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Linear and Nonlinear Guided-Wave Photonics

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Linear and Nonlinear Guided-Wave Photonics

from waveguides to optical solitons

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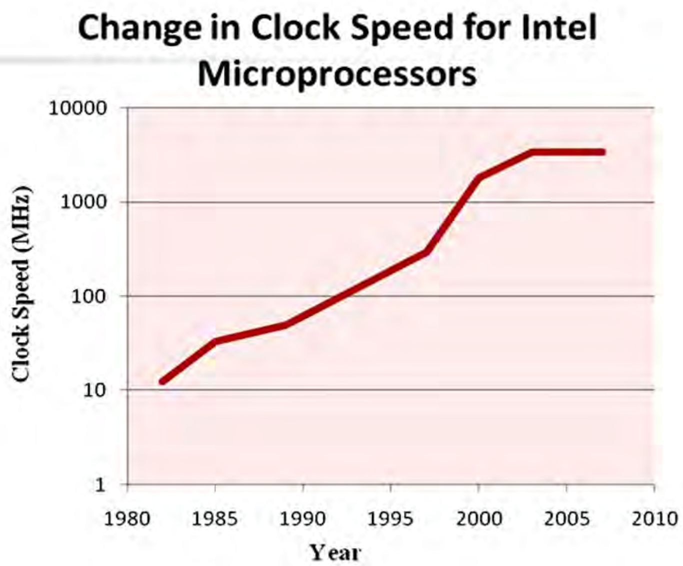
From electronics to photonics

- Electronic components

 Speed of processors is saturated due to
 high heat dissipation
 frequency dependent attenuation,
 crosstalk, impedance matching, etc.



- Photonic integration

 Light carrier frequency is 100,000 times higher, therefore a potential for faster transfer of information

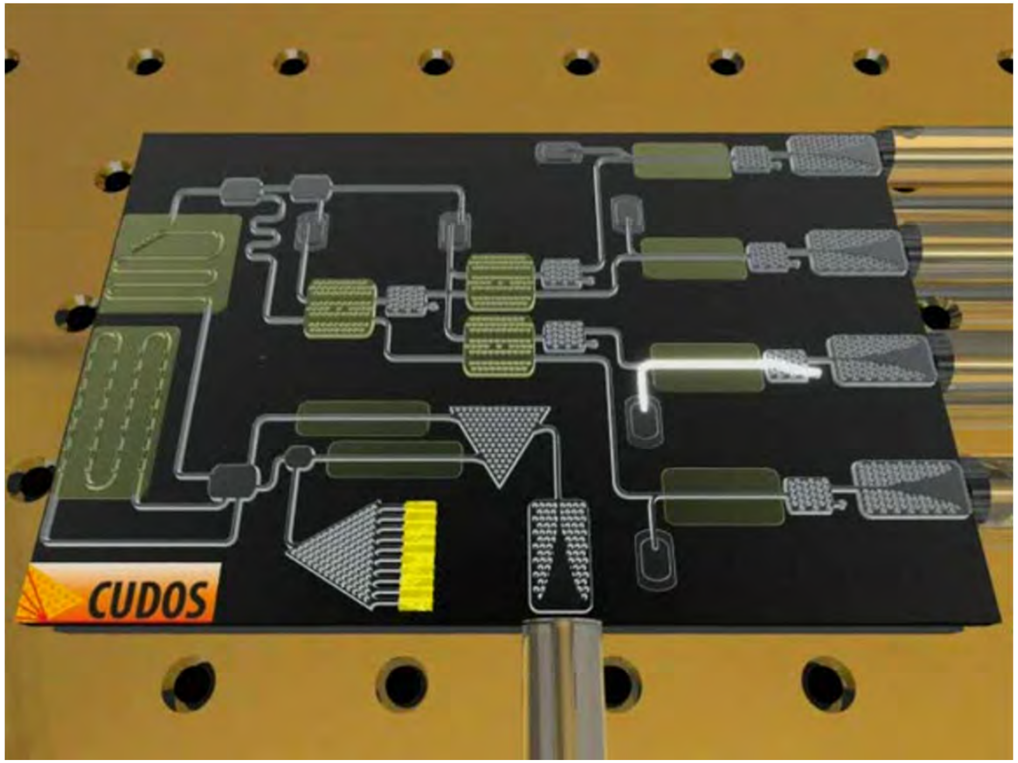
- Photonic interconnects

 already demonstrate advantages of photonics for passive transfer of information

The photonic chip

Processing of the information all-optically

Need to scale down
the dimensions



http://www.cudos.org.au/cudos/education/Animation.php

Photonic chip components

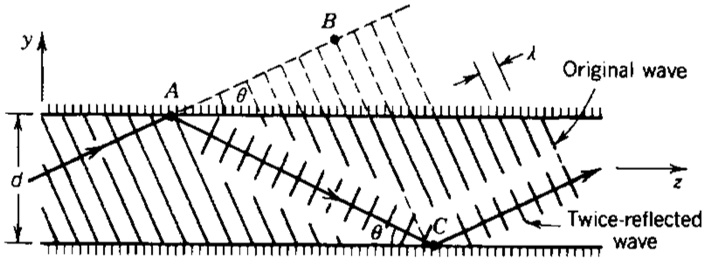
1. Light sources (lasers).
2. Waveguides (photonic wires).
3. Functionalities (ability of waveguides to process information)
   -- electro-optical;
   -- all-optical.
4. Detectors (photo diodes).

Lecture #1

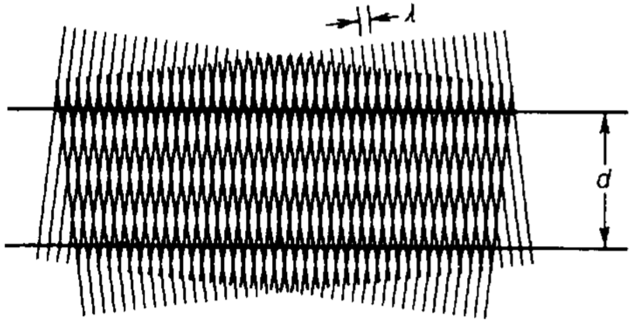
- Waveguides
- Waveguide dispersion
- Pulse propagation in waveguides
- Nonlinearities and optical nonlinearities
- Self-phase modulation
- Optical solitons

Waveguides: photonic wires

Waveguide guiding



Modes of a waveguide



The incident and reflected wave create a pattern that does not change with z – wg mode

Photonic elements

What happens to the light in a waveguide

- Waveguide propagation losses

  Light can be dissipated or scattered as it propagates

- Dispersion

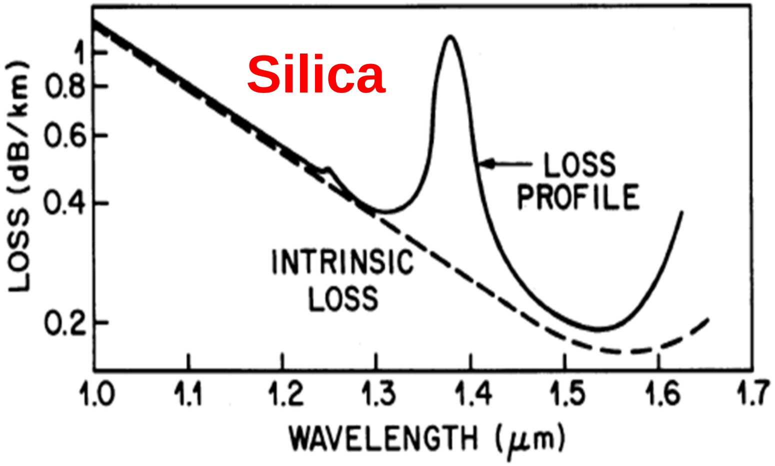
  Different colours travel with different speed in the waveguide

- Nonlinearities at high powers

  At high power, the light can change the refractive index of the material that changes the propagation of light.

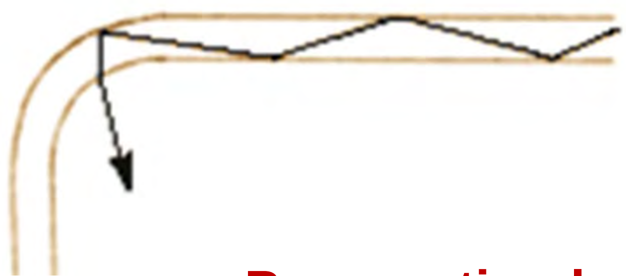
Waveguide loss: mechanisms

- Intrinsic/material



- Scattering due to inhomogeneities:
  - Rayleigh scattering: $\alpha_R \sim \lambda^{-4}$;
  - Side wall roughness

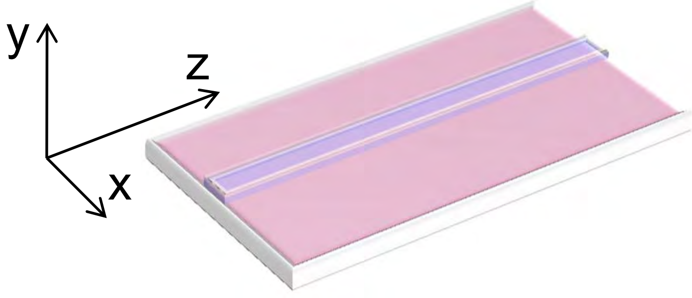
- Waveguide bending



Propagation loss

- **Coupling loss**

Waveguide loss: description



$$P(z)=P_0\exp(-\alpha z)$$

α [cm^{-1}] – attenuation constant

$$\alpha_{dB}=-10\log_{10}\left(\frac{P(z)}{P_0}\right)$$

3 dB loss = 50% attenuation

Often propagation loss is measured in dB/cm

$$\alpha_{dB/cm}=-\frac{10}{L}\log_{10}\left(\frac{P(L)}{P_0}\right)=4.343\alpha$$

Typical loss for waveguides 0.2 dB/cm
for fibres 0.2 dB/km

Dispersion - Mechanisms

- Material (chromatic)
- Waveguide
- Polarisation
- Modal

Material dispersion

- Related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons.

Sellmeier equation
(far from resonances)

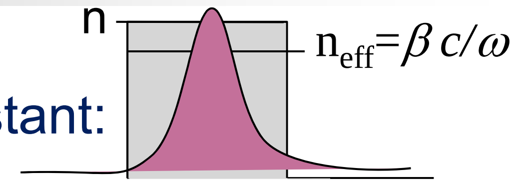
$$n^2(\lambda) = 1 + \sum_{j=1}^{m} \frac{B_j \lambda^2}{\lambda^2 - \lambda_j^2},$$

where λ_j are the resonance wavelengths and B_j are the strength of jth resonance

- For short pulses (finite bandwidth): different spectral components will travel with different speed c/n(λ) giving rise to Group Velocity Dispersion (GVD).

Group velocity dispersion

Accounted by the dispersion of the propagation constant:



$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \cdots,$$

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m}\right)_{\omega=\omega_0} \qquad (m = 0, 1, 2, \ldots).$$

$$\beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c}\left(n + \omega\frac{dn}{d\omega}\right),$$

v_g is the group velocity, n_g is the group index

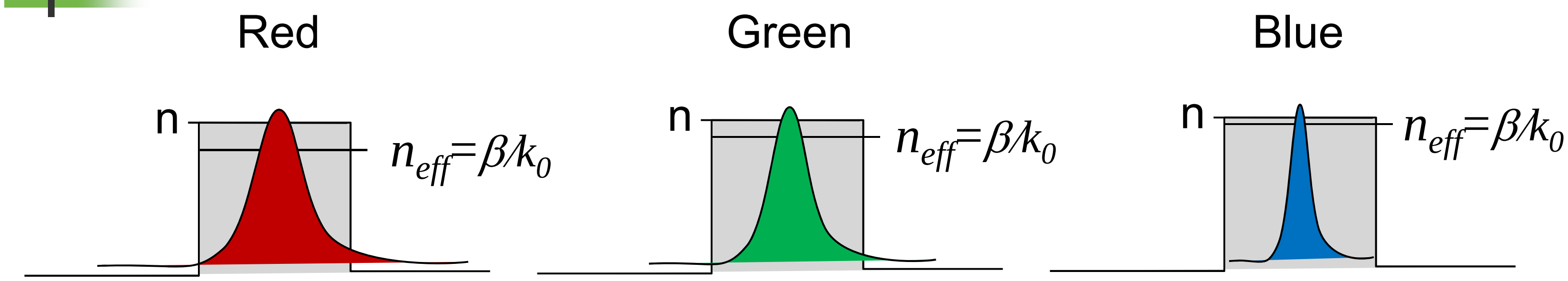
GVD is quantified by the dispersion parameter

$$D = \frac{d\beta_1}{d\lambda} \approx \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

measured in [ps/(km nm)]

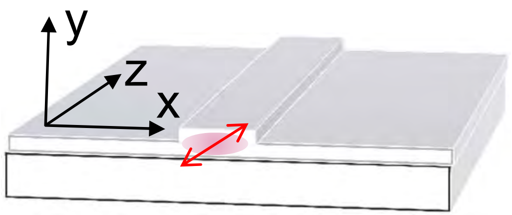
D>0 – anomalous dispersion; D<0 – normal dispersion

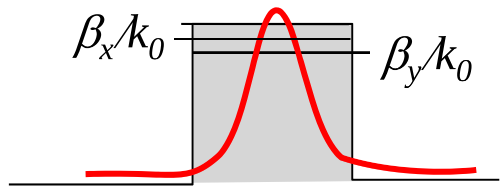
Waveguide dispersion



At different wavelengths the mode has a different shape.
This geometrical consideration leads to shift in the dispersion curves.
The effect is more pronounced in high index and narrow waveguides, e.g. photonic nanowires.

Polarisation-mode dispersion





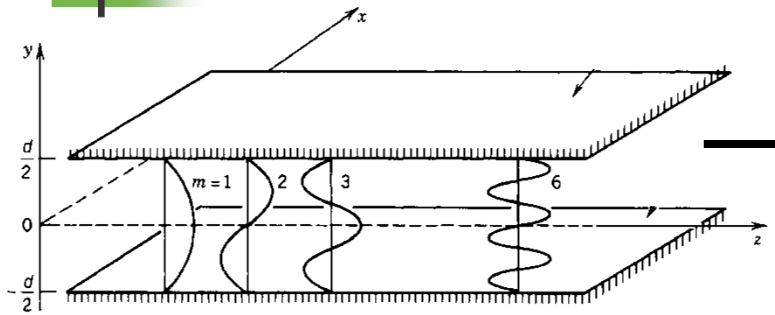
- Usual waveguides are strongly birefringent, therefore the propagation constants for x and y polarisation will be different.

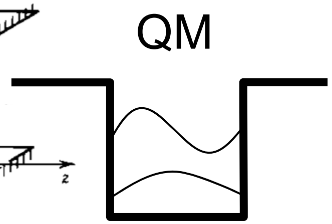
- The two polarisations will travel with different speed inside the waveguide

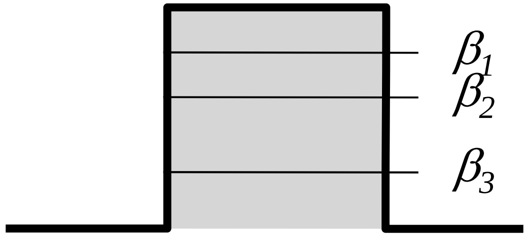
$$\Delta T = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| = L|\beta_{1x} - \beta_{1y}|$$

Time delay between two pulses of orthogonal polarisation

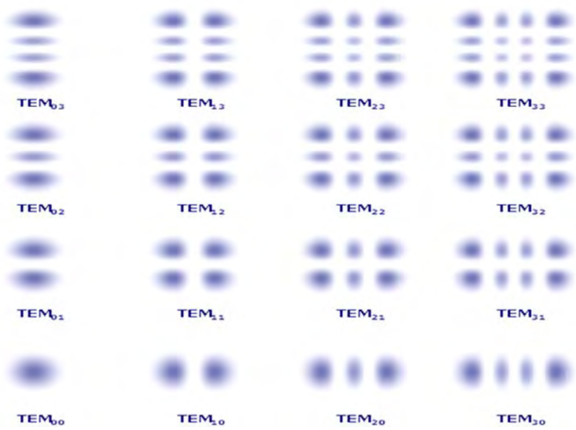
Modal dispersion





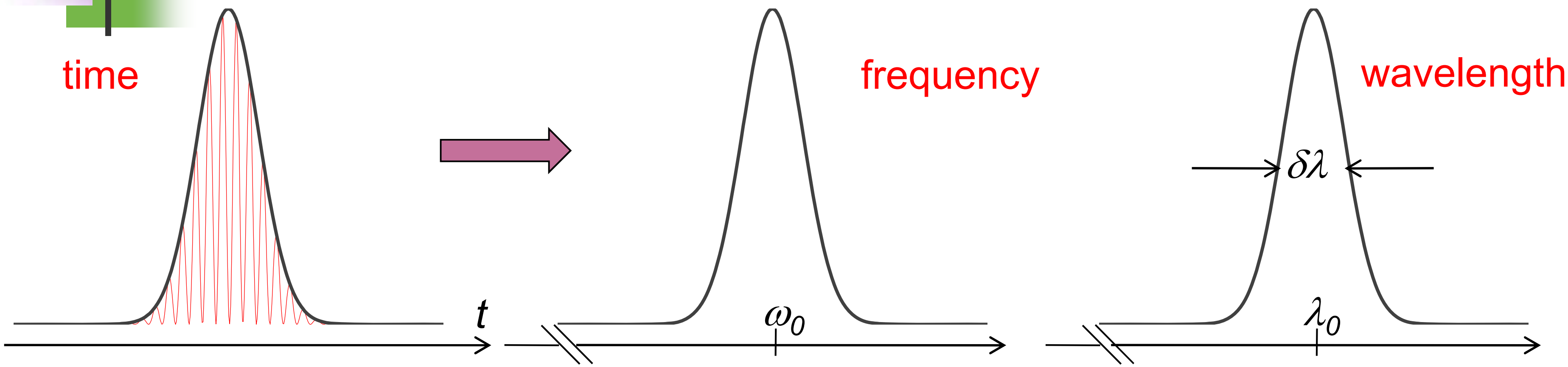


Modes of a square waveguide



- The different modes have different propagation constants and will travel with different velocities in side the waveguide.
- Need to make the waveguide single mode or excite one mode only.

Pulses: time and frequency



A pulse is a superposition (interference) of monochromatic waves:

$$A(z,t) = \int_{-\infty}^{\infty} A(z,\omega)\exp(i\omega t)d\omega$$

Each of these components will propagate with slightly different speed, but also their phase will evolve differently and the pulse will be modified:

velocity $\neq$ ph. velocity and duration (profile) will change

Group velocity

- As a result of the dispersion, the pulse (the envelope) will propagate with a speed equal to the group velocity

$$v_g \equiv \frac{d\omega}{dk} = \frac{c}{n}\left(1 - \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$$

Possibility for slow, superluminal, or backward light

- One can define a group index as $n_g = c/v_g$

$$n_g = \frac{c}{v_g} = \left(n - \lambda \frac{dn}{d\lambda}\right)$$

Index which the pulse will feel

Pulse broadening

- The finite bandwidth ($\delta\lambda$) of the source leads to a spread of the group velocities δv_g

$$\delta v_g = \frac{dv_g}{d\lambda}\delta\lambda = \frac{c\lambda}{n^2}\left(\frac{d^2n}{d\lambda^2} - \frac{2}{n}\left(\frac{dn}{d\lambda}\right)^2\right)$$

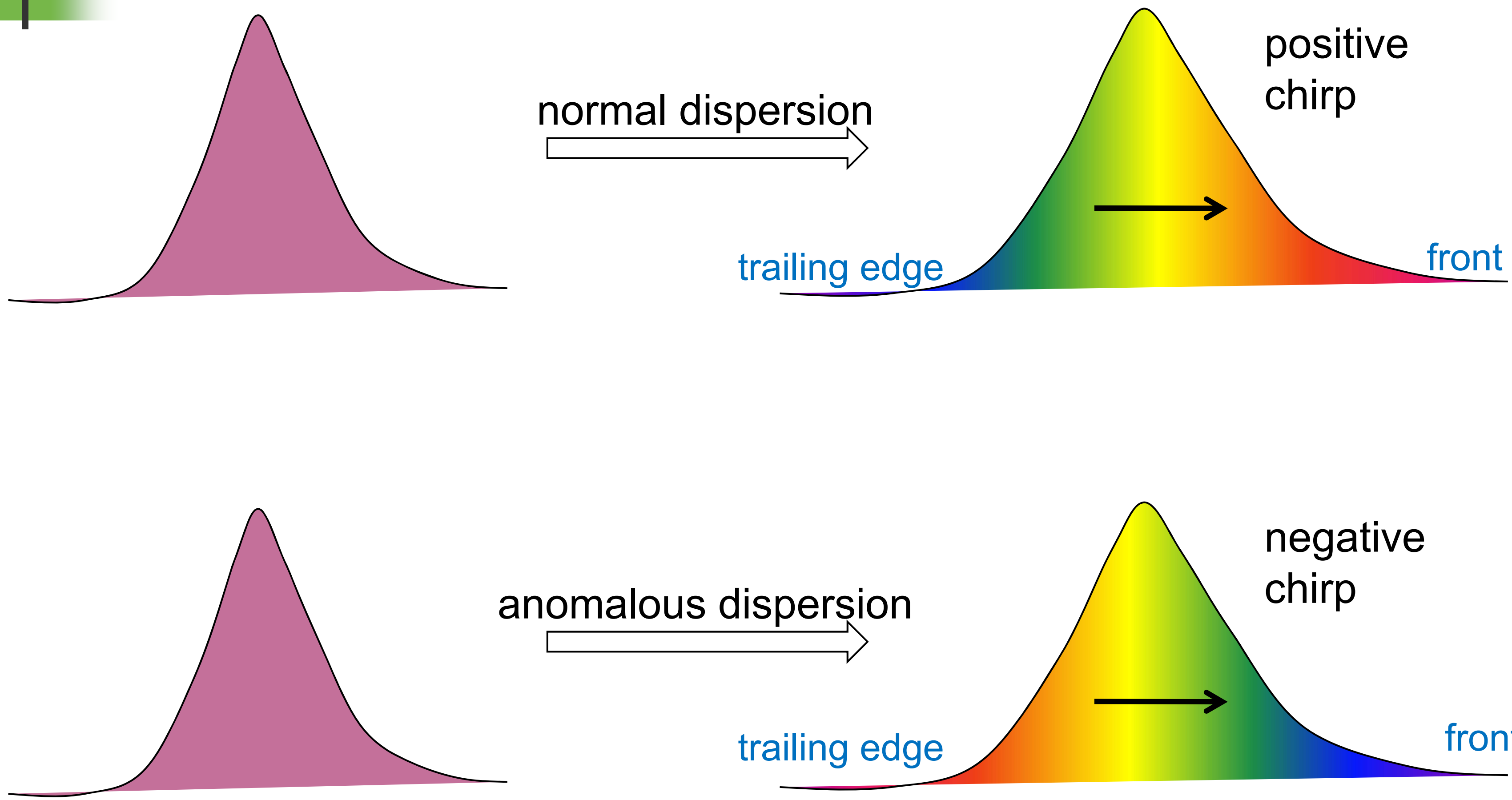
- Then a short pulse will experience a broadening δt after propagation L in the material:

$$\delta t = \frac{L}{v_g}\frac{\delta v_g}{v_g} = LD\delta\lambda$$

where $D = \frac{\lambda}{c}\left(\frac{d^2n}{d\lambda^2}\right).$

Dispersion coefficient

Pulse chirp



normal dispersion
positive chirp
trailing edge
front
anomalous dispersion
negative chirp
trailing edge
front

Short pulse propagation in dispersive media

The propagation of pulses is described by the propagation equation:

$$i\frac{\partial A}{\partial z}-\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}=0, \qquad \text{where} \quad \beta_2=-\frac{\lambda^2}{2\pi c}D$$

This is a partial differential equation, usually solved in the frequency domain.

$$i\frac{\partial \tilde{A}}{\partial z}+\frac{\beta_2}{2}\omega^2\tilde{A}=0, \qquad \Rightarrow \qquad \tilde{A}(z,\omega)=\tilde{A}(0,\omega)\exp\left(\frac{\beta_2}{2}\omega^2 z\right),$$

Important parameter:

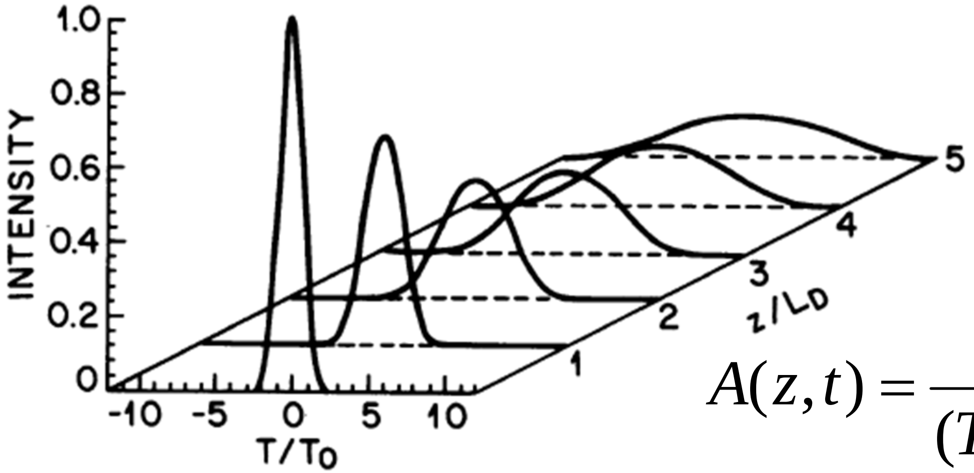
Dispersion length

$$L_D=\frac{T_0^{\,2}}{|\beta_2|}$$

The length at which the dispersion is pronounced

T_0 pulse width

Example: Gaussian pulse



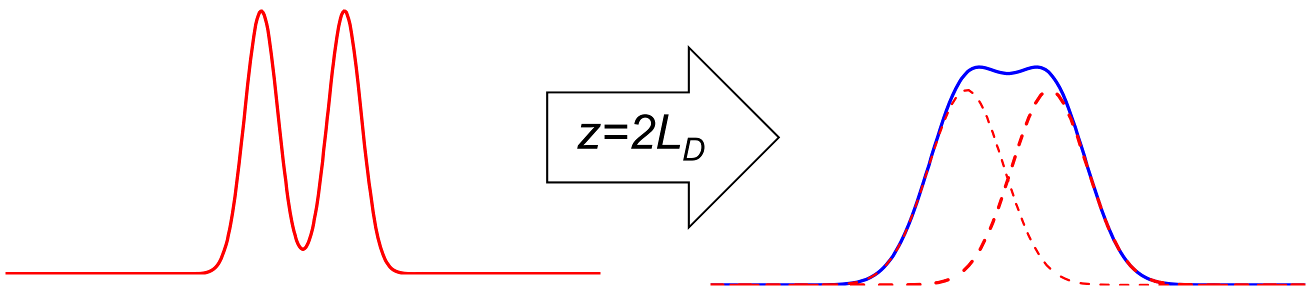
$$A(0,t) = \exp\left(-\frac{t^2}{2T_0^2}\right)$$

$$A(z,t) = \frac{T_0}{(T_0 - i\beta_2 z)^{1/2}} \exp\left(-\frac{t^2}{2(T_0^2 - i\beta_2 z)}\right)$$

A Gaussian pulse maintain its shape with propagation, but its width increases as

$$T(z) = T_0[1 + (z / L_D)^2]^{1/2}$$

How to compensate the spreading due to dispersion?



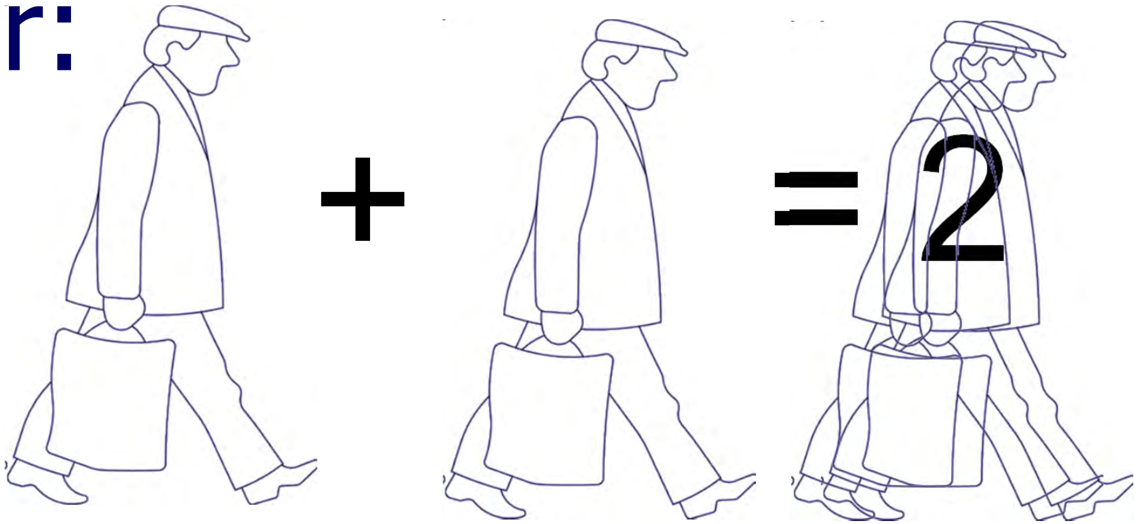
The dispersion needs to be compensated or close wavepackets will start overlapping.

This is usually done by dispersion compensator devices placed at some distances in the chip, or through proper dispersion management

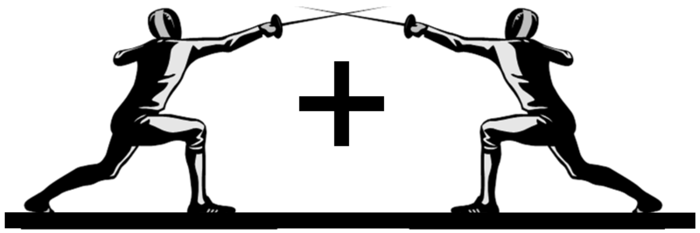
Material nonlinearity can balance the dispersion and pulses can propagate with minimum distortion.

What is nonlinearity?

Linear:



Nonlinearity: interaction

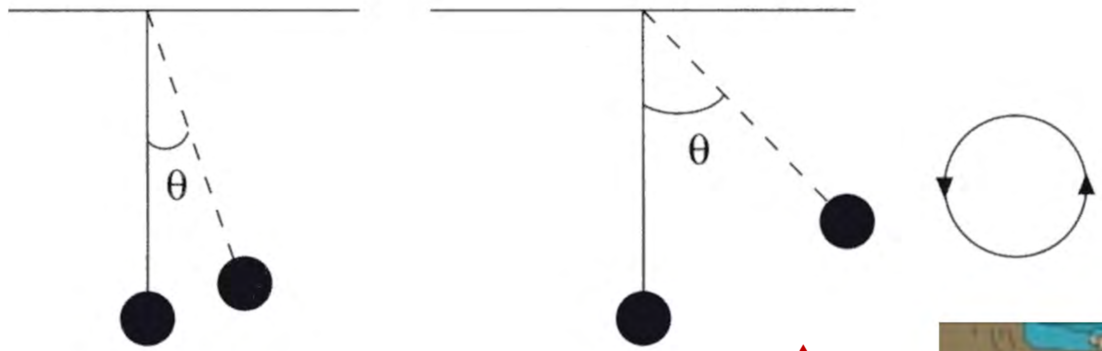


= ? 0,1,2

Nonlinearity: Examples

- Nonlinearity is present to many different systems in nature, including social sciences, biology, atmospheric physics, hydrodynamics, solid-state physics, and of course optics

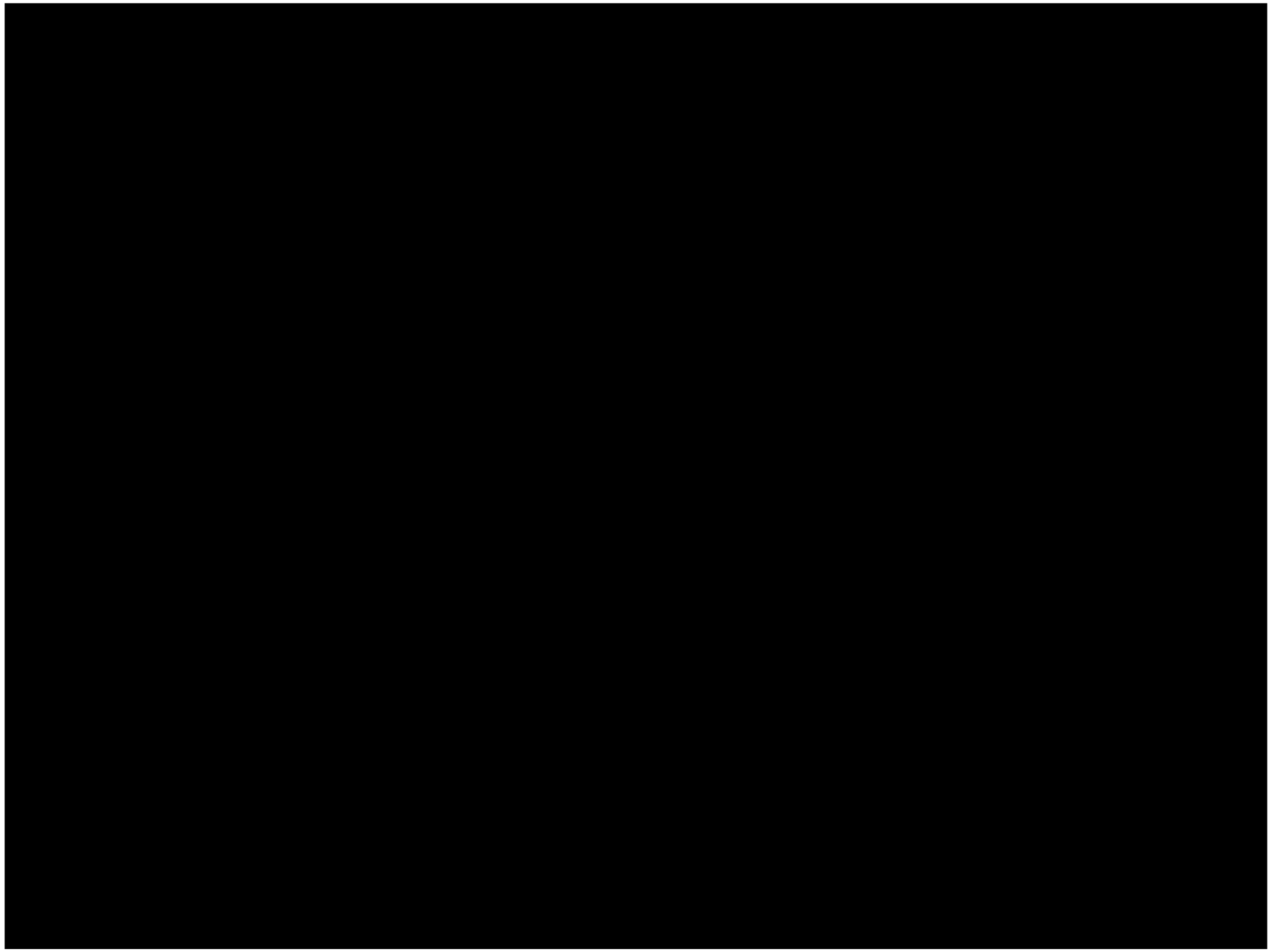
Physics: Pendulum



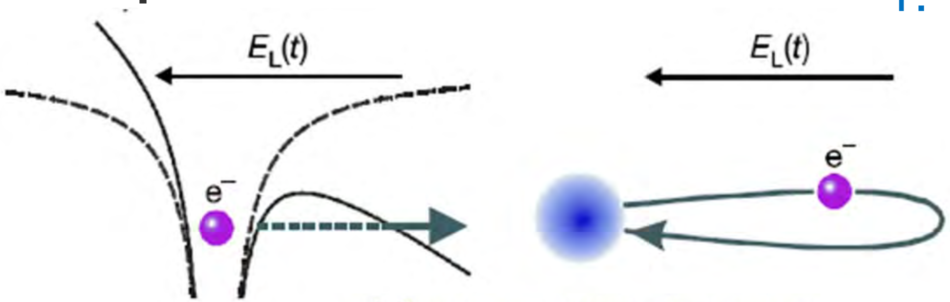
Large amplitude oscillations of a pendulum

The force is no more linear with the amplitude

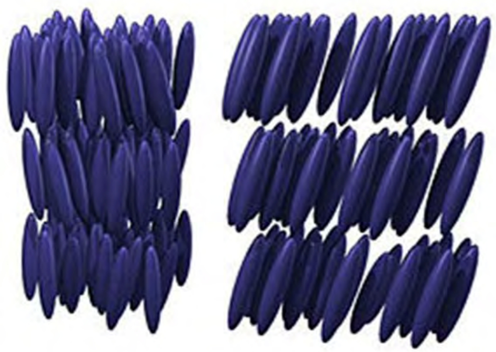
Extreme nonlinearities

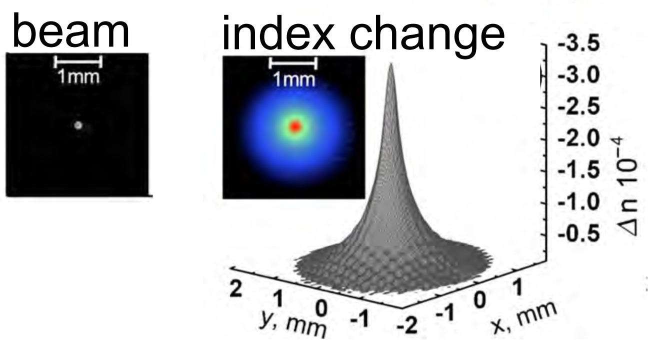


Origin of nonlinearities



Liquid crystals





1. Electronic
   The light electric field distorts the clouds displacing the electrons. Due to anharmonic motion of bound electrons (Similar to the nonlin. pendulum). *Fast response (10fs), high power kW - GW*
2. Molecular orientation
   due to anisotropic shape of the molecules they have different refractive index for different polarisation. The light field can reorient the molecules.
   Response 1ps – 10ms, 1kW – 1mW
3. Thermal nonlinearities
   due to absorption the material can heat, expand, and change refractive index (thermo-optic effect) *1-100ms, 1mW*

Origin of nonlinearities

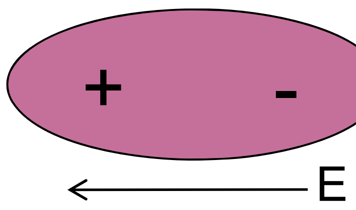
4. Photorefractive
   due to photo-excitation of charges, their separation in the material and electro-optic effect, 1-10s, <1μW
5. Atomic
   due to excitation of atomic transitions
6. Semiconductor
   due to excitation of carriers in the conduction bands
7. Metal
   due to deceleration of the free electrons next to the surface

- Classification:
  Non-resonant and resonant nonlinearities
  depending on the proximity of resonances

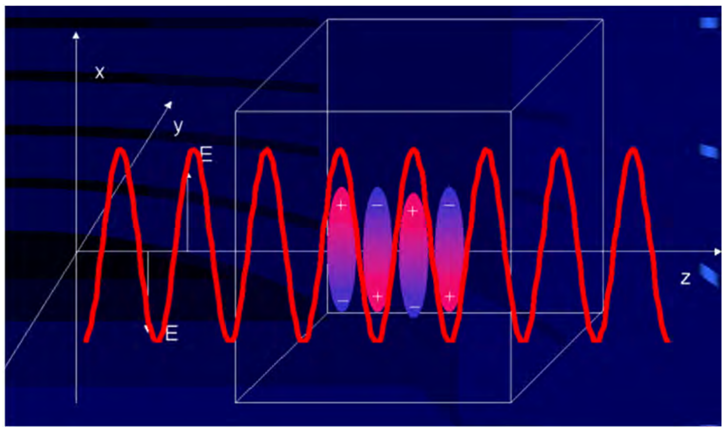
Optics: Medium polarisation

- Separation of charges gives rise to a dipole moment
  (model of bound electron clouds surrounding nucleus)
- Dipole moment per unit volume is called Polarisation

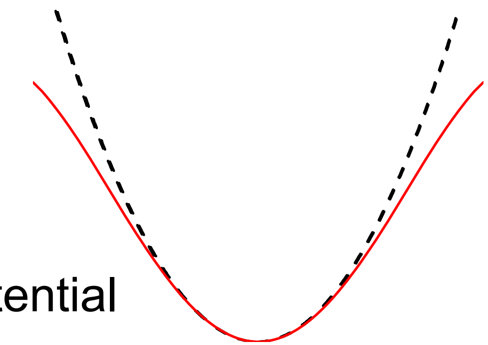
This is similar to a mass on a spring



$F=-kx$



When the driving force is to strong
the oscillations become anharmonic



Polarisation: description

$$P=\chi\varepsilon_0 E$$

χ is the dielectric susceptibility, ε_0 is the vacuum permittivity

- In isotropic materials, the above relationship may be scalar and $P||E$
- In general, however the relation between P and E is tensor
- At high electric field, the susceptibility becomes a nonlinear function of the electric field.

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{EE} + \chi^{(3)} \vdots \mathbf{EEE} + \cdots \right)$$

- This expansion valid for a non-resonant nonlinearity.
For electronic nonlinearity $E << E_{internal} = 10^{11}$ V/m; At optical wavelengths, an intensity of 10^{12} W/m^2 corresponds to $E \sim 10^9$ V/m

Polarisation: description

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{EE} + \chi^{(3)} \vdots \mathbf{EEE} + \cdots \right)$$

1 2 3

- $\chi^{(j)}$ (j=1,2,...) is j^{th} order susceptibility;
- $\chi^{(j)}$ is a tensor of rank j+1;
- for this series to converge $\chi^{(1)}E >> \chi^{(2)}E^2 >> \chi^{(3)}E^3$

- $\boldsymbol{\chi^{(1)}}$ is the linear susceptibility (dominant contribution). Its effects are included through the refractive index (real part) and the absorption α (imaginary part).

Nonlinear refraction

- The refractive index is modified by the presence of optical field:

$$n(\lambda, I) = n_0(\lambda) + n_2 I$$

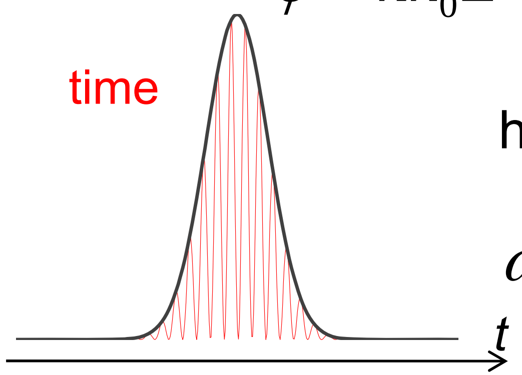
where $n_0(\lambda)$ is the linear refractive index,
$I=(nc\varepsilon_0/2)|E|^2$ is the optical intensity,
$n_2 = 12\pi^2\chi^{(3)}/n_0c \quad 3\chi^{(3)}/4\varepsilon_0 n_0^2 c$
is the nonlinear index coefficient

- This intensity dependence of the refractive index leads to a large number of nonlinear effects with the most widey used:
  - Self-phase modulation
  - Cross phase modulation

Self phase modulation

- SPM – self-induced phased shift experienced by the optical pulse with propagation

$\phi = nk_0L = (n_0 + n_2 I)k_0 L$ where $k_0=2\pi/\lambda$ vacuum wavenumber, L is the propagation length

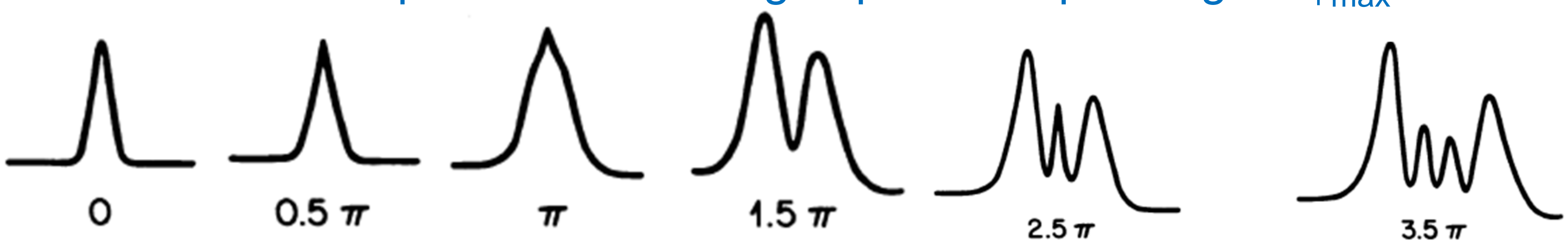


however $I=I(t)$ hence $\phi=\phi(t)$ What does this mean?

$$\omega(t) = \omega_0 + \delta\omega = \omega_0 - \frac{d\phi}{dt}$$

Generation of new frequencies
Spectral broadening

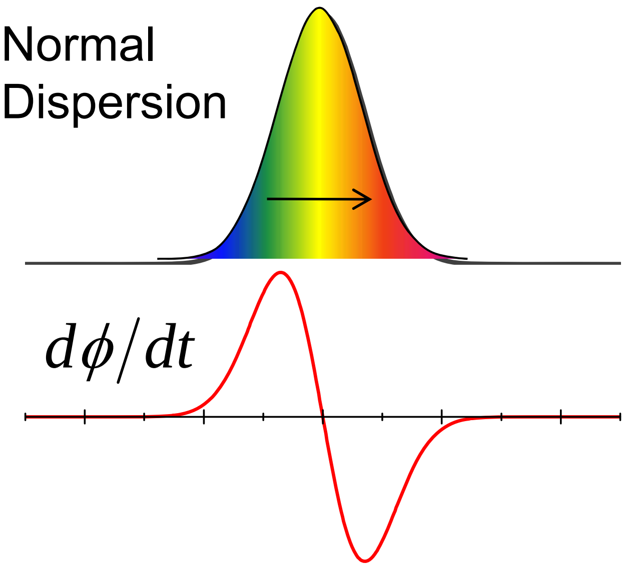
Measured spectral broadening of pulses depending on ϕ_{max}



Optical solitons

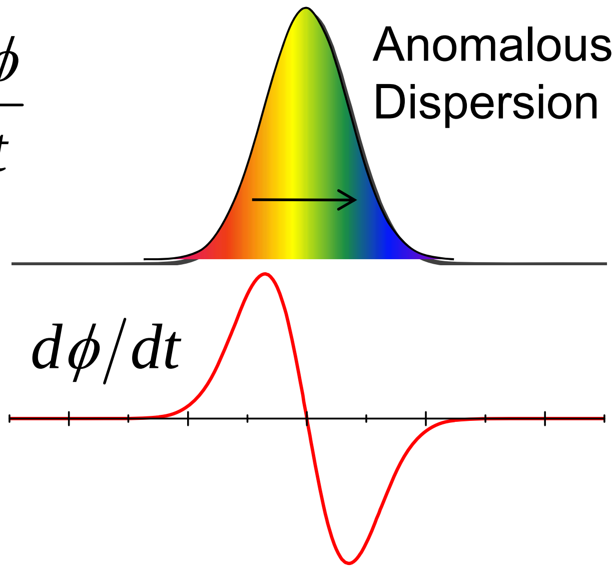
- What happens to intense pulses in dispersive media?

Normal Dispersion



$$\omega(t) = \omega_0 - \frac{d\phi}{dt}$$

Anomalous Dispersion

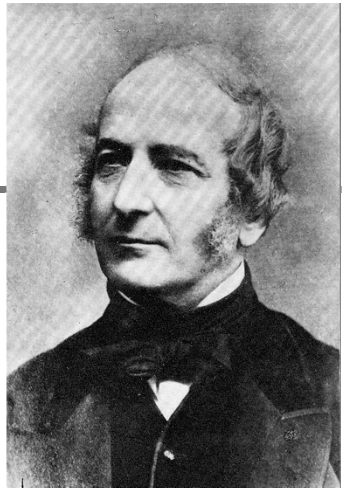


Nonlinearity increases the dispersion

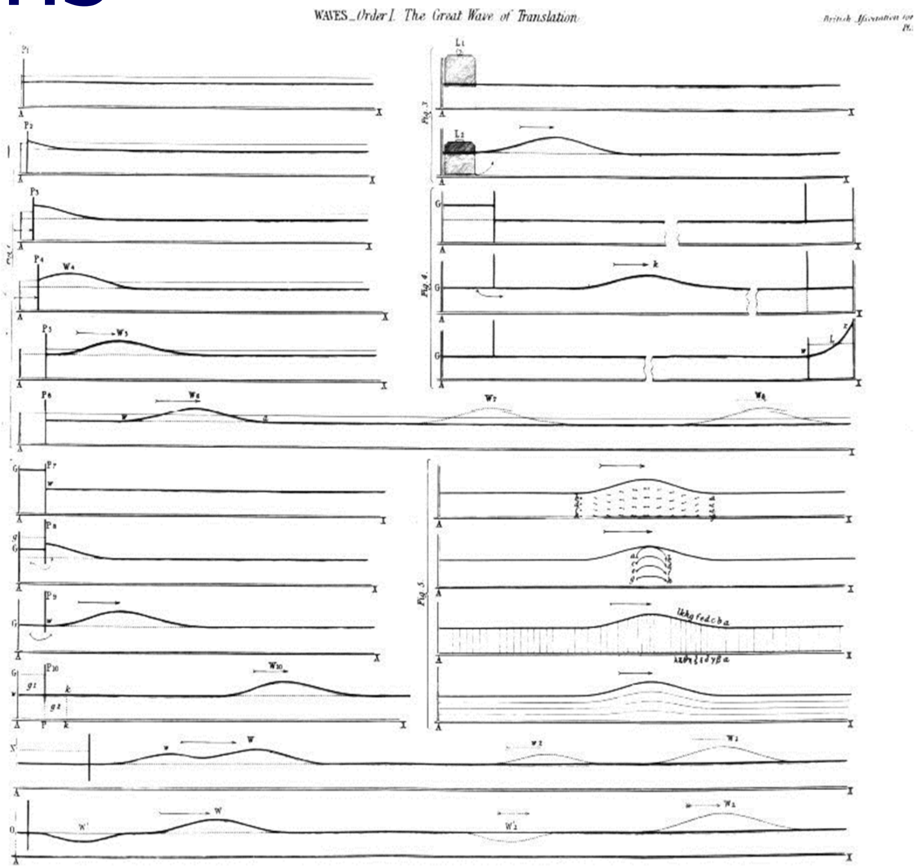
Nonlinearity counteract the dispersion

- Nonlinearity can fully balance the dispersion: **Optical Soliton**

Discovery of solitons



John Scott Russell (1808-1882)

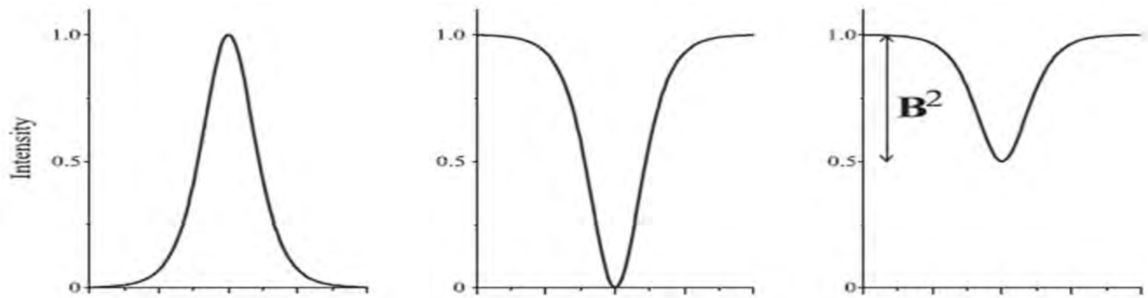


Very bright engineer: invented an improved *steam-driven road carriage* in 1833. ``Union Canal Society'' of Edinburgh asked him to set up a navigation system with steam boats

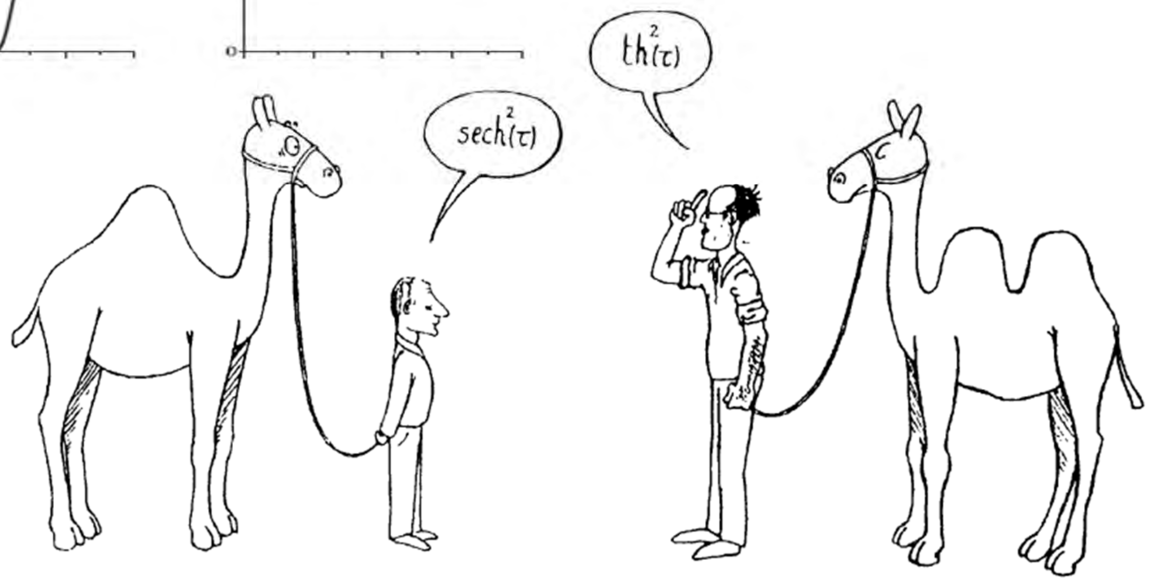
During his investigations, 6 miles from the centre of Edinburgh, he observed a *soliton* for the first time in August 1834

Types of solitons

Dark –normal dispersion

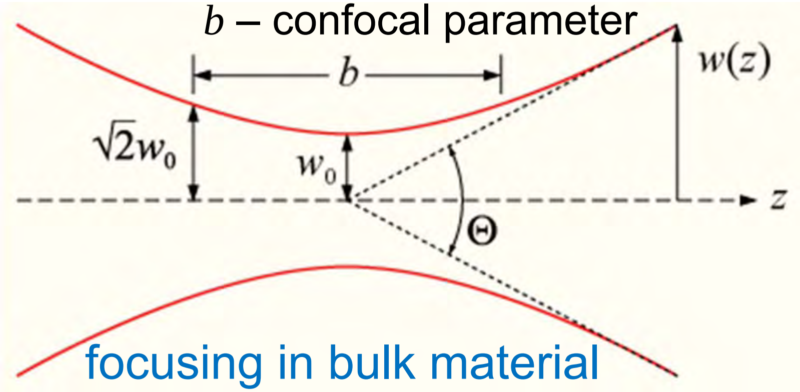


Bright
anomalous
dispersion



Nonlinear effects in waveguides

w – beam waist
b – confocal parameter



focusing in bulk material

A figure of merit for the efficiency of a nonlinear process: $I\,L_{eff}$

$$(IL_{\text{eff}})_{\text{bulk}} = \left(\frac{P}{\pi w_0^2}\right)\frac{\pi w_0^2}{\lambda} = \frac{P}{\lambda}$$

$$(IL_{\text{eff}})_{\text{wg}} = \int_0^L I(z)\exp(-\alpha z)\,dz = \frac{P}{\pi w_0^2 \alpha}[1-\exp(-\alpha L)].$$

$$F = \frac{(IL_{eff})_{wg}}{(IL_{eff})_{bulk}} = \frac{\lambda}{\pi w_0{}^2 \alpha}$$

propagation length is only limited by the absorption

for λ=1.55μm, w_0=2μm,
α=0.046cm^{-1} (0.2dB/cm) → ***F*~2** x **10^4**

Summary of the second part

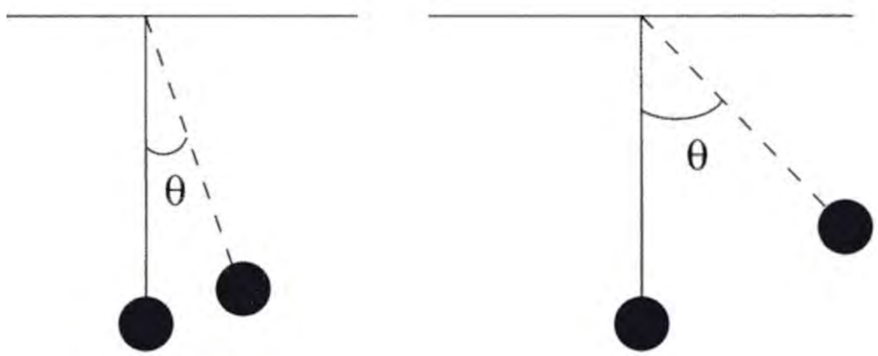
- Pulses propagate with a group velocity and spread due to GVD
- There exist different type of nonlinearities, but the main effects remain similar
- Description of nonresonant nonlinear polarisation: nonlinear SPM and CPM
- Nonlinearity can fully balance the anomalous dispersion in the form of solitons

Lecture #2

- Nonlinear optics emerges
- $\chi^{(2)}$ parametric processes
- Phase matching
- Four-wave mixing and applications
- Plasmonic waveguides

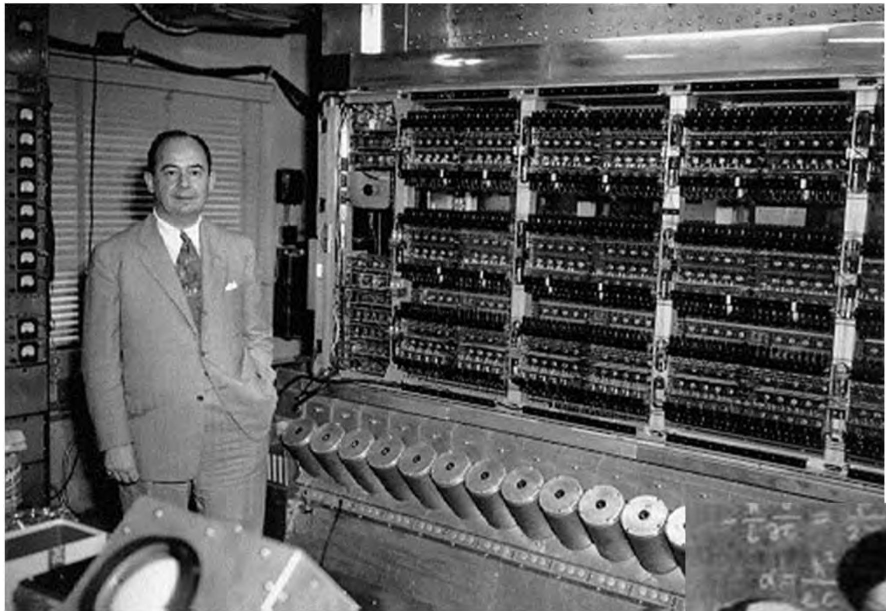
Nonlinearity: Examples

Nonlinearity is present in many different systems in nature, including social sciences, biology, chemistry, atmospheric physics, hydrodynamics, solid-state physics, and optics



The force is no more linear with the amplitude

The birth of Nonlinear Physics:
Fermi-Pasta-Ulam Problem

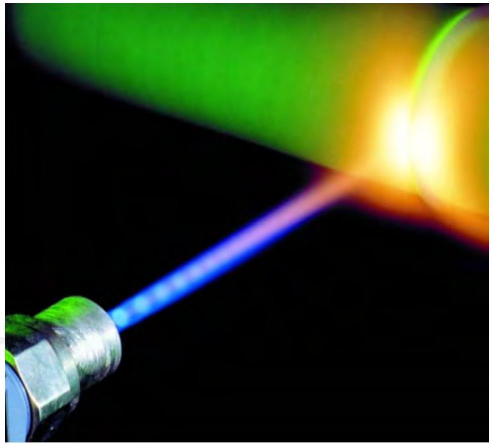


MANIAC I and Von Neumann

MANIAC: Mathematical Analyzer, Numerator, Integrator, and ***Computer***

Los Alamos -1950s

Nonlinear optics



1958-60: Invention of the laser

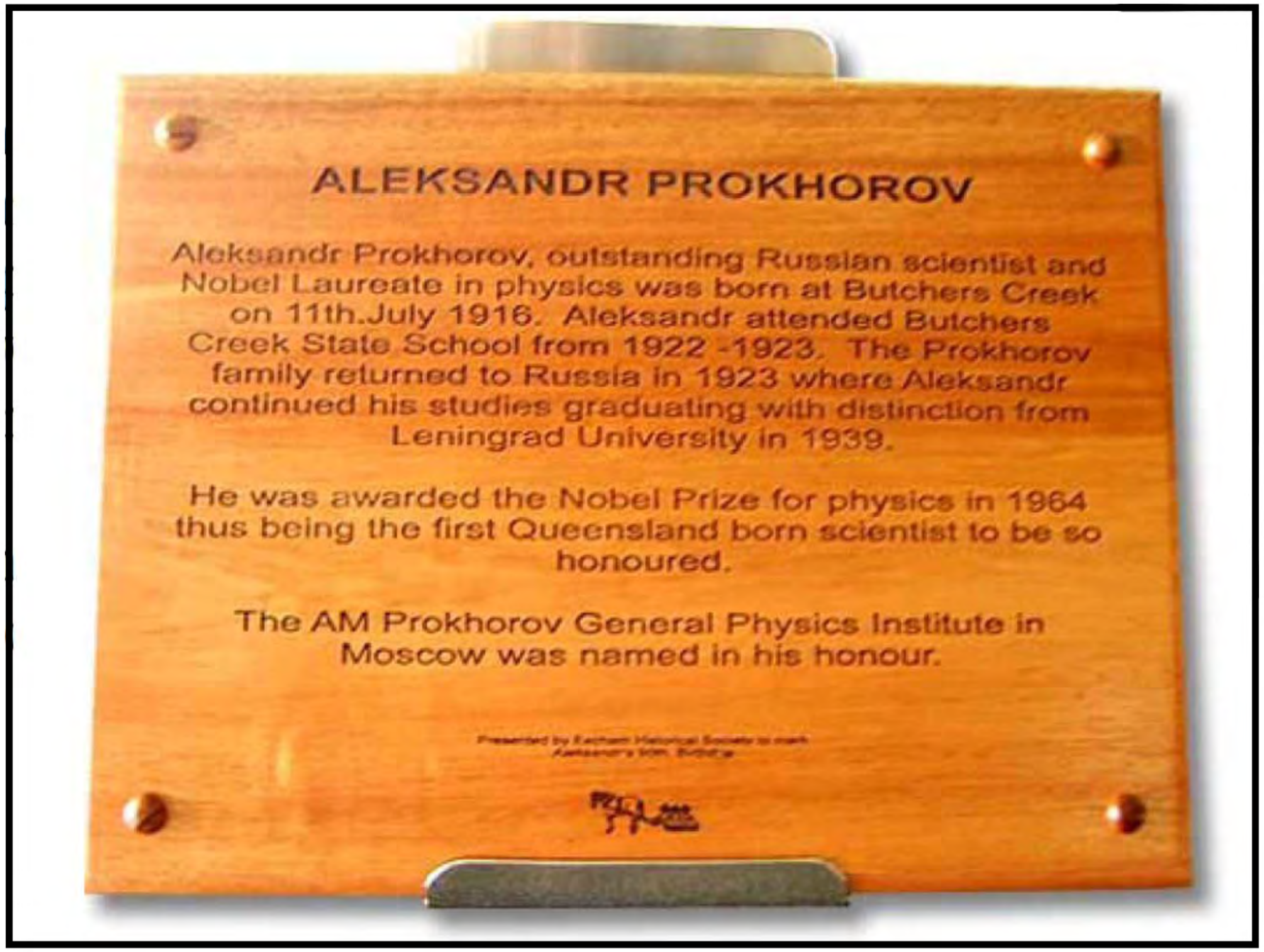
1964: Townes, Basov and Prokhorov shared the **Nobel prize** for their fundamental work leading to the construction of lasers

1981: Bloembergen and Schawlow received the **Nobel prize** for their contribution to the development of laser spectroscopy. One typical application of this is *nonlinear optics* which means methods of influencing one light beam with another and permanently joining several laser beams

Nicolaas Bloembergen

Arthur Leonard Schawlow

A.M. Prokhorov



Polarisation: description

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{EE} + \chi^{(3)} \vdots \mathbf{EEE} + \cdots \right)$$

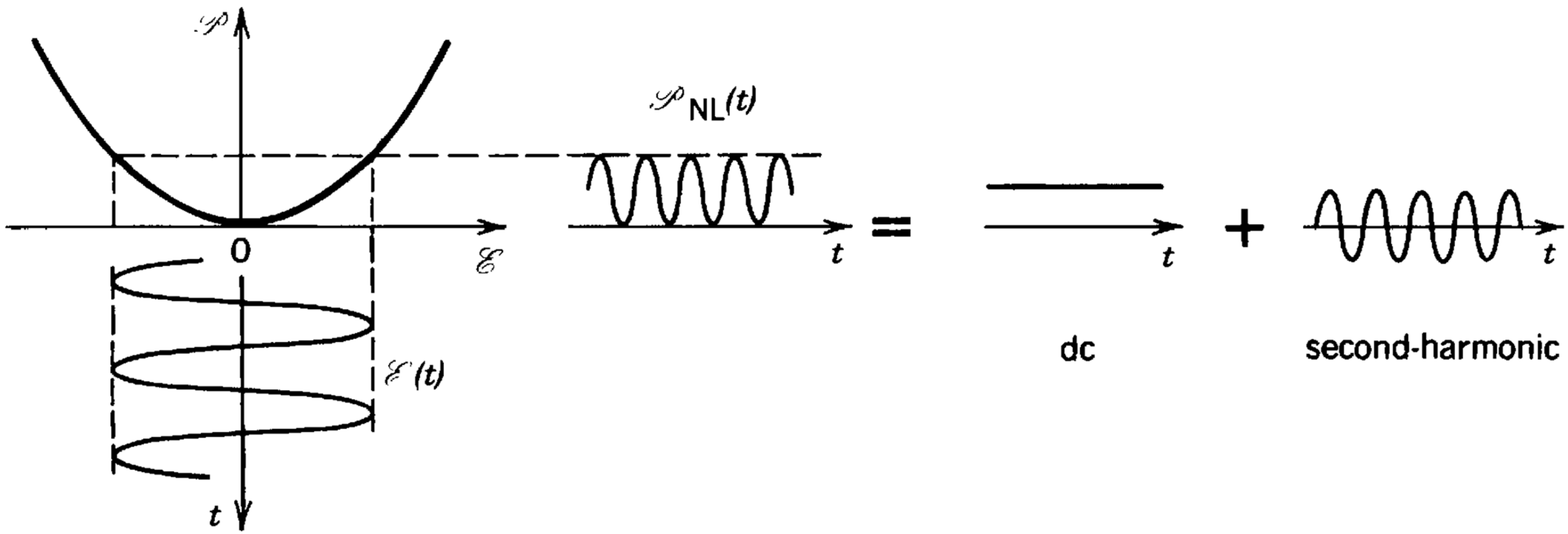
1 2 3

- $\chi^{(j)}$ (j=1,2,...) is jth order susceptibility;
- $\chi^{(j)}$ is a tensor of rank j+1;
- for this series to converge $\chi^{(1)}E >> \chi^{(2)}E^2 >> \chi^{(3)}E^3$

- $\chi^{(1)}$ is the linear susceptibility (dominant contribution). Its effects are included through the refractive index (real part) and the absorption α (imaginary part).

Nonlinearity in noncentrosymmetric media

$P^{(2)} = \chi^{(2)} E E$

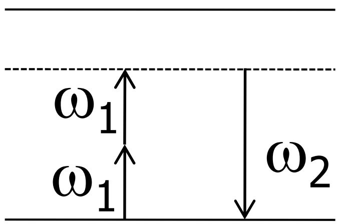


Nonlinear frequency conversion

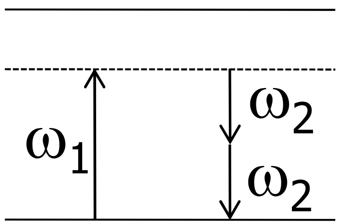
Can use $\chi^{(2)}$ or $\chi^{(3)}$ nonlinear processes. Those arising from $\chi^{(2)}$ are however can be achieved at lower powers.

Frequency mixing

Three wave mixing

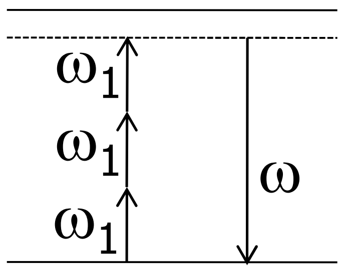


Sum frequency generation

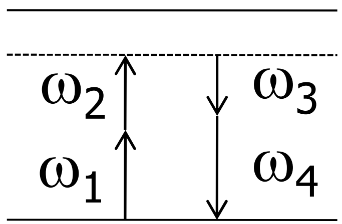


Difference freq. generation

Four wave mixing



THG



FWM

$\chi^{(2)}$ parametric processes

- Anisotropic materials: crystals (..................)

$$P_i = \sum_{jk} \chi^{(2)}_{ijk} E_j^{\omega_a} E_k^{\omega_b} \qquad E^{\omega_a} = E_0 \sin(\omega_a t), \quad E^{\omega_b} = E_0 \sin(\omega_b t)$$

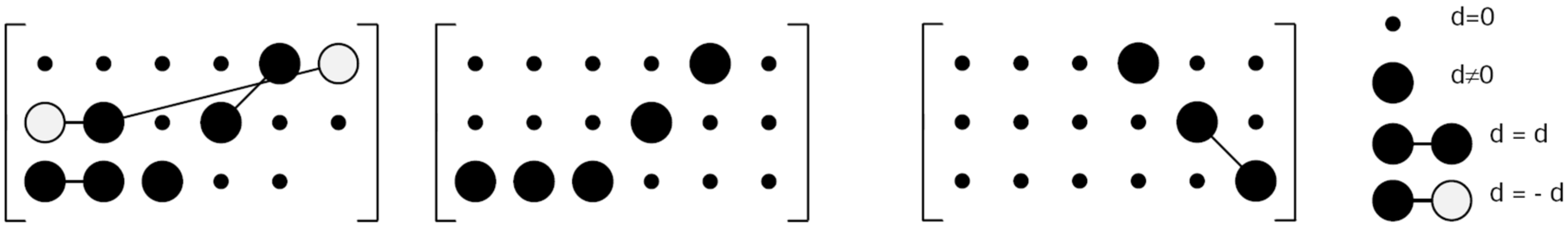
$$P_i \propto E_j^{\omega_a} \sin(\omega_a t) \times E_k^{\omega_b} \sin(\omega_b t) \quad \Rightarrow \quad \begin{array}{ll} \sin[(\omega_a + \omega_b)t] & \text{SFG} \\ \sin[(\omega_a - \omega_b)t] & \text{DFG} \end{array}$$

- Due to symmetry and when $\chi^{(2)}$ dispersion can be neglected, it is better to use the tensor $d_{ijk} = \frac{1}{2}\chi^{(2)}_{ijk}$

- In lossless medium, the order of multiplication of the fields is not significant, therefore $d_{ijk}=d_{ikj}$. (only 18 independent parameters)

Crystalline symmetries

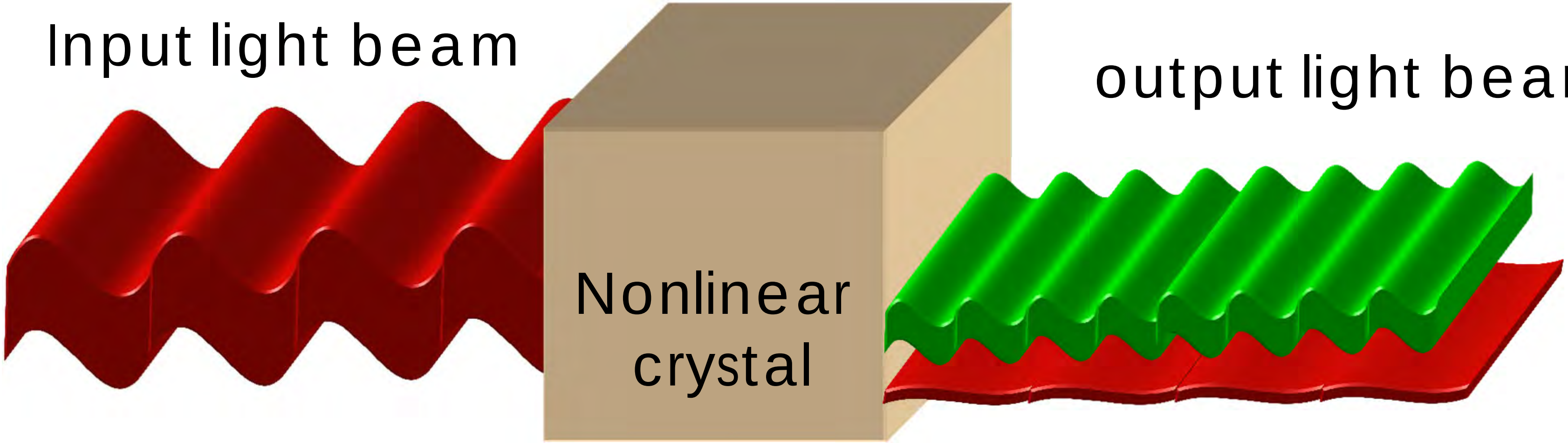
$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix} \begin{pmatrix} E_x(\omega_1)E_x(\omega_2) \\ E_y(\omega_1)E_y(\omega_2) \\ E_z(\omega_1)E_z(\omega_2) \\ E_y(\omega_1)E_z(\omega_2)+E_z(\omega_1)E_y(\omega_2) \\ E_x(\omega_1)E_z(\omega_2)+E_z(\omega_1)E_x(\omega_2) \\ E_x(\omega_1)E_y(\omega_2)+E_y(\omega_1)E_x(\omega_2) \end{pmatrix}$$



3m eg. Quartz, $LiNbO_3$ — mm2 eg. KTP, LBO — $\bar{4}2m$ eg. KDP

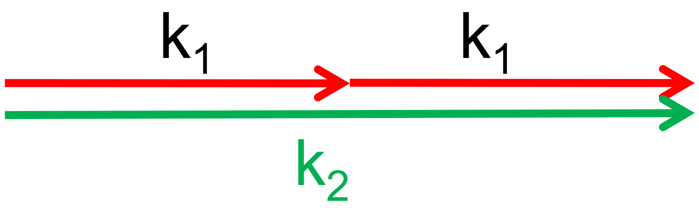
The generated light can have different polarisation than the incident fields

Second harmonic generation



1. Energy conservation $\omega_1 + \omega_1 = \omega_2$

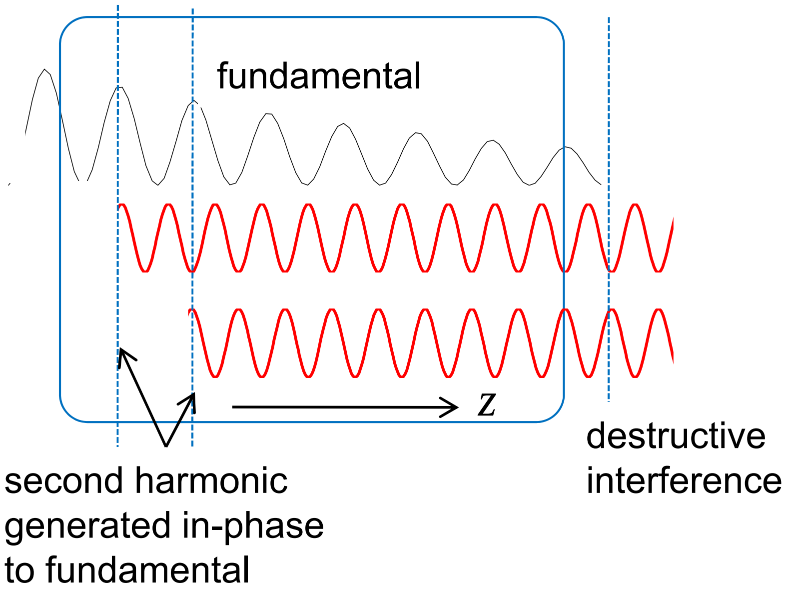
2. Momentum conservation
Phase matching

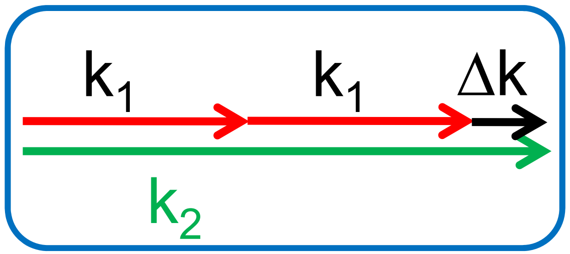


$k_1 + k_1 = k_2$

$n_1 = n_2$

Phase matching: SHG



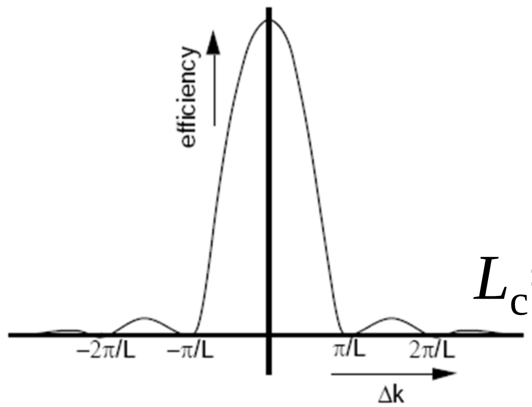


At all z positions, energy is transferred into the SH wave. For a maximum efficiency, we require that all the newly generated components interfere constructively at the exit face.
(the SH has a well defined phase relationship with respect to fundamental)

The efficiency of SHG is given by:

$$SH \propto L^2 \frac{\sin^2(\Delta k L/2)}{(\Delta k L/2)^2}$$

$$\Delta k = k_2 - 2k_1$$

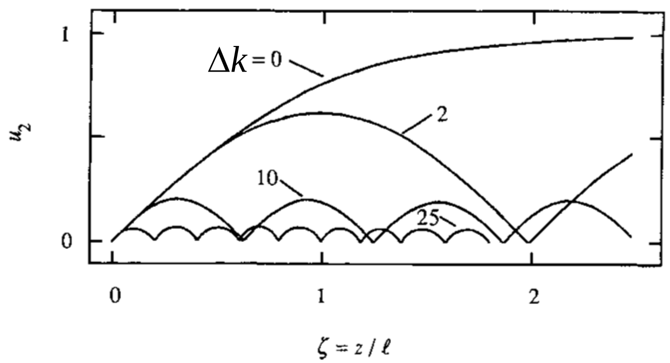


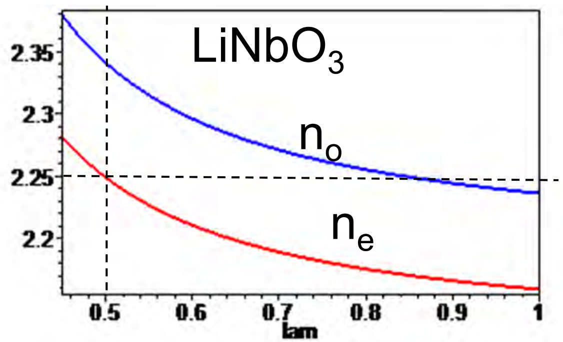
$$L_c = \pi/\Delta k$$

Coherence length:
SH is out-of-phase

Methods for phase matching

- In most crystals, due to dispersion of phase velocity, the phase matching can not be fulfilled.
  Therefore, efficient SHG can not be realised with long crystals.

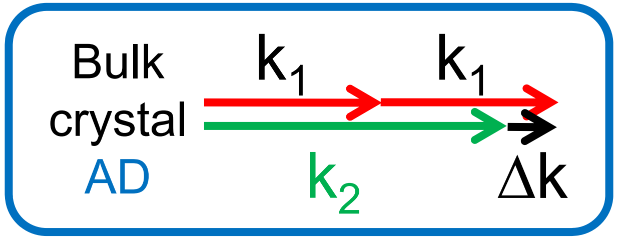


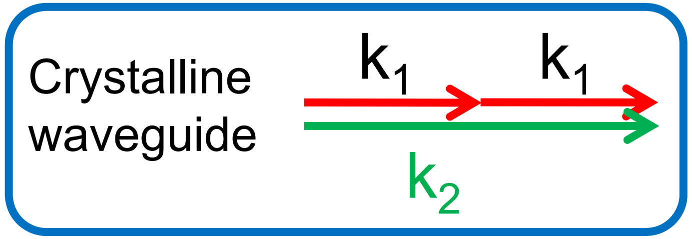


- Methods for achieving phase matching:
  - dielectric waveguide phase-matching (*difficult*)
  - non-colinear phase-matching
  - birefringent phase-matching
  - quasi phase-matching

Phase matching

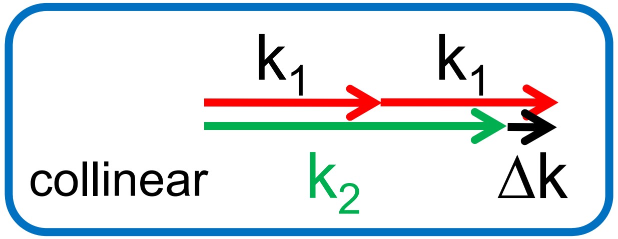
1. **Waveguide phase matching:** $n^{SH}_{eff}=n^{FF}_{eff}$; $n_{eff}=\beta/k_0$
   usually $n^{SH}_{eff}>n^{FF}_{eff}$ due to waveguide dispersion (*see slide 16/1*)

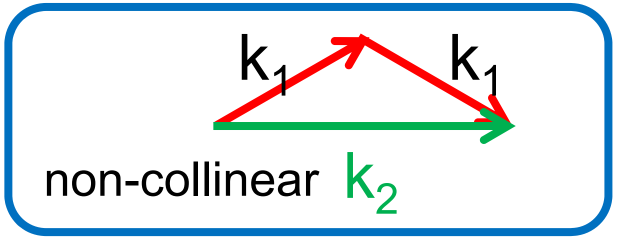




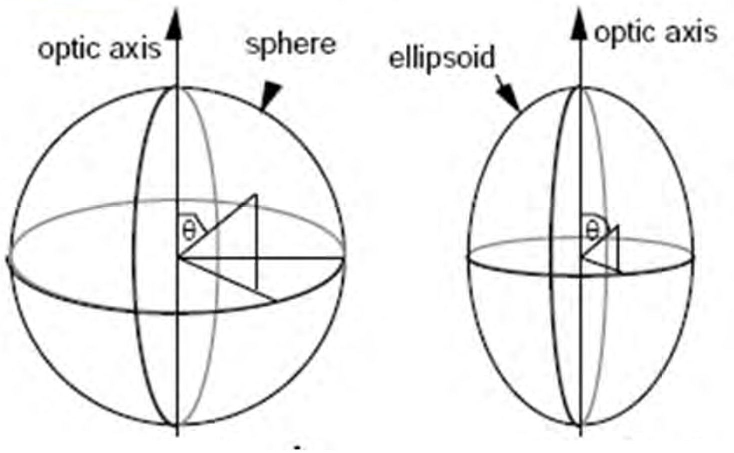
   Need to take care of the overlap of the modes of the FF and SH.

2. **Non-collinear phase matching:** (not suitable in waveguide geometry)

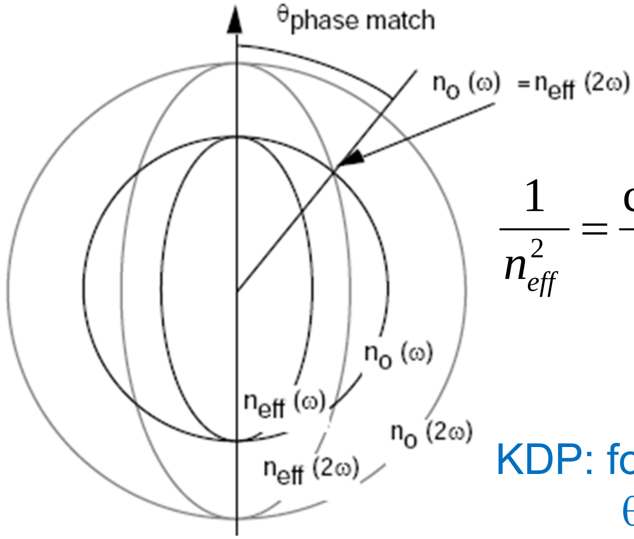




3. Birefringent phase-matching



o-ray e-ray ($n_e > n_o$)

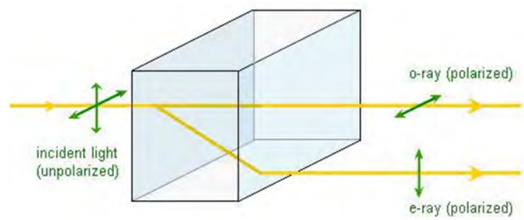


$$\frac{1}{n_{eff}^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

KDP: for FF@1064nm
θ = 42.4°

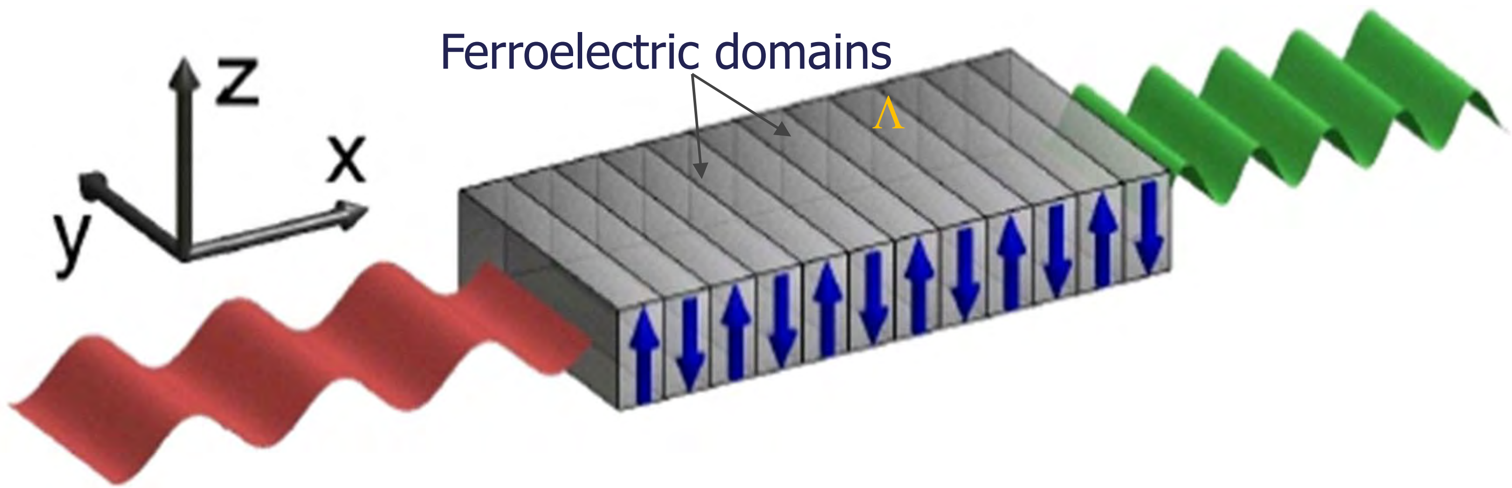
Can have different types of interactions: Type I: oo-e; Type II: oe-e

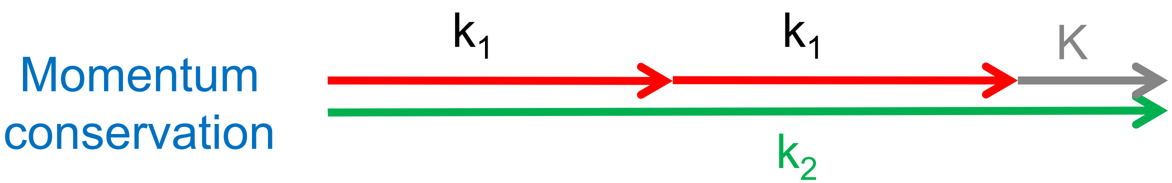
Problems: need of various materials/crystals:
birefringence (beams are not overlapping)



4. Quasi-phase matching

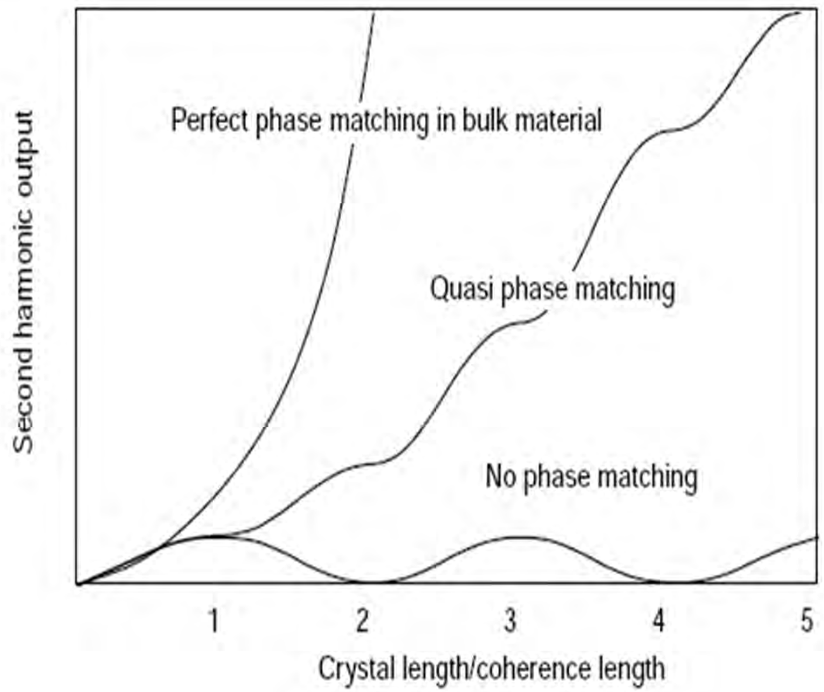
The ferroelectric domains are inverted at each L_c. Thus the phase relation between the pump and the second harmonic can be maintained.



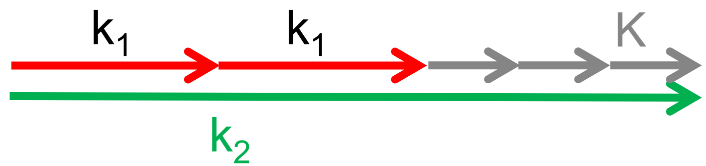


$k_1 + k_1 = k_2 + K$ $K = 2\pi/\Lambda$

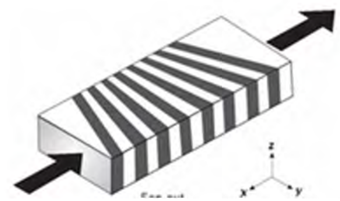
Quasi-phase matching: advantages



- Use any material
  smallest size Λ=4μm
- Multiple order phase-matching

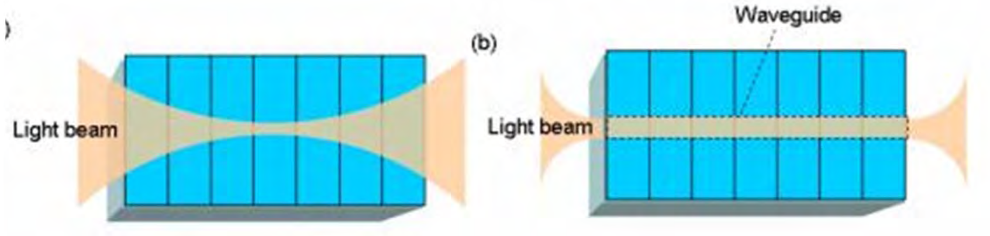


- Noncritical phase-matching
  propagation along the crystalline axes
- Complex geometries
  chirped or quasi-periodic poling for multi-wavelength or broadband conversion

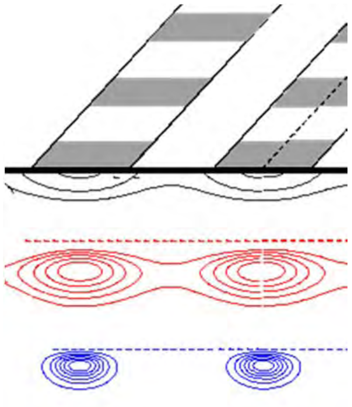


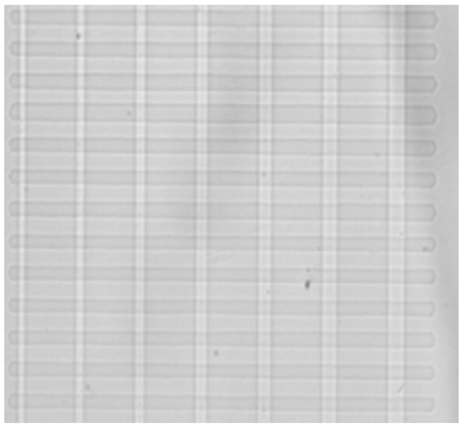
Waveguides for frequency conversion

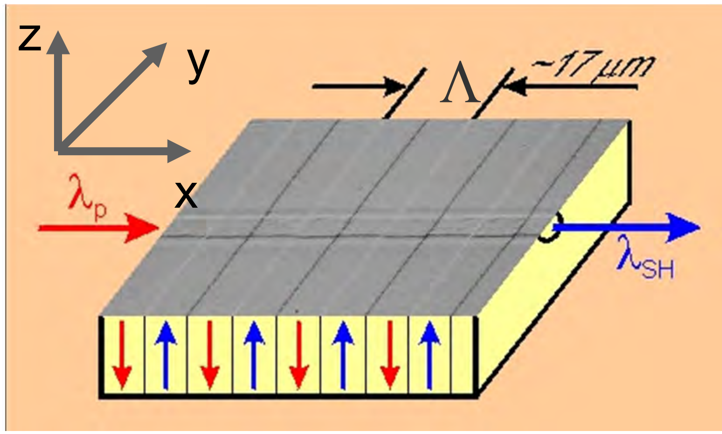
- Increased interaction length
  Can achieve >99% conversion of 1μW over 1cm.



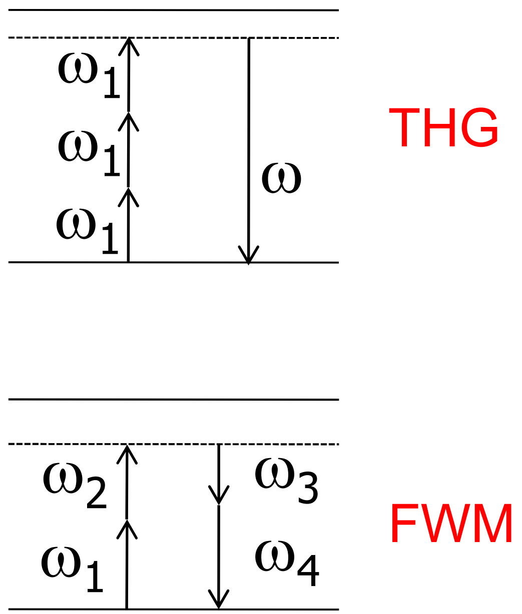
- Most commonly used crystals for poling are lithium niobate ($LiNbO_3$) and stoichiometric lithium tantalate (SLT).
- Periodically poled $LiNbO_3$ (PPLN) is most suitable for waveguides.







Four wave mixing (FWM)



- In isotropic materials, the lowers nonlinear term is the cubic $\chi^{(3)}$
- It also exist in crystalline materials.
- NL Polarization:

$$\mathbf{P}_{\mathrm{NL}} = \varepsilon_0 \chi^{(3)} \vdots \mathbf{EEE}$$

FWM: Description

- Four waves ω_1, ω_2, ω_3, ω_4, linearly polarised along x

$$\mathbf{E} = \frac{1}{2}\hat{x}\sum_{j=1}^{4} E_j \exp[i(k_j z - \omega_j t)] + \text{c.c.}$$ where $k_j = n_j\omega_j/c$ is the wavevector

$$\mathbf{P}_{\text{NL}} = \frac{1}{2}\hat{x}\sum_{j=1}^{4} P_j \exp[i(k_j z - \omega_j t)] + \text{c.c.}$$

SPM CPM

$$P_4 = \frac{3\varepsilon_0}{4}\chi^{(3)}_{xxxx}[|E_4|^2 E_4 + 2(|E_1|^2 + |E_2|^2 + |E_3|^2)E_4 + 2E_1E_2E_3\exp(i\theta_+) + 2E_1E_2E_3^*\exp(i\theta_-) + \cdots]$$

FWM-SFG FWM-DFG

$$\theta_+ = (k_1 + k_2 + k_3 - k_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t,$$
$$\theta_- = (k_1 + k_2 - k_3 - k_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t.$$

FWM- Phase matching

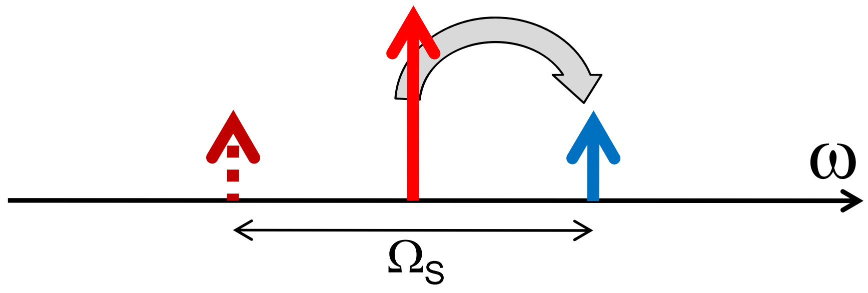
- Linear PM: $\Delta k = k_3 + k_4 - k_1 - k_2$
- However, due to the influence of SPM and CPM, Net phase mismatched: $\kappa = \Delta k + \gamma(P_1 + P_2)$

$$\gamma_j = n_2' \omega_j / (cA_{\text{eff}}) \approx \gamma.$$

- Phase matching depends on power.
- For the degenerate FWM: $\kappa = \Delta k + 2\gamma P_0$
- Coherence length: $L_{\text{coh}} = 2\pi / |\kappa|$

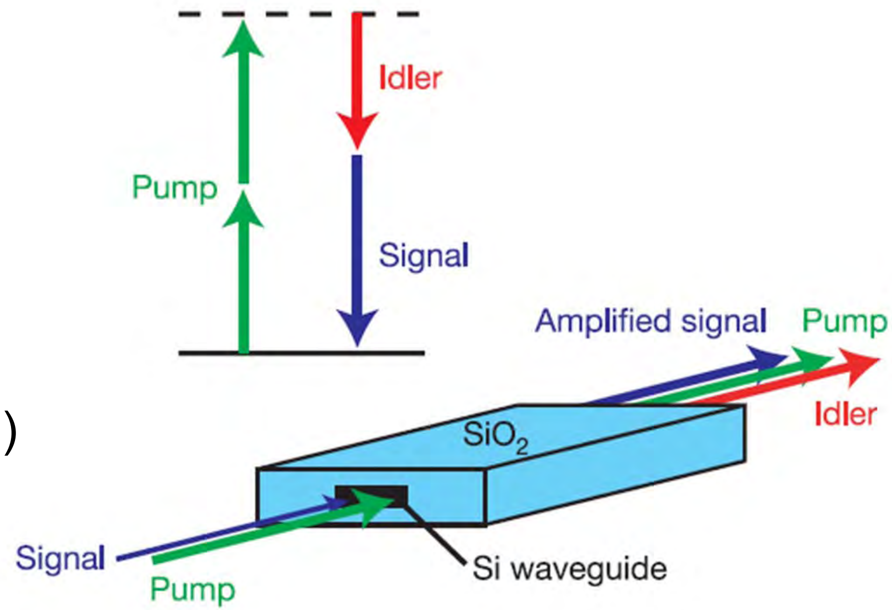
FWM applications

- Amplification:
- Twice the Raman ampl.



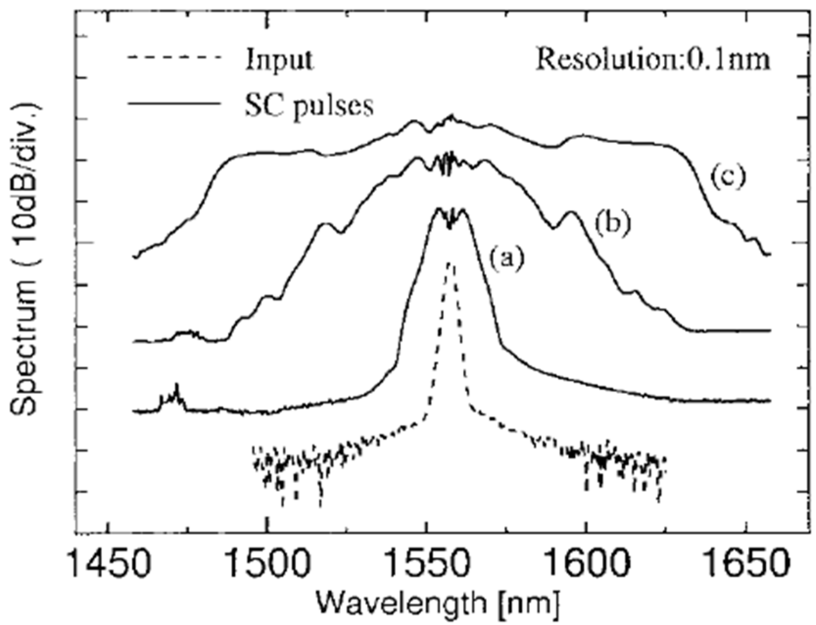
- Demonstrated broadband amplifier in Si nanowires (2006)

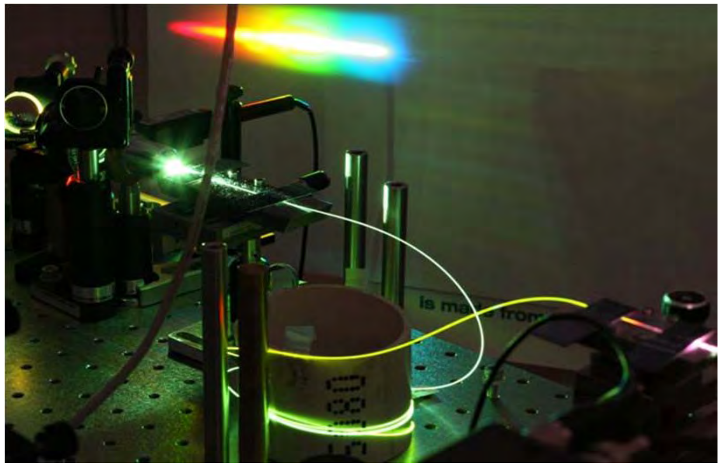
Foster, et al. Nature 441, 960 (2006)



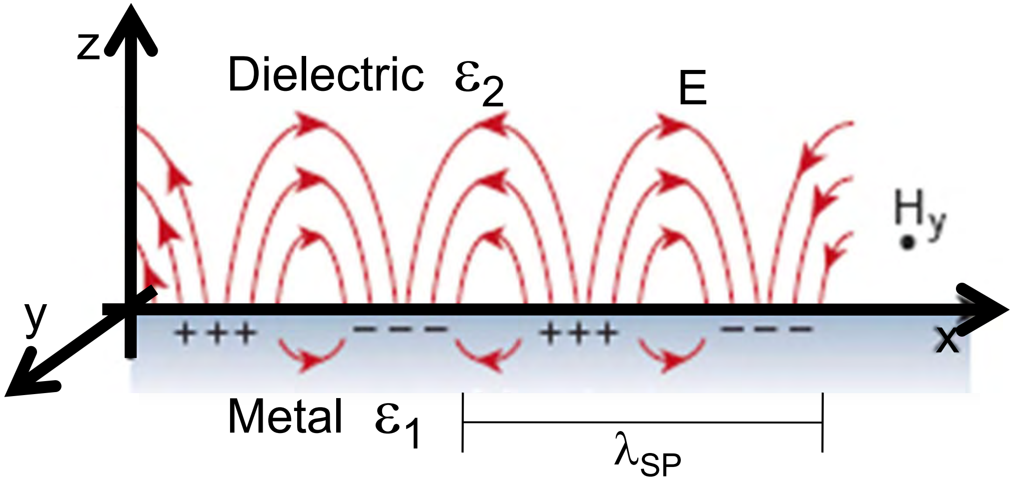
FWM applications

- Supercontinuum generation: Due to the combined processes of cascaded FWM, SRS, soliton formation, SPM, CPM, and dispersion





Metal-dielectric interface



$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon_1(\omega)\varepsilon_2}{\varepsilon_1(\omega)+\varepsilon_2}}$$

Dispersion relation for TM waves

Boundary conditions TM (p) wave

$$H_{y1}=H_{y2}$$

$$\varepsilon_1 E_{z1}=\varepsilon_2 E_{z2}$$

$$z>0: H = A_1 e^{i\beta x} e^{-k_2 z}$$

$$z<0: H = A_2 e^{i\beta x} e^{k_1 z}$$

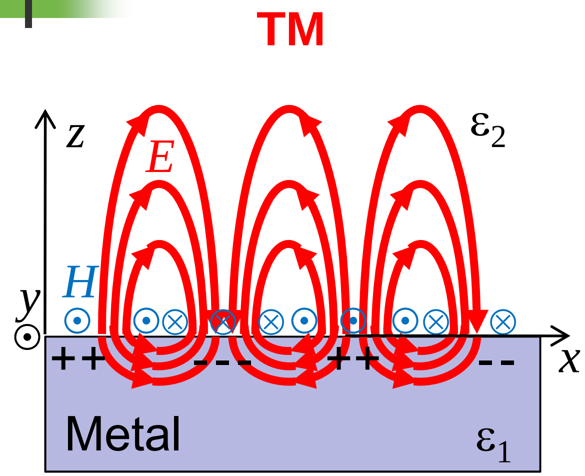
$$\Rightarrow \frac{k_2}{k_1} = -\frac{\varepsilon_2}{\varepsilon_1}$$

TM equation

$$\frac{\partial^2 H_y}{\partial z^2} + (k_0^2\varepsilon - \beta^2) H_y = 0$$

Im(β) defines the propagation;
k_1 and k_2 define the penetration

SPP at metal-dielectric interface



Boundary conditions

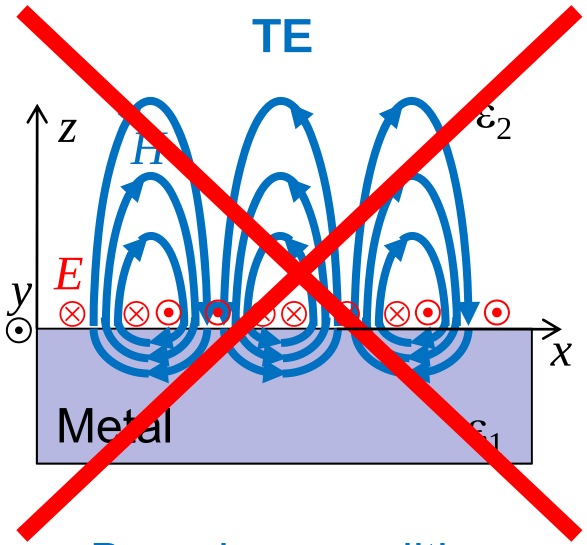
E_x – discontinuous

(electric charges)

$H_{y1}=H_{y2}$

$\varepsilon_1 E_{z1}=\varepsilon_2 E_{z2}$

$\varepsilon_1 < 0$



Boundary conditions

H – continuous

(no magnetic charges)

but sign(H_{y1})=-sign(H_{y2})

No **TE** Surface plasmon polaritons

Plasmonics

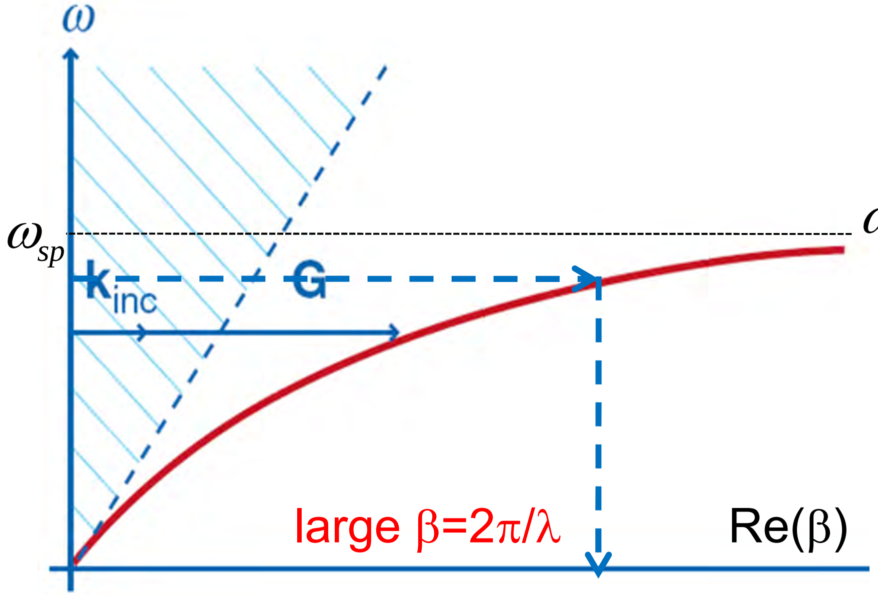
Plasmon: quantum of plasma oscillations
Photon
Surface plasmon polariton

Lycurgus cup: made by Roman glass blowers, 4th century AD

Reflection

Transmission

Dispersion relation of SPP



$$\omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_2}}$$ Surface plasmon frequency

$$\ell_{SP} = \frac{1}{2\,\mathrm{Im}(k_{SP})} = \frac{2}{[\mathrm{Re}(k_{SP})]^3}\frac{[2\pi \mathrm{Re}(\varepsilon_m)]^2}{\lambda^2 \mathrm{Im}(\varepsilon_m)}$$

1. Large wavevector, short λ: Optical frequencies, X-ray wavelengths. Sub-wavelength resolution!

2. The maximum propagation and maximum confinement lie on opposite ends of Dispersion Curve

Example:
air-silver interface

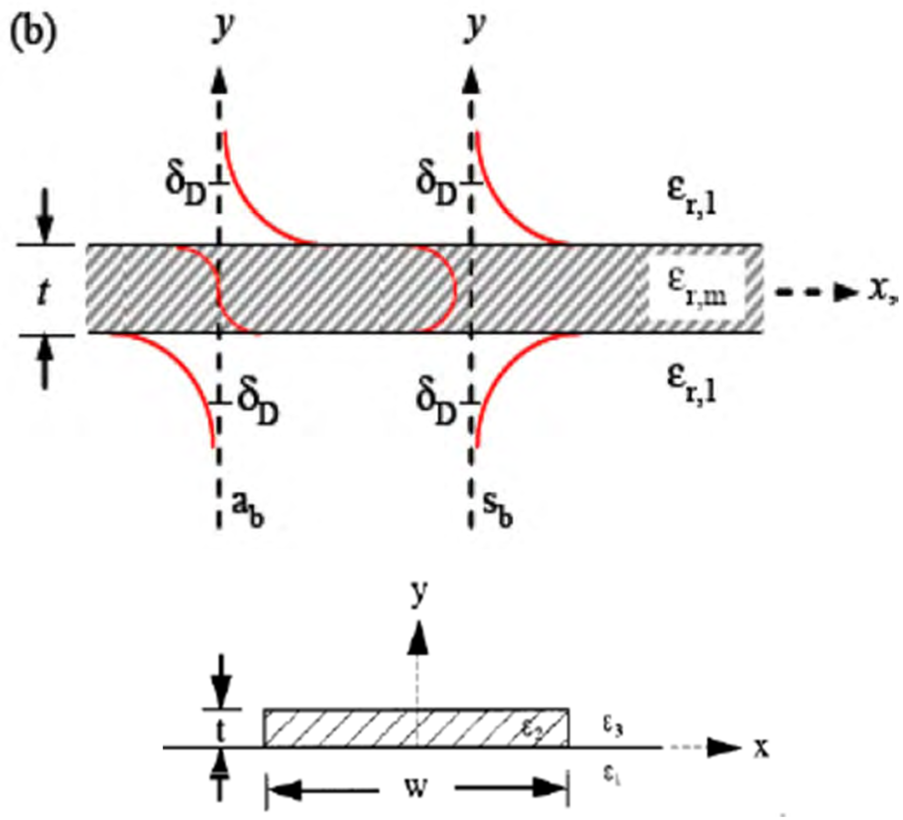
$\lambda_0 = 450nm$ $L \approx 16\ \mu m$ and $z \approx 180$ nm.

$\lambda_0 = 1.5\mu m$ $L \approx 1080\ \mu m$ and $z \approx 2.6\ \mu m$.

$z \approx 20$ nm metal

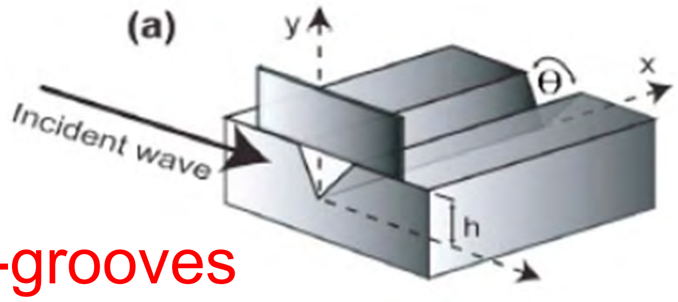
SPP waveguides

- – SPPs at either surface couple giving symmetric and anti-symmetric modes
- – Symmetric mode pushes light out of metal: lower loss
- – Anti-symmetric mode puts light in and close to metal, higher loss
- Metal strips: Attenuation falls super-fast with *t*, so does confinement

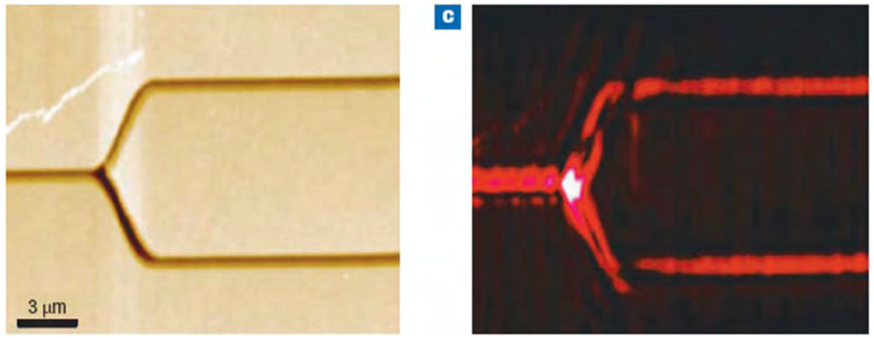


Plasmonic waveguides

- To counteract the losses while keeping strong confinement (100nm), new designs are explored:

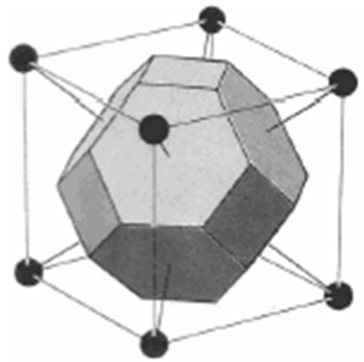


V-grooves

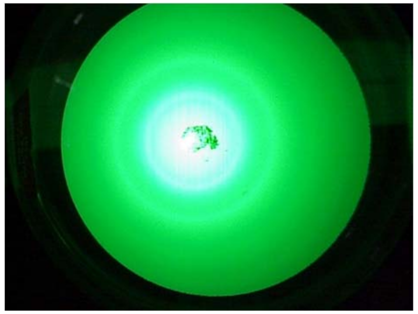


Slot-waveguides

Electrons in crystals: history

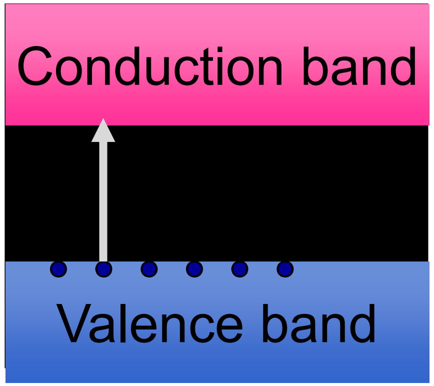


Crystals: A regular periodic array of atoms, molecules, or ions



Thomson & Davisson (1937)
Nobel prize – Discovery of diffraction of electrons

Semiconductors: Solid crystalline materials with special conducting properties



Crystal triode
The transistor
Bell Labs, Dec. 1947

Microelectronics

Braggs vs. Resonant Reflection

W.H. Bragg

W.L. Bragg

born in 1890 in Adelaide

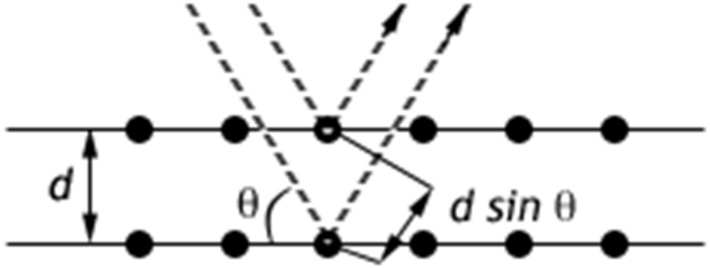
(Nobel Prize in Physics 1915)

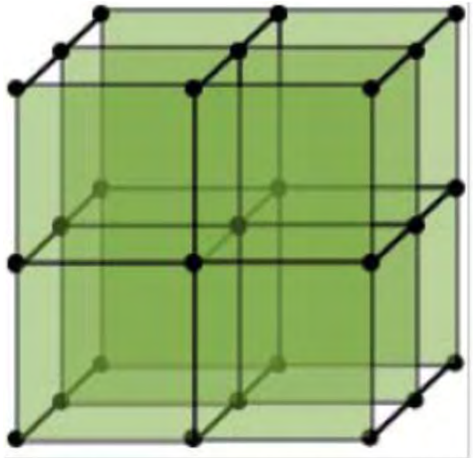
WILLIAM LAWRENCE BRAGG

The diffraction of X-rays by crystals

*Nobel Lecture, September 6, 1922**

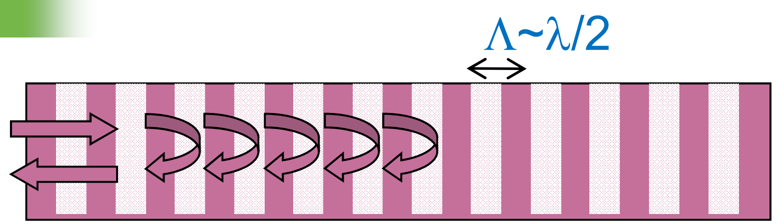
$$2d\sin\theta = n\lambda$$





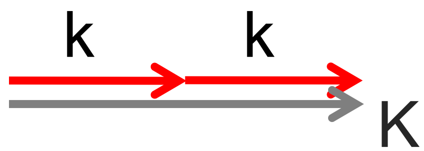
1a) Normal Plane NaCl Crystal.

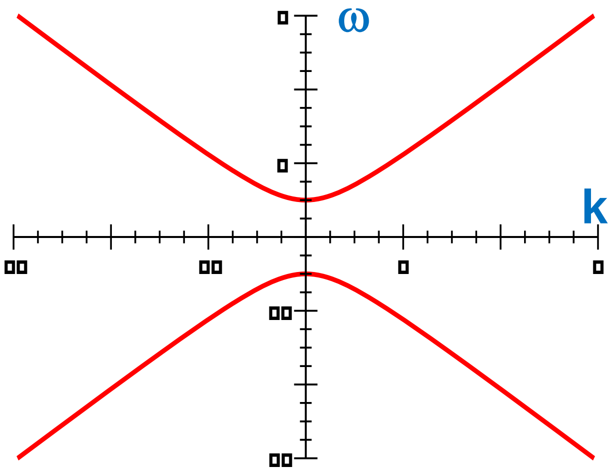
1D PC: Bragg grating

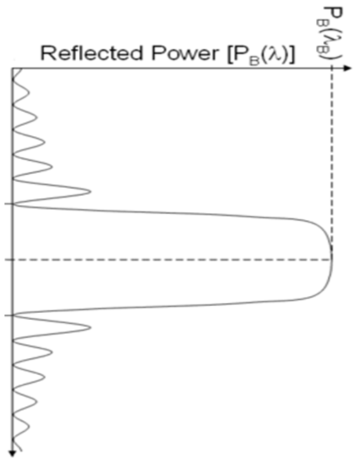


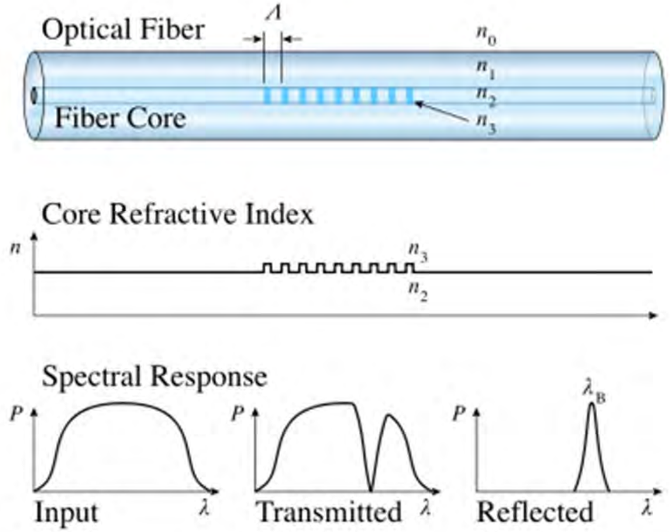
Bragg condition: $\lambda_B = 2n\,\Lambda/m$,
where $n = (n_1 + n_2)/2$

m=1



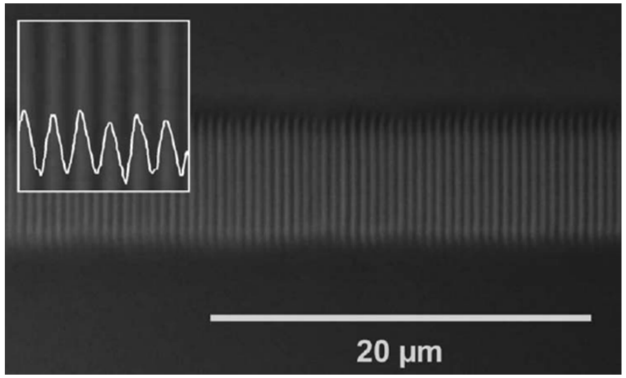






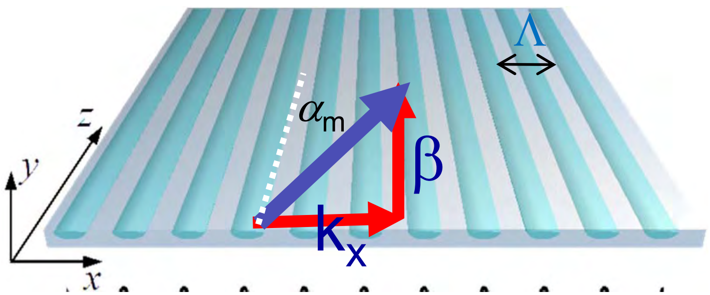
The reflections from the periodic layers results in a formation of a photonic bandgap

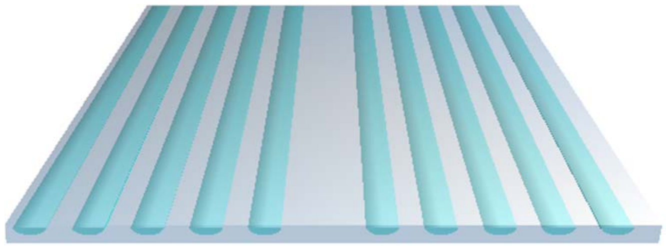
Bragg grating



Bragg grating in a waveguide written in glass by direct laser writing
MQ University (2008)

1D photonic crystals



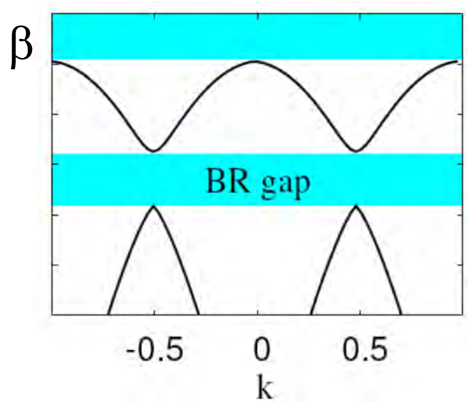


Bragg condition

$$\lambda_B = \Lambda \sin \alpha_m / m$$

- Period~5μm
- Δn~0.5

In PCFs the Bragg reflections are realised for small angles and light propagates along z axis freely. The reflection is negligible.

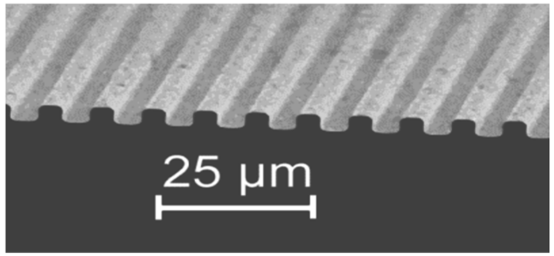


Bragg reflection gap, where waves are reflected.

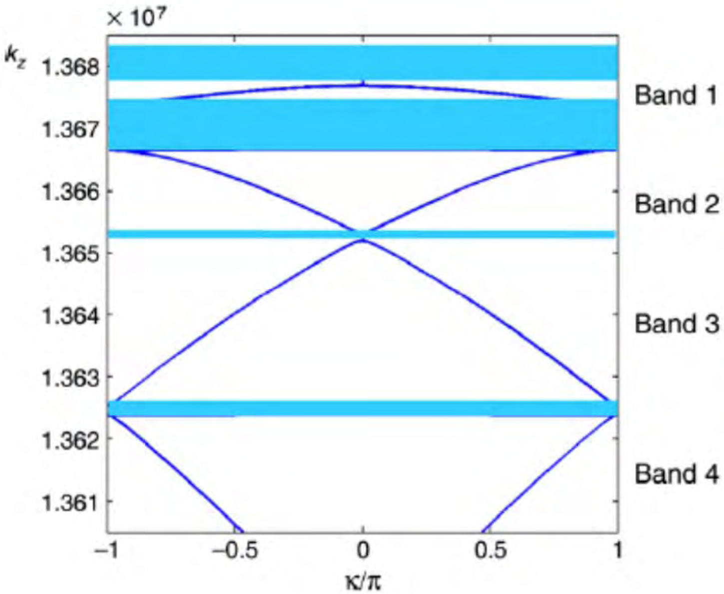
A defect, where waves with certain propagation constant can propagate, but they are reflected by the surrounded by two Bragg reflectors.

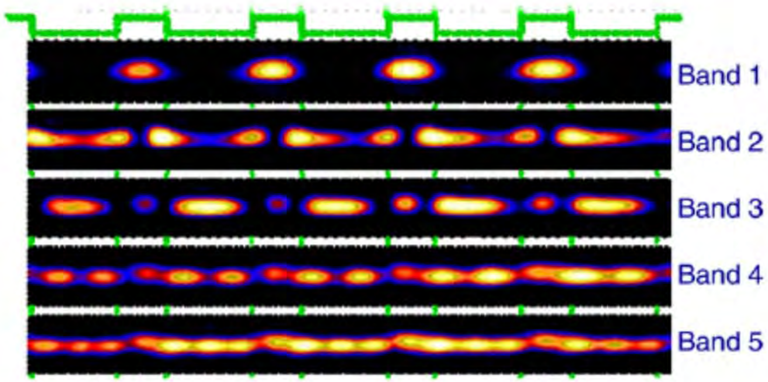
Band structure of a continuous lattice

$$i\frac{\partial E}{\partial z}+\frac{1}{2k}\nabla_{\perp}^{2}E+V(x)E=0$$



$$V(x+D)=V(x)$$





Linear waveguide arrays

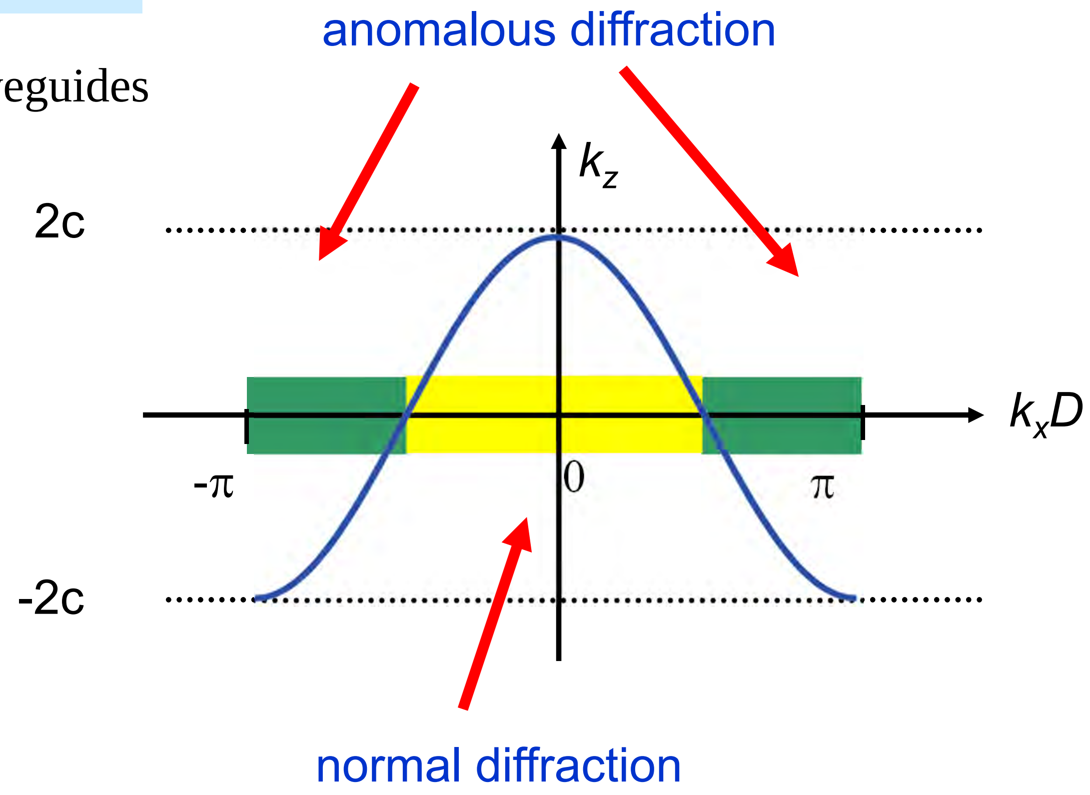
Dispersion relation

$$k_z = 2c\cos(k_x D)$$

$$i\frac{dE_n}{dz} + c(E_{n+1} + E_{n-1}) = 0$$

D: distance between waveguides

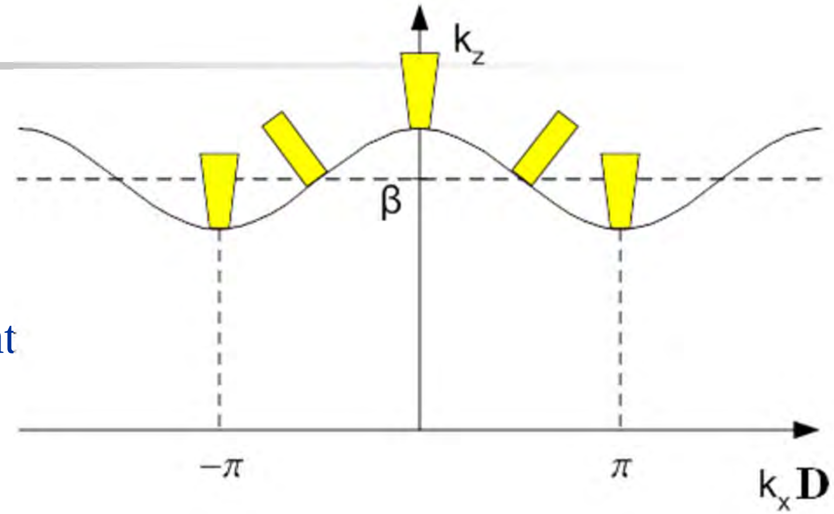
First Brillouin zone



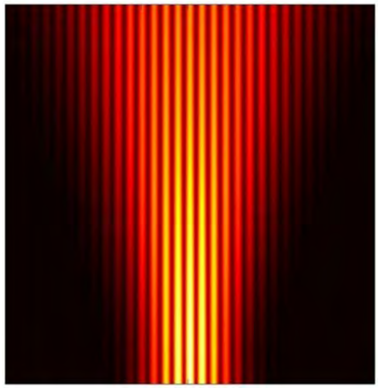
Waveguide Array Diffraction

Assuming a discrete Floquet-Bloch function) : $a_n = \exp[i(k_z z + nk_x D)]$

$$k_z = 2c \cdot \cos(k_x D)$$

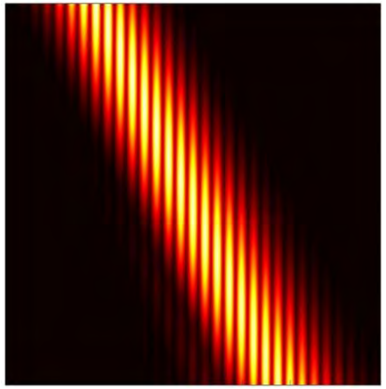


- Relative phase difference between adjacent waveguides determines discrete diffraction
- Dispersion relation is periodic



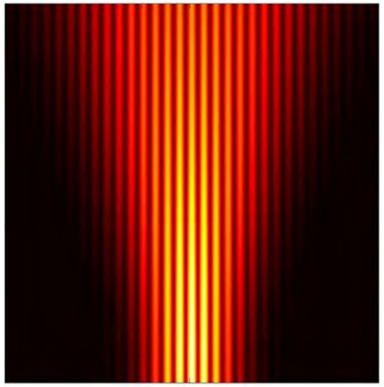
$k_x D = 0$

normal diffraction



$k_x D = \pi/2$

zero diffraction

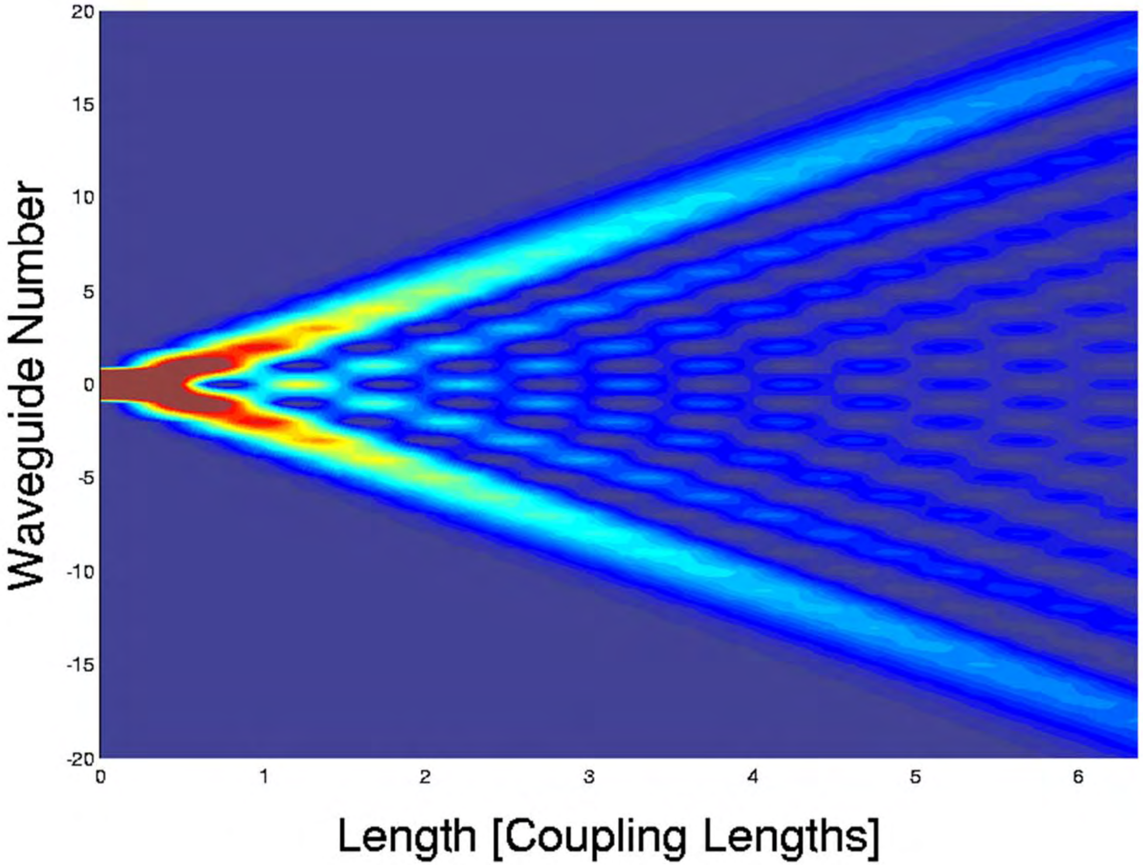


$k_x D = \pi$

anomalous diffraction

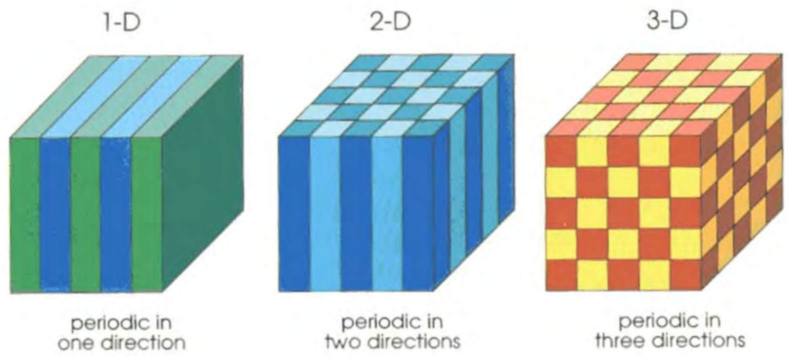
Discrete diffraction

$$E_n = A_0 (i)^n J_n(2cz)\exp(i\beta z)$$

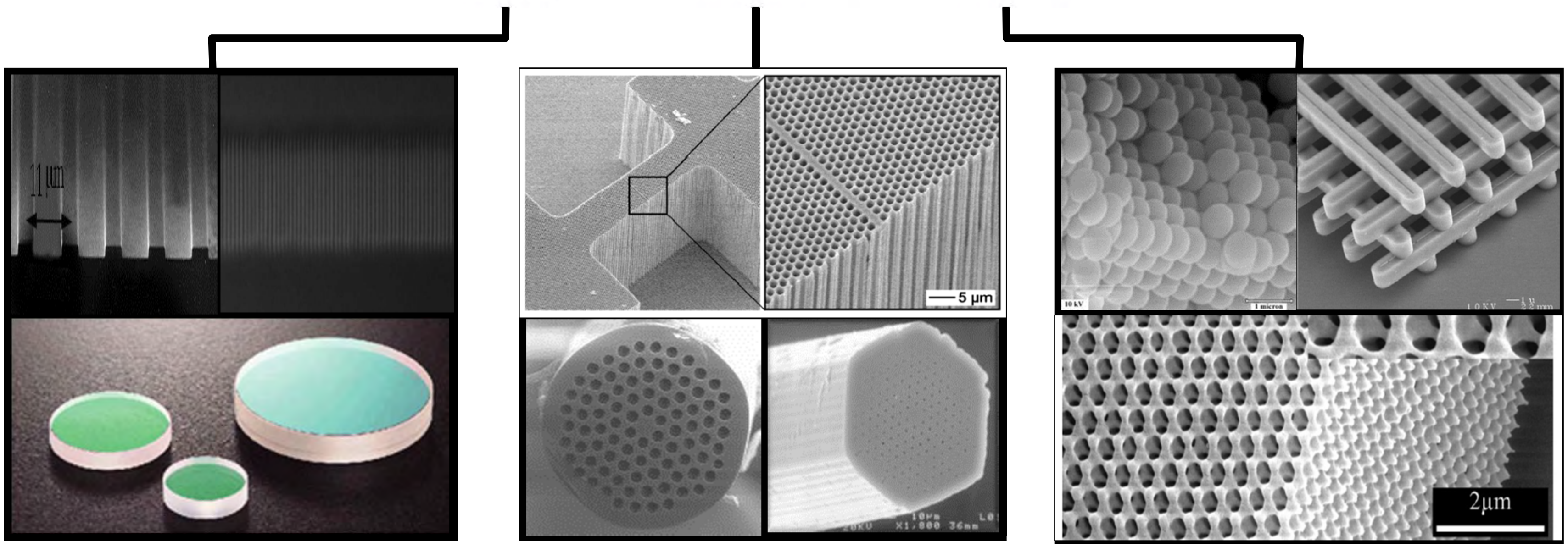


Photonic Crystals: examples

Bragg: 1915
Nobel prise -
X-ray diffraction



PRL 58 (1987):
Sajeev John;
Eli Yablonovitch



Manipulation of light in direction of periodicity: dispersion, diffraction, emission