Global existence and optimal decay rate of solutions for the degenerate quasilinear wave equation with a strong dissipation

by

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Abstract: We will study the initial boundary value problem of the degenerate quasilinear wave equation with a strong dissipation of the form

$$u'' - \phi\left(\int_\Omega |\nabla_x u|^2\, dx\right)\Delta_x u - \sigma(t)\Delta_x u' + |u|^\alpha u = 0 \quad in \;\; \Omega \times \mathbb{R}_+.$$

We prove global existence of solutions in Sobolev spaces and general stability estimates using multiplier method and general weighted integral inequalities. Without imposing any growth condition at the origin ϕ, we show that the energy of the system is bounded above by a quantity, depending on σ and ϕ, which tends to zero (as time goes to infinity). We also prove the optimality of decay rate of the energy for $\sigma \equiv 1$ and ϕ is slowly degenerate. These estimates allow us to consider large class of functions σ and ϕ with general growth at the origin. We give many significant examples to illustrate how to derive from our general estimates the polynomial, exponential or logarithmic decay.

Keywords and phrases: Degenerate quasilinear wave equation, Global existence, Strong dissipative term, Multiplier method, Integral inequalities.