



2292-6

School and Conference on Analytical and Computational Astrophysics

14 - 25 November , 2011

The role of turbulence in the solar wind

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Why turbulence in astrophysics and space physics?

- Cross scale couplings
 - Dynamical couplings across wide ramge of space and time
- Space weather and prediction
 - Stochastic behavior
 - Spatial and temporal complexity
- Mapping of field lines and transport of charged particles:
 - Influence of randomness
 - Influence of structure
 - nonGaussian statistics (rare events)
- Variability in space and time: unavoidable influence on predictability
 - Finite correlation time and correlation length
 - Deterministic chaos sensitivity to ICs, BCs and driving
 - "1/f noise"
- Modification of coupling strengths, rates, diffusion coefficients
 - Heating
 - Mixing
 - Drag
 - Transport across boundaries (e.g., reconnection)

I: Solar wind turbulence: context

Mean flow and fluctuations

- In turbulence there can be great differences between mean state and fluctuating state
- Example: Flow around sphere at R = 15,000





Instantaneous flow

Mean flow

Activity in the solar chromosphere and corona: SOHO spacecraft

Origin of the solar wind

UV spectrograph: EIT 340 A

White light coronagraph: LASCO C3





Large scale features of the Solar Wind: Ulysses



- High latitude
 - Fast
 - Hot
 - steady
 - Comes from coronal holes
- Low latitude
 - slow
 - "cooler" (40,000 к @ 1 AU)
 - nonsteady
 - Comes from streamer belt j

II. Turbulence

•Hydrodynamics

wide range of scales --Reynolds number energy decay – vonKarman energy cascade – Kolmogorov\ Oubukov

• MHD

Energy, cross helicity, magnetic helicity (at least TWO fields)
Parameter space- - alfven ratio; kinetic helicity,
Multiple characteristic scales (correlation scales, etc)

•Plasma & kinetic effects

•Hall, aniso pressure, kinetic damping...

Hydrodynamic turbulence

- Incompressible
- Homogeneous
- Fourier decomposition

Turbulence: nonlinearity and cascade

 $\frac{\partial V}{\partial t} \sim V \cdot \nabla V$ Fourier $\frac{\partial V(k)}{\partial t} \sim \sum_{r+p=k}^{\infty} V_p(r) V_s(p)$ "Triad INTERACTION" ORDER - CHAOS ergodicity, MIXING CHAOS - ORDER coherent structures Intermitteney CHARACTERISTIC PROCESSES line stretching, vortex coalescence, reconnection

Turbulence and examples of fluid plasmas

hydro \rightarrow MHD \leftarrow liquid metals MHD \leftarrow plasma (collisions,

or $\Omega_{\text{cyclotron}} >> \Omega_{\text{plasma}}$

2D Hydro \leftarrow plasma ($\Omega_{\text{plasma}} << \Omega_{\text{cyclotron}}$)

solar corona \rightarrow MHD solar wind \rightarrow MHD interstellar medium \rightarrow MHD lonospheric plasma (e.g., auroral) \rightarrow 2D hydro

- Variables velocity v magnetic field $B = B_0 + b$ = B_0 mean
- Incompressible for "same reasons" as in hydro $\rho = const.$ $\nabla \cdot v = 0$
- Lorentz force $(\nabla \times B) \times B = J \times B$ in momentum equation
- Ohm's Law $E = -\nu \times B + \mu J + ...$ (plasma...caution!)

Distinctive effects in MHD

• Two fields and multiplicity of length scales

The incompressible MHD model, in terms of the fluid velocity **u** and the magnetic field **B**, involves the momentum equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \frac{1}{4\pi\rho} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{u} \quad (1)$$

and the magnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \mu \nabla^2 \mathbf{B}.$$
 (2)

• Anistropy/propagation

$$\frac{\partial \mathbf{z}_{\pm}}{\partial t} \mp \mathbf{V}_{\mathbf{A}} \cdot \nabla \mathbf{z}_{\pm} = -\mathbf{z}_{\pm} \cdot \nabla \mathbf{z}_{\pm} - \frac{1}{\rho} \nabla P + \mu \nabla^2 \mathbf{z}_{\pm},$$

- Multiple ideal invariants/direct/inverse cascades (see also "quasiinvariants")
- Dimensionless parameters

There are numerous reasons to doubt that MHD turbulence admits the same sort of "universality" that hydro does.



III. Similarity decay of energy: simple but powerful theories:

Energy decay in turbulence



Energy decay (decay of direct cascaded quantity)

- vK-H ideas
 - energy dissipation independent of R, Rm
 - 3rd order law (Pouquet and Politano, 1998)
 NB mean magnetic field does not appear in VKH eqns!
- `Similarity decay equations'
 - Isotropic
 - Anisotropic:

4 length scales, and it appears they cannot be indedendent

$$\begin{aligned} \frac{dZ_{+}^{2}}{dt} &= -\alpha_{+} \frac{Z_{+}^{2} Z_{-}}{L_{+}}; \qquad \frac{dZ_{-}^{2}}{dt} = -\alpha_{-} \frac{Z_{+} Z_{-}^{2}}{L_{-}} \\ \frac{dL_{+}}{dt} &= \beta_{+} Z_{-}; \qquad \frac{dL_{-}}{dt} = \beta_{-} L_{+} \end{aligned}$$

Wan et al, JFM 2011 submitted

Isotropic MHD-vKH decay (2 length scales)

• An example

Set of simulations results



Energy→

IV. Correlations and spectra

- Hydro and MHD naturally give rise to a *heirarchy* of correlations
- 1st order -- Means
- 2nd order correlations, spectra, structure fns.
- Higher order correlations (structure fns...)

Basic relationships:

- Ergodic theorems: replace ensemble averages by space and/or time averages
- Fourier transform relationship between correlation fns. & spectra

"Required reading": G. K. Batchelor, Theory of Homogeneous Turbulence Monin & Yaglom: Statistical Fluid Dynamics v 1&2

Kolmogorov 1941

- Widely separated energy containing scale L and dissipation scale η L/η >>1
- Energy flux across scales should be independent of scale in this "inertial range"
- u_k^2 = energy per unit mass near wavenumber k
- $t_{nl}(k) = \text{nonlinear time scale at wavenumber } k \rightarrow 1/(k u_k)$
- ε_k = energy flux across shell of radius k $\rightarrow \varepsilon_k = \frac{u_k^2}{t_{nl}(k)} = \varepsilon$

Indepdendent of k!

Then...
$$E(k) = u_k^2 / k = C \epsilon^{2/3} k^{-5/3}$$

... the Kolmogorov spectrum



Solar wind magnetic field autocorrelation function at 1 AU: 1 s/c & frozein in flow



Spectral method simulations of MHD turbulence

- Fourier spectral methods provide high order accuracy for computing accurate cascade dynamics (homogenous Turbulence
- Examples of 2D incompressible MHD at varying resolution and Reynolds number



TABLE I: Parameters for the first set of simulations with $k_0 = 15$ and initial k-band [8,40].

Run	Grid	$^{ u,\eta}$	Peak time	K_{max}/K_{diss}	Х
Run 1	2048^{2}	0.00045	0.14	3.1	963
Run 2	4096^{2}	0.00015	0.20	3.0	1279
Run 3	8192^{2}	0.000055	0.30	3.0	2971
Run 4	16384^{2}	0.000022	0.35	3.1	7945

• Theory

K41 & I-Kr65

- Simulation \rightarrow
- Solar wind observation





MHD spectra are highly variable!

- Lee at al (PRE, 2010) showed that initial conditions with same E, Hc and Hm, and same initial spectra can evolve very differently →
- The reason for this is not agreed upon
- May point to role of the structure of 4th order moments (Wan et al JFM 2011, submitted); if these are nonuniversal then MHD is not universal.
- NB: B0 and 4th order correlations do not enter third order law, so anisotropy at 3rd and 4th order may be very important



In simulations AND solar wind More than one cascaded quantity – when more than one ideal quadratic invariant (conserved flux in k-space)

- Direct and inverse cascades
- Signaled by "Bose condensation' in modified thermodynamic limit in Gibbs ensemble statistical mechanics of the Fourier-Galerkin model
- 2D hydro: E inverse; enstrophy direct (Kraichnan PoF 1967)
- 3D MHD: Hm inverse; energy direct (Frisch et al) [Hc 'hybrid'] (Stribling and Matthaeus, 1991)
- 2D MHD: mean square magnetic potential inverse; energy direct (Fyfe&Montgomery, 1976)
- 2D Guiding center plasma: electric potential inverse; mean square charge density direct Seyler et al PoF (1975, 1976

Solar wind: indications of both turbulence and wave-like properties:



"standard" turbulence spectrum



- **Dissipation:** conversion of (collective) fluid degrees of freedom into motions into kinetic degrees of freedom
- **Heating:** increase in random kinetic energy
- **Entrop**y increase: irreversible heating

V. Turbulence gives heating

Solar wind proton temperatures: nonadiabatic and anisotrpic (fast wind)



Richardson and Paularena, GRL, 1995 IMP, Voyager temperatures (faraday cup)



Marsch, Helios proton distributions From L.Rev Solar Phys. 2006

- The solar wind is "too hot" at 1 AU
- The solar wind is "too hot" at 30 AU
- The corona is "too hot" at 2 Rs

Coleman, 1968 Tu et al, 1988

Matthaeus et al., 1999

McKenzie, Axford et al 1996 Dmitruk et al, 2002 + ... Phenomenological decay models with cross helicity (for use in dynamic alignment regimes)

•MHD phenomenologies: decay of Elsasser energi

•Kolmogoroff-like	$\frac{dZ_+^2}{dt} = -$	$-\alpha \frac{Z_+^2 Z}{\lambda}$	$\frac{dZ_{-}^{2}}{dt} = -$	$-\alpha \frac{Z^2 Z_+}{\lambda}$
•Kraichnan-like	$\frac{dZ_+^2}{dt} = -$	$-\alpha \frac{Z_+^2 Z^2}{V_A \lambda}$	$\frac{dZ_{-}^{2}}{dt} = -$	$-\alpha \frac{Z_+^2 Z^2}{V_A \lambda}$

 $Z_{\pm}^{2} = \langle |\mathbf{u} \pm \mathbf{b}|^{2} \rangle$

•Embed the turbulence phenomenology in a non-WKB transport theory (energy, cross helicity, correlation scale) with expansion, reflection ("mixing"), driving by shear and pickup ions. Feed turbulent dissipation into internal energyequation.

•See Breech et al, 2008 and references therein; Ng and Bhattacharjee, 2007

•For electrons and protons separately – see Breech et al, 2009; Cranmer et al,, 2009

Transport model:

low latitude wind and comparison with Voyager data

Csh = 1.5



Transport model: high latitude parameters and Ulysses data



Matches data fairly well.

VI. Relaxation and correlations

Turbulence causes/processes alfvenic fluctuations – and other relaxation processes


$\sigma_c = 2$ Hc /E tends to increase in time

Theory based on Kraichnan 65 ideas: Dobrowolny et al, 1980



2D: Matthaeus et al, 1983



FIG. 1. Evolution of the correlation coefficient ρ defined in Eq. (1.1). Dashed line, random initial conditions (same conditions as in Fig. 2); solid line, Orszag-Tang vortex (conditions described in Fig. 3).

3D: Pouquet et al 1986

Decrease of Alfvenicity with heliocentric distance



Roberts et al, 1987b

σ_c does not always increase!

General picture of global turbulent relaxation: A balance of competing processes



Simulations in which cascade is driven by large scale shear show decreasing cross helicity



ALFVENIC FLUCTUATIONS: Global and local relaxation: Beltrami, force free, Alfven alignments:

$$\delta \int \left[\left(|\mathbf{v}|^2 + |\mathbf{b}|^2 \right) - 2\alpha \mathbf{v} \cdot \mathbf{b} - \phi \mathbf{a} \cdot \mathbf{b} \right] d^3 x = 0$$

in which constant magnetic helicity and constant cross helicity are imposed constraints. Here, α and ϕ are Lagrange multipliers, **a** the potential vector, and $\mathbf{b} = \nabla \times \mathbf{a}$. The Euler-Lagrange equations imply that

$$\mathbf{v} = \alpha \mathbf{b} = \frac{\alpha \phi}{1 - \alpha^2} \mathbf{j} = \frac{\phi}{1 - \alpha^2} \omega \tag{1}$$

Local rapid relaxation causes several types of correlation

- non-Gaussian statistics, intermittency
- coherent structures/ discontinuities

→ PATCHES

e.g. local alfvenic alignment







local relaxation and suppression of nonlinearity: Beltrami, force free, Alfven alignment:

$$\mathbf{v} = \alpha \mathbf{b} = \frac{\alpha \phi}{1 - \alpha^2} \mathbf{j} = \frac{\phi}{1 - \alpha^2} \omega$$

Local rapid relaxation implies:

- several types of correlation
- suppression of nonlinearity
- non-Gausian statistics □ intermittency
- coherent structures/ discontinuities



Hydrodynamic antecedent: "local Beltramization" :

R.H. Kraichnan and R. Panda, Phys. Fluids (1988); R.B. Pelz, V. Yakhot, S.A. Orszag, L. Shtilman and E. Levich, Phys. Rev. Lett. (1985); R.M. Kerr, Phys. Rev. Lett. (1987).

- Analysis of patches of Alfvenic correlations- SIMs and SW
- Distributions of $\cos(\theta_{vb})$
- Global statistics & statistics of linear subsamples (~1-2 correlation scales)
- SW and 3D MHD SIM (512^3)
- Global Alfvenicity $\sigma_c \approx 0.3$



For a specified sample size, can get highly variable
Alfvenicity (see Roberts et al. 1987a,b)
Same effect in SW and in SIMs!



Turbulence causes distinctive correlations

• E.g., nonequipartition

Alfven ratio $rA(k) = E_v(k)/E_b(k)$

10¹







Fig. 2-8. The Alfvén ratio (for the non-radial components of the velocity and magnetic field fluctuations) as a function of heliocentric distance. The data used are from Helios 2, 1977 near 0.4 AU, from the Voyager 2, 1977 near 2 AU, and from Voyager 2, 1985 near 20 AU (adopted from Roberts *et al.*, 1990).

Fig. 2-6. The inverse of the ratio $r_A = E_v(f)/E_b(f)$ versus frequency as computed at different heliocentric distances for Helios 1 (upper panel) and Helios 2 (lower panel). The distances are indicated at each curve (adopted from Bruno *et al.*, 1985).

Suggestion: rA < 1 due To current sheet formation

VII. Turbulence produces anisotropy

- Variance anisotropy
- Spectral anisotropy



Spectral anisotropy in MHD



Solar wind fluctuation geometry

- "Maltese cross" two component model
- Slab + 2D
- Cosmic ray scattering parallel mean free paths → 20% slab - 80% 2D
- NI MHD Theory –20% 80%
- Direct measurement \rightarrow 20 % 80%

Maltese: Matthaeus et al, 1990 Cos Ray: Bieber et al, 1994 NI MHD: Zank and Matthaeus, 1991 Direct: Bieber et al, 1996



A significant fraction (~80%) of the fluctuation energy is in highly oblique (70+ deg) modes

Spectral/correlation anisotropy

- Theory (Shebalin et al, 1983; Oughton et al 1994)
- Simulation
- Observation in SW

r,

2D axisymmetric magnetic field correlation fn. from ~2 years Of ISEE-3 data

"Maltese Cross"

Mathaeus et al 1990



Spectral method simulation with strong B0=10 br ms

Parallel

Direction \rightarrow

Perp

plane



Dmitruk + whm, 2004

 \mathbf{r}_{\perp}

VIII. Turbulence causes complexity in the structure of the magnetic field

Nature of magnetic flux surfaces depends on turbulence, scale and local topology



IX. Turbulence accounts for discontinuities

3D MHD compressible simulation with mean B_0



PDF of component variances

- Variances are approx. log-normal
 - → Suggests independent (scale invariant) distribution of coronal sources



0

b,

10

5

0.00

-10

-5

 When normalized to remove variability of mean and variance, component distributions are close to <u>Gaussian</u>

Pdfs of increments- one magnetic field component



Statistics of the induced electric field

Milano et al, PRE, 2002

- For Gaussian v, b \Rightarrow Induced E is exponential or exponential-like
- Ind. E is localized but not as localized as the reconnection zones
- Kurtosis 6 to 9

Dashed lines are theoretical Values for Gaussian v, b



Spectral MHD simulation t = 3

30 years of 1 hour SW data



Comparison of waiting times and increment PDFs from SW-ACE and CHMHD turbulence simulation





X. Turbulence regulates reconnection rates

Electric fields in turbulence and near reconnection sites





Large number of X points and O points in a small fraction of a large 2D MHD simulation At moderately high Rm

FIG. 6: PDF of the out-of-plane electric field contributions: total electric field given by the Ohms' law in the Eq. (7) (res solid-line), advective component (green dot-dashed line), diffusive (blue dotted line) part, reconnection rate (magenta solid-dotted line) and the electric field at the X-points evaluated from A_z^{Gauss} (azure dotted line). In the inset, a zoom in the core of these distributions is shown.

BIG electric fields are random inductive and away from Rec. Regions!

XI. Turbulence drives dissipation and small scale kinetic processes

Solar Wind Dissipation

 ION SCALES: steepening near 1 Hz (at 1 AU) -- breakpoint scales best with ion inertial scale; Helicity signature → proton gyroresonant contributions ~50%; both Kpar and Kperp involved, oblique current sheets

 BETWEEN ION AND ELECTRON SCALES: steepening continues, dispersion range, kinetic Alfven waves? second inertial range?, subsequent steepening at electron scales



dissipation mechanisms can be...

Homogenous (e.g., cyclotron damping, Landau damping)
or
Inhomogeneous (current sheets, reconnection, vortices → coherent structures)

Linear (linear Vlasov theory, instabilities...)

or

Nonlinear (turbulence, nonlinear kinetic processes, particle acceleration...

Kinetic heating of ions: MHD and kinetic scale hybrid simulation (Orszag-Tang vortex)



T. N. Parashar, M. A. Shay, P. A. Cassak,^{a)} and W. H. Matthaeus

Dissipation "channels"

- Homogeneous
 - landau, cyclotron (dissipation?)

- collisions

- Inhomogeneous
 - Structures + kinetic response (e.g., reconnection)
 - Current channels, reconnection, vortices
 (Markovsky, Hollweg)

Coherent structures are associated with enhanced heating

Red pdfs have PVI $\geq 4 \leftarrow \rightarrow$ Region 3

Each of the heating diagnostics is conditionally sampled so that the values associated with each of the three regions can be plotted as a separate PDF.



The red PDFs have probability densities that vanish for the lower values and are enhanced for the highest values.

Current Sheet Hierarchy



Organization of SW in ($\beta \parallel$, T \perp /T \parallel) plane: discontinuities are hotter and are found at limiting parameters



Distribution of mean *PVI* in (β_{\parallel} , T_{\perp}/T_{\parallel}) plane



From Kasper, Maruca & Bale, 2011



From Osman et al, 2011

More detailed turbulence spectrum



- **Cascade:** progressively enhances nonGaussian character
- Generation of **coherent structures** and patchy correlations
- Coherent structures are sites of **enhanced dissipation**

XII. Turbulence accelerates particles producing distinctive anisotropies

Test particles in MHD: distributions at short times (<crossing time of Lc)



Dmitruk et al, 2004
XIII. Turbulence destroys wave behavior and is of a "zero frequency" nature

Are there any kind of recognizable "waves" in turbulence?

- Simulations of driven dissipative MHD with imposed DC magnetic field of varying strength show little indication of power in "waves" at frequencies that solve the dispersion relations
 - for ANY value of imposed magnetic field B0 !
- Shown are Eulerian frequency spectra (one point) with B0=8, for :
 - driven steady case
 - decaying (energy
 - renormalized) turbulence
- Varying dB/B0 one find no more than ~16% energy in the dispersion relation peaks,
 With maximum at dB/B0 ~ ½
- See Dmitruk and Matthaeus, Phys Plasmas 2008



Eulerian frequency spectra

SUMMARY: We are only beginning to understand the subject of turbulence in space physics and astrophysics1

There are many outstanding issues/questions involving turbulence that need to be addressed using a broad range of methodologies and approaches:

- heating of the corona
- Distributed heating of the solar wind
- Origin of the kinetic signatures
- Scattering of energetic particles
- Role/relationship to MHD scale turbulence
 - Cascade
 - Coherent structures
- Applicability of wave theory
- Applicability of linear vlasov theory
- Homogenous vs inhomogeneous dissipation
- Contributions from proton, electron and inter-p-e scales