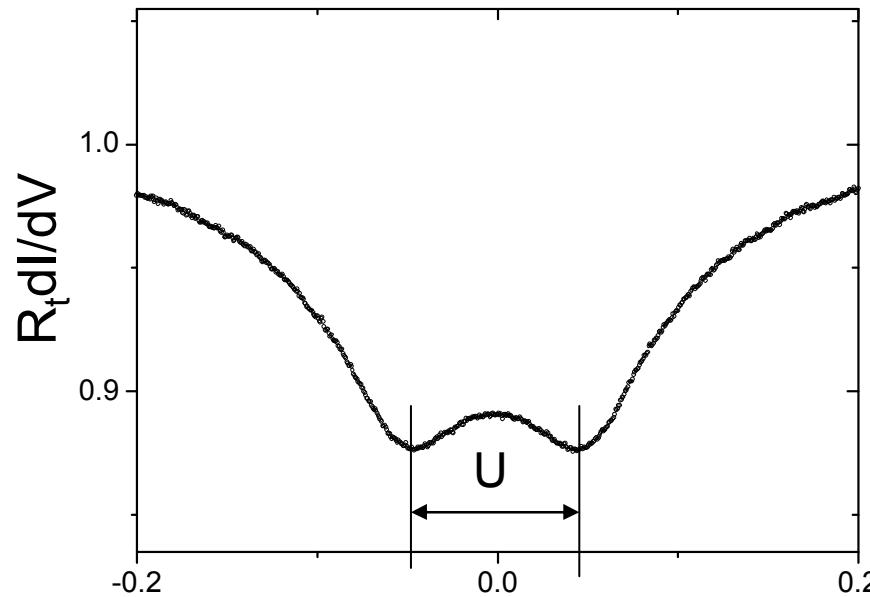
implanted

$U = 0.1 \text{ mV}$
 $B = 0.3 \text{ T}$



strong interaction

bare



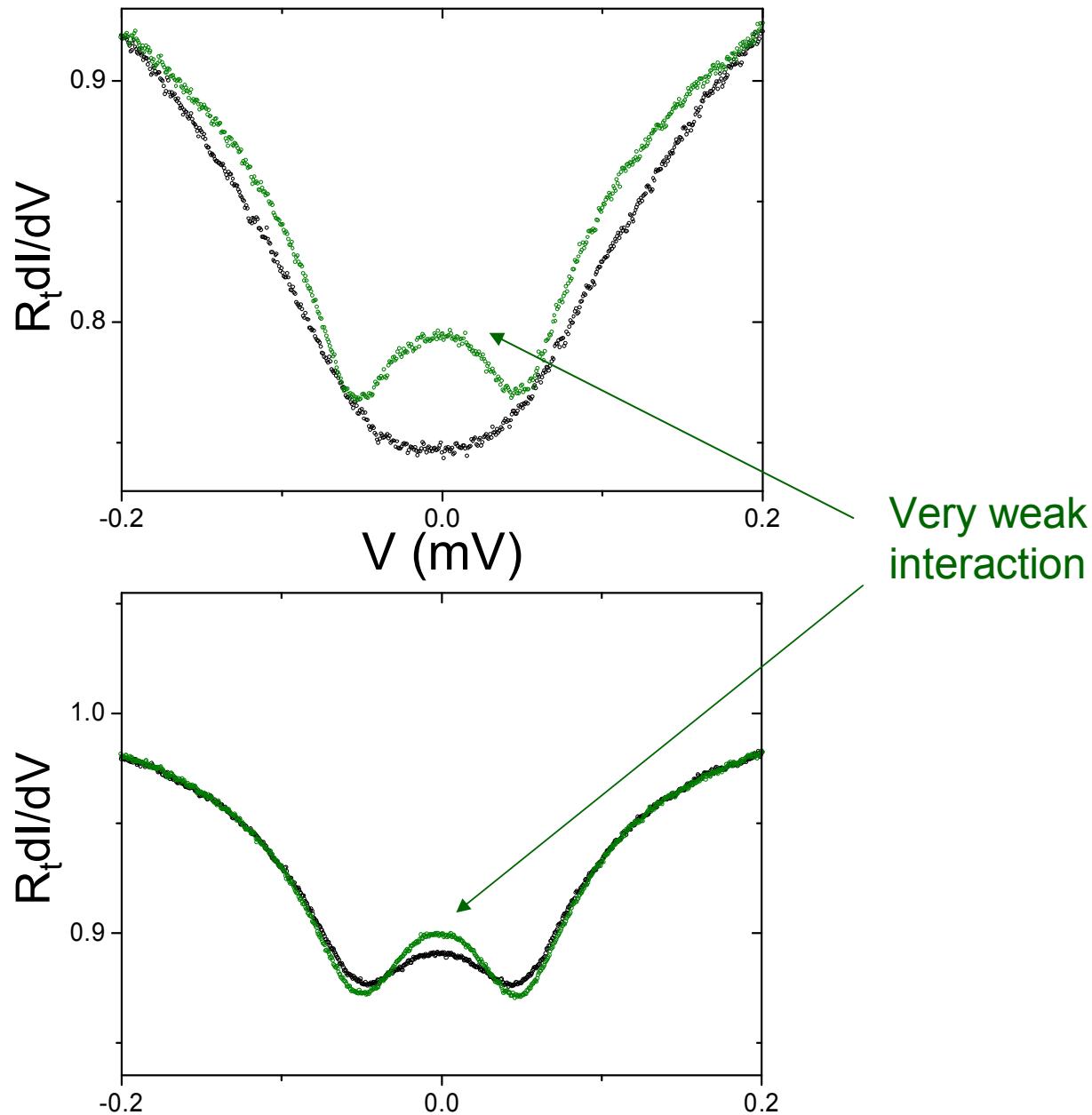
weak interaction

Experimental data at weak and at strong B

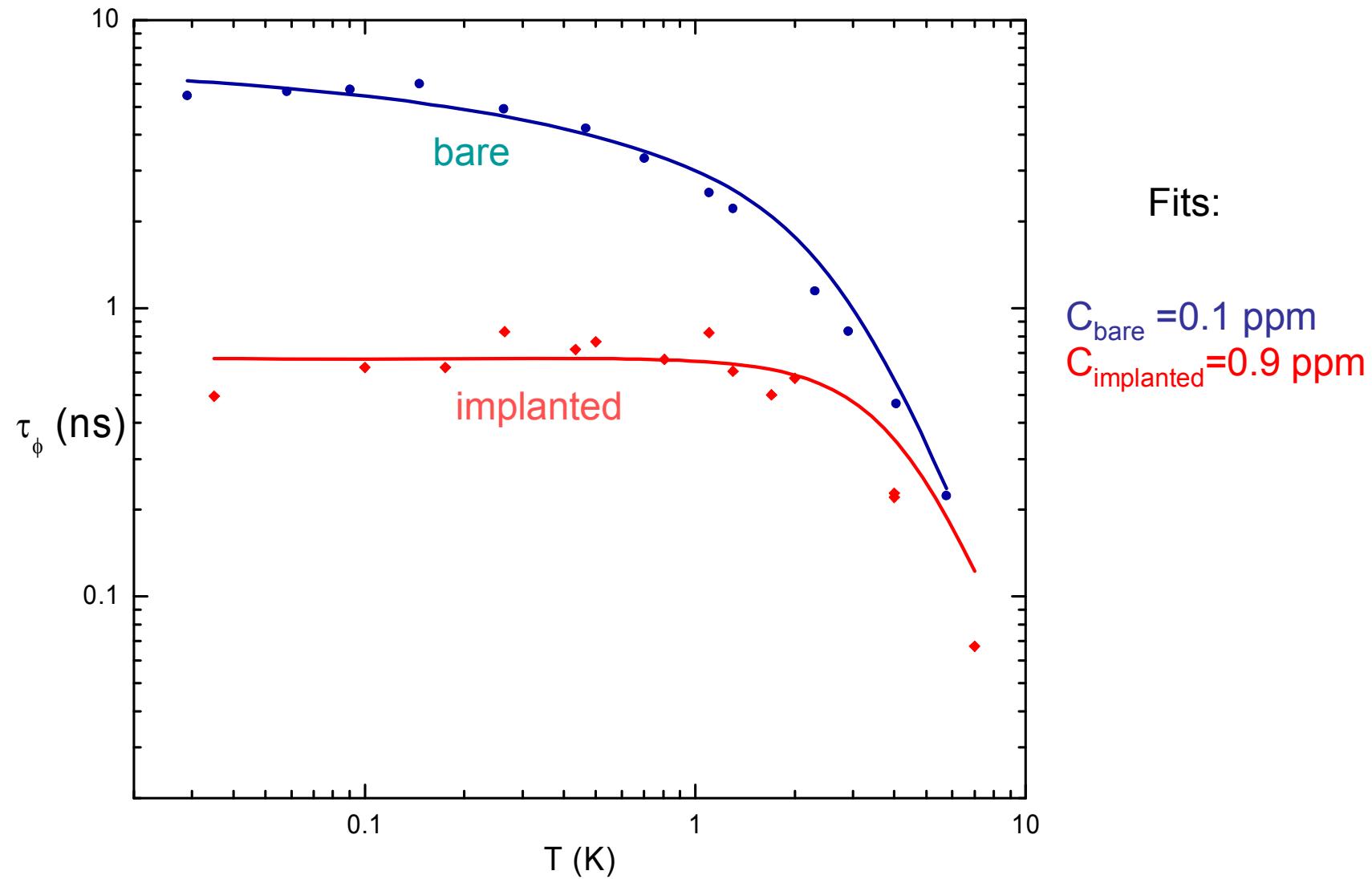
implanted

$U = 0.1 \text{ mV}$
 $B = 0.3 \text{ T}$
 $B = 2.1 \text{ T}$

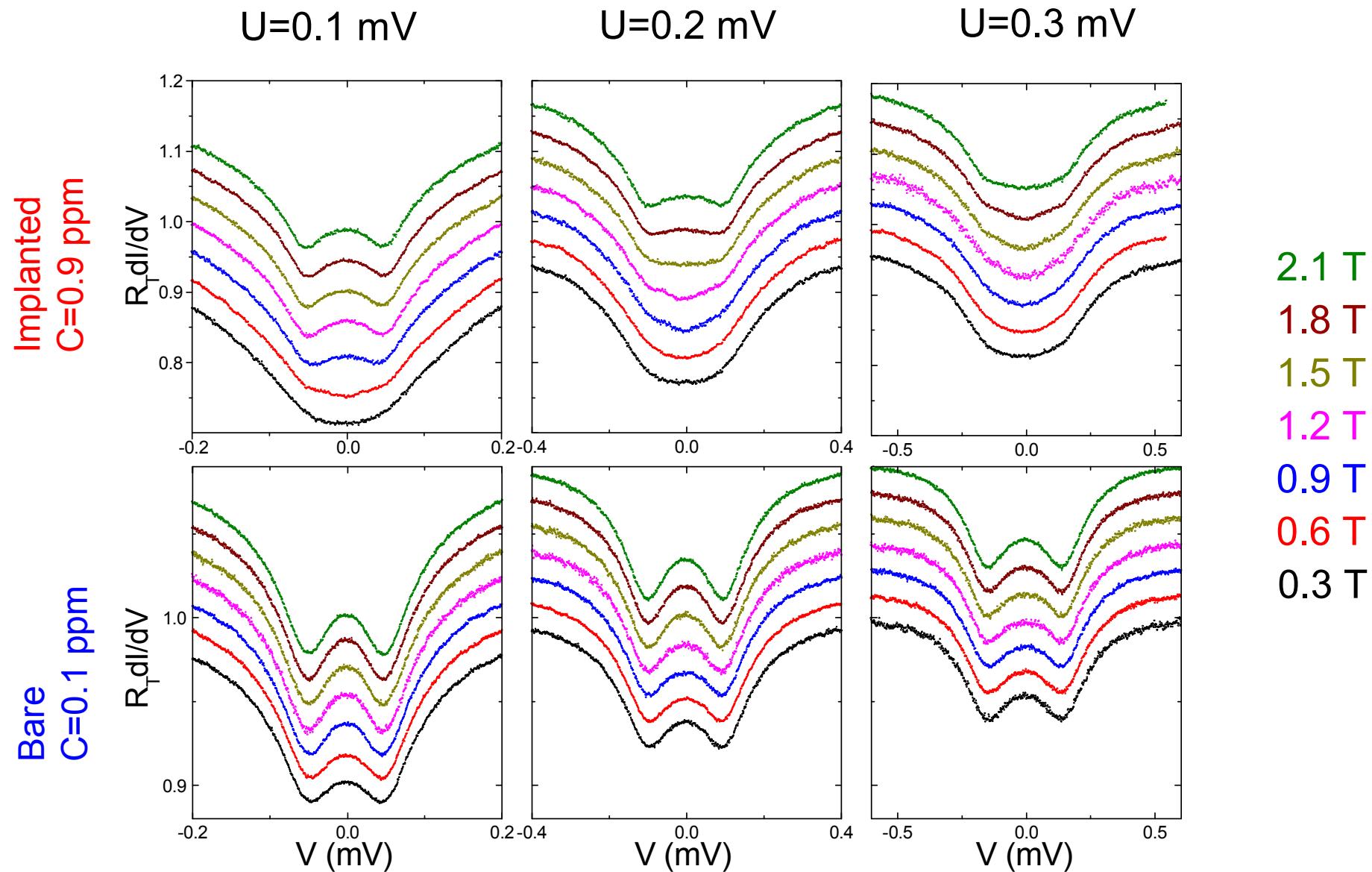
bare



Coherence time measurements on the same 2 samples



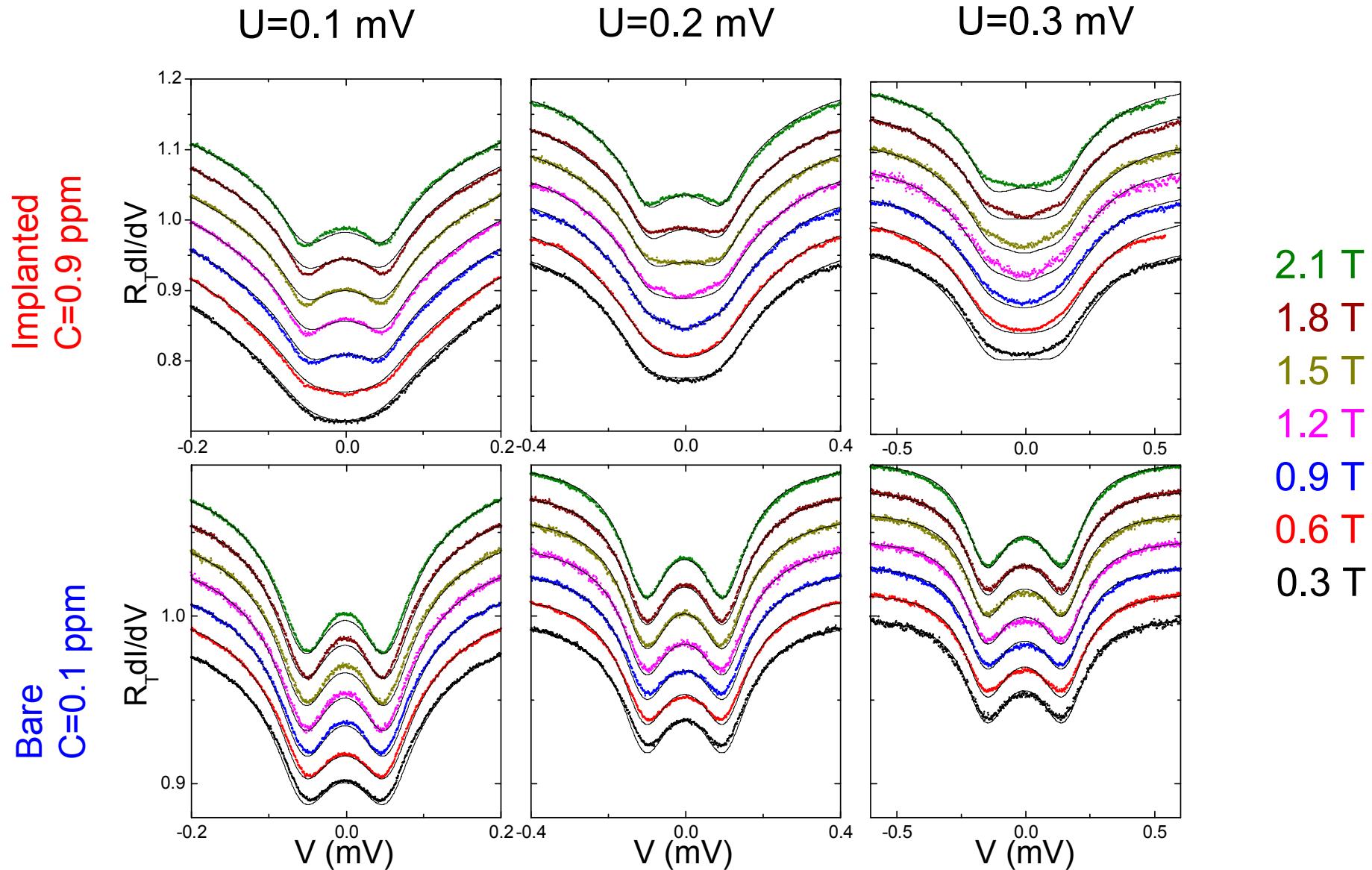
Full U,B dependence



Comparison with theory

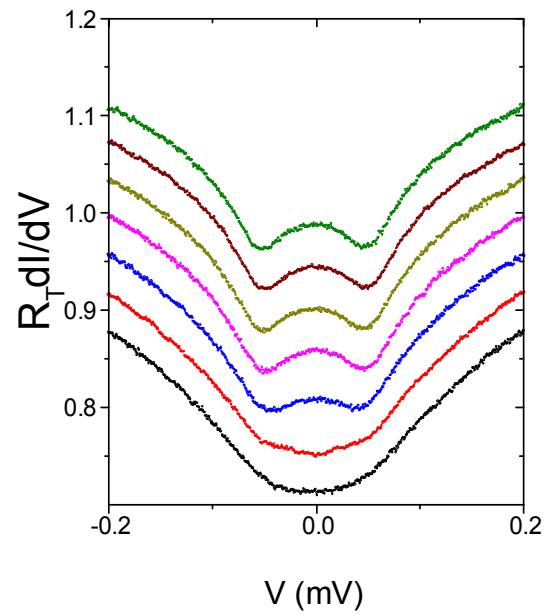
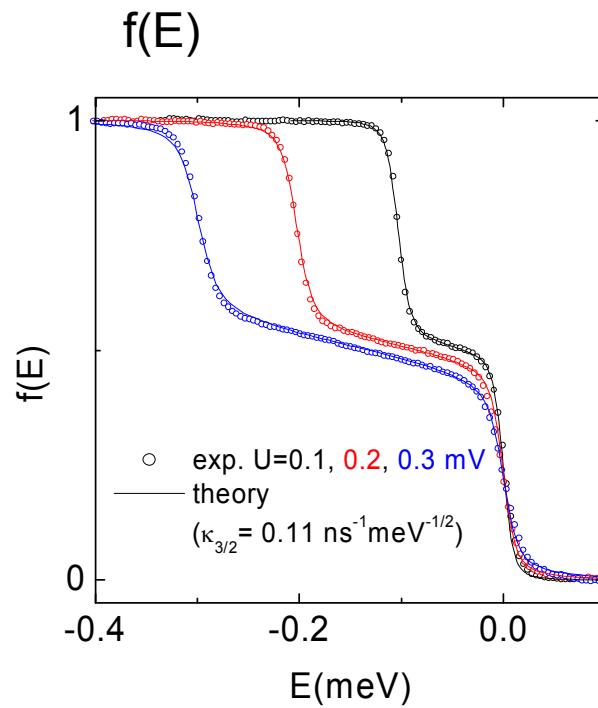
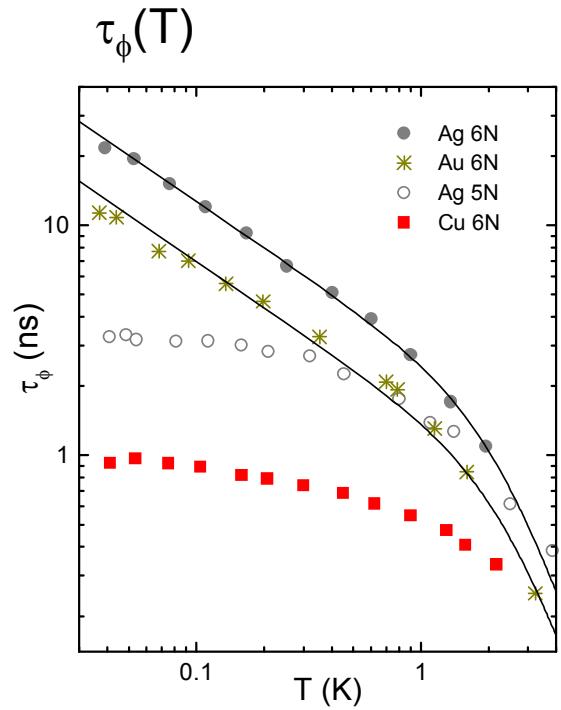
$$\left(s = \frac{1}{2} \right)$$

Goeppert, Galperin, Altshuler and Grabert, PRB 64, 033301 (2001)



Conclusions

Two methods to investigate interactions in wires



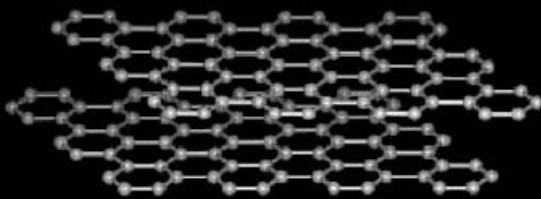
Moral of the story: even at concentrations as low as 1 ppm, magnetic impurities have a large influence on dephasing and energy exchange in metals at low-temperature.

Outline

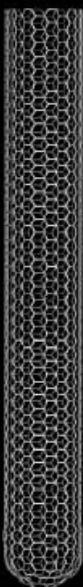
- Part 1: Diffusive Metal Wires
 - The distribution function, $f(E)$, in nonequilibrium situations
 - How to measure $f(E)$ using tunnel spectroscopy
 - First measurements of $f(E)$ in diffusive metal wires
 - Magnetic impurities: theory (Kaminsky & Glazman)
 - Magnetic impurities: experiments
- Part 2: Carbon Nanotubes
 - Introduction to carbon nanotubes
 - $f(E)$ in ballistic vs. diffusive wires
 - Experimental issues with nanotubes
 - First measurements of $f(E)$ in single-wall tubes
 - Future prospects



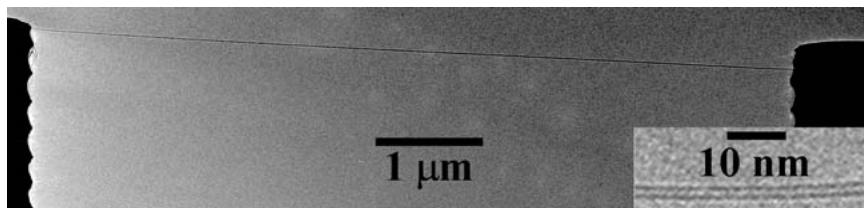
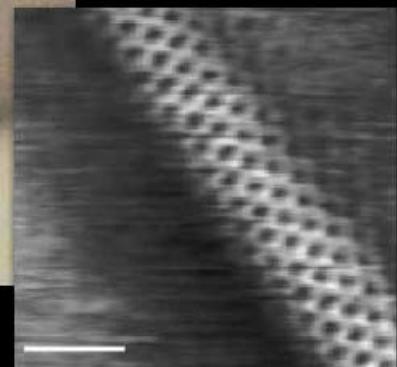
DIAMOND



GRAPHITE



NANOTUBE



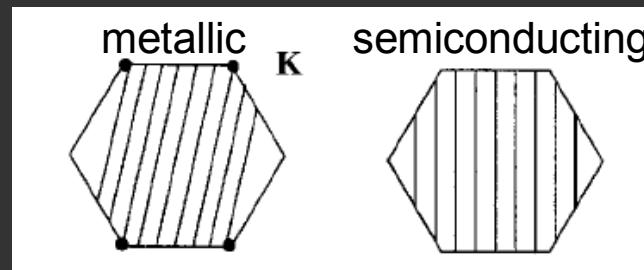
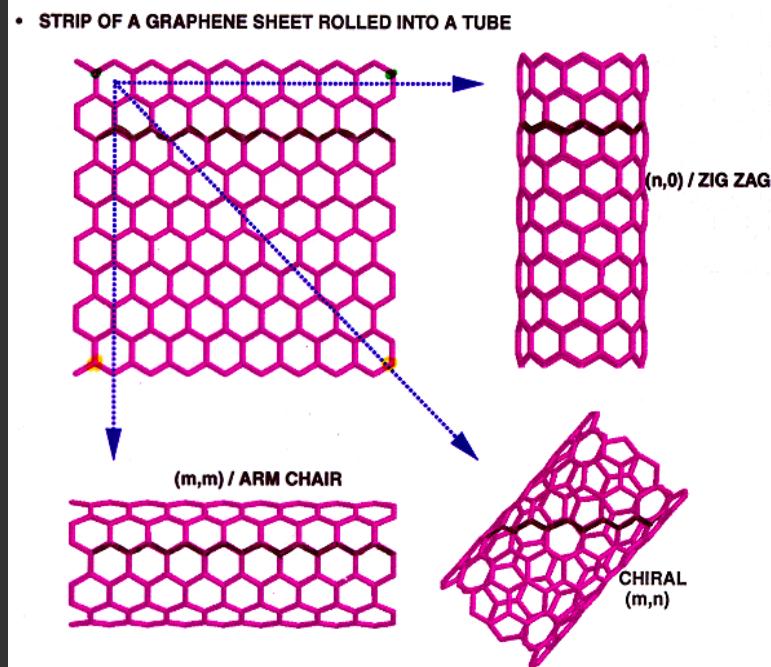
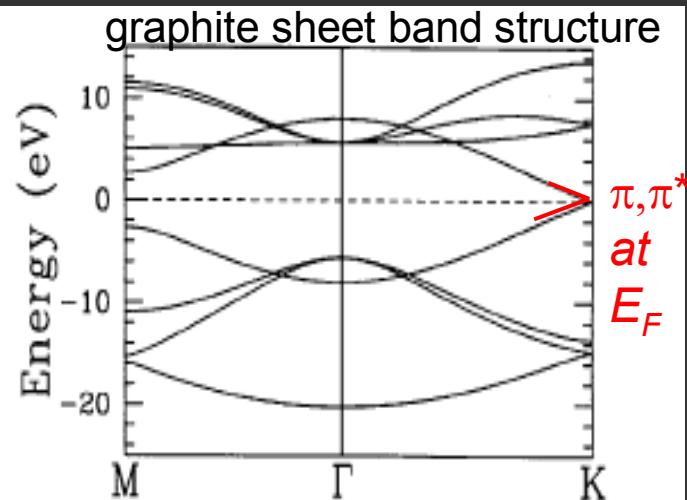
Carbon Nanotubes

Discovered in 1991 by Iijima (NEC, Japan)

Stronger than steel,
lightweight, flexible like straws

diameter < 1nm,
length > 1mm

Electrical Properties

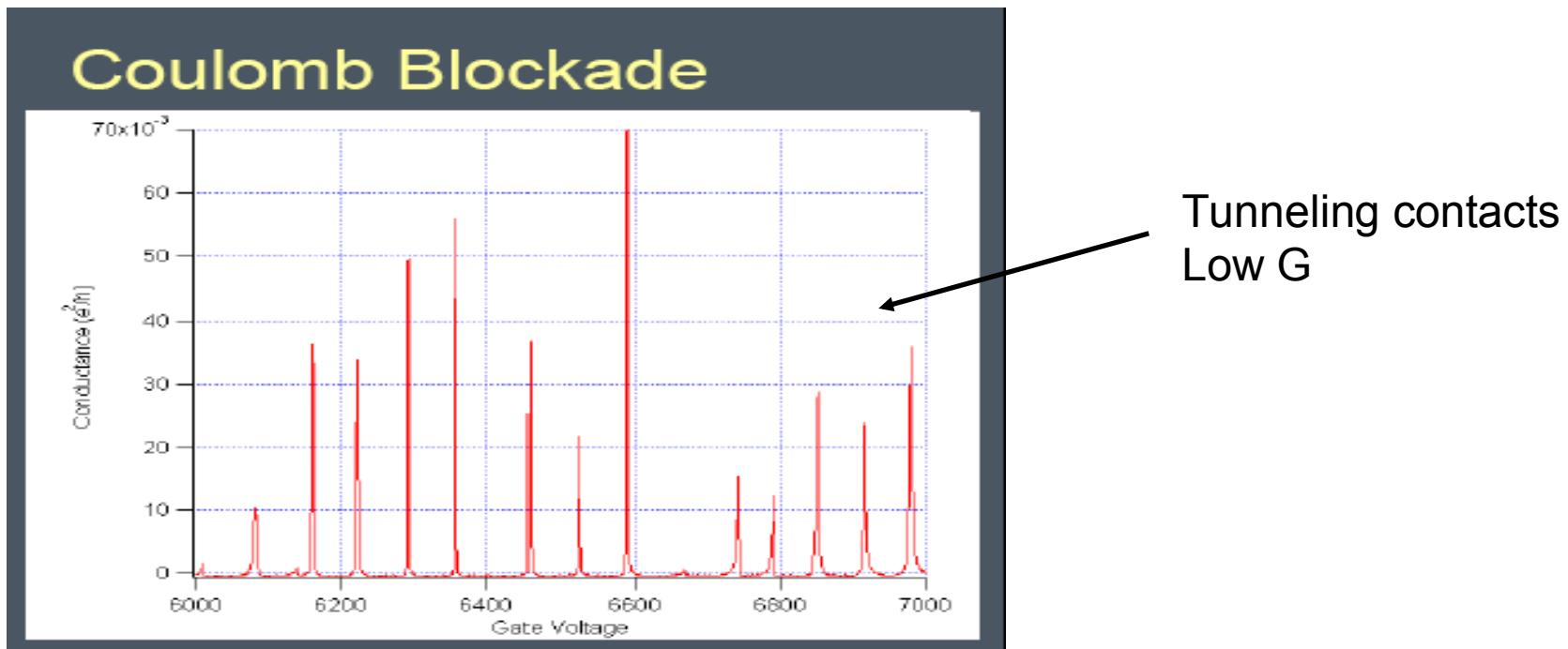


$$n-m=3j \rightarrow \text{metallic}$$

Metallic nanotubes as 1D conductors

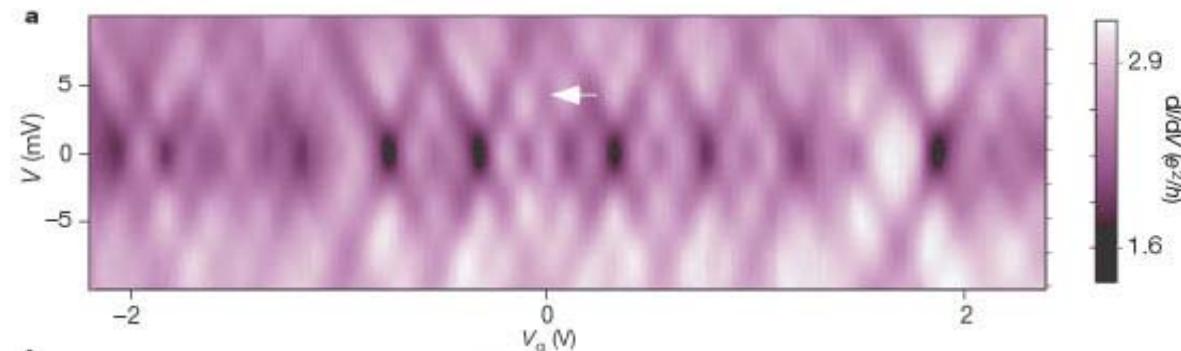
- 2 channels $\Rightarrow G_{\text{ideal}} = 4e^2/h$
- Long mean-free paths
- 1D band structure \Rightarrow Luttinger liquid physics?

Nanotube conductance regimes



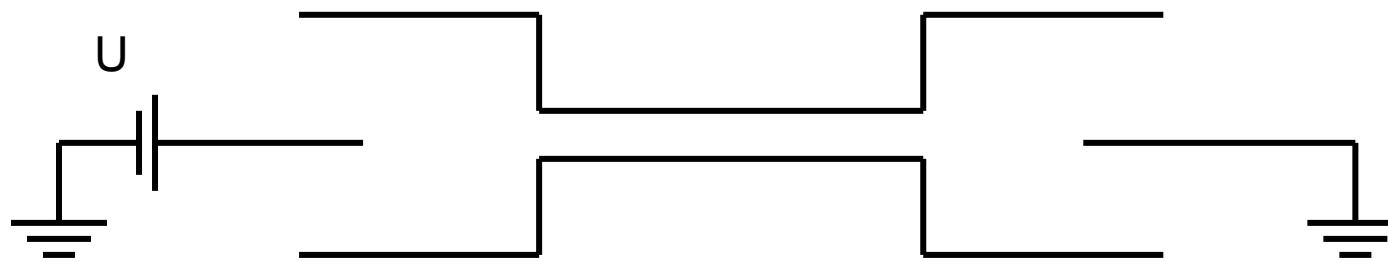
Ohmic contacts
High G \Rightarrow
Fabry-Perot
interference

Liang et al., Nature 2001

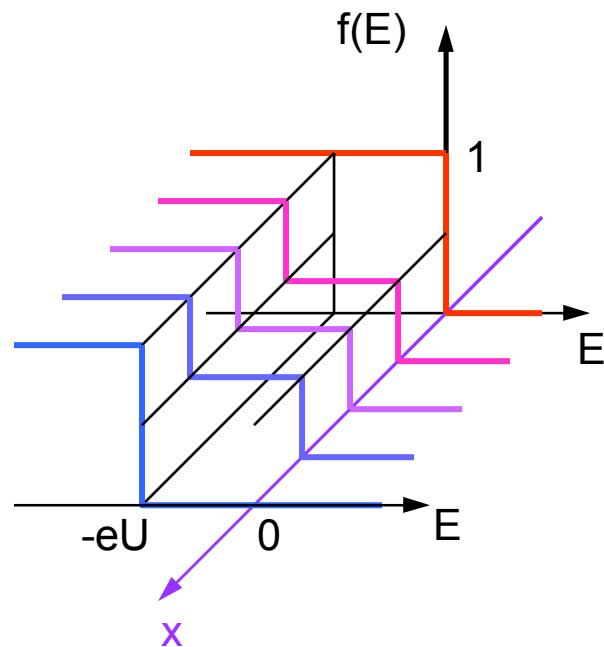


$f(E)$ in ballistic vs. diffusive wires

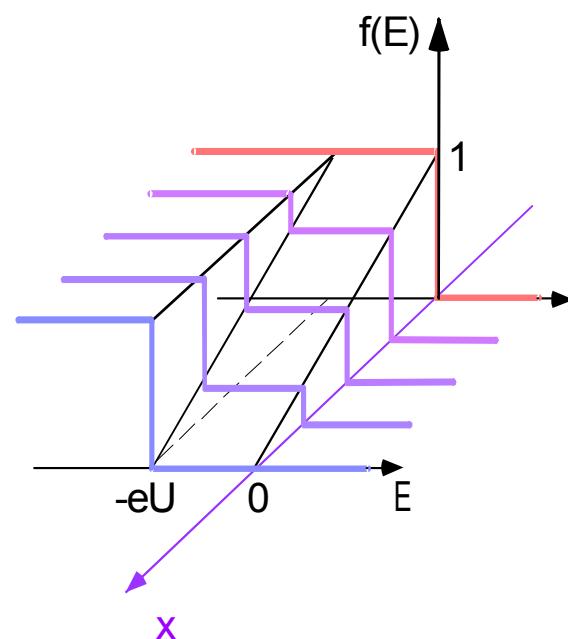
free electrons, no e-e interactions



ballistic



diffusive

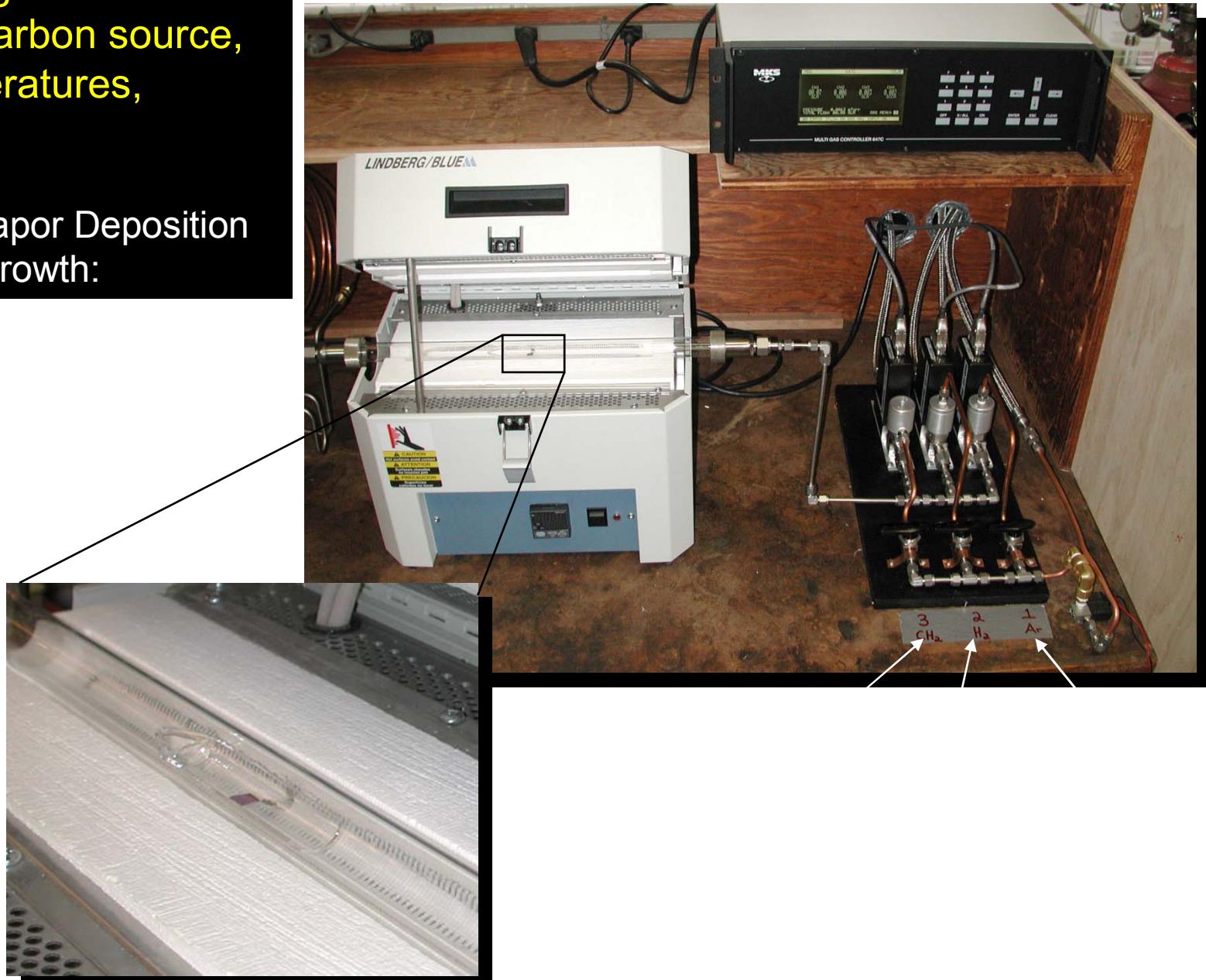


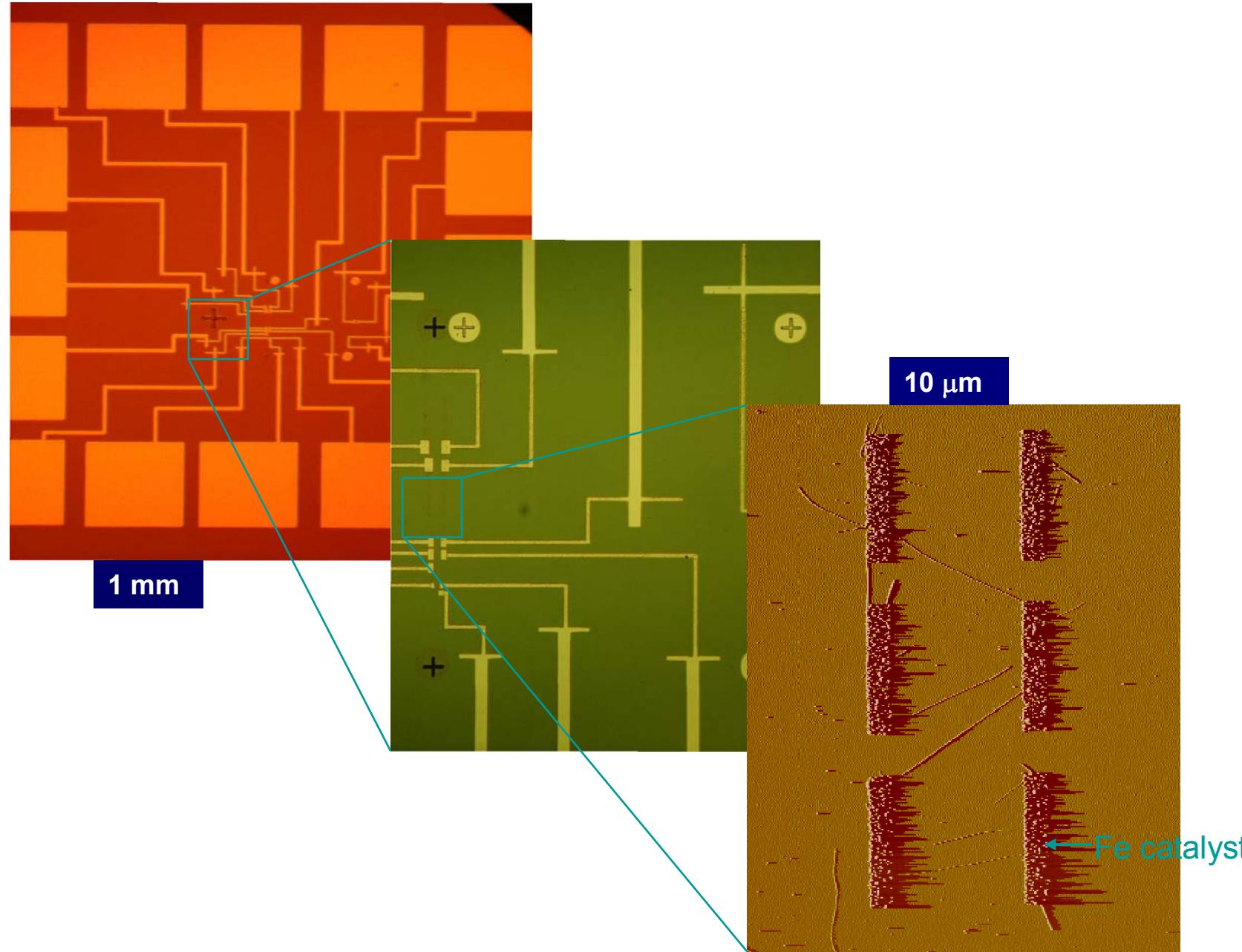
Motivation for $f(E)$ measurement

- Nanotubes are (nearly) ideal 1D conductors
- Are e-e interactions strong (Luttinger liquid) or weak (long mean-free path)?
- Desired: direct measurement of interactions between quasiparticles
- Test ballistic vs. diffusive behavior of $f(E)$

Nanotube growth
requires: carbon source,
high temperatures,
catalyst

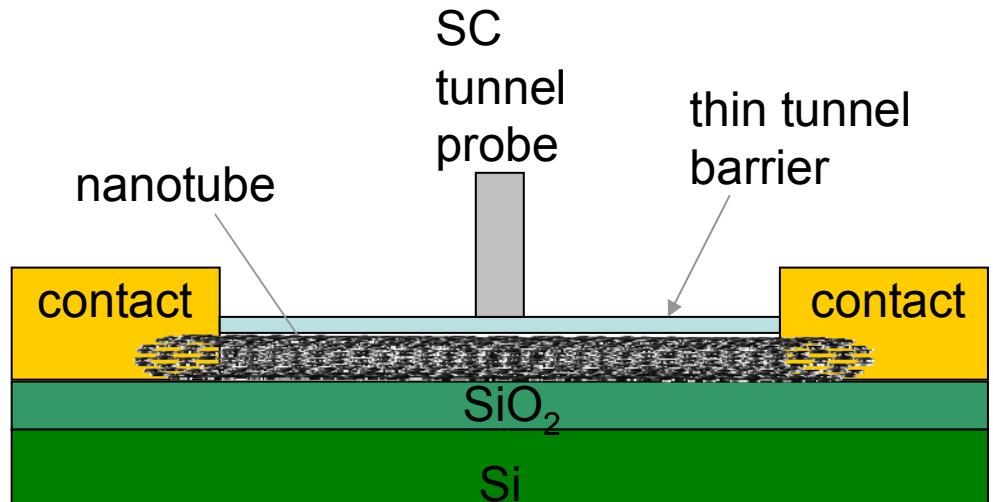
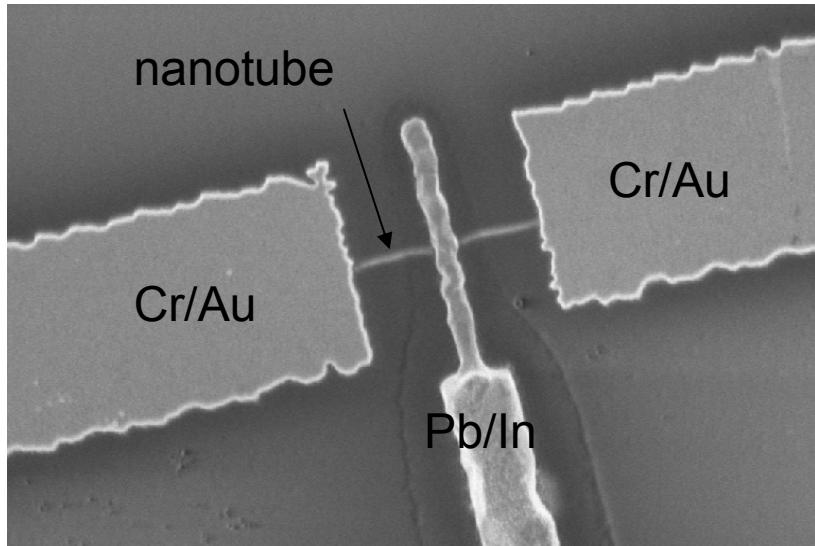
Chemical Vapor Deposition
Nanotube Growth:





Device fabrication

- CVD tube growth
- Cr/Au end contacts (low resistance)
- Atomic Layer Deposition of AlOx
- Pb/In top probes

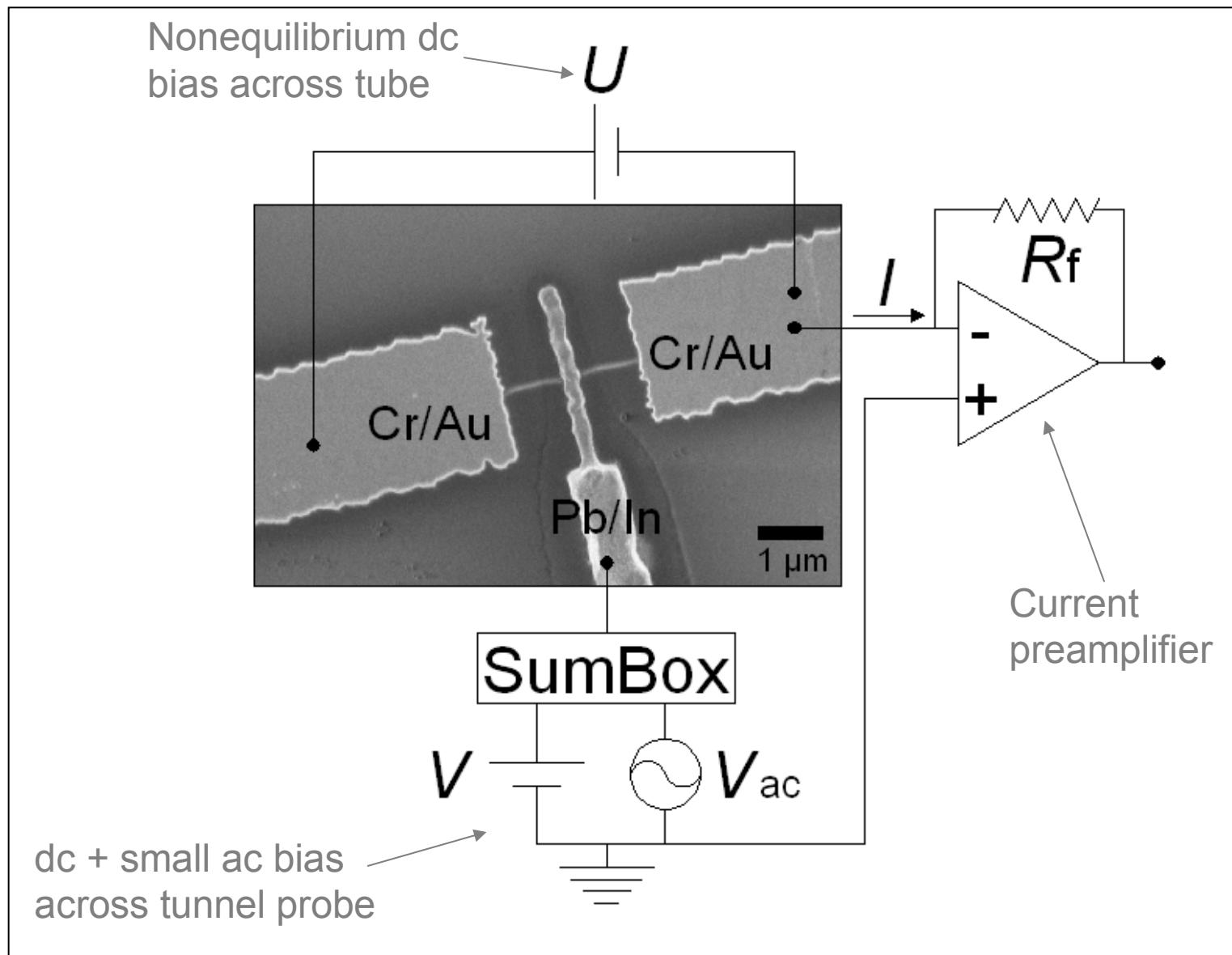


Room temperature resistances:

$$R_{\text{tube}} = 20\text{--}100 \text{ k}\Omega \quad R_{\text{tunnel}} = 1\text{--}5 \text{ M}\Omega$$

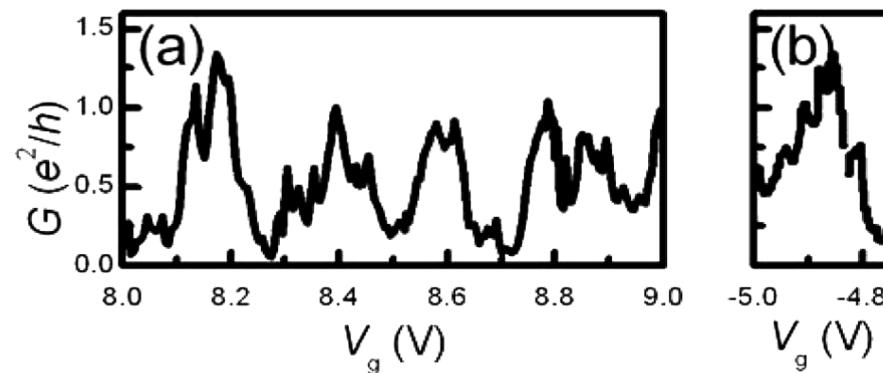
$$\text{cf: } R_{\text{ideal}} = 6.5 \text{ k}\Omega$$

Measurement Scheme



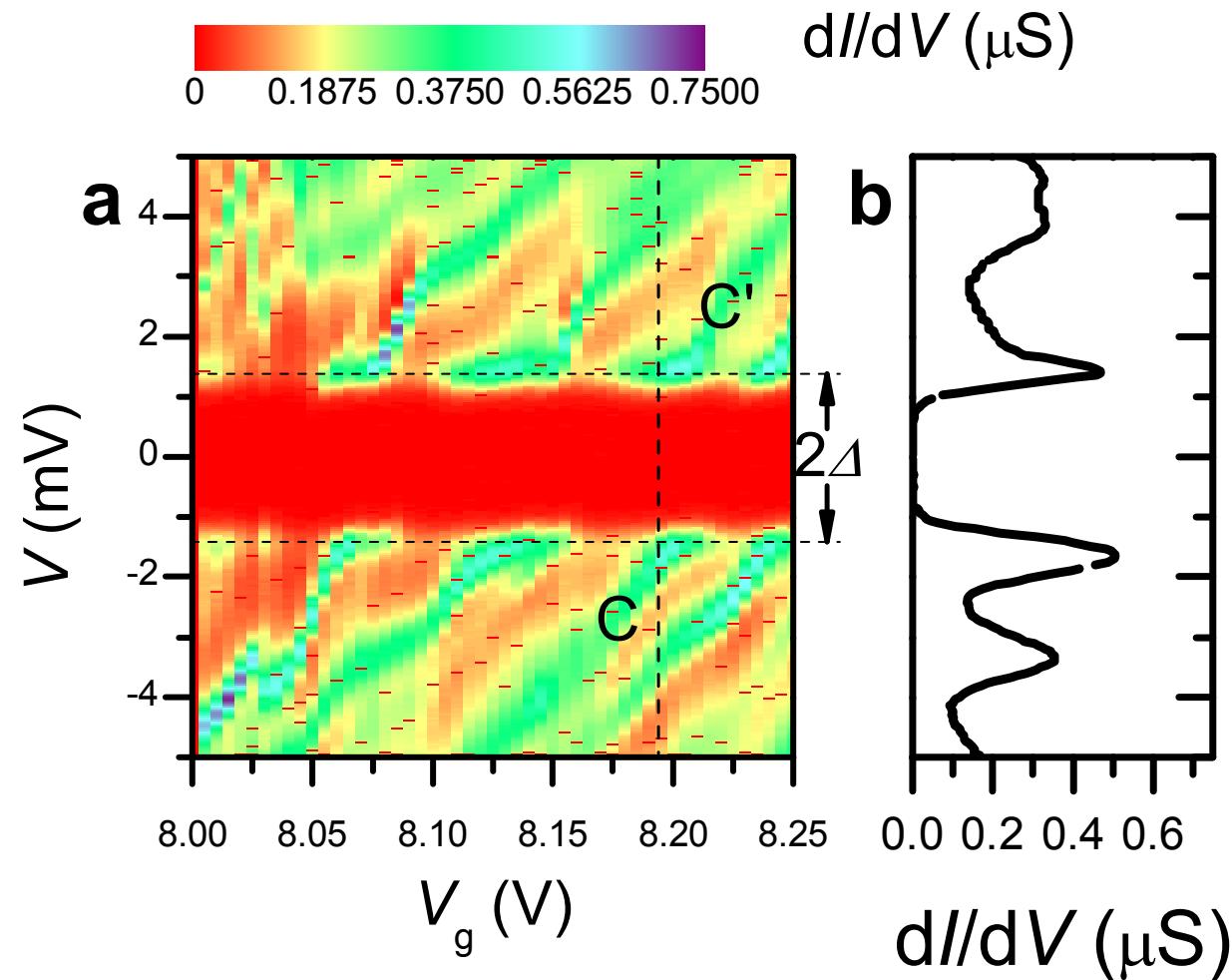
Sample characterization

End-to-end conductance vs. gate voltage:



$G \approx e^2/h$, broad peaks \Rightarrow strong coupling between nanotube and contacts

Characterization of Tunnel Probe (equilibrium)

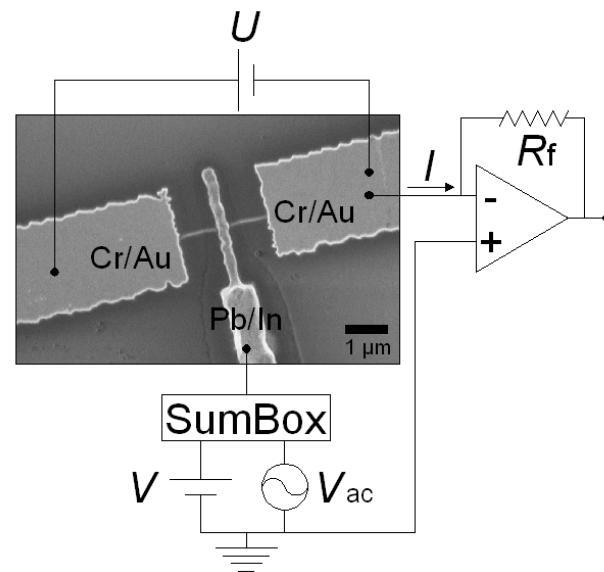
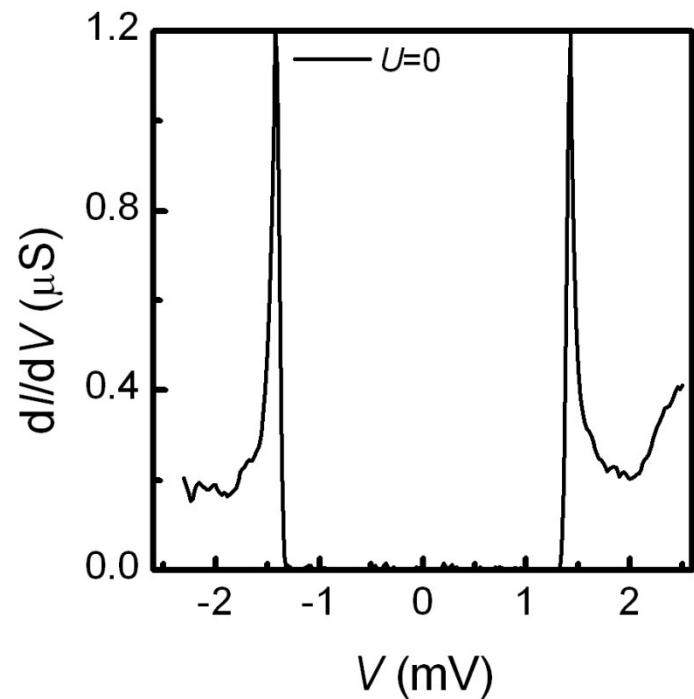


Clear superconducting gap of Pb electrode

Non-equilibrium tunneling spectroscopy

Sample B, $T = 53$ mK

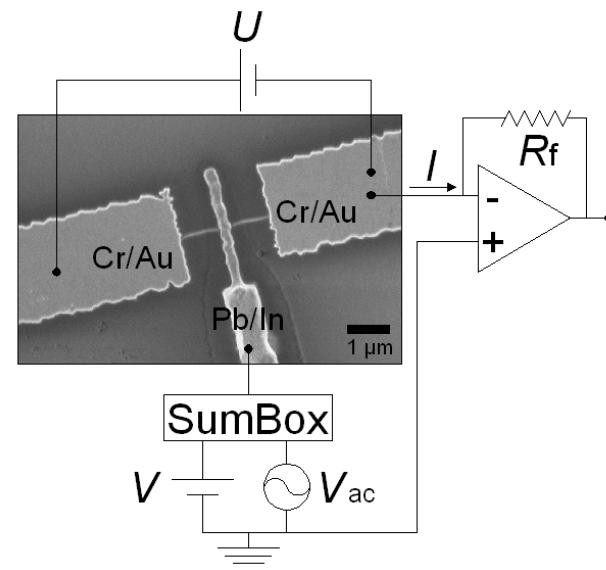
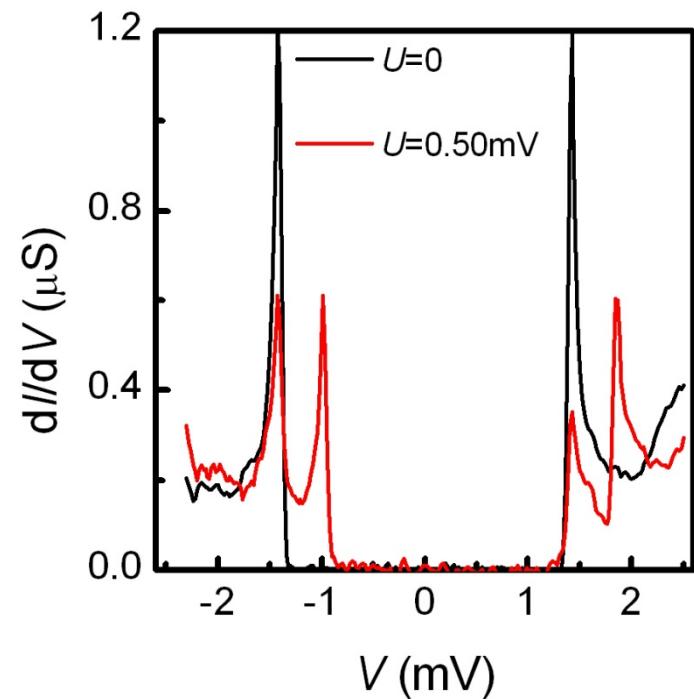
$(k_B T \ll eU)$



Non-equilibrium tunneling spectroscopy

Sample B, $T = 53$ mK

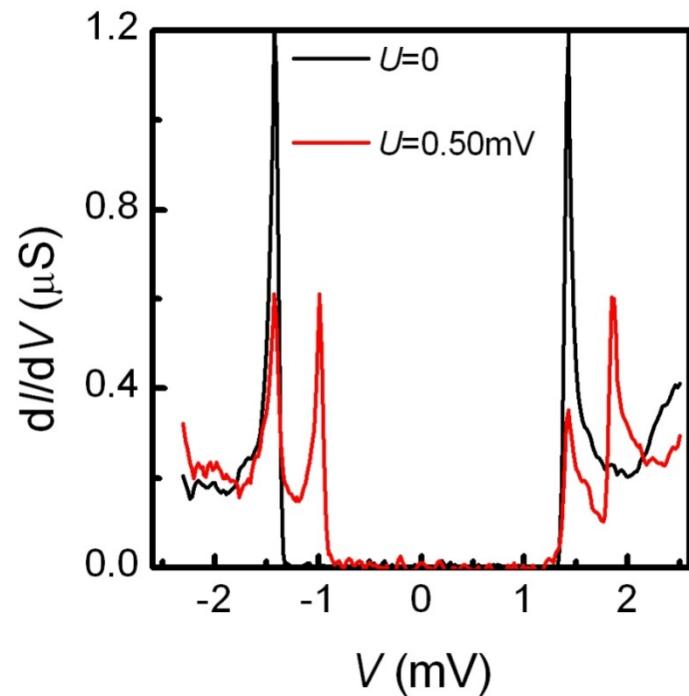
$(k_B T \ll eU)$



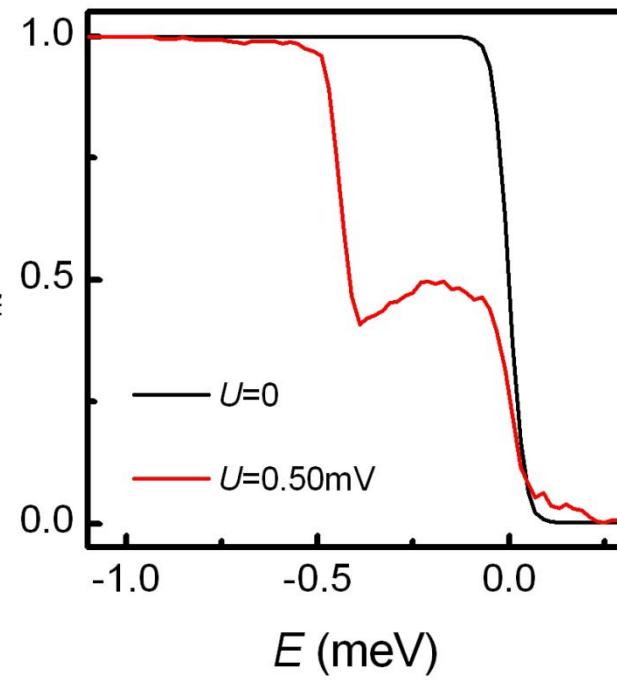
Non-equilibrium tunneling spectroscopy

Sample B, $T = 53 \text{ mK}$

$(k_B T \ll eU)$



numerical
deconvolution f_{nt}

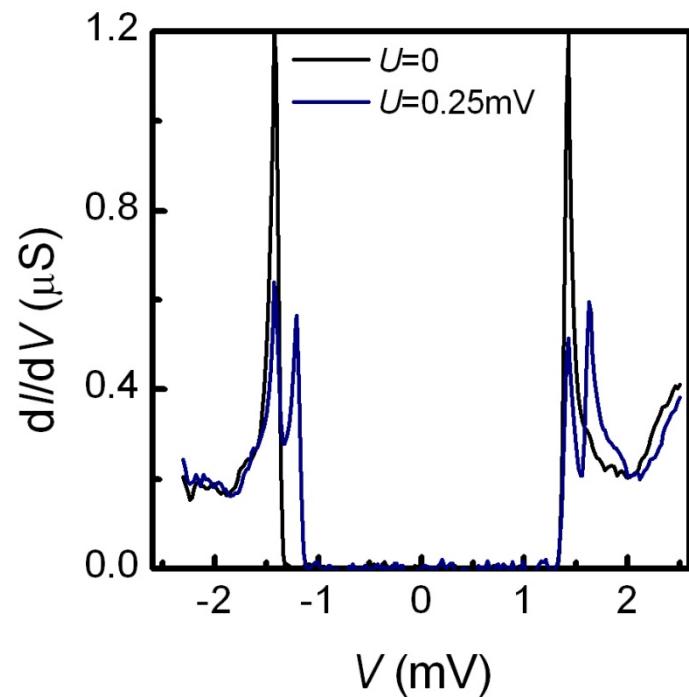


$$I(V) = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE n_S(E + eV) n_{nt}(E) (f_{nt,U}(E) - f_S(E + eV))$$

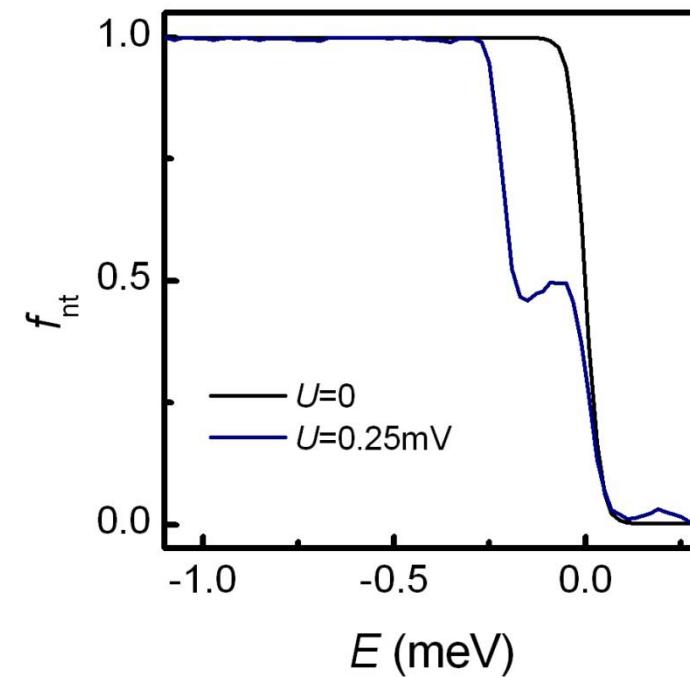
Non-equilibrium tunneling spectroscopy

Sample A, $T = 53 \text{ mK}$

$(k_B T \ll eU)$



numerical
deconvolution

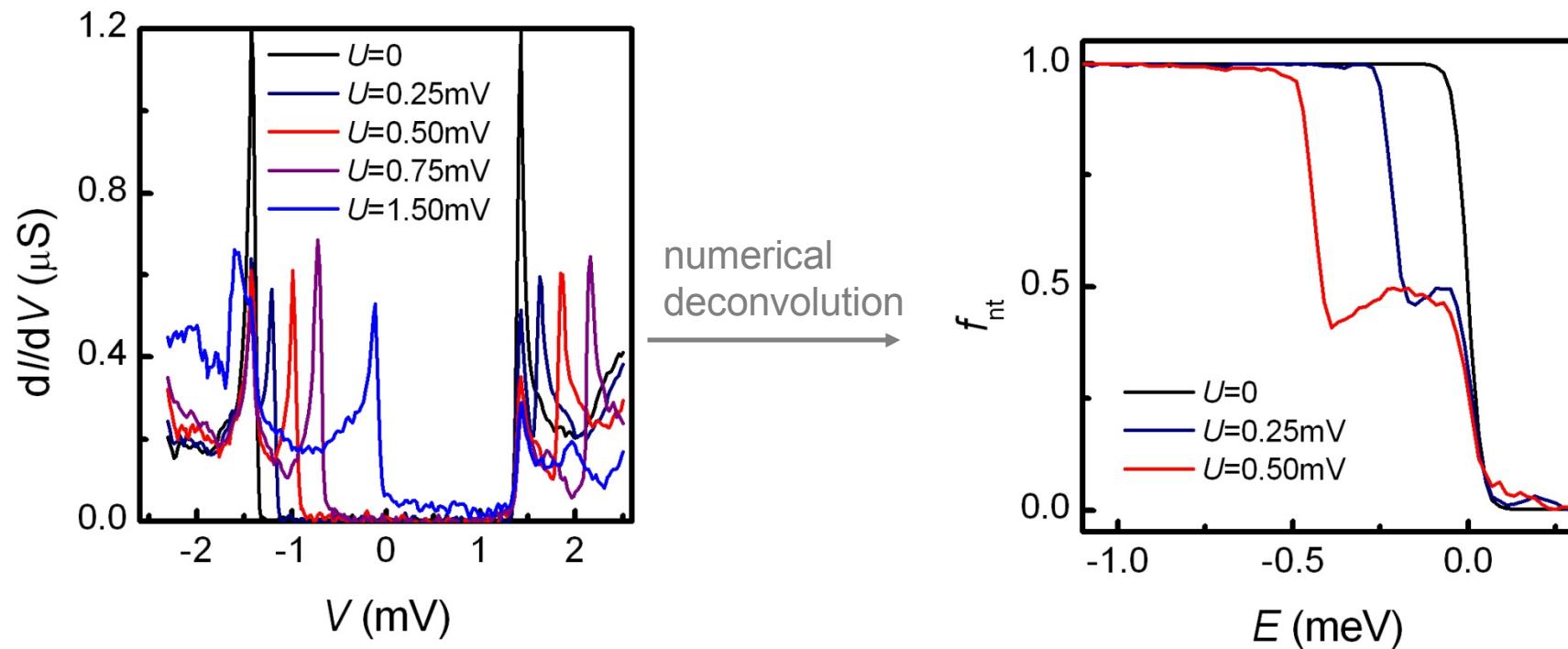


$$I(V) = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE n_S(E + eV) n_{nt}(E) (f_{nt,U}(E) - f_S(E + eV))$$

Non-equilibrium tunneling spectroscopy

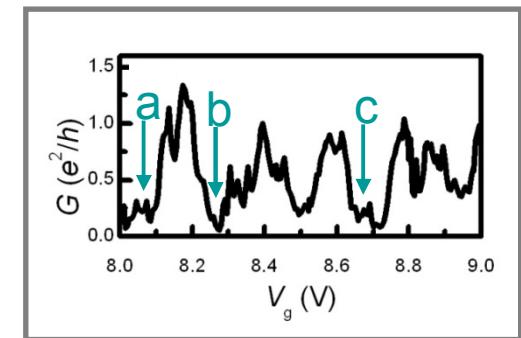
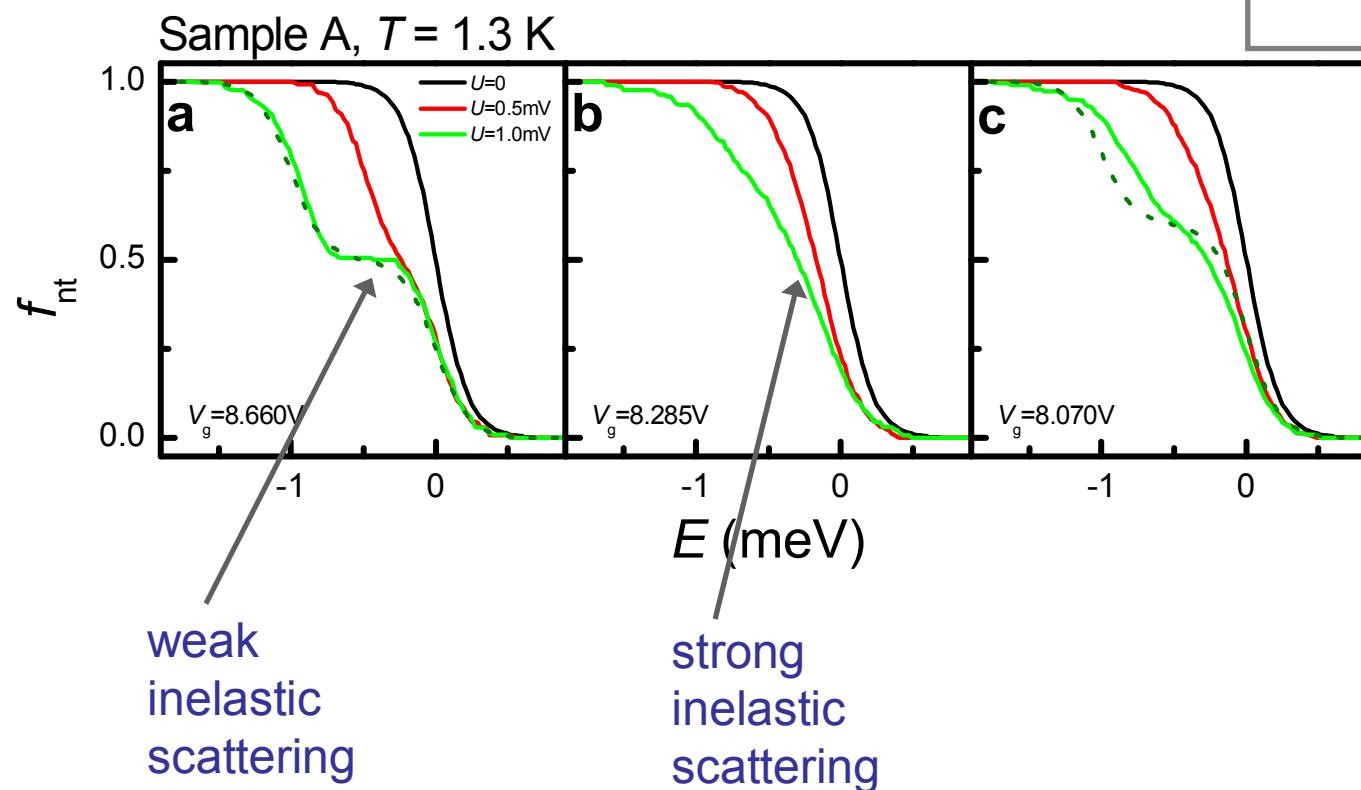
Sample B, $T = 53$ mK

$(k_B T \ll eU)$



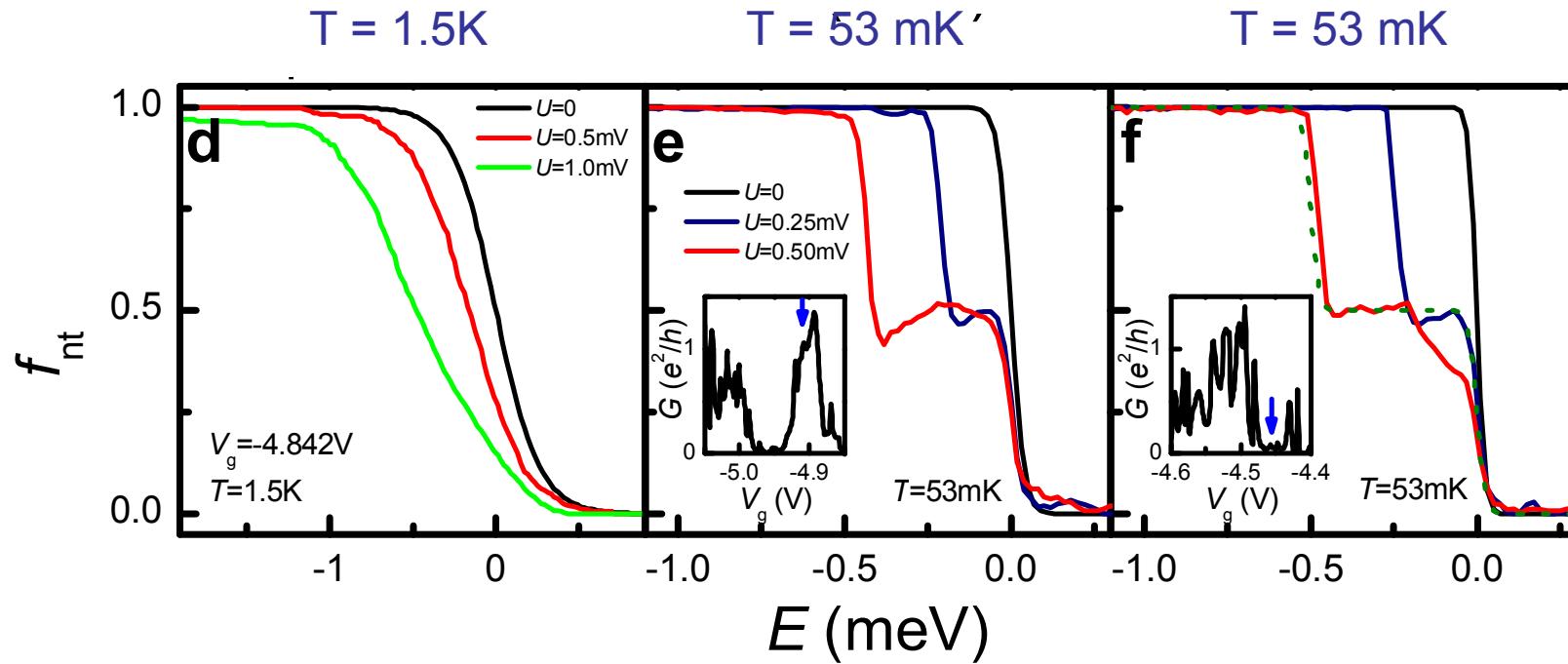
- electrons maintain energy over ~ 1 micron
- inelastic scattering can be weak (two-steps in $f(E)$)

$f(E)$ tuned with gate voltage



Shape of $f(E)$ not correlated with end conductance

No smearing at low temperatures



Somewhat consistent with theoretical predictions ...

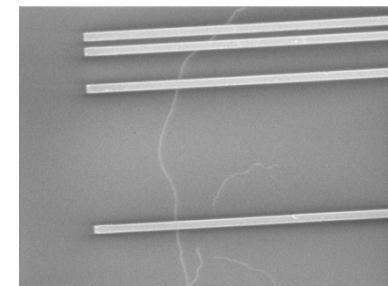
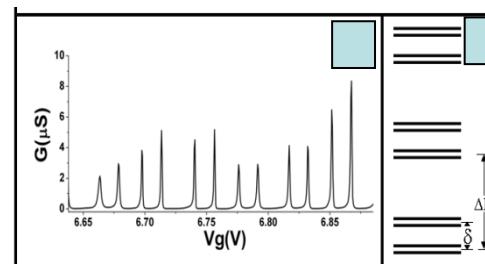
- * no energy relaxation in Luttinger Liquid [Khodas et al, PRB (2007); Gutman et al, PRL (2008)]
 - * energy relaxation only if there's disorder [Bagrets et al, arXiv:0809.3166v1 (2008)]
 - * energy relaxation channel opens as T increases [Levchenko, Matveev]
- Data consistent with limited energy relaxation at low T
- Why gate dependence, more smearing (or disorder) at higher T?

Parameters that may matter ...

- Disorder
- Dwell time: $\tau = L/(v_F * t) \sim 50\text{--}400 \text{ ps}$ for $R_{\text{end}} \sim 100 \text{ K}\Omega - 1\text{M}\Omega$
- 1D → 0D crossover: Thouless energy $\hbar v_F / L \sim 0.26 \text{ meV}$
(compare to $kT \sim .03\text{mV}$, $U \sim 1\text{mV}$)

In Progress:

- Cleaner tubes
- Length dependence of $f(E)$

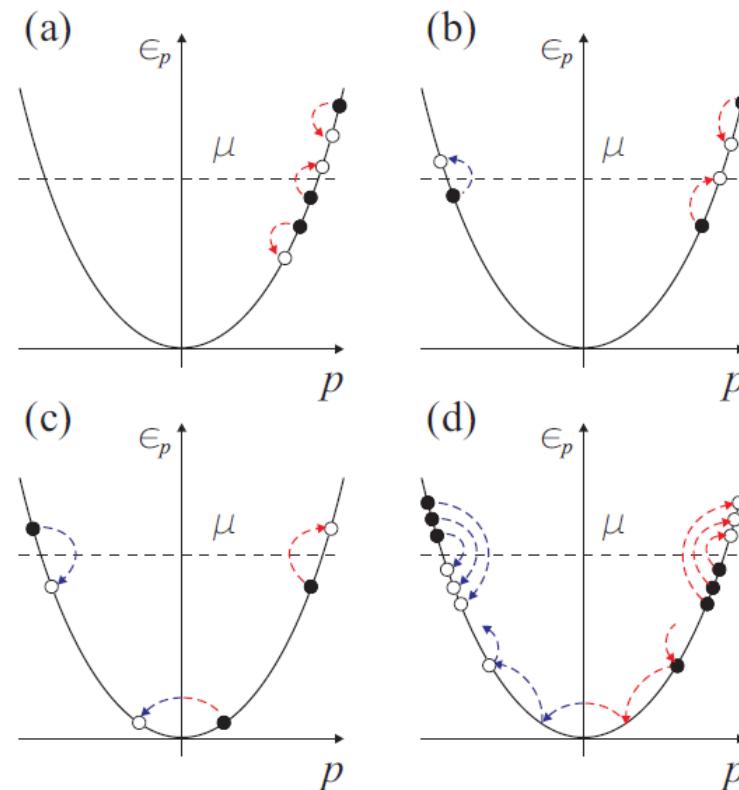


Energy Relaxation and Thermalization of Hot Electrons in Quantum Wires

Torsten Karzig,¹ Leonid I. Glazman,² and Felix von Oppen¹

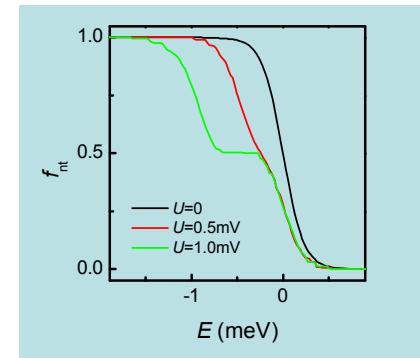
Interaction effects on thermal transport in quantum wires

Alex Levchenko,¹ Tobias Micklitz,² Zoran Ristivojevic,³ and K. A. Matveev¹



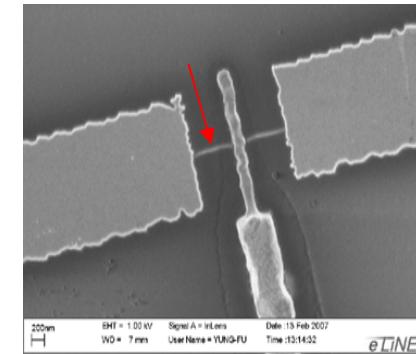
Summary

- First measurements of $f(E)$ in CNTs
- Fermi distr. at low T, sometimes smeared at high T
- Low T: consistent with LL
- Gate and Temp dependence: not consistent w/ LL?

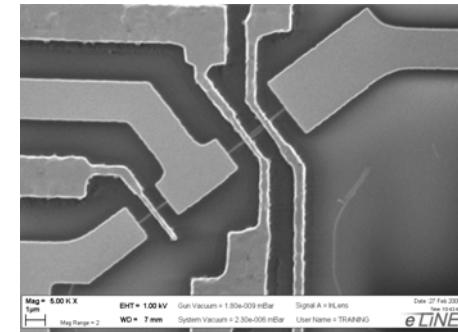


In Progress:

- Cleaner tubes
- Length dependence of $f(E)$
- Multiple tunnel probes
- SC, FM contacts



Superconducting Tunneling Spectroscopy
opens new avenues to studying electron
behavior in 1D!



References

- Energy Exchange in diffusive metal wires
 - Pothier, Gueron, Birge, Esteve, Devoret, Phys. Rev. Lett. **79**, 3490 (1997).
 - Pothier, Gueron, Birge, Esteve, Devoret, Z. Physik. B **104**, 178 (1997).
 - Anthore, Pierre, Pothier, Esteve, Phys. Rev. Lett. **90**, 076806 (2003).
 - Huard, Anthore, Birge, Pothier, Esteve, PRL **95**, 036802 (2005).
- Energy exchange in carbon nanotubes
 - Chen, Dirks, Al-Zoubi, Birge, and Mason, PRL **102**, 036804 (2009).
- Dephasing (not discussed in this lecture):
 - Pierre, Gougam, Anthore, Pothier, Esteve, and Birge, Phys. Rev. B **68**, 085413 (2003).