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Elementary Introduction to Accretion Discs

Andria Rogava

Georgian National Astrophysical Laboratory, Tbilisi
Georgia

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

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Andria Rogava
Centre for Theoretical Astrophysics
Institute of Theoretical Physics
Ilia State University

The Big Dipper

1. Introductory heuristics

1. The matter accreting onto a compact object will have a significant angular momentum and may form a disc;
2. The gas elements in the disc lose angular momentum due to friction between adjacent layers;
3. A part of the released gravitational energy increases the kinetic energy – gas particles spiral inwards;
4. The other part is converted into thermal energy which is being radiated from the disc surfaces;
5. Viscosity converts gravitational potential energy into radiation.

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Noel Carboni, NCarboni@att.net
Canon EOS-20D and 17-40 zoom lens at f/4 piggybacked on 10" LX200 GPS UHTC
15 x 30 second ISO 1600 exposures

Early Studies of disc accretion

- **Von Weizsaker:** *'The rotation of cosmic gas masses'*, Z. Naturforsch, **3a,** 524 (1948)
- **Lust:** Z. Naturforsch, **7a,** 87 (1952)
- **Prendergast & Burbridge:** *'On the nature of some galactic X-ray sources'* Ap.J. Lett., **151,** L83 (1968).
- **Shakura: Astron. Zh. 49,** 921 (1972).
- **Pringle & Rees: Astron. Astrophys. 21,** 1 (1972).
- **Shakura & Sunyaev: Astron. Astrophys. 24, 337** (1973).

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Where do they occur?

- Active galaxies
- Galactic X-ray sources
- Cataclysmic variables
- Young stellar objects

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Basic build-up of the model

- (r, ϕ, z) cylindrical coordinates are used; the z-axis chosen as the axis of rotation: z=0 is the central, equatorial plane of the disc;
- the disc is assumed to be **axisymmetric**:

$$\partial_{\phi} = 0$$

- The disc is supposed to be **geometrically thin**:

$$h(r) \ll r$$

- the **total pressure** is the sum of the **gas** and **radiaton pressures**:

$$P_{tot} = P_g + P_{rad} = \frac{k}{\mu m_H}\rho T + \frac{a}{3}T^4$$

- the **specific internal energy** is:

$$\varepsilon = C_V T + \frac{a T^4}{\rho}$$

1. Mass conservation

- The continutiy equation:

$$\partial_t \rho + \frac{1}{r}\partial_r (r\rho v_r) + \partial_z (\rho v_z) = 0$$

After we define the **Surface density** as:

$$S(r,t) \equiv \int \rho dz$$

leads to:

$$\partial_t S + \frac{1}{r}\partial_r (rSv_r) = 0$$

2. Viscosity tensor

- In the standard disc model it is assumed that the main (only?) nonzero component of the viscosity tensor is:

$$t_{r\varphi} = \eta r \partial_r \left(\frac{v_\varphi}{r} \right) = \eta r \frac{d\Omega}{dr}$$

And its vertically averaged value is:

$$W_{r\varphi} \equiv \int t_{r\varphi} dz$$

3. Angular momentum conservation

- From the **azimuthal component** of the equation of motion, taking into account the continuity equation and integration over z we derive:

$$S\left[\partial_t(rv_\varphi)+v_r\partial_r(rv_\varphi)\right]=\frac{1}{r}\partial_r\left(r^2W_{r\varphi}\right)$$

4. Radial momentum conservation

Initial equation has the form:

$$\rho\left(\partial_t v_r + v_r \partial_r v_r\right) = \rho\left(\frac{v_\varphi^2}{r} - \partial_r \Phi\right) - \partial_r P$$

After z-integration it reduces to:

$$S\left(\partial_t v_r + v_r \partial_r v_r\right) = S\left(\frac{v_\varphi^2}{r} - \frac{GM}{r^2}\right) - \partial_r W$$

where

$$W \equiv \int P dz$$

5. Energy conservation

- Initial equation is:

$$\rho\left(\partial_t + v_r\partial_r\right)\left[\frac{v_r^2}{2} + \frac{v_\varphi^2}{2} + h + \Phi\right] = \partial_t P + \frac{1}{r}\partial_r\left(r t_{r\varphi} v_\varphi\right) - \partial_z F$$

after integration leading to:

$$S\left(\partial_t + v_r\partial_r\right)\left[\frac{v_r^2}{2} + \frac{v_\varphi^2}{2} + (A+1)\frac{W}{S} + \Phi\right] = \partial_t W + \frac{1}{r}\partial_r\left(r W_{r\varphi} v_\varphi\right) - Q^-$$

- the last term on the left hand side is the cooling rate
- Note that the dissipation function is defined as:

$$\mathrm{T} \equiv 2 t_{r\varphi}\theta_{r\varphi} = t_{r\varphi} r \frac{d\Omega}{dr}$$

and consequently:

$$Q^+ \equiv \int \mathrm{T} dz = W_{r\varphi} r \frac{d\Omega}{dr}$$

is the energy produced per unit area ('heating rate').

6. Vertical hydrostatic balance

- **Vertical hydrostatic balance** in the disc:

$$\partial_z P = -\rho \frac{GM}{r^2} \frac{z}{r}$$

- from this equation it follows that **circular motion in the accreton disc is highly supersonic**!

$$z_0 / r \approx c_s / v_\varphi$$

- In the **standard disc model** it is assumed that the energy dissipated into heat is totally radiated in the vertical direction: $\partial_z F = \mathrm{T} = t_{r\varphi} r \partial_r \Omega$
- Which, in other words, means **exact balance of cooling and heating rates**:

$$Q^+ = Q^-$$

7. Keplerian limit

- **Keplerian limit**
- $v_z \ll v_r \ll v_{phi}$
- **Circular** and **angular velocities**:

$$v_\varphi = r\Omega \qquad \Omega \equiv \left(\frac{GM}{r^3}\right)^{1/2}$$

- **Angular momentum conservation** reduces to:

$$\dot{M}\,\Omega r^2 + 2\pi r^3 \frac{d\Omega}{dr}\int \eta\, dz = const$$

- In this approximation cooling and heating rates are indeed equal to each other.

8. Steady discs

- Continuity:

$$\dot{M} \equiv -2\pi r S v_r = const$$

- Angular momentum:

$$\dot{M}\left[I(r) - I(r_0)\right] = -2\pi r^2 W_{r\varphi} = 3\pi I(r)\int \eta dz$$

- From this equation we find remarkably simple equation for the vertically integrated viscosity coefficient:

$$\int \eta dz = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{r_0}{r}\right)^{1/2}\right]$$

9. Cooling rate and luminosity

- **Cooling rate** is equal to:

$$Q^{-} = \frac{3GM\dot{M}}{4\pi r^{3}}\left[1-\left(\frac{r_0}{r}\right)^{1/2}\right]$$

- While for the **luminosity** we calculate:

$$L_D = 2\pi\int_{r_0}^{\infty} Q^{-} r\,dr = \frac{1}{2}\frac{GM}{r_0 c^2}\dot{M}c^2$$

- It means that **half** of the potential energy is radiated away. The other half is in the form of kinetic energy situated outside the boundary layer.

10. Standard ‘α–model’

- **Detailed disc models can be built only if we know the viscosity law!**
- Accretion disc flow is **extremely complex**:

(a) Highly supersonic;

(b) Strongly sheared;

(c) Radiative;

(d) Large Reynolds numbers

- **Molecular viscosity** is not sufficient to generate the intense X-ray emission!

$\mathbf{v = \alpha\, C_s\, z_0}$ or alternatively: $\mathbf{W_{r\phi} = \alpha\, W}$