The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

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2292-23

School and Conference on Analytical and Computational Astrophysics

14 - 25 November , 2011

Coronal Loop Seismology - Basics

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Coronal Loop Seismology - Basics

Mag Selwa

ICTP School and Conference on Analytical and Computational Astrophysics, Trieste, 23.11.2011

Outline

- ✓Simple model of coronal loop
- ✓The concept of coronal seismology
- ✓MHD waves
- ✓Observations of coronal loops oscillations as an input for coronal seismology:
  - ➢Magnetic field from kink oscillations
  - ➢Magnetic field from slow waves

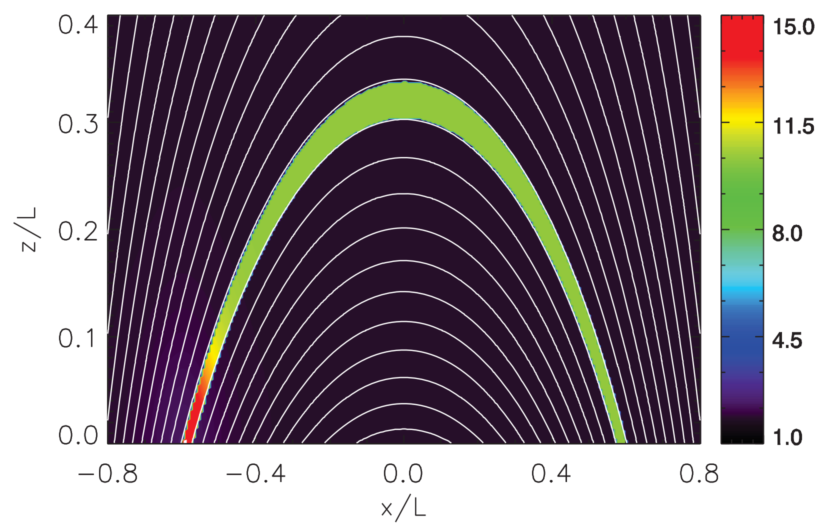
2D curved loop model

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right)=\nabla p+\frac{1}{\mu}(\nabla\times\mathbf{B})\times\mathbf{B}+F$$

Variables: ρ, p, **V**, **B**
EOS: T(p) -> ρ, c_s(T), V, V_A

$$c_s^2=\frac{\gamma p}{\rho}$$

$$V_A^2=\frac{B^2}{4\pi\rho}$$



- ✓ Numerical model
- ✓ Boundary conditions
- ✓ Initial conditions

$$(\nabla\times\mathbf{B})\times\mathbf{B}=0$$

$$\nabla\times\mathbf{B}=0$$

$$A=B\Lambda_B\cos(x/\Lambda_B)\exp(-z/\Lambda_B)$$

$$\mathbf{B}=\left[B\cos(x/\Lambda_B),-B\sin(x/\Lambda_B),\right]\exp(-z/\Lambda_B)$$

$$\Lambda_B=\frac{2L}{\pi}$$

$$\rho(z)=\rho_0$$

2D curved loop model

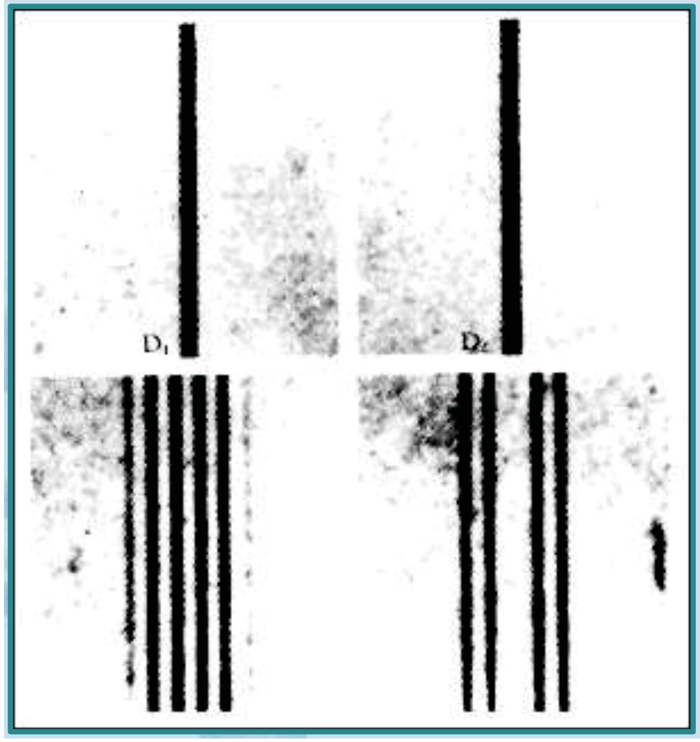
Variables: ρ, p, **V**, **B**
EOS: T(p) -> ρ, c_s(T), V, V_A

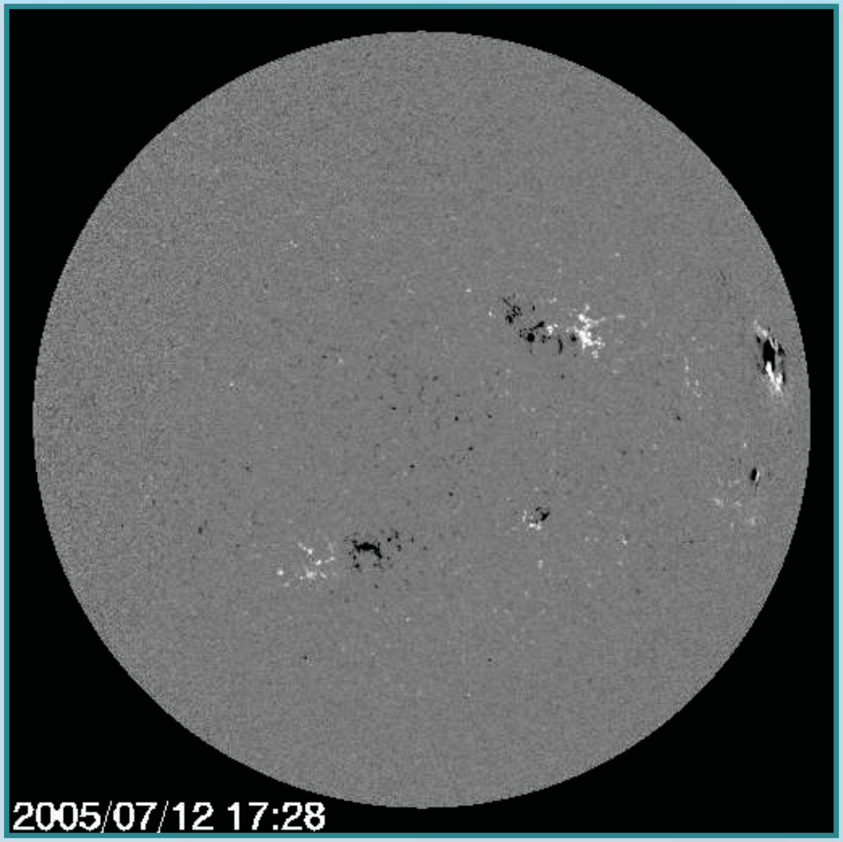
Equilibrium:

✓**V**=0
✓T – peak temperature
TRACE: 171 Fe IX/X 1MK
195 Fe XII 1.5MK
284 Fe XV 2MK
SUMER(SOHO) Fe XIX 6.3 MK
Fe XX 8MK
✓ ρ – observed brightness
+ theory of Thomson scattering
+ scale height
- spectroscopy
✓ B - ? -> β regime $\rho\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right)=\nabla p+\frac{1}{\mu}(\nabla\times\mathbf{B})\times\mathbf{B}\ ;\ \beta=\frac{p}{B^2/2\mu}=\frac{2}{\gamma}\frac{c_s^2}{V_A^2}$

Magnetic field measurements

Magnetogram - refers to a pictorial representation of the spatial variations in strength of the solar magnetic field. Magnetograms are often produced by exploiting the Zeeman effect (1896).

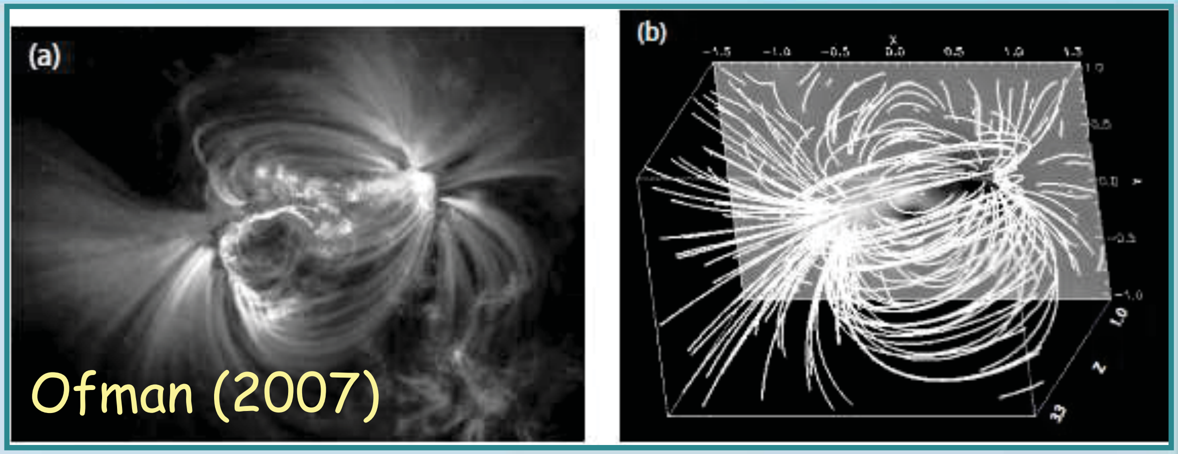




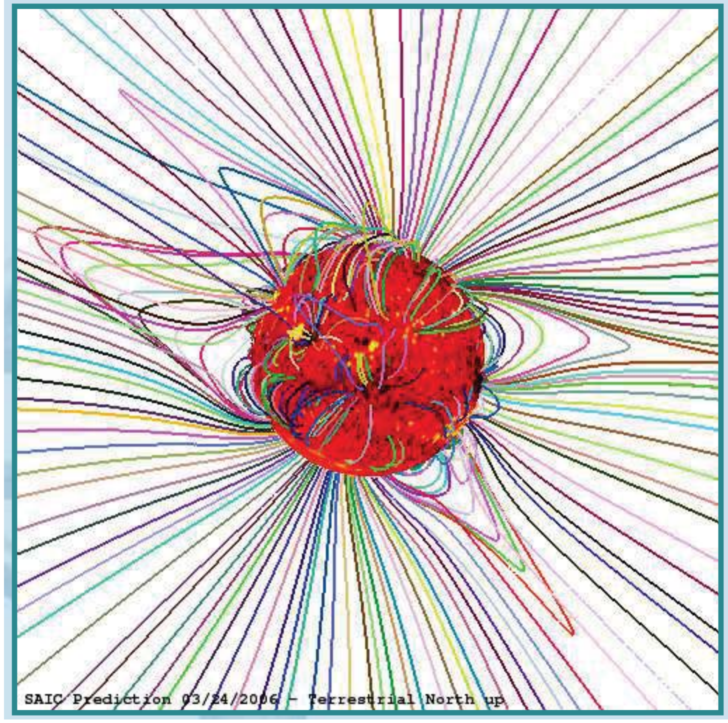
Magnetic field measurements

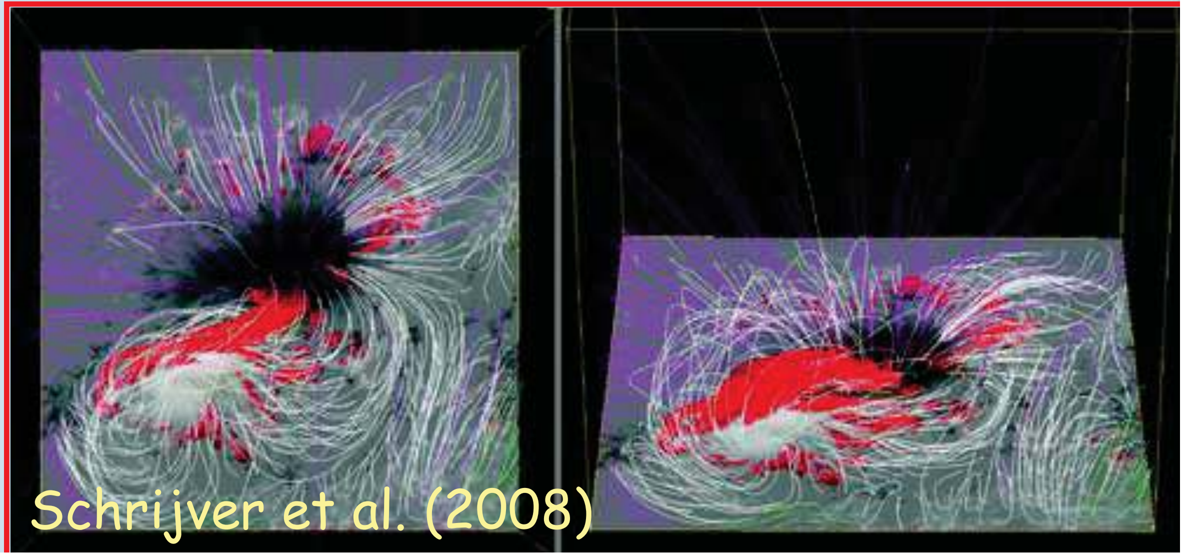
Photospheric magnetogram

✓Potential extrapolation
✓NLFF model
✓Potential field source surface (PFSS)

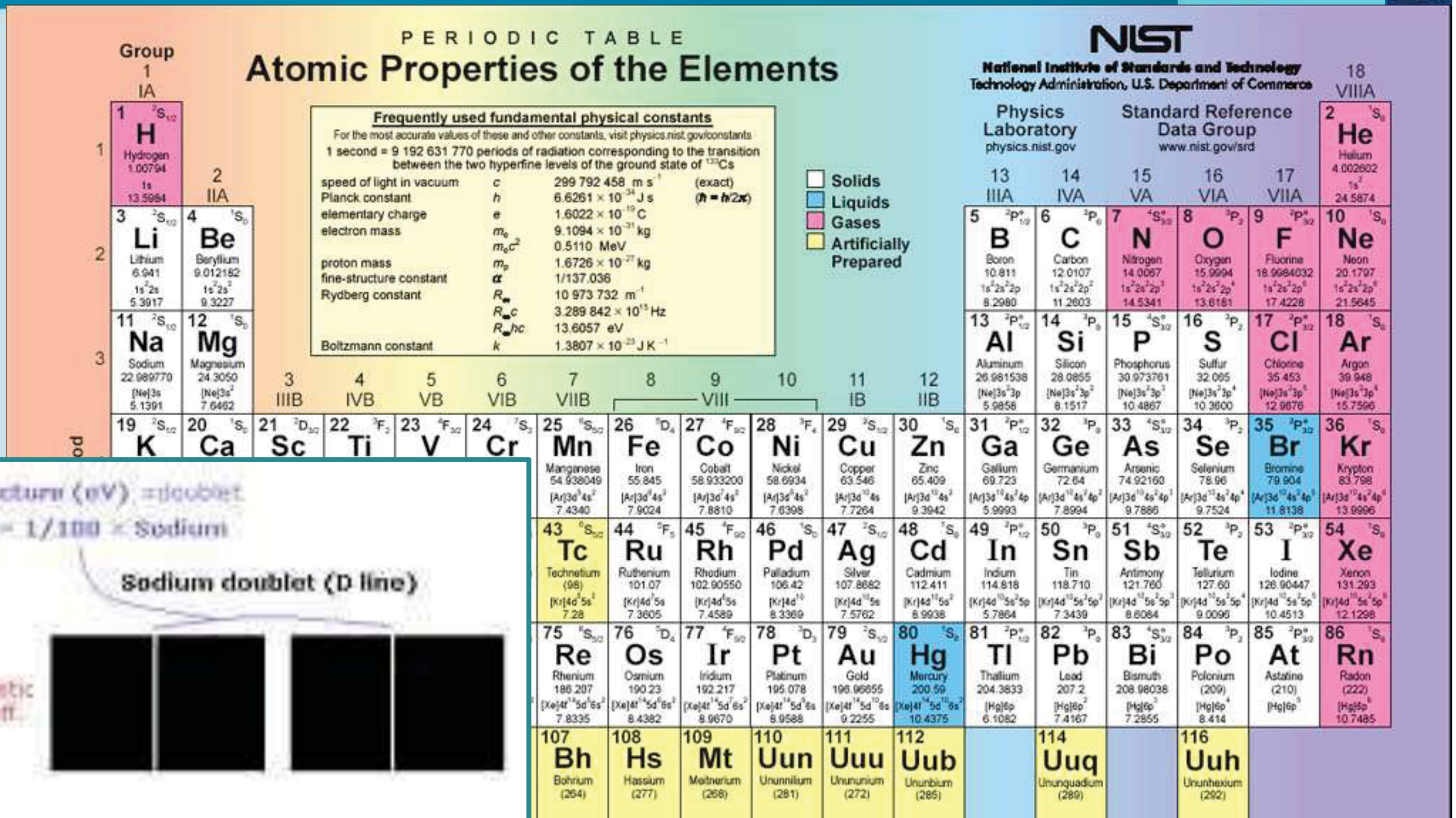


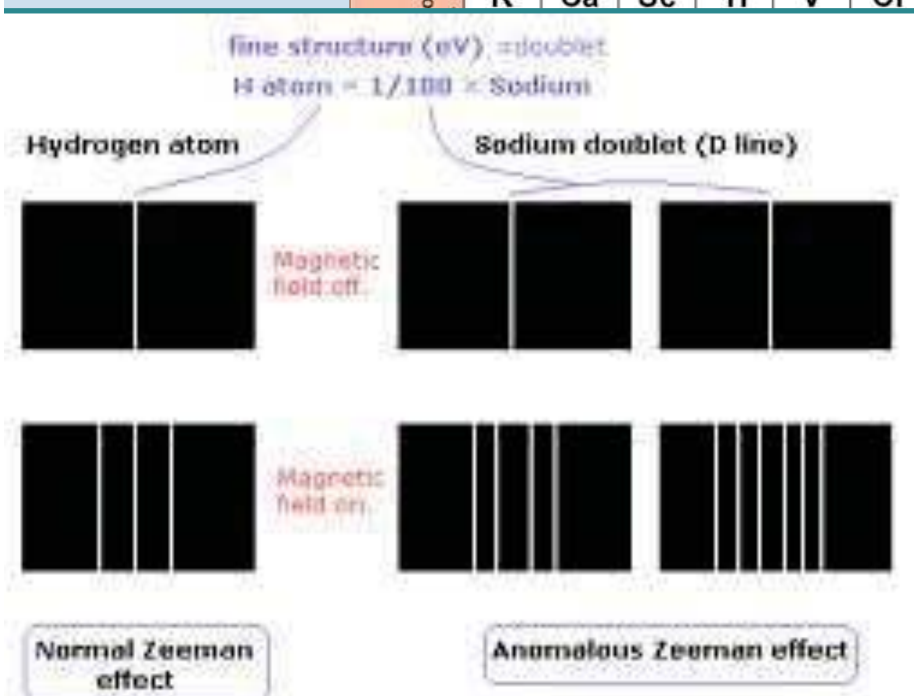
Ofman (2007)




Schrijver et al. (2008)

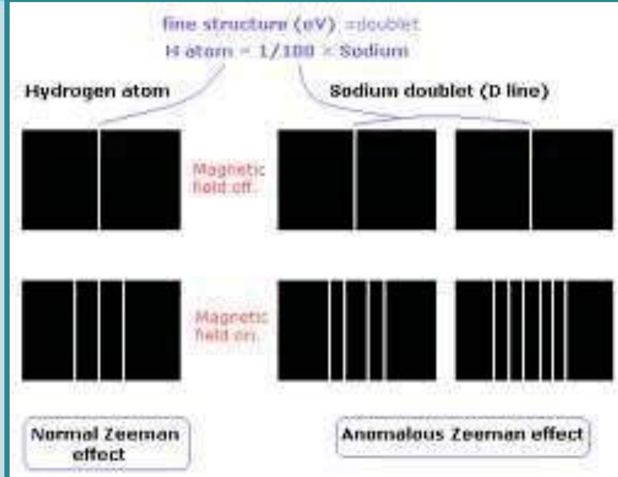
Zeeman effect





Zeeman effect

Quantum numbers:
- ✓Principal $n \geq 1$
- ✓Azimuthal (angular momentum) $0 \leq l \leq n-1$
- ✓Magnetic (projection of angular momentum) $-l \leq m_l \leq l$
- ✓Spin $\pm\frac{1}{2}$



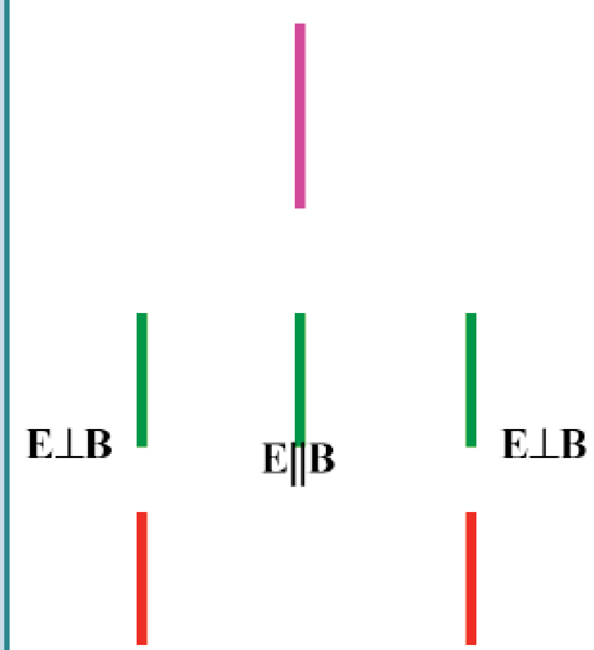
Energy of magnetic moment

✓Normal $\Delta E = -\boldsymbol{\mu} \cdot \mathbf{B} = -\mu_z B = m_l \mu_B B = m_l \frac{e\hbar}{2m_e} B$

✓Anomalous $\Delta E = g m_j \mu_B B$

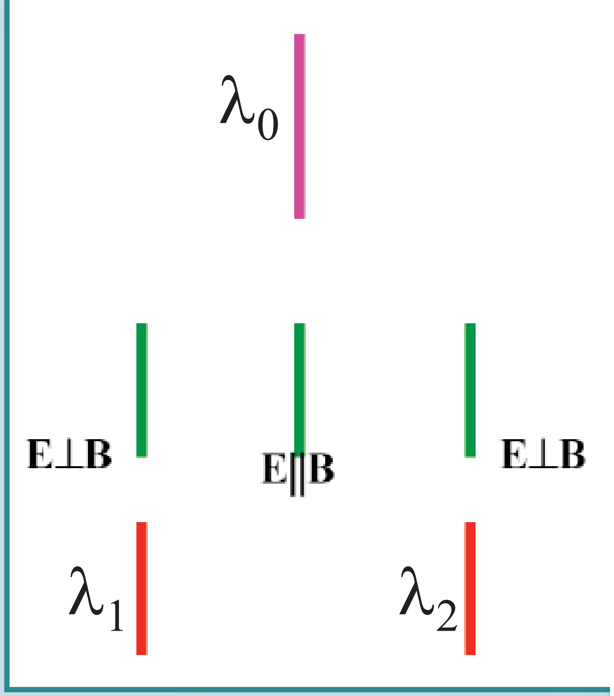
$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$



Zeeman effect

$\lambda_0, \lambda_1, \lambda_2 \,(\Delta\lambda = \lambda_1 - \lambda_2) \rightarrow$ B=?



$$\Delta E_{1,2} = g_{1,2} m_{j1,2} \mu_B B = g_{1,2} m_{j1,2} \frac{e\hbar}{2m_e} B$$

$$\Delta E = \left(g_1 m_{j1} - g_2 m_{j2}\right)\mu_B B$$

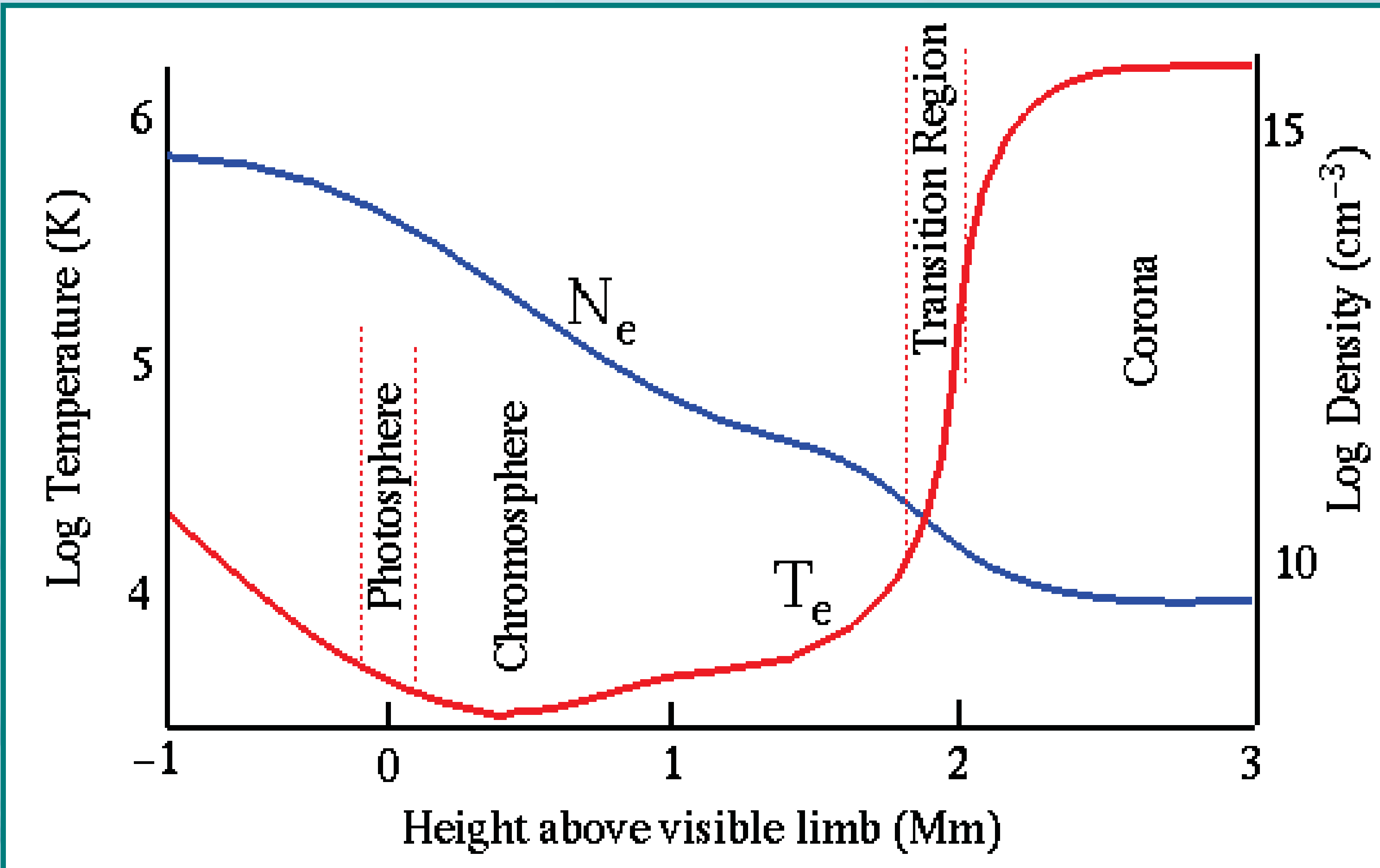
$$E = h\nu = \frac{hc}{\lambda_0}$$

$$\Delta\lambda = -\frac{hc}{E^2}\Delta E$$

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

$$B = \frac{\Delta E}{\left(g_1 m_{j1} - g_2 m_{j2}\right)\mu_B} = -\frac{E^2 \Delta\lambda}{hc\left(g_1 m_{j1} - g_2 m_{j2}\right)\mu_B} = -\frac{4\pi m_e c \Delta\lambda}{\lambda_0^2 \left(g_1 m_{j1} - g_2 m_{j2}\right)e}$$

Zeeman effect



(Coronal) Seismology

Seismology is the scientific study of earthquakes and the propagation of elastic waves through the Earth or through other planet-like bodies. The field also includes studies of earthquake effects, such as tsunamis as well as diverse seismic sources such as volcanic, tectonic, oceanic, atmospheric, and artificial processes (such as explosions).

- The study of waves with the aim to get the information about the local medium of propagation (remote diagnosis).

Coronal seismology is a technique of studying the plasma of the Sun's corona with the use of magnetohydrodynamic (MHD) waves and oscillations.

- The study of coronal waves, in order to measure physical quantities in the solar corona.

Source: Wikipedia

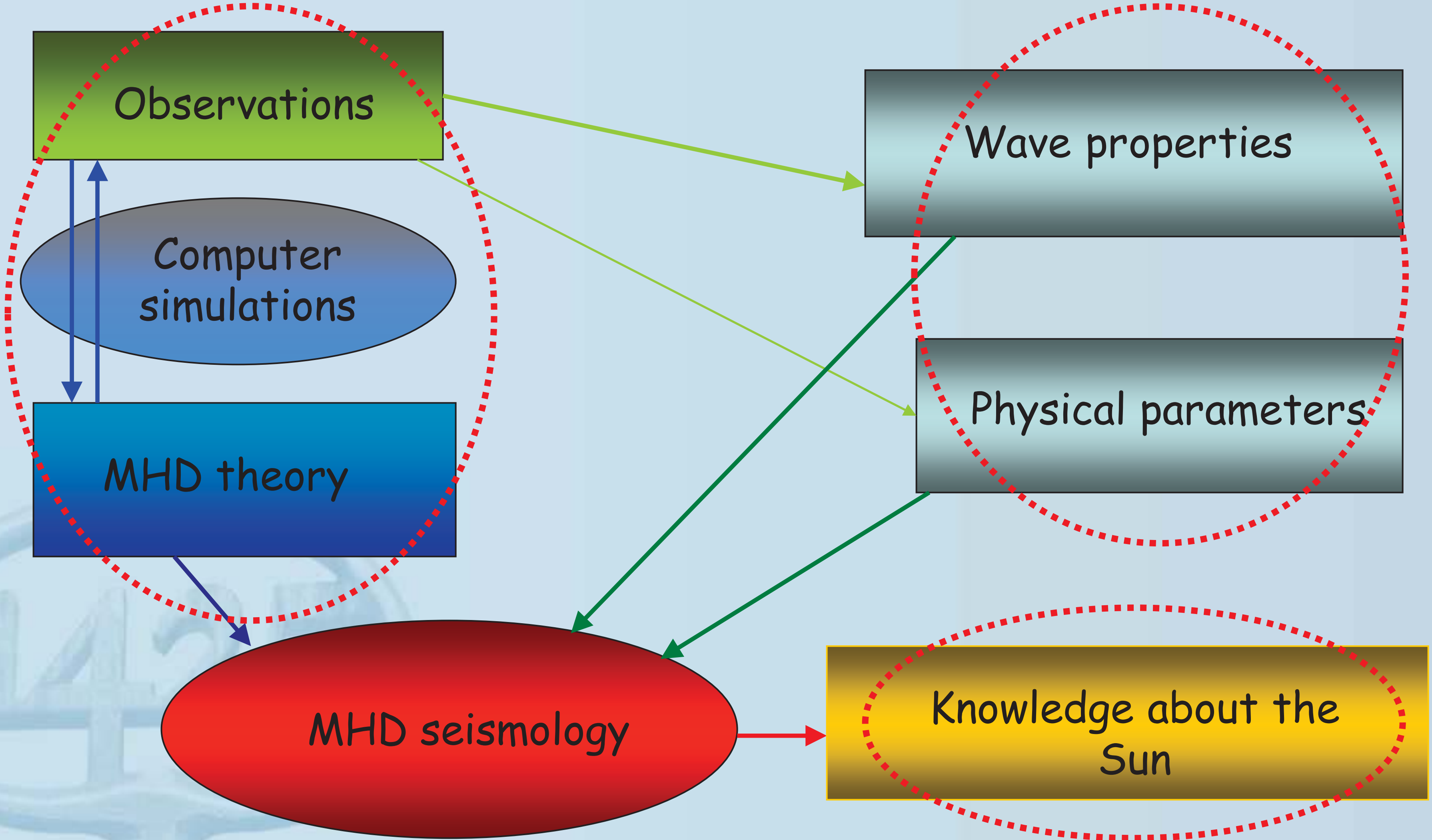
Why Coronal Seismology?

- ✓ To understand wave processes in plasma environments.
- ✓ To measure physical properties in the solar corona, that are difficult to measure otherwise. Often localized in space, so natural fine spatial resolution.
- ✓ To test physical models for coronal structures.

Works well for the description of a number of plasma structures:

- ✓ that can be described by a straight cylinder (structures non-uniform in the transverse direction and extending along the magnetic field, that can act as a waveguide) and
- ✓ that are observed in solar corona (e.g. coronal loops).

Coronal Seismology



History of Coronal Seismology?

- ✓ Uchida (1970) – "on the basis of the density distribution in the corona, this seismological diagnosis reveals the distribution of magnetic field"

- ✓ Roberts et al. (1984) – "magnetoacoustic oscillations provide a potentially useful diagnostic tool for determining physical conditions in the inhomogeneous corona"

Coronal loops

Coronal loops form the basic structure of the lower corona and transition region of the Sun. These highly structured and elegant loops are a direct consequence of the twisted solar magnetic flux within the solar body.

Source: Wikipedia

Theory of MHD waves

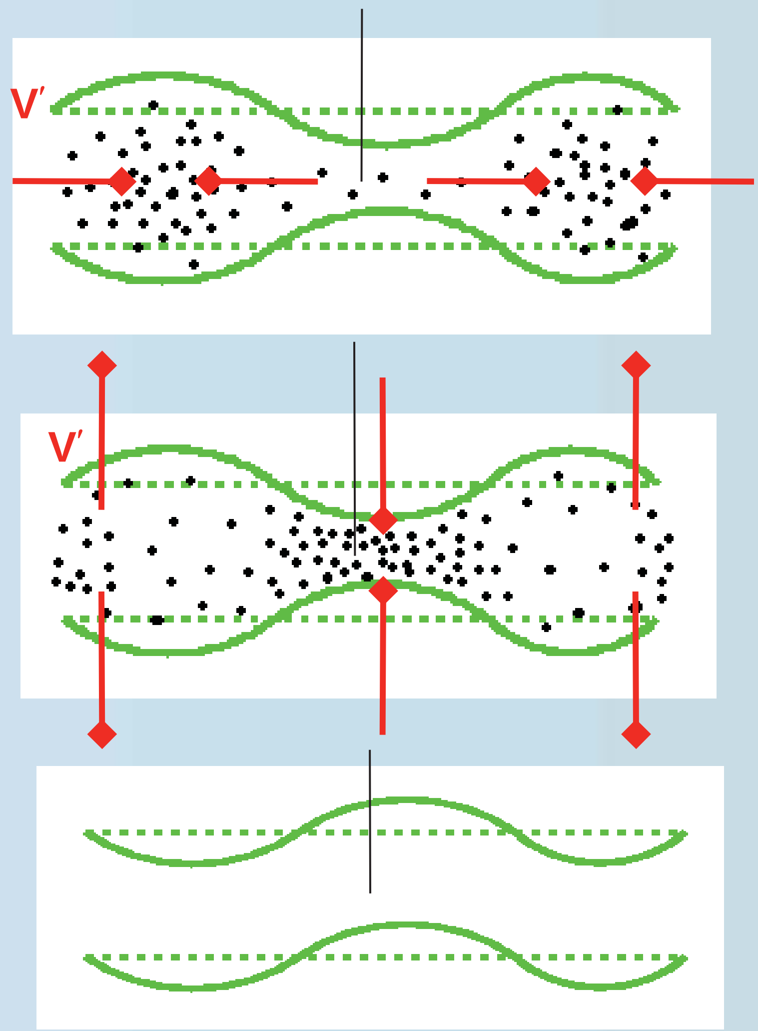
Edwin & Roberts (1983); Roberts et al. (1984)

Slow waves

Fast waves

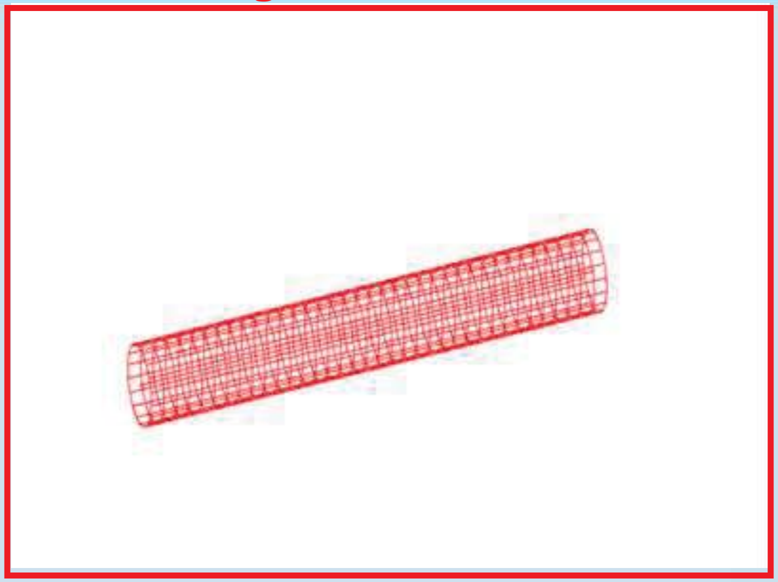
sausage

kink



Theory of MHD waves

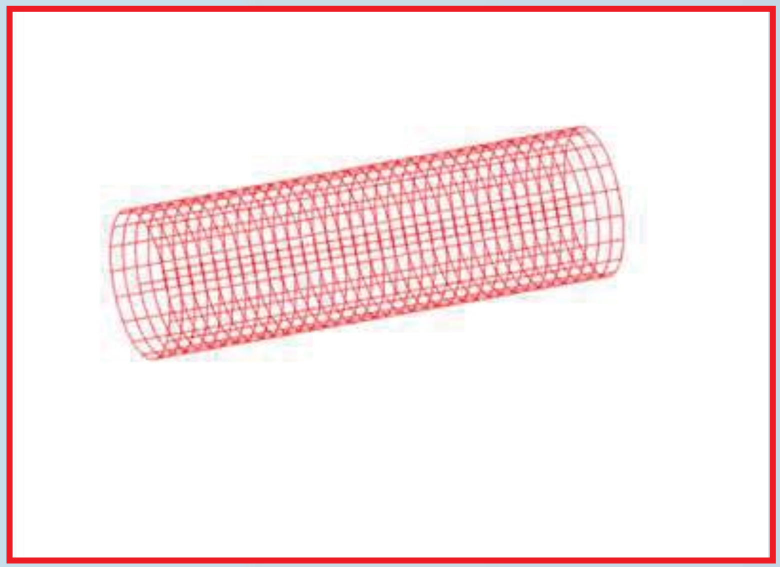
Fast sausage wave



Fast kink wave

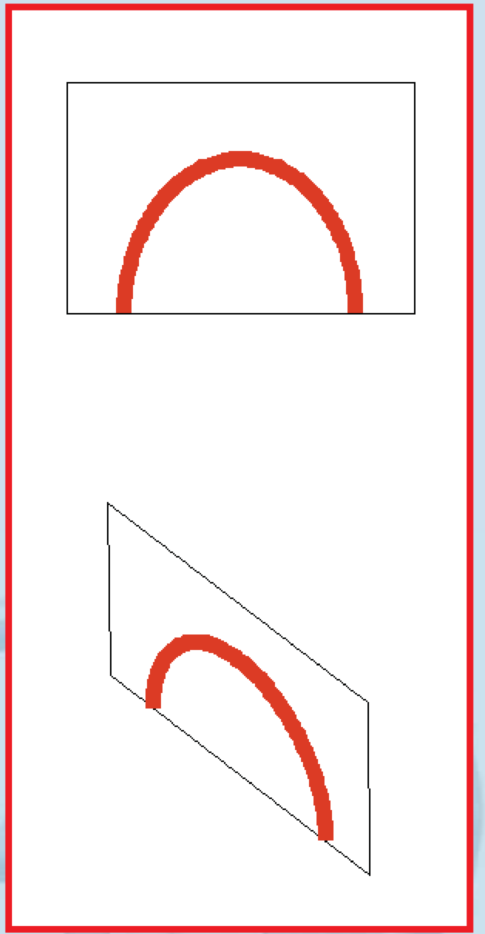
Slow wave

Alfvèn wave



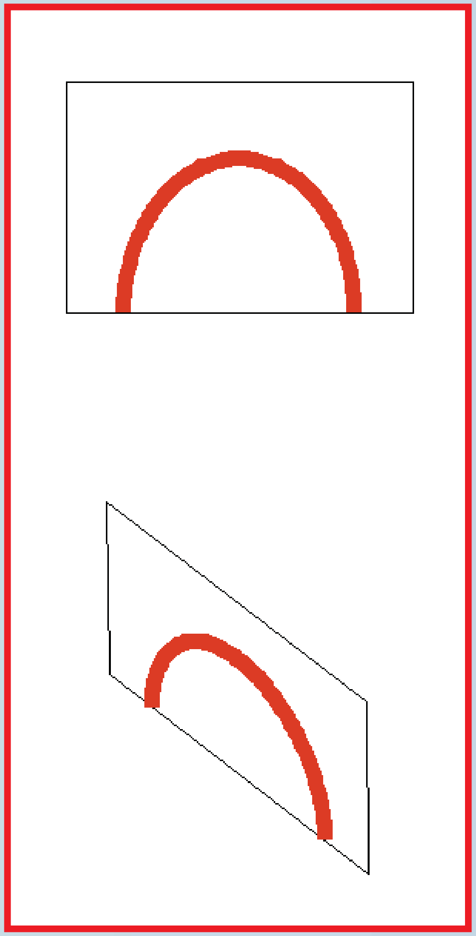
Kink waves

Horizontal mode



Aschwanden et al. (2002)

Vertical mode



Wang & Solanki (2004)

Van Doorsselaere et al. (2004)

-> curvature does not select a preferential oscillation direction

Theory of MHD waves

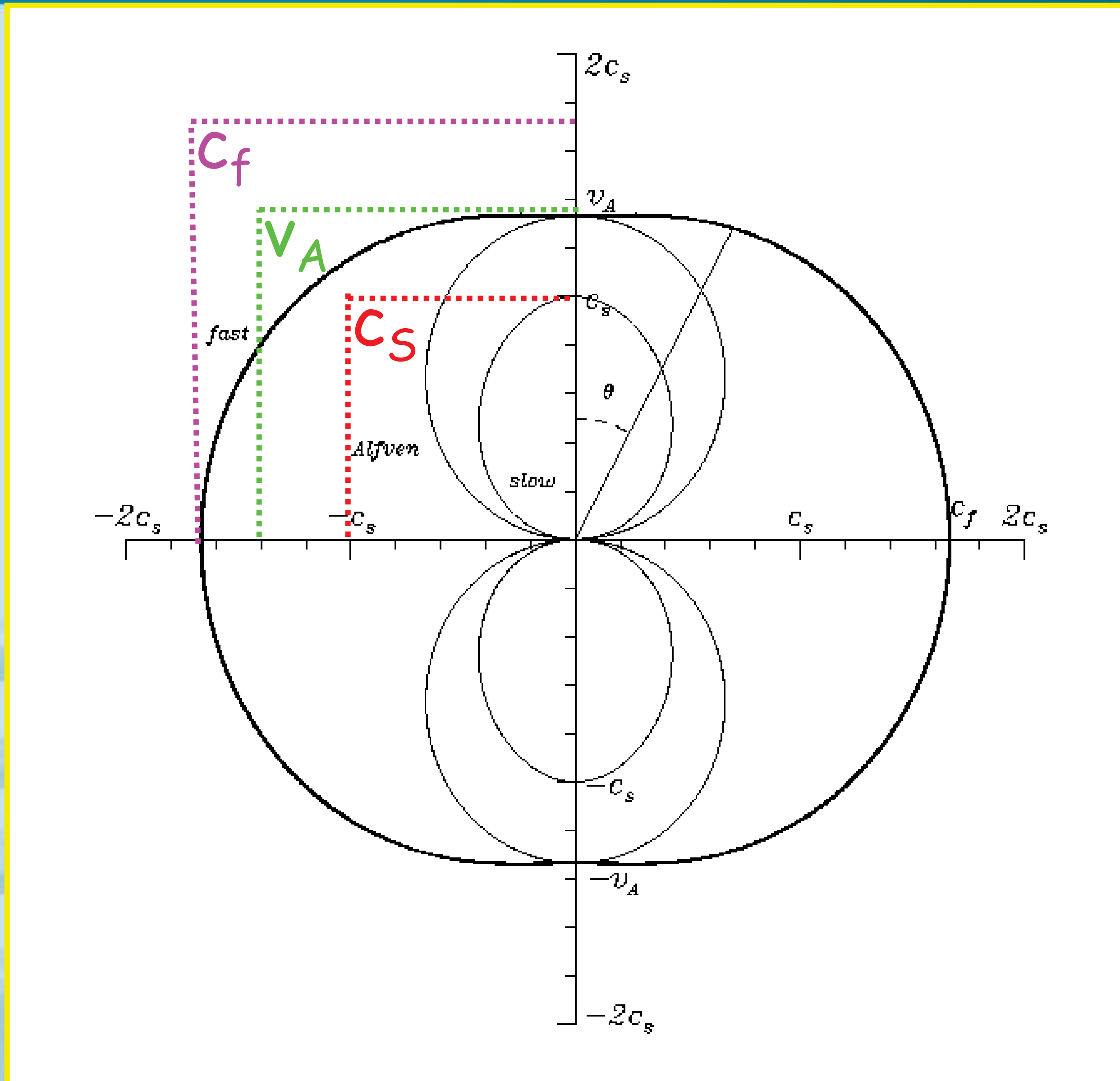
Roberts, Edwin, & Benz (1984)

TABLE III

Pulse periods of MHD waves

Standing MHD waves		
Slow modes	$T_0 = 2 \times 10^6$ K	$\tau_s \approx 850$ s
$\tau_s = \frac{2L}{kc_T} = \frac{1.2 \times 10^{-4} L}{jT_0^{1/2}} \left(1 + \left(\frac{c_0}{v_A}\right)^2\right)^{1/2}$	$L = 10^{10}$ cm $j = 1, c_0 \ll v_A$	
Fast kink mode	$N_0 = 10^9$ cm^{-3}	$\tau_s \approx 50$ s
$\tau_f = \frac{2L}{kc_k} = \frac{4\pi^{1/2} L}{j} \left(\frac{\rho_0 + \rho_e}{B_0^2 + B_e^2}\right)^{1/2}$	$L = 10^{10}$ cm $B_0 = 40$ G, $\rho_0 \gg \rho_e$	
Fast sausage mode	$N_0 = 10^9$ cm^{-3}	$\tau_s \approx 1.5$ s
$\tau_{f'} = \frac{2\pi a}{c_k} = 4\pi^{3/2} a \left(\frac{\rho_0 + \rho_e}{B_0^2 + B_e^2}\right)^{1/2}$	$a/L = 10^{-2}$ $B_0 = 40$ G, $\rho_0 \gg \rho_e$	
Propagating MHD waves		
$\tau_c = \frac{2\pi a}{j_{0,e} v_A} \left(1 - \frac{\rho_e}{\rho_0}\right)^{1/2} \approx 2.6 \left(\frac{a}{v_A}\right)$	$a = 2 \times 10^8$ cm, $s = 1$ $N_0 = 10^9$ cm^{-3} $B_0 = 40$ G, $\beta \ll 1$	$\tau_c \approx 2$ s

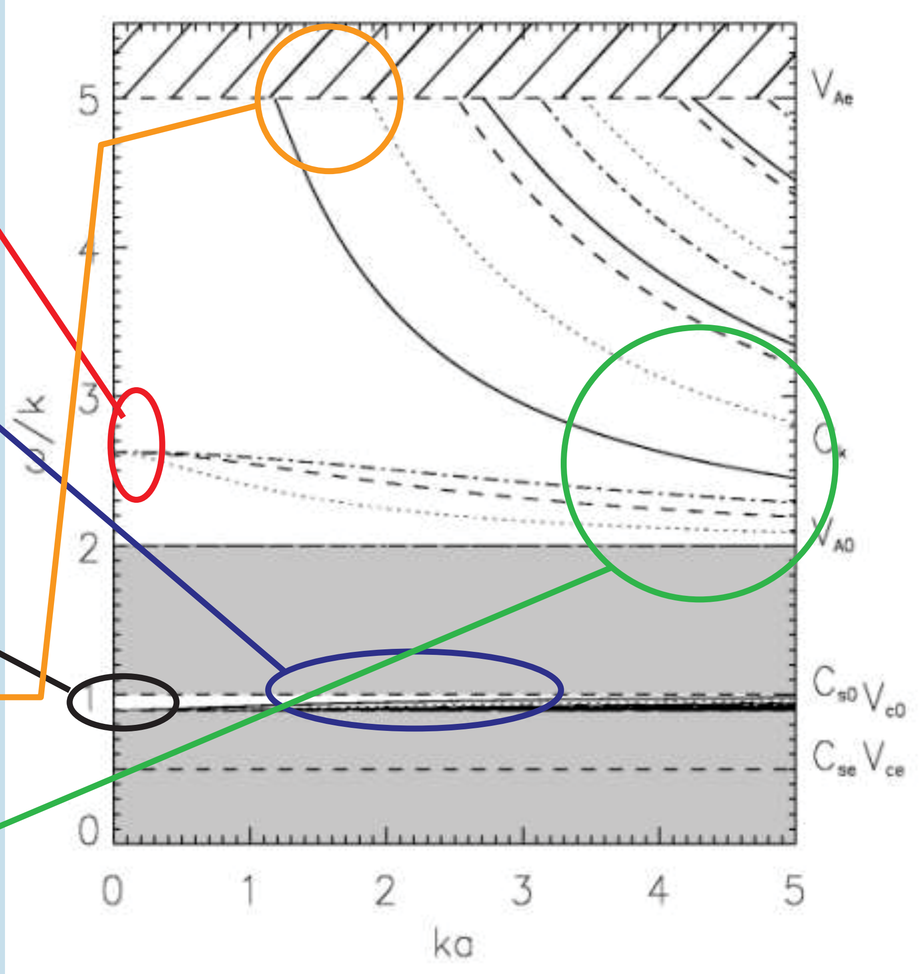
Theory of MHD waves



Breakthrough (SOHO & TRACE)

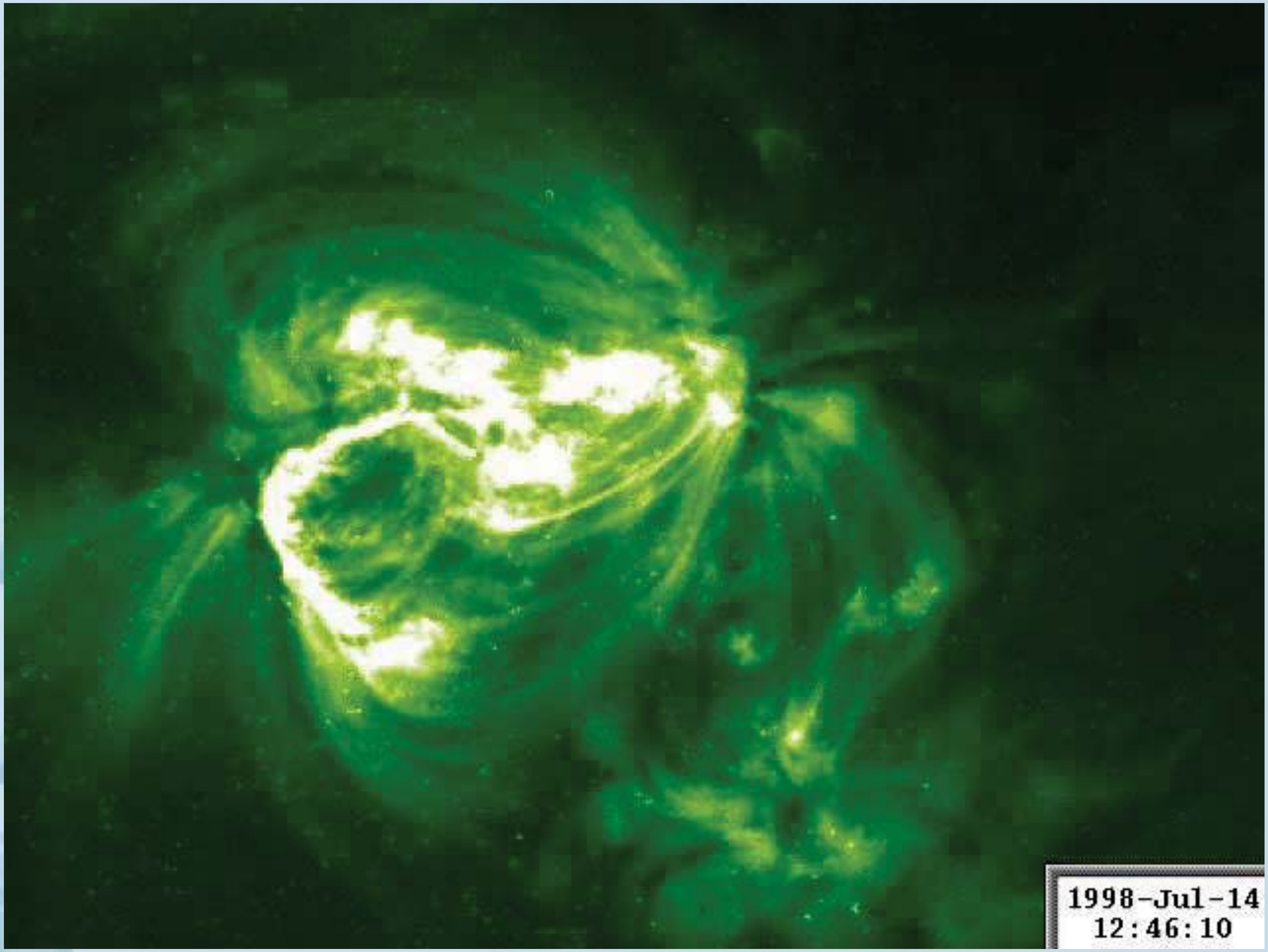
1. Kink oscillations of coronal loops (Aschwanden et al. 1999, 2002; Nakariakov et al. 1999; Verwichte et al. 2004)
2. Propagating longitudinal waves in polar plumes and near loop footpoints (Ofman et al. 1997-1999; DeForest & Gurman, 1998; Berghmans & Clette, 1999; Nakariakov et al. 2000; De Moortel et al. 2000-2004)
3. Standing longitudinal waves in coronal loops (Kliem at al. 2002; Wang & Ofman 2002)
4. Global sausage mode (Nakariakov et al. 2003)
5. Propagating fast wave trains. (Williams et al. 2001, 2002; Cooper et al. 2003; Katsiyannis et al. 2003; Nakariakov et al. 2004, Verwichte et al. 2005)

Already identified coronal MHD modes:



B from kink oscillations

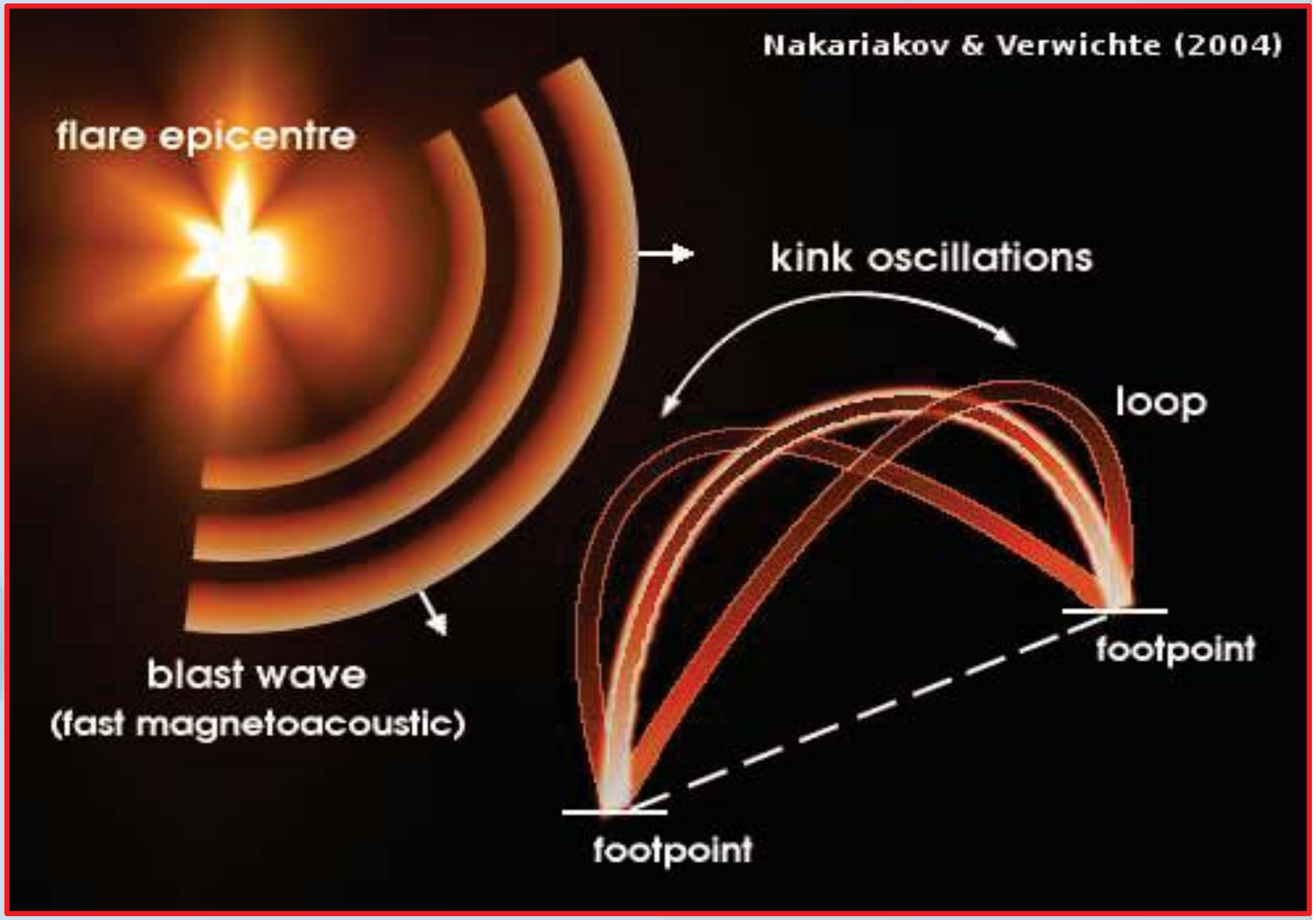
Kink oscillations



B from kink oscillations

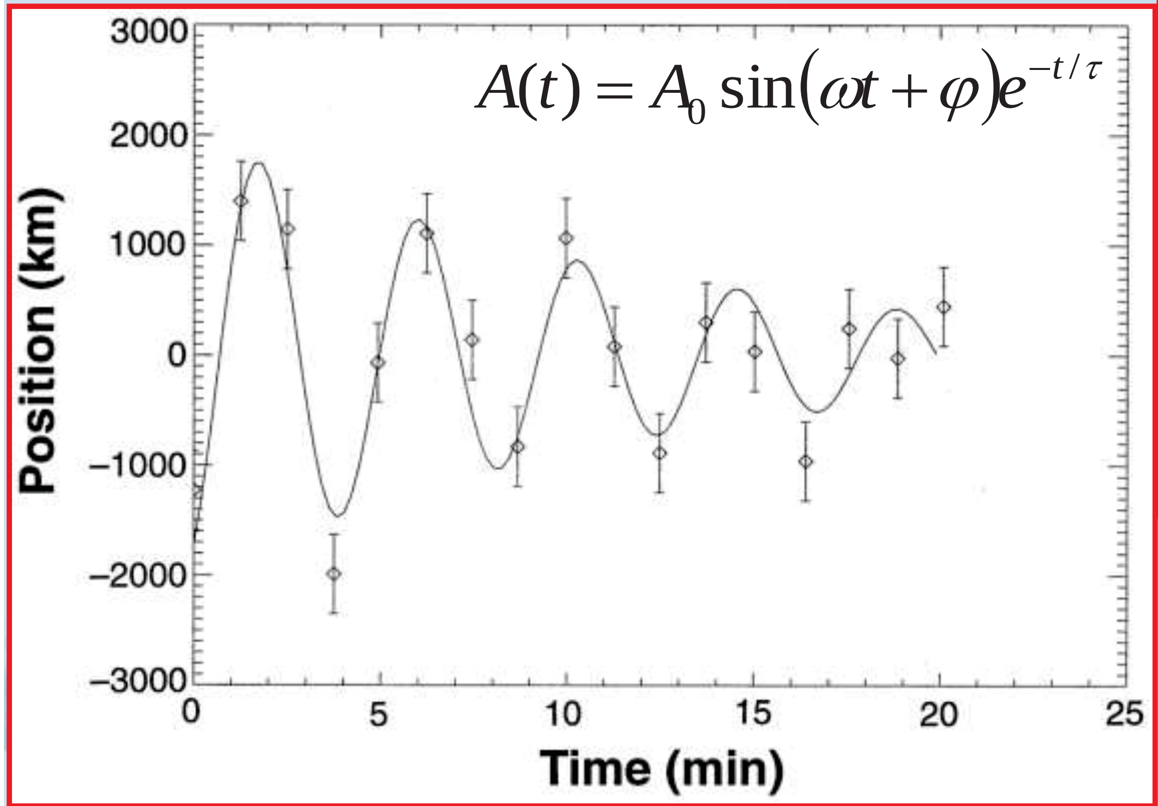
Kink oscillations

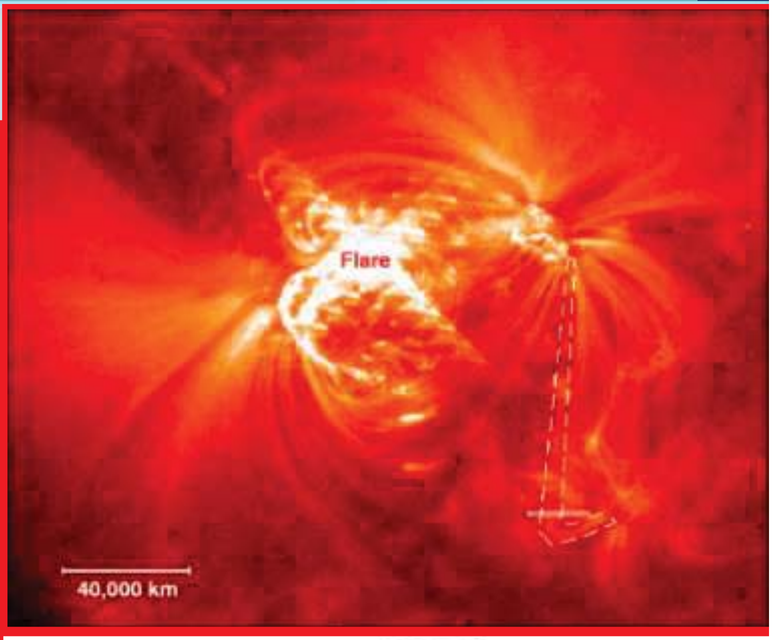
- ✓ Periods: 1s - 1h
- ✓ Generation: granules or flares
- ✓ Quasiperiodicty
- ✓ Fast generation of standing waves and strong damping

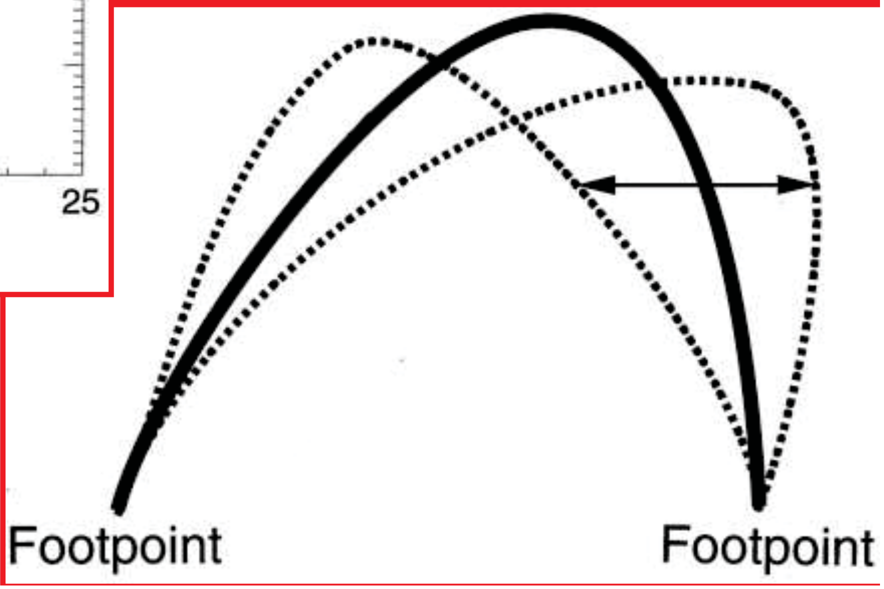


B from kink oscillations

Case 14 July 1998 12:11 UT





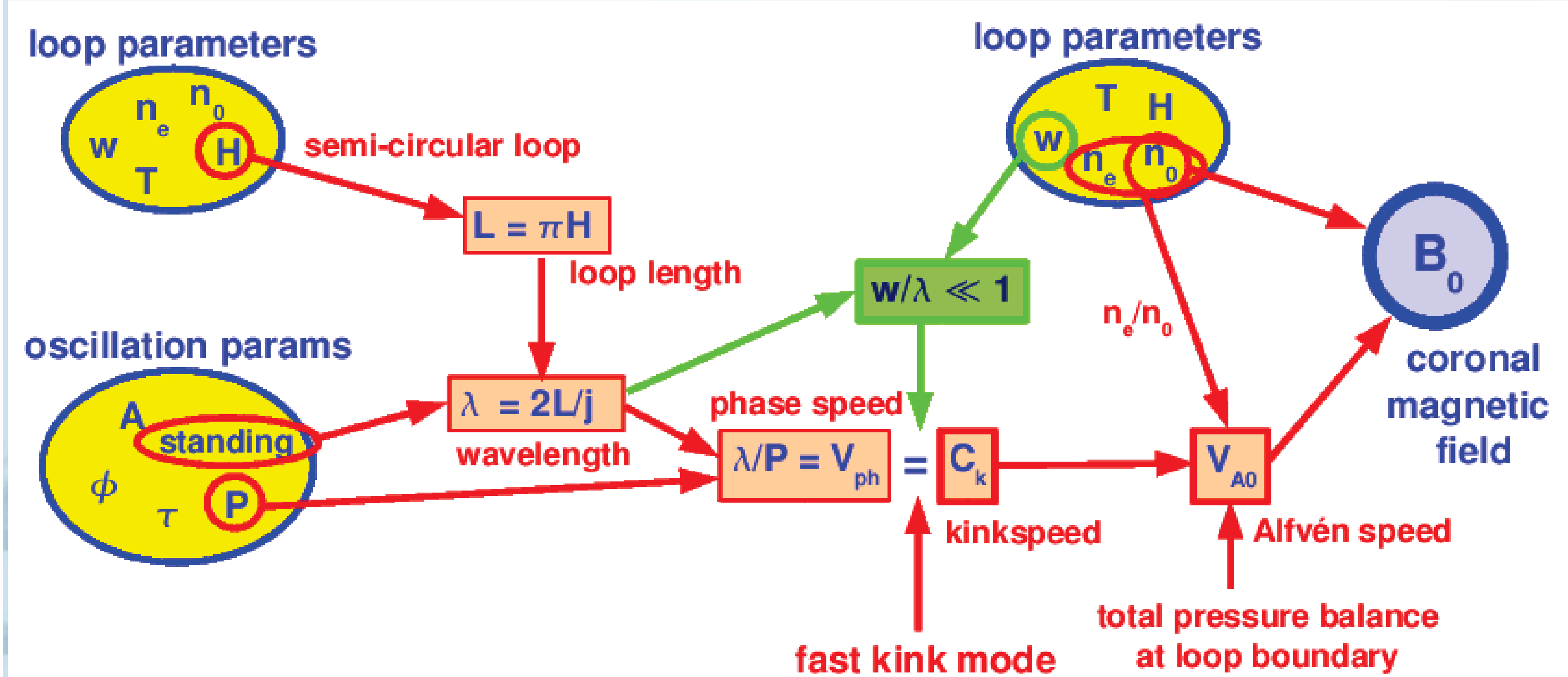


frequency of 3.90 ± 0.13 mHz
decay time 12.1 ± 6.7 min

Nakariakov et al. (1999)

B from kink oscillations

Scheme of the method



Nakariakov, Ofman (2001)

B from kink oscillations

Details:

1. Distance between footpoints -> semi-circular loop length L=πR
2. Observed period and length -> phase speed $V_{ph}=2L/P$
3. Phase speed=kink speed -> $V_{ph}=c_k$
4. Kink speed -> Alfvén speed $c_k = V_A\sqrt{\dfrac{2}{1+\rho_e/\rho_i}}$
5. Alfvén speed and density -> magnetic field $V_A^2 = \dfrac{B^2}{4\pi\rho_i}$

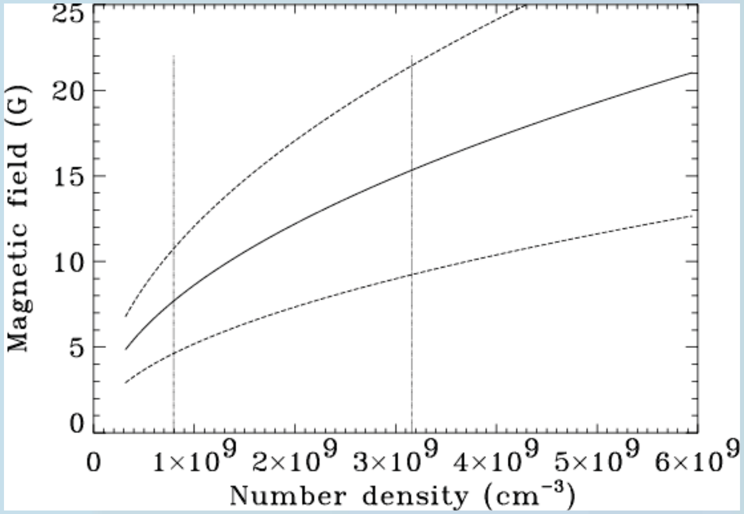
$$B = V_A\sqrt{4\pi\rho_i} = \frac{\sqrt{2}\pi^{3/2}L}{P}\sqrt{\rho_i\left(1+\rho_e/\rho_i\right)}$$

For $\rho = 1\times10^9\ \mathrm{cm^{-3}}$ to $\rho = 6\times10^9\mathrm{cm^{-3}}$ ->B=4-30G

6. From detailed emission measure studies

$\rho=10^{9.2\pm0.3}\ \mathrm{cm^{-3}}$ to $\rho = 6\times10^{9.3\pm0.3}\mathrm{cm^{-3}}$ –

-> B=13±9G



Nakariakov, Ofman (2001)

B from kink oscillations

Error analysis:

1. Errors in determination of loop length (due to projection and different than circular shape, e.g. elliptic -> 15%).
2. The error in the determination of the oscillation period (sensitive to the condition of the specific observation, here 3% as many periods were observed).
3. The accuracy of determination of the densities depends upon the specific method applied and is about 50% (e.g. Mason et al. 1999 and references therein). The determination of the density ratio possibly has even larger error, but as the ratio is assumed to be small, one can neglect its value.
4. The relative error of the method can be estimated as

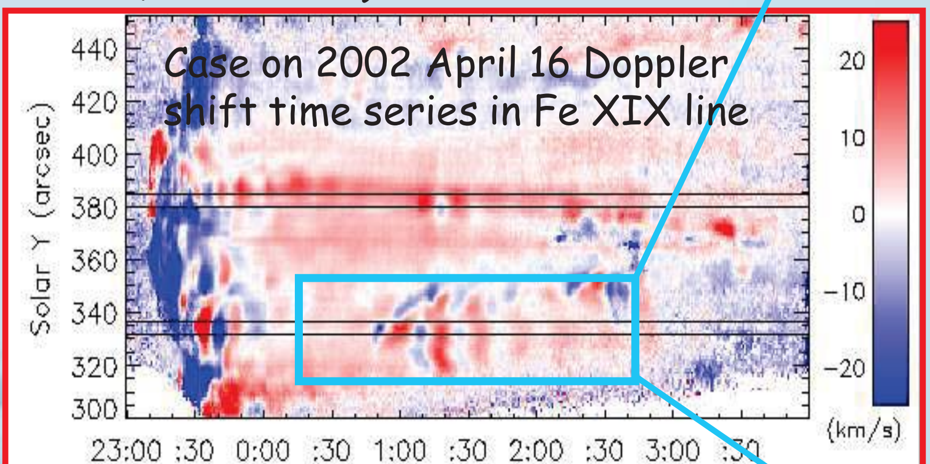
$$\delta B = \sqrt{(\delta L)^2 + (\delta P)^2 + (\delta \rho_i / 2)^2}$$

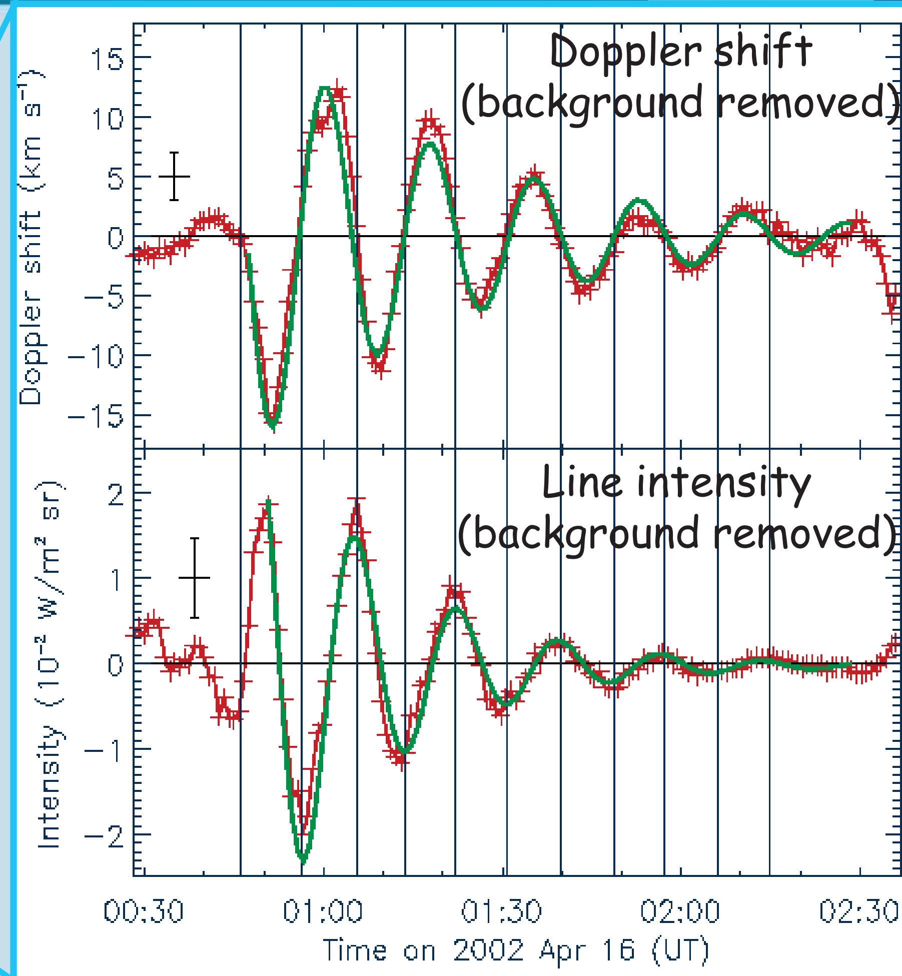
For δL=10%, δP=3%, δq=50% -> δB=30%

Nakariakov, Ofman (2001)

B from slow oscillations

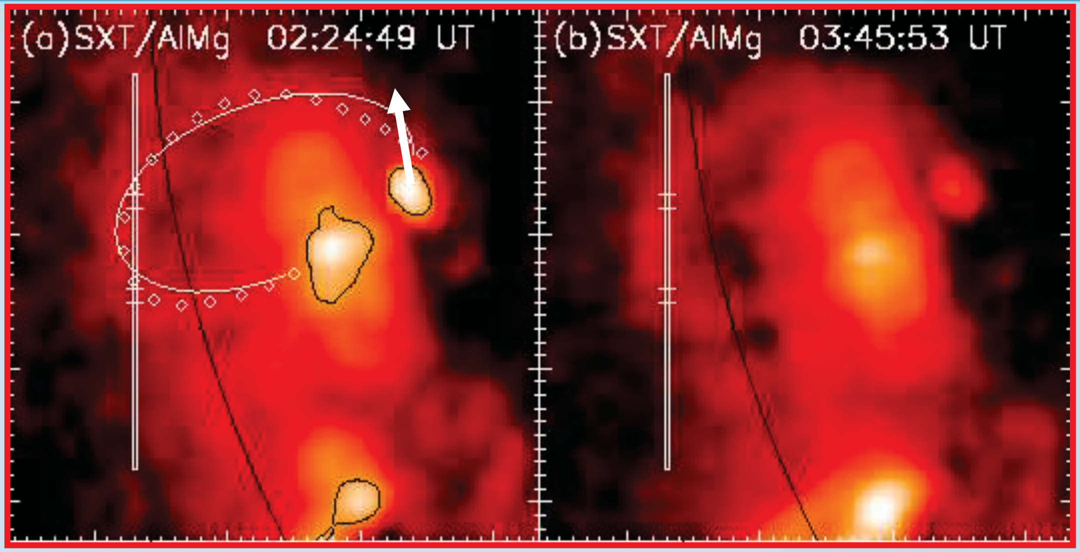
Wang, Solanki, et al. (2002, 2003)
SUMER observations
of loop oscillations above limb,
in flare loops with temperatures
T>6 MK (Fe XIX, T=6.3 MK;
Fe XX, T=8 MK)



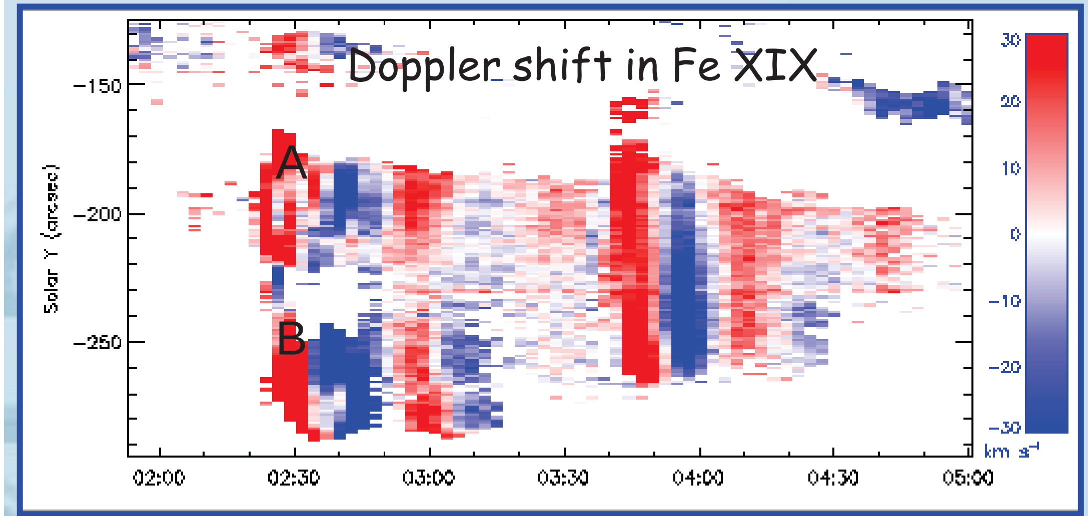


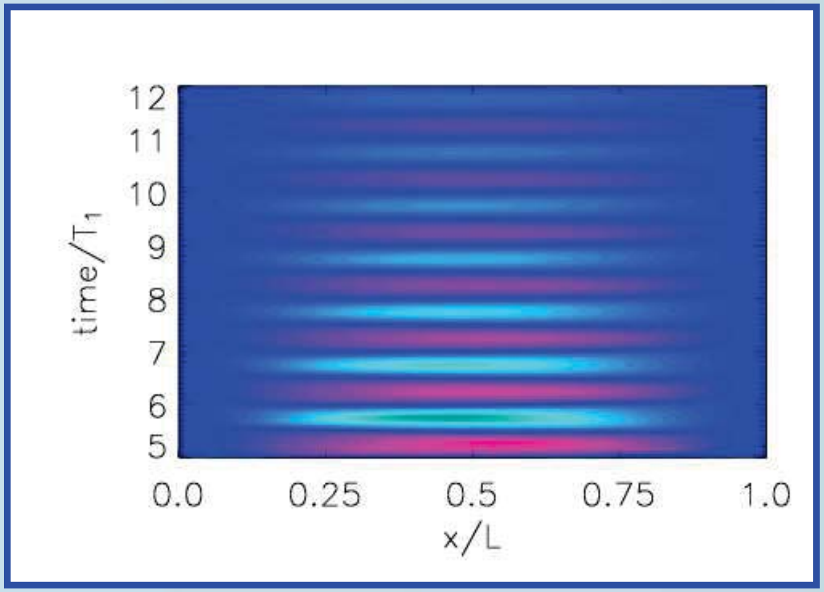
- ✓ Periods are 10-20 min
- ✓ Velocity and intensity have $\frac{1}{4}$ period phase shift
- ✓ Strong damping τ=12-19 min, τ/P~1

B from slow oscillations



Trigger of oscillations: initiation associated with a footpoint brightening





Case on 2000 September 29

Wang et al. (2003)

Selwa, Murawski, Solanki (2005)

B from slow oscillations

Method:

1. Distance between footpoints -> elliptical loop length L
2. Observed period and length -> tube speed c_t=2L/P, $c_t = V_A \sqrt{\frac{1}{c_s^2 + V_A^2}}$
3. Slow speed (measured T) and Alfvén speed -> plasma β

$$c_s^2 = \frac{\gamma p}{\rho} = \sqrt{\gamma k_B T / \mu m_p} \quad ; \; V_A^2 = \frac{B^2}{4\pi \rho_i} \quad \rightarrow \quad \beta = \frac{2}{\gamma} \frac{c_s^2}{V_A^2}$$

4. Thus magnetic field can be estimated as

$$B = \left(\frac{n_9}{C_1} \right)^{1/2} \left(\frac{P^2}{4L^2} - \frac{1}{C_2 T_6} \right)^{-1/2}$$

For C_1 =4.8x10^3, C_2 =2.3x10^4, B[G], n_9[10^{9}cm^{-3}], P[s], L[km], T[10^{6}K]

6. Results: β: 0.15-0.91 (mean 0.33±0.26),

excluding 1 loop 0.15-0.33 (mean 0.24±0.08),

B: 21-61 G (mean 34±14G)

Wang et al. (2007)

B from slow oscillations

Error analysis:

1. Errors in determination of electron density (less significant as f_n is the constant number)
2. Errors in determination of temperature
3. Errors in determination of the loop length
4. Errors in determination of the oscillation period
5. The relative error of the method can be estimated as

$$\delta B = \sqrt{f_n^2(\delta n)^2 + f_T^2(\delta T)^2 + f_L^2(\delta L)^2 + f_P^2(\delta P)^2}$$

For $\delta n=\delta L=\delta T=5\%$

-> $\delta B=40\%$ with $f_T \sim 2.4$ and $f_L=f_P \sim 5.9$

$$f_n = \frac{\partial B}{\partial n} \Big/ \frac{B}{n} = \frac{1}{2},$$

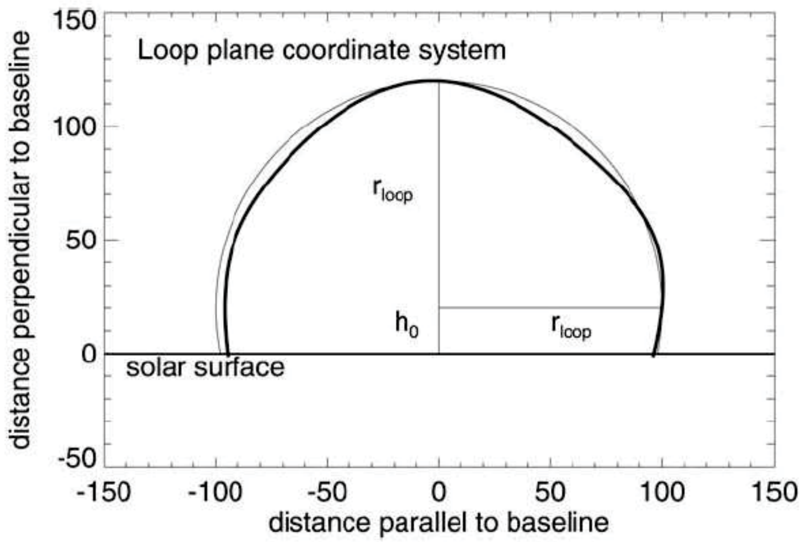
$$f_T = \frac{\partial B}{\partial T} \Big/ \frac{B}{T} = \left(\frac{C_2 P^2 T_6}{2L^2} - 2\right)^{-1} = \frac{1}{\gamma\beta},$$

$$f_L = \frac{\partial B}{\partial L} \Big/ \frac{B}{L} = \left(1 - \frac{4L^2}{C_2 T_6 P^2}\right)^{-1} = 1 + \frac{2}{\gamma\beta},$$

$$f_P = \frac{\partial B}{\partial P} \Big/ \frac{B}{P} = f_L = 1 + 2f_T.$$

Wang et al. (2007)

B from kink oscillations



Average and ranges of physical parameters of 26 oscillating loops[a].

Parameter	Average	Range
Loop half length L	110 ± 53 Mm	37–291 Mm
Loop width w	8.7 ± 2.8 Mm	5.5–16.8 Mm
Oscillation period P	321 ± 140 s	137–694 s[b]
	5.4 ± 2.3 min	2.3–10.8 min[b]
Decay time t_d	580 ± 385 s	191–1246 s[c]
	9.7 ± 6.4 min	3.2–20.8 min[c]
Oscillation duration d	1392 ± 1080 s	400–5388 s
	23 ± 18 min	6.7–90 min
Oscillation amplitude A	2200 ± 2800 km	100–8800 km
Number of periods	4.0 ± 1.8	1.3–8.7
Electron density of loop $n_{\rm loop}$	$(6.0 \pm 3.3)10^8$ cm^{-3}	$(1.3–17.1) \times 10^8$ cm^{-3}
Maximum transverse speed v_{max}	42 ± 53 km s^{-1}	3.6–229 km s^{-1}
Loop Alfvén speed $v_{\rm A}$	2900 ± 800 km s^{-1}	1600–5600 km s^{-1}
Mach factor $v_{\rm max}/v_{\rm sound}$	0.28 ± 0.35	0.02–1.53
Alfvén transit time $t_{\rm A}$	150 ± 64 s	60–311 s
Duration/Alfvénic transit $d/t_{\rm A}$	9.8 ± 5.7	1.5–26.0
Decay/Alfvénic transit $t_d/t_{\rm A}$	4.1 ± 2.3	1.7–9.6[c]
Period/Alfvénic transit $P/t_{\rm A}$	2.4 ± 1.2	0.9–5.4[b]

[a] All Alfvénic speeds and times are calculated for a magnetic field of $B = 30$ G, so scaling to other magnetic field values are $v_{\rm A}(B) = v_{\rm A}(B/30\ G)$ and $t_{\rm A}(B) = t_{\rm A}(B/30\ G)^{-1}$.
[b] The most extreme period of $P = 2004$ s (case 9a) is excluded in the statistics.
[c] Only the 10 most reliable decay times t_d (with no parentheses in Table II) are included in the statistics.

Aschwanden et al. (2002)

Conclusions

Coronal seismology;

- ✓ Observe and model coronal waves
- ✓ Measure physical parameters

- ✓ Magnetic field measure in photoshpere (Zeeman effect)

- ✓ Magnetic field determined from kink and slow loop oscillations