The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2291-15

Joint ICTP-IAEA Course on Science and Technology of Supercritical Water Cooled Reactors

27 June - 1 July, 2011

INTRODUCTION TO REACTOR PHYSICS

Prepared by

D. HUMMEL and David NOVOG

McMaster University Faculty of Engineering
Department of Engineering Physics, 1280 Main Street West
John Hodgins Engineering Building
Hamilton L8S4L7 Ontario
CANADA

Introduction to Reactor Physics

Prepared by D. Hummel

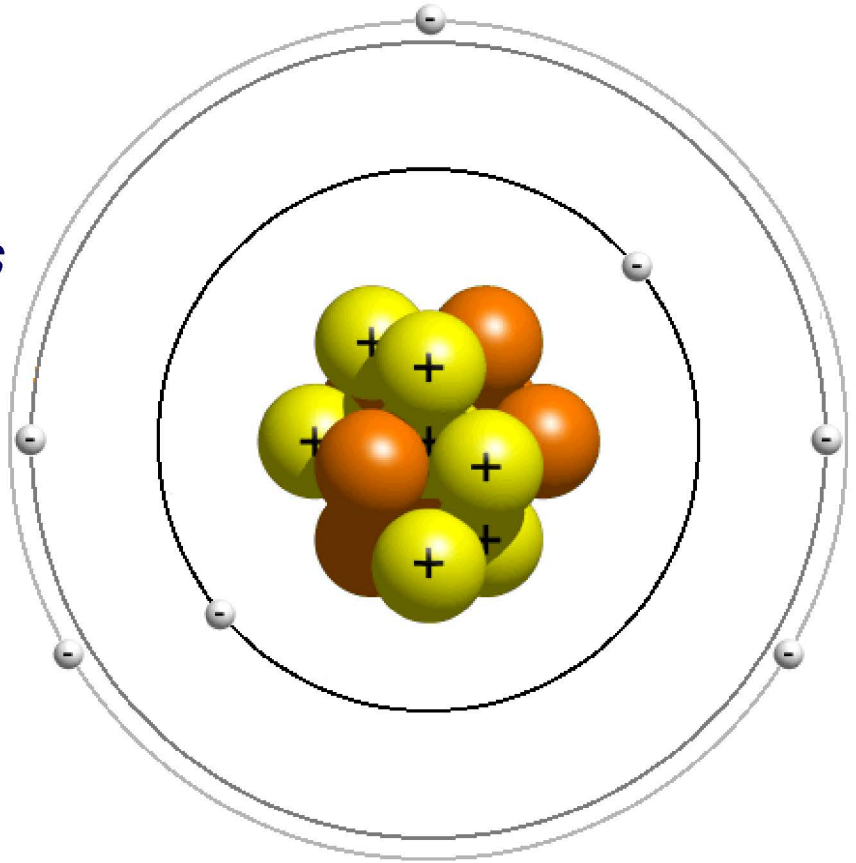
David Novog (Ph.D., P.Eng.)

Associate Professor

McMaster University (Canada)

The Atom

- Nucleus consists of:
  - *protons* (positive charge)
  - *neutrons* (no charge)
- Surrounded by cloud of *electrons*
  - negative charge
  - $m_e << m_p, m_n$
- # of protons (Z) define the *element*
- # of protons + # of neutrons (N) define the *isotope* (A)
- Notation: ${}^{A}_{Z}X$ ⟵ chemical symbol



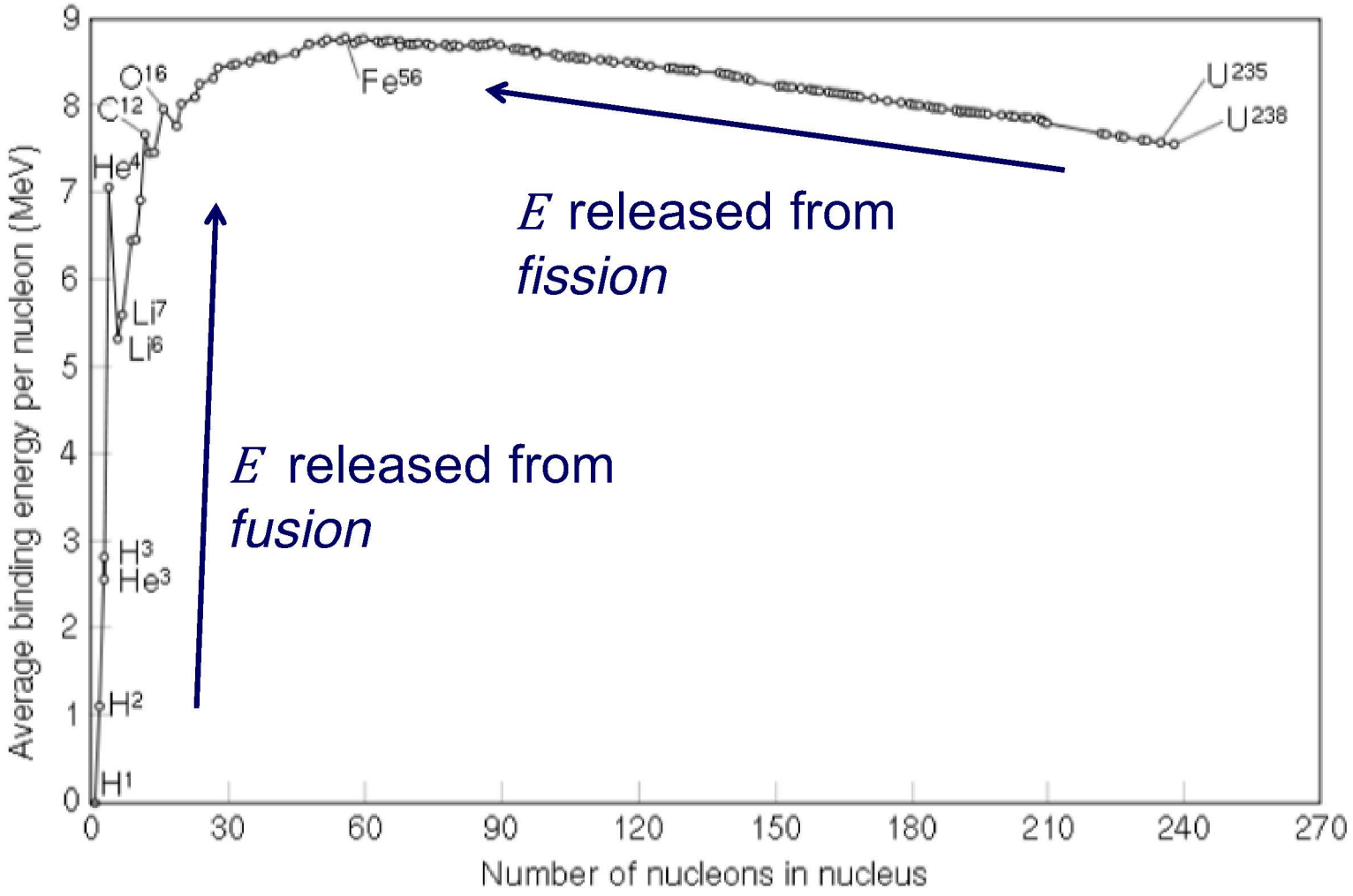
The Mass Deficit

- The observed mass of a nucleus is smaller than the sum of its parts:

$$\Delta = \left[Z m_p + (A - Z) m_n \right] - {}^{A}m_Z$$

- The mass deficit (Δ) has an equivalent energy (from $E = mc^2$) called the *binding energy* ($B = \Delta c^2$)
- Nuclear reactions that result in a net *release* of energy (B) include:
  - *fusion* of two small nuclei
  - ***fission*** of a large nucleus

Nuclear Binding Energy



Original Image Source: Adriaan Buijs, *EP704 Advanced Reactor Physics*, Course Notes, McMaster University, 2009.

Induced Fission

- It is possible for a heavy nucleus to fission on its own, but it is very rare (low probability of occurrence)
- Many elements fission readily when the nucleus absorbs an additional neutron
- Classify these materials as:
  - ***fissile*** : fissions readily with a low energy neutron e.g.:

$$^{233}_{92}U \quad ^{235}_{92}U \quad ^{239}_{94}Pu$$

  - *fissionable* : fissions with a high energy neutron e.g.:

$$^{238}_{92}U$$

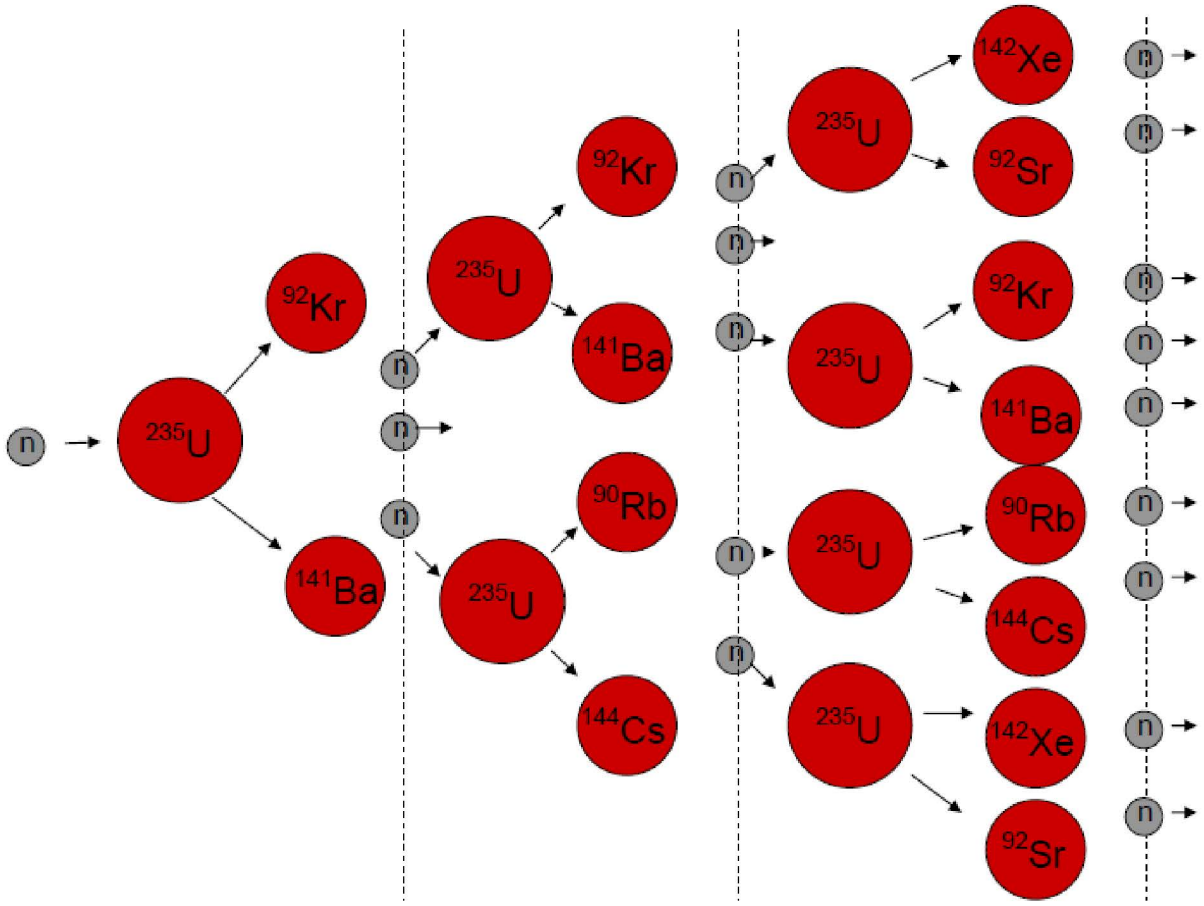
  - *fertile* : absorbs a neutron to become a fissile material e.g.:

$$^{232}_{90}Th \rightarrow ^{233}_{92}U \qquad ^{238}_{92}U \rightarrow ^{239}_{94}Pu$$

Neutron-Nucleus Interactions

- A neutron may undergo several different reactions with a nucleus, including:
  - *Scattering* (elastic or inelastic): there is a transfer of energy between the neutron and the nucleus
  - *Absorption*: the neutron is absorbed into the nucleus and lost
  - ***Fission***: the neutron causes the nucleus to fission, *releasing additional neutrons and fission products*
- The likelihood of an interaction occurring is represented with a microscopic *cross section* (σ) [1 barn (b) = 10^{-28} m^2]
  - Dependent on the *isotope* of the interacting material (and its temperature)
  - Dependent on the incident *neutron energy*

Fission Chain Reaction



Energy Dependence

- Neutron energies cover ~10 orders of magnitude:
  - Fission spectrum
  - Delayed spectrum
  - Moderation
- Interaction cross sections may change by 5+ orders of magnitude over this range of energy.
- Need to solve neutron evolution over these ranges.
- OPTIONS →
  - Full transport (absorption, moderation, fission) at each “point” in the reactor (i.e., continuous energy solution).
  - Transport over “Groupwise” energies (i.e., “multi-group cross sections).
  - Diffusion over “Few” groups (i.e., 2-group diffusion solvers such as SCALE)
  - What is the relative calculation times for these approaches?

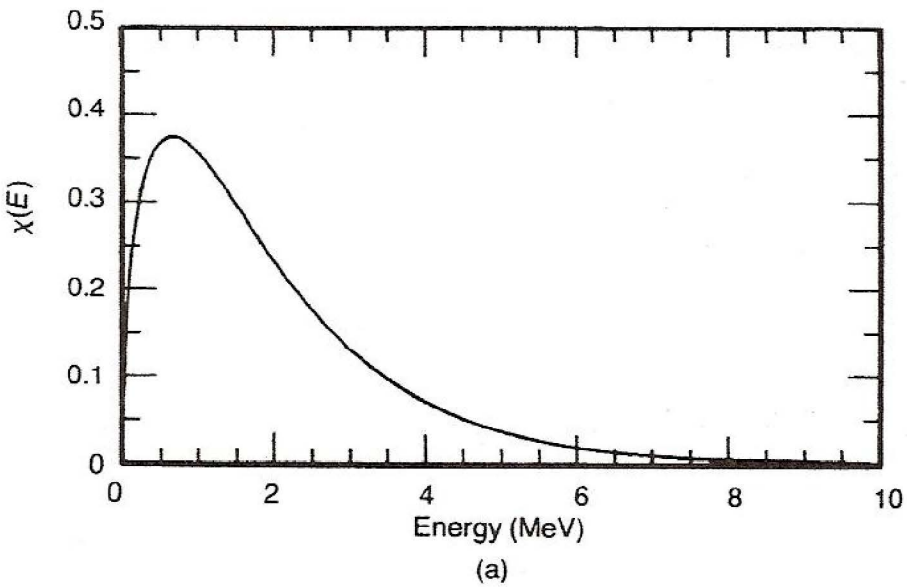
The Fission-Neutron-Energy Range

- the energies range up to several MeV, with a maximum around 0.7 MeV.
- The fission-neutron spectrum has the form

$$\chi(E) = 0.453 e^{-1.036E} \sinh\sqrt{2.29E} \quad (1)$$

where E is in MeV

(Note: this is a distribution in number of neutrons, not flux)

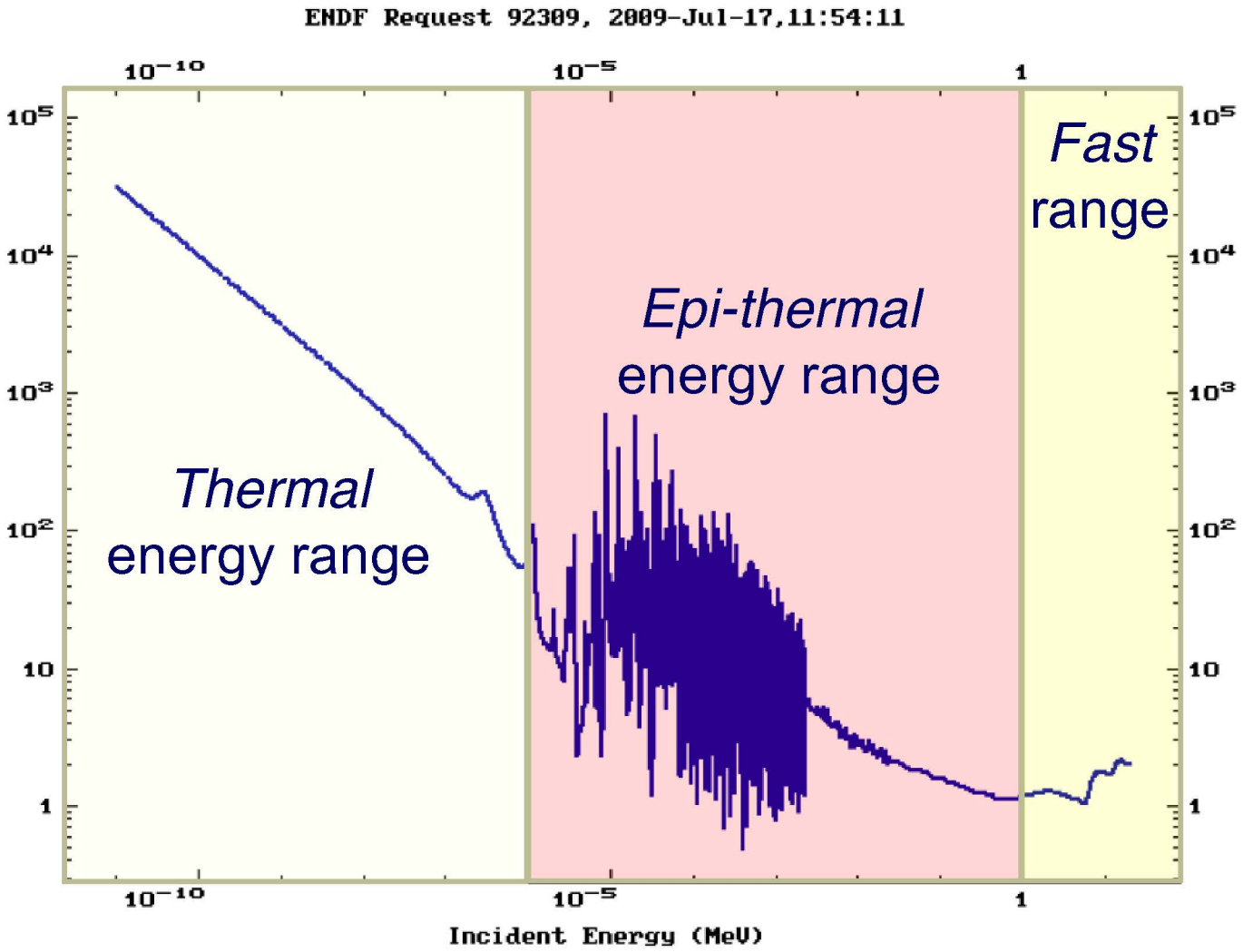


Energy Distribution of Fission Neutrons

Neutron Energies

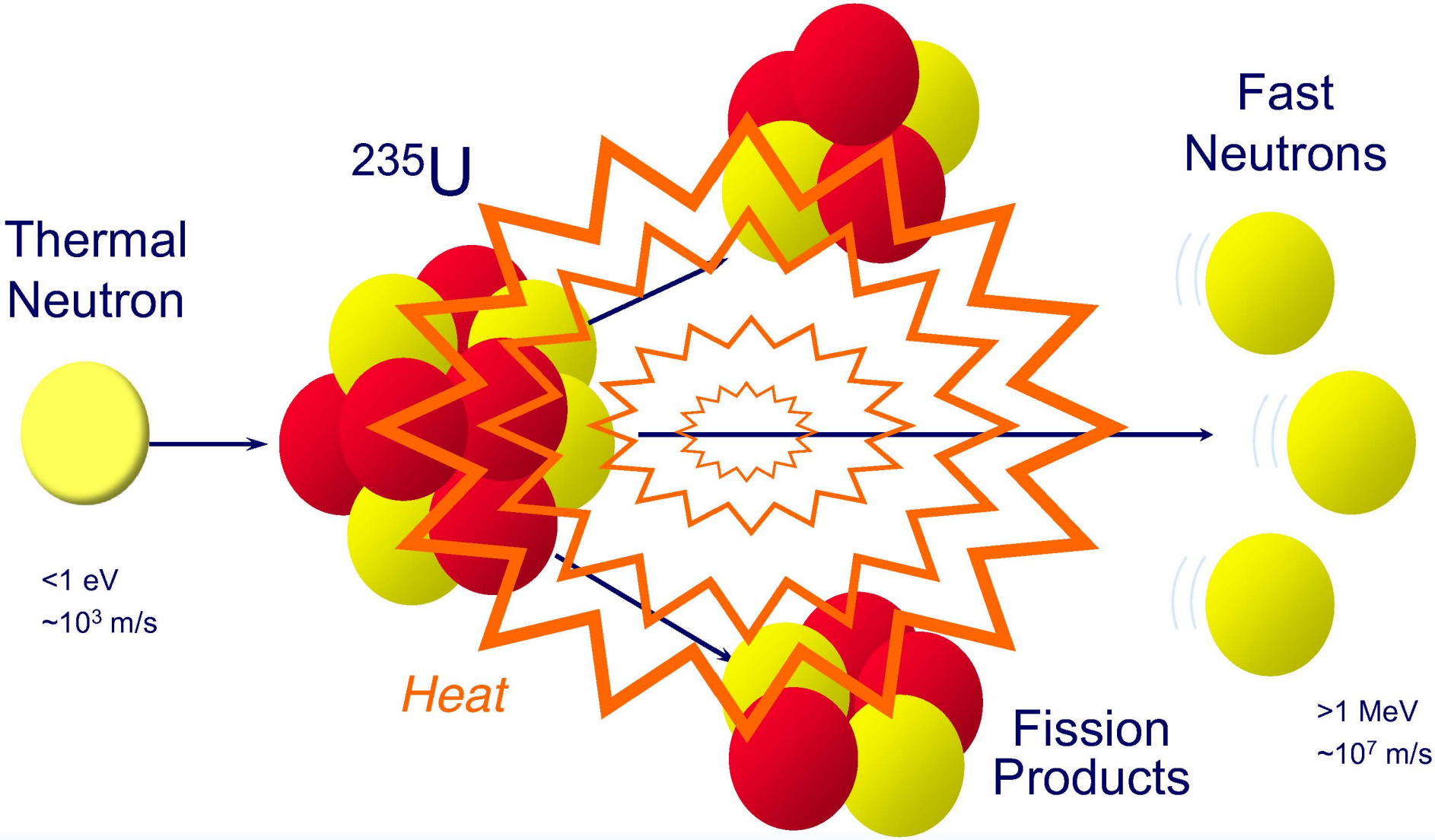
- Neutronic energy distribution can be classified as:
  - the fission spectrum at energies above about 50-100 keV
  - the slowing-down spectrum to about 1 eV
  - the Maxwellian spectrum at thermal energies, below about 1 eV

^{235}U Fission Cross Section



Original Image Source: Adriaan Buijs, *EP704 Advanced Reactor Physics*, Course Notes, McMaster University, 2009.

Thermal Fission of ^{235}U

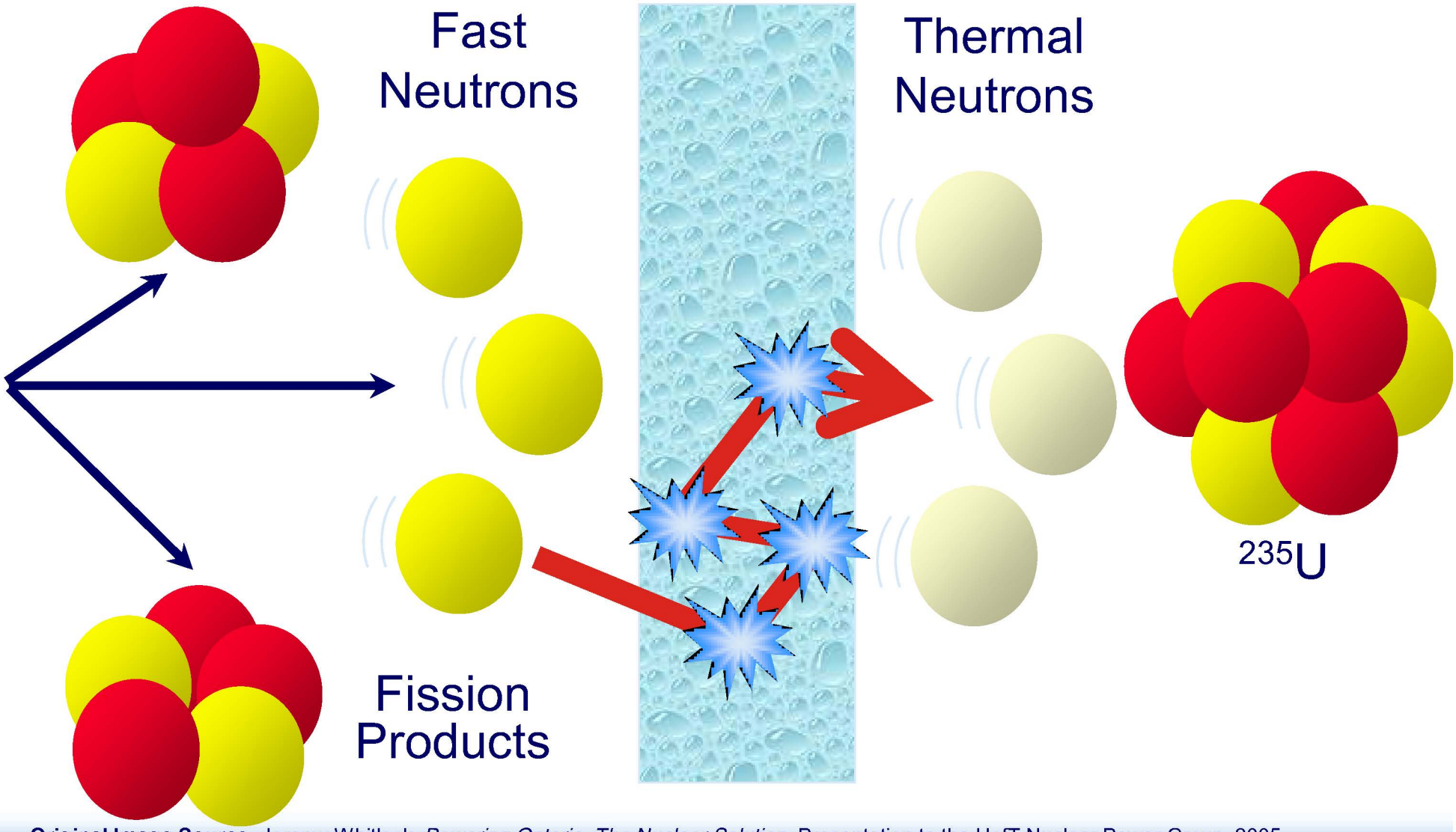


Original Image Source: Jeremy Whitlock, *Powering Ontario: The Nuclear Solution*, Presentation to the UofT Nuclear Power Group, 2005.

Neutron Moderation
A KEY CONSIDERATION FOR SCWR

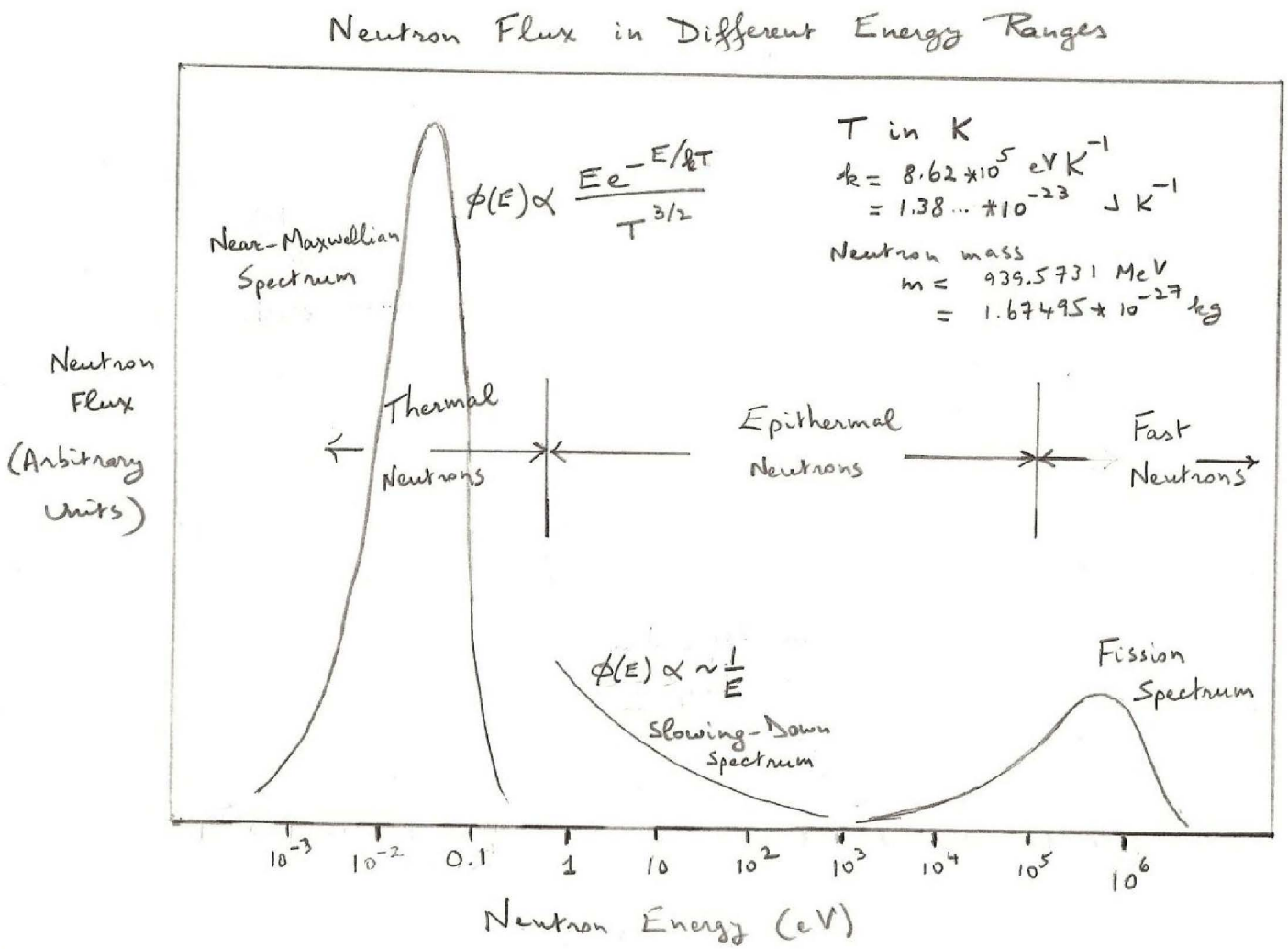
- Most neutrons born from fission are in the fast range (high energy)
- To sustain a fission chain reaction, the fast neutrons must be brought down to a lower energy (where $\sigma_{fission}$ is higher) via interaction with a *moderator*
  - *Thermal reactor*
- Neutrons transfer their "excess" energy to the moderator through series of scattering interactions / collisions
- Good moderators have:
  - Low absorption cross sections $\sigma_{absorption}$
  - Low atomic masses (to maximize ΔE in a single interaction)

Neutron Moderation in Water



Original Image Source: Jeremy Whitlock, *Powering Ontario: The Nuclear Solution*, Presentation to the UofT Nuclear Power Group, 2005.

Full Neutron Energy Spectrum



Criticality

- The reactor is *critical* when the number of neutrons produced in each generation is equal to the number lost
- The *multiplication factor* is defined as:

$$k \equiv \frac{rate\ of\ neutron\ production}{rate\ of\ neutron\ loss}$$

  - $k < 1$: the reactor is *subcritical*
  - $k = 1$: the reactor is *critical*
  - $k > 1$: the reactor is *supercritical*

Prompt and Delayed Neutrons

- Neutrons that are released immediately after the fission occurs are referred to as ***prompt*** neutrons
- Most fission products are unstable nuclei that undergo *radioactive decay*
- Following radioactive decay, some daughter nuclei may have sufficient energy to release additional neutrons called ***delayed*** *neutrons*
  - Time constants for release of delayed neutrons are dominated by the half life of the unstable fission product
- Delayed neutrons must be included in analysis

Importance of Delayed Neutrons

- Direct fission neutrons have a “lifetime”
  - Neutrons born in fission interact with
    - moderator (scatter/absorption)
    - core materials (absorption).
    - Fast fission materials
  - Their “lifetime” is very short.
  - Control of such a system is very difficult (mechanical and I&C systems cannot respond on this timeframe).
- Delayed neutrons have time scales much longer (order of seconds).
  - A thermal reactor is designed such that the reactor is slightly “subcritical” based on direct fission neutrons alone.
  - The delayed neutrons provide the remaining neutrons to make the core critical.
- Therefore control of the reactor can be achieved through changes in the delayed neutron absorption.

Reactivity

- Define *reactivity* as the relative distance from criticality:

$$\rho = 1 - \frac{1}{k}$$

  - $\rho < 0$: the reactor is *subcritical*
  - $\rho = 0$: the reactor is *critical*
  - $\rho > 0$: the reactor is *supercritical*
- Units of reactivity [$1\ \text{mk} = 0.001$, or $1\ \text{pcm} = 0.01\ \text{mk}$] are typically viewed as being *added* or *removed* from the reactor

Reactor Physics

The goal of reactor physics calculations is to track neutrons as they evolve in *space*, *energy* and *time*.

This allows predictions of power, radiation levels, decay heat…. etc..

- Fundamental assumptions of most reactor physics analysis:
  - *The average behaviour of neutrons is descriptive*
  - *Neutrons do not interact with one another*

4 Factor Formula

- To study the fission process, many physical features can be understood by examining the 4-factor formula:

$$k_\infty = \varepsilon p f \eta$$

- Which can be derived form the diffusion equation.

$$\varepsilon = \frac{\nu\Sigma_{f1}\Phi_1 + \nu\Sigma_{f2}\Phi_2}{\nu\Sigma_{f2}\Phi_2} = \frac{total\ \ fission\ rate}{thermal\ \ fission\ rate}$$

$$f = \frac{\Sigma_{a2}^{fuel}\Phi_2}{\Sigma_{a,}\Phi_,} = \frac{rate\ of\ \ thermal\ \ absorption\ \ in\ \ fuel}{total\ \ rate\ of\ \ thermal\ \ absorption\ s}$$

$$p = \frac{\Sigma_{a2}\Phi_2}{\Sigma_{a1}\Phi_1 + \Sigma_{s1\to 2}\Phi_1} = \frac{\Sigma_{s1\to 2}\Phi_2}{\Sigma_{a1}\Phi_1 + \Sigma_{s1\to 2}\Phi_1}$$

$$= \frac{rate\ of\ \ slowing\ down}{rate\ of\ \ slowing\ down + absorptions}$$

$$\eta = \frac{\nu\Sigma_{f2}\Phi_2}{\Sigma_{a2}^{fuel}\Phi_2}$$

$$= \frac{rate\ of\ neutron\ production\ through\ thermal\ \ fission}{rate\ of\ thermal\ absorption}$$

Angular Neutron Flux Density

- Fundamental quantity is the *angular neutron flux density* in $\Phi(r, E, \vec{\Omega}, t)$
  - Space (position r)
  - Energy (E)
  - Direction (solid angle Ω)
  - Time (t)
- Also expressible in terms of the *neutron density*

$$\Phi(r, E, \vec{\Omega}, t) = v\, n(r, E, \vec{\Omega}, t)$$

neutron velocity $v = \sqrt{\frac{2E}{m_n}}$

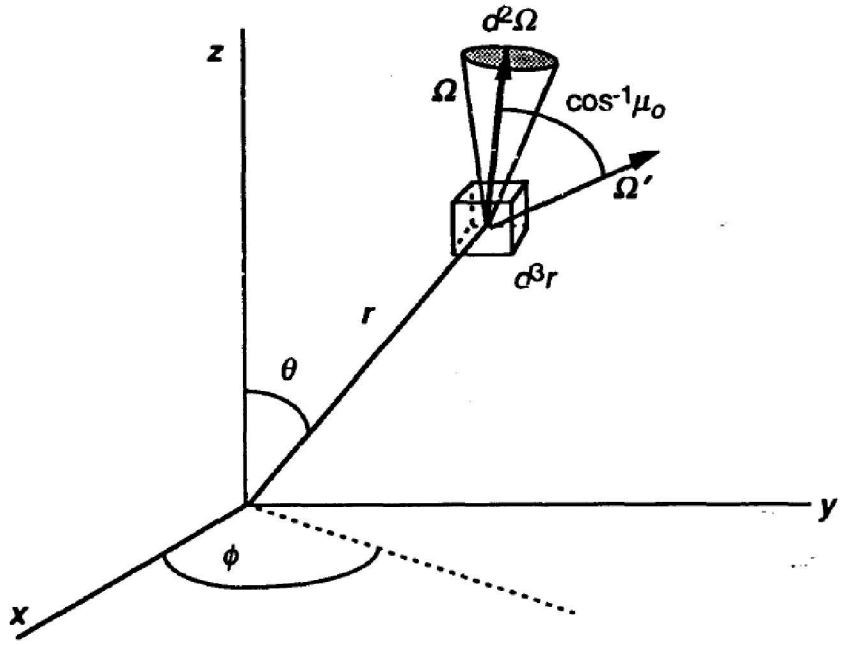


Image Source: Daniel Rozon, "Chapter 2: The Diffusion Equation and the Steady State" in *Introduction to Nuclear Reactor Kinetics*, École Polytechnique de Montréal, 1998.

Macroscopic Cross Sections

- Define the *macroscopic cross section* as:

$$\Sigma_{(r,E,t)} = N_{(r,t)}\sigma_{(E)} \qquad [\mathrm{cm}^{-1}]$$

density of nuclei in a volume

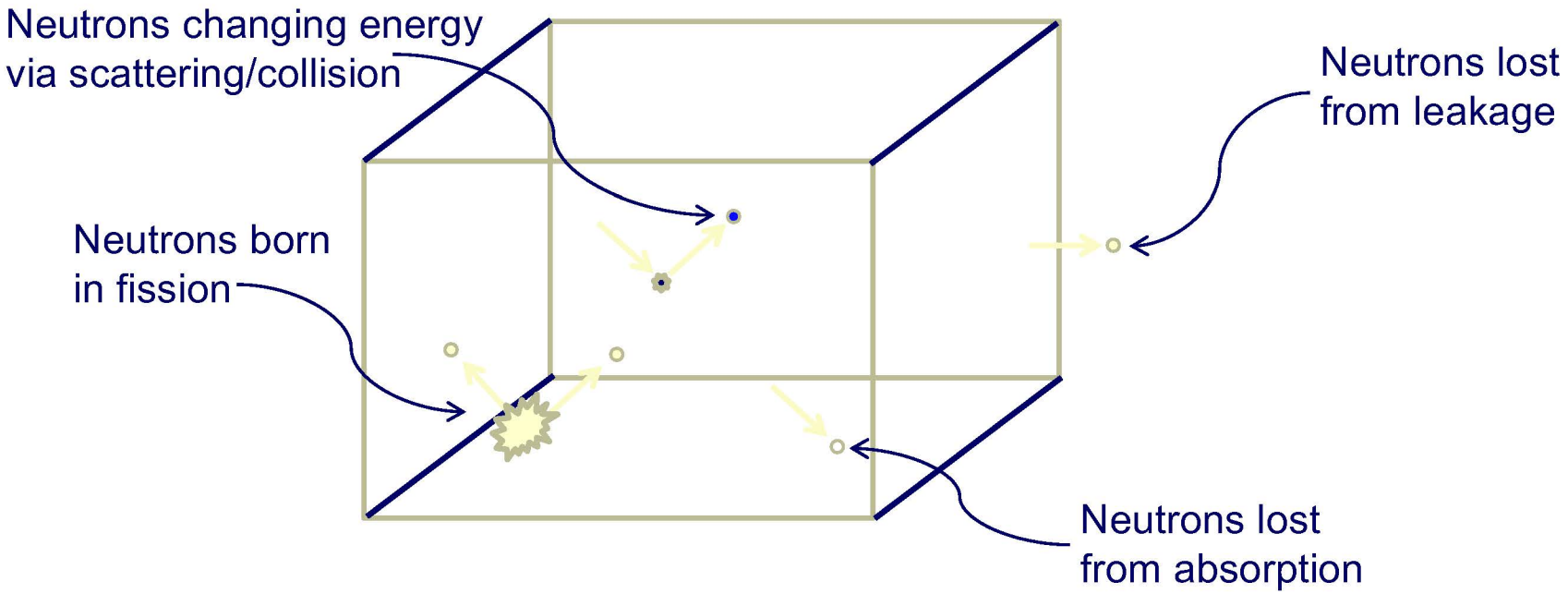
microscopic cross section

- Σ represents the probability of a reaction taking place
- The *reaction rate* for any given reaction is simply:

$$R = \Sigma\Phi$$

Neutron Conservation

- Posit that the rate of change of the neutron density in a volume is the sum of all neutron sources and sinks/losses



$$\Phi_{(r,E,\vec{\Omega},t)} = v\, n_{(r,E,\vec{\Omega},t)}$$

Neutron Transport Equation

__angular__ neutron flux density

neutrons lost via scattering and absorption

neutrons lost via leakage

$$\frac{1}{v}\frac{\partial}{\partial t}\Phi_{(r,E,\bar{\Omega},t)} = -\Sigma_{(r,E,t)}\Phi_{(r,E,\bar{\Omega},t)} - \bar{\Omega}\cdot\bar{\nabla}\Phi_{(r,E,\bar{\Omega},t)}$$

neutrons gained via scattering →

$$+\int_0^\infty \Sigma_{s,(r,E'\to E,t)}\Phi_{(r,E,\bar{\Omega},t)}\,dE'$$

neutrons born in fission →

$$+\chi_{P(E)}\int_0^\infty \nu_{p(E')}\Sigma_{f(r,E',t)}\Phi_{(r,E,\bar{\Omega},t)}\,dE'$$

$$+S_{d_{(r,E,t)}} + S_{(r,E,t)}$$

delayed neutron source

other sources

Solving the Neutron Transport Equation

- The neutron transport equation can be solved with appropriate selection of initial and boundary conditions
  - Some initial neutron flux distribution
  - Vacuum, reflective, white or periodic boundaries
- Typical approaches to solution are:
  - ***Deterministic***: discretization in space and energy with direct numerical solution (e.g. WIMSD, DRAGON)
  - ***Stochastic***: solution via Monte Carlo methods (e.g. MCNP, KENO, SERPENT)
- Often looking for steady state flux distributions
  - *isotope depletion* (burnup) evolved separately in time
  - Used an initial condition for kinetics calculations.

Lattice Cells

- Neutron transport solutions are computationally intensive
- Solution normally constrained to two dimensional models of single fuel assemblies or *lattice cells*
- Can also create three dimensional models of several lattice cells (called *supercells*)
  - Useful for finding reactivity worth of control devices in the reactor

PWR MOX fuel assembly

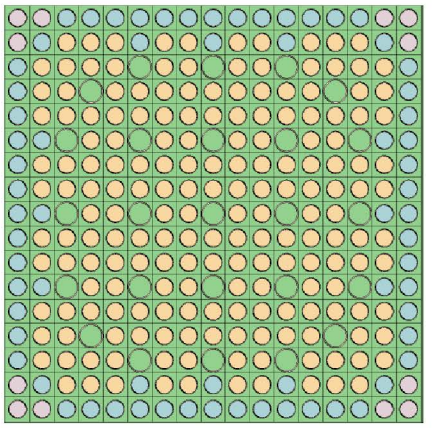


Image Source: *SERPENT 1.1.7*, VTT Technical Research Centre of Finland, 2010.

Typical CANDU Supercell

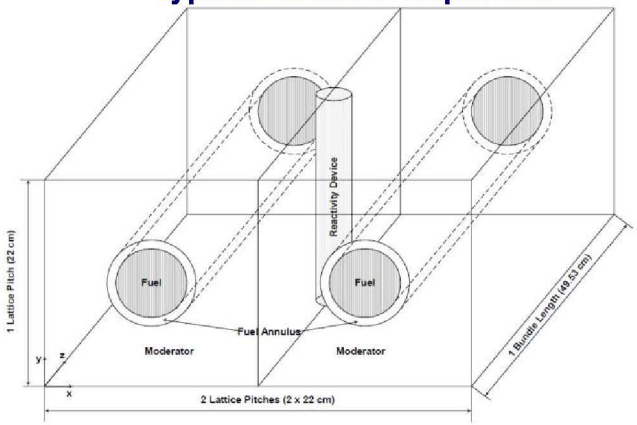


Image Source: Ben Rouben, "CANDU Fuel Management" in *EP6D03: Course Notes*, McMaster University, 2009.

The Diffusion Approximation

- The quantity necessary to calculate the actual reactor power is the *scalar neutron flux density*

$$\phi(r, E, t) = \int_0^{4\pi} \Phi(r, E, \vec{\Omega}, t) d^2\vec{\Omega} = vn(r, E, t)$$

- To solve the new equation we approximate the leakage term as a *diffusion* process

$$\int_0^{4\pi} \left(-\vec{\Omega} \cdot \vec{\nabla}\Phi(r, E, \vec{\Omega}, t)\right) d^2\vec{\Omega} \rightarrow +\vec{\nabla} \cdot D(r, E)\vec{\nabla}\phi(r, E, t)$$

The Diffusion Equation

scalar *neutron flux density*

neutrons lost via scattering and absorption

neutrons lost via leakage

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(r,E,t) = -\Sigma(r,E,t)\phi(r,E,t) + \vec{\nabla}\cdot D(r,E)\vec{\nabla}\phi(r,E,t)$$

neutrons gained via scattering $\longrightarrow$

$$+\int_0^\infty \Sigma_s(r,E'\rightarrow E,t)\phi(r,E,t)dE'$$

neutrons born in fission $\longrightarrow$

$$+\chi_P(E)\int_0^\infty \nu_p(E')\Sigma_f(r,E',t)\phi(r,E,t)dE'$$

$$+S_d(r,E,t) + S(r,E,t)$$

delayed neutron source

other sources

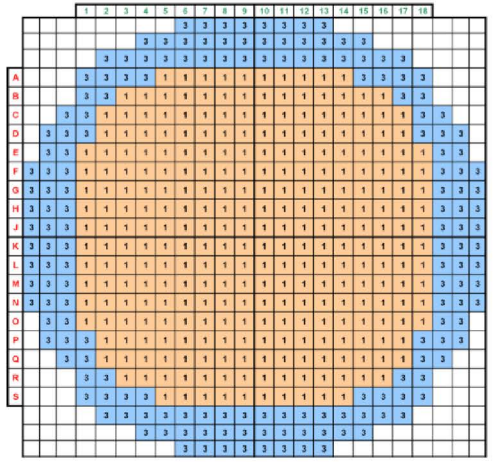
Consequence of the Diffusion Approximation

- By assuming that leakage out of the unit cell is describable by *diffusion*, we've assumed a high level of *isotropy* in the neutron flux
- This assumption breaks down:
  - At material boundaries and the external boundary of the domain
  - Near localized sources
  - In highly absorbing materials
- The high fidelity of a neutron transport solution is lost
  - Must use unit cells much larger than the *mean free path* of a neutron with *homogenized* properties within the cell.
  - Homogenization ➔ "representative" properties for a "large cell" that give similar results as a more detailed calculation.

Solving the Diffusion Equation

- Codes that solve the diffusion equation allow three dimensional simulation of entire reactor cores
  - e.g. PARCS, DONJON
  - Suitable for accident analysis and refuelling calculations
- Typically neutron energy is collapsed into few characteristic *groups*
  - e.g. fast/thermal or more
- Neutron transport codes are used to supply the collapsed few group *homogenized* cross sections in unit cells suitable for the diffusion solution

3D Neutron Flux Map for CANDU



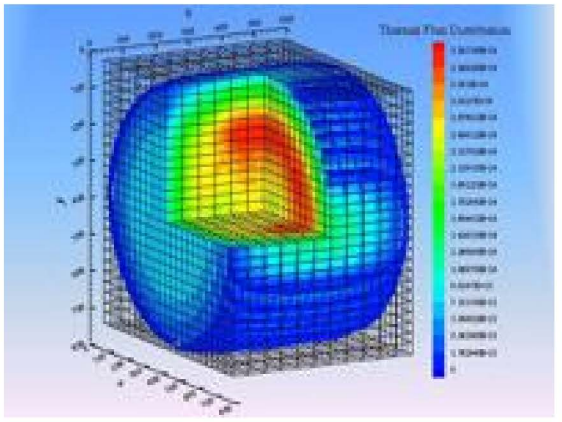


Image Source: D. Sekki et al, *A User Guide for DONJON Version 4*, École Polytechnique de Montréal, 2011.

Kinetics vs. Depletion

- Kinetics calculations are used to:
  - Assess the neutron (power) behaviour in space and time over relatively shorter time scales
  - Linkage to thermalhydraulics changes
  - Safety and stability analyses.
  - Point vs. 3-D reactor kinetics.
- Depletion calculations:
  - Determine the slower evolution of physics → determine the change in composition of the fuel with burn-up.

The "Point Kinetics" Approximations

- In "point kinetics", only the behaviour of the reactor as a whole (i.e. a "point") is considered
- Furthermore, only "short time" phenomena are relevant
  - Neither the core composition or flux "shape" are expected to change quickly relative to the total power or "amplitude"

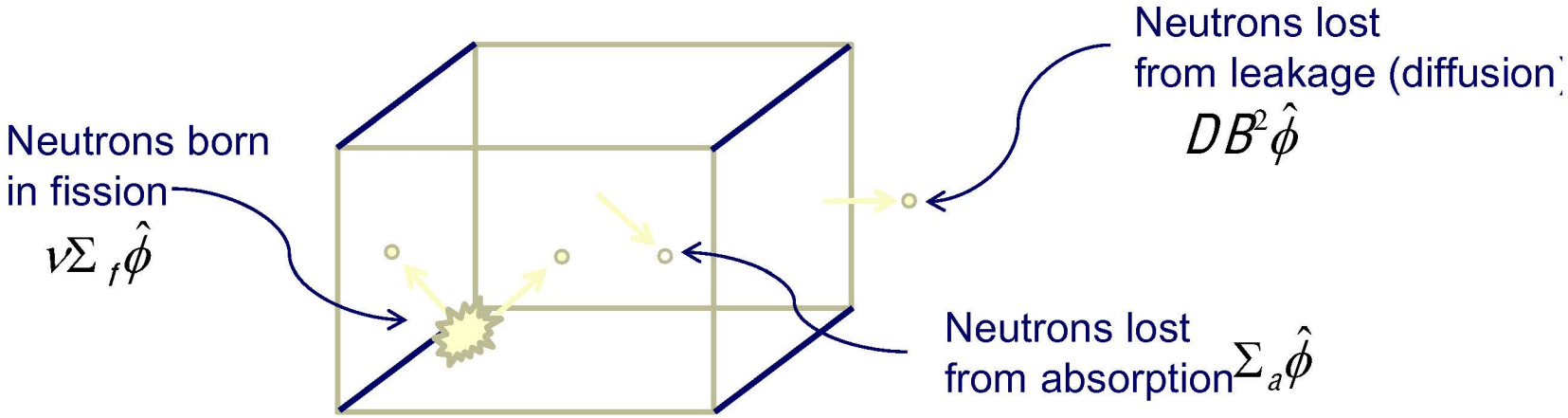
  - The flux and macroscopic cross-sections over the entire reactor are thus homogenized or averaged as:

$$\phi(r, E, t) \Rightarrow \hat{\phi}(t) = n(t)\overline{v}$$

$$\Sigma(r, E, t) \Rightarrow \Sigma\,(\text{constant})$$

Neutron Conservation in Point Kinetics

- Again, express the rate of change of the neutron density as the sum of all neutron sources and sinks/losses



$$\frac{dn}{dt} = \nu\Sigma_f\hat{\phi} - \left(\Sigma_a + DB^2\right)\hat{\phi}$$

$$\Downarrow$$

$$\frac{1}{\bar{v}}\frac{d\hat{\phi}}{dt} = \left(\nu\Sigma_f - \Sigma_a - DB^2\right)\hat{\phi}$$

Delayed Neutrons in Point Kinetics

- The delayed neutron precursors are homogenized into a finite number of “groups” with characteristic yields and decay constants:

$$\frac{1}{\bar{v}}\frac{d\hat{\phi}}{dt} = \left(\nu_p \Sigma_f - \Sigma_a - DB^2\right)\hat{\phi} + \sum_k \lambda_k C_k$$

- The concentration of each group evolves over time as:

$$\frac{dC_k}{dt} = -\lambda_k C_k + \nu_{dk}\Sigma_f \hat{\phi}$$

Delayed Neutrons in Point Kinetics

- For the thermal fission of ^{235}U typically use $k = 6$

Fractional Group Yield β_k/β	Decay Constant λ_k [s^{-1}]	Total Delayed Yield
0.0380	0.0133	ν_d = 0.0166
0.1918	0.0325	
0.1638	0.1219	
0.3431	0.3169	β = 0.00682
0.1744	0.9886	
0.0890	2.9544	

Physically Significant Parameters

- Effective multiplication factor

$$k_{eff} = \frac{neutrons\ created}{neutrons\ lost} = \frac{\nu\Sigma_f}{\Sigma_a + DB^2} \qquad \nu = \nu_p + \nu_d$$

- Reactivity (or “distance from criticality”)

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\nu\Sigma_f - \Sigma_a - DB^2}{\nu\Sigma_f}$$

- Prompt neutron generation time

$$\Lambda = \frac{1}{\overline{V}\nu\Sigma_f}$$

- Delayed neutron fraction

$$\beta = \sum_{k=1}^{K} \beta_k \qquad \beta_k = \frac{\nu_{dk}\Sigma_f}{\nu\Sigma_f}$$

The Point Kinetics Equations

- With some rearrangement, we get the classical form of the point kinetics equations:

$$\frac{1}{\bar{V}}\frac{d\hat{\phi}}{dt} = \left(\nu_p \Sigma_f - \Sigma_a - DB^2\right)\hat{\phi} + \sum_k \lambda_k C_k$$

$$\Downarrow$$

$$\frac{1}{\bar{V}\nu\Sigma_f}\frac{d\hat{\phi}}{dt} = \left[\frac{\nu\Sigma_f - \Sigma_a - DB^2}{\nu\Sigma_f} - \frac{\nu_d\Sigma_f}{\nu\Sigma_f}\right]\hat{\phi} + \frac{1}{\nu\Sigma_f}\sum_k \lambda_k C_k$$

$$\Downarrow$$

$$\frac{d\hat{\phi}}{dt} = \frac{(\rho-\beta)}{\Lambda}\hat{\phi} + \sum_k \lambda_k C_k; \qquad \frac{dC_k}{dt} = -\lambda_k C_k + \frac{\beta_k}{\Lambda}\hat{\phi}$$

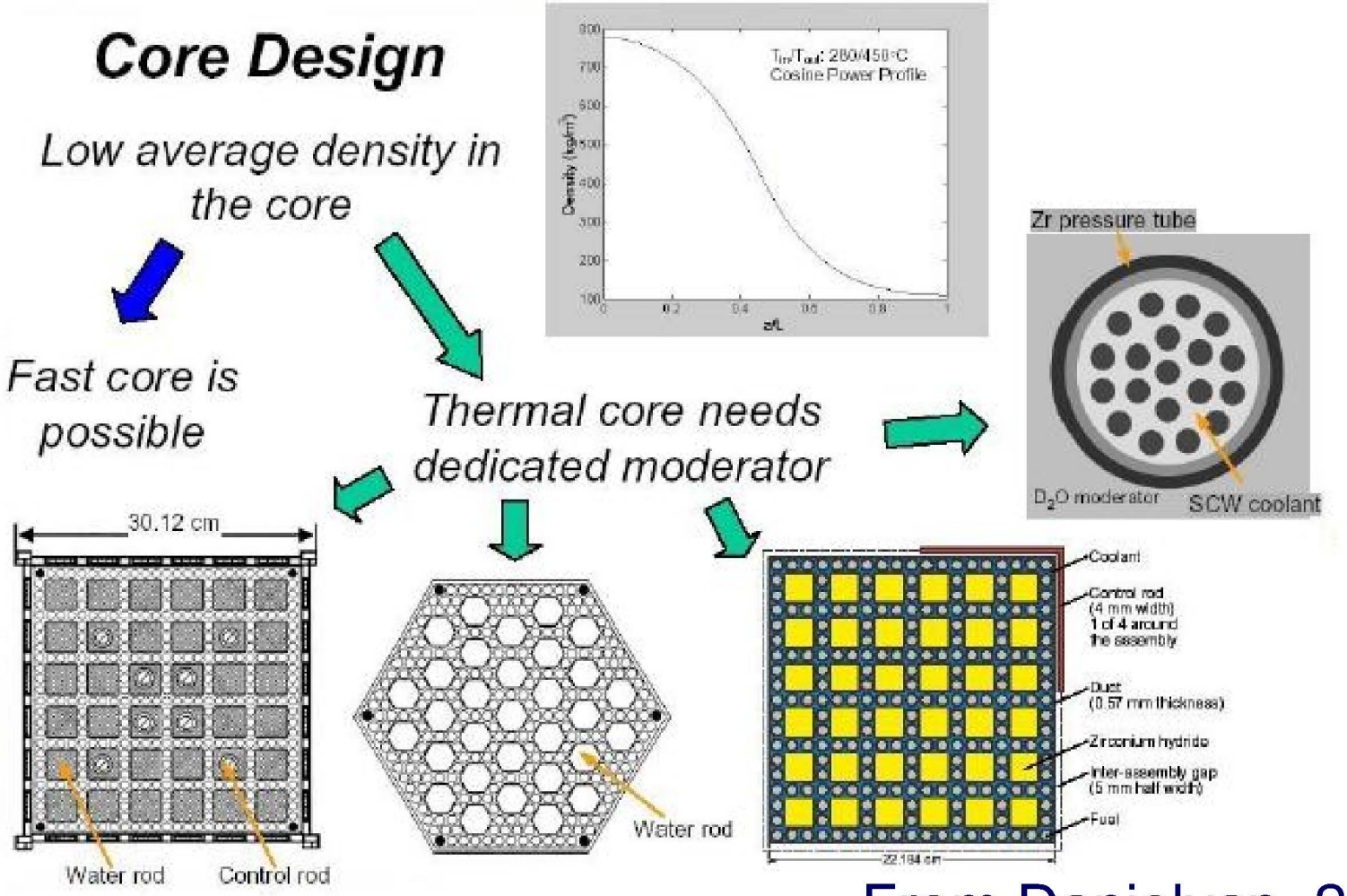
Using Point Kinetics

- The point kinetics equations are relatively easily solved as a system of $k+1$ equations
- Point kinetics is typically used to easily determine the relative change in reactor power from insertions of reactivity
- Many heat transport system thermal-hydraulic codes (e.g. RELAP5, TRACE) contain point kinetics solvers

3D Kinetics

- Often in Safety Analysis, the flux distribution can evolve very quickly (e.g. rod ejection event), and hence the neutronic behaviour may show “local” effects.
- In such cases treating the core as a “point” in point kinetics may not be sufficiently accurate.
- Need for 3D kinetics calculations
  - Solve the diffusion equations in space and time.
- See Lecture of TH-RP coupling

Core Design



From Danielyan, 2003.

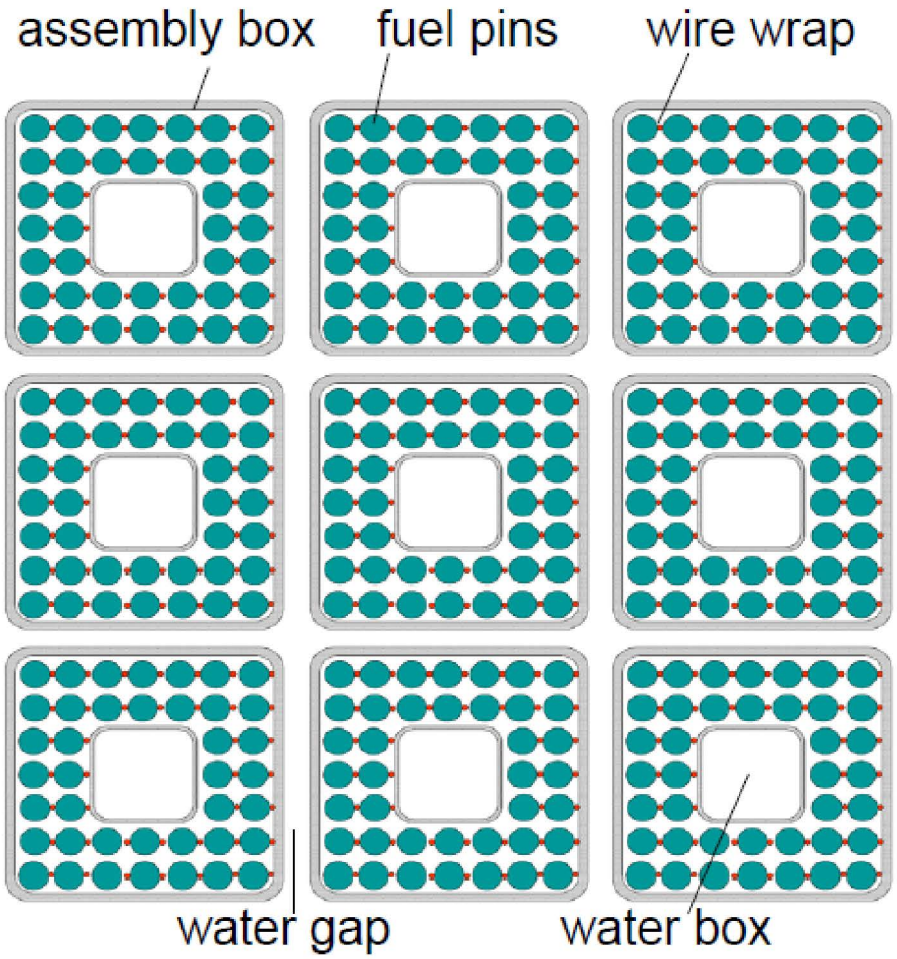
SCWR Neutronics

- A majority of work on SCWR is centered around a THERMAL DESIGN.
- Important features:
  - Fuel
  - Moderator Structures
- The SCWR coolant can be classified as either
  - Gas like
  - Liquid like
  - The differences greatly affect the absorption/moderation characteristics.
- Main difference in physics issues
  - Doppler broadening of resonances
  - Coolant-density effects
  - “Harder” Neutronic spectrum as compared to typical thermal reactors.
  - Moderation

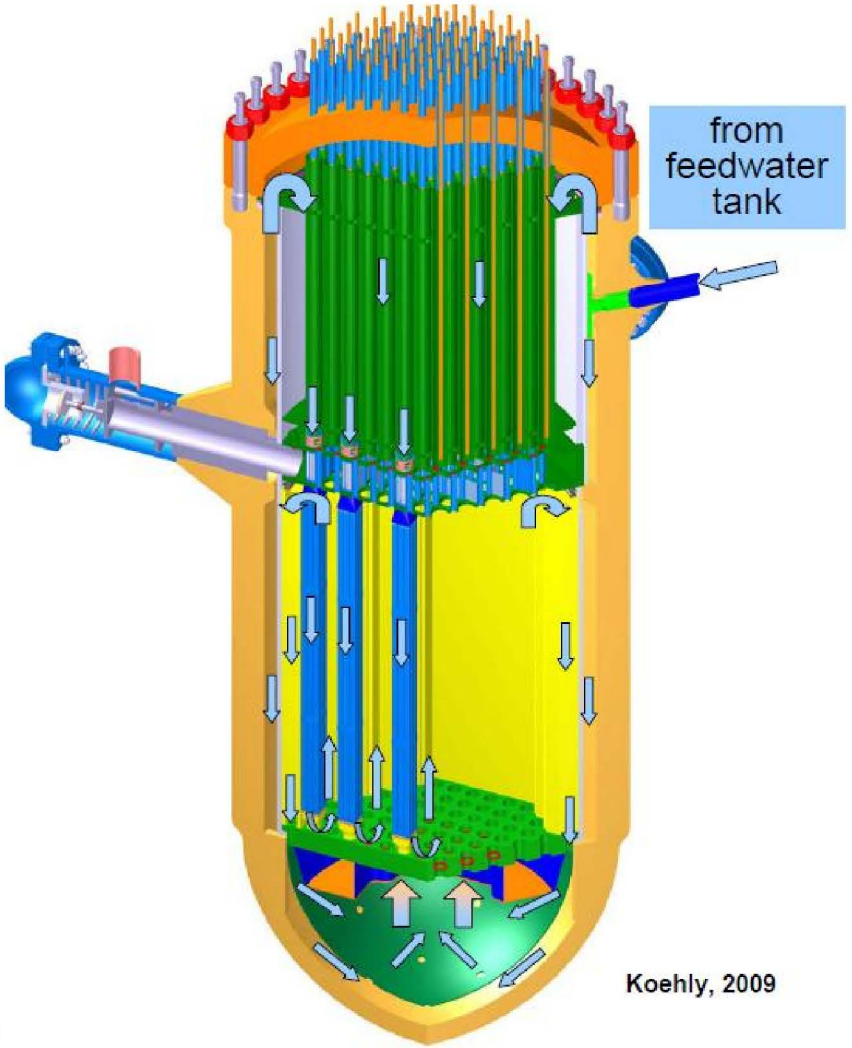
Moderation in SCWR Approach 1

- In many designs (e.g., HPLWR), the coolant enters the core in its “liquid” like state.
  - Part of this “liquid like” coolant is first directed through “water boxes”.
  - The water boxes provide moderation to the surrounding assemblies
- After passing through the water boxes, the coolant then travels through the fuel assemblies to remove heat.
  - As its enthalpy rises in the fuel region → transitions to the “gas like” state.
  - Rapid change in moderation/absorption cross section due to the rapid change in the density.
  - Density feedback effects → Need for coupled neutronics-thermalhydraulics.

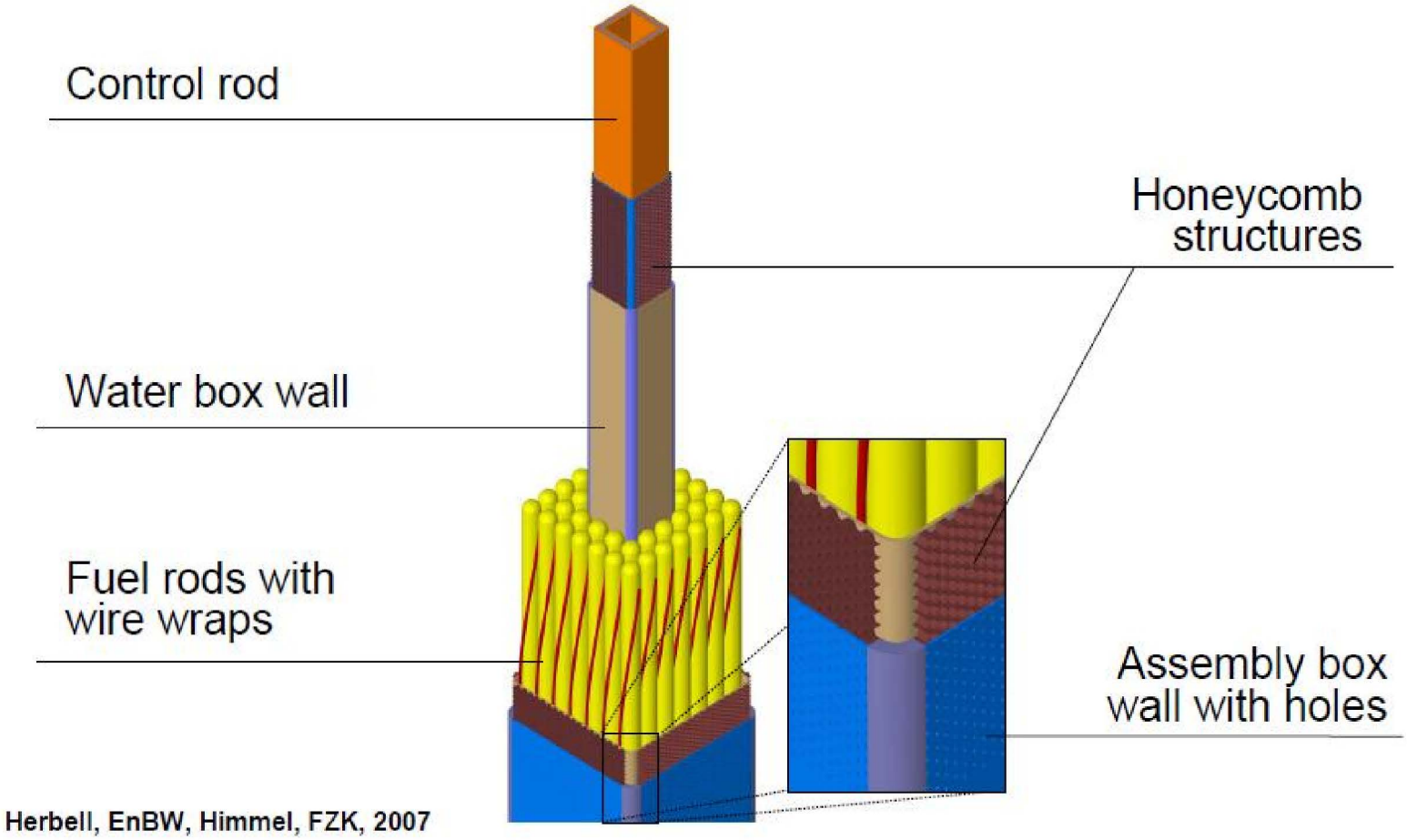
Water Box Moderation



J. Starflinger, FZK, Presentation at McMaster University, June 9, 2009



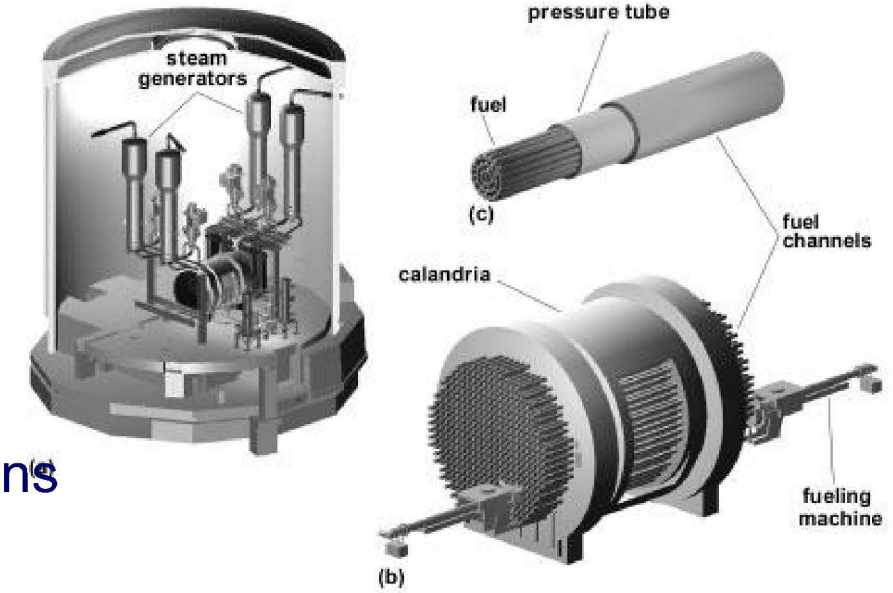
HPLWR Rod Assembly



Herbell, EnBW, Himmel, FZK, 2007

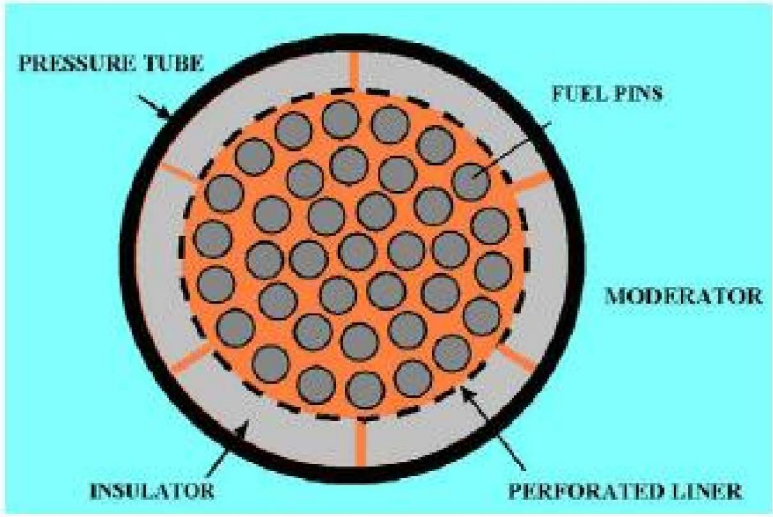
Moderation in SCWR Approach 2

- The heavy water type thermal reactors utilize separate moderator – coolant systems (e.g., CANDU-SCWR)
  - Allows for moderator to be in a subcritical fluid regime.
  - No mixing of coolant and moderator
  - Control and safety devices are not inserted into the high pressure regions of the core.
  - Cold moderator (60C)
- Pressure tube isolates the coolant/fuel region from the moderator tank.

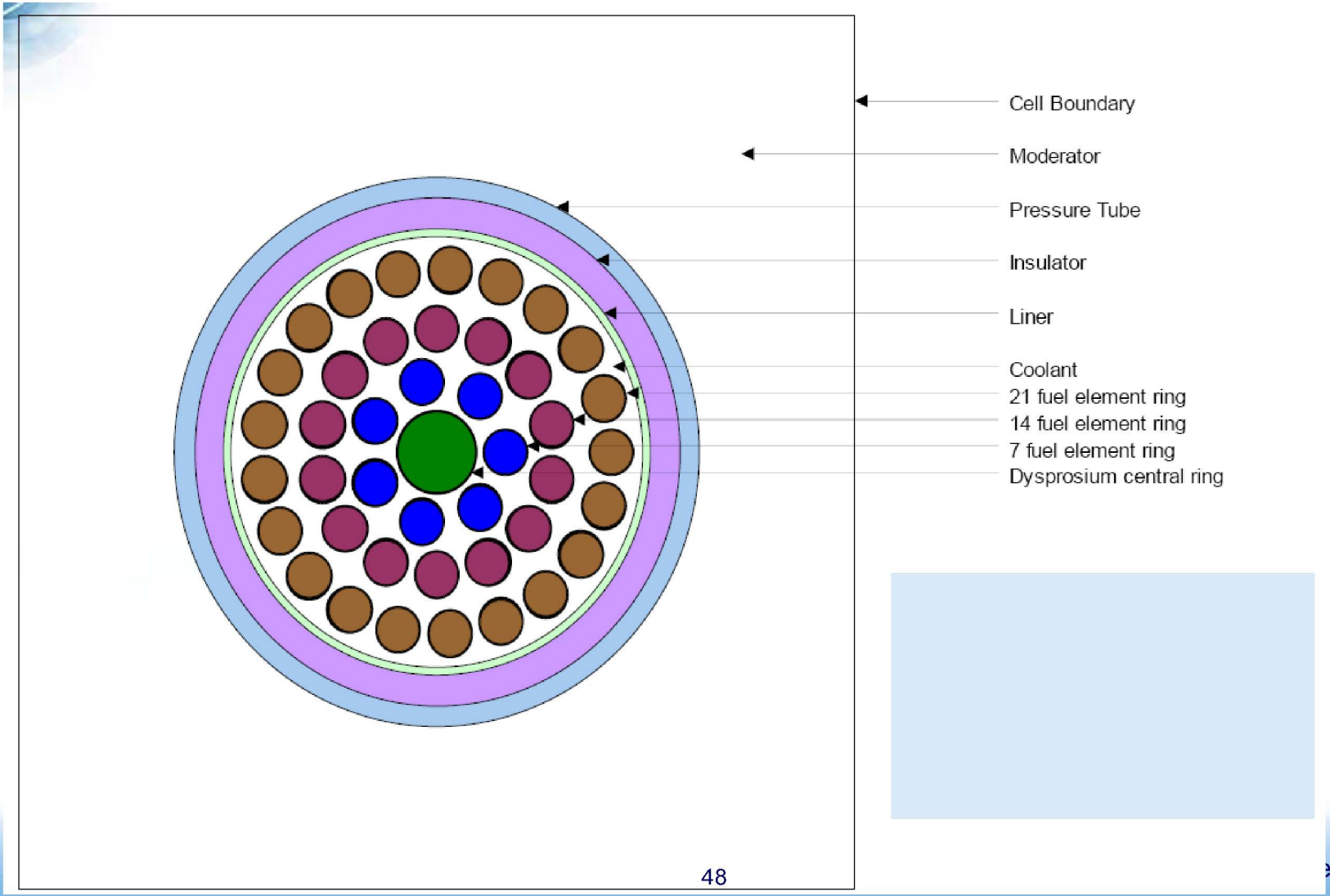


General SCWR Pressure Tube Concept

- Insulate pressure tube on the inside.
- Remove calandria tube.
- Insulator thickness optimized to obtain
  - Usual heat loss by conduction/convection to the moderator under normal operation.
  - Sufficient heat rejection by radiation/conduction/ convection under accident conditions.

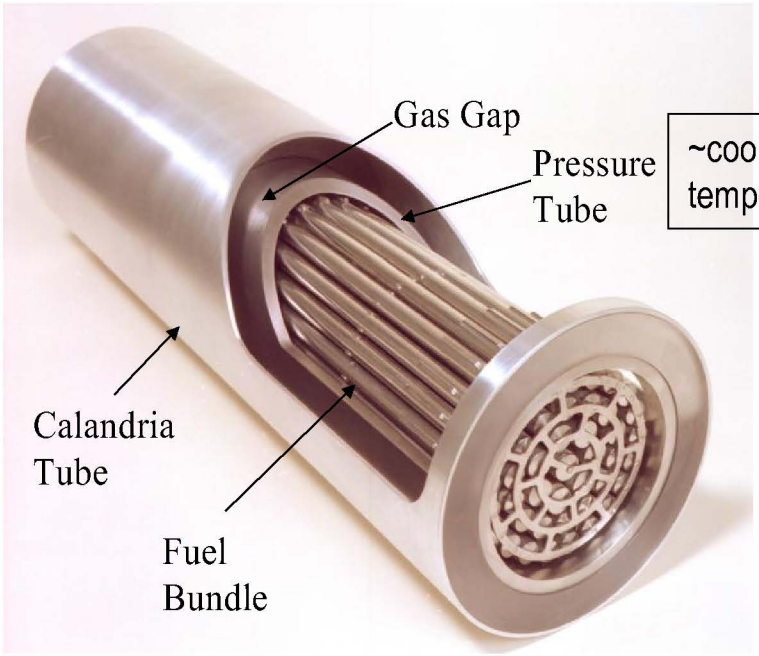


Fuel and PT Design

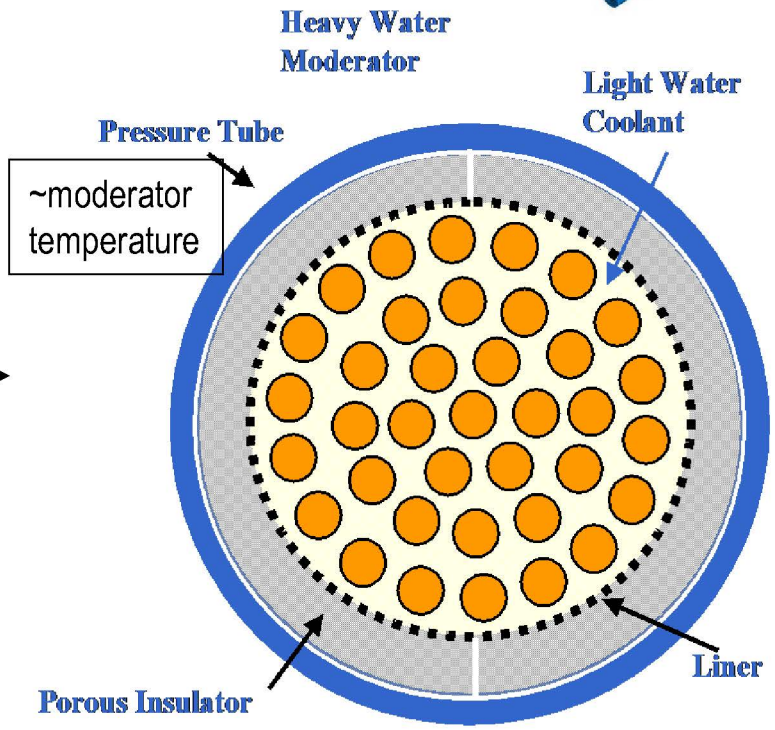


Advanced Fuel Channel:

- Eliminate the calandria tube
- Insulate the inside of the pressure tube



~coolant temperature



Heat deposited in moderator ~5% (neutron/gamma heating); ~0.1% through gas gap

Heat deposited in moderator ~5% (neutron/gamma heating + 1% through insulator

Concluding Remarks

- SCWR neutronics has no “new” or unique physics to be modelled.
- Rather the geometry, temperatures, properties are different.
- While many physics code can perform calculations under these conditions → the accuracy and validity of the results has not been rigorously demonstrated.
  - Higher fuel and moderator temperatures
  - Harder neutronic spectrums
- In addition to more calculations, there is a need for validation.