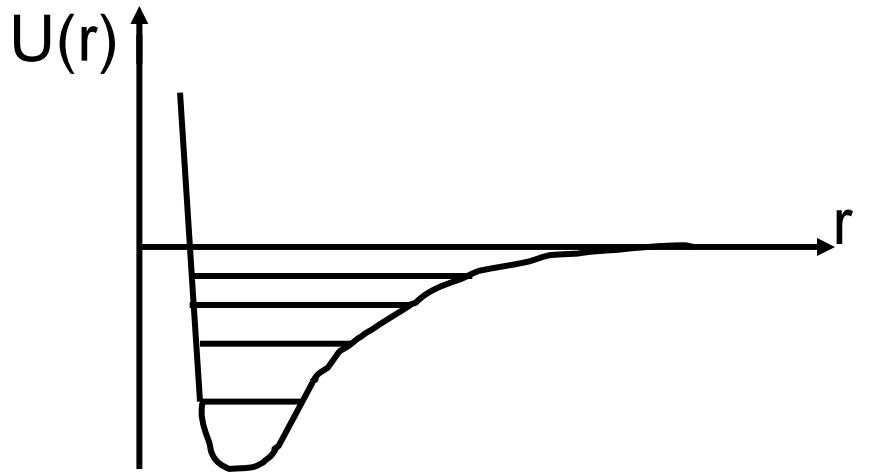


2: Tailored Condensates

Control of interactions between atoms
Control of trapping potential
rotating potential
spatially periodic potential

Atom-atom interactions



The magnitude and sign of a depend sensitively on the detailed shape of long range potential
Importance of position of last bound state

At low temperature,
only s wave collisions, $l=0$

$$\psi(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} - \frac{a}{r} e^{ikr}$$

$$a = -\lim_{k \rightarrow 0} \frac{\tan \delta_0(k)}{k}$$

a : scattering length
 $|a| \sim 1$ to 10 nm

$$V(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2 a}{m} \delta(\vec{r}_1 - \vec{r}_2)$$

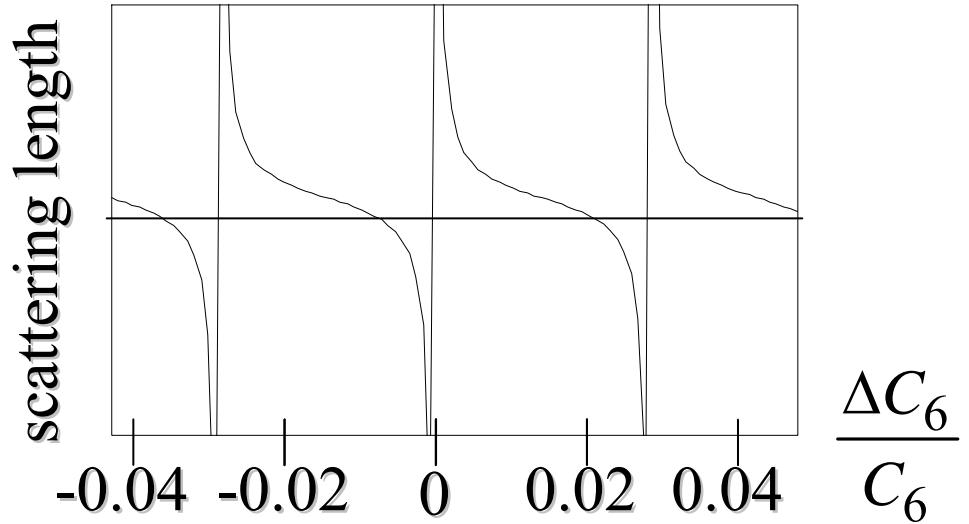
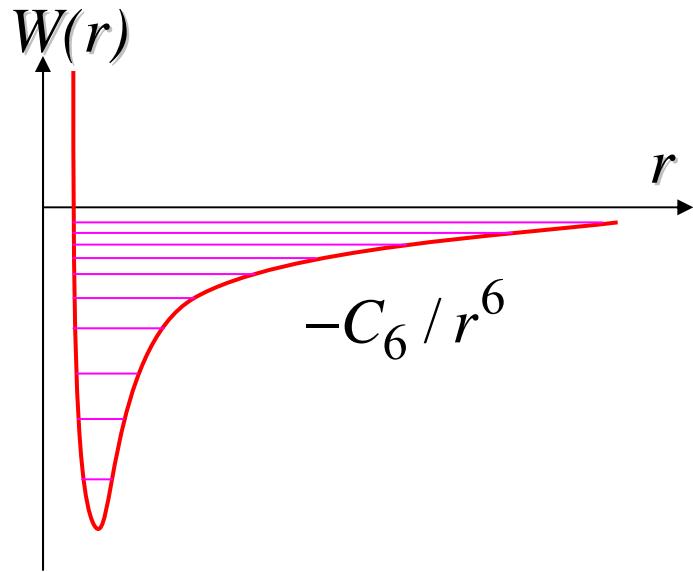
$a > 0$: effective repulsive interaction
 $a < 0$: effective attractive interaction

Control of interactions in a condensate

Condensate dynamics governed by Gross-Pitaevski equation :

$$\left(-\frac{\hbar^2}{2m} \Delta + V(\vec{r}) + N g |\psi(\vec{r})|^2 \right) \psi(\vec{r}) = \mu \psi(\vec{r}) \quad g = \frac{4\pi\hbar^2 a}{m}$$

a : scattering length

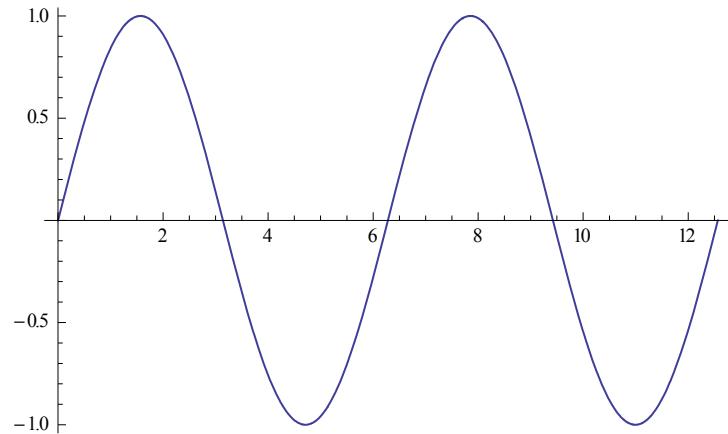


$|a|$ is typically between 1 and 10 nanometers (Bohr radius : 0.05 nm)

atom-atom interactions

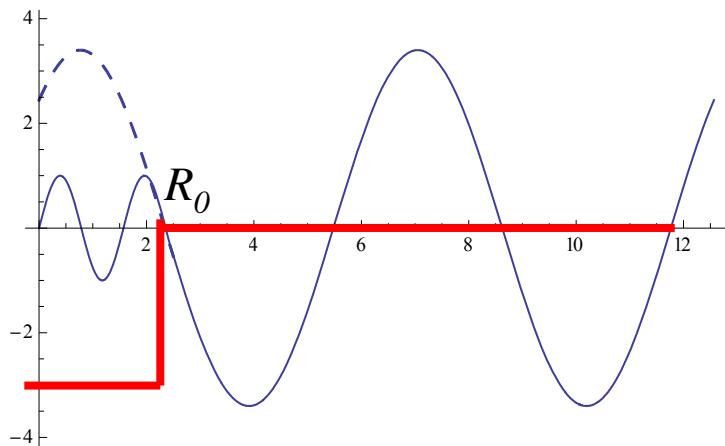
Free particle

$$u(r) = r\psi(r)$$



$$u(r) = \sin(kr)$$

Finite range potential



$$u(r > R_0) \sim \sin(k(r - a))$$

Bethe-Peierls condition

Replace the true potential by the boundary condition

$$\Psi(r \rightarrow 0) = A \left(\frac{1}{r} - \frac{1}{a} + o(1) \right)$$

a is called the *scattering length*

➤ Scattering amplitude: $f(k) = \frac{-a}{1 + ika}$

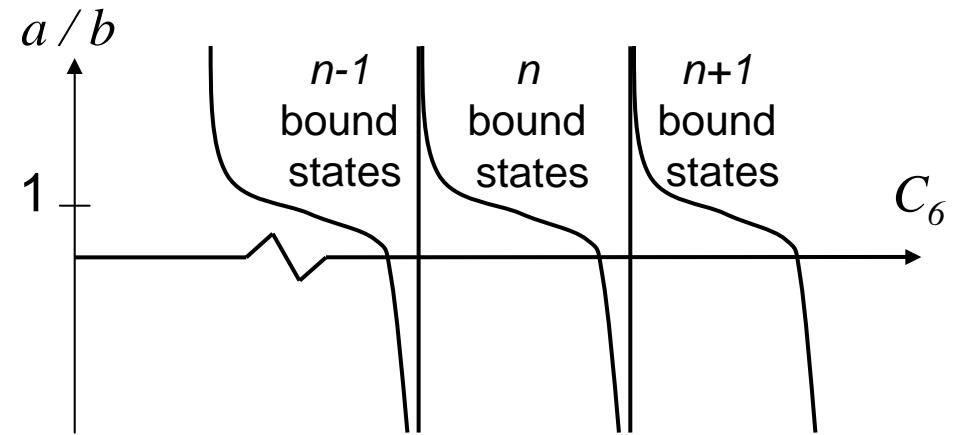
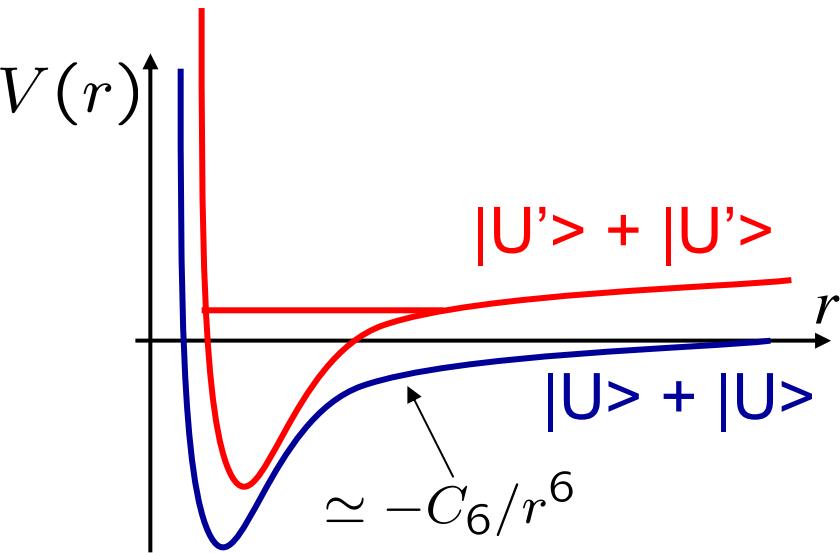
➤ Bound state for $a > 0$: $E = -\frac{\hbar^2}{ma^2}$

Fano-Feshbach resonance

At low T , interactions are characterized by the s wave scattering length a

$$V(\vec{r}_1 - \vec{r}_2) = g \delta(\vec{r}_1 - \vec{r}_2)$$

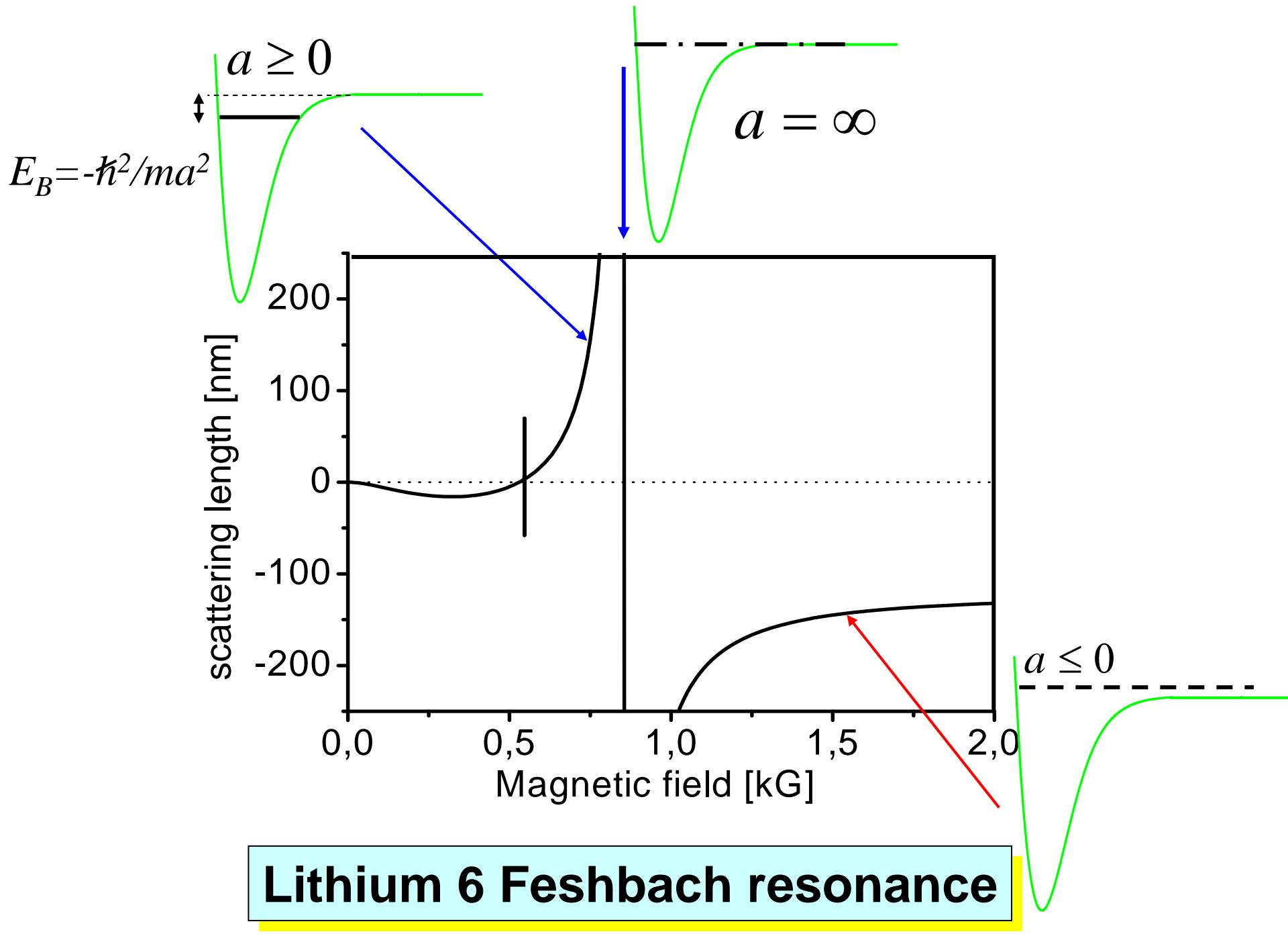
$$g = \frac{4\pi\hbar^2 a}{m}$$



$$b = (mC_6/\hbar^2)^{1/4}$$

$$|a| \sim b \sim 1 - 10 \text{ nm}$$

A Fano-Feshbach resonance brings a new (closed) channel in the collision process, and it “mimicks” the entrance of a new bound state.



interactions in a condensate

- The scattering length, a , is used to describe elastic and inelastic collisions in cold gases

- Scattering crosssection, disting. particles $\sigma = 4 \pi a^2$

- Mean field: gas density $n(r)$

$$U(r) = \frac{4 \pi \hbar^2 n(r) a}{m} \quad \text{NL GPE}$$

- Example: mean field in a BEC

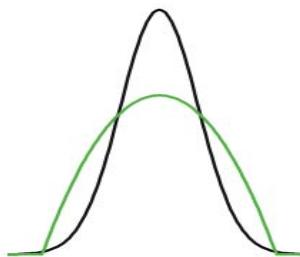
Ideal gas : $a=0$

Repulsion: $a>0$

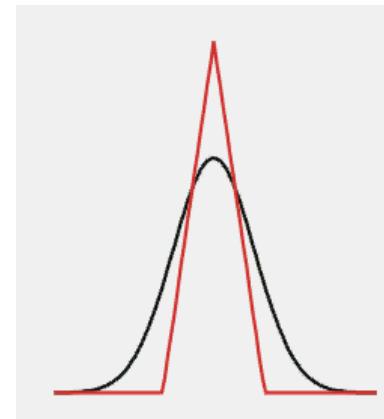
Attraction : $a<0$



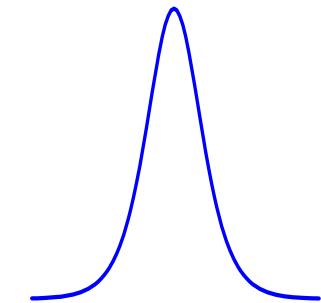
Gaussian



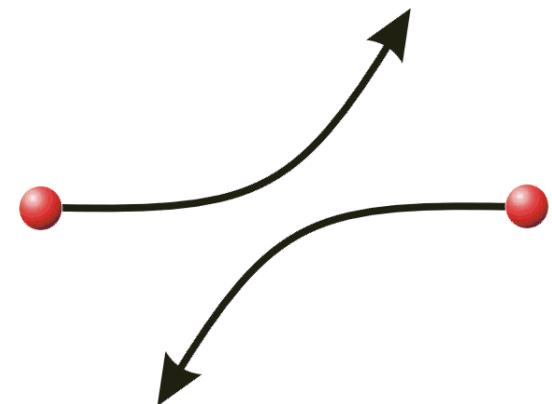
Parabola



Collapse (for $N>N_{\text{crit}}$)



Soliton

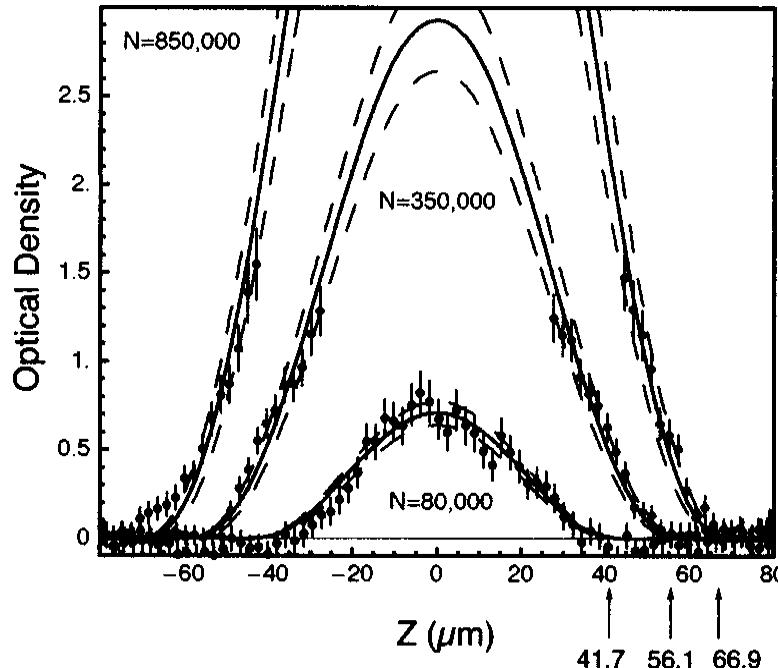


interactions in a condensate $a>0$

$$\left(-\frac{\hbar^2}{2m} \Delta + V(\vec{r}) + Ng |\psi(\vec{r})|^2 \right) \psi(\vec{r}) = \mu \psi(\vec{r})$$

Thomas Fermi approximation

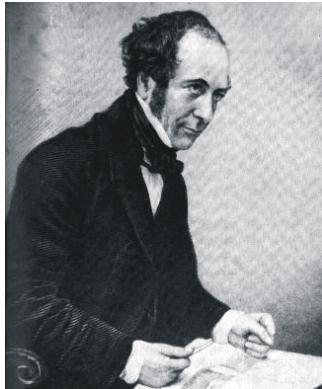
$$|\psi(r)|^2 = \frac{\mu - V(r)}{Ng} = \frac{\mu - 1/2m\omega^2 r^2}{Ng}$$



L. Hau et al.,
Phys Rev A 58, R54, 1998

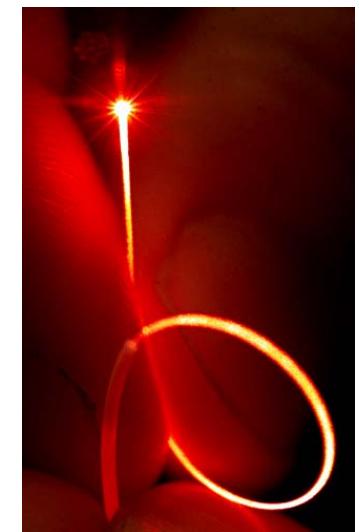
a < 0:Soliton

- Dispersion counterbalanced by non linear interaction
- Discovered 1834 by Scott Russell in water



- Used in optical fibers for telecommunication
Non linear Schroedinger equation

Appears in many fields of Science and Technology !



Formation of Matter Wave Solitons

For negative a , a soliton is a stable self-interacting quantum system which propagates over large distances without attenuation nor dispersion

For positive a , a soliton is an excited state of a BEC which can be excited by engineering a special phase and amplitude upon the BEC ,
Hannover, NIST, Harvard, JILA

Another elegant method : band soliton (Heidelberg, M. Oberthaler)
Shift the sign of the mass
In a lattice, the effective mass can be negative !

Matter wave soliton

Assumption: start with a BEC such that $\hbar\omega_{\perp} \gg N_0 |g| |\phi|^2$ No collapse

Condensate wavefunction $\phi(x, y, z) = \psi(z)\chi(x)\chi(y)$

with harmonic oscillator ground state wave function along x and y

Gross-Pitaevskii energy functional

$$E_{GP}(\psi) = N_0 \int dz \left[\frac{\hbar^2}{2m} \left| \frac{d\psi}{dz} \right|^2 + \frac{1}{2} m \omega_z^2 z^2 |\psi(z)|^2 + \frac{1}{2} N_0 g_{1D} |\psi(z)|^4 \right]$$

with

$$g_{1D} = g \frac{m \omega_{\perp}}{2\pi\hbar} = 2a(\hbar\omega_{\perp})$$

$$g_{1D} \leq 0$$

Soliton (2)

What happens if one turns down slowly ω_z ?

Well known bright soliton solution for $\omega_z = 0$

$$\psi(z) = \frac{1}{(2l)^{1/2}} \frac{1}{\cosh(z/l)}$$

Spatial size of soliton:

$$l = -\frac{2\hbar^2}{N_0 mg_{1D}}$$

Trade-off between minimization of kinetic energy $\frac{\hbar^2}{ml^2}$

and interaction energy per particle: $N_0 \frac{g_{1D}}{l}$

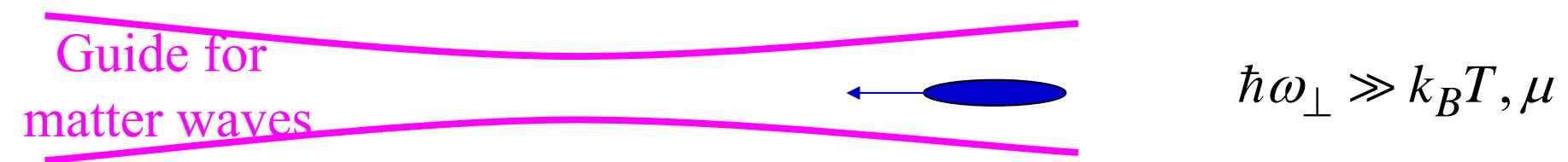
Chemical potential: $\mu = -\frac{1}{8} N_0^2 \frac{mg_{1D}^2}{\hbar^2}$

With, of course, $\mu \leq \hbar\omega_\perp$

manipulation of interactions : soliton production

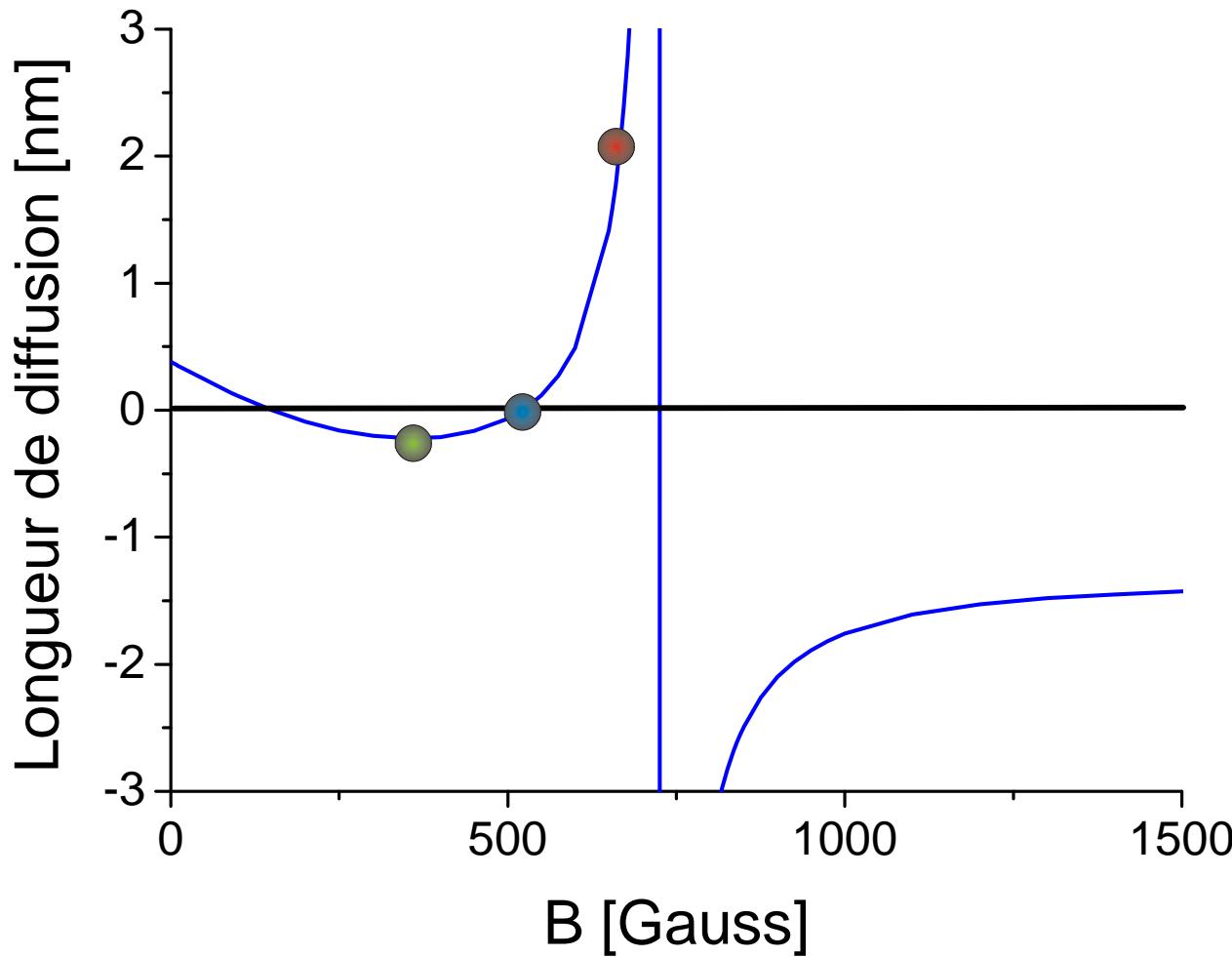
Prepare a lithium 7 condensate with repulsive interaction, $a > 0$

Transfer into a 1D geometry:



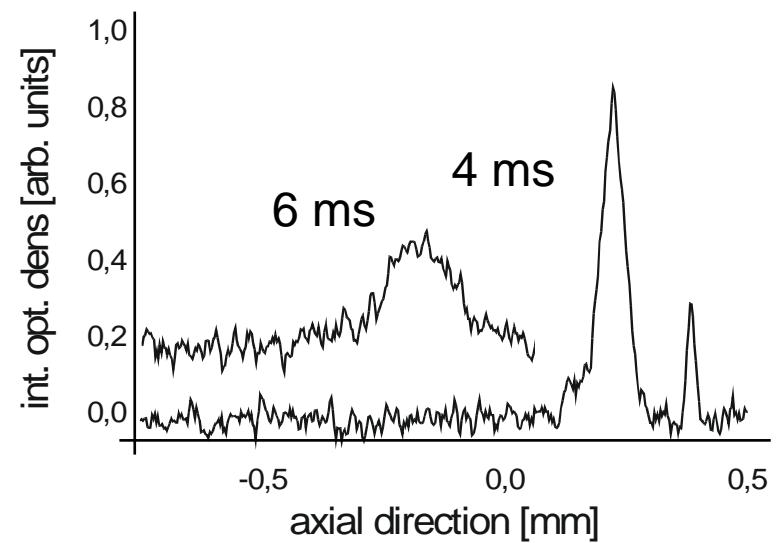
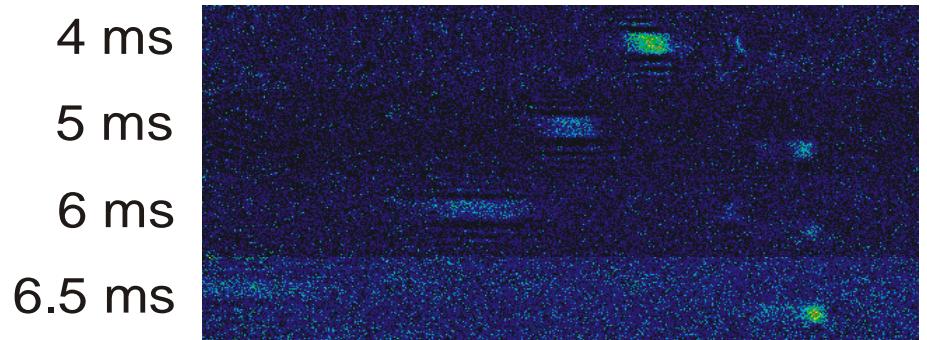
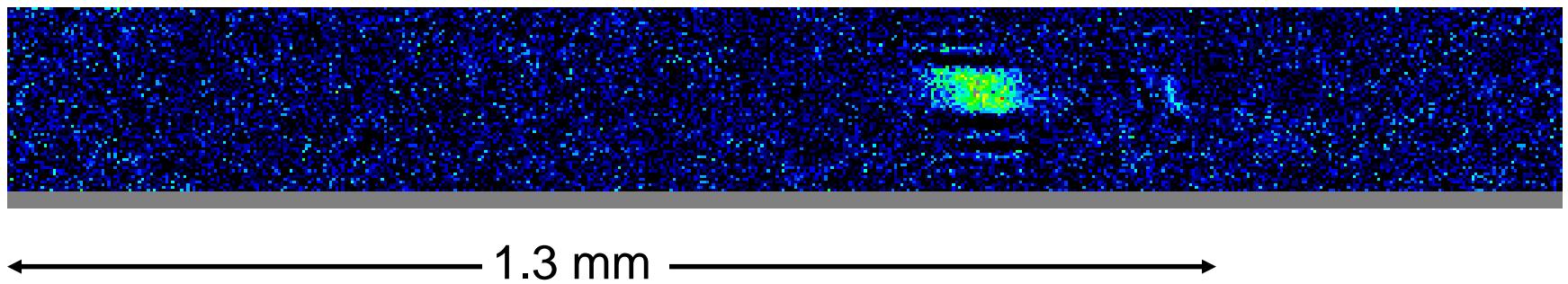
Switch interactions towards $a = 0$ (ideal gas) or $a < 0$ (attraction)

Dynamic control of interactions : soliton production



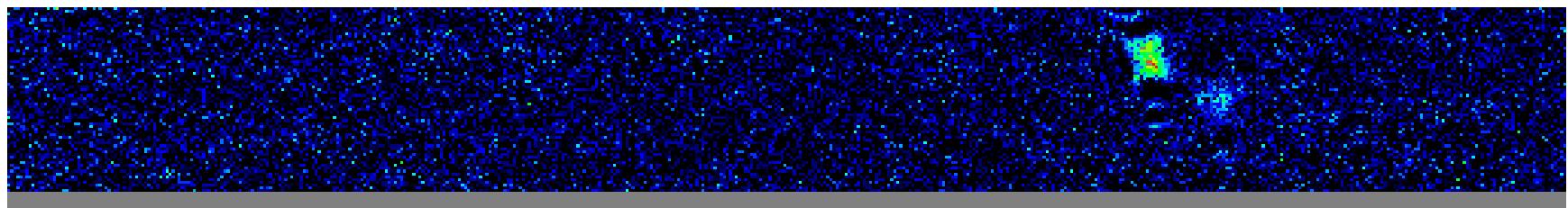
- Evaporation
- ideal gas
- Soliton : $a < 0$

Ideal gas in 1D waveguide: $a \sim 0$

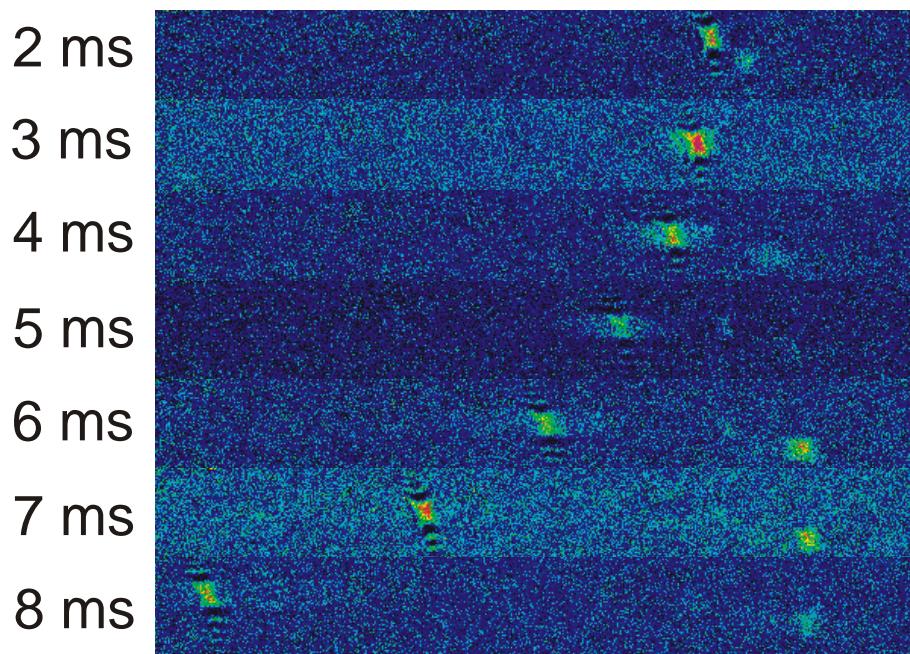


Dispersion of non interacting matter waves

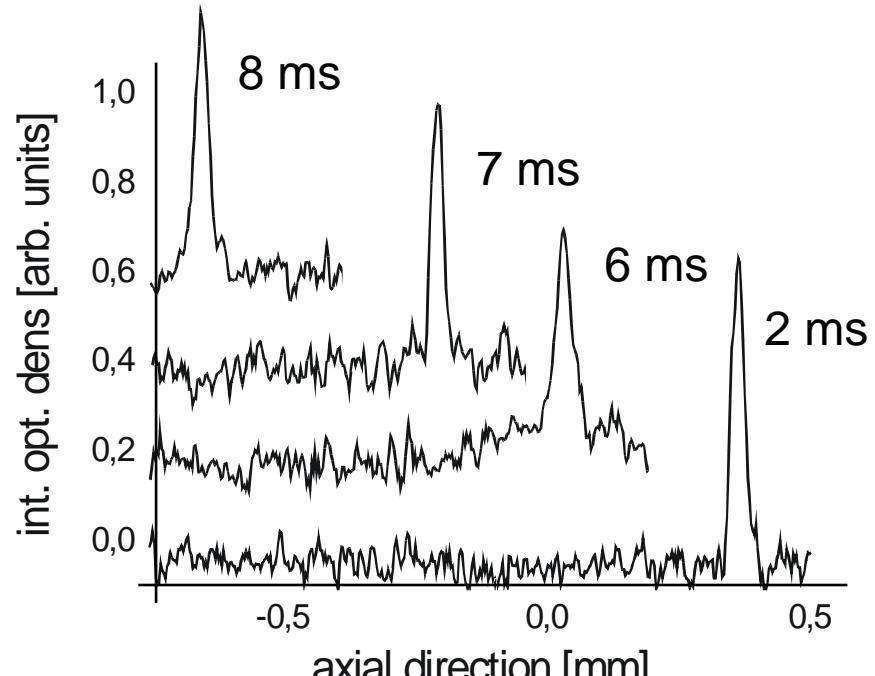
Soliton : $a < 0$



← 1.3 mm →



Propagation without spread of
 $N = 6(2) \times 10^3$ atoms

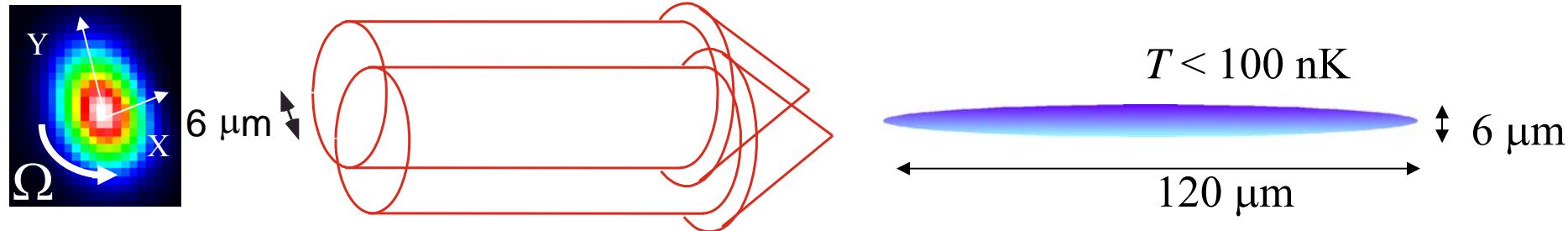


L. Khaykovich et al., Science 296,
1290, 2002

**Second example:
Can one rotate a condensate ?
apply rotating trap anisotropy**

Rotating bucket experiment with a condensate

Anisotropic laser beam: rotating potential in xy plane



magnetic potential : $\frac{1}{2}m\omega_{\perp}^2(x^2 + y^2) + \frac{1}{2}m\omega_z^2z^2$

Laser potential : $\delta U(\vec{r}) = \frac{1}{2}m\omega_{\perp}^2(\varepsilon_x X^2 + \varepsilon_y Y^2)$ $\varepsilon_x = 0.03$, $\varepsilon_y = 0.09$

Can the condensate rotate ?

$$\psi(\vec{r}) = \sqrt{\rho(\vec{r})} e^{iS(\vec{r})} \xrightarrow{\text{In a point where } \rho \neq 0} \vec{v} = \frac{\hbar}{m} \vec{\nabla} S \quad \oint \vec{v} \cdot d\vec{r} = \frac{nh}{m}$$

incompatible with rigid rotation

$\vec{v} = \vec{\Omega} \times \vec{r}$ *of a classical fluid*

From single vortex to vortex lattices

As for liquid helium, there exists a critical rotation velocity, above which a first vortex appears:

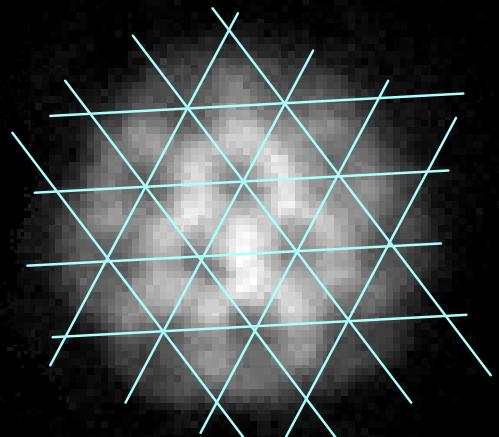
ENS

*Just below
critical
frequency*

*Just above
critical
frequency*

*Notably above
critical
frequency*

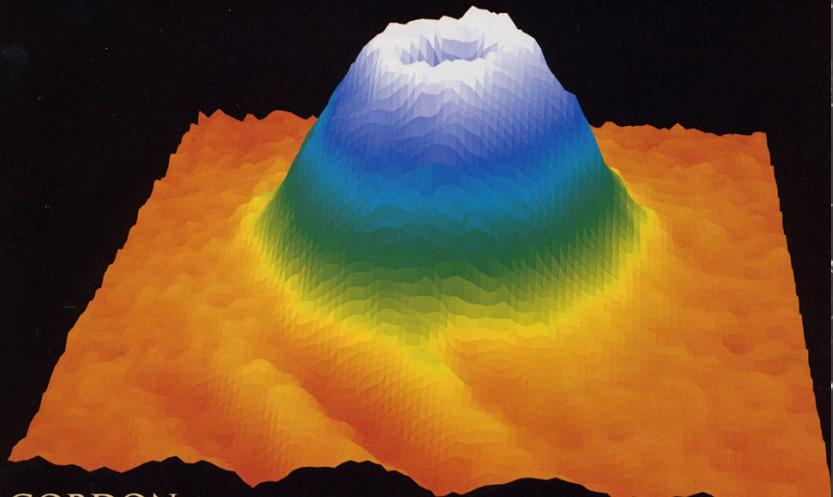
*Larger atom numbers
Abrikosov lattice*



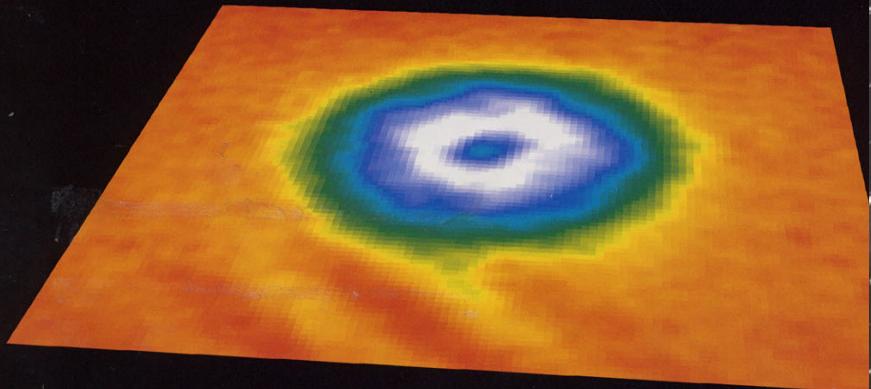
Science

23 February 2001

Vol. 291 No. 5508
Pages 1435–1650 \$9



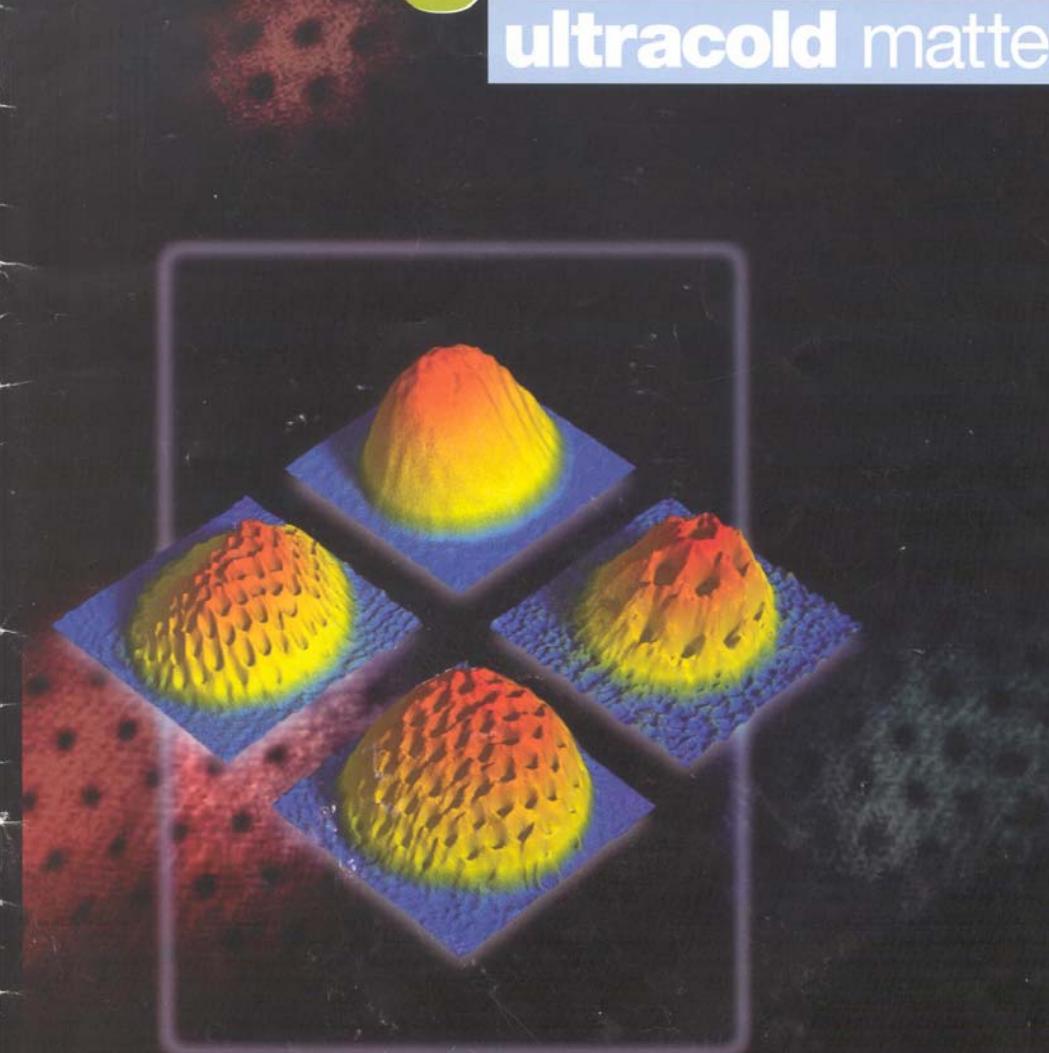
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nature insight

Reprinted from Vol. 416, no. 6877, 14 March

ultracold matte

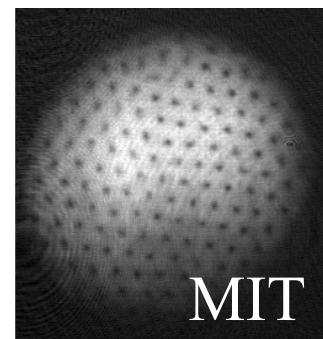
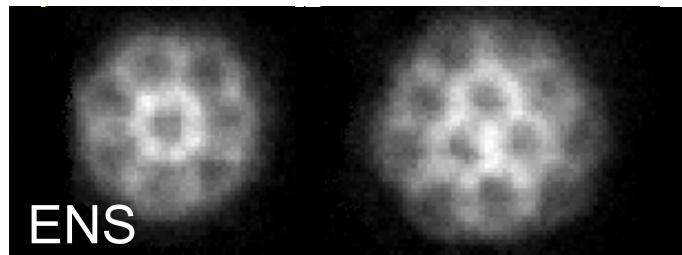
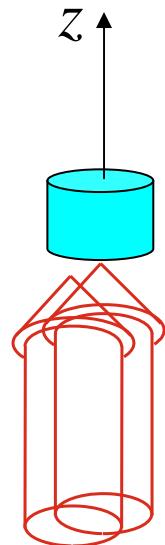


AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

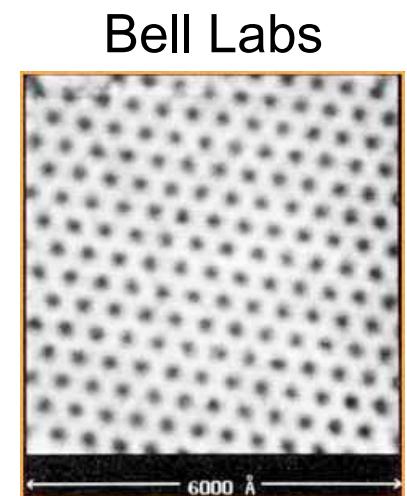
“Orbital magnetism” with neutral atoms

At the origin of orbital magnetism, Lorentz force: $\vec{F} = q \vec{v} \times \vec{B}$

A way to simulate it: stir the neutral superfluid and work in the rotating frame, where the Coriolis force reads: $\vec{F} = 2m \vec{v} \times \vec{\Omega}$



vortex lattices in rotating condensates

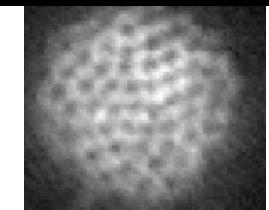


vortex lattices in
superconductors
in a magnetic field

Vortices now all have the same sign, imposed by the external rotation

Reaching the quantum Hall regime with atoms?

Equivalent of the filling factor: $f = \frac{N_{\text{atoms}}}{N_{\text{vortices}}}$



10^5 atoms
 10^2 vortices

QH-like effects are expected for f of order unity

One experiment so far in this regime: Chu's group (Stanford)

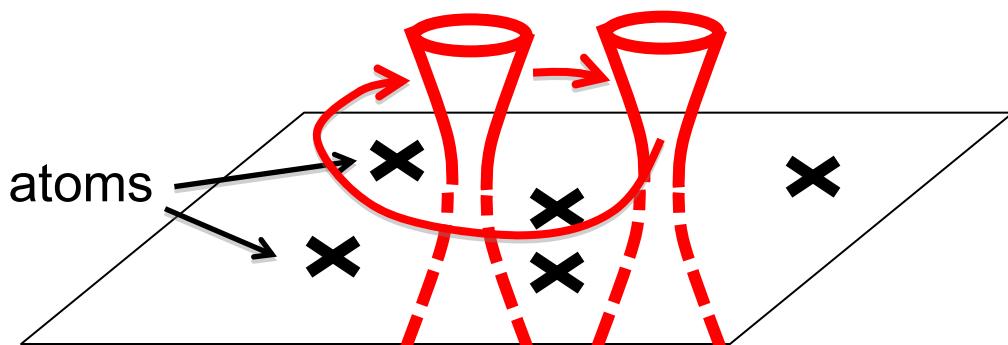
arXiv:1007.2677

$$N_{\text{atoms}} \sim N_{\text{vortices}} \sim 5 - 10$$

Evidence for a correlated state

How could one detect anyons in such systems?

Paredes *et al.*, 2001



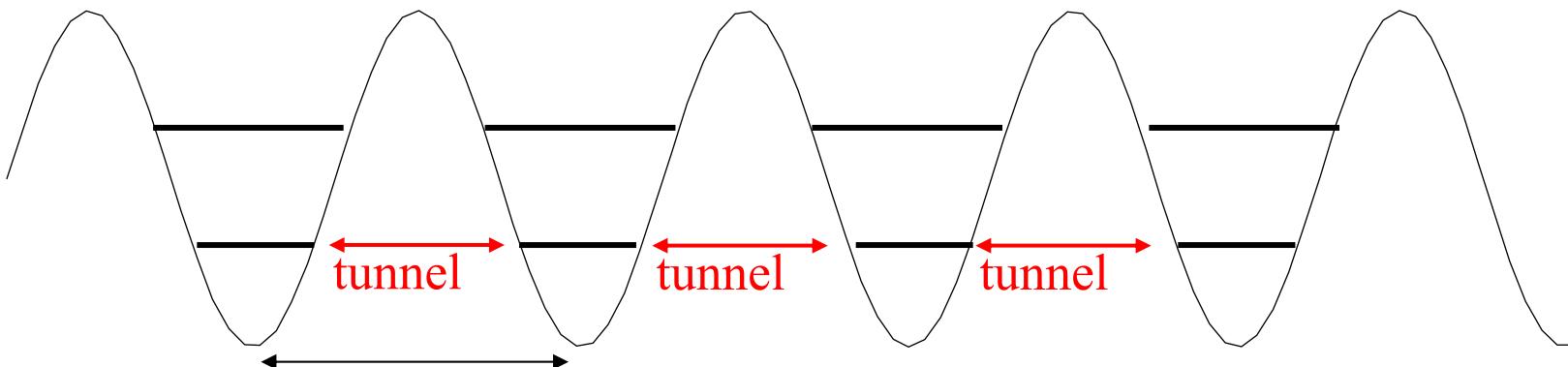
- Create two excitations by focusing two laser beams
- Rotate one excitation around the other
- Detect the phase acquired by the many-body system

Tailored Condensates

Artificial crystals

Condensate in periodic potential

Potential is created by a laser standing wave :

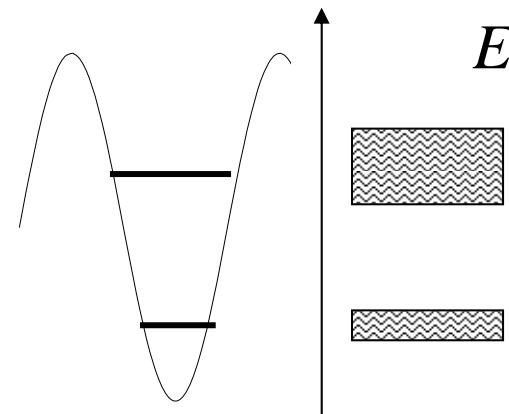


$$\lambda_{\text{laser}} / 2 = \pi / k \sim 0.3 \mu\text{m}$$

Energy scale:

$$E_R = \frac{\hbar^2 k^2}{2m} \sim 0.2 \mu\text{K}$$

1-particle Hamiltonian:
Energy states are energy bands



Condensate loaded in fundamental band

Momentum distribution measured by time of flight

diffraction profile indicates the coherence of the atomic state over different sites

$$|\Psi\rangle = \left(\sum_{i=1}^M \hat{a}_i^\dagger \right)^N |\text{vac}\rangle$$

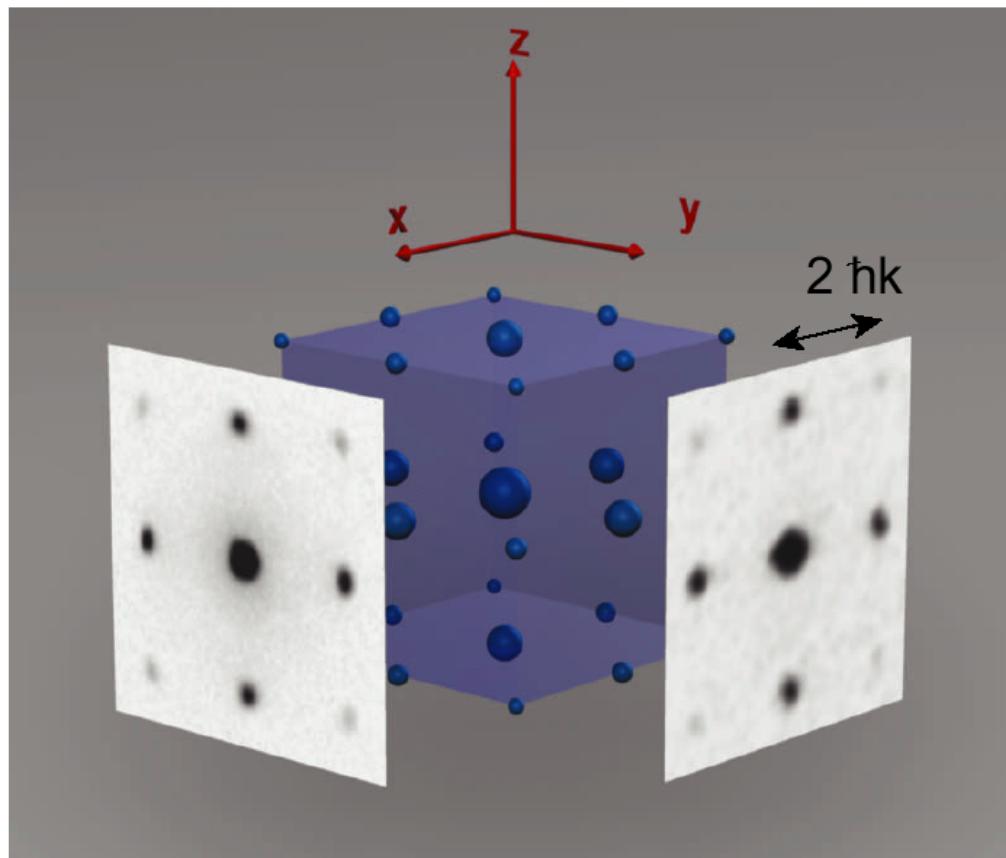
\hat{a}_i^\dagger : creates one atom on site i

M : number of sites

N : number of atoms

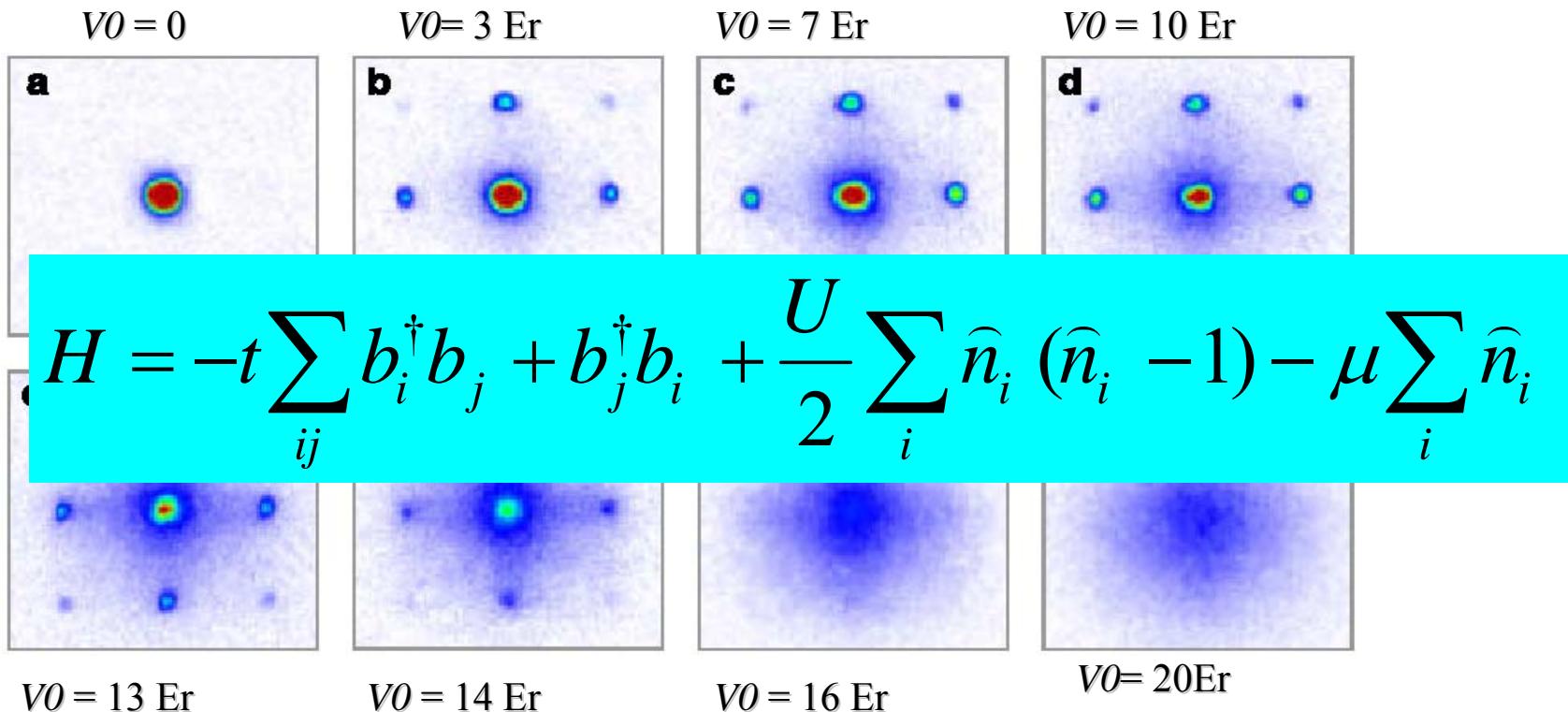
typically $n=N/M \approx 1$ to 3

Weak lattice



The Mott-Insulator transition

Competition between tunnel effect that tends to maintain coherence and repulsion between atoms that favor a fixed number of atoms per site



$$|\Psi\rangle = \left(\sum_{i=1}^M \hat{a}_i^\dagger \right)^N |\text{vac}\rangle$$

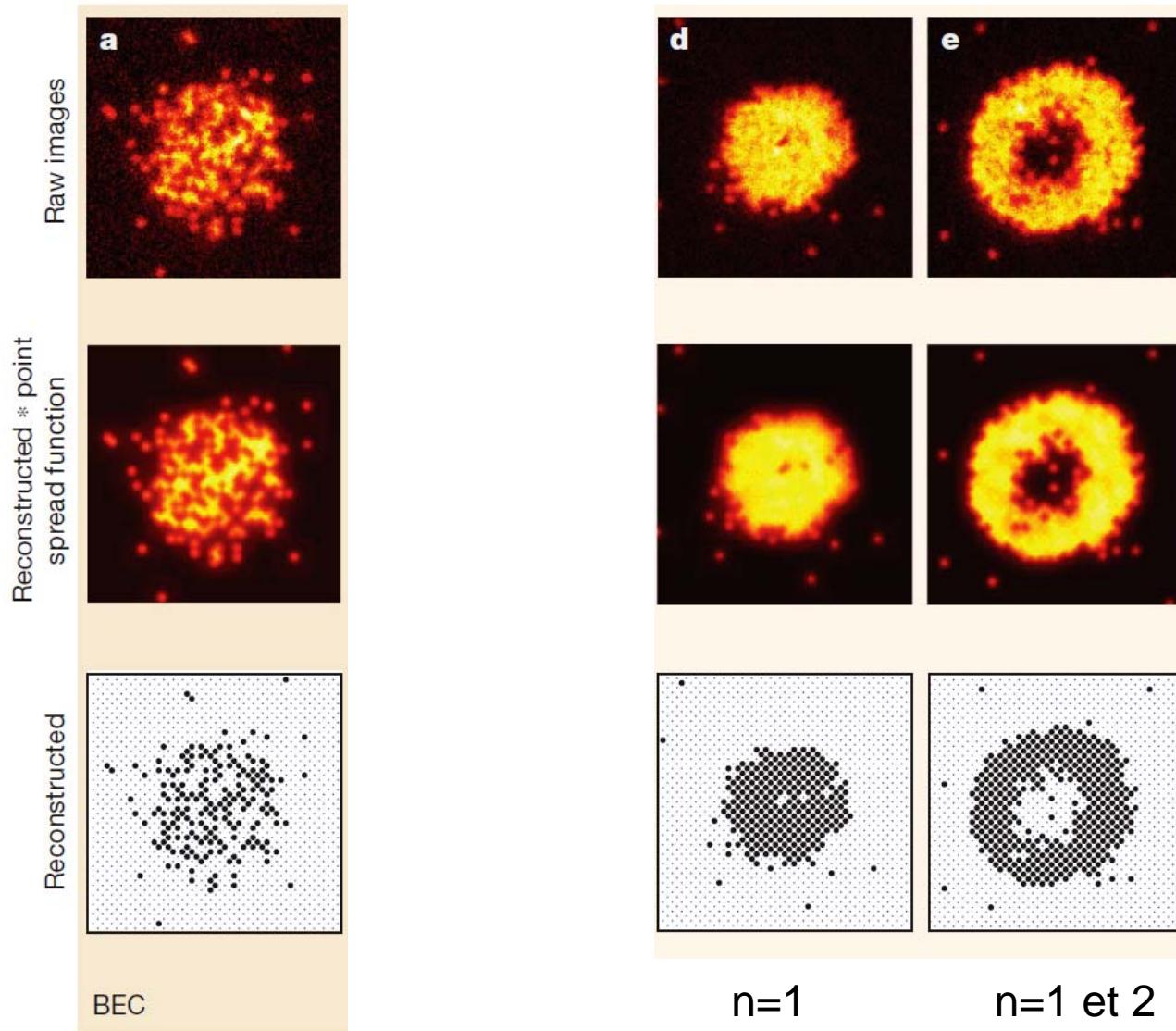


$$|\Psi\rangle \propto \prod_{i=1}^M \left(\hat{a}_i^\dagger \right)^n |\text{vac}\rangle$$

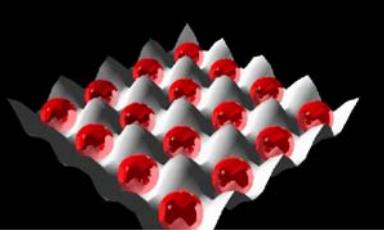
er et al.,
15,
2002)

Observation of Mott insulator with single site - single atom optical resolution

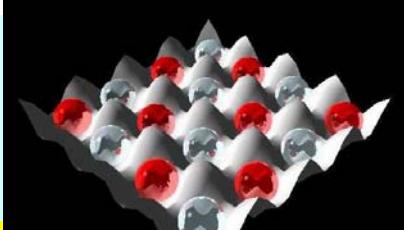
J. Sherson, C. Weitenberg, M. Endres, M. Cheneau, I. Bloch and S. Kuhr, Nature, 467, 2010



Also M. Greiner et al.
Harvard 2010



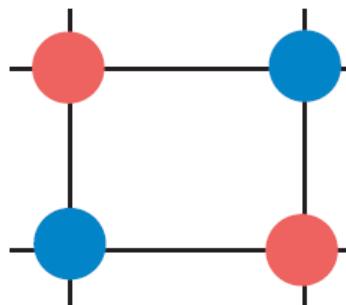
Next challenge: produce antiferromagnetic order



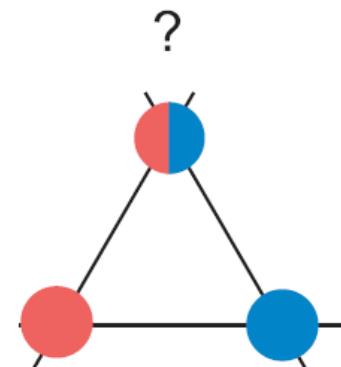
- First step: generate an interaction $\alpha \vec{S}_i \cdot \vec{S}_j$ between adjacent sites?
 - Use atoms with a large dipole (chromium, Stuttgart)
 - Use ground state polar molecules
 - Use Pauli principle + on-site interactions: super-exchange

Observed in a double well potential by the Mainz group (2007)

- Second step: achieve a low enough temperature in a lattice: $k_B T < \alpha$



Square lattice:
Néel ordering



Triangular lattice:
frustration

Perspectives

- Mott insulator transition seen also for fermions in 2008, ETH, Munich
- Towards quantum magnetism
- Two-dimensional Bose fluids and gauge fields,

Further reading:

- Two-dimensional Bose fluids :an atomic physics perspective, Z. Hadzibabic, J. Dalibard, ArXiv0912.1490
- BOSE-EINSTEIN CONDENSATION IN ATOMIC GASES, Proceedings of the International School of Physics « Enrico Fermi », Course CXL, ed. M. Inguscio, S. Stringari, C.E. Wieman, IOS Press, 1999.
- ULTRACOLD FERMI GASES
Proceedings of the International School of Physics « Enrico Fermi », Course CLXIV, ed. M. Inguscio, W. Ketterle, C. Salomon, IOS Press, 2007.