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Introduction to Accretion Phenomena in Astrophysics - Spherical Accretion

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Elementary Intrtoduction to Accretion Flows Lecture II: Spherical Hydrodynamic Accretion

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Abstract

In this lecture, after specifiying main theoretical aspects of accretion flow models in astrophysics, I consider hydrodynamic spherical accretion problem (Bondi, 1952) that allows exact analytical inspection. Main features of this model and its solutions are derived and discussed.

1 Introduction

- **Accretion** the process by which compact stars gravitationally capture ambient matter;
- Accretion of gas on a **compact** ($M \sim M_\odot = 2 \times 10^{30} kg$) **star** likely source of energy in the binary X-ray sources;
- **Quasars and AGN's** same process onto **supermassive black holes with** $M \simeq 10^6 - 10^9 M_\odot$;
- Isolated compact stars: with $M \sim M_\odot$: accrete gas as they wander through the interstellar medium;
- General Relativistic **Accretion: probably the most efficient cosmic energy source in the Universe!**
- **WHY?!** Because falling through the steep gravitational potential of a neutron star or a black hole $\sim 1/10$ of the accreted rest-mass energy may be converted into radiation;
- If the effective mean free path for particles, ℓ, is much shorter than the accretion flow spatial length scale, L:

$$\ell << L, \tag{1}$$

  In this case the accretion flow is hydrodynamical in nature;
- **BUT**: Calculations of the accretion flow onto a compact star and the emitted radiation is terribly difficult;

- **WHY?** The difficulties are related with the following issues:

  1. One must determine the **flow geometry**. If the gas possesses intrinsic angular momentum the flow will be two-dimensional (2D) or three-dimensional (3D), depending on the flow symmetry. In the simplest cases: the flow may be spherical (no mean motion of gas far from an accreting star); disc-like flow (in axisymmetric ∂_φ=0 case). Both these assumptions simplify calculations enormously!
  2. What are **dominant heating and cooling mechanisms**? If the gas is *optically thick* radiation has to be determined *self-consistently*;
  3. The possible **role of magnetic fields**: might be significant both for dynamics and radiation field;
  4. The effect of the **radiation pressure**;
  5. Understanding the flow **boundary conditions**: both outer ones (where the accreting gas joins on to the ambient medium) and at the inner boundary, at the stellar surface, where the gas merges smoothly into the star;
  6. **CONCLUSION**:In the general accretion case one must deal with time-dependent, multidimensional, relativistic, MHD equations with coupled radiation transfer; No wonder: the problem of gas accretion onto compact star has been solved only in a few idealized cases;

- Strangely enough: even the **most idealized models**, based on most over-simplified set of assumptions, yield results in qualitative agreement with observational data;

- **Idealized accreion models**: black hole accretion absence of the stellar surface and magnetic filed, the presence of the event horizon ("vacuum cleaner" boundary conditions); Usually self-gravity and slow increase of the central mass are also neglected.

2 Spherical, hydrodynamic accretion

- It is expected that both in interstellar medium and in the plasma transferred between binary stars **the accretion flow onto compact objects will be hydrodynamical in nature**.

- **What couples particles? Collisions, macroscopically weak magnetic fields, plasma instabilities**, other plasma collective effects - keep the effective particle mean free path small, ensuring hydrodynamical flow.

- Let us consider accretion of ambient gas onto a stationary star of mass M in the fluid limit.

- The gas flow is assumed to be **adiabatic**: i.e., **entropy losses due to radiation are neglected**.

- **Main Assumptions:**

  1. **Stationary flow**:
  $$\partial_t = 0, \tag{2}$$

2. **Spherical symmetry**:

$$\partial_\varphi = \partial_\theta = 0, \tag{3}$$

- **Boundary conditions**

$$V_\infty = 0; \qquad P_\infty \neq 0, \qquad \rho_\infty \neq 0, \qquad C_\infty \neq 0, \tag{4}$$

- **Equation of state**:

$$P = K\rho^\Gamma, \qquad K, \Gamma \quad constants, \tag{5}$$

- The **sound speed** is defined as:

$$\mathcal{C} \equiv \left[\frac{dP}{d\rho}\right]^{1/2} = \left[\Gamma\frac{P}{\rho}\right]^{1/2}, \tag{6}$$

- Essential characteristics of the flow are quite well described within the feamework of **Newtonian gravity**, especially good this approximation is at distances as large as:

$$r \gg GM/c^2, \tag{7}$$

- The peculiar feature of accretion onto a black hole: the black hole imposes distinct regularity conditions at small radii near $r = 2GM/c^2$. These conditions are evaluated by using **General Relativistic** theory. This solution gives the maximum **accretion rate** $\dot{M}$.

- For stars with **surface -** the accretion rate depends on the boundary conditions near the surface;

- The flow is governed by the followng pair equations:

1. **The continuity equation:**

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\rho u\right) = 0, \tag{8}$$

which can be integrated, yielding the equation for the accretion rate:

$$\dot{M} \equiv 4\pi r^2 \rho u = const, \tag{9}$$

alternatively (8) can be written also as:

$$\frac{2}{r} + \frac{\rho'}{\rho} + \frac{u'}{u} = 0, \tag{10}$$

2. **The Euler equation:**

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2}, \tag{11}$$

Taking into account (5) we ca rewrite this equation as:

$$uu' + \mathcal{C}^2\frac{\rho'}{\rho} + \frac{GM}{r^2} = 0, \tag{12}$$

Besides, (11) can be integrated and reduced to the following algebraic equation:

$$\frac{u^2}{2} + \frac{1}{\Gamma - 1}\mathcal{C}^2 - \frac{GM}{r} = \frac{1}{\Gamma - 1}\mathcal{C}^2_\infty. \tag{13}$$

- From (10) and (12) we can derive the following interesting equations:

$$u' = \frac{D_1}{D}, \tag{14}$$

$$\rho' = -\frac{D_2}{D}, \tag{15}$$

where

$$D_1 \equiv \frac{2\mathcal{C}^2/r - GM/r^2}{\rho}, \tag{16}$$

$$D_2 \equiv \frac{2u^2/r - GM/r^2}{u}, \tag{17}$$

$$D \equiv \frac{u^2 - \mathcal{C}^2}{\rho u}, \tag{`18}$$

From these equations it follows that in order to ensure the smooth, monotonic increase of u and ρ with decreasing r the solution has to pass through the critical, **"sonic point"**:

$$D_1 = D_2 = D = 0, \quad at \quad r = r_s, \tag{19}$$

and r_s the **critical point**, which corresponds to the **transsonic distance**, where the flow radial velocity equals the sound speed, we have:

$$u_s^2 = \mathcal{C}_s^2 = \frac{GM}{2r_s}, \tag{20}$$

- We can connect physical variables at the sonic point with the known value of the sound speed at infinity.

$$\mathcal{C}_s^2 = u_s^2 = \left(\frac{2}{5-3\Gamma}\right)\mathcal{C}_\infty^2, \tag{21}$$

$$r_s = \left(\frac{5-3\Gamma}{4}\right)\frac{GM}{\mathcal{C}_\infty^2}, \tag{22}$$

- Let us determine how the density at the sonic point depends on the density at infinity. We obtain:

$$\rho_s = \rho_\infty \left(\frac{\mathcal{C}_s}{\mathcal{C}_\infty}\right)^{2/(\Gamma-1)}, \tag{23}$$

- It is important to calculate the accretion rate and determine how it depends on the position of the sonic point. Straightforward but tedious calculations lead to:

$$\dot{M} = 4\pi\lambda_s \left(\frac{GM}{\mathcal{C}_\infty^2}\right)^2 \rho_\infty \mathcal{C}_\infty, \tag{24}$$

where:

$$\lambda_s \equiv \left(\frac{1}{2}\right)^{\frac{\Gamma+1}{2(\Gamma-1)}} \left(\frac{5-3\Gamma}{4}\right)^{-\frac{(5-3\Gamma)}{2(\Gamma-1)}}, \tag{25}$$

The transsonic flow asymptotical behaviour is easy to calculate:

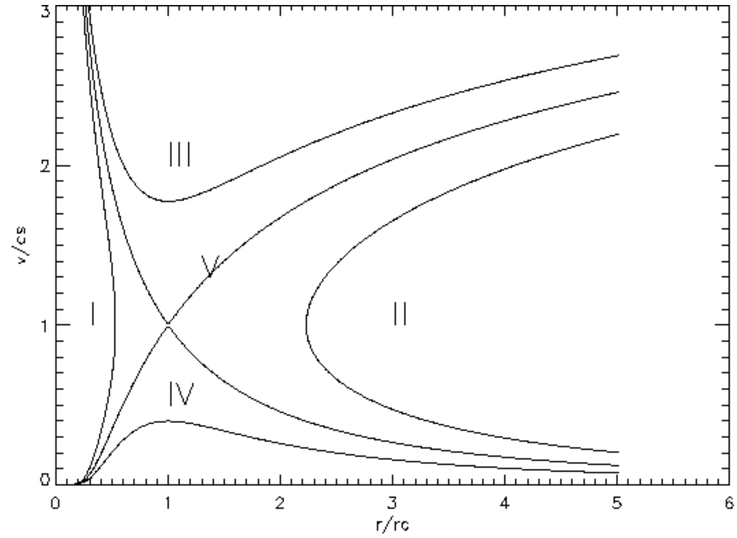

Figure 1: Different regimes of spherical accretion and stellar wind.

– At $r \gg r_s$ - the gravitational potential is hardly felt at large distances and thermodynamic quantities are approximately equal to their asymptotic values at infinity:

$$\rho \approx \rho_\infty, \qquad T \approx T_\infty, \qquad \mathcal{C} \approx \mathcal{C}_\infty, \tag{26}$$

for the radial velocity we derive the following asymptotic expression:

$$\frac{u}{\mathcal{C}_\infty} \approx \lambda_s \left(\frac{GM}{\mathcal{C}_\infty^2}\right)^2 r^{-2}, \tag{27}$$

– Inside the sonic surface, at $r \ll r_s$, close enough to the central object, gravitational forces are significant and in (13) the gravitational potential term on the left hand side is much larger than the $\mathcal{C}^2/(\Gamma - 1)$ term. Therefore the decceleration due to gas pressure is negligible and the inflow speed approaches the free fall limit for these particles:

$$u \approx \left(\frac{2GM}{r}\right)^{1/2}, \tag{28}$$