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Dynamics of charged particles in space
The fundamental problem

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Dynamics of charged particles in space

The fundamental problem

$$\frac{d}{dt}\mathbf{r}_i = \mathbf{v}_i(t)$$

$$\frac{d}{dt}\mathbf{v}_i = \frac{1}{m_i} e_i \left[\mathbf{E}(\mathbf{r}_i(t)) + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}(\mathbf{r}_i(t))\right]$$

$$\frac{\partial}{\partial t}\mathbf{E} = c\nabla \times \mathbf{B} - 4\pi \mathbf{J}$$

$$\frac{\partial}{\partial t}\mathbf{B} = -c\nabla \times \mathbf{E}$$

$$\nabla \bullet \mathbf{B} = 0$$

$$\nabla \bullet \mathbf{E} = 4\pi\rho_q$$

$$\rho_q(\mathbf{r},t) = \sum_{i=1}^{N} e_i\, \delta(\mathbf{r} - \mathbf{r}_i(t))$$

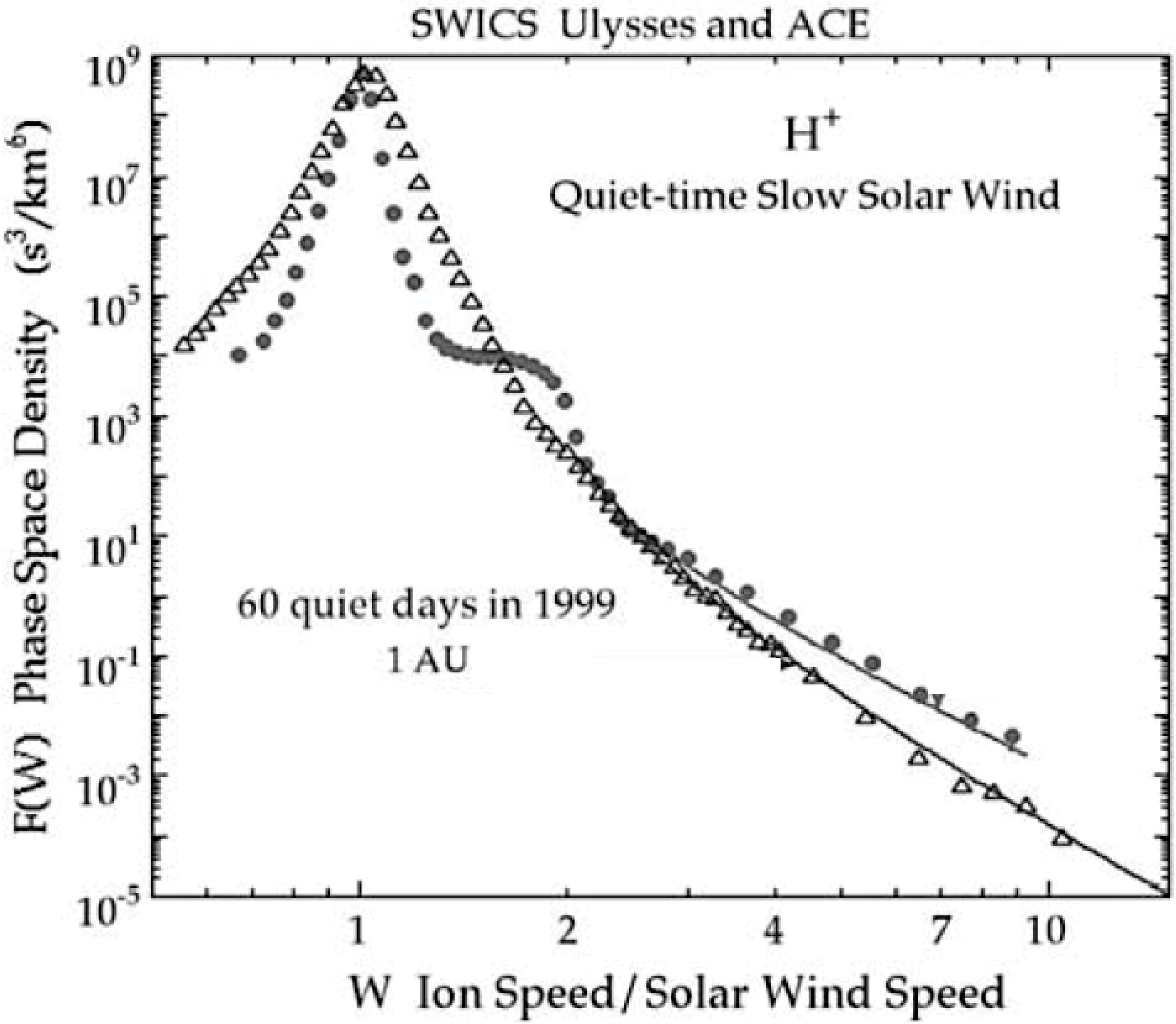
$$\mathbf{J}(\mathbf{r},t) = \sum_{i=1}^{N} e_i\, \mathbf{v}_i(t)\, \delta(\mathbf{r} - \mathbf{r}_i(t))$$

Too many particles to follow detailed trajectory in the phase space !

⇓

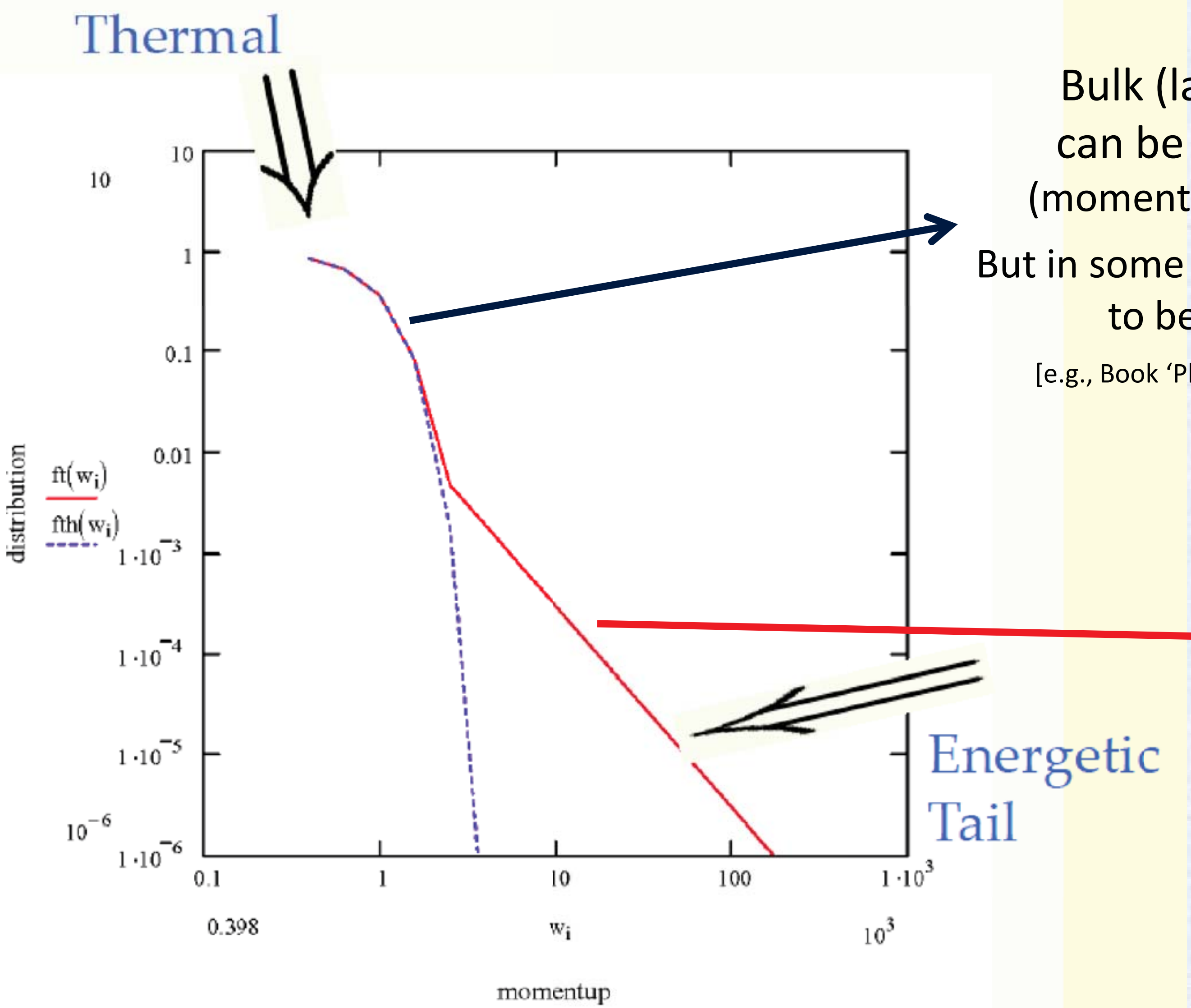
Statistic (how many in the vicinity of a state): Kinetic Theory

Typical plasma distribution function of H+ in the interplanetary space



Adapted, from McComas et al., 2007, Rev of Geophys.

Typical plasma distribution function of H+ in the interplanetary space



Bulk (large and slow) scales
can be modeled using **MHD**
(moments of *f*(*v*) and truncate KT)

But in some point, the truncation needs
to be 'repaired' (a patch)

[e.g., Book 'Plasma Astrophysics' by Somov, 2006]

Simplified KT
+
to know the bulk
behavior (scenario)

MagnetoHydrodynamics (the groundwork)

- From merging fluids and electromagnetism
- In general, valid for slow motions and smooth and large spatial scales

$$\frac{\partial}{\partial t}\rho + \nabla \bullet (\rho \mathbf{U}) = 0$$

Mass conservation

$$\rho \frac{d}{dt}\mathbf{U} = -\nabla p + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \mathbf{g} + \rho \nu \nabla^2 \mathbf{U}$$

Linear momentum conservation

$$\left(\beta = \frac{8\pi nkT}{B^2} \right)$$

$$\frac{d}{dt}(p\rho^{-\gamma}) = 0$$

Energy (simplified): ideal gas, thermodynamics $\rightarrow$ polytropic ($\gamma = C_p/C_v$ for adiabatic, $\gamma=1$ for isothermic)

More general cases can be used in MHD

$$\nabla \bullet \mathbf{B} = 0$$

No magnetic monopoles

$$\frac{\partial}{\partial t}\mathbf{B} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} + \overset{?}{\ldots}, \quad \eta = \frac{c^2}{4\pi\sigma}$$

Magnetic induction equation (from simplified Ohm´s law and Faraday)

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}$$

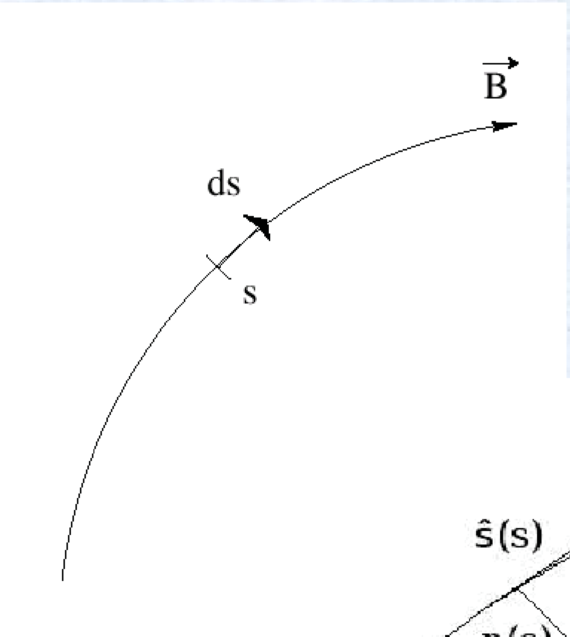
Ampere: **J** is decoupled, can be computed a posteriori, from **B**

And electric field 'hided', can be computed from the Ohm law

Magnetic forces: Pressure and tension

Magnetic pressure

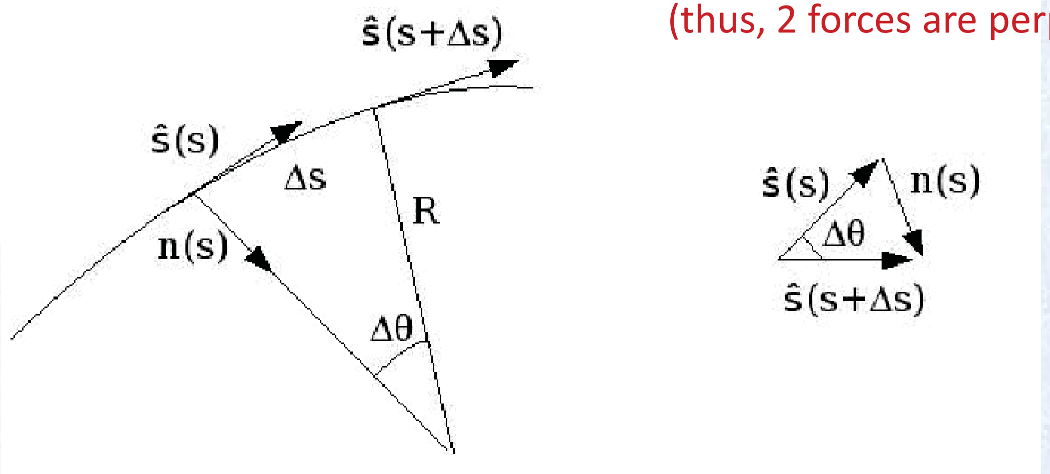
$$\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} = \frac{1}{4\pi}[\underbrace{(\mathbf{B}\bullet\nabla)\mathbf{B}} - \overbrace{\nabla(B^2/2)}]$$



Magnetic tension (restitutive force)

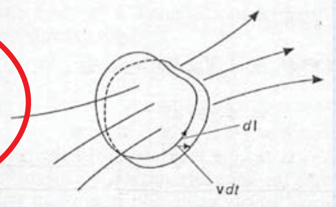
$$B\frac{d}{ds}(B\hat{\mathbf{s}}) = \frac{d}{ds}(B^2/2)\hat{\mathbf{s}} + B^2\frac{\hat{\mathbf{n}}}{R_c}$$

Cancellation of pressure force along **B** (thus, 2 forces are perpendicular to **B**)



Main ideal MHD Invariants

$$\Phi = \iint_S \mathbf{B} \bullet d\mathbf{s} \qquad \frac{d}{dt}\Phi = -\frac{c}{\sigma}\oint_l \mathbf{J} \bullet d\mathbf{l}$$



moving slice of a magnetic flux tube

But turbulent dissipation

$$E = \int_V dV \left(\tfrac{1}{2}\rho U^2 + \frac{B^2}{8\pi} + \rho\phi_g \right), \qquad \mathbf{g} = -\nabla\phi_g$$

$$\frac{dE}{dt} = -\int_V dV \left(\tfrac{1}{\sigma} J^2 + \rho\nu\omega^2 \right), \qquad \boldsymbol{\omega} = \nabla \times \mathbf{U}, \quad \omega = |\boldsymbol{\omega}|$$

Main mechanisms for energy transference:

- $E_u \rightarrow E_B$ (dynamo)
- $E_B \rightarrow E_U$+diss(reconnection)
- spatial scales (turbulence)

But turbulent dissipation

$$H^c = \iiint_V dV\, \mathbf{U} \bullet \mathbf{B}$$

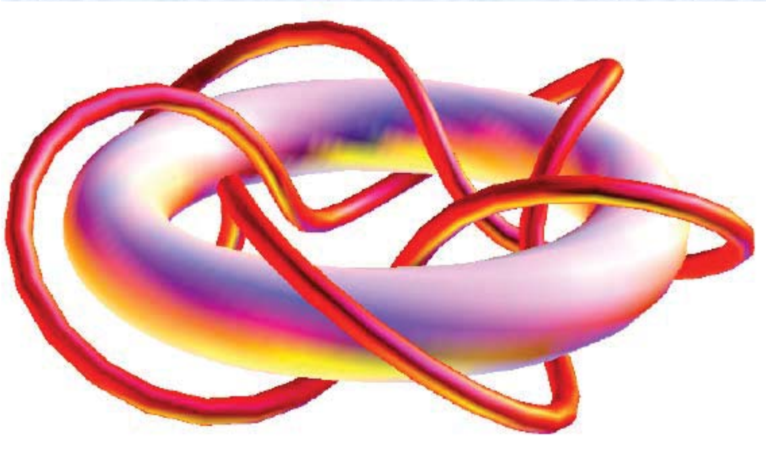
$$\frac{dH^c}{dt} = -(\eta + \rho\nu)\iiint_V dV\, \partial_i B_j \partial_i U_j \xrightarrow{\text{for } \nabla \bullet \mathbf{U}=0} -(\eta + \rho\nu)\iiint_V dV\, \mathbf{J} \bullet \boldsymbol{\omega}$$

Magnetic Helicity

$$H = \int_V dV\ \mathbf{A} \bullet \mathbf{B} \quad (\mathbf{B} = \nabla \times \mathbf{A})$$

Turbulent dissipation ?

$$\frac{dH}{dt} = -\eta \int_V dV\ \mathbf{J} \bullet \mathbf{B}$$



•H is gauge invariant for closed systems

•H needs to be defined very carefully in open systems because potential vector is not physical, but a proper reference field can help to do it [e.g., Berger & Field'84]

•H involves the topology of **B** field lines

•When dissipation, H decays slower than E !

•In turbulence: Inverse cascade (self organization of large scale structures)

•In turbulence: Asymptotic Taylor states are expected (minimum energy keeping H constant)

Helicity represents a 'Gauss linking number' for field lines

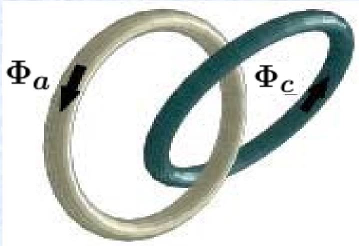
$$H^{\text{closed}} = \int_{\Phi}\int_{\Phi} \mathcal{L}_{a,c}^{\text{closed}}\, \mathrm{d}\Phi_a\, \mathrm{d}\Phi_c$$

Gauss linking number

$$\mathcal{L}_{a,c}^{\text{closed}} = \frac{1}{4\pi}\oint_a\oint_c \hat{\mathrm{t}}(\vec{x}_a)\times\hat{\mathrm{t}}(\vec{x}_c)\cdot\frac{(\vec{x}_a-\vec{x}_c)}{|\vec{x}_a-\vec{x}_c|^3}\,\mathrm{d}l_a\,\mathrm{d}l_c$$

Summation over all the field flux tubes pairs

Simple example:
Two inter-linked flux tubes



$$H^{\text{closed}} = -2\,\Phi_a\,\Phi_c$$

(Elsasser 1956)

$$H^{\text{closed}} = \int_{\mathcal{V}} \vec{A}\cdot\vec{B}\,\mathrm{d}V \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

Well defined only for open **B**:
B_n= **B•n**=0 ∀ $S(V)$

(Berger & Fields 1984; Finn & Antonsen 1985)

$$H = \int_{\mathcal{V}} (\vec{A}+\vec{A}_p)\cdot(\vec{B}-\vec{B}_p)\,\mathrm{d}V$$

$$\mathbf{B}_p\bullet\mathbf{n}\big|_{S(V)} = \mathbf{B}\bullet\mathbf{n}\big|_{S(V)}$$
$$\nabla\times\mathbf{A}_p = \mathbf{B}_p$$
$$\nabla\times\mathbf{B}_p = 0$$

The Solar Wind

MHD

Exospheric models

(e.g., Book “Basic of the Solar Wind” by Meyer-Vernet, 2007)

Basal Parker's model (HD) for the solar wind [1958]

- Stationary & spherical symmetry. Radial flow
- Only force balance between gravity and fluid pressure
- Isothermal and ideal gas

$$U\frac{dU}{dr} = -\frac{1}{n\,m_H}\frac{dp}{dr} - \frac{GM_{Sun}}{r^2}$$

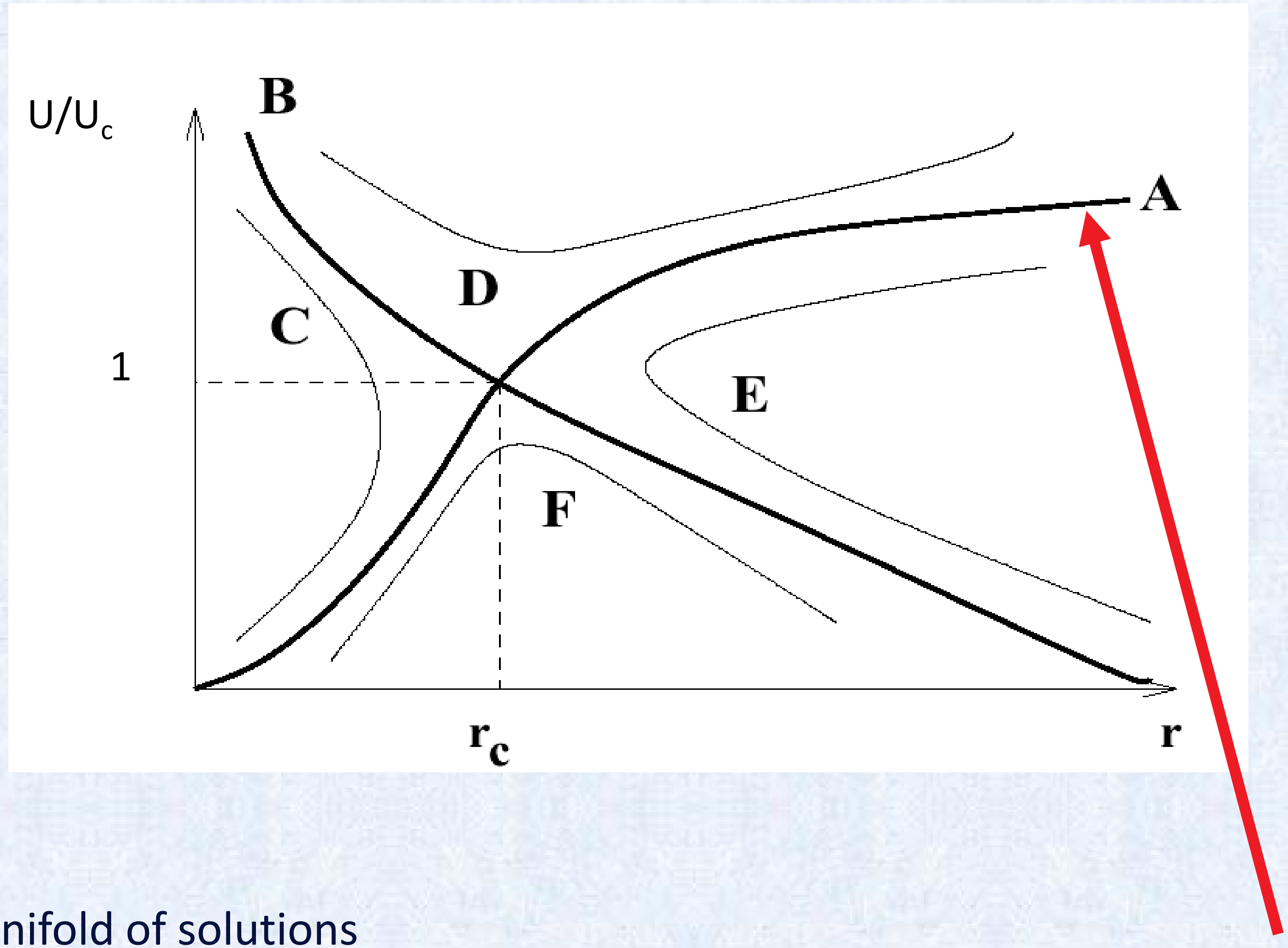
$$r_c = \frac{GM_{Sun}m_H}{4kT} \quad \text{(critical point)}$$

$$\frac{dp}{dr} = 2kT\frac{dn}{dr} \qquad (p = p_p + p_e)$$

$$U_c \doteq U(r_c) = \sqrt{\frac{2kT}{m_H}} \quad \text{(critical speed)}$$

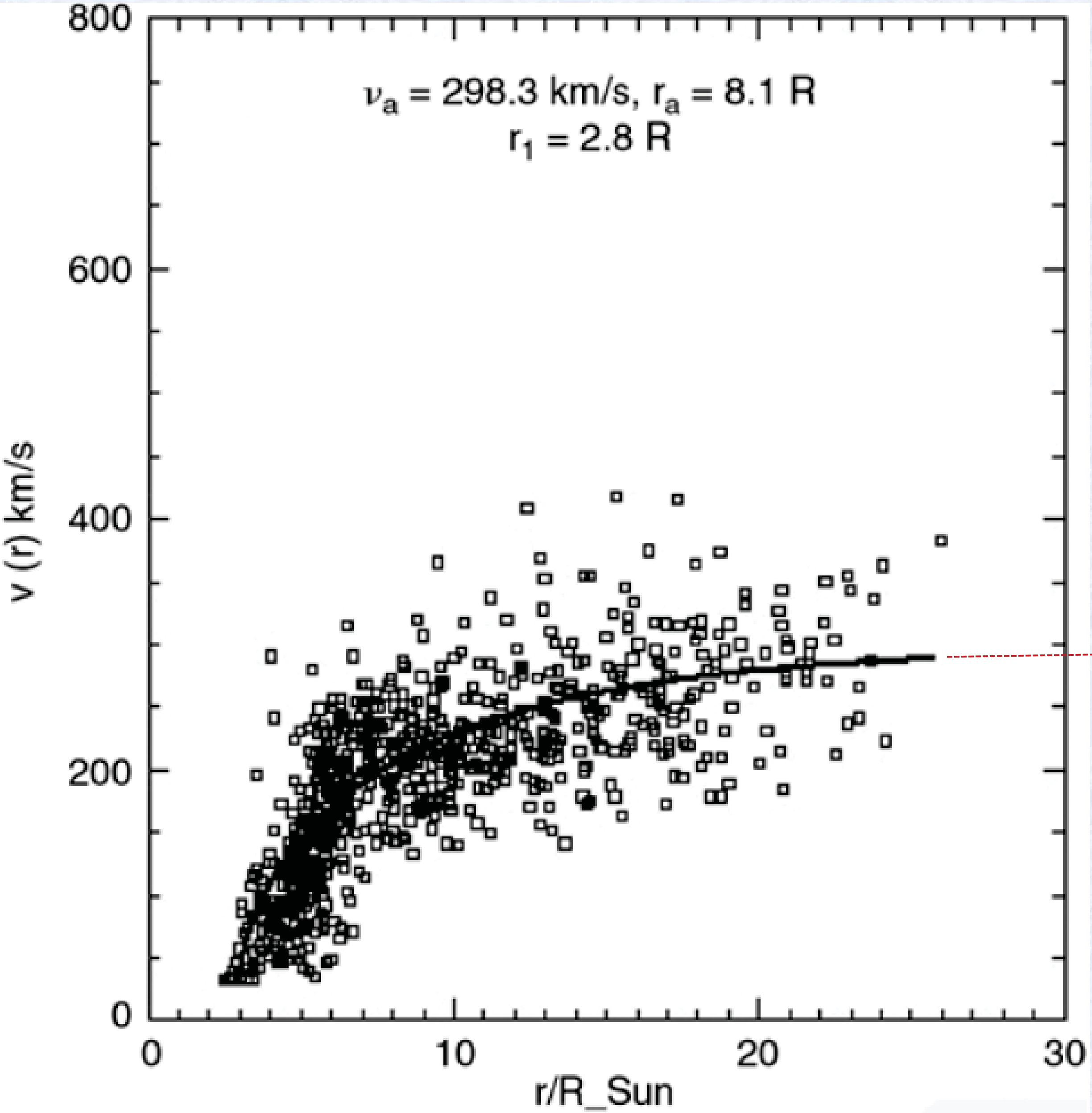
$$\nabla \bullet (n\mathbf{U}) = 0 \longrightarrow nUr^2 = const.$$

$$\ln\left(U^2\right) - \frac{U^2}{U_C^2} + 4\left(\frac{r_c}{r} + \ln r\right) = C$$



- Manifold of solutions
- U monotonically from sub-sonic to super-sonic in SW → branch **A**
- For the SW: r_c ~ 10 R_{Sun} ~ 0.05 AU
- U → constant for $r >> r_c$

Observed profile of SW speed



Observations
(difference images from
SOHO LASCO)
of plasma traces

Fitted model given by
Sheeley et al.[1997]

Figure from
McComas et al.,
Rev. of Geophysics [2007]

Velocities in agreement
with in situ observations
(Helios S/C) near 0.3 AU

A polytropic model ($p\rho^{\gamma}$=const) gives a similar result for $U(r)$, but also provides $T(r)$

Some asymptotic results ($r \longrightarrow \infty$):

$$U \sim \text{const}, \quad \rho \sim r^{-2}, \quad T \sim r^{2(1-\gamma)}, \quad p \sim r^{-2\gamma}$$

The real SW is not isothermal (it gets cooler while it expands)

And it is not adiabatic (a parcel of fluid increase internal energy)

Main mechanisms to increase SW internal energy:

-Turbulent dissipation

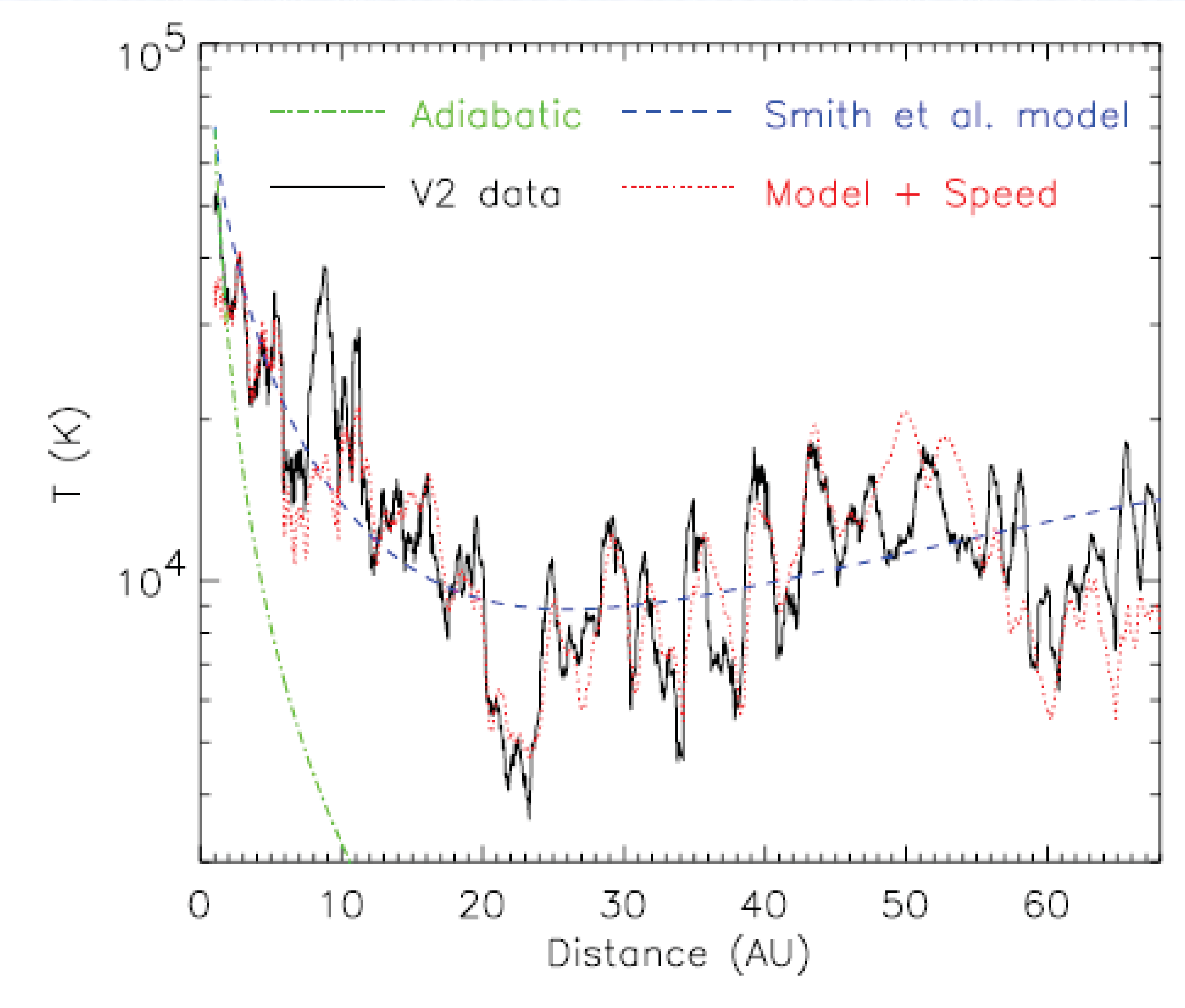
-Electron heat fluxes ($v_{th,e}$>>Vsw, while $v_{th,p}$<<Vsw)

Then, $1 < \gamma < 5/3$ is expected

isothermal

adiabatic

Local heating in the Solar Wind



[From Richardson & Smith, GRL, 2003]

Typical parameters of the SW at 1AU near solar minimum (from in situ observations) [Schwenn, 1990]

	Low speed wind (LSM)	Fast wind (HSS)
Flow speed v_p	$250-400$ km s^{-1}	$400-800$ km s^{-1}
Proton density n_p	10.7 cm^{-3}	3.0 cm^{-3}
Proton flux density $n_p\ v_p$	3.7×10^8 cm^{-2} s^{-1}	2.0×10^8 cm^{-2} s^{-1}
Proton temperature T_p	3.4×10^4 K	2.3×10^5 K
Electron temperature T_e	1.3×10^5 K	1×10^5 K
Momentum flux density	2.12×10^8 dyne cm^{-2}	2.26×10^8 dyne cm^{-2}
Total energy flux density	1.55 erg cm^{-2} s^{-1}	1.43 erg cm^{-2} s^{-1}
Helium content n_p/n_{He}	2.5%, variable	3.6%, stationary

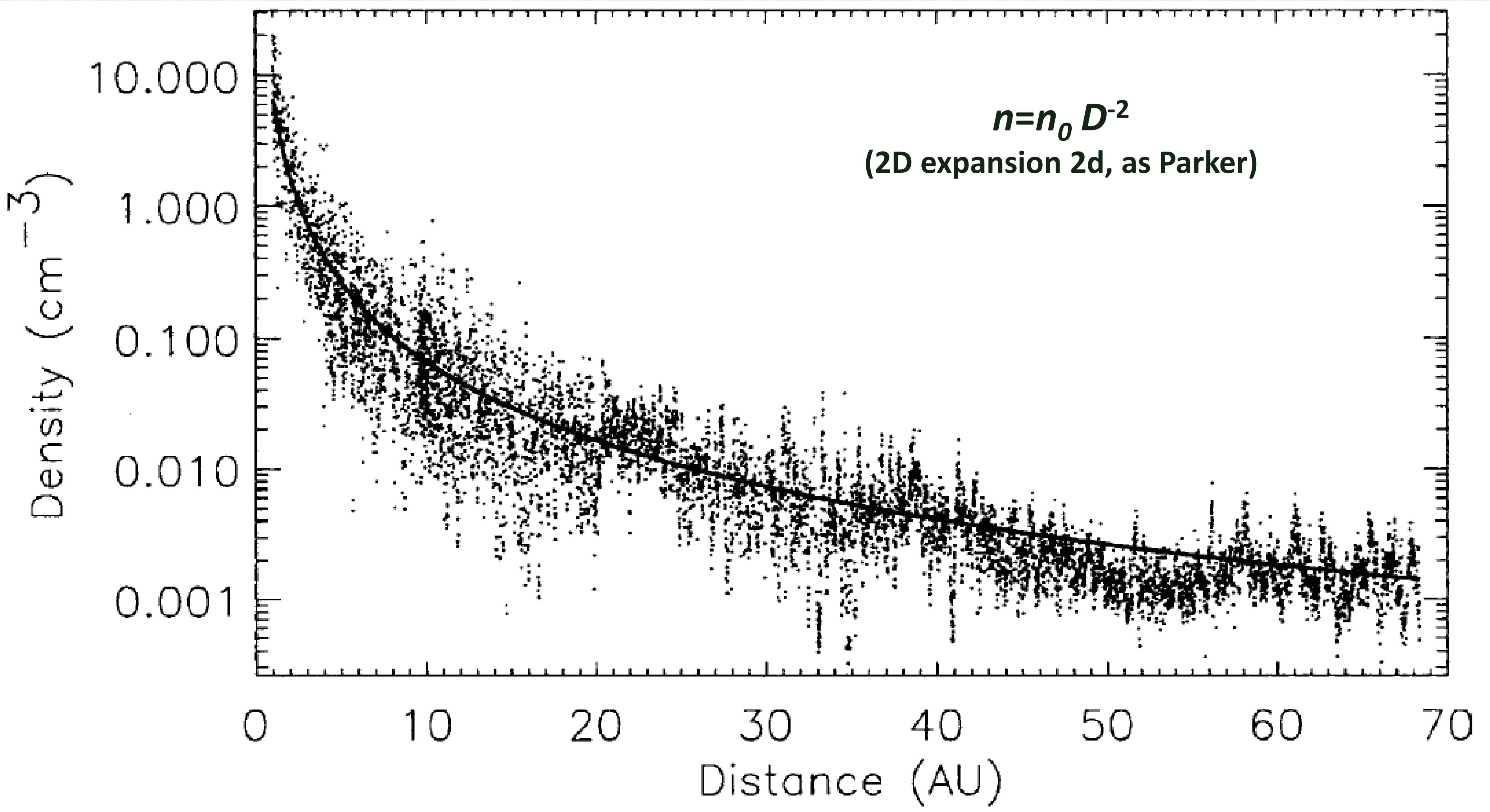
Stationary simplified Solar Wind

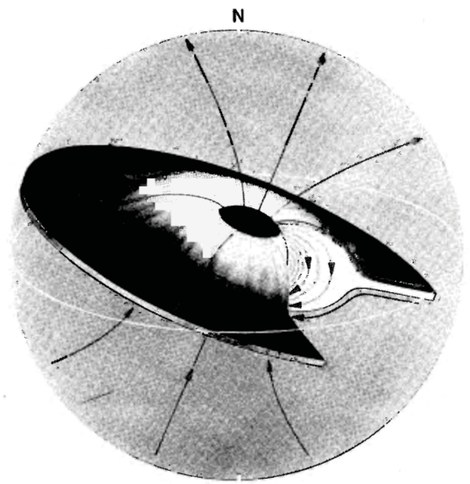
Assuming V_r=cte (2D expansion)

Conservation of mass → $n_p \sim D^{-2}$

[From Richardson et al., ASR, 2004]

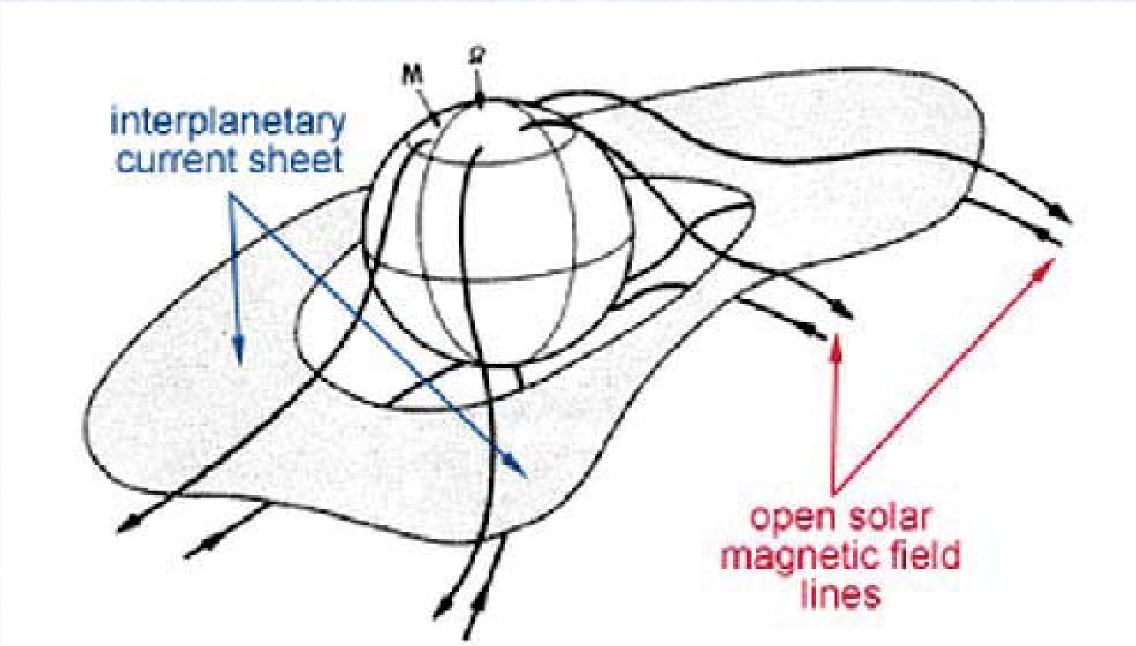
Observations from Voyager



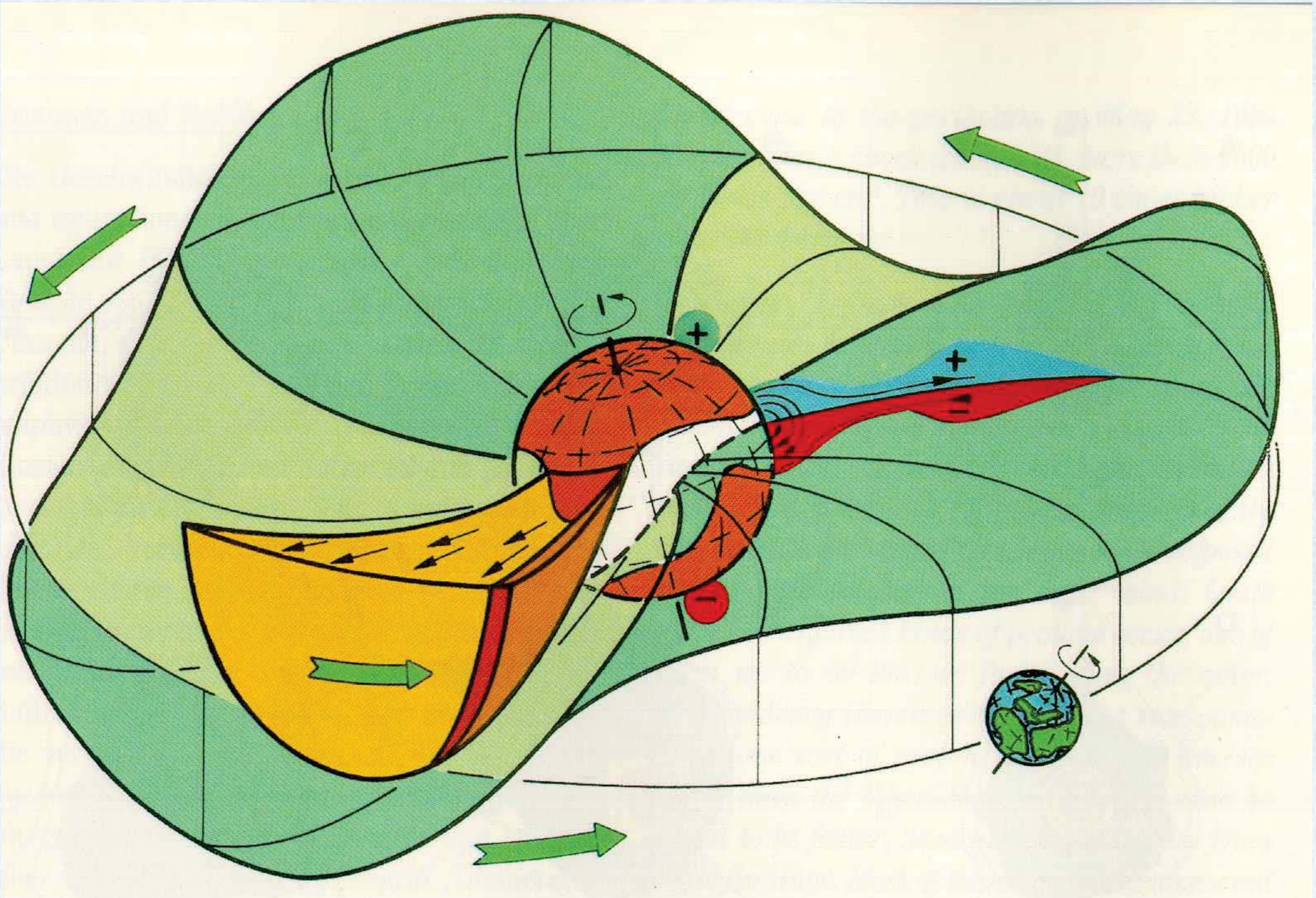


Distorted (advected) dipolar configuration

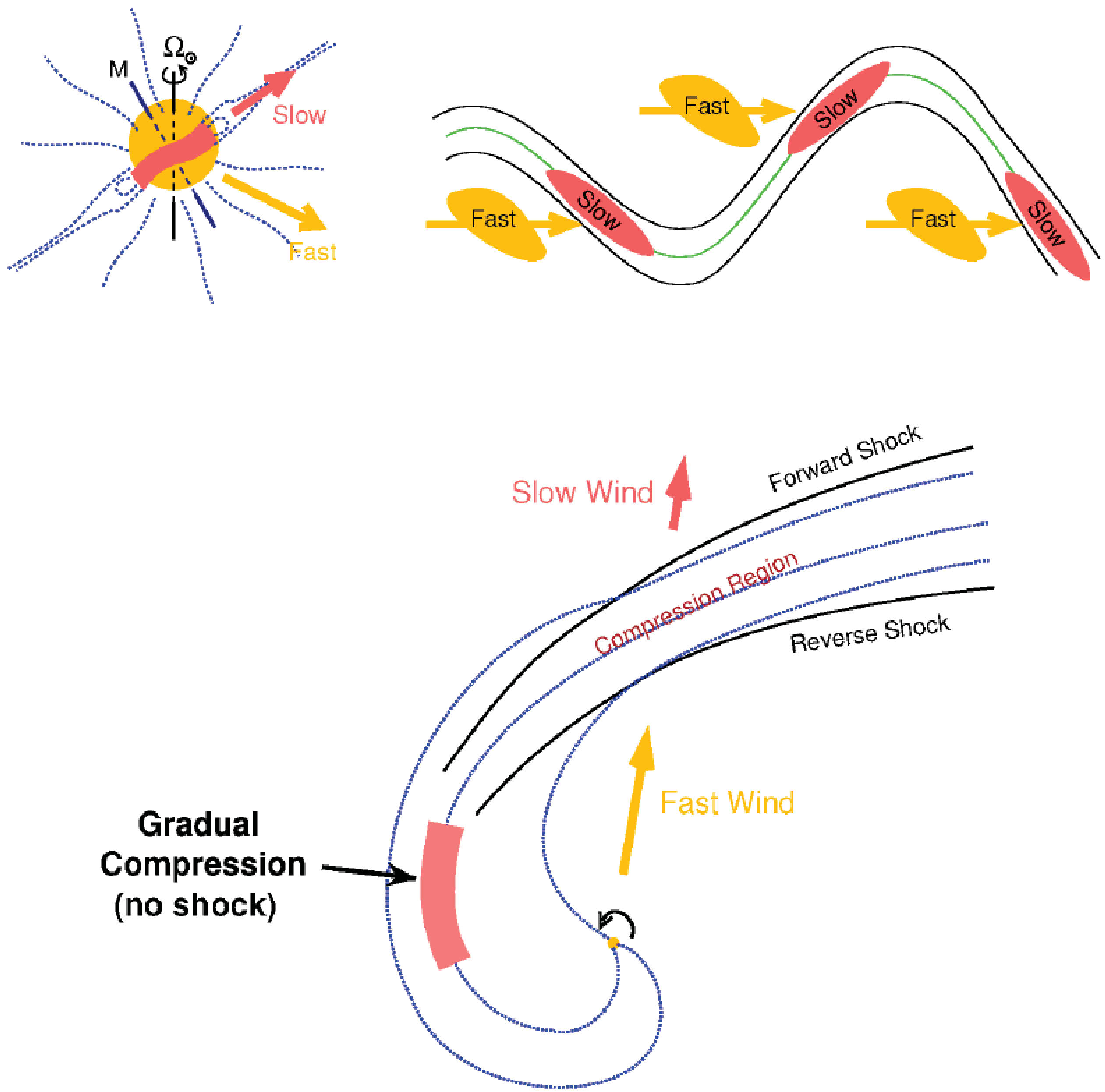
The Sun rotates and there is a tilt between magnetic dipole and Ω

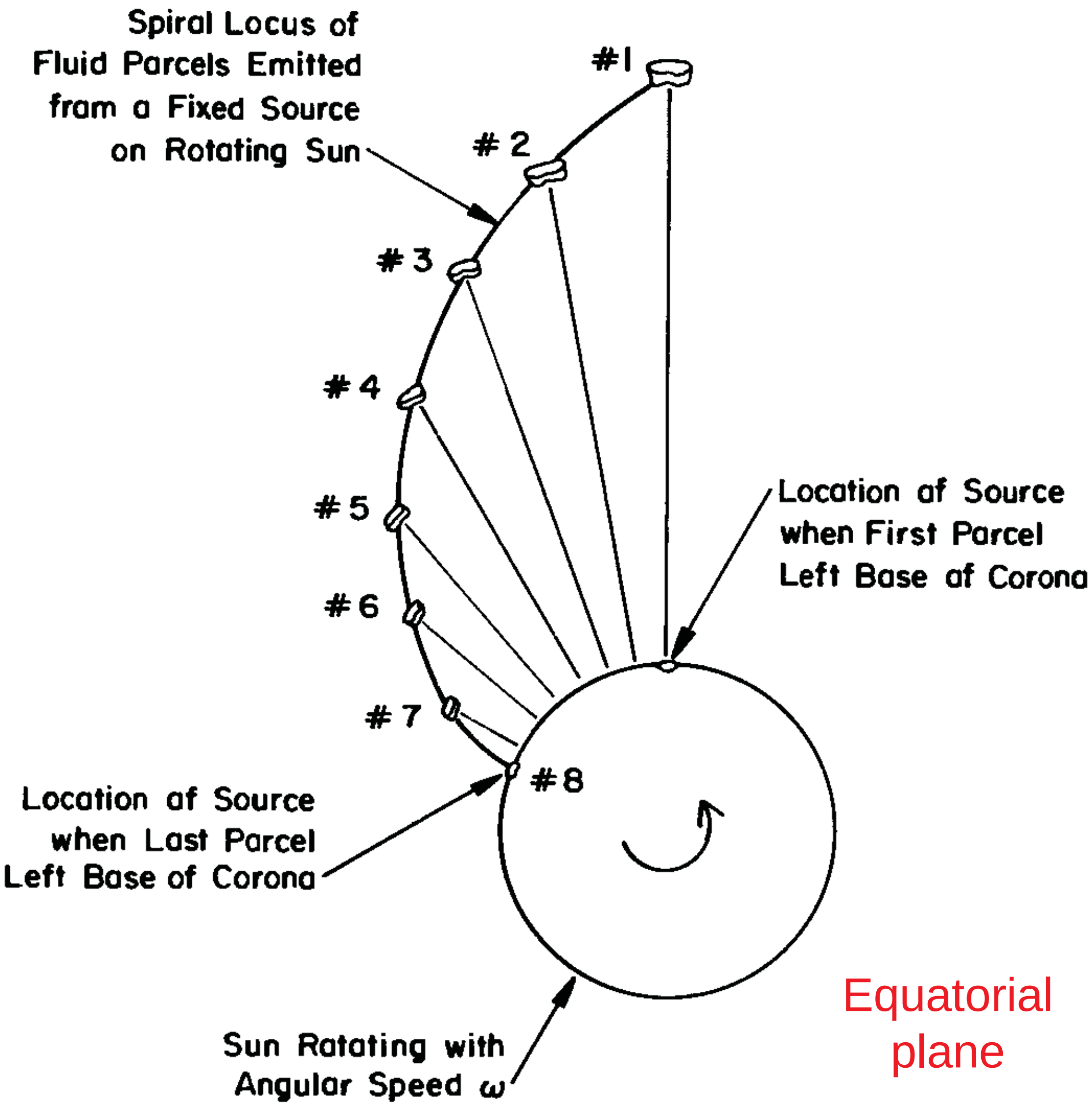


Corotating Interaction Regions



Co-rotating Interaction Regions



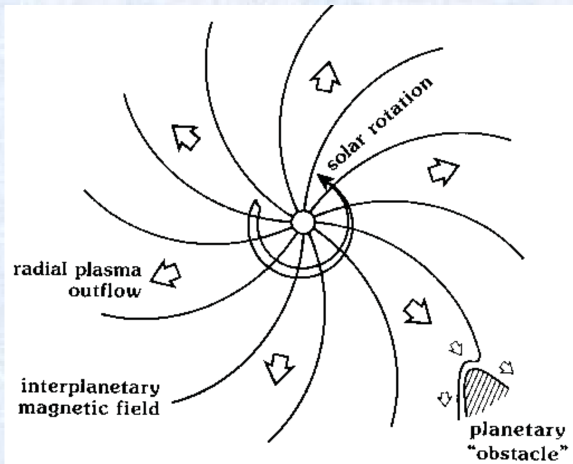


Equatorial plane

Parker Spiral from:

- Solar rotation
- Radial flow
- 'Frozen-in' condition

α~45° for 1AU



$$\frac{B_\phi}{B_r} = \tan(\alpha) = \frac{\Omega\, D}{U_{sw}}$$

Stationary simple Solar Wind at ecliptic
Assuming ideal MHD and V_r=constant

Conservation of magnetic flux in an elementary fluid parcel

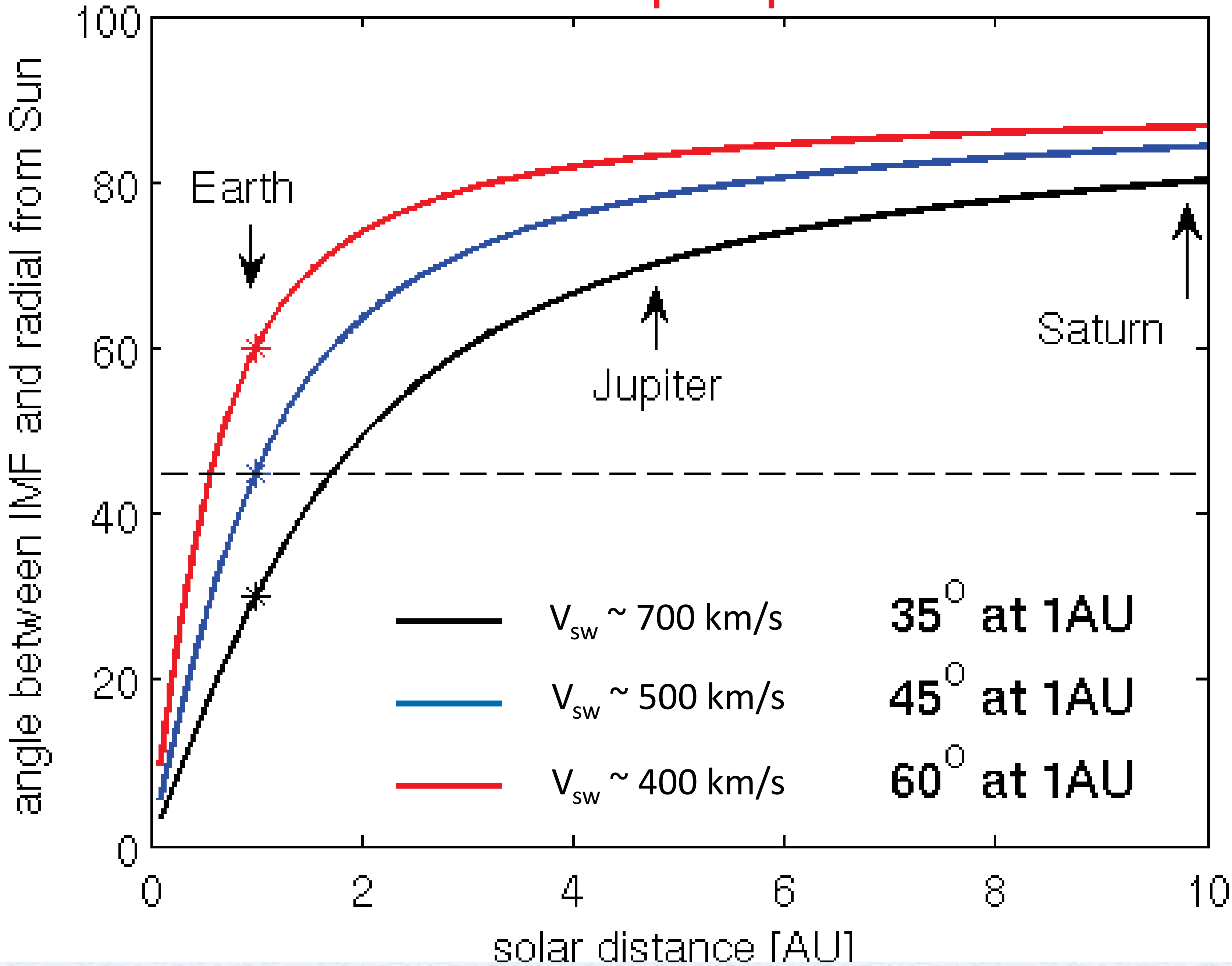
→ $B_r \sim D^{-2}$ & $B_\phi \sim D^{-1}$

Then $B \sim \mathrm{sqrt}(D^{-2} + D^{-1})$

Spiral angle: $\tan(\alpha) = B_\phi / B_r \sim D$

Note that at ecliptic plane $B_z \sim 0$

At the ecliptic plane

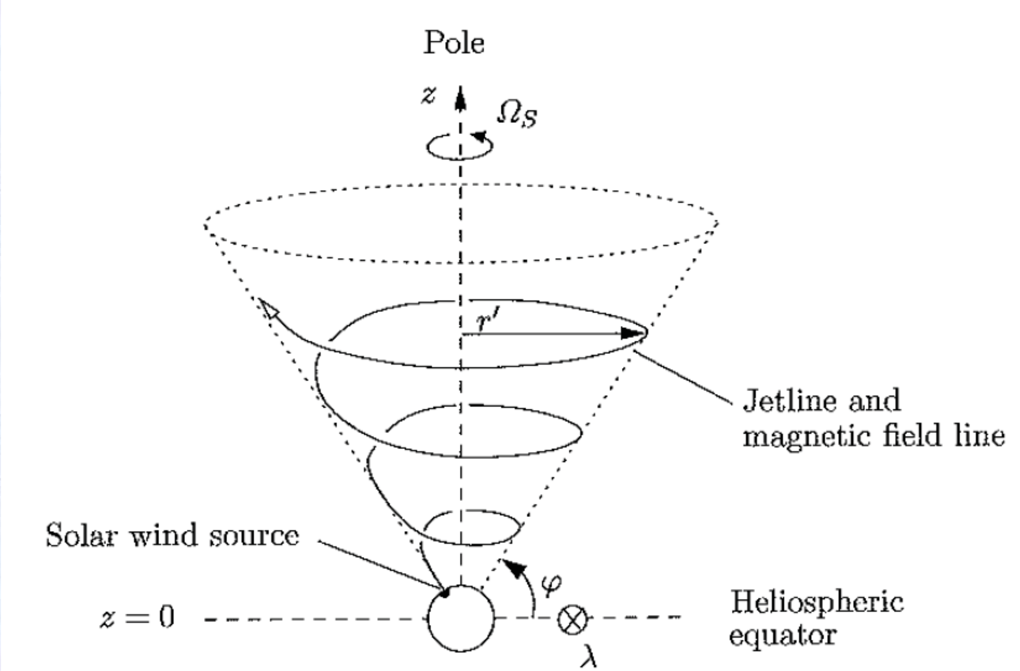


Interplanetary Magnetic Field (3D) in stationary Solar Wind (simple model)

$$\mathbf{B} = B_r(r)\hat{\mathbf{r}} + B_\lambda(r)\hat{\boldsymbol{\lambda}}$$

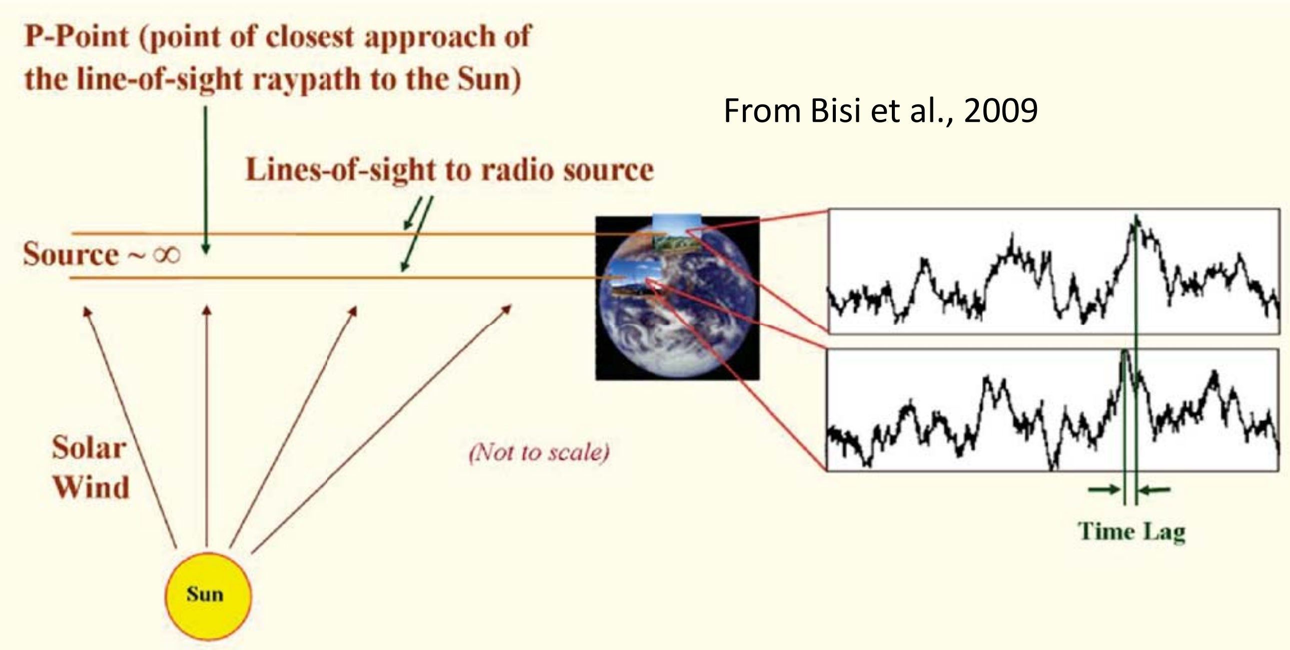
$$B_r(r) = B_0\left(\frac{r_0}{r}\right)^2$$

$$B_\lambda(r) = -B_0 \frac{r_0\,\Omega_{Sun}}{U_{sw}}\left(\frac{r_0}{r}\right)\cos(\varphi)$$



$$B(r) = B_0\left(\frac{r_0}{r}\right)^2\sqrt{1+\frac{r^2\,\Omega_{Sun}{}^2\cos^2(\varphi)}{U_{sw}{}^2}} \xrightarrow[r>>1AU]{} \frac{B_0 r_0{}^2\Omega_{Sun}\cos(\varphi)}{rU_{sw}}$$

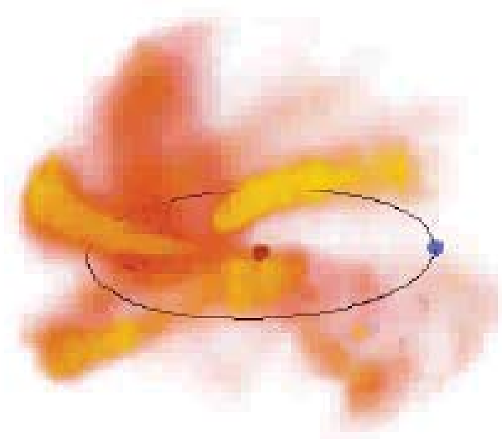
Radio IP scintillation (IPS)



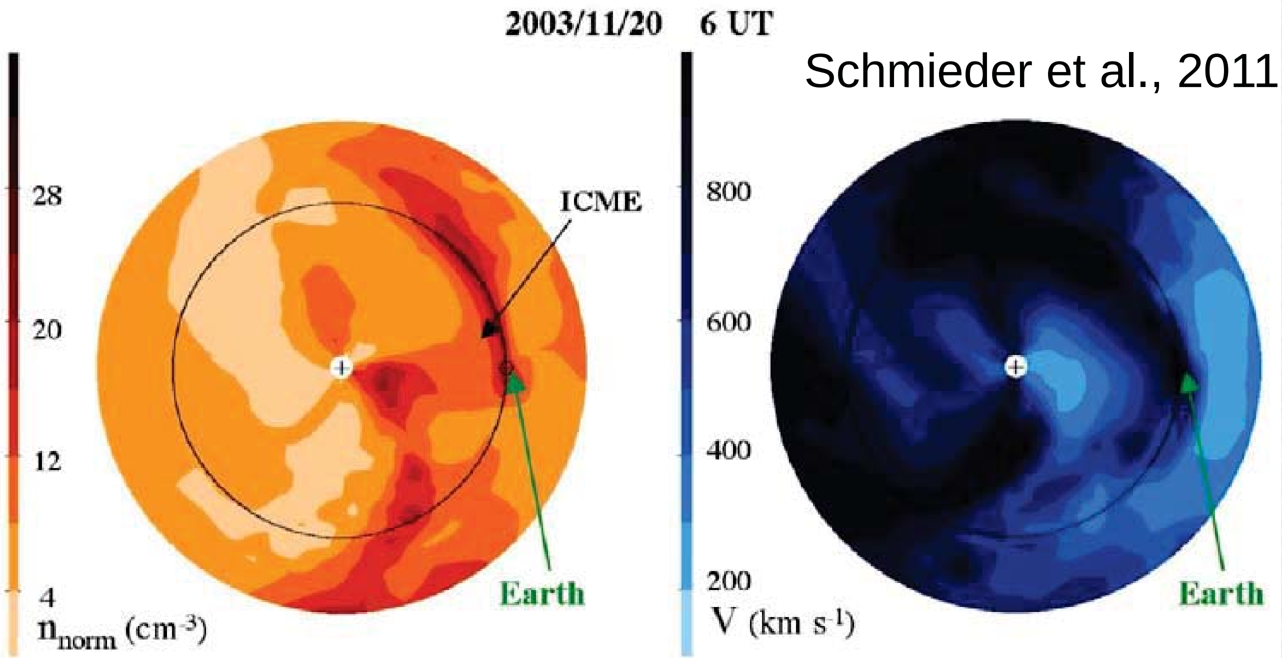
From Bisi et al., 2009

Courtesy from Bernie Jackson

June 23, 1994 to July 20, 1994
(Carrington Rotation 1884)

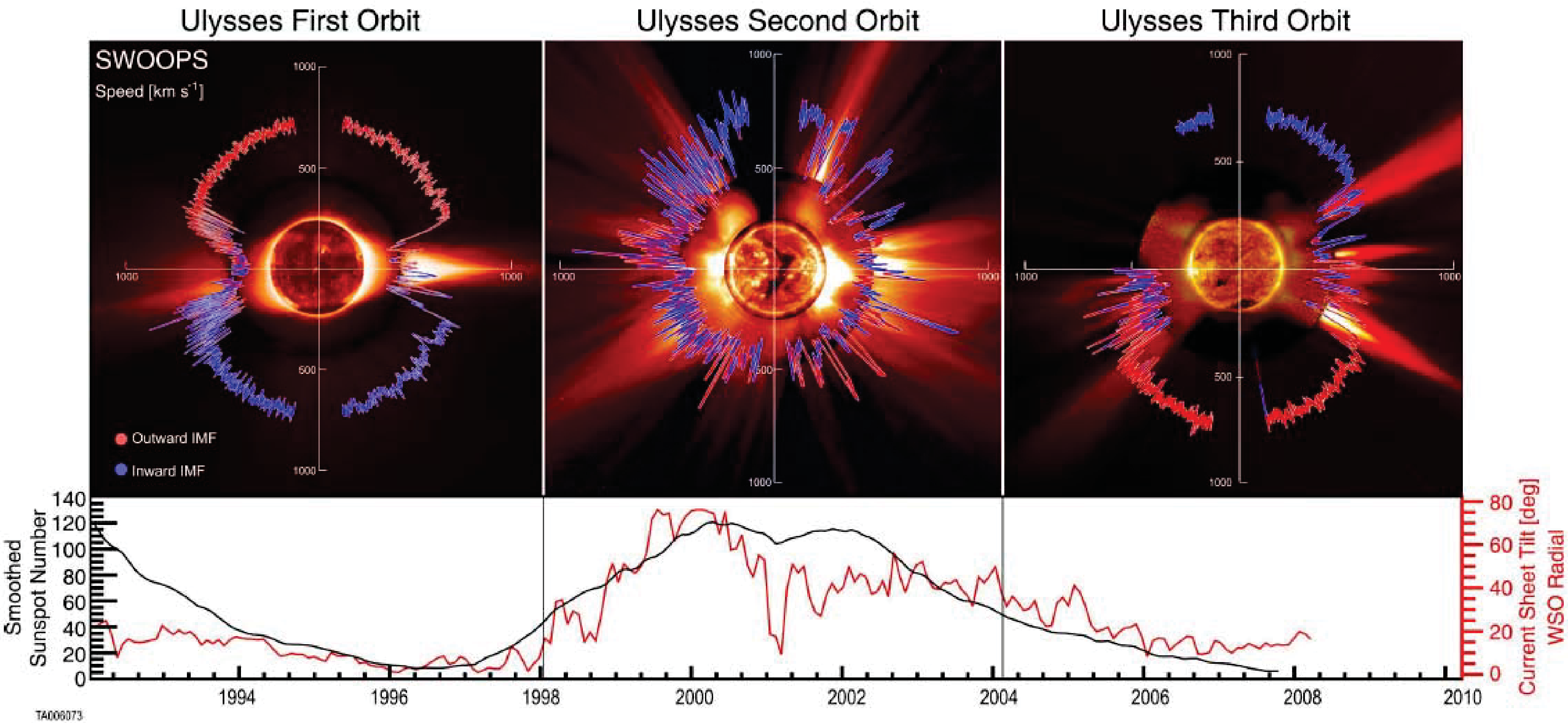


Parker spiral from compression (increase of mass density)



Schmieder et al., 2011

The Solar Wind along the solar cycle



From Gosling, 2010