



Noise of quantum coherent conductors

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Autumn College on Non-Equilibrium Quantum Systems
2-13 de May 2011, Departamento de Fisica, FCEyN, Buenos Aires, Argentina

School Topics

Autumn College on Non-Equilibrium Quantum Systems

2-13 de May 2011,

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Covering,

- Many-body localization
- Topological quantum computations
- Quantum noise: Theory and experiment
- Non-equilibrium physics in nano-structures
- 1D fermions beyond the Luttinger liquid theory
- Ultra-cold atomic gases out of equilibrium
- Methods of non-equilibrium many-body theory
- Non-equilibrium spintronics

Lecture Topics

Noise of quantum coherent conductors

I Noise of quantum coherent conductors: Introduction

II Noise of quantum coherent conductors: correlations

III Noise of quantum coherent conductors: current topics

Three one-hour lectures

Fundamental sources of noise

Buttiker, PRB 46, 12485 (1992)

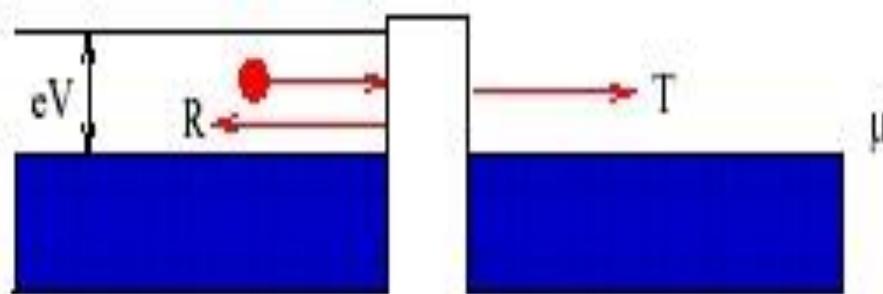
1. Thermal fluctuations of occupation numbers in the contacts

$$\Delta n(E) = n(E) - \langle n(E) \rangle; \quad f(E) = \langle n(E) \rangle$$

$$\langle (\Delta n)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = f - f^2 = f(1-f) = -kT df/dE$$

\Rightarrow Nyquist-Johnson noise

2. Quantum partition noise: $kT = 0$



occupation numbers:

n_I : incident beam

n_T : transmitted beam

n_R : reflected beam

averages: $\langle n_I \rangle = 1$; $\langle n_T \rangle = T$; $\langle n_R \rangle = R$;

Each particle can only be either transmitted or reflected:

$$\langle n_T n_R \rangle = 0; \Rightarrow$$

$$\langle (\Delta n_T)^2 \rangle = \langle (\Delta n_R)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = TR = T(1-T)$$

Lecture Topics

Noise of quantum coherent conductors

I. Conductance from Transmission

1. Single channel conductors
2. Two-probe multichannel conductors
3. Multiprobe conductors

II. Current noise in mesoscopic conductors

1. Basics
2. Equilibrium noise
3. Shot noise: two probe conductors
4. Shot noise: Correlations

III. Current noise: frequency dependence

1. Frequency dependent noise
2. Quantum noise
3. Charge relaxation

Books

Electronic Transport in Mesoscopic Systems
S. Datta, Cambridge University Press, 1995

Introduction to Mesoscopic Physics,
Y. Imry, Oxford University Press, 1997.

Mesoscopic Physics of Electrons and Photons

E. Akkermans and G. Montambaux, Cambridge University Press, 2007

Quantum Transport

Y. Nazarov and Ya. M. Blanter, Cambridge University Press, 2009

Review Articles

Quantum Transport in Semiconductor Nanostructures

C.W. J. Beenakker , H. van Houten, Solid State Physics 44, 1 (1991)

Random-matrix theory of quantum transport

C. W. J. Beenakker, Rev. Mod. Phys. 69, 731 (1997)

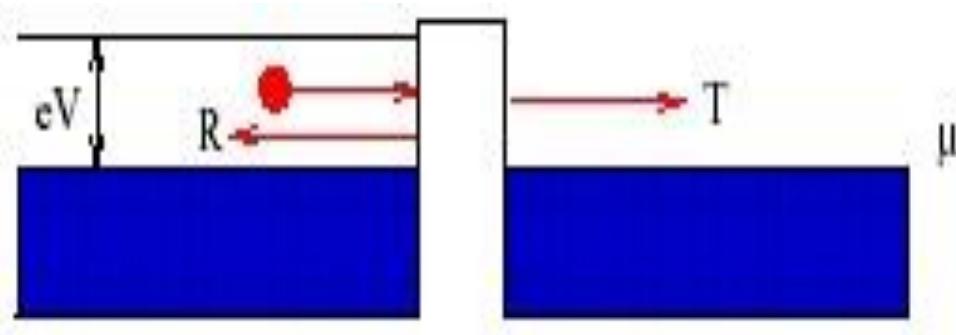
Shot Noise in Mesoscopic Conductors

Ya. M. Blanter, M. Buttiker , Phys. Rep. 336, 1 (2000).

Conductance from Transmission

1. Single channel conductors

Conductance from transmission



$$G = dI/dV = \frac{e^2}{h} T$$

$$\mathcal{R} = dV/dI = \frac{h}{e^2} \frac{1}{T}$$

conductance quantum

$$\frac{e^2}{h}$$

resistance quantum

$$\frac{h}{e^2} \approx 24 \text{ kOhm}$$

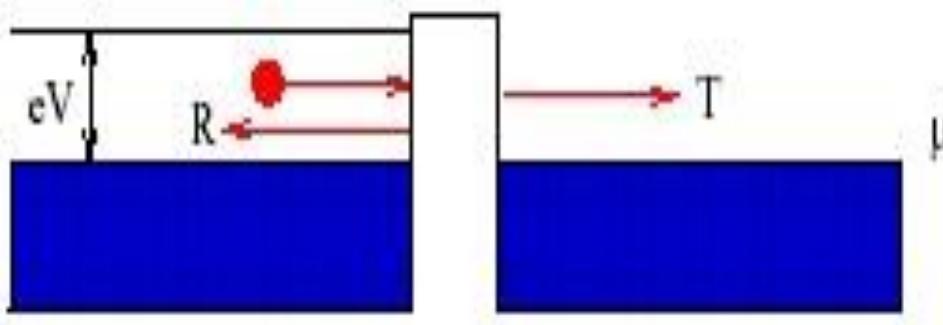
dissipation and irreversibility

$$W = IV = GV^2$$

boundary conditions

Transmission probability and conductance 9

Heuristic discussion



Fermi energy left contact $\mu + eV$
Fermi energy right contact μ ,
applied voltage eV ,
transmission probability T ,
reflection probability R ,

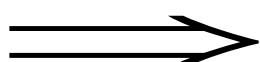
incident current

$$I_{in} = ev_F \Delta\rho$$

density

$$\Delta\rho = (d\rho/dE) eV$$

density of states $d\rho/dE = (d\rho/dk) (dk/dE) = (1/2\pi) (1/\hbar v_F)$



$$I_{in} = (e/h)eV \quad \text{independent of material !!}$$

$$I = (e/h)TeV \quad \Longrightarrow$$

$$G = dI/dV = \frac{e^2}{h} T \quad \text{« Landauer formula »}$$

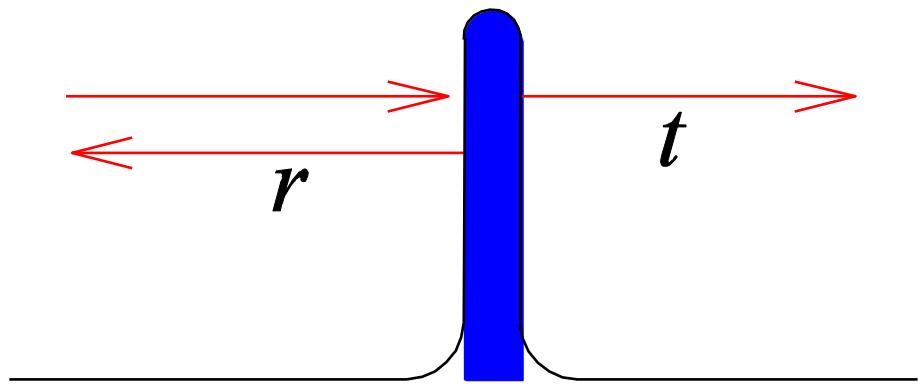
Scattering matrix

scattering state

$$|\Psi\rangle_{inc} = e^{ikx}$$

$$|\Psi\rangle_{ref} = r e^{-ikx}$$

$$|\Psi\rangle_{tra} = t e^{ikx}$$



scattering matrix

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

current conservation \Rightarrow S is a unitary matrix

In the absence of a magnetic field S is an orthogonal matrix

$$t' = t$$

Magnetic field symmetry

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = s(B) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

B-invariant if momenta and magnetic field are reversed

$$\begin{pmatrix} a_1^* \\ a_2^* \end{pmatrix} = s(-B) \begin{pmatrix} b_1^* \\ b_2^* \end{pmatrix} \Rightarrow \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = s^*(-B) \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Rightarrow s^*(-B)s(B) = 1 \Rightarrow s^\dagger(B) = s^*(-B) \Rightarrow$$

$$s^T(B) = s(-B)$$

$$t'(B) = t(-B) \Rightarrow T'(B) = T(-B)$$

but $T'(B) = T(B) \Rightarrow T(B) = T(-B)$

$$G = dI/dV = \frac{e^2}{h} T \quad \text{is an even function of magnetic field}$$

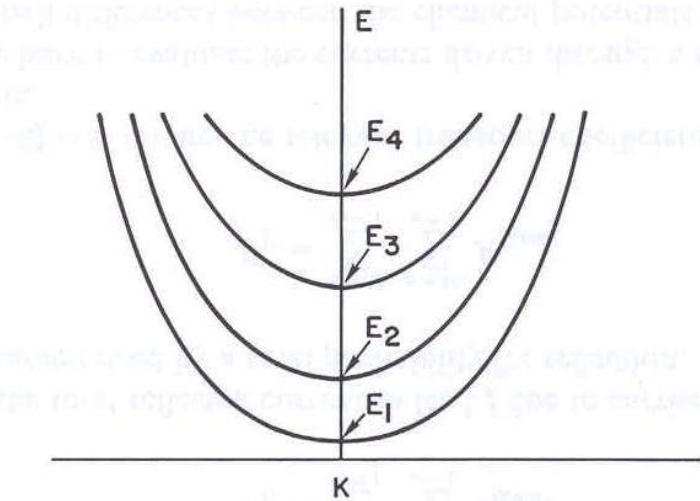
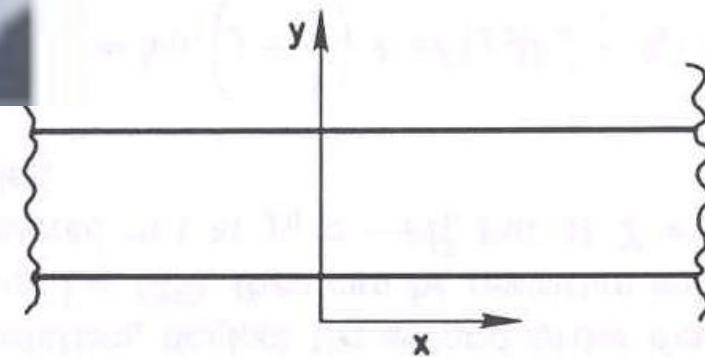
Conductance from Transmission

2. Two-probe multi-channel conductors

Multi-channel conductance: leads



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asymptotic perfect translation invariant potential

$$V(x, y) = V(y) \implies$$

separable wave function

$$\phi_{\alpha n}^{\pm}(\mathbf{r}, E) = e^{\pm i k_n(E) x} \chi_{\alpha n}(y)$$

energy of transverse motion E_n channel threshold

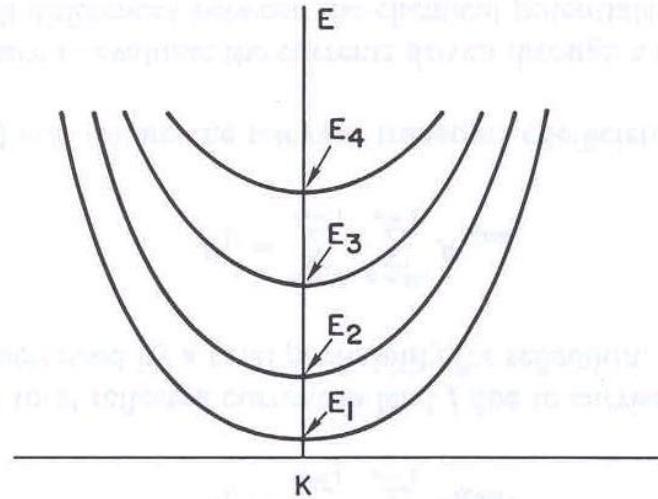
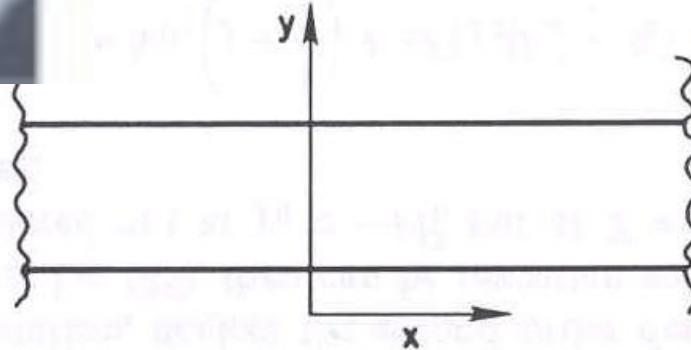
energy for transverse and longitudinal motion

$$E = E_n + \hbar^2 k^2 / 2m \iff \text{scattering channel}$$

Lead: wave function



@Bouchiat



Example: hard wall; N channel lead

$$\Psi(\mathbf{r}, E) = \sum_{n=1}^{n=N} \frac{1}{\sqrt{h\nu_n}} [a_n e^{ik_n(E)x} + b_n e^{-ik_n(E)x}] \chi_n(y)$$

transverse wave function

$$\chi_n(y) = \left(\frac{2}{w}\right)^{1/2} \sin \frac{n\pi y}{w}$$

wave vector in channel n

$$E = \frac{\hbar^2 k_n^2}{2m} + \frac{\hbar^2}{2m} \left(\frac{n\pi}{w}\right)^2$$

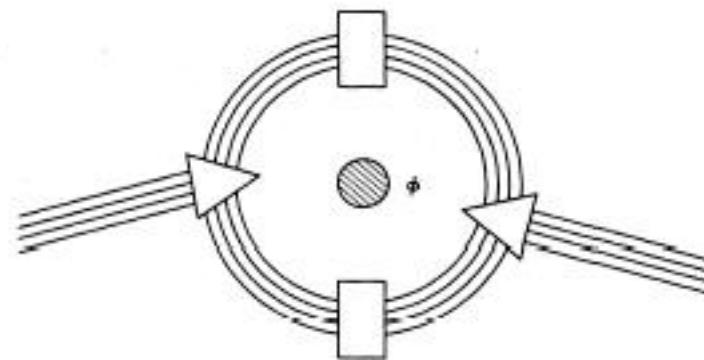
particle flux towards scatterer

$$|a_n|^2$$

particle flux moving away from scatterer

$$|b_n|^2$$

Scattering matrix



leads $\alpha = 1, 2, \dots$ with N_α channels

$|a_{\alpha n}|^2$ particle flux towards scatterer in lead α

$|b_{\alpha n}|^2$ particle flux moving away from scatterer in lead α

Scattering matrix s

$$b_{\beta m} = \sum_{\alpha} \sum_n s_{\alpha \beta m n} a_{\alpha n}$$

$|s_{\alpha \beta m n}|^2$ Probability that a carrier incident in lead α in channel n leaves the conductor in lead β in channel m

Scattering matrix: properties

Scattering matrix s

$$b_{\beta m} = \sum_{\alpha} \sum_n s_{\alpha \beta mn} a_{\alpha n}$$

Particle flux (current conservation) implies that s is unitary:

$$s^\dagger s = 1$$

Microreversibility in presence of a magnetic field B:

$$s(B) = s^T(-B)$$

$$s_{\alpha \beta mn}(B) = s_{\beta \alpha nm}(-B)$$

Probability to scatter from lead α in channel n into lead β in channel m in a magentic field B is the same as th probability to scatter from lead β in channel m into lead α in channel n in a magentic field - B

Eigen channels

$$T = \sum_{mn} T_{\beta\alpha,mn} = \sum_{mn} |s_{\beta\alpha,mn}|^2 = Tr[s_{\alpha\beta}^\dagger s_{\alpha\beta}] = Tr[t^\dagger t]$$

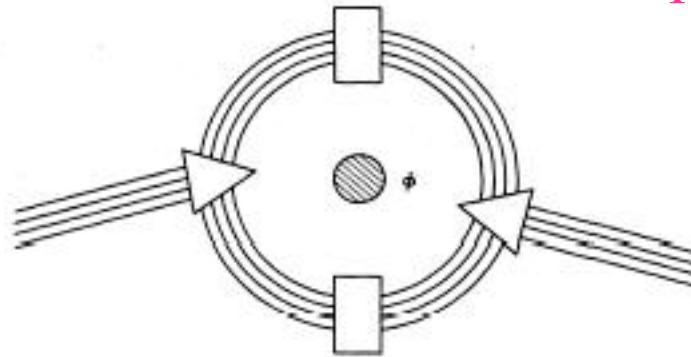
$t^\dagger t$ hermitian matrix; real eigenvalues T_n

$r^\dagger r$ hermitian matrix; real eigenvalues R_n

$$T = Tr[t^\dagger t] = \sum_n T_n$$

$$G = \frac{e^2}{h} \sum_n T_n$$

T_n are the genetic code of mesoscopic conductors !!



Many single channel conductors in parallel.

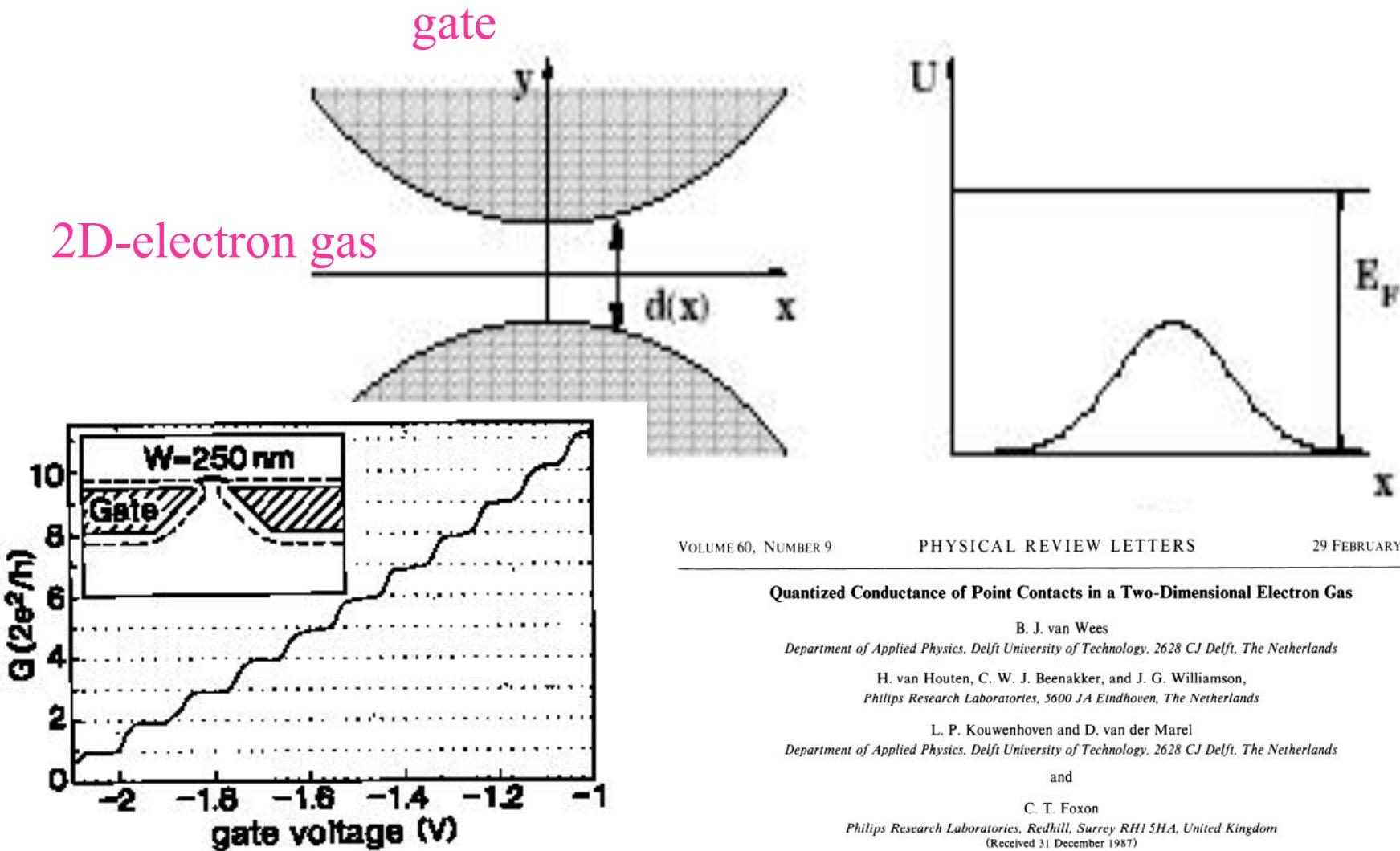
All the properties we discussed for single-channel two-probe conductors apply equally to many-channel multi-probe conductors: in particular

$$G(B) = G(-B)$$

Quantum Point Contact

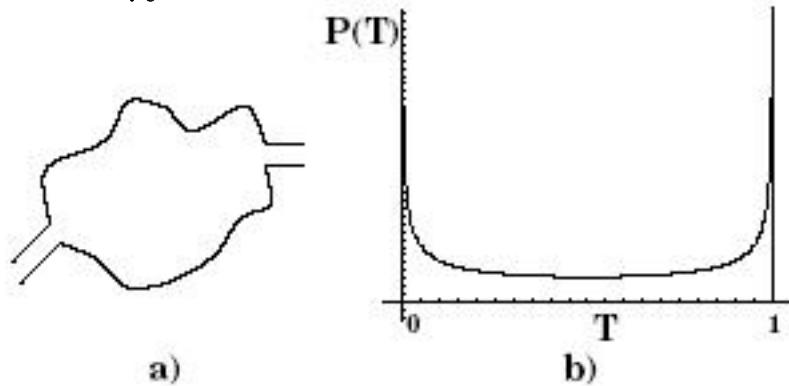
van Wees et al., PRL 60, 848 (1988)

Wharam et al, J. Phys. C 21, L209 (1988)



Chaotic cavity

$$G = \frac{e^2}{h} \sum_n T_n$$



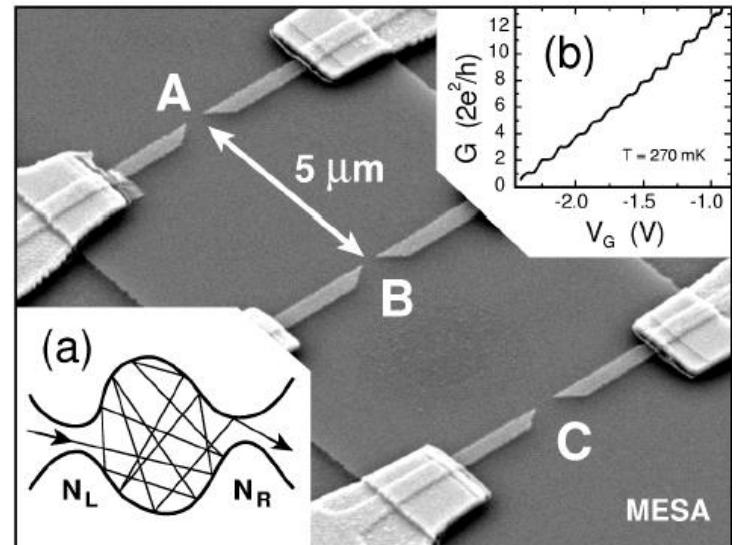
$$p(T) = \frac{1}{\pi} \frac{1}{\sqrt{T(1-T)}} \quad \text{for symmetric cavity with} \quad N_1 = N_2 = N/2 \gg 1$$

\Rightarrow

$$\langle G \rangle = \frac{e^2 N}{h} \frac{1}{4}$$

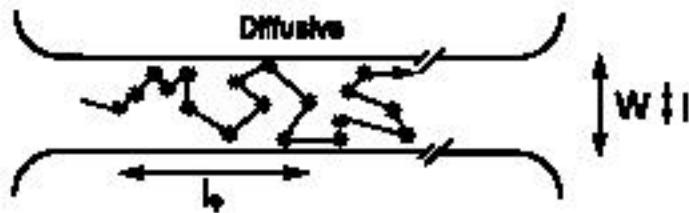
asymmetric cavity including weak localization:

$$\langle G \rangle = \frac{e^2}{h} \left[\frac{N_2 N_1}{N} - \frac{(2-\beta) N_2 N_1}{\beta N^2} + O(1/N^3) \right]$$



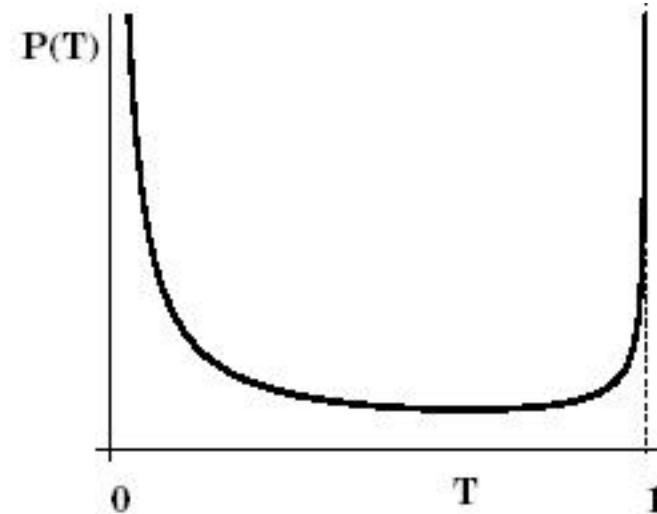
Baranger and Mello, 1994

Diffusive wire



$$p(T) = \frac{l_e}{2L} \frac{1}{T\sqrt{(1-T)}}$$

$$\langle G \rangle = \frac{e^2}{h} N \frac{l_e}{L}$$

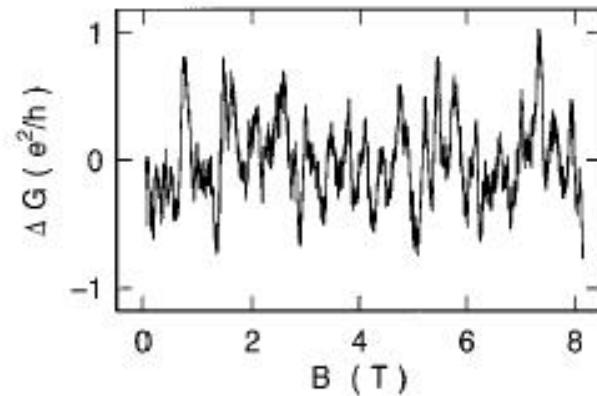


Dorokhov-Mello-Pereyra-Kumar

Universal conductance fluctuations

$$\langle (\Delta G)^2 \rangle = \frac{2}{15\beta} \frac{e^2}{h}$$

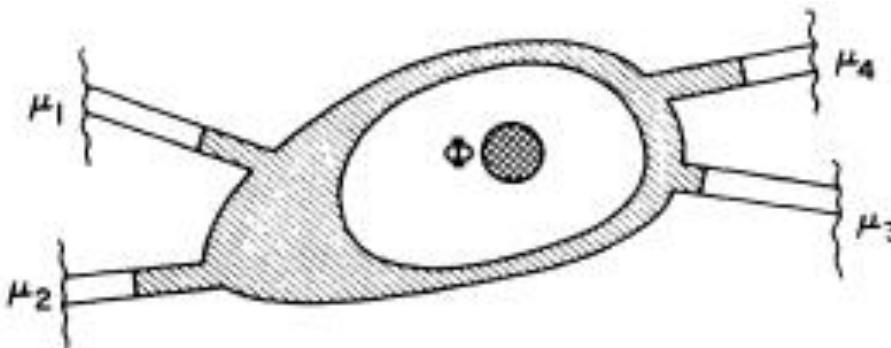
Stone and Lee, Altschuler



Conductance from Transmission

3. Multi-probe conductors

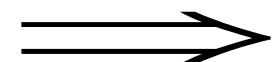
Multi-probe conductors



$$\mu_\alpha = \mu_0 + eV_\alpha$$

$$I_\alpha = \frac{e}{h} [(N_\alpha - R_{\alpha\alpha}) \mu_\alpha - \sum_{\beta \neq \alpha} T_{\alpha\beta} \mu_\beta]$$

Conductance matrix



$$G_{\alpha\alpha} = dI_\alpha/dV_\alpha = \frac{e^2}{h} (N_\alpha - R_{\alpha\alpha}) = \frac{e^2}{h} \sum_{\beta \neq \alpha} T_{\alpha\beta}$$

$$G_{\alpha\beta} = dI_\alpha/dV_\beta = -\frac{e^2}{h} T_{\alpha\beta}$$

$$I_\alpha = \sum_\beta G_{\alpha\beta} V_\beta ; \quad \sum_\alpha G_{\alpha\beta} = 0 ; \quad \sum_\beta G_{\alpha\beta} = 0$$

Four-probe resistance

$$\mathcal{R}_{\alpha\beta,\gamma\delta} = \frac{V_\gamma - V_\delta}{I} = \frac{G_{\gamma\alpha} G_{\delta\beta} - G_{\gamma\beta} G_{\delta\alpha}}{\mathcal{D}}$$

Gives the time-averaged currents: dissipative! Fluctuations!!

Current Noise in Mesoscopic Conductors

1. Basics

Fundamental sources of noise

Buttiker, PRB 46, 12485 (1992)

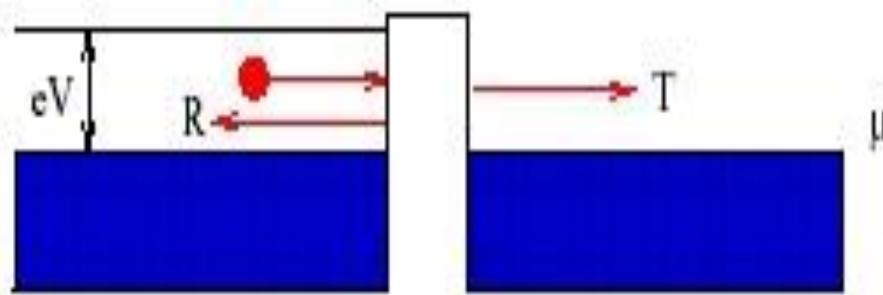
1. Thermal fluctuations of occupation numbers in the contacts

$$\Delta n(E) = n(E) - \langle n(E) \rangle; \quad f(E) = \langle n(E) \rangle$$

$$\langle (\Delta n)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = f - f^2 = f(1-f) = -kT df/dE$$

\Rightarrow Nyquist-Johnson noise

2. Quantum partition noise: $kT = 0$



occupation numbers:

n_I : incident beam

n_T : transmitted beam

n_R : reflected beam

averages: $\langle n_I \rangle = 1$; $\langle n_T \rangle = T$; $\langle n_R \rangle = R$;

Each particle can only be either transmitted or reflected:

$$\langle n_T n_R \rangle = 0; \Rightarrow$$

$$\langle (\Delta n_T)^2 \rangle = \langle (\Delta n_R)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = TR = T(1-T)$$

Occupation numbers and current amplitudes

Buttiker, PRB 46, 12485 (1992)

Incident current at $kT = 0$

$$I_{in} = (e/h)eV$$

Incident current at $kT > 0$

$$dI_{in} = (e/h) f(E) dE$$

Occupation number

$$f(E) = \langle n(E) \rangle$$

$\langle \rangle$ = statistical average

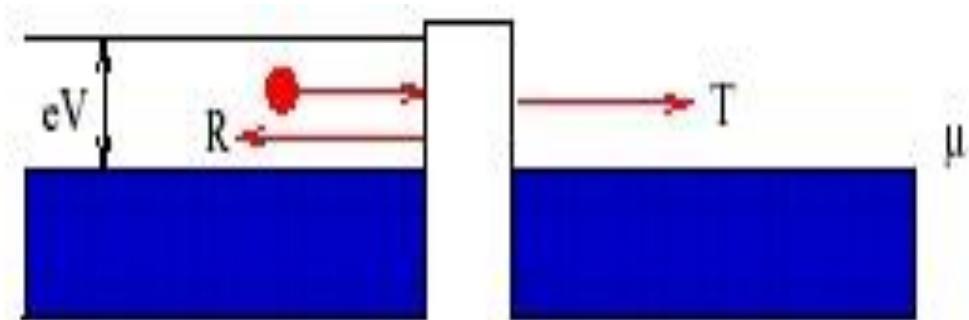
Creation and annihilation operators

$$\langle \hat{a}^\dagger(E) \hat{a}(E') \rangle = f(E) \delta(E - E')$$

«Incident current» «Current amplitude» $\hat{a}(E)$

$$\hat{I}_{in}(t) = (e/h) \int dE \int dE' \hat{a}^\dagger(E) \hat{a}(E') e^{i(E-E')t/\hbar}$$

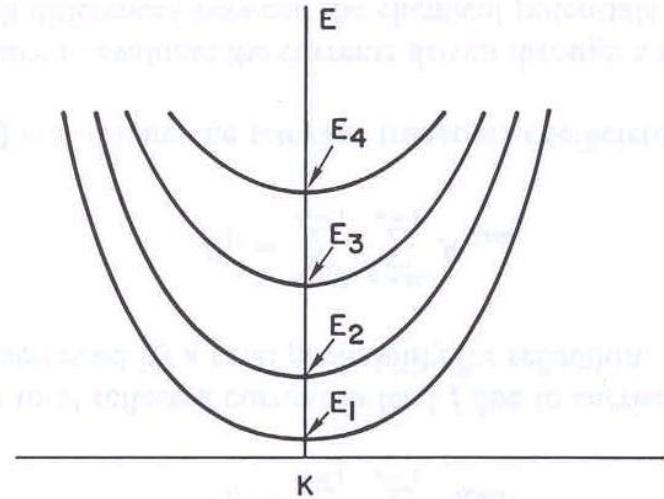
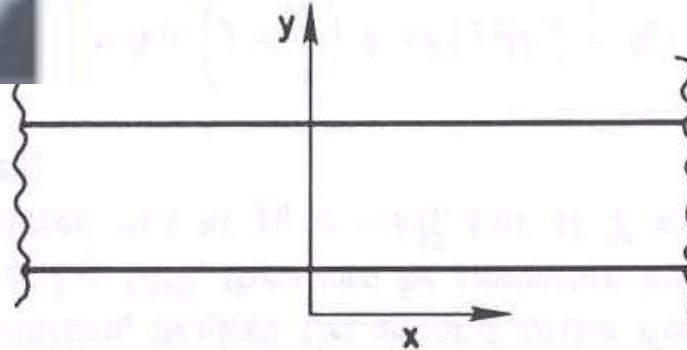
$$\hat{I}_{in}(t) = (e/h) \int dE \hat{n}(E, t)$$



Fermi field



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Example: hard wall; N channel lead

$$\hat{\Psi}(\mathbf{r}, t) = \int dE \sum_{n=1}^{n=N} \frac{1}{\sqrt{hv_n(E)}} [\hat{a}_n(E) e^{ik_n(E)x} + \hat{b}_n(E) e^{-ik_n(E)x}] \chi_n(y)$$

transverse wave function $\chi_n(y) = (\frac{2}{w})^{1/2} \sin \frac{n\pi y}{w}$

$$\text{wave vector in channel } n \quad E = \frac{\hbar^2 k_n^2}{2m} + \frac{\hbar^2}{2m} \left(\frac{n\pi}{w} \right)^2$$

annihilation operator (in-moving particle at energy E) $\hat{a}_n(E)$

annihilation operator (out-moving particle at energy E) $\hat{b}_n(E)$

$$[\hat{a}_n^\dagger(E), \hat{a}_m(E')] = \delta_{nm} \delta(E - E') \implies [\hat{b}_n^\dagger(E), \hat{b}_m(E')] = \delta_{nm} \delta(E - E')$$

Current operator

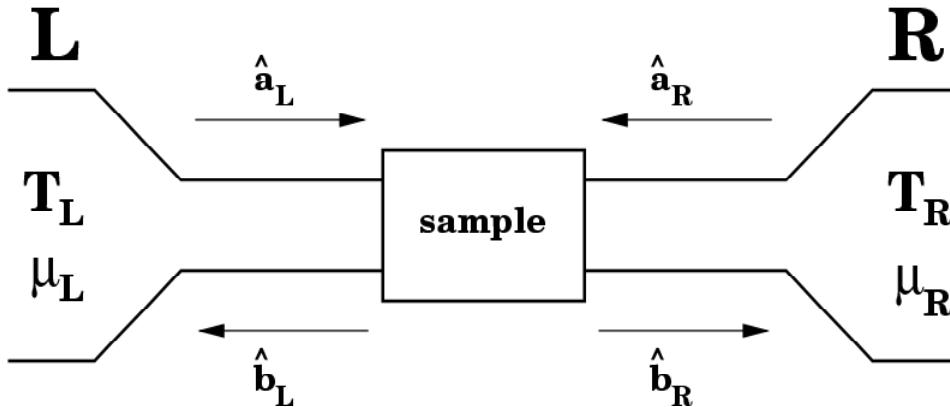
Buttiker, PRL 65, 2901 (1990)

Current in contact α single channel result

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE [\hat{n}_{\alpha,in}(E, t) - \hat{n}_{\alpha,out}(E, t)]$$

current amplitude: $\hat{a}_\alpha(E)$ (incoming) $\hat{b}_\alpha(E)$ (outgoing)

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE' dE [\hat{a}_\alpha^\dagger(E') \hat{a}_\alpha(E) - \hat{b}_\alpha^\dagger(E') \hat{b}_\alpha(E)] e^{i(E'-E)t/\hbar}$$



Current in contact α multi-channel channel result

$$\hat{I}_\alpha(t) = \frac{e}{h} \sum_n \int dE' dE [\hat{a}_{\alpha n}^\dagger(E') \hat{a}_{\alpha n}(E) - \hat{b}_{\alpha n}^\dagger(E') \hat{b}_{\alpha n}(E)] e^{i(E'-E)t/\hbar}$$

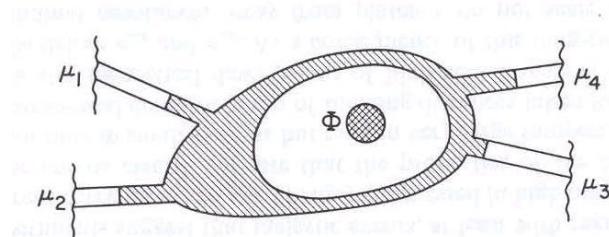
Spatial oscillations of the current as a function of r are suppressed (screening)

Current operator: scattering matrix

Buttiker, PRB 46, 12485 (1992)

$$\hat{b}_\alpha = \sum_\beta s_{\alpha\beta} \hat{a}_\beta$$

Elimination of out-going annihilation operators



$$I_\alpha(t) = \frac{e}{h} \int dE' dE [a_\alpha^\dagger(E') a_\alpha(E) - b_\alpha^\dagger(E') b_\alpha(E)] e^{i(E'-E)t/\hbar}$$

$$I_\alpha(t) = \frac{e}{h} \int dE' dE \sum_{\beta,\gamma} a_\beta^\dagger(E') A_{\beta\gamma}(\alpha, E', E) a_\gamma(E) e^{i(E'-E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E') s_{\alpha\gamma}(E)$$

Average currents

Quantum-statistical expectation values

$$\langle a_{\alpha m}^\dagger(E') a_{\beta n}(E) \rangle = \delta(E' - E) \delta_{\alpha\beta} \delta_{mn} f(E)$$

Review: Conductance (finite temperature)

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE' dE \sum_{\beta, \gamma} \hat{a}_\beta^\dagger(E') A_{\beta\gamma}(\alpha, E', E) \hat{a}_\gamma(E) e^{i(E' - E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E') s_{\alpha\gamma}(E)$$

quantum statistical average

$$\langle \hat{a}_\beta^\dagger(E) \hat{a}_\gamma(E') \rangle = \delta_{\beta\gamma} \delta(E - E') f_\beta(E) \quad \Rightarrow$$

$$I_\alpha = \frac{e}{h} \int dE \left[(N_\alpha - R_{\alpha\alpha}) f_\alpha - \sum_{\beta \neq \alpha} T_{\alpha\beta} f_\beta \right]$$

$$R_{\alpha\alpha} = \text{Tr}(s_{\alpha\alpha}^\dagger s_{\alpha\alpha}) \quad T_{\alpha\beta} = \text{Tr}(s_{\alpha\beta}^\dagger s_{\alpha\beta})$$

$$f_\alpha(\mu_\alpha) = f(\mu_0) - (df/dE)eV_\alpha + .. \quad \Rightarrow$$

$$I_\alpha = \frac{e}{h} \sum_\beta G_{\alpha\beta} V_\beta \quad \sum_\alpha G_{\alpha\beta} = 0 \quad \sum_\beta G_{\alpha\beta} = 0$$

$$G_{\alpha\alpha} = \frac{e^2}{h} \int dE (-df/dE) (N_\alpha - R_{\alpha\alpha}); \quad G_{\alpha\beta} = -\frac{e^2}{h} \int dE (-df/dE) T_{\alpha\beta}$$

Normal and exchange pairing

$$\langle a_{\delta k}^\dagger(E''') a_{\gamma l}(E'') a_{\beta m}^\dagger(E') a_{\alpha n}(E) \rangle =$$

Normal pairing

$$\delta = \gamma, \ k = l, \ \beta = \alpha, \ m = n, \ E''' = E'', \ E' \stackrel{\Rightarrow}{=} E$$

$$= \delta_{\delta\gamma} \delta_{kl} \delta_{\beta\alpha} \delta_{mn} \delta(E''' - E'') \delta(E' - E) f_\delta(E) f_\beta(E'')$$

\Rightarrow product of average currents

Exchange pairing

$$\delta = \alpha, \ k = n, \ \beta = \gamma, \ m = l, \ E''' = E, \ E' = E''$$

$$= \delta_{\delta\alpha} \delta_{kn} \delta_{\beta\gamma} \delta_{ml} \delta(E''' - E) \delta(E' - E'') f_\alpha(E) (1 - f_\beta(E'))$$

 noise

Noise spectral density

Spectral density S (noise power)

$$(1/2)\langle I_\alpha(\omega)I_\beta(\omega') + I_\beta(\omega')I_\alpha(\omega) \rangle = 2\pi S_{\alpha\beta}(\omega)\delta(\omega+\omega')$$

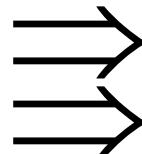
quantum statistical average of four creation and annihilation op. \Rightarrow

zero-frequency spectrum (white noise limit)

$$S_{\alpha\beta} = 2\frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)]f_\gamma(E)(1-f_\delta(E))$$

$$A_{\beta\gamma}(\alpha, E) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E) s_{\alpha\gamma}(E)$$

equilibrium



fluctuation-dissipation theorem
shot-noise

non-equilibrium

Buttiker, PRL 65, 2901 (1990); PRB 46, 12485 (1992)

Current Noise in Mesoscopic Conductors

2. Equilibrium Noise

Thermal current fluctuations

Use

$$S_{\alpha\beta} = 2 \frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)] f_\gamma(E) (1 - f_\delta(E))$$

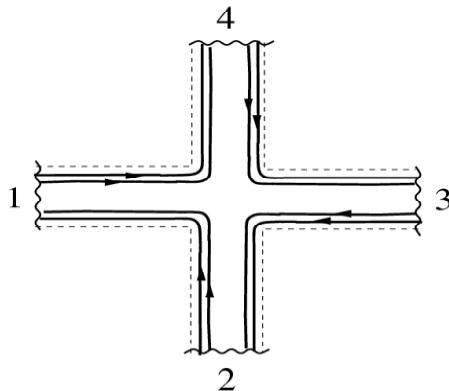
with $f_\alpha(E) = f(E)$ for all $\alpha = 1, 2, 3, \dots \Rightarrow$

auto-correlation $\langle I_\alpha^2 \rangle_\nu$

$$S_{\alpha\alpha} = 2kT G_{\alpha\alpha} = 2kT \frac{e^2}{h} \int dE (-df/dE) (N_\alpha - R_{\alpha\alpha}) ;$$

cross-correlation $\langle I_\alpha I_\beta \rangle_\nu$

$$S_{\alpha\beta} = kT [G_{\alpha\beta} + G_{\beta\alpha}] = -kT \frac{e^2}{h} \int dE (-df/dE) [T_{\alpha\beta} + T_{\beta\alpha}]$$



QHE-plateau N:

$$S_{\alpha\alpha} = 2kT \frac{e^2}{h} N ;$$

$$S_{\alpha+3,\alpha} = -kT \frac{e^2}{h} N ;$$

$$S_{\alpha+2,\alpha} = S_{\alpha+1,\alpha} = 0$$

Thermal voltage fluctuations

infinite impedance external circuit $I_\alpha = 0$

$$I_\alpha = \frac{e}{h} \sum_\beta G_{\alpha\beta} V_\beta + dI_\alpha$$

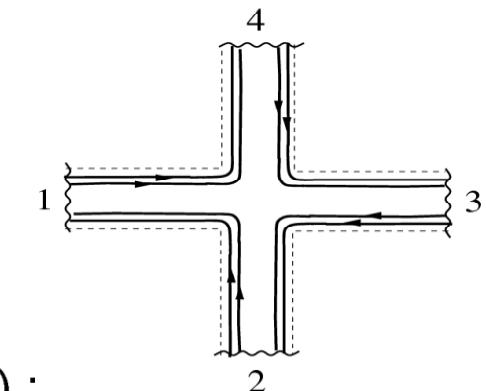
dI_α Langevin sources with

$$S_{\alpha\alpha} = 2kT G_{\alpha\alpha} = 2kT \frac{e^2}{h} \int dE (-df/dE) (N_\alpha - R_{\alpha\alpha});$$

$$S_{\alpha\beta} = kT [G_{\alpha\beta} + G_{\beta\alpha}] = -kT \frac{e^2}{h} \int dE (-df/dE) [T_{\alpha\beta} + T_{\beta\alpha}]$$

$$\langle (V_\alpha - V_\beta)^2 \rangle = 2kT \mathcal{R}_{\alpha\beta, \alpha\beta}$$

$$\langle (V_\alpha - V_\beta)(V_\gamma - V_\delta) \rangle = kT (\mathcal{R}_{\alpha\beta, \gamma\delta} + \mathcal{R}_{\gamma\delta, \alpha\beta})$$



QHE plateau:

$$\langle (V_\alpha - V_\beta)(V_\gamma - V_\delta) \rangle = 2kT R_L \quad \text{for longitudinal resistance}$$

$$\langle (V_\alpha - V_\beta)(V_\gamma - V_\delta) \rangle = 0 \quad \text{for Hall resistance}$$

Current Noise in Mesoscopic Conductors

3. Shot Noise: Two-probe conductors

Shot-Noise: Two-terminal

$$S_{\alpha\beta} = 2 \frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)] f_\gamma(E) (1 - f_\delta(E))$$

Consider $kT = 0$, $V > 0$, and a two-terminal conductor:

$$S = S_{11} = -S_{12} = -S_{21} = S_{22};$$

Quantum partition noise

$$S = 2 \frac{e^2}{h} |eV| Tr[tt^\dagger rr^\dagger] = 2 \frac{e^2}{h} |eV| \sum_n T_n (1 - T_n)$$

If all $T_n \ll 1 \Rightarrow$

$$S = 2e \left(\frac{e^2}{h} \sum_n T_n \right) |V| = 2e |I| \quad \text{Shottky (Poisson)}$$

Fano factor

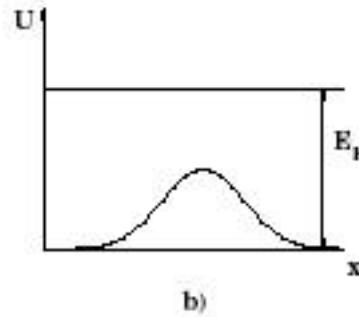
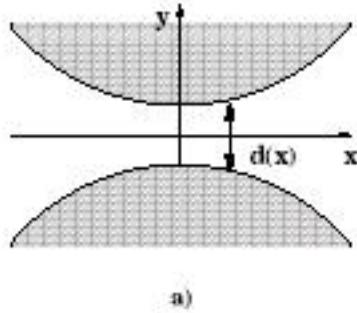
$$F = \frac{S}{S_P} = \frac{\sum_n T_n (1 - T_n)}{\sum_n T_n}$$

Khlus (1987)

Lesovik (1989)

Buttiker (1990)

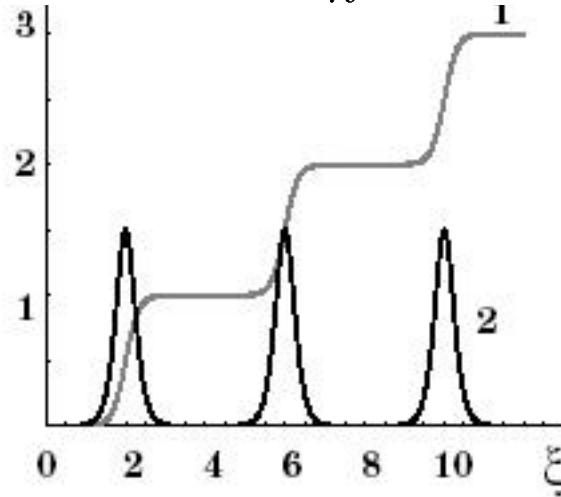
Shot-Noise: Quantum Point Contact



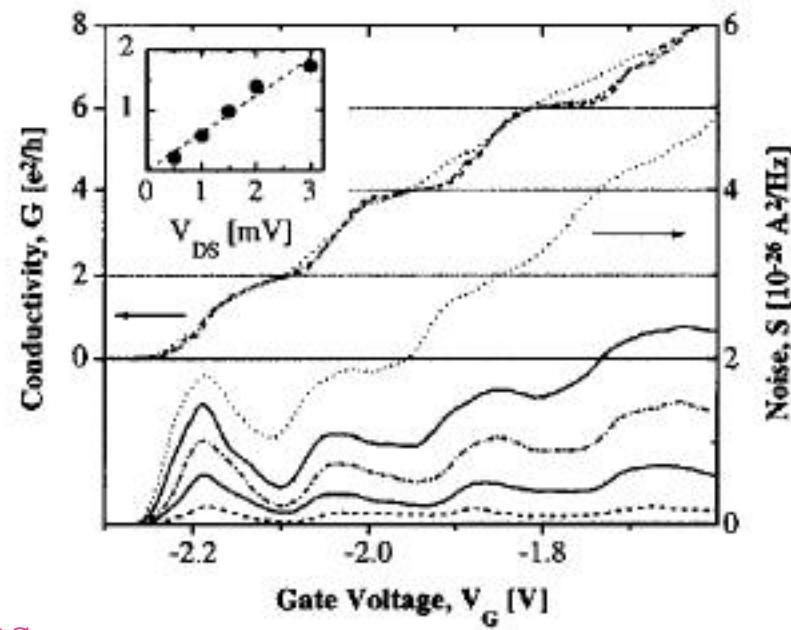
- Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, B. Etienne, PRL 76, 2778 (1996)

- M. I. Reznikov, M. Heiblum, H. Shtrikman, D. Mahalu, PRL 75, 3340 (1996)

$$S = 2 \frac{e^2}{h} |eV| \sum_n T_n (1 - T_n)$$



Ideally only one channel contributes



Shot-Noise: Metallic diffusive wire

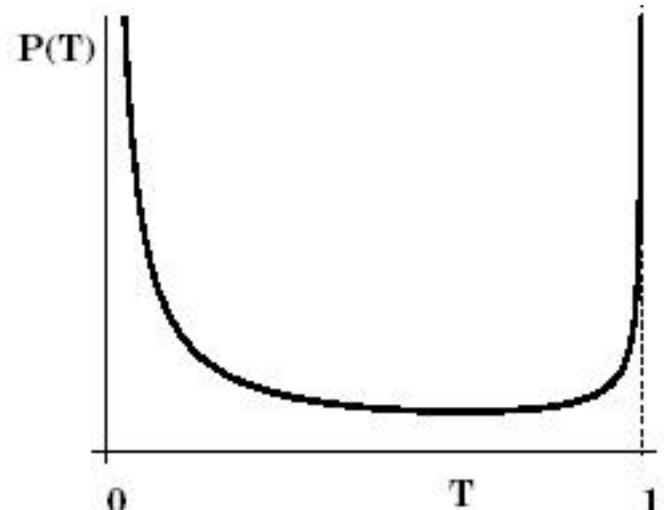
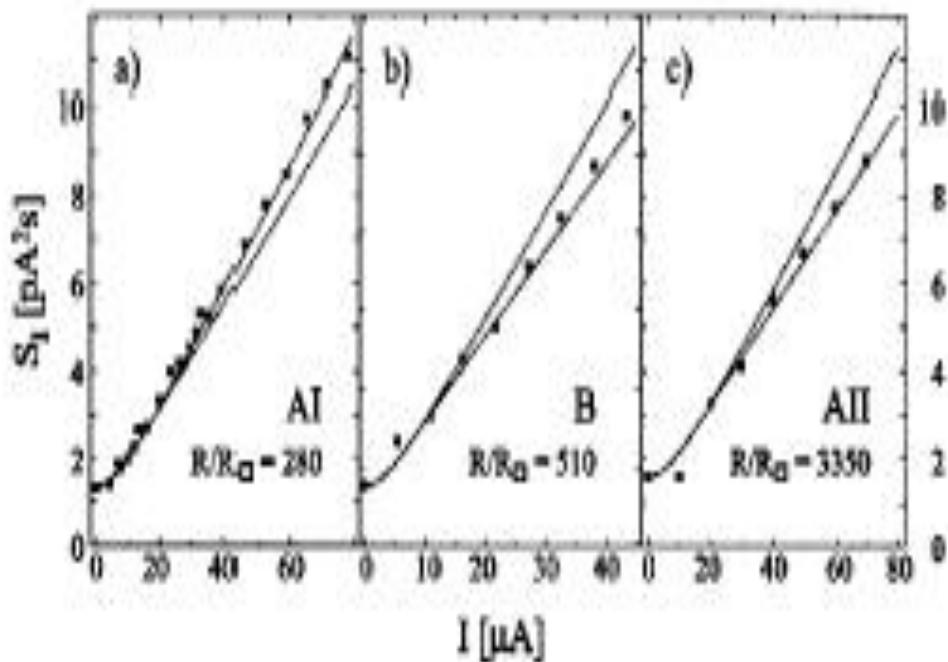
Beenakker and Buttiker, PRB 46, 1889 (1992), [K. Nagaev, Phys. Lett. A 169, 103 (1992).]

$$G = \frac{e^2}{h} \sum_n T_n$$

$$S = 2 \frac{e^2}{h} |eV| \sum_n T_n (1 - T_n)$$

[Steinbach, Martinis, Devoret, PRl (1996)].

Henny et al. PRB 59, 2871 (1999)



$$p(T) = \frac{l_e}{2L} \frac{1}{T \sqrt{(1-T)}}$$

$$\langle G \rangle = \frac{e^2}{h} N \frac{l_e}{L}$$

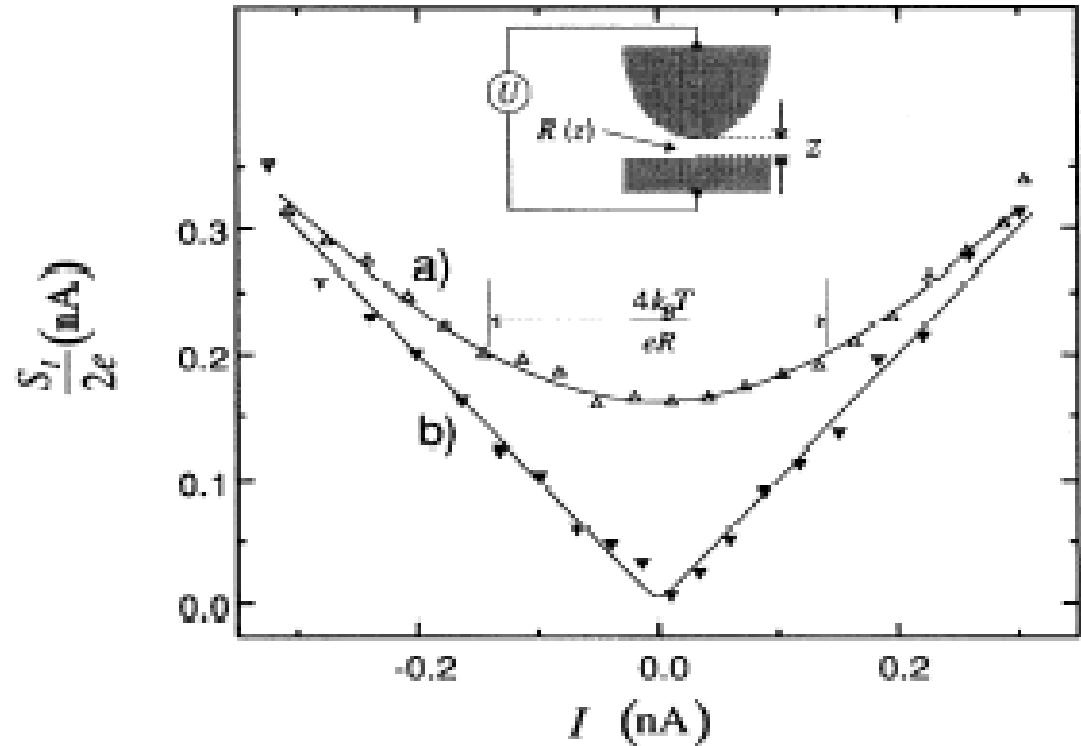
$$\langle S \rangle = \frac{1}{3} 2e|I|$$

Crossover from thermal to shot noise

$$S = 2 \frac{e^2}{h} \sum_n \int dE [T_n f_1(1-f_1) + T_n f_2(1-f_2) + T_n R_n (f_1 - f_2)^2]$$

tunnel junction $T_n \ll 1$

$$S = 2 \frac{e^2}{h} \sum_n T_n |eV| \coth\left(\frac{eV}{2kT}\right) = 2e|I| \coth\left(\frac{eV}{2kT}\right)$$



H. Birk et al., PRL 75, 1610 (1995)

Current Noise in Mesoscopic Conductors

4. Shot Noise: Correlations

Shot noise: correlations

$$S_{\alpha\beta} = 2 \frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)] f_\gamma(E) (1 - f_\delta(E))$$

Consider multi-terminal conductor at $kT = 0$,

M source contacts with distribution f voltage

All other contacts grounded at f_0 voltage

Correlation measured between two grounded contacts:

$$S_{\alpha\beta} = -2 \frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[B_{\alpha\beta}^\dagger B_{\alpha\beta}] ; B_{\alpha\beta} = \sum_{\gamma=1}^M s_{\alpha\gamma}^\dagger s_{\beta\gamma} (f - f_0)$$

$M = 1$, partition noise

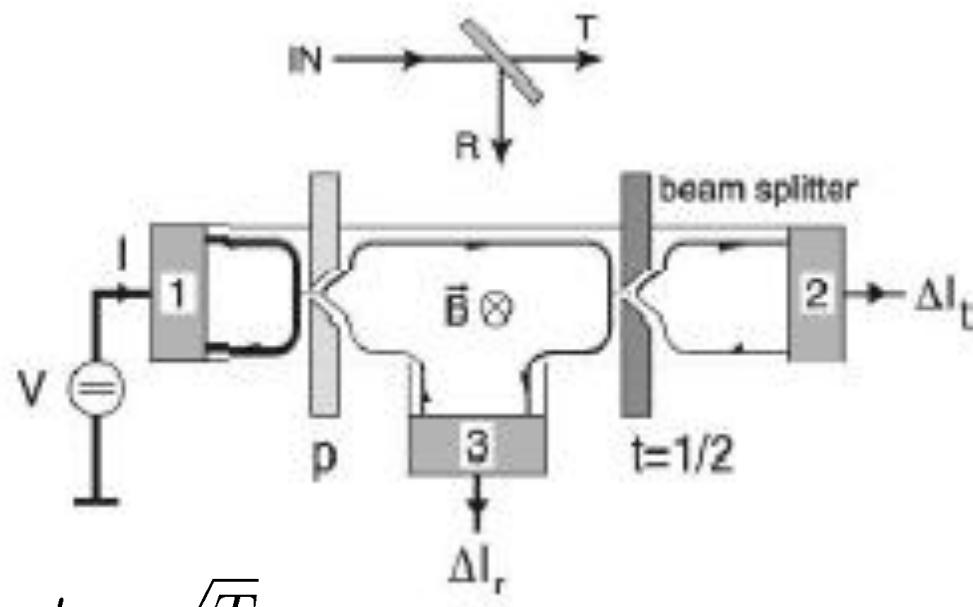
$M = 2$, exchange effects,

two particle Aharonov-Bohm effect,

orbital entanglement, violation of Bell inequality

Beam splitter with noisy input state

Oberholzer et al. Physica E6, 314 (2000)



Here $p = \kappa$, $t = \sqrt{T}$

Bias configuration: $\mu_1 = \mu_0 + eV$, $\mu_2 = \mu_3 = \mu_0$

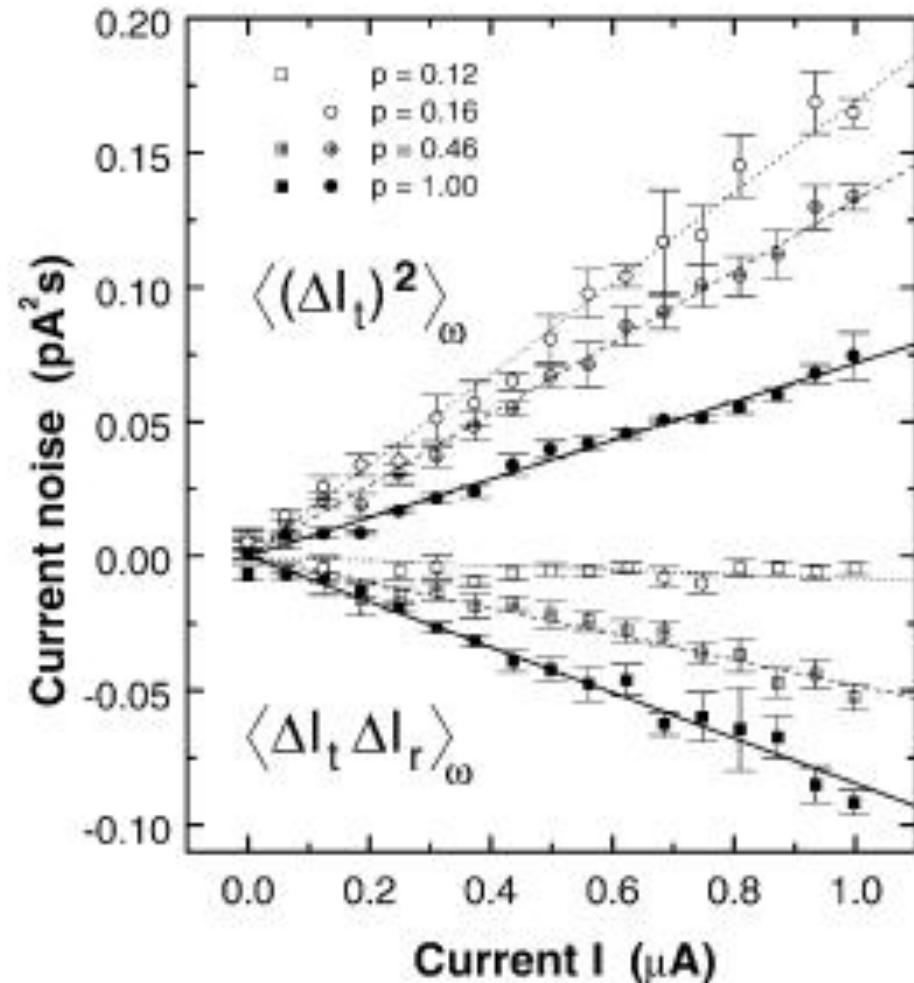
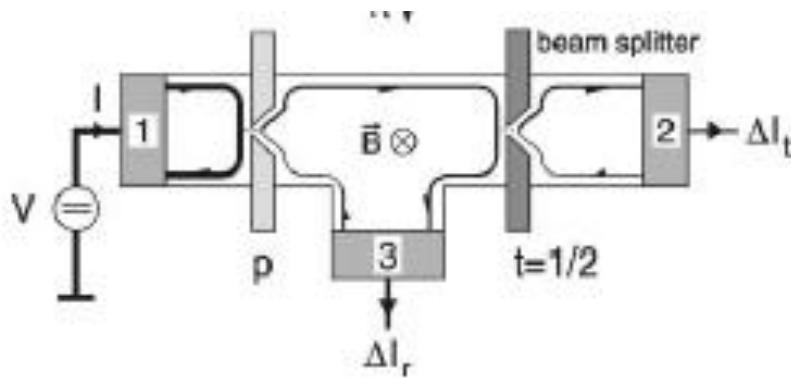
$$S_{23} = -2\frac{e^2}{h}|eV|\kappa^2 TR$$

$$S_{22} = 2\frac{e^2}{h}|eV|\kappa T(1 - \kappa T)$$

$$S_{33} = 2\frac{e^2}{h}|eV|\kappa R(1 - \kappa R)$$

Experiment of Oberholzer et al.

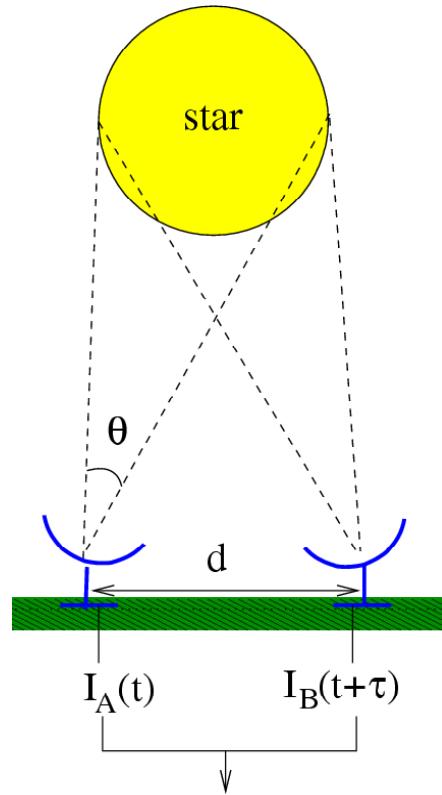
Oberholzer et al, Physica E6, 314 (2000)



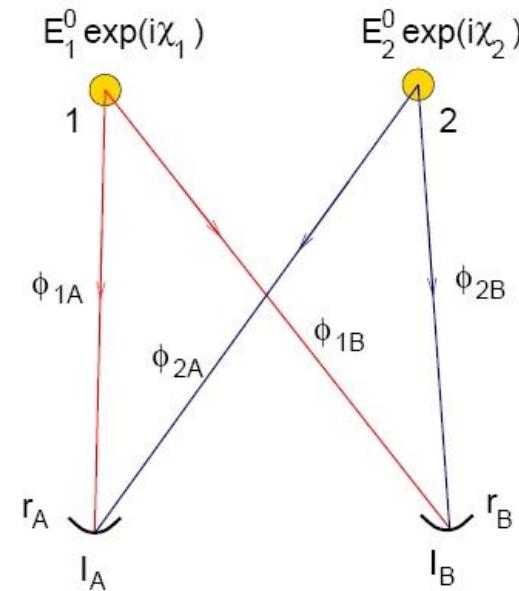
See also: Henny, et al., Science 284, 296 (1999); Oliver et al. Science 284, 299 (1999)

Hanbury Brown Twiss

Hanbury Brown and Twiss,
Nature 177, 27 (1956)



Mandel,
Phys. Rev. A 28, 929 (1983)

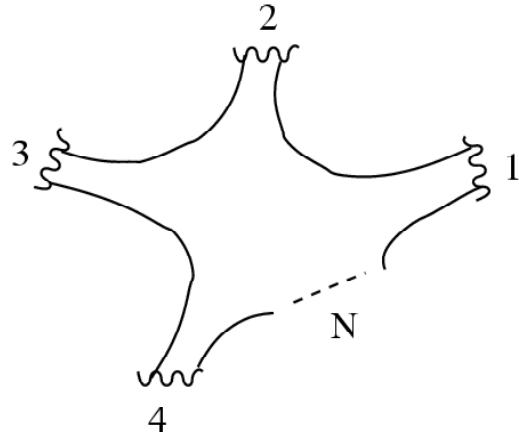


Interference of independent photons:

- R. Kaltenbach et al. (Zeilinger), PRL 96, 240502 (2006) (synchronization)
- M. Halder, et al. (Gisin) Nature Physics 3, 692 (2007) (down conversion)

Exchange interference

Mesoscopic conductor with N contacts



$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & \dots & s_{1N} \\ s_{21} & s_{22} & s_{23} & \dots & \\ s_{31} & s_{32} & s_{33} & \dots & \\ \vdots & \vdots & & & \\ s_{N1} & & & & s_{NN} \end{pmatrix}$$

At $kT = 0$,

$$G_{\alpha\beta} = -\frac{e^2}{h} \text{Tr} [s_{\alpha\beta}^\dagger s_{\beta\alpha}]$$

$$S_{\alpha\beta} = 2 \int dt \langle \Delta \hat{I}_\alpha(t) \Delta \hat{I}_\beta(0) \rangle$$

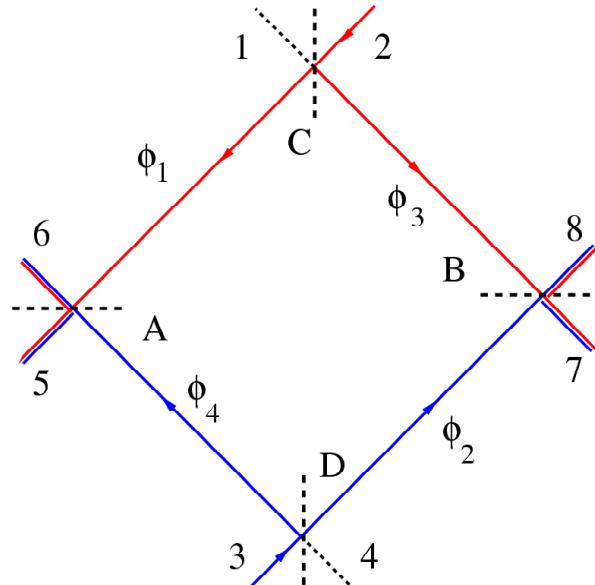
At $kT = 0$, M contacts $f_\gamma = f$, N-M contacts at $f_\delta = f_0$

$$S_{\alpha\beta} = -2 \frac{e^2}{h} \int dE \text{Tr} [B_{\alpha\beta}^\dagger B_{\beta\alpha}], \quad B_{\alpha\beta} = \sum_{\gamma=1}^M s_{\alpha\gamma} s_{\beta\gamma}^\dagger (f_\gamma - f_0)$$

$M > 1$, relative phase of scattering matrix elements becomes important exchange interference effects: Buttiker, PRL 68, 843 (1992)

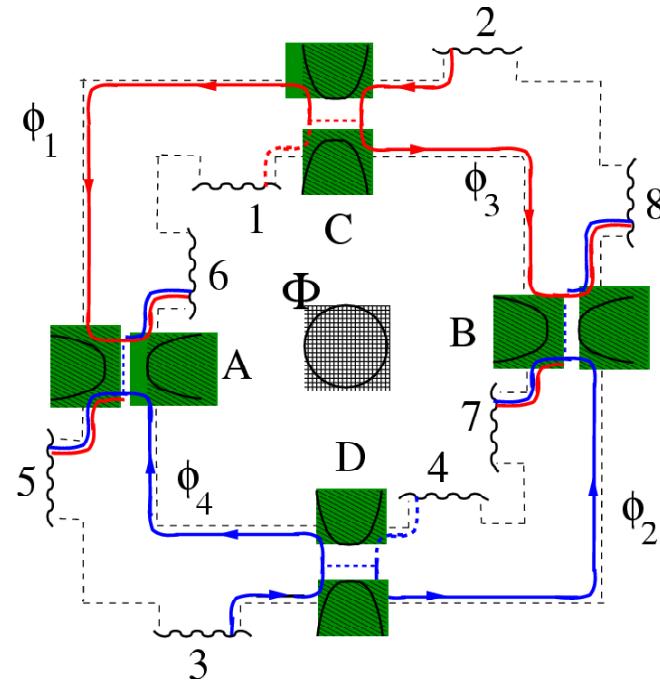
Two-particle interferometer

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



Yurke and Stoler, PRA 46, 2229 (1992)

$$s_{52} = T_A^{1/2} e^{i(\phi_1 + \chi_1)} T_C^{1/2} \Rightarrow$$

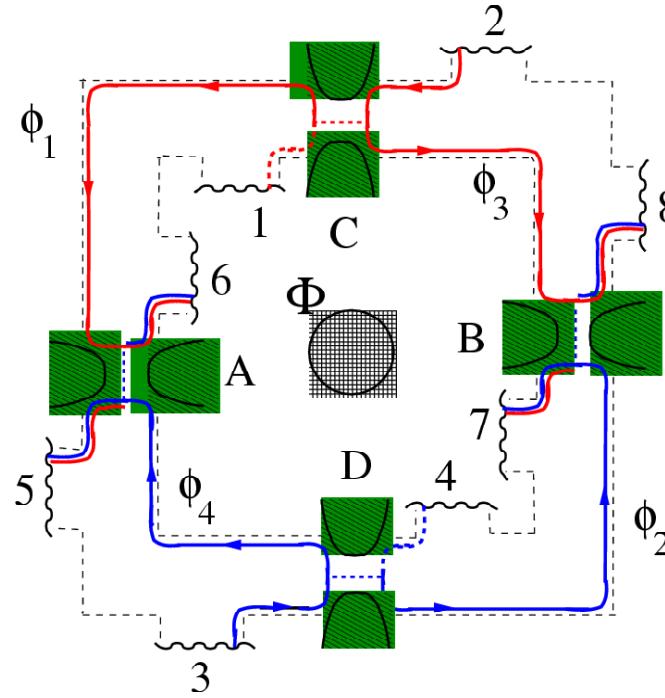
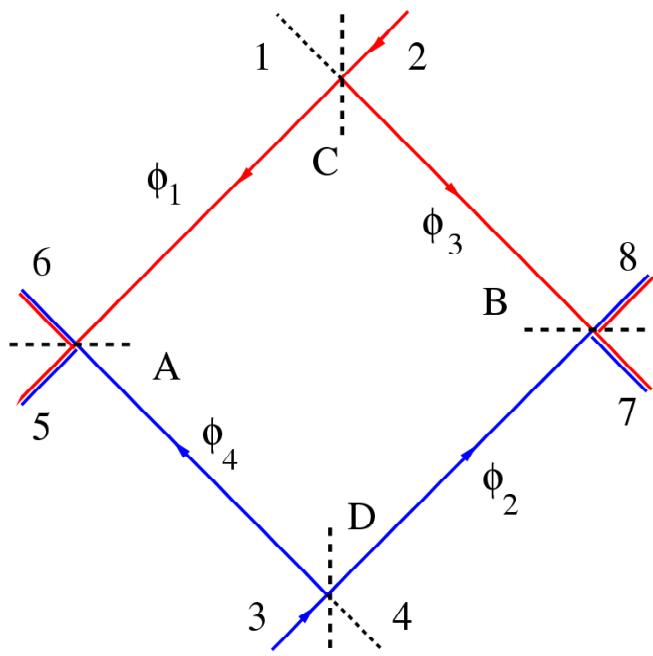


$$G_{52} = -\frac{e^2}{h} T_A T_C$$

All elements of the conductance matrix are independent of AB-flux

Two-particle Aharonov-Bohm effect

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



$$S_{\alpha\beta} = 2 \int dt \langle \Delta I_\alpha(0) \Delta I_\beta(t) \rangle$$

$$S_{58} = -2 \frac{e^2}{h} \int dE |s_{52}^* s_{82} + s_{53}^* s_{83}|^2 (f - f_0)^2$$

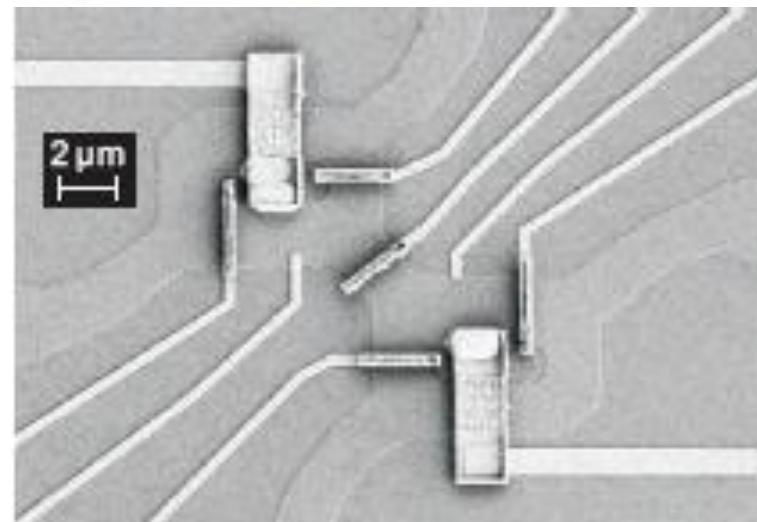
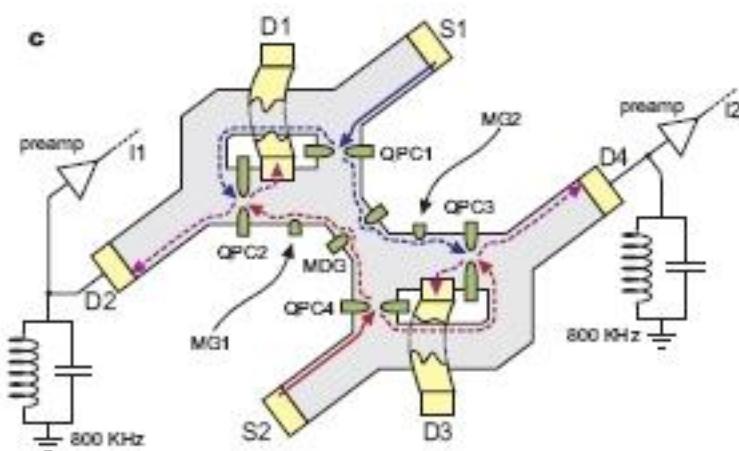
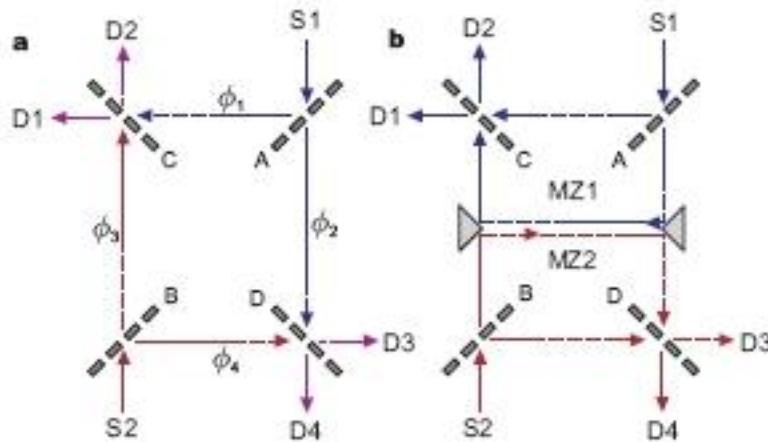
For $T_A = T_B = T_C = T_D = 1/2$;

$$S_{58} = -\frac{e^2}{4h} |eV| \left[1 + \cos \left(\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi \frac{\Phi}{\Phi_0} \right) \right]$$

[N > 2 ; Sim and Sukhorukov, PRL 96, 020407 (2006)]

Two-particle Aharonov-Bohm effect

I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu and V. Umansky,
Nature 448, 333 (2007)



$$\nu = 0.8 \text{ MHz}$$

$$\Delta\nu = 60 \text{ kHz}$$

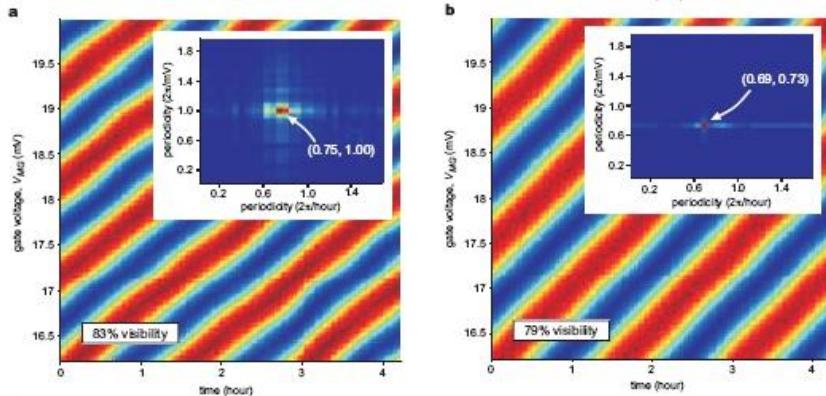
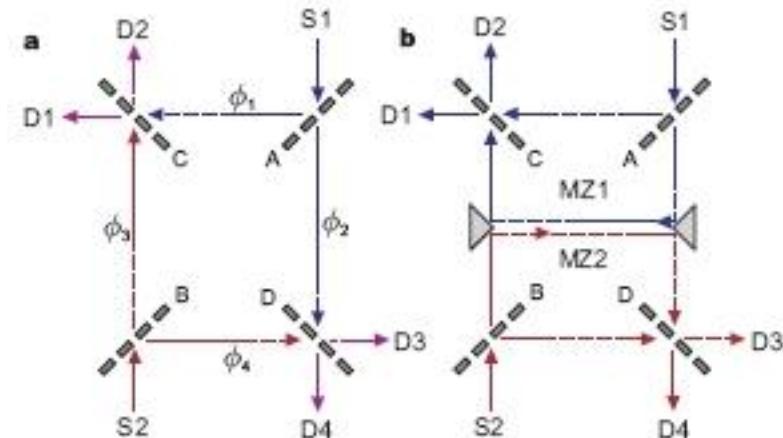
$$\Delta V = 8 \mu\text{V}$$

$$kT = 10 \text{ mK}$$

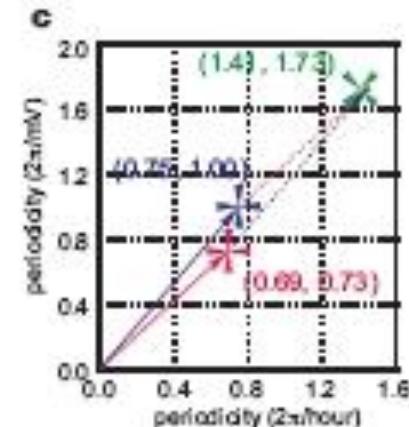
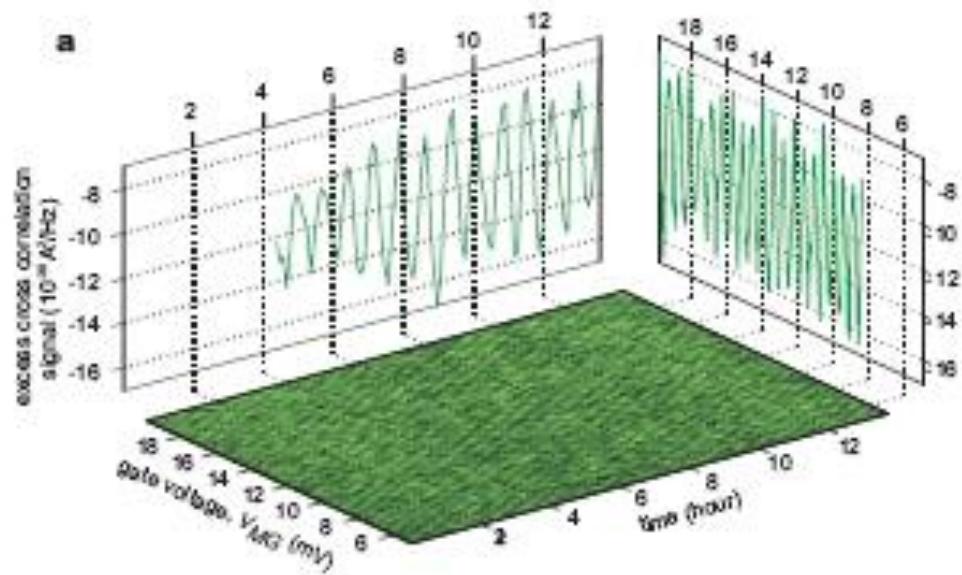
$$\mu = 5 \times 10^6 \text{ cm}^2/\text{V sec}$$

Two-particle Aharonov-Bohm effect

I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu and V. Umansky,
Nature 448, 333 (2007).



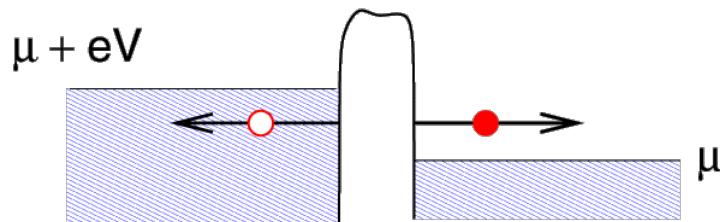
Shot noise correlation



Two-particle orbital entanglement

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)

Electron-hole picture



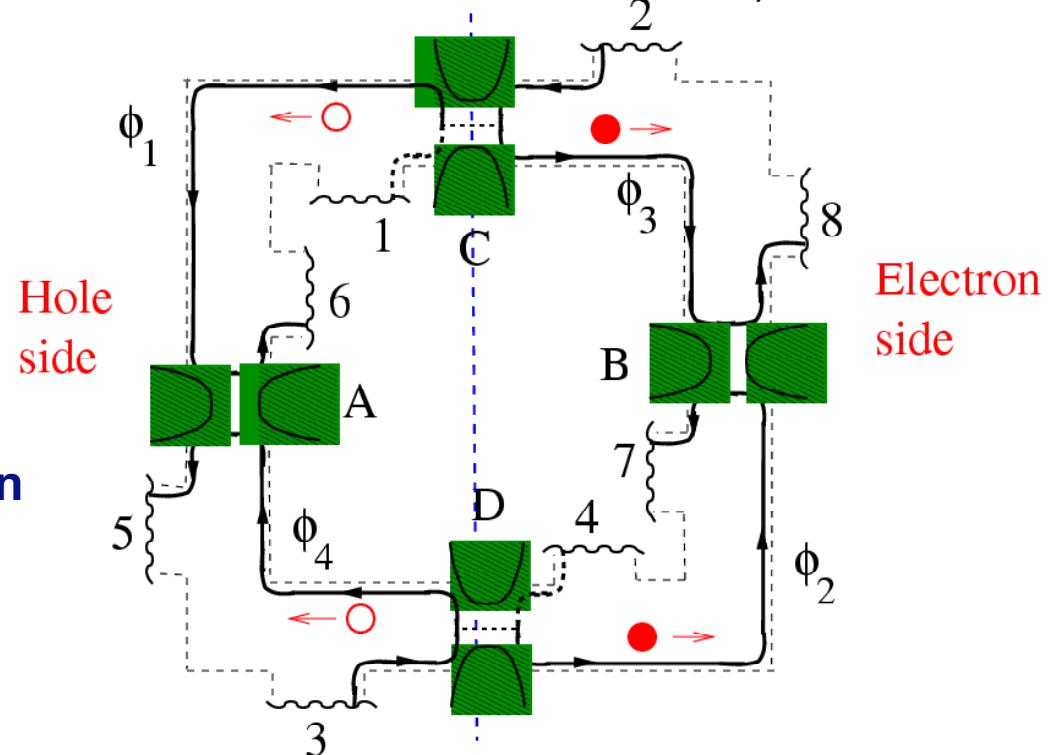
Beenakker, Emery, Kindermann, van Velsen, PRL 91, 147901 (2003)

$$\begin{bmatrix} c_2^\dagger = t_C c_{2A}^\dagger + r_C c_{2B}^\dagger \\ c_3^\dagger = r_D c_{3A}^\dagger + t_D c_{3B}^\dagger \end{bmatrix}$$

$R_C = T_D = R \ll 1$; $\tau_C = \hbar/eV$; $\tau \sim \hbar/eVR$, tunneling limit

$$|\Psi_{in}\rangle = \prod_{0 < E < eV} c_2^\dagger(E) c_3^\dagger(E) |0\rangle \quad \text{incident state}$$

$$|\Psi\rangle = |\bar{0}\rangle + \sqrt{R} \int_0^{eV} dE [c_{3B}^\dagger c_{3A} + c_{2B}^\dagger c_{2A}] |\bar{0}\rangle + O(R) \quad \text{orbitally entangled e-h-state}$$



Current noise: frequency dependence

1. Frequency dependent noise

Frequency dependent noise spectrum

$$I_\alpha(t) = \frac{e}{\hbar} \int dE' dE \sum_{\beta, \gamma} a_\beta^\dagger(E') A_{\beta\gamma}(\alpha, E', E) a_\gamma(E) e^{i(E' - E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E') s_{\alpha\gamma}(E)$$

Spectral density S (noise power)

$$(1/2)\langle I_\alpha(\omega)I_\beta(\omega') + I_\beta(\omega')I_\alpha(\omega) \rangle = 2\pi S_{\alpha\beta}(\omega) \delta(\omega + \omega')$$

$$S_{\alpha\beta}(\omega) = 2\frac{e^2}{\hbar} \sum_{\gamma, \delta} \int dE Tr[A_{\gamma\delta}(\alpha)(E, E + \hbar\omega) A_{\delta\gamma}(\beta)(E + \hbar\omega, E)] F_{\gamma\delta}(E, E + \hbar\omega)$$

$$A_{\beta\gamma}(\alpha, E, E + \hbar\omega) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E) s_{\alpha\gamma}(E + \hbar\omega)$$

$$F_{\alpha\delta}(E, E + \hbar\omega) = f_\gamma(E)(1 - f_\delta(E + \hbar\omega)) + f_\delta(E + \hbar\omega)(1 - f_\gamma(E))$$

Quantum noise: detector is important: non-symmetrized spectra

Non-interacting theory: does not provide current conservation

Current noise: frequency dependence

2. Quantum noise

Absorption and Emission Noise

G.B. Lesovik and R. Loosen, JETP Lett. 65, 295 (1997)

R. Aguado and L. P. Kouwenhoven, PRL 84, 1986 (2000)

Non-symetrized noise

$$\begin{aligned} S_{\alpha\alpha}(\omega) &= \int d\tau e^{-i\omega\tau} \langle I_\alpha(0)I_\alpha(\tau) \rangle \\ &= 2\frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[A_{\gamma\delta}(\alpha)(E, E+\hbar\omega) A_{\delta\gamma}(\alpha)(E+\hbar\omega, E)] f_\gamma(E) (1-f_\delta(E+\hbar\omega)) \end{aligned}$$

$\omega > 0$ emission noise

Example: QPC at voltage V, $kT = 0$, no energy dependence,

Emission noise

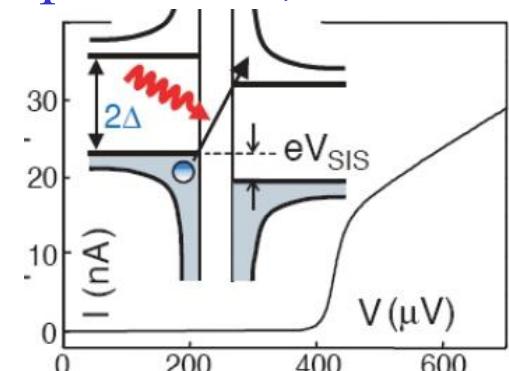
$$S(\omega) = 0, \quad eV \leq \hbar\omega$$

$$S(\omega) = \frac{2e^2}{h} \sum T_n (1-T_n) (eV - \hbar\omega), \quad 0 \leq \hbar\omega \leq eV$$

Absorption noise

$$S(\omega) = \frac{2e^2}{h} [\sum T_n (e|V| - \hbar\omega) - \sum T_n^2 (e|V| + \hbar\omega)], \quad -e|V| \leq \hbar\omega \leq 0$$

$$S(\omega) = \frac{4e^2}{h} \sum T_n \hbar\omega, \quad \hbar\omega \leq -e|V|$$



Current noise: frequency dependence

3. Charge relaxation

Noise spectrum: single lead conductor

$$S_{\alpha\beta}(\omega) = 2\frac{e^2}{h} \sum_{\gamma\delta} \int dE Tr[A_{\gamma\delta}(\alpha)(E, E+\hbar\omega) A_{\delta\gamma}(\beta)(E+\hbar\omega, E)] F_{\gamma\delta}(E, E+\hbar\omega)$$

$$A_{\beta\gamma}(\alpha, E, E+\hbar\omega) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E) s_{\alpha\gamma}(E+\hbar\omega)$$

$$F_{\alpha\delta}(E, E+\hbar\omega) = f_\gamma(E)(1-f_\delta(E+\hbar\omega)) + f_\delta(E+\hbar\omega)(1-f_\gamma(E))$$

New energy scale: modulation quantum $\hbar\omega$. Compare to $E_T, k_B T, eV$

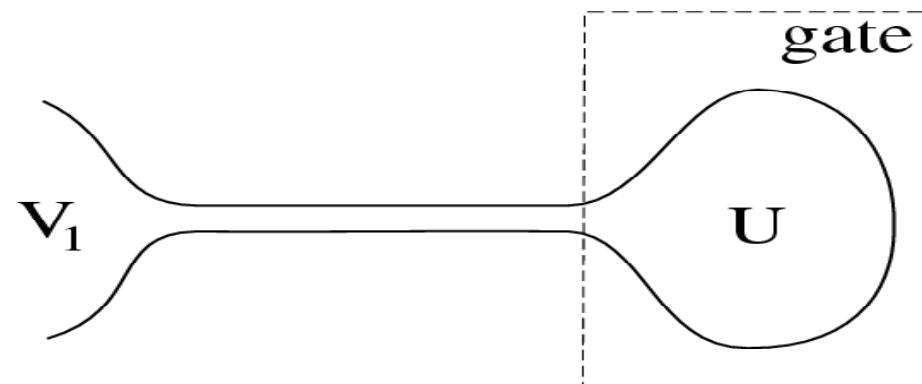
Most of the literature assumed energy independent transmission and reflection probabilities: $\hbar\omega, k_B T, eV \ll E_T$

Here

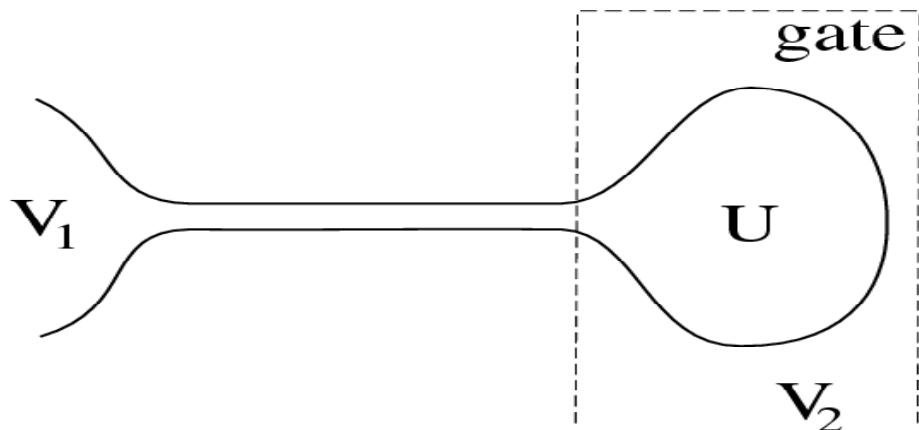
$$k_B T, eV \ll E_T \approx \hbar\omega$$

Simplest case:

A « one » lead conductor



Noise spectrum: single lead conductor



S-matrix: reflection

Only one reservoir with Fermi distribution f

$$S(\omega) = 2 \frac{e^2}{h} \int dE \text{Tr}[2 - s^\dagger(E)s(E + \hbar\omega) - s^\dagger(E + \hbar\omega)s(E)]F(E, E + \hbar\omega)$$

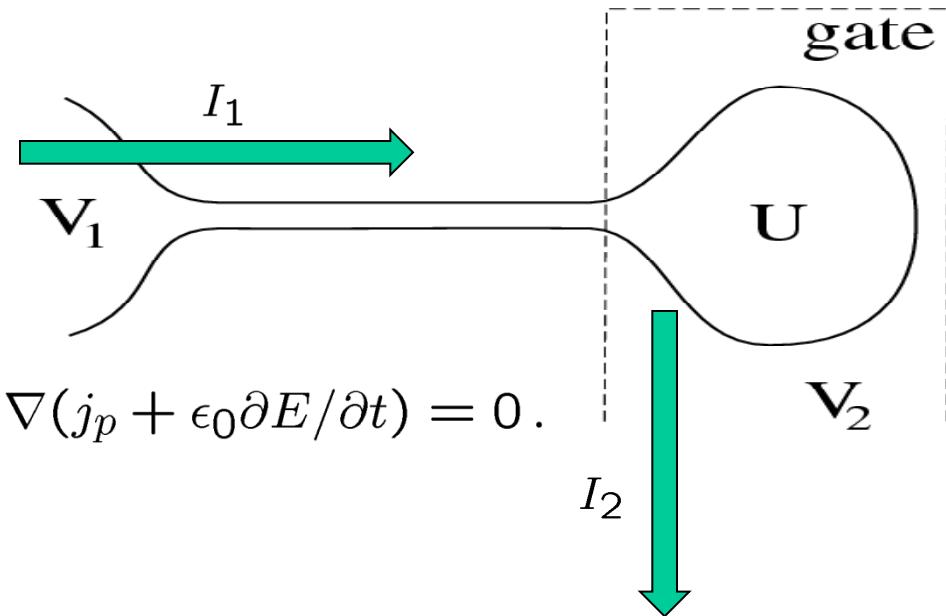
$$F(E, E + \hbar\omega) = 2 \frac{f(E) - f(E + \hbar\omega)}{\hbar\omega} \epsilon(\hbar\omega, kT)$$

Linear response

$$G(\omega) = \frac{e^2}{h} \int dE \text{Tr}[1 - s^\dagger(E)s(E + \hbar\omega)] \frac{f(E) - f(E + \hbar\omega)}{\hbar\omega}$$

Self-consistent response

Buttiker, Thomas, Prêtre, Phys. Lett. A 180, 364 (1993)



$$\nabla(j_p + \epsilon_0 \partial E / \partial t) = 0.$$

$$G^{ext}(\omega) = \frac{e^2}{h} \int dE \text{Tr}[1 - s^\dagger(E)s(E + \hbar\omega)] \frac{f(E) - f(E + \hbar\omega)}{\hbar\omega}$$

External + internal response = charging permitted by Coulomb

$$I_1(\omega) = G^{ext}(\omega) dV_1 + i\omega \Pi(\omega) dU = -i\omega C (dU - dV_2)$$

$$I_2(\omega) = -i\omega C (dV_2 - dU)$$

Invariance under arbitrary potential shift $\Rightarrow i\omega \Pi = -G^{ext}$

$$G^{-1}(\omega) = (-i\omega C)^{-1} + (G^{ext}(\omega))^{-1}$$

Capacitance and charge relaxation resistance

Buttiker, Thomas, Prêtre, Phys. Lett. A180, 364 (1993)

$$G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + ..$$

electrochemical capacitance

$$C_\mu^{-1} = C^{-1} + (e^2 Tr[N])^{-1}$$

charge relaxation resistance

$$R_q = \frac{h}{2e^2} \frac{Tr[N^\dagger N]}{(Tr[N])^2}$$

Eigen channels of s; $\exp(i\phi_n)$; $n = 1, 2, , \Rightarrow$

$$\nu(E) = Tr[N] = \frac{1}{2\pi i} Tr[s^\dagger \frac{ds}{dE}] = \frac{1}{2\pi} \sum_n \frac{d\phi_n}{dE} \quad \text{N Wigner-Smith matrix}$$

$$Tr[N^\dagger N] = (\frac{1}{2\pi})^2 Tr[\frac{ds^\dagger}{dE} \frac{ds}{dE}] = (\frac{1}{2\pi})^2 \sum_n (\frac{d\phi_n}{dE})^2$$

$$R_q = \frac{h}{2e^2} \frac{\sum_n (d\phi_n/dE)^2}{(\sum_n d\phi_n/dE)^2} \quad \text{For n = 1: } R_q = \frac{h}{2e^2} \quad \text{Universal !}$$

Equilibrium noise of a mesoscopic capacitor

Linear response

$$G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + ..$$

Current fluctuations: fluctuation-dissipation theorem

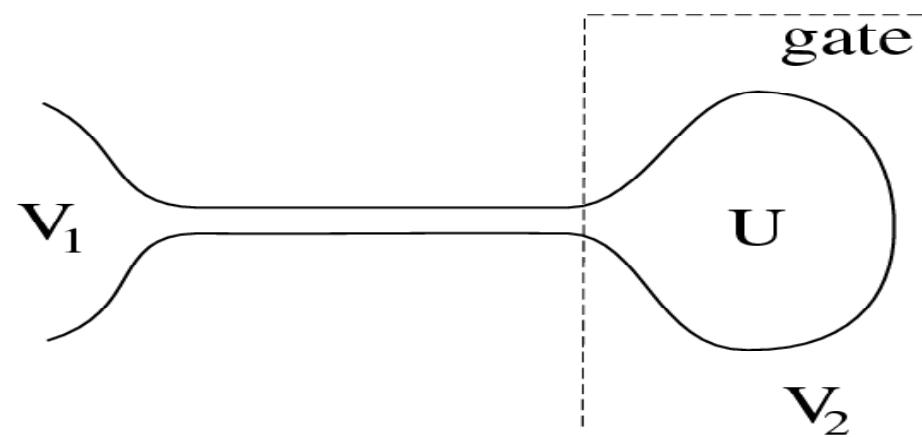
$$\langle (\Delta I)^2 \rangle = 2 \omega^2 C_\mu^2 R_q kT + ..$$

Charge fluctuations

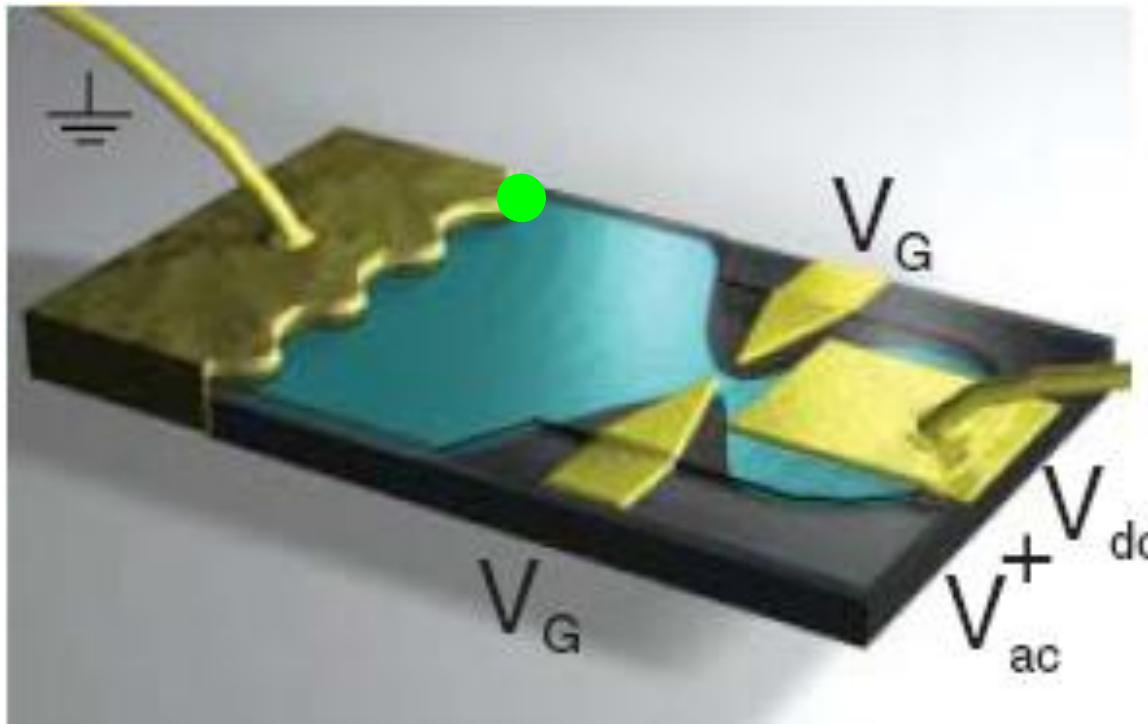
$$\langle (\Delta Q)^2 \rangle = 2 C_\mu^2 R_q kT + ..$$

Internal potential fluctuations

$$\langle (\Delta U)^2 \rangle = 2 \left(\frac{C_\mu}{C} \right)^2 R_q kT + ..$$



Mesoscopic capacitor: experiments



Quantized charge relaxation resistance:

J. Gabelli, G. Fèvre, J.-M. Berroir, B. Plaçais, A. Cavanna, B. Etienne, Y. Jin, D.C. Glattli, SCIENCE 313, 499 (2006)

Single charge emitter:

G. Fèvre, A. Mahé, J.-M. Berroir, T. Kottos, B. Plaçais, D. C. Glattli, A. Cavanna, B. Etienne, Y. Jin, SCIENCE 316, 1169 (2007)

Noise of a single charge emitter :

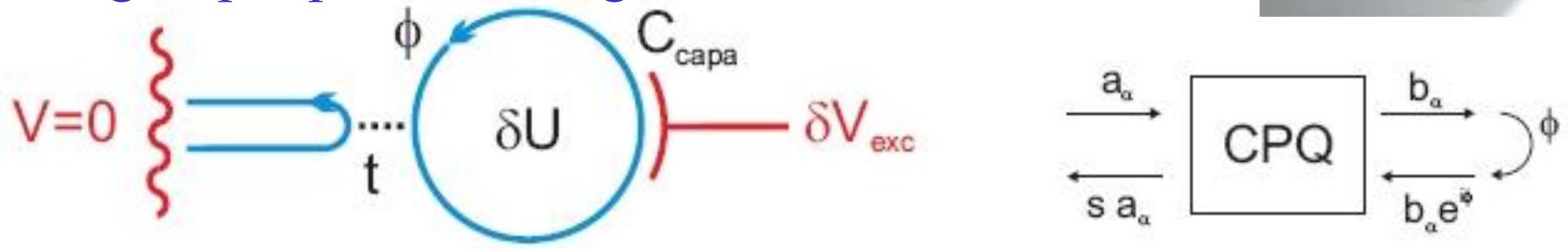
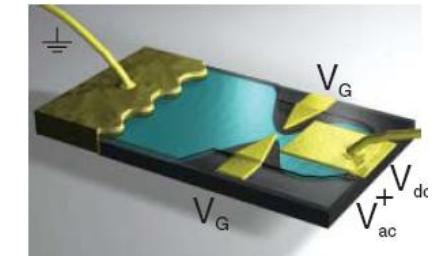
A. Mahé, F. D. Parmentier, E. Bocquillon, J. -M. Berroir, D. C. Glattli, T. Kontos, B. Plaçais, G. Fèvre, A. Cavanna, Y. Jin, Phys. Rev. B 82, 201309 (2010).

Capacitor: edge state model

Gabelli et al, Science 313, 499 (2006)

Pretre, et al., Phys. Rev. B 54, 8130 (1996).

Single spin polarized edge state



$$\begin{pmatrix} sa \\ b \end{pmatrix} = \begin{pmatrix} r & -t \\ t & r \end{pmatrix} \begin{pmatrix} a \\ \exp(i\phi)b \end{pmatrix} \Rightarrow s(\epsilon) = -e^{i\phi} \frac{1 - r e^{-i\phi}}{1 - r e^{i\phi}}$$

density of states

$$\nu = \frac{1}{2\pi i} s^\dagger \frac{ds}{d\epsilon} = \frac{1}{2\pi i} s^\dagger \frac{ds}{d\phi} \frac{d\phi}{d\epsilon} = \frac{1}{2\pi} \frac{d\phi}{d\epsilon} \frac{1 - r^2}{1 - 2r \cos(\phi) + r^2}$$

assumption 1: uniform level spacing

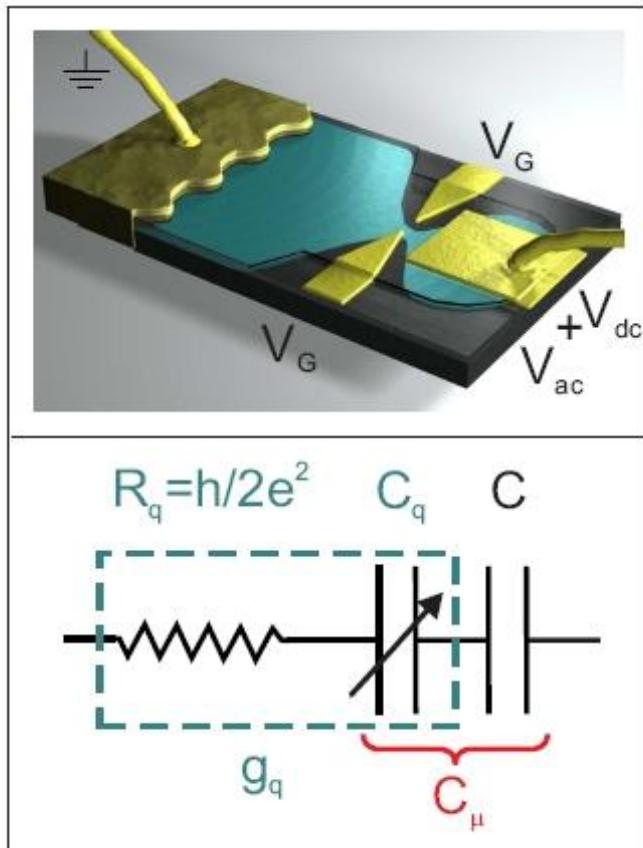
$$\phi = 2\pi\epsilon/\Delta$$

assumption 2: voltage dependence of transmission through QPC

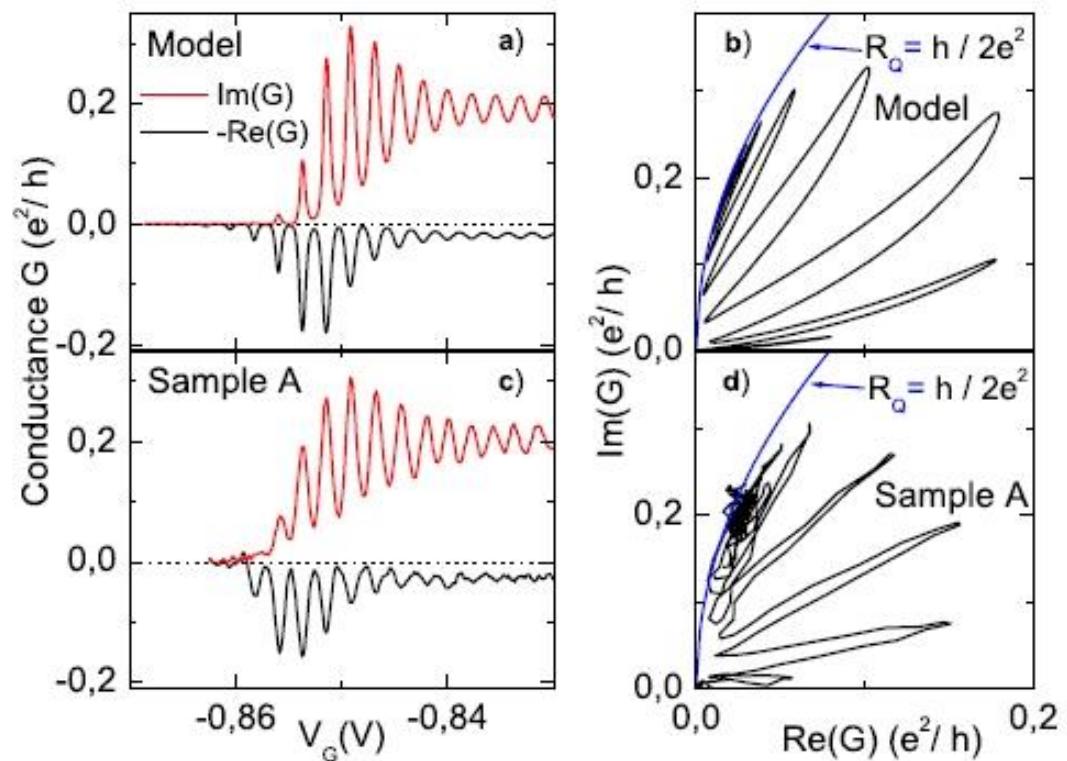
$$t^2 = 1 / (1 + \exp(-(V_{QPC} - V_0)/\Delta V_0))$$

Mesoscopic capacitor: experiment

Gabelli, Feve, Berroir, Placais, Cavanna, Etienne, Jin, Glattli, Science 313, 499 (2006).



$$k_B T < \hbar\omega < E_c < \Delta$$

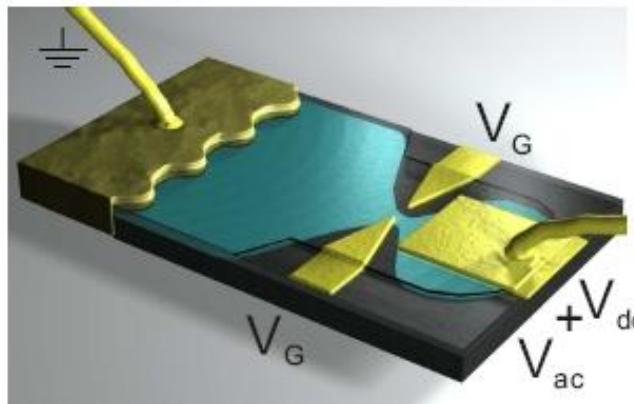
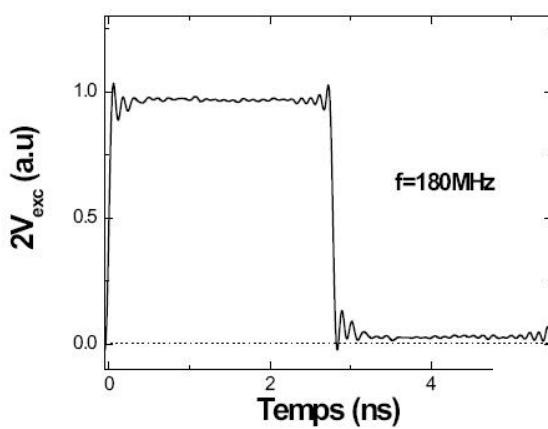


$$\nu = 1.2 \text{ GHz} \quad T = 100 \text{ mK} \quad C = 4 \text{ fF} \quad C_\mu = 1 \text{ fF} \quad B = 1.3 \text{ T}$$

Outlook

Single particle emitter at GHz-frequency

G.Fèvre, A. Mahé, J.-M.Berroir, T. Kottos, B.Plaçais, D. C., Glattli, A. Cavanna, B.Etienne, Y.Jin, SCIENCE 316, 1169 (2007)

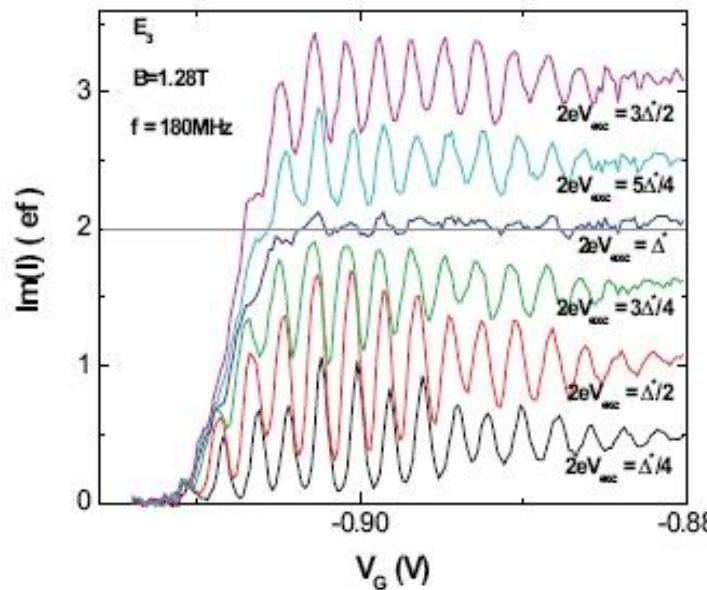
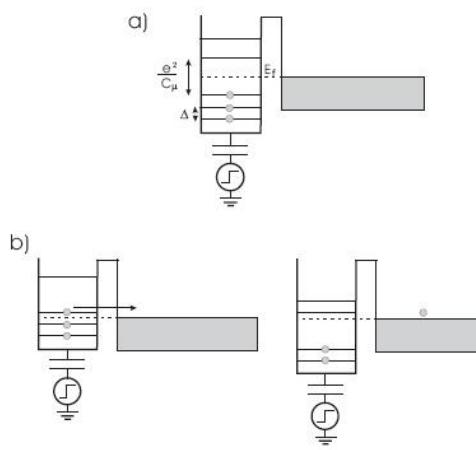


Novel electron source

Accuracy?

Noise ?

Few electron
Experiments?



Summary

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