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Angular momentum transport in accretion disks

Gianluigi Bodo

Osservatorio Astronomico, Torino
Italy

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

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Gianluigi Bodo
INAF Astrophysical Observatory of Torino

Standard model of thin accretion disks

The problem of angular momentum transport

Angular momentum transport by waves

Angular momentum transport by outflows

Angular momentum transport by magnetorotational turbulence

Standard model of thin disks

Thermal energy is radiated efficiently and the disk is cold and geometrically thin

$$c_s << v_\phi \qquad h/r << 1$$

c_s sound speed

v_ϕ rotation velocity

h Disk thickness

r radius

We can decouple radial and vertical structureand we can neglect the pressure gradient term with respect to gravity and inertia forces

Radial equilibrium is a balance between the gravitational (due to the central object) and centrifugal forces.

The angular velocity in the disk is equal to the Keplerian value

$$\Omega = \Omega_k = \left(\frac{GM}{r^3}\right)^{1/2}$$

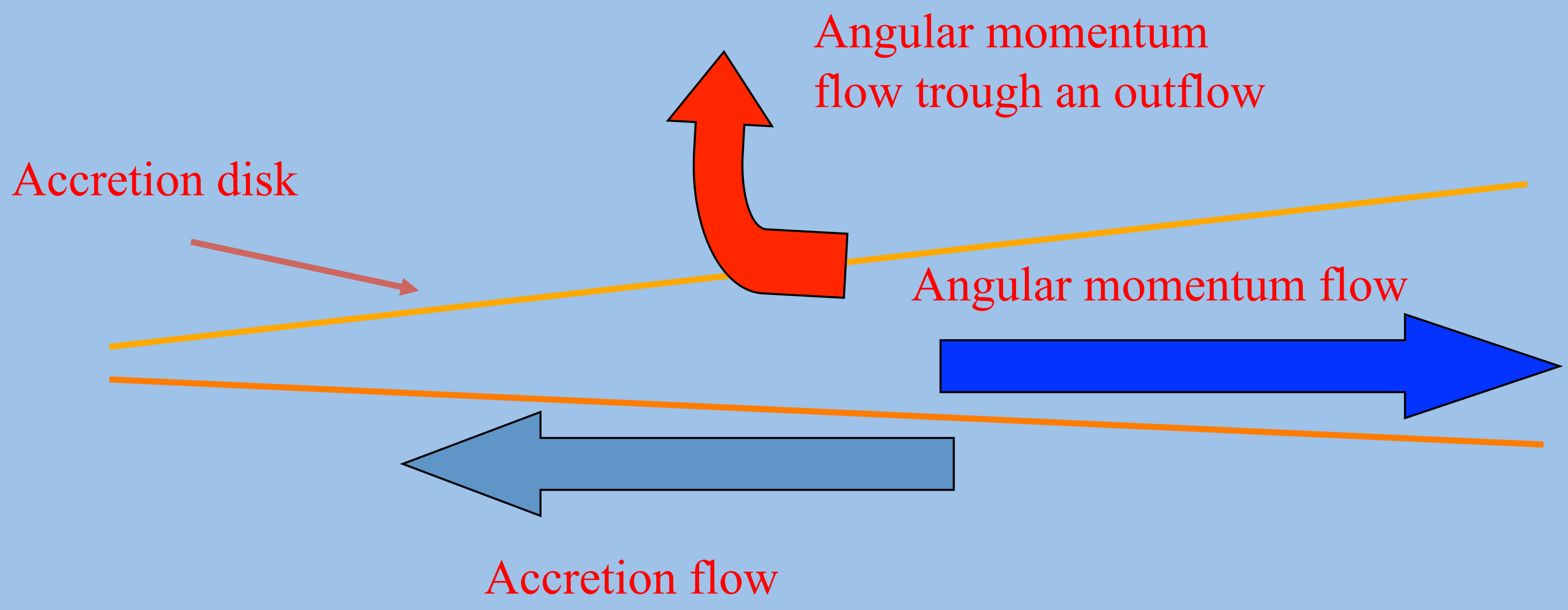
The equilibrium in the vertical direction is determined by the balance between pressure gradient and the vertical component of gravity

$$\frac{dP}{dz} = -\rho\frac{GMz}{r^3} \quad \Rightarrow \quad h/r \sim c_s/v_k$$

The distribution of specific angular momentum in the disk is

$$l_k = (GMr)^{1/2}$$

In order to accrete matter has to loose angular momentum



Viscous disks

Since the disk is accreting, in addition to the keplerian velocity it has a radial velocity component. Viscous time scale much longer than the dynamical time scale

$$v_r << c_s << v_k$$

We integrate the equations of mass and momentum conservation in the vertical direction

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Sigma v_r) = 0$$

$$\frac{\partial}{\partial t}(\Sigma r^2 \Omega) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\Sigma v_r r^2 \Omega - \nu \Sigma r^3 \frac{d\Omega}{dr}\right) = 0$$

Angular momentum conservation

Advection flux

Viscous torque

Σ surface density ν kinematic viscosity

In steady state

$$\frac{\partial}{\partial r}\left(\Sigma r^3 \Omega v_r - \nu \Sigma r^3 \frac{d\Omega}{dr}\right) = 0$$

Integrating and assuming that the stresses vanish at the inner edge $\frac{d\Omega}{dr} = 0$

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - (r_i/r)^{1/2}\right]$$

Viscosity also dissipates kinetic energy
Dissipation rate per unit area per unit time

$$Q = \frac{1}{2}\nu\Sigma\left(r\frac{d\Omega}{dr}\right)^2$$

$$Q = \frac{3GM\dot{M}}{8\pi r^3}\left[1 - (r_i/r)^{1/2}\right]$$

There is no explicit dependence on viscosity
But mass accretion rate depends on viscosity

Viscous time scale $\sim \frac{r}{v_r} \sim \frac{r^2}{\nu}$

Standard molecular viscosity $\nu \sim \lambda c_s$

$$\frac{t_{visc}}{t_{dyn}} \sim \frac{r^2\Omega}{\lambda c_s} \sim \frac{r^2}{\lambda H} \sim Re$$

Typical Reynolds number is huge

$r \sim 10^{10} cm \qquad T \sim 10^4 K \qquad n \sim 10^{16} cm^{-3} \qquad \lambda \sim 10^{-3} cm \qquad c_s \sim 10^3 cm^2 s^{-1}$

$$\nu \sim 10^3 cm^2 s^{-1} \qquad t_{visc} \sim 3 \times 10^9 yrs$$

Molecular viscosity is too low

What is the origin of viscosity?

Shakura & Sunyaev 1973 → α disk

Since Reynolds number is huge we can assume that the flow is turbulent
The dominant process for redistributing angular momentum is turbulence, we can then define an eddy viscosity

$$\nu \sim lv$$

l is the typical size and v the typical turnover velocity of the largest eddies

We take $l \sim H$ and $v \sim c_s$

$$\nu \sim \alpha H c_s$$

All our ignorance is then concentrated in the parameter α

Angular momentum transport

MHD momentum equation

$$\rho \frac{\partial \mathbf{v}}{\partial t} + (\rho \mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \left(P + \frac{B^2}{8\pi} \right) - \rho \nabla \Phi + \left(\frac{\mathbf{B}}{4\pi} \cdot \nabla \right) \mathbf{B} - \eta \nabla^2 \mathbf{v}$$

Azimuthal component in angular momentum conservation form

$$\frac{\partial}{\partial t} (\rho r v_\phi) + \nabla \cdot \left[r \left(\rho v_\phi \mathbf{v} - \frac{B_\phi}{4\pi} \mathbf{B_p} + \left(p + \frac{B_p^2}{8\pi} \right) \hat{\mathbf{e}}_\phi - \eta r^2 \nabla (v_\phi / R) \right) \right]$$

We separate the circular basic flow from the fluctuations

$$\mathbf{v} = r \Omega \hat{\mathbf{e}}_\phi + \mathbf{u}$$

Averaging over the azimuthal direction

$$\frac{\partial}{\partial t} < \rho r^2 \Omega > + \nabla \cdot \left[r \left(< \rho r \Omega \mathbf{u_p} > + \mathbf{T} \right) \right] = 0$$

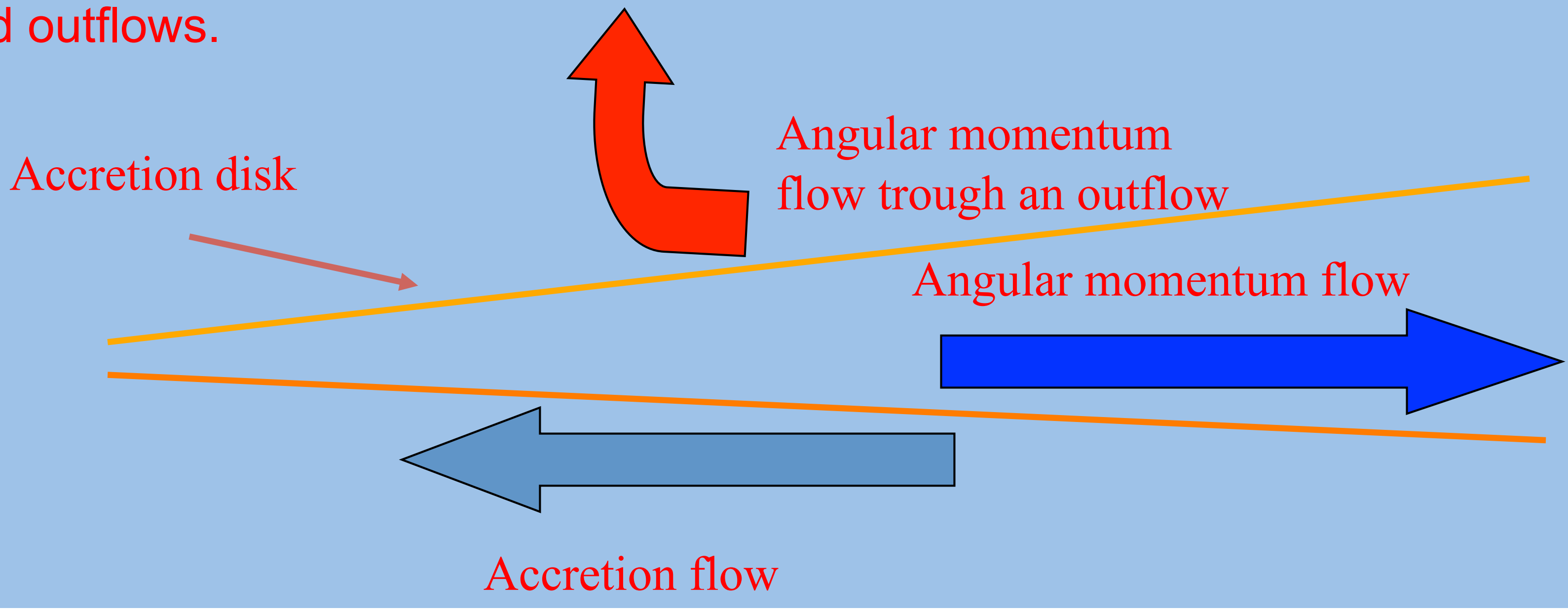
$$\mathbf{T} = < \rho u_\phi \mathbf{u_p} - \frac{B_\phi \mathbf{B_p}}{4\pi} >$$

Reynolds stresses

Maxwell stresses

The transport of angular momentum depends on the correlations between poloidal and azimuthal components of velocity and magnetic field.

This is true not only for a turbulent situation, but also in the case of waves and outflows.



Energy equation averaged over ϕ

Angular momentum transport and energy extraction from the mean flow

$$\frac{\partial E}{\partial t} + \nabla \cdot F_E = - < \left(\rho u_r u_\phi - \frac{B_r B_\phi}{4\pi} \right) > \frac{d\Omega}{dr}$$

$$E =< \frac{1}{2}\rho \left(u^2 + \Phi_{eff}\right) + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi} > \qquad \Phi_{eff} = \Phi - \int r\Omega^2 dr$$

$$F_E =< \mathbf{u} \left(\frac{1}{2}\rho u^2 + \frac{\gamma p}{\gamma - 1} + \rho \Phi_{eff} \right) + \frac{\mathbf{B}}{4\pi} \times (\mathbf{u} \times \mathbf{B}) >$$

Different mechanisms for angular momentum transport

1) Waves

   Density spiral waves

2) Outflows

   Jet acceleration

3) Turbulence

   Magnetorotational instability (MRI) driven turbulence

Waves

Consider an unmagnetized disk – Polytropic equation of state

Enthalpy function

$$\mathcal{H} = \int \frac{dp}{\rho} = \frac{c_s^2}{\gamma - 1}$$

The gas rotates (angular frequency Ω) in the gravitational field of a central mass, Axisymmetric equilibrium, use cylindrical coordinates, consider small perturbation.

A perturbed flow quantity has the form

$$\delta X = \delta X(r, z) \exp(im\phi - i\omega t)$$

Linearized equations

$$-i\tilde{\omega}\delta v_r - 2\Omega\delta v_\phi = -\frac{\partial\delta\mathcal{H}}{\partial r}$$

$$-i\tilde{\omega}\delta v_\phi + 2\frac{k^2}{2\Omega}\delta v_r = -i\frac{m}{r}\delta\mathcal{H}$$

$$-i\tilde{\omega}\delta v_z = -\frac{\partial\delta\mathcal{H}}{\partial z}$$

Momentum equation

$$-i\tilde{\omega}\frac{\delta\rho}{\rho} + \frac{1}{\rho}\nabla\cdot(\rho\delta\mathbf{v}) = 0$$

Continuity equation

$$\tilde{\omega} = \omega - m\Omega$$

$$k^2 = 4\Omega^2 + \frac{d\Omega^2}{d\ln r}$$

Epicyclic frequency

$$\delta\mathcal{H} = c_s^2\frac{\delta\rho}{\rho}$$

Negative epicyclic frequency = hydrodynamic instability

We can solve for the three velocity components

$$\delta v_r = \frac{i}{D}\left[\tilde{\omega}\frac{\partial \delta\mathcal{H}}{\partial R} - \frac{2\Omega m}{r}\delta\mathcal{H}\right]$$

$$\delta v_\phi = \frac{i}{D}\left[\frac{k^2}{2\Omega}\frac{\partial \delta\mathcal{H}}{\partial R} - \frac{m\tilde{\omega}}{r}\delta\mathcal{H}\right]$$

$$\delta v_z = -\frac{i}{\tilde{\omega}}\frac{\partial \delta\mathcal{H}}{\partial z}$$

and get the wave equation

$$D = k^2 - \tilde{\omega}^2$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\rho}{D}\frac{\partial}{\partial r}\right) - \frac{1}{\tilde{\omega}^2}\frac{\partial}{\partial z}\left(\rho\frac{\partial}{\partial z}\right) - \frac{m}{r^2}\frac{\rho}{D} + \frac{1}{r\tilde{\omega}}\frac{\partial}{\partial r}\left(\frac{2\Omega m\rho}{D}\right) + \frac{\rho}{c_s^2}\right]\delta\mathcal{H}$$

WKB expansion

Leading order

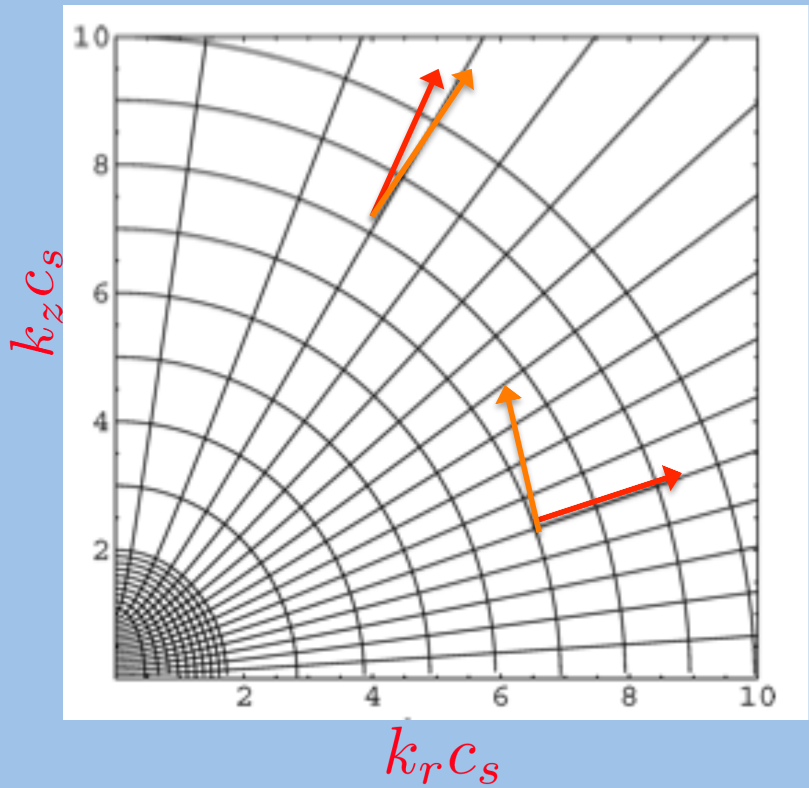
$$\frac{k_z^2}{\tilde{\omega}^2} + \frac{k_r^2}{\tilde{\omega}^2 - k^2} = \frac{1}{c_s^2}$$

Density spiral waves

Inertial waves

Wave vector

Group velocity



Equicontours

$\tilde{\omega}$

$$U_r \equiv \frac{\partial \tilde{\omega}}{\partial k_r} = -\frac{k_r/\tilde{\omega} D}{(k_r/D)^2 + (k_z/\tilde{\omega}^2)^2} \qquad U_z \equiv \frac{\partial \tilde{\omega}}{\partial k_z} = -\frac{k_z/\tilde{\omega}^3}{(k_r/D)^2 + (k_z/\tilde{\omega}^2)^2}$$

Dispersion relation

$$\tilde{\omega}^4 - \left[(k_r^2 + k_z^2)c_s^2 + k^2\right]\tilde{\omega}^2 + k^2 k_z^2 c_s^2 = 0$$

High frequency → density waves

$$\tilde{\omega}^2 = (k_r^2 + k_z^2)c_s^2 + k^2$$

Valid in the limits

$$(k_z^2 + k_r^2)c_s^2 >> k^2$$

Acoustic waves

$$k_z << k_r$$

1D density waves

Low frequency → inertial waves

$$\tilde{\omega}^2 = \frac{k_z^2}{k_r^2 + k_z^2}k^2$$

Valid in the limits

$$k_z c_s >> k$$

incompressible

Essential vertical component of wave vector
Restoring force = Coriolis force → horizontal motions
Incompressible flows → v perpendicular wave vector

Second order WKB → conservation of wave action

$$\nabla \cdot \left[\rho A^2 \tilde{\omega} \left(k_r^2/D^2 + k_z^2/\tilde{\omega}^4\right) \mathbf{U}\right] = 0$$

Wave action density

Wave action not wave energy is a conserved quantity

Angular momentum flux

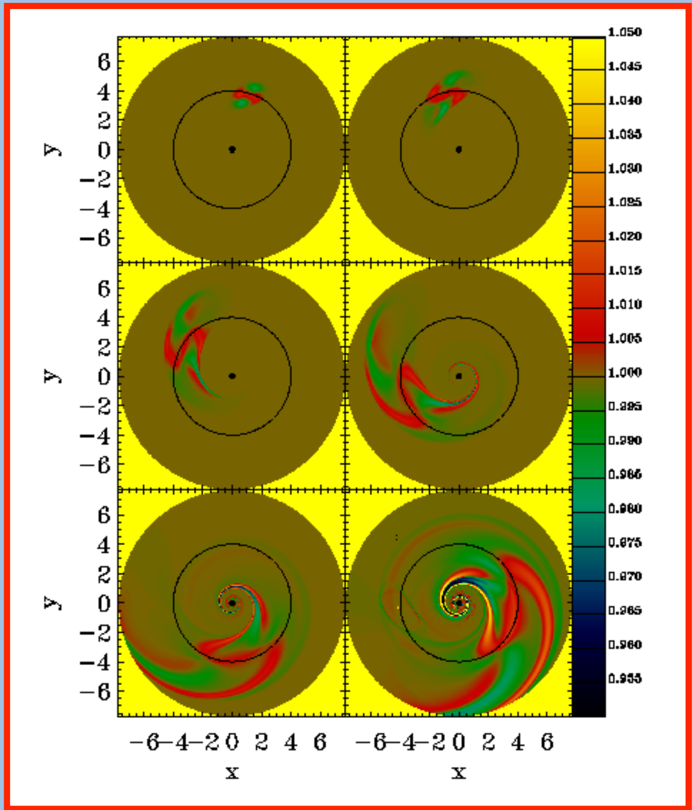
$$F_J = \rho r < \delta v_\phi \delta \mathbf{v} >$$

$$F_J = \frac{m \rho k_r}{\tilde{\omega}^2 - k^2} \frac{A^2}{2}$$

Proportional to wave action

volumetric energy exchange rate = $-\rho < \delta v_r \delta v_\phi > \dfrac{d\Omega}{d \ln r}$

In any shearing disk local nonaxisymmetric disturbances evolve toward a trailing configuration, when there is a decreasing rotation profile



Evolution of a density wave in a rotating flow
Anticlockwise rotation

m and k_r have the same sign

$$F_J = \frac{m \rho k_r}{\tilde{\omega}^2 - k^2} \frac{A^2}{2}$$

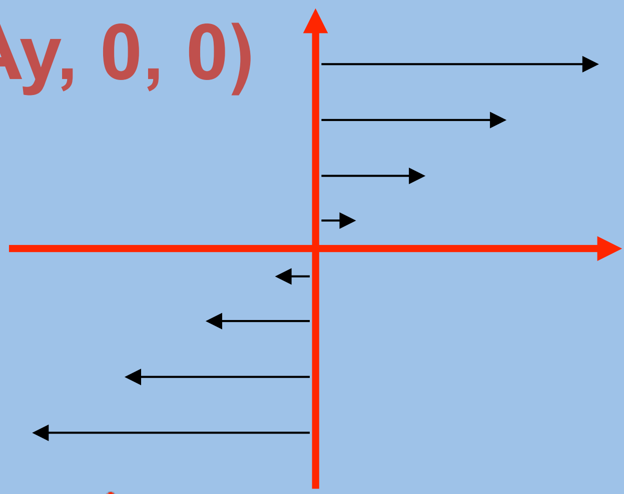
Density waves carry angular momentum outward – inertial waves carry angular momentum inward

Generation of density waves: planets - vortices

Vortex-wave conversion in compressible shear flows

Planar compressible shear flow with linear velocity profile

U = (Ay, 0, 0)



Consider the linearized equations

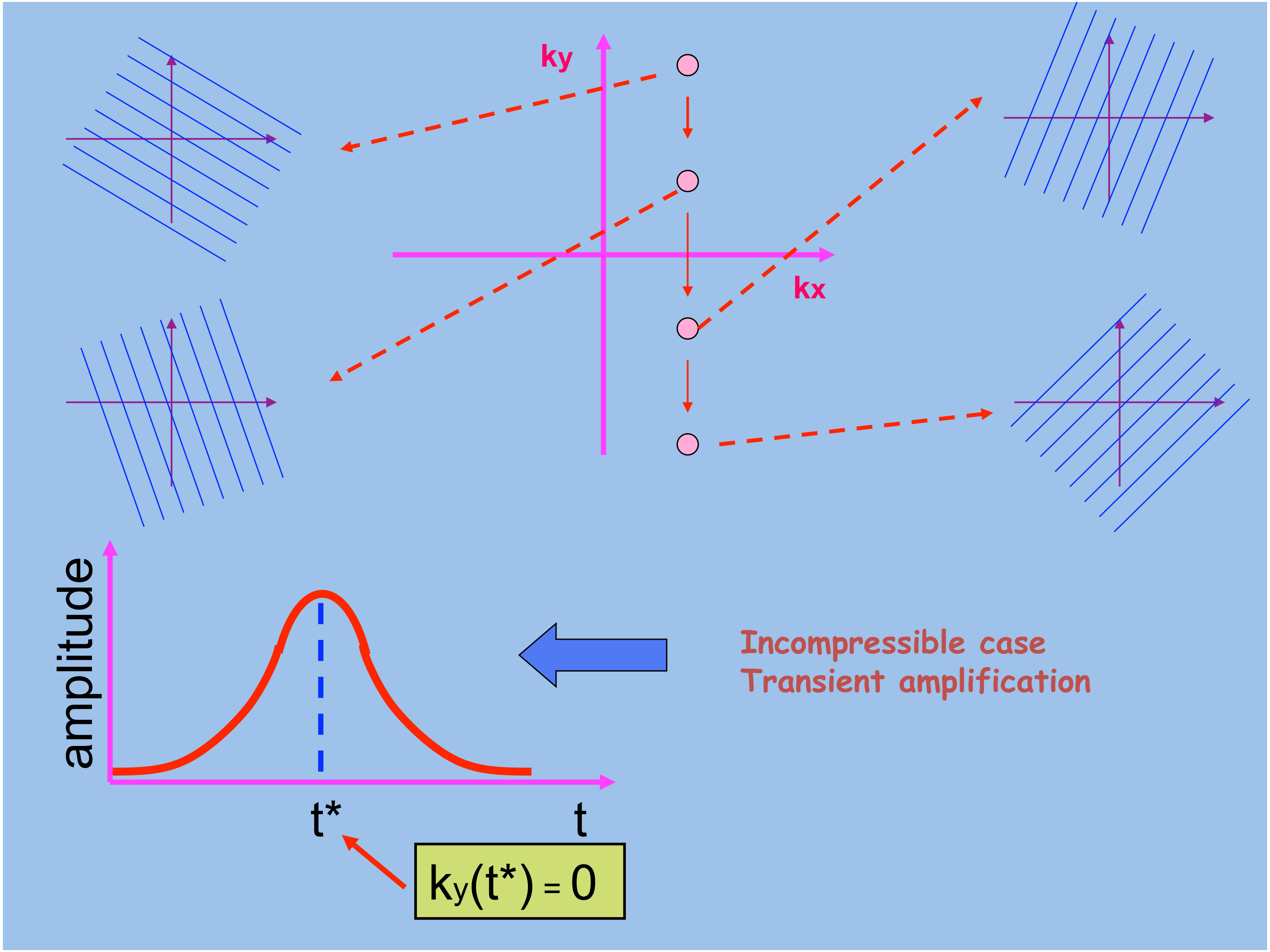
Non-modal analysis:

Change of variables: $x' = x - Ayt \; ; y' = y \; ; \tau = t$

We study the temporal evolution of a spatial Fourier harmonics (SFH) perturbation

Drift in Fourier space: $\propto F(t) \exp(ik_x x + ik_y(t)t)$

$$k_y(t) = k_{y0} - Ak_x t$$



ky
kx
amplitude
t*
t
$k_y(t^*) = 0$
Incompressible case
Transient amplification

$$\frac{dD}{d\tau} = k_x v_x + k_y(\tau) v_y \qquad D \equiv i\frac{\rho}{\rho_0}$$

$$\frac{v_x}{d\tau} = -Av_y - k_y c_s^2 D$$

$$\frac{dv_y}{d\tau} = -k_y(\tau) c_s^2 D$$

$$I = k_y(t) v_x - k_x v_y + AD$$

Invariant

No dissipation

Conservation of potential vorticity

In the compressible case we get the following second order ODE:

$$\frac{d^2 v_x}{dt^2} + \omega^2(t) v_x = k_y(t) c_s^2 I$$

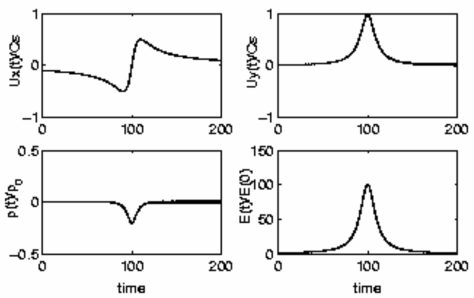
invariant

$$\omega^2(t) = (k_x^2 + k_y^2(t)) c_s^2$$

The equation describe two modes: vortex and sound wave

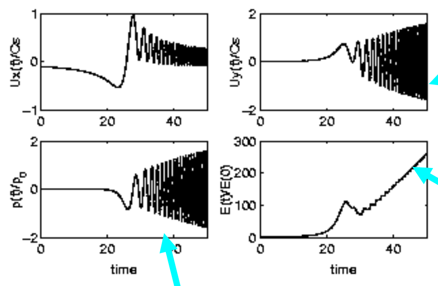
The dynamics is determined by the parameter

$$R = \frac{A}{k_x c_s}$$



R << 1

Incompressible dynamics



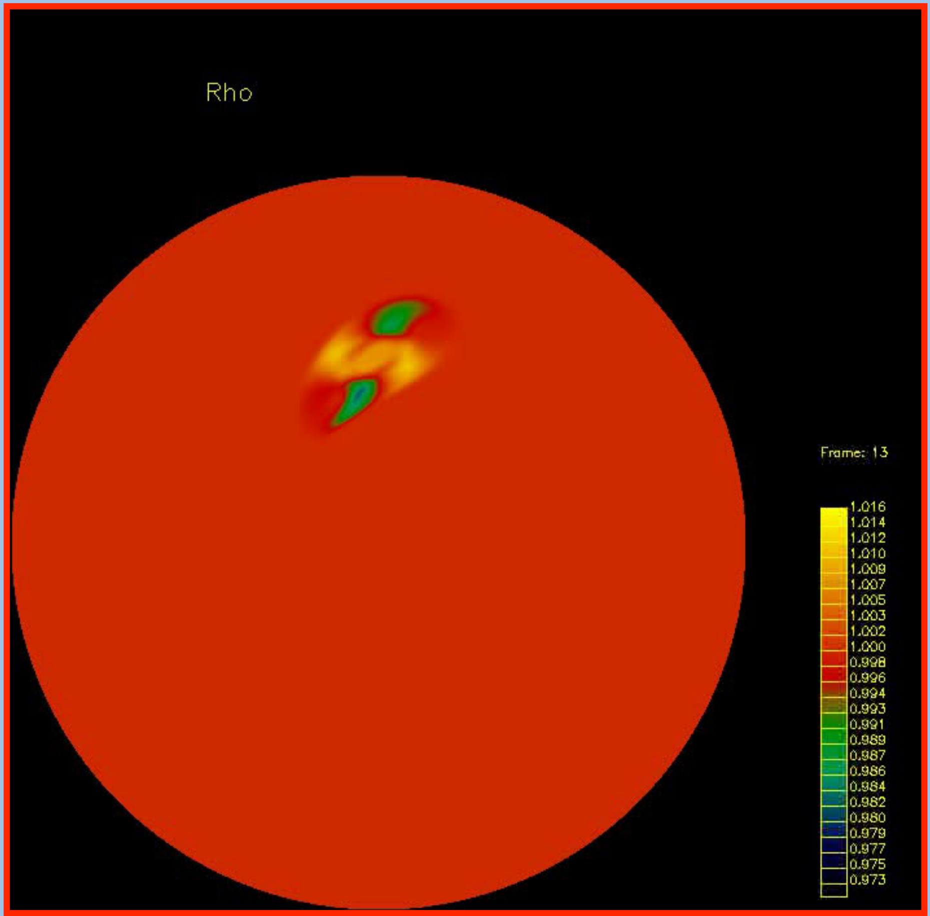
R = 0.3

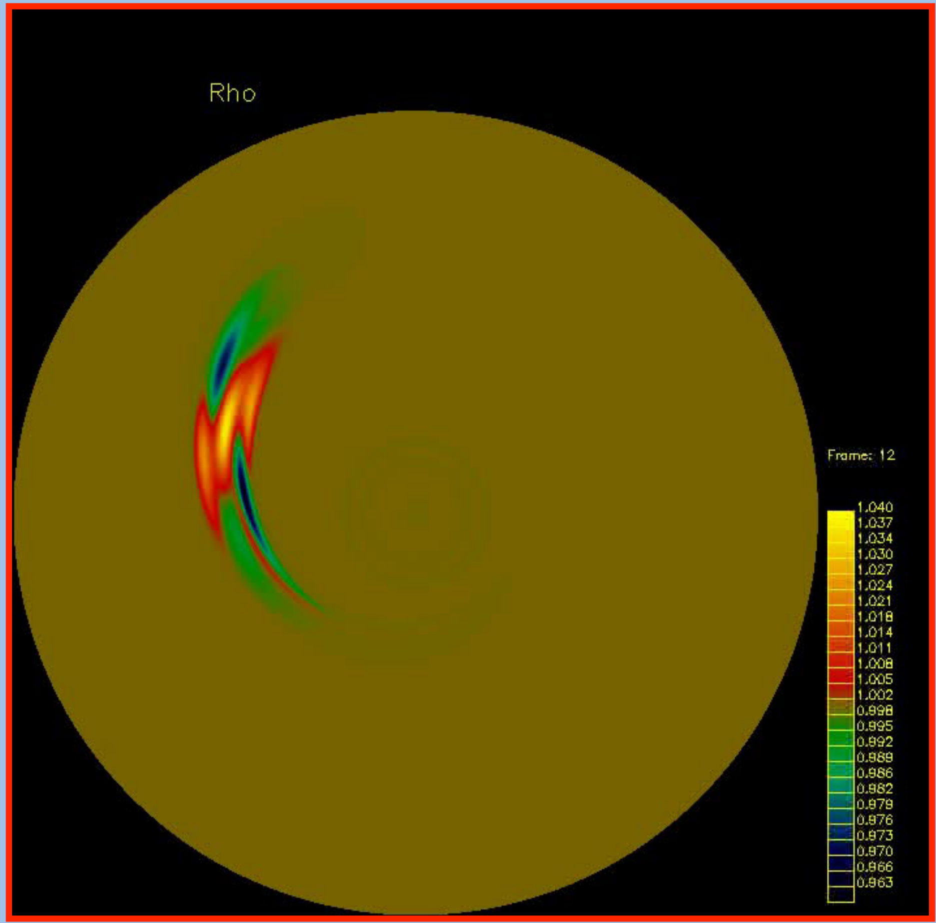
The wave extracts energy from the mean flow

Evolution of an initial vortex SFH

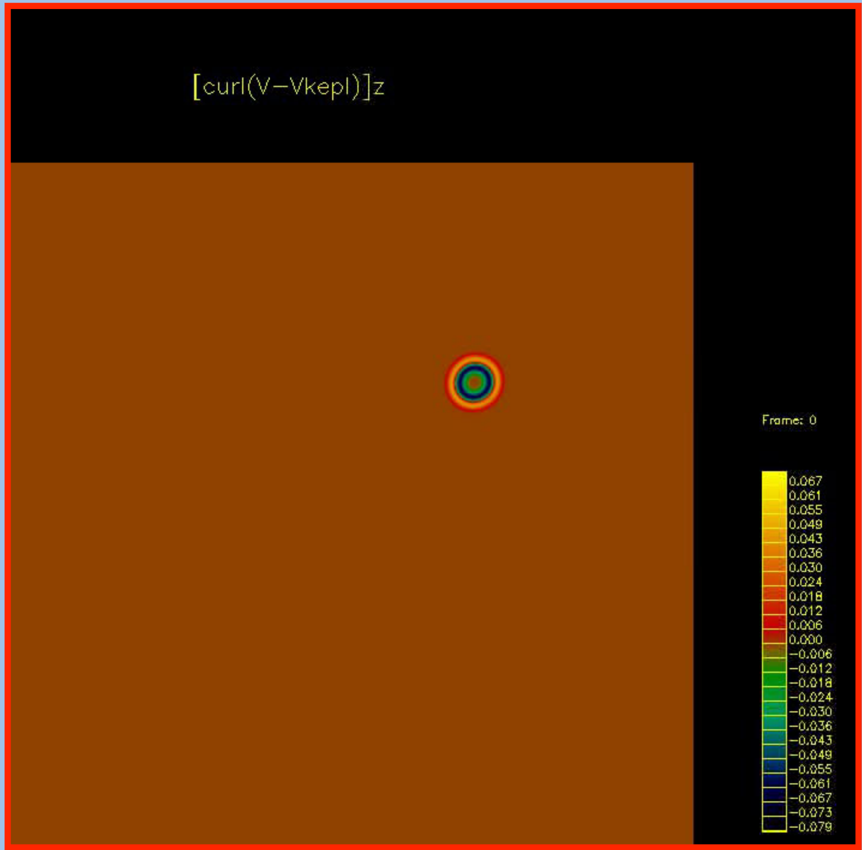
Wave generation

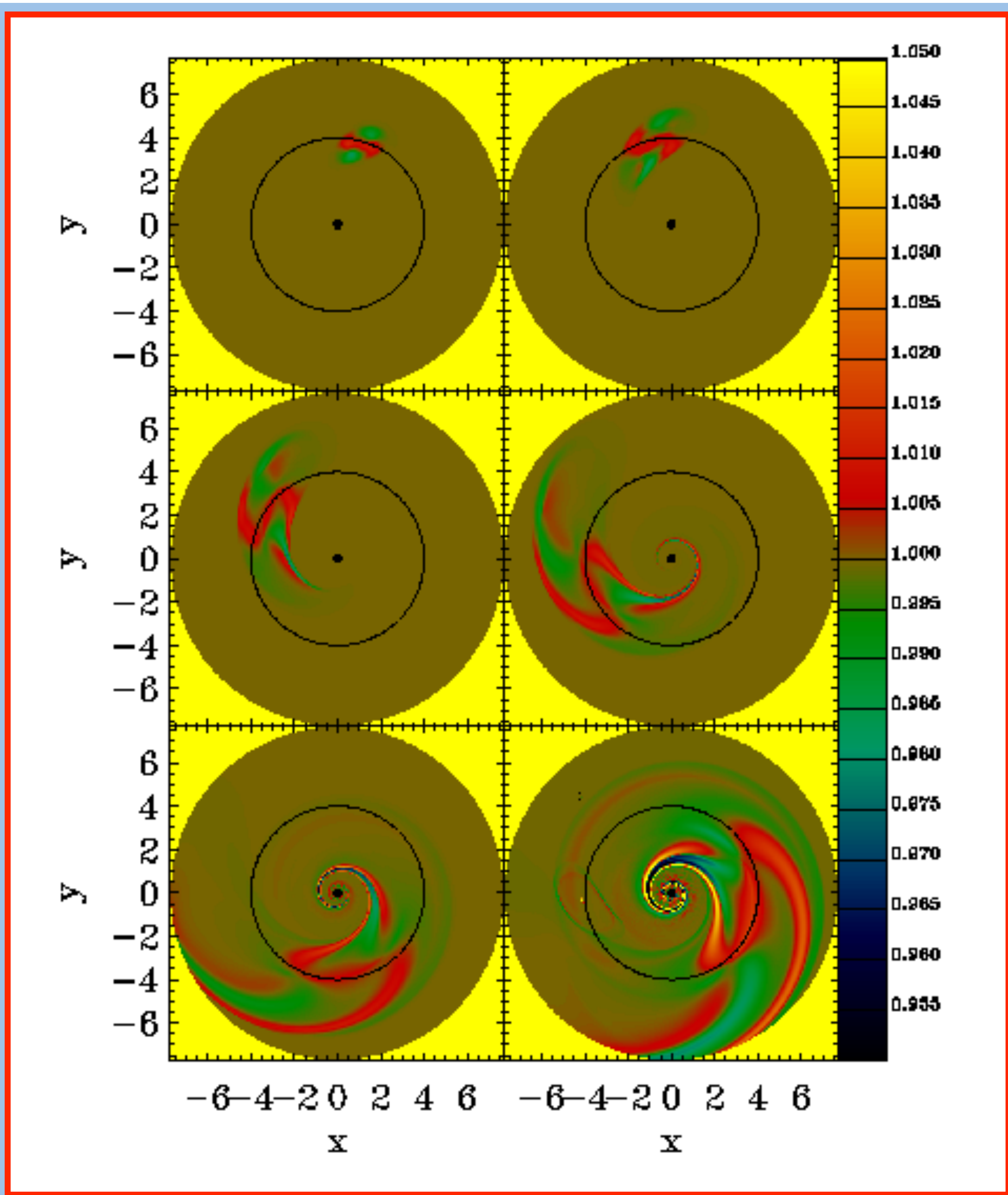
Wave generation: detail





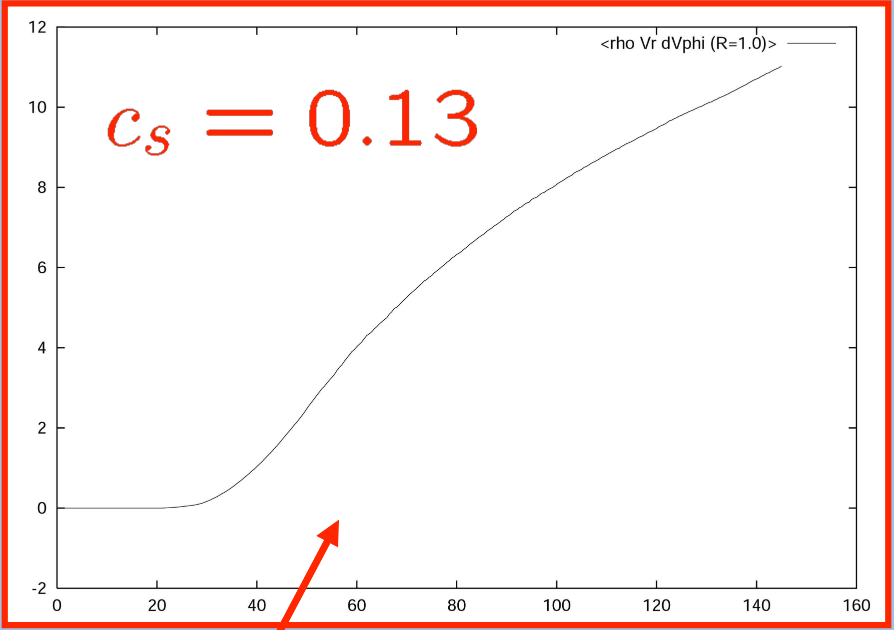
Ring vortex

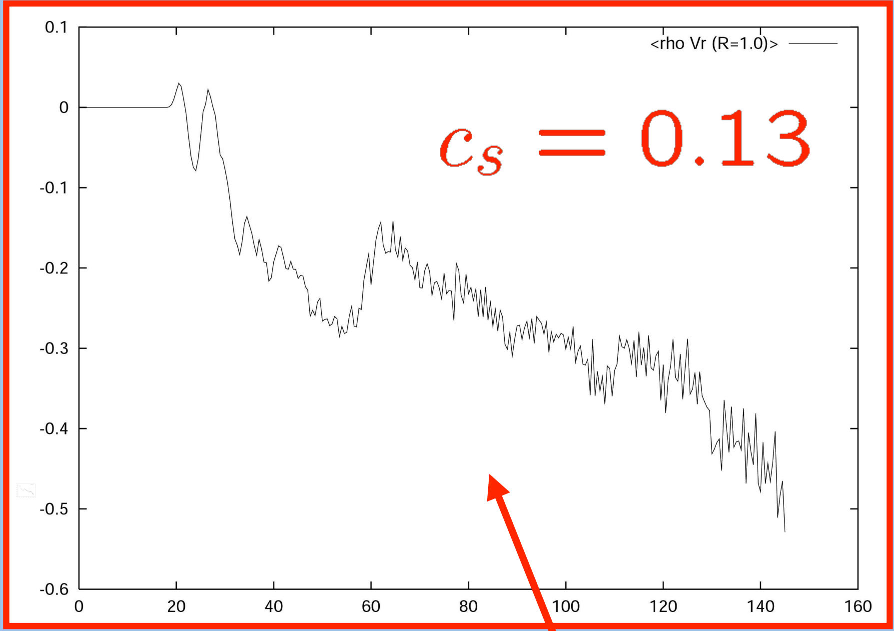




Above: initial vorticity perturbation; left: density (6 different times) Note the emergence of the wave (2nd panel) and the triggering of the global spiral mode.

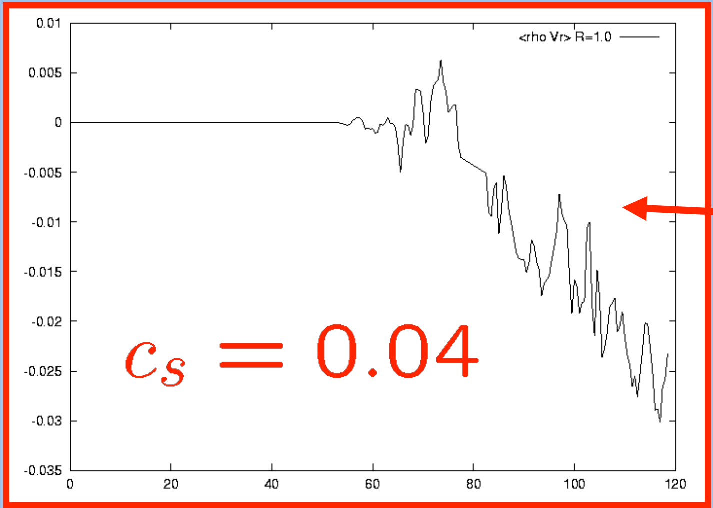
Mass inflow driven by the wave





Reynolds stresses

Mass inflow



Angular momentum transport by outflows

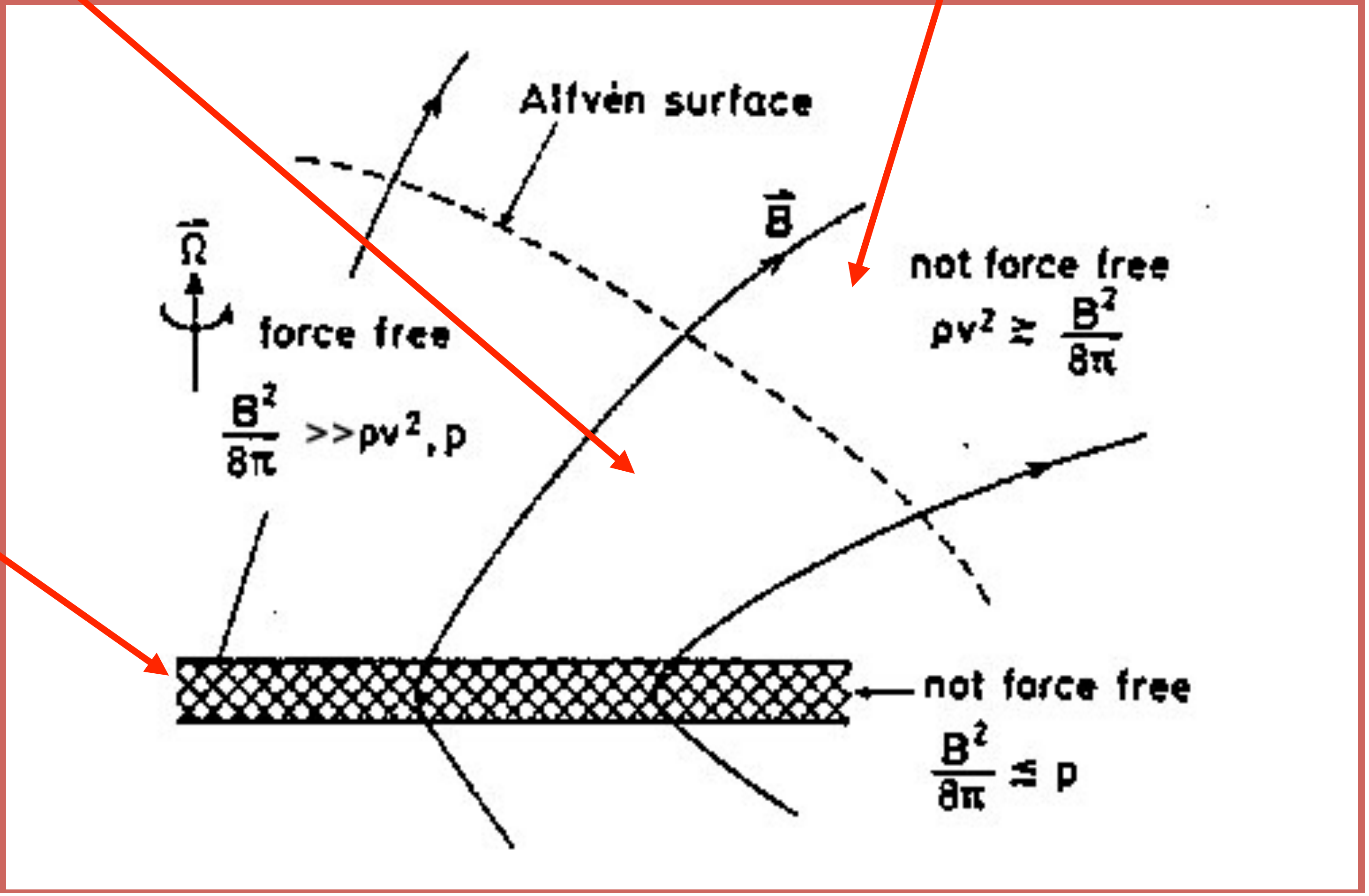
Jets are connected to accretion disks in many astrophysical environments
Magnetocentrifugal mechanism for acceleration

Physics of magnetic acceleration
Interplay between magnetic field and rotation

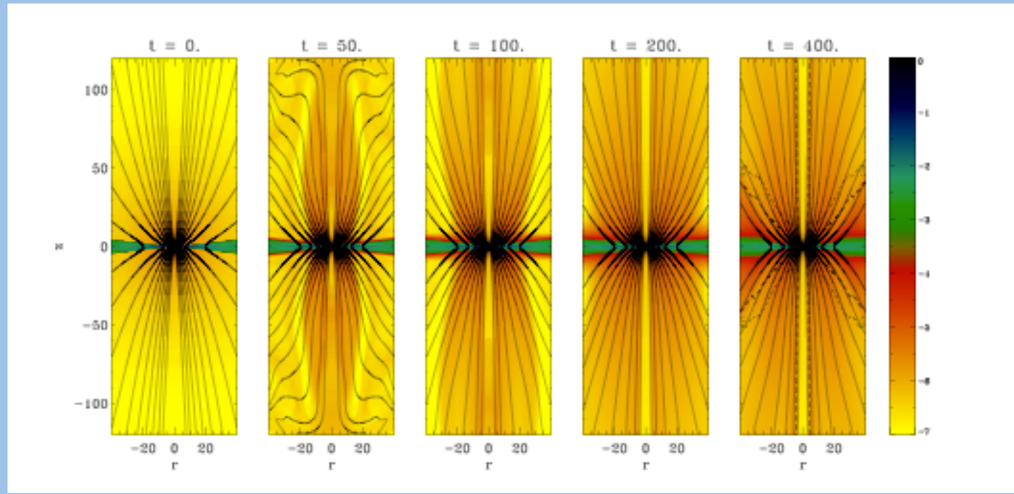
No corotation

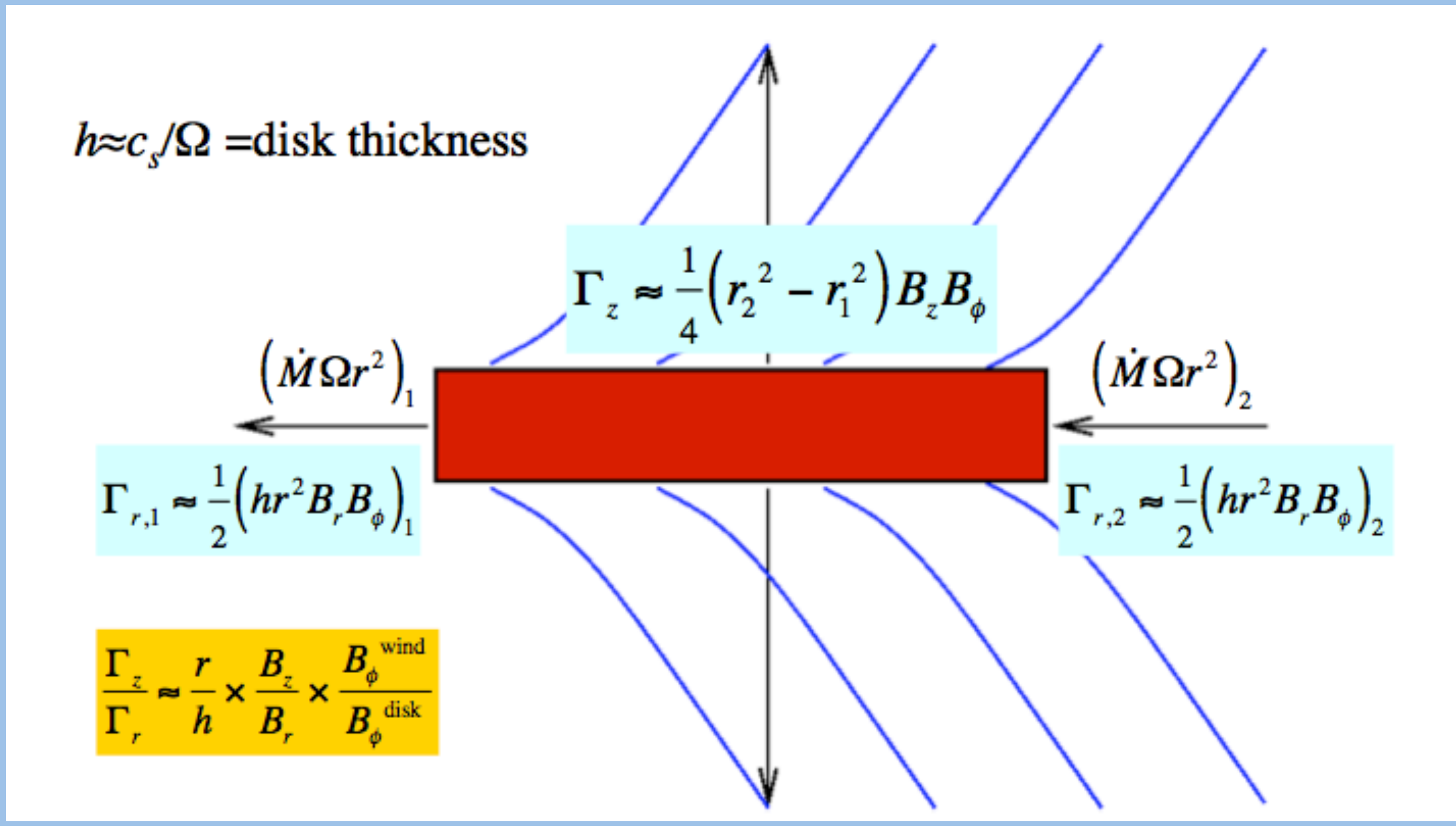
Corotation

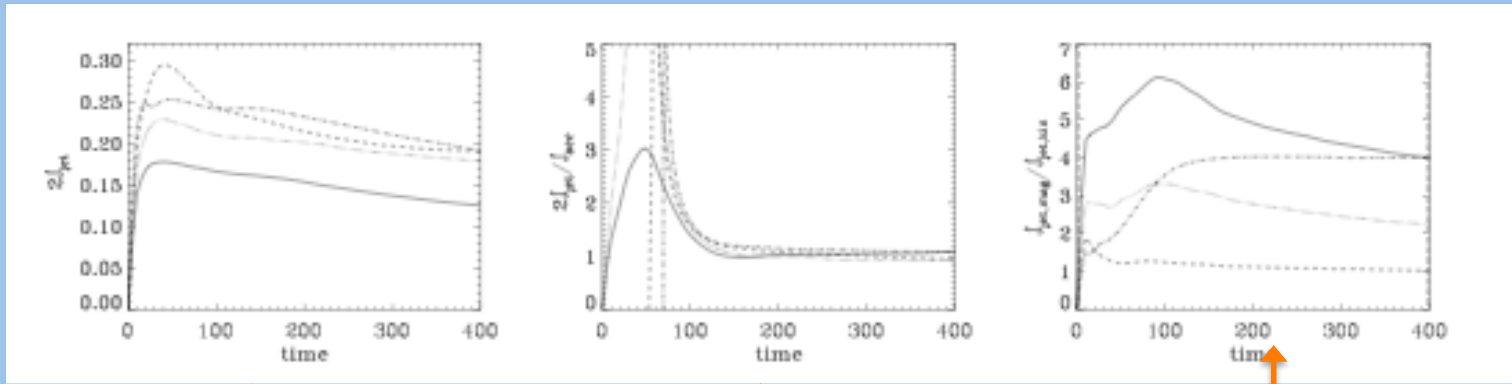
Thin disk
(cool)



Result of simulation



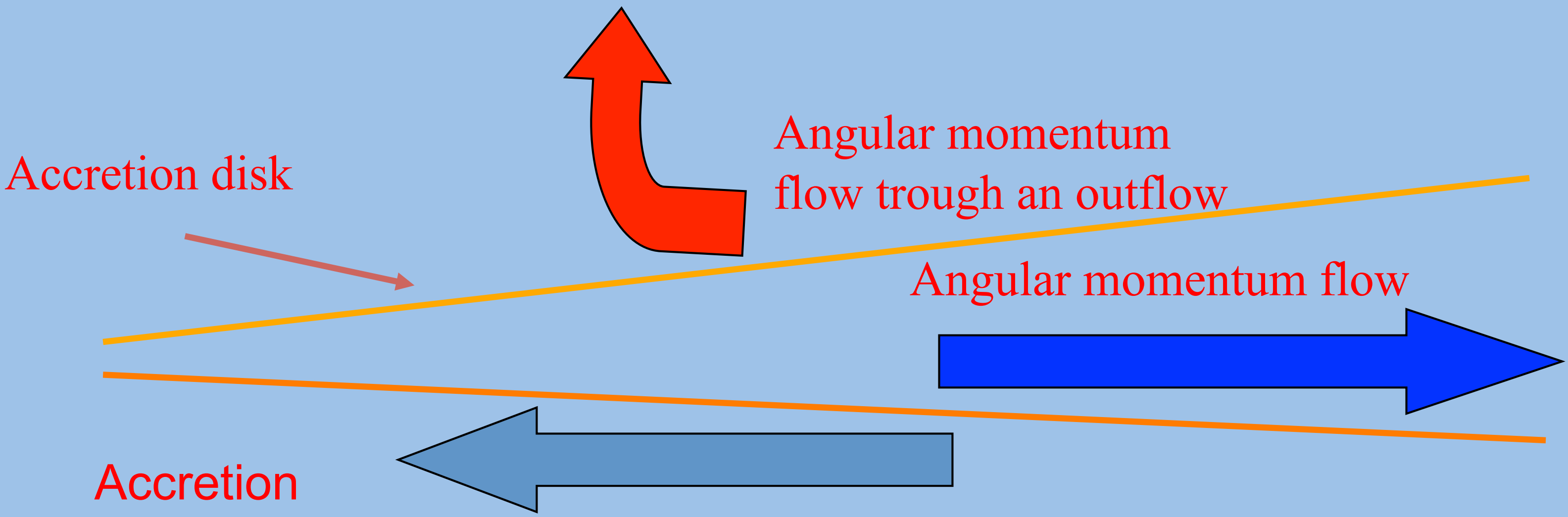




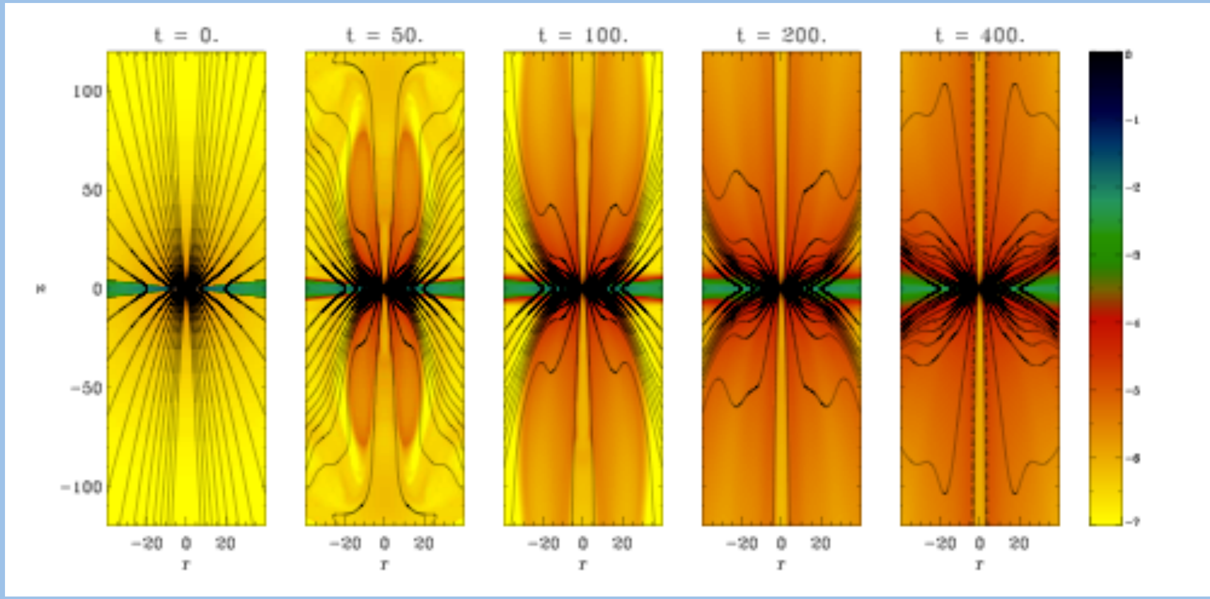
Temporal variation of the vertical angular momentum flux carried by the jet

Ratio between the jet angular momentum flux and momentum flux in the disk carried by accretion

Ratio between magnetic and kinetic components of angular momentum flux in the jet



The problem is however that in order to have steady accretion magnetic field cannot be frozen in the disk, matter must be able to flow trough the field



Enhanced resistivity

If accretion disks are turbulent what is the mechanism that leads to turbulence?

Hydrodynamic disks are linearly stable
Rayleigh criterion → Stability for angular momentum growing outward

Finite amplitude instability?

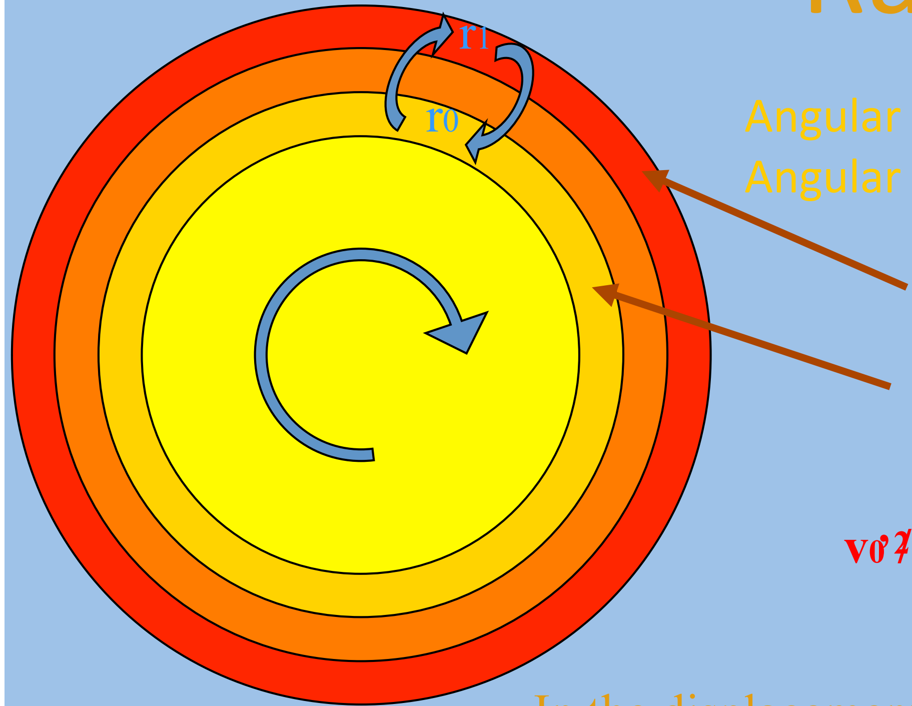
Numerical simulation show that turbulence is not sustained, but Reynolds number of simulations quite low.

Laboratory experiments do not show a transition to turbulence (Ji et al 2006)

Could be turbulent at high Reynolds numbers but not efficient in transfering angular momentum (Lesur & Longaretti 2005)

Magnetic field may change the situation

Rayleigh criterion



Angular velocity distribution Ω(r)

Angular momentum distribution j(r)

v_1^2/r

Centripetal acceleration

v_0^2/r

$v_0'^2/r \;<\; v_1^2/r$ **Stability**

In the displacement angular momentum kept constant $v_0' = v_0 \frac{r_0}{r_1}$

$$\frac{v_0'^2}{r_1} = \frac{v_0^2 r_0^2}{r_1^3} < \frac{v_1^2}{r_1} \;\Rightarrow\; (\Omega_0 r_0^2)^2 < (\Omega_1 r_1^2)^2$$

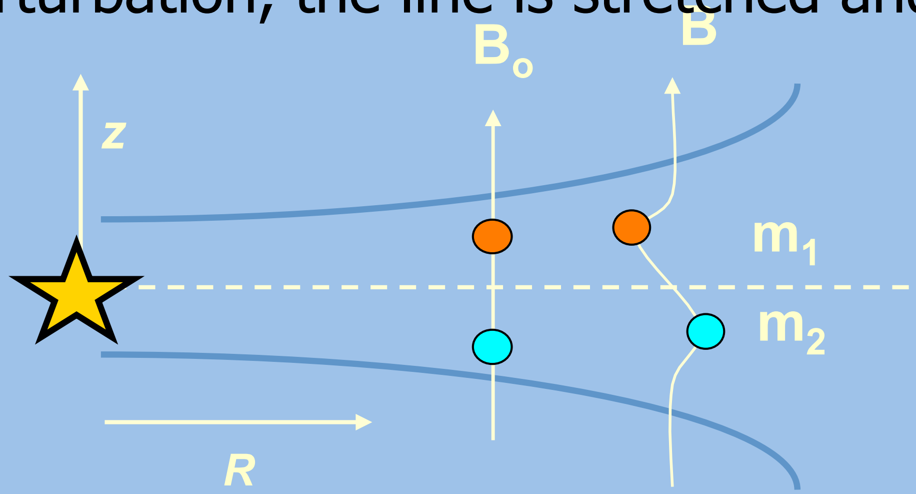
$$j_0^2 < j_1^2 \;\Rightarrow\; \frac{dj^2}{dr} > 0$$ for stability

$$j_k \propto r^{1/2}$$

Magnetorotational instability

Instability first discussed by Velikhov (1959) and Chandrasekhar (1961) and applied in the context of disks by Balbus & Hawley (1991)

- Two fluid elements, are joined by a vertical field line (B_o). The tension in the line is negligible.
- Introducing a perturbation, the line is stretched and develops tension.



- The tension reduces the angular momentum of m_1 and increases that of m_2. This further increases the tension and the process "runs away".

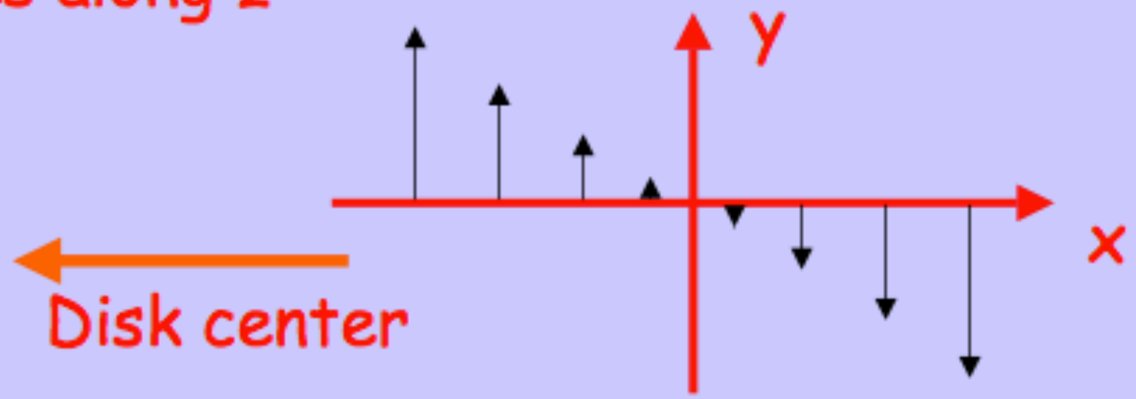
Consider the simplest case of a thin disk threaded by a vertical field

Consider length scales of interest that are small compared to the disk size

$$\lambda < h << r$$

In this case we can make use of a local analysis shearing sheet aproximation
We consider a small patch of disk around a reference radius r_0
The patch is corotating with the disk at the reference position r_0
The shearing sheet is based on a first order expansion and make use a system of local cartesian coordinates

The x, y and z direction are respectively the radial, azimuthal and vertical direction .
In the analysis that follows we neglect gravity, so there is no variation of the equilibrium quantities along z



Ideal MHD equations for the shearing sheet are

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}+2\mathbf{\Omega}\times\mathbf{v}=\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho}+\frac{1}{\rho}\nabla\left(p+\frac{B^2}{8\pi}\right)-\nabla(2A\Omega x^2)$$

Coriolis term

Tidal expansion at first order of gravity + centrifugal force

The local angular velocity Ω and shear rate A are treated as constant

$$A \equiv \frac{r}{2}\frac{\partial \Omega}{\partial r} = -3/4\Omega$$

We take an incompressible fluid

$$\nabla \cdot \mathbf{v} = 0$$

Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} - \mathbf{B} \cdot \nabla \mathbf{v}$$

In equilibrium the velocity field in the shearing sheet has a linear variation with x

$$v_y = 2Ax$$

Equilibrium is maintained by the balance of Coriolis force and tidal term.

Equilibrium magnetic field is uniform and directed along z

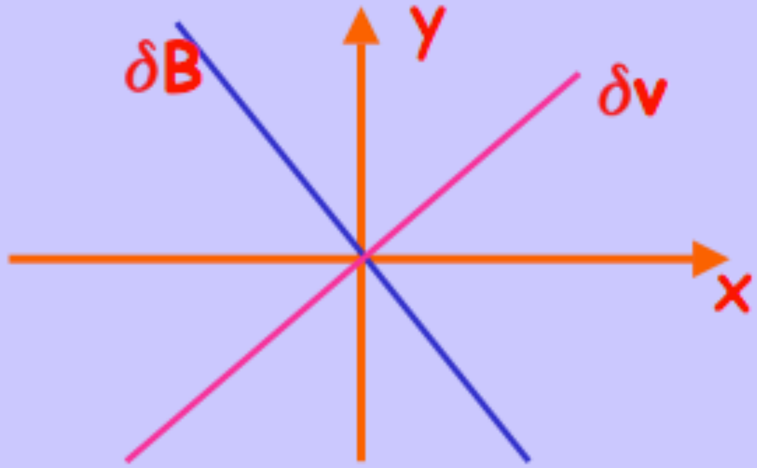
We will see that we are interested in values of plasma $\beta >> 1$

The sound speed is much larger than the Alfven speed this justify the incompressibility assumption

Consider axisymmetric modes, i.e. with wavevector directed along z
It is the simplest case and gives the highest growth rate
We look for solution of the form

$$(B_x, B_y, B_z) = \bar{b}e^{st} \cos Kz(\sin\gamma, -\cos\gamma, 0)B_0 + (0, 0, B_0) ,$$
$$(v_x, v_y, v_z) = \bar{b}e^{st} \sin Kz(\cos\gamma, \sin\gamma, 0)v_0 + (0, 2Ax, 0) .$$

The field lines lie in fixed vertical planes



Eulerian $\delta \mathbf{v}$ Perpendicular to $\delta \mathbf{B}$ but Lagrangian perturbation parallel

Substituting in the shearing sheet equations (note that as we will discuss below we don't need to linearize) we get the following relations

$$s = -A\sin(2\gamma)$$

Growth rate vanishes for purely azimuthal or purely radial perturbations, it is maximum for $\gamma = \pi/4$

For each pitch angle γ we get a wavenumber K

$$(Kv_A)^2 = -4A\left(\Omega + A\cos^2\gamma\right)\sin^2\gamma$$

When the perturbation is purely azimuthal, K = 0
The maximum value of K is reached when the perturbation is purely radial and the growth rate vanishes

$$K_{max} = \frac{-4A\Omega}{V_A}$$

Combining all the equations we get the dispersion relation

$$s^4 + [\kappa^2 + 2(KV_A)^2]s^2 + (KV_A)^2[(KV_A)^2 + 4A\Omega] = 0$$

where $k^2 = 4\Omega(\Omega + A)$

Condition for instability $\dfrac{d\Omega}{dr} < 0$

Rayleigh criterion angular momentum has to decrease outward

Magnetorotational instability, angular velocity has to decrease outward

The unstable modes have a minimum wavelength

$$\lambda_{min} = \frac{2\pi v_A}{3\Omega}$$

Increasing the strength of the magnetic field the minimum wavelength increases (increase of magnetic tension)
For having instability in an accretion disk this minimum wavelength has to be smaller that the disk height h

$$\frac{2\pi v_a}{3\Omega r} < \frac{h}{r} = \frac{v_s}{v_\phi}$$

$$\frac{v_s}{v_A} > \frac{2\pi}{3}$$

We can observe that the nonlinear terms of the form $\mathbf{a} \cdot \nabla \cdot \mathbf{b}$ vanish if we take a single Fourier harmonic and if $\nabla \cdot \mathbf{a} = 0$

$$\mathbf{a} \cdot \nabla \mathbf{b} = i\,(\mathbf{k} \cdot \mathbf{a})\,\mathbf{b} = 0$$

There is a nonlinearity in the magnetic pressure but it has no effect in the incompressible dynamics. The pressure gradient serves only to enforce the condition $\nabla \cdot \mathbf{v} = 0$

The solution above is an exact solution of the nonlinear equations (Goodman & Xu 1994)
The perturbations grow exponentially even in the nonlinear regime

If we want to understand the process of angular momentum transport we need now to understand how MRI is saturated

We have to use numerical simulations:

Local shearing sheet simulations (we can afford larger resolution)

Global whole disk simulations (overcome the limitations of a local analysis, effect of boundary conditions)

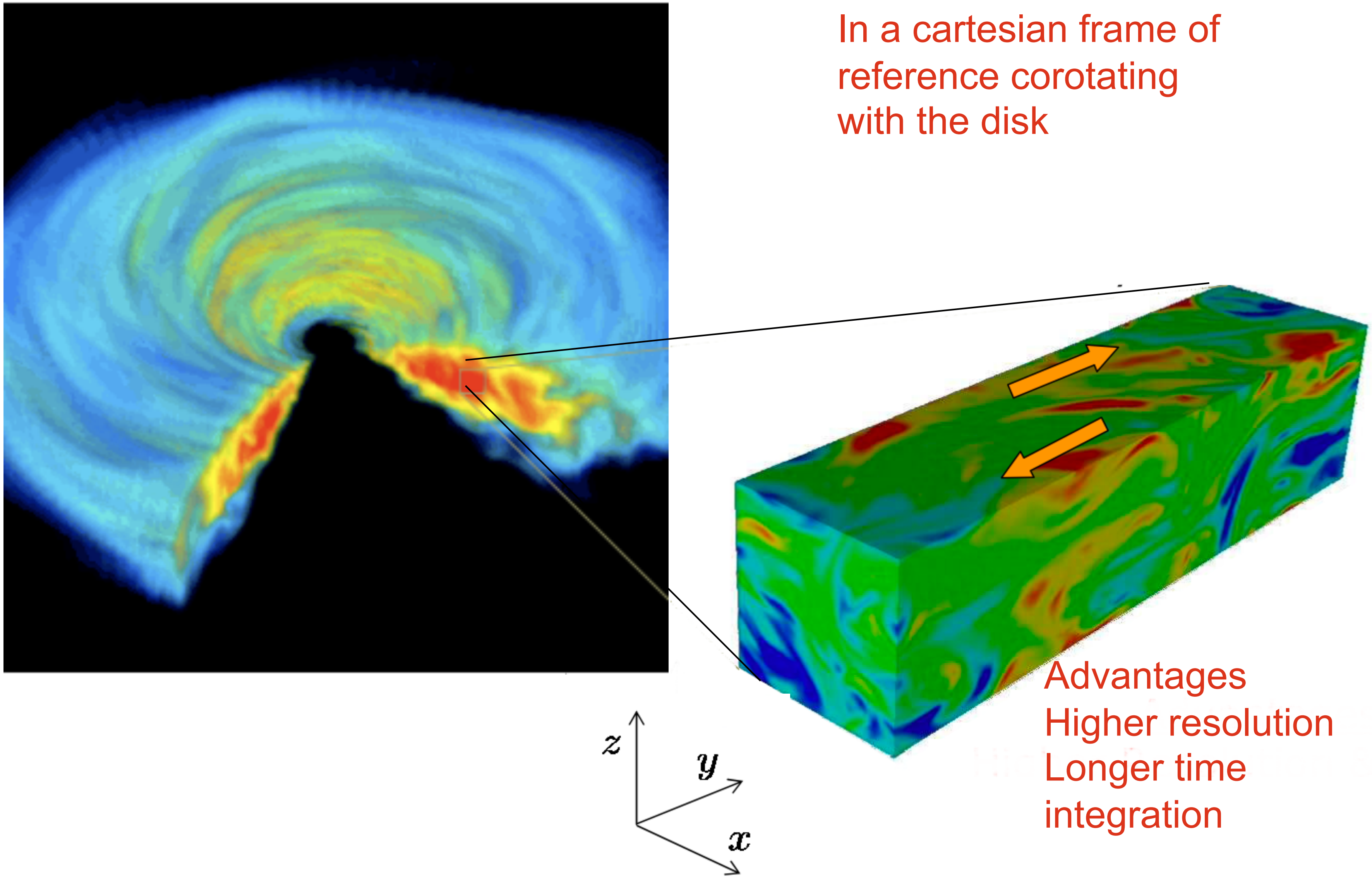
The exponential growth of the solutions creates growing gradients of velocity and magnetic field, eventually it becomes unstable.

Goodman & Xu (1994) analysis of secondary instabilities of this growing solution
Two types of instabilities, Kelvin-Helmholtz and related to tearing

One point to stress is that the essence of Magnetorotational instability is the angular momentum transport..
What we discussed above was
Instability → turbulence → angular momentum transport
Here it is the instability itself that transfers angular momentum

Local versus global simulations

In a cartesian frame of reference corotating with the disk

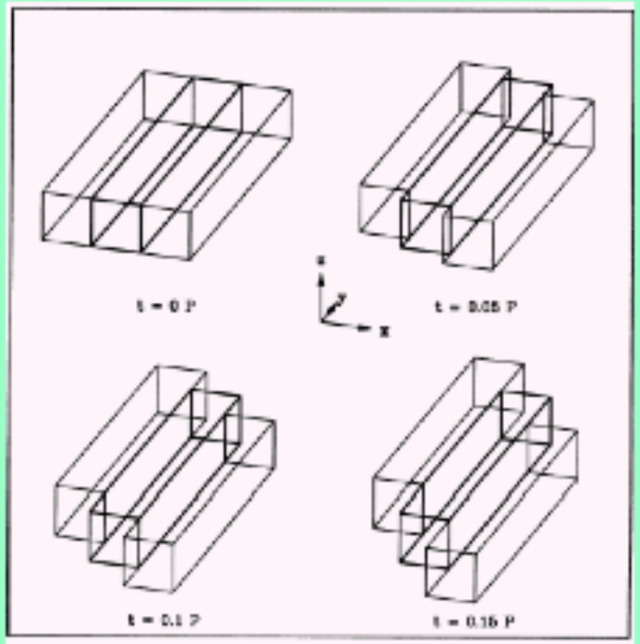


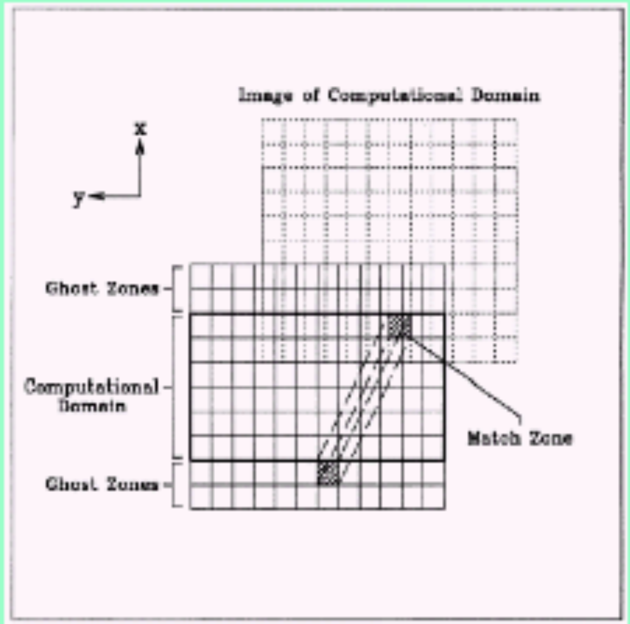
Advantages

Higher resolution

Longer time integration

Simulations in the shearing sheet aproximations



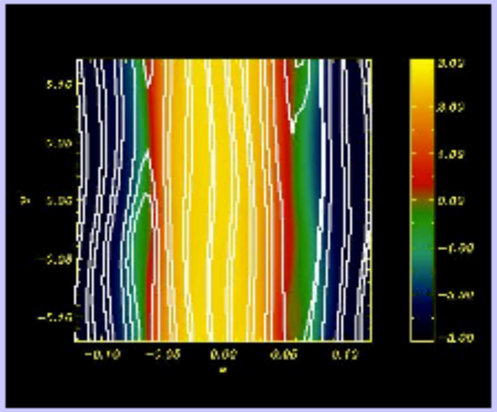


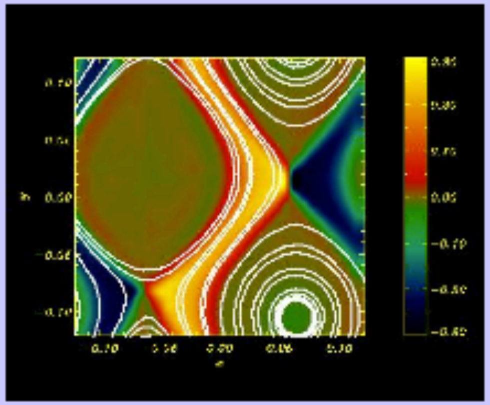
Small piece of a disk, cartesian geometry, Coriolis forces, tidal stresses

2D Simulations

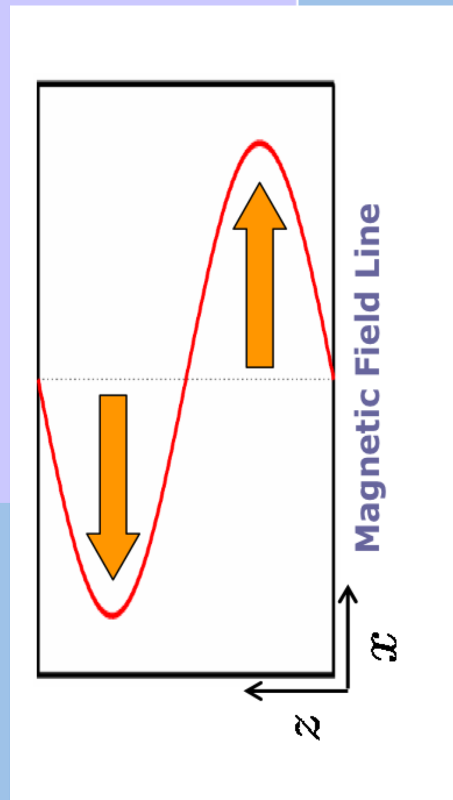
We have only axisymmetric modes

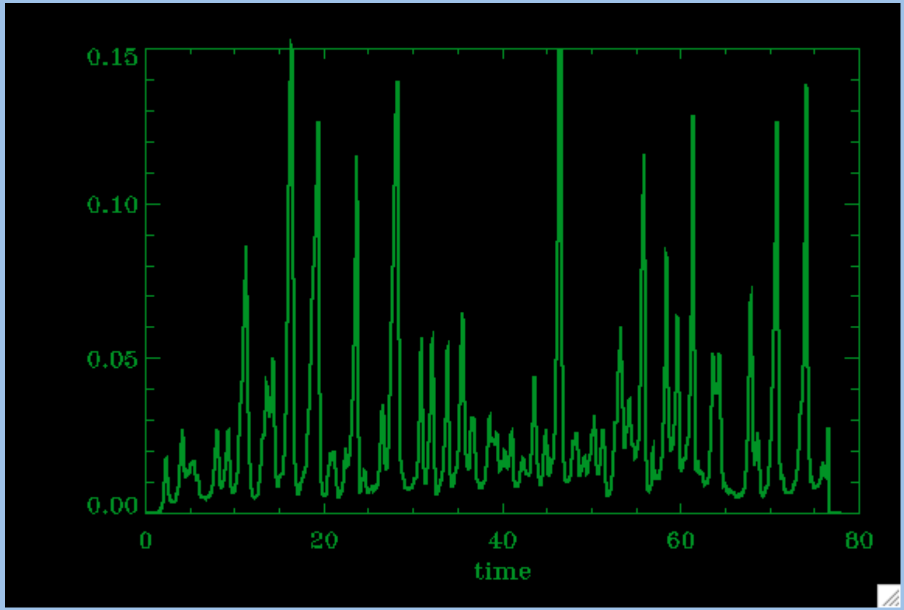
Cyclic behavior: quasi periodic transition between two states

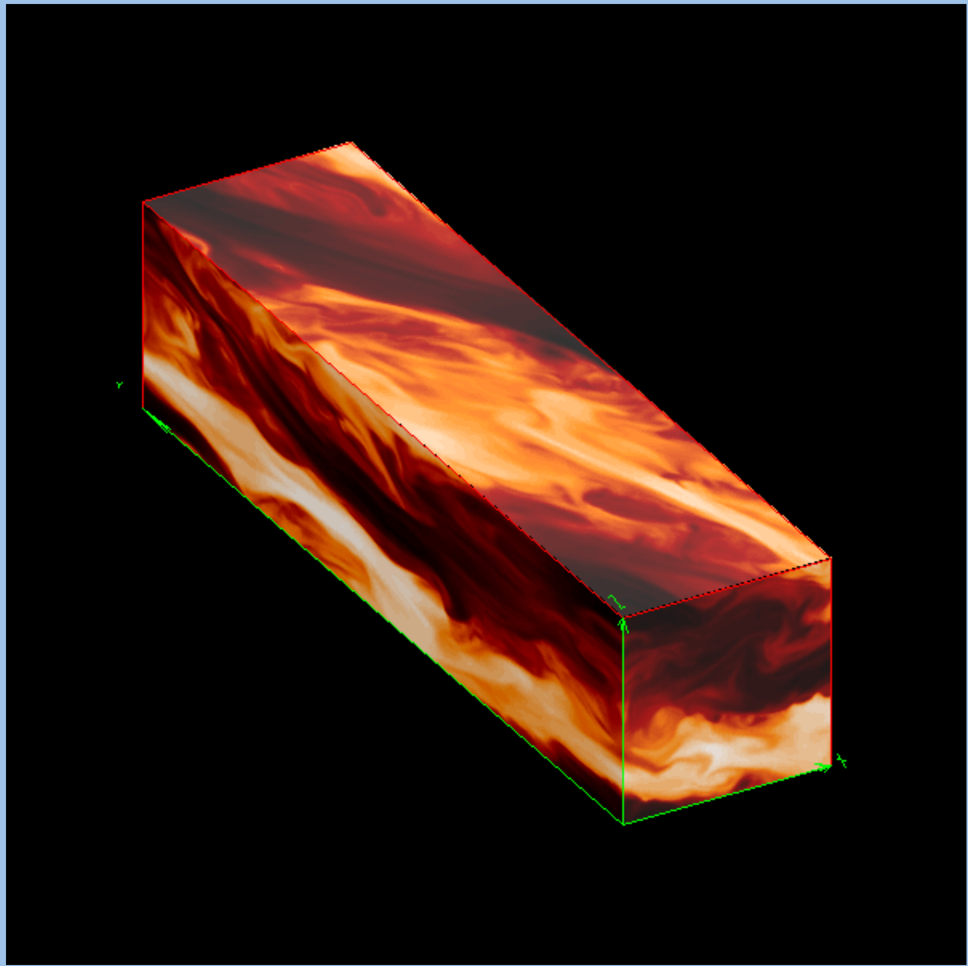




Channel flow
MRI mode
growing exponentially



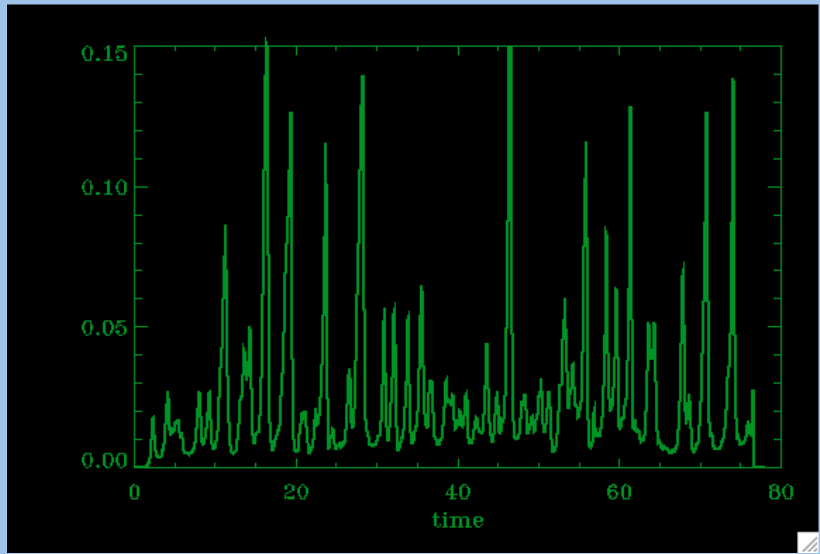




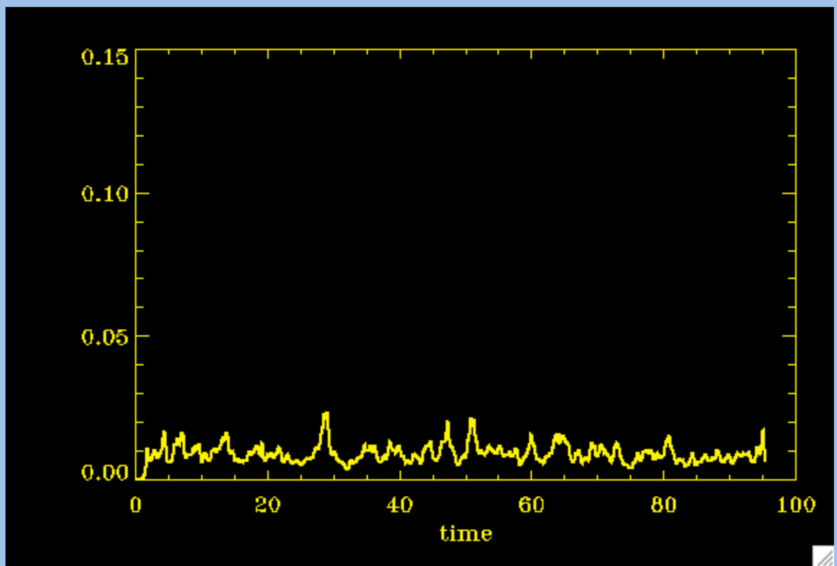
Channel solution
in correspondence
of peaks of
Maxwell stresses

(Sano & Inutsuka 2001)

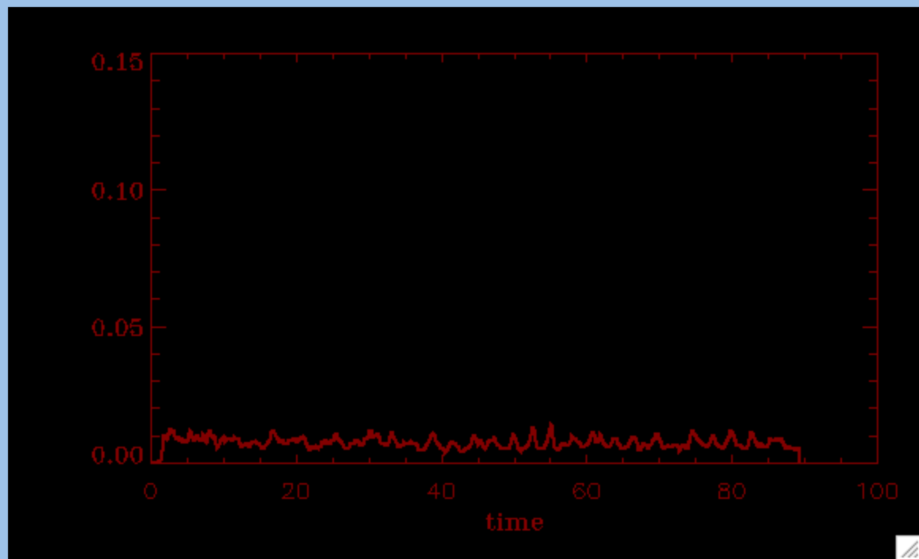
Aspect Ratio = 1



Asp. Ratio = 4



Asp. Ratio = 8



How efficient is the transport

We can get a measure in terms of an effective α

We have defined α through the definition of an eddy viscosity

$$\nu \sim \alpha H c_s$$

α can be defined in terms of the rϕ component of the stress tensor normalized to pressure

When looking at the results of simulations, to get an efficiency of transport one measure some average of the rϕ component of the stress tensor (xy component in the local frame)

$$< -\frac{B_x B_y}{4\pi} + \rho v_x \delta v_y >$$

What do we get from simulations? Is consistent with what is required by observations?
How α depends on the disk parameters

“Measured” values of α

Numerical simulations of MRI *varies with large-scale field, dissipation terms*	10^{-3}-10^{-1}
Protostellar disks *based on disk masses, temperatures, accretion rates, and lifetimes*	10^{-2}-10^{-3}
Cataclysmic variables *based on models of “dwarf nova” outbursts*	10^{-3}-10^{0}
AGN *direct observational constraints are few to none*	?

Distinction between simulations with a net magnetic flux and simulations with a zero net magnetic flux

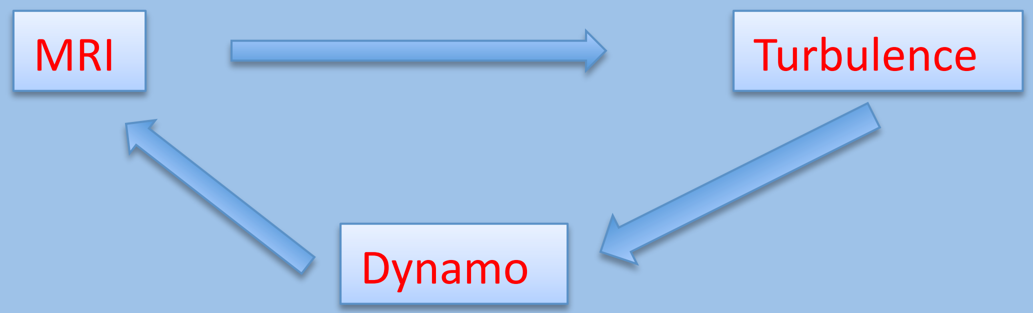
In the net flux case there is always a magnetic field for driving the instability

Large scale field threading the disk

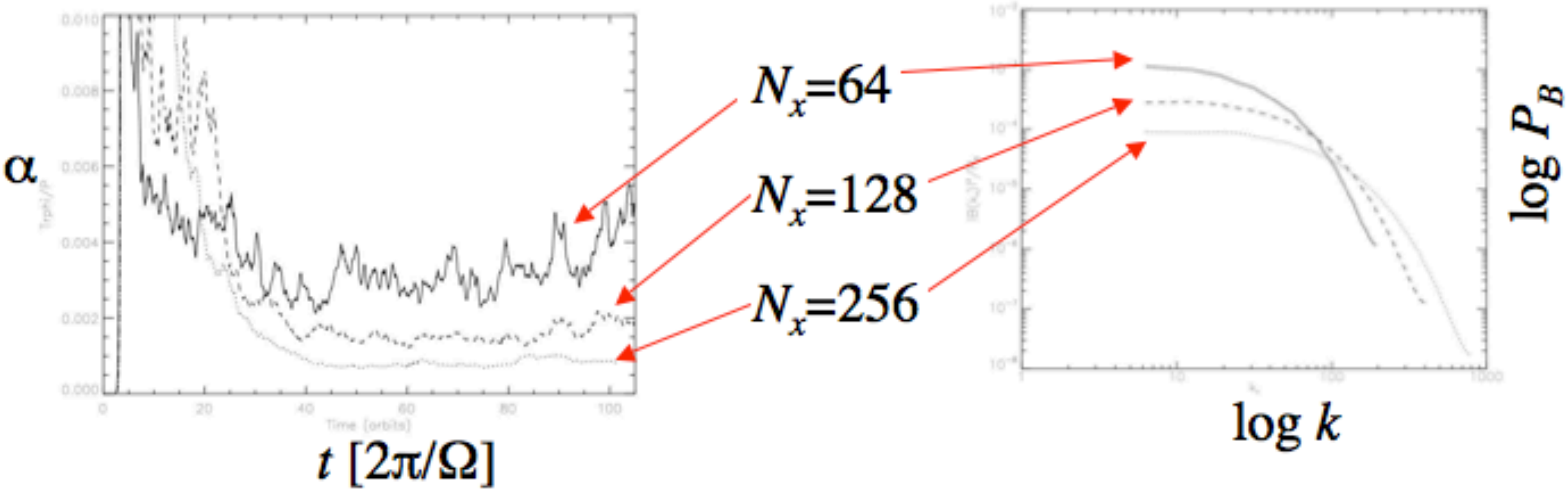
Transport depends on the intensity of the background field

Zero net flux case → dynamo mechanism to sustain the field

Universal behavior of disks



Saturation depends on grid scale when $<B_z> = 0$!



Both the level & the dominant lengthscale of the turbulence
decrease approximately linearly with the grid scale!

Fromang & Papaloizou (2007)

Where is the problem?

$$\frac{D\mathbf{u}}{Dt} + Ax\frac{\partial \mathbf{u}}{\partial y} + Au_x\mathbf{e}_y + 2\Omega \times \mathbf{u} = \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho} - \frac{1}{\rho}\nabla\left(\frac{\mathbf{B}^2}{8\pi} + p\right) + \nu\nabla^2\mathbf{u},$$

$$\frac{D\mathbf{B}}{Dt} + Ax\frac{\partial \mathbf{B}}{\partial y} - AB_x\mathbf{e}_y - \mathbf{B}\cdot\nabla\mathbf{u} = \eta\nabla^2\mathbf{B},$$

$$\nabla\cdot\mathbf{u} = \nabla\cdot\mathbf{B} = 0,$$

We non dimensionalize the equations using these units

$$\tau = \frac{1}{\Omega}; \qquad \mathcal{L} = l_D \equiv \sqrt{\frac{\nu}{\Omega}}; \qquad u^* = \sqrt{\nu\Omega}; \qquad B^* = \sqrt{\rho\nu\Omega}.$$

$$\frac{D\hat{\mathbf{u}}}{D\hat{t}} - \frac{3}{4}\hat{x}\frac{\partial\hat{\mathbf{u}}}{\partial\hat{y}} - \frac{3}{4}\hat{u}_x\mathbf{e}_y + 2\mathbf{e}_z \times \hat{\mathbf{u}} = \frac{1}{4\pi}\hat{\mathbf{B}}\cdot\hat{\nabla}\hat{\mathbf{B}} - \hat{\nabla}\left(\frac{\hat{\mathbf{B}}^2}{8\pi} + \hat{p}\right) + \hat{\nabla}^2\hat{\mathbf{u}}$$

$$\frac{D\hat{\mathbf{B}}}{D\hat{t}} - \frac{3}{4}\hat{x}\frac{\partial\hat{\mathbf{B}}}{\partial\hat{y}} + \frac{3}{4}\hat{B}_x\mathbf{e}_y - \hat{\mathbf{B}}\cdot\nabla\hat{\mathbf{u}} + \frac{1}{P_m}\hat{\nabla}^2\hat{\mathbf{B}} = 0$$

Except for the magnetic Prandtl number no parameters

There is an external parameter, no physical significance
Low wavenumber cutoff: the computational box size

$$R = L/\lambda_D$$

Stresses have the dimension of the square of the velocity

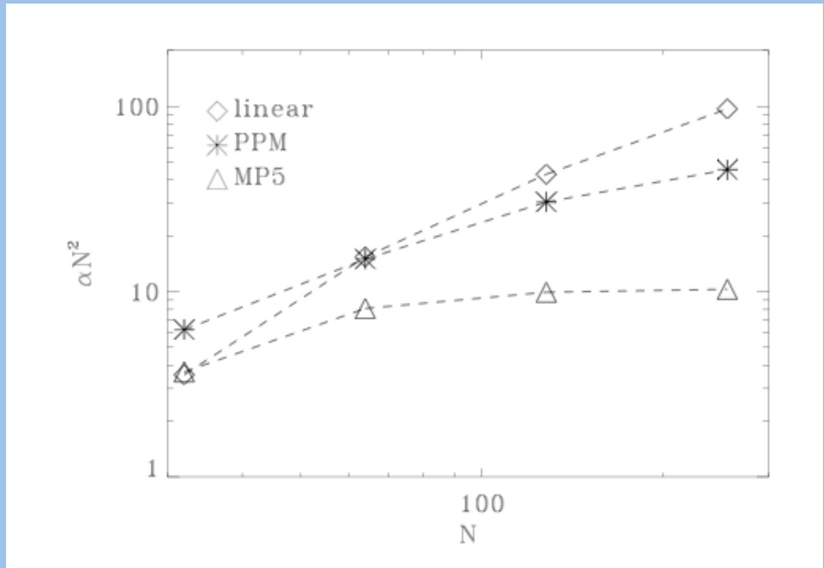
$$\Sigma \equiv \overline{< u_x u_y - \frac{B_x B_y}{4\pi\rho} >}$$

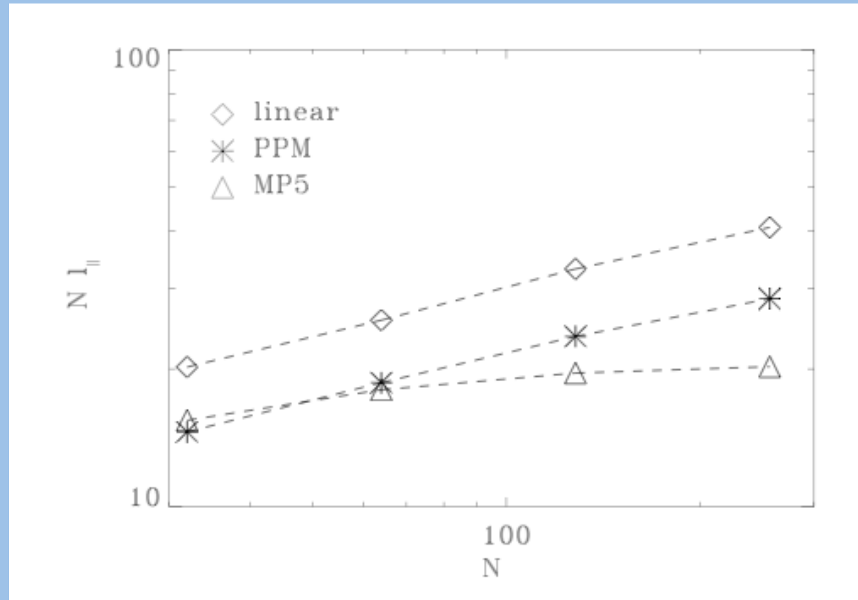
$$\Sigma \sim f(R)\Omega^2 l_D^2 = f(R)\Omega\nu.$$

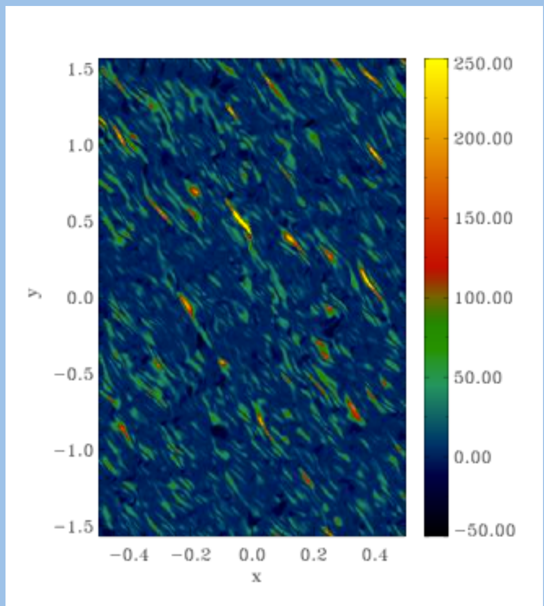
What happens when R>>1, there is still a dependence on the box size or the dependence disappears? In the second case the stresses decrease with viscosity.

$$\alpha \sim \frac{1}{R^2}$$

Results of simulations with no explicit diffusivities, only numerical Bodo et al 2011







$$l_{\parallel} = \left(\frac{< |\mathbf{B}|^4 >}{< |\mathbf{B} \cdot \nabla \mathbf{B}|^2 >} \right)^{1/2};$$

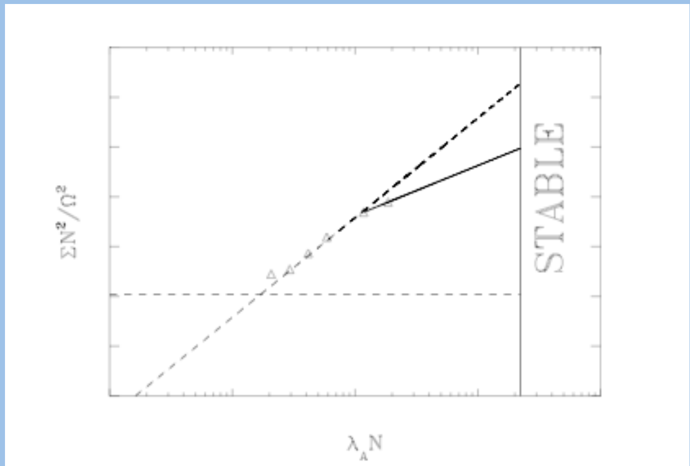
Scaling in the net flux case

New scale related to background field $\lambda_A = \frac{1}{\Omega}\frac{B_0}{\sqrt{4\pi\rho}}.$

For $$\lambda_D << \lambda_A << L$$

We can expect stresses to be independent both from the dissipation acale and from the box size and therefore

$$\Sigma \sim \lambda_A^2 \Omega^2$$



Recent results by Fromang (2010) seem to indicate that with physical dissipation the strsses tand to become indipendent of dissipation

Still not in asymptotic regime?

If true need to form structures of the scale of the box → large scale dynamo

In any case there are problems with the shearing box approach

Either gives values of the transport astrphysically irrelevant or formation of scales of the order of the box size invalidate its use.

Problem of convergence related to shearing box approach

→Global disk simulations

→ first step introduce stratification in the vertical direction

First results (Davis et al. 2010, Shi et al. 2010) show some indication of convergence but still inconclusive